

The aim of this project is to explore and develop fast and stable algorithms, standard reference algorithms, and measurement procedures for non-linear geometrical Gaussian and spline filtration. Focusing on:

- Exploration of suitable numeric models for robustness and stability of non-linear filters.
- Creation of fast algorithms for the efficient implementation of non-linear Gaussian and spline filters.
- Development of standard reference algorithms implementing the definitions according to the ISO 16610 series of standards.

$$\sum_{k=0}^{n-1} \rho(z_k - s(x_k)) + \lambda \int_{x_0}^{x_{n-1}} \left\{ \frac{d^2 s(x)}{dx^2} \right\}^2 dx \rightarrow \text{Min}_{s(x_k)}$$

L2	$\rho(x) = x^2/2$	$\psi(x) = x$	$w(x) = 1$	linear
L1	$\rho(x) = x $	$\psi(x) = \text{sgn}(x)$	$w(x) = \frac{1}{ x }$	Nonlinear Robust
Huber	$\rho(x) = \begin{cases} x^2/2, & x \leq k \\ k(x - k/2), & x > k \end{cases}$	$\psi(x) = \begin{cases} x, & x \leq k \\ k \text{sgn}(x), & x > k \end{cases}$	$w(x) = \begin{cases} 1, & x \leq k \\ k/ x , & x > k \end{cases}$	
Cauchy	$\rho(x) = \frac{c^2}{2} \log\left(1 + \left(\frac{x}{c}\right)^2\right)$	$\psi(x) = \frac{x}{1 + (x/c)^2}$	$w(x) = \frac{1}{1 + (x/c)^2}$	
Tukey	$\rho(x) = \begin{cases} \frac{c^2}{6} \left[1 - \left(1 - (x/c)^2\right)^3\right], & x \leq c \\ \frac{c^2}{6}, & x > c \end{cases}$	$\psi(x) = \begin{cases} x \left[1 - (x/c)^2\right]^2, & x \leq c \\ 0, & x > c \end{cases}$	$w(x) = \begin{cases} \left[1 - (x/c)^2\right]^2, & x \leq c \\ 0, & x > c \end{cases}$	

Generalised Spline filter

$$\int_0^l \rho\left(z(\xi) - w(x) - \beta_1(x)(\xi - x) - \beta_2(x)(\xi - x)^2\right) s(\xi - x) d\xi$$

→ Min_{w(x), β₁(x), β₂(x)}

Generalised higher order gaussian regression filter for 2D Profile

$$\int_0^{ly} \int_0^{lx} \rho\left\{ \begin{aligned} & z(\xi, \eta) - \beta_{10}(x, y)(\xi - x) - \beta_{01}(x, y)(\eta - y) - \beta_{20}(x, y)(\xi - x)^2 \\ & - \beta_{11}(x, y)(\xi - x)(\eta - y) - \beta_{02}(x, y)(\eta - y)^2 - w(x, y) \end{aligned} \right\} \cdot s(\xi - x, \eta - y) d\xi d\eta$$

⇒ Min_{w(x, y), β₁₀(x, y), β₀₁(x, y), β₂₀(x, y), β₁₁(x, y), β₀₂(x, y)}

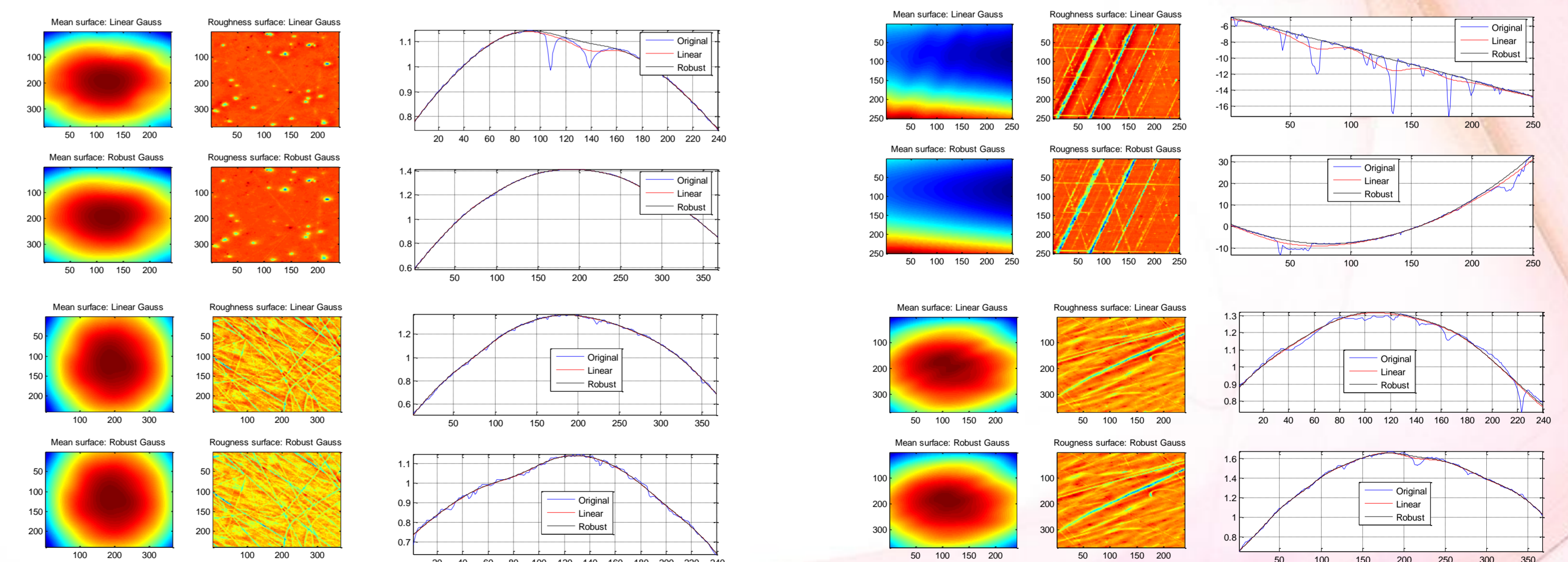
Generalised higher order gaussian regression filter for 3D surface

Fast algorithms:

1. Convolution to FFT;
2. Pre-calculation;
3. Separable in rows and cols

Significant speed improvement:

For a typical 60,000 pts data, 100 ms is needed compared with tradition algorithms need a few hours



Research Festival
23 March ~ 2 April
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