



# University of HUDDERSFIELD

## University of Huddersfield Repository

Zhang, Zonghua, Huang, Shujun, Gao, Nan, Gao, F. and Jiang, Xiang

Full-Field 3D Shape Measurement of Specular Object Having Discontinuous Surfaces

### Original Citation

Zhang, Zonghua, Huang, Shujun, Gao, Nan, Gao, F. and Jiang, Xiang (2017) Full-Field 3D Shape Measurement of Specular Object Having Discontinuous Surfaces. In: International Conference on Optical and Photonic Engineering (icOPEN 2017), 5-7 April 2017, Singapore.

This version is available at <http://eprints.hud.ac.uk/id/eprint/33181/>

The University Repository is a digital collection of the research output of the University, available on Open Access. Copyright and Moral Rights for the items on this site are retained by the individual author and/or other copyright owners. Users may access full items free of charge; copies of full text items generally can be reproduced, displayed or performed and given to third parties in any format or medium for personal research or study, educational or not-for-profit purposes without prior permission or charge, provided:

- The authors, title and full bibliographic details is credited in any copy;
- A hyperlink and/or URL is included for the original metadata page; and
- The content is not changed in any way.

For more information, including our policy and submission procedure, please contact the Repository Team at: [E.mailbox@hud.ac.uk](mailto:E.mailbox@hud.ac.uk).

<http://eprints.hud.ac.uk/>

# Full-Field 3D Shape Measurement of Specular Object Having Discontinuous Surfaces

Zonghua Zhang<sup>1,2,\*</sup>, Shujun Huang<sup>1</sup>, Nan Gao<sup>1</sup>, Feng Gao<sup>2</sup>, Xiangqian Jiang<sup>2</sup>

<sup>1</sup>School of Mechanical Engineering, Hebei University of Technology, Tianjin, China.  
300130

<sup>2</sup>Centre for Precision Technologies, University of Huddersfield, Huddersfield, HD1  
3DH, UK

\*zhzhang@hebut.edu.cn; zhzhangtju@hotmail.com

## Abstract

This paper presents a novel Phase Measuring Deflectometry (PMD) method to measure specular objects having discontinuous surfaces. A mathematical model is established to directly relate the absolute phase and depth, instead of the phase and gradient. Based on the model, a hardware measuring system has been set up, which consists of a precise translating stage, a projector, a diffuser and a camera. The stage locates the projector and the diffuser together to a known position during measurement. By using the model-based and machine vision methods, system calibration is accomplished to provide the required parameters and conditions. The verification tests are given to evaluate the effectiveness of the developed system. 3D (Three-Dimensional) shapes of a concave mirror and a monolithic multi-mirror array having multiple specular surfaces have been measured. Experimental results show that the proposed method can obtain 3D shape of specular objects having discontinuous surfaces effectively.

**Keywords:** specular object 3D measurement, phase deflectometry, system calibration, discontinuous surface measurement, phase calculation

## 1. INTRODUCTION

With the development of 3D (Three-Dimensional) optical shape measurement technique, it has been widely applied in the fields of reverse engineering, biological recognition and digitalization of cultural relics, etc., because of the advantages of high speed acquisition, non-contact sensing, and high measurement precision. However, most of the existing techniques are applied to measure objects with diffused surfaces. There are a large number of transparent, black, and reflective objects in actual industrial applications. The research of shape measurement for these kinds of objects is still in the early stage. The main methods in industry are using coordinate measuring machine [1] or changing the surface characteristics by spraying paint [2]. Therefore, it is vital to study a direct optical shape measurement method for specular objects.

PMD (Phase Measuring Deflectometry) has been widely studied to test specular free-form surfaces [3-4] because of its advantages of non-contact operation, full-field measurement, fast acquisition, high precision and automatic data processing qualities. PMD has been applied to measure aspheric mirror [5], dynamic specular surface [6], subsurface crack detection [7], and from micro-size [8] to large specular surface [9]. This kind of technique needs to display straight sinusoidal fringe patterns on a screen or to project the structured pattern onto a ground glass. From another viewpoint, the reflected fringe patterns via the tested surface appear deformed with regard to the slope variation of the surface and the modulated fringe patterns are recorded by an imaging device, such as a CCD (Charge-Coupled Device)

camera. Phase information in the deformed fringe patterns is demodulated to obtain the slope of the measured specular surface and then 3D shape of the tested surface can be reconstructed by integrating the gradients [10].

All the existing PMD methods just measure the local slope of smooth surfaces [11], instead of the actual 3D shape information. In order to achieve the final goal, a 2D (two-dimensional) integration procedure is necessary to reconstruct the shape from the measured derivatives, which is incapable of measuring multiple discontinuous surfaces. However, there are many complicated specular components during the intelligent manufacturing, for example, isolated and/or discontinuous surfaces on multi-mirror arrays [12]. Therefore, it is a challenging problem to fast full-field measure the 3D shape of specular objects having discontinuous surfaces.

However, few researches have been done to measure specular objects having discontinuous surfaces. Based on ray intersection, Petz et al proposed a method of RGP (Reflection Grating Photogrammetry) to measure discontinuous specular surfaces [3]. The tested points are independent from the others since this method employs ray intersection instead of integration. Xiao et al presented a FRP (Fringe Reflection Photogrammetry) method by importing the constraint bundle adjustment into RGP [13]. Though absolute coordinates of discontinuous specular surfaces can be obtained by these two methods [3,13], they do not give a direct relationship between height and phase information and require spatial geometry computation. Moreover, the translated distance by a translating stage has great effects on the measured accuracy, especially when the axis of translation and surface normal of the reference plane are not parallel. A zonal wave-front reconstruction algorithm is implemented to realize three-dimensional, highly reflected and specular surface reconstruction [14]. Although gauge blocks with two different heights were tested successfully, it is difficult to measure multiple discontinuous surfaces since this method needs each measured surface of the step object having one perturbed stripe.

This paper presents a novel PMD method to measure specular objects having isolated and/or discontinuous surfaces by directly building up the relationship between the absolute phase and depth. When a diffuser film having the same fringe patterns is located at two known different positions, the phase can directly relate to depth, instead of the slope of the measured specular surface. Sinusoidal fringe patterns having the optimum fringe numbers are generated by software and projected onto a diffuser surface through a DLP (Digital Light Processing) projector. A CCD camera captures the two sets of deformed color fringe pattern images from a reflected viewpoint. Wrapped and absolute phase maps are calculated by using the four-step phase-shifting algorithm and the optimum three-fringe number selection method, respectively. After calibrating the system, depth information of a specular object can be directly obtained from the calculated absolute phase map. Initial experimental results on a concave mirror and a monolithic multi-mirror array having multiple specular surfaces show that the proposed measuring method can obtain the 3D shape of specular objects with isolated and/or discontinuous surfaces effectively.

The following Section introduces the principle of the proposed method. Section 3 presents calibration to obtain the system parameters, mainly the distance  $d$  between reference mirror and the diffuser, the translated distance  $\Delta d$  of the diffuser during measurement. Some initial experiments on measuring specular objects having discontinuous surfaces are demonstrated in Section 4. Section 5 gives the conclusions and future directions.

## 2. MEASUREMENT PRINCIPLE

When the same sinusoidal fringe pattern sets are generated by software and displayed on a LCD (Liquid Crystal Display) screen at two different locations, they are reflected and deformed by the reference plane and the specular object surface under test. The deformed fringe patterns are captured by a CCD camera from another viewpoint for post processing. The corresponding phase of each point on the reflected surface can be determined by the multiple-step phase-shifting or Fourier transform

algorithm [15]. Considering the speed and accuracy, the four-step phase-shifting algorithm plus the optimum three-fringe number selection method [16,17] will be used to calculate the wrapped phase data and the absolute phase value pixel by pixel, respectively. After calibrating the parameters of the system, depth information can be directly obtained from the calculated absolute phase map.

## 2.1 Geometric Relationship between Absolute Phase and Depth

In order to directly obtain 3D shape of specular objects from the deformed fringe patterns, the displayed fringe patterns need to be reflected from two known positions, as illustrated in Fig. 1. A diffuser, instead of a LCD screen, has been used to display fringe patterns and then reflected by the specular object surface because of the following two reasons. On one hand, the LCD screen is for the purpose of displaying so it does not have high accurate flatness; on the other hand, the effect of the refraction that occurs in the transparent layers of the LCD screen when reflecting fringe patterns. A DLP projector will be used to project the generated fringe patterns onto the diffuser surface. To simplify the relationship, the DLP projector has been omitted in the schematic diagram.

$\Delta d$  is the distance between diffuser1 (D1) and diffuser2 (D2) and  $d$  is the distance between the reference plane and the diffuser1 (D1). Assuming the imaging system is a pinhole projection, two rays of light are displayed and reflected into the CCD camera through a tested surface and a reference plane, as illustrated in Fig. 1. The two incident rays correspond to the same reflection light. The phase of the two incident rays is  $\varphi_{r1}$  and  $\varphi_{r2}$  on the reference plane and  $\varphi_{m1}$  and  $\varphi_{m2}$  on the measured freeform surface.  $\theta$  is the angle between one incident ray and normal vector of the reference plane, and  $\theta + \varphi$  is the angle between the other incident ray and normal vector of the reference plane. The period of the projected fringe pattern on the diffuser is  $q$ .  $\Delta l$  is the distance on D1 between the two incident rays because of height and gradient of the measured surface. Parameter  $h$  stands for height of the measured

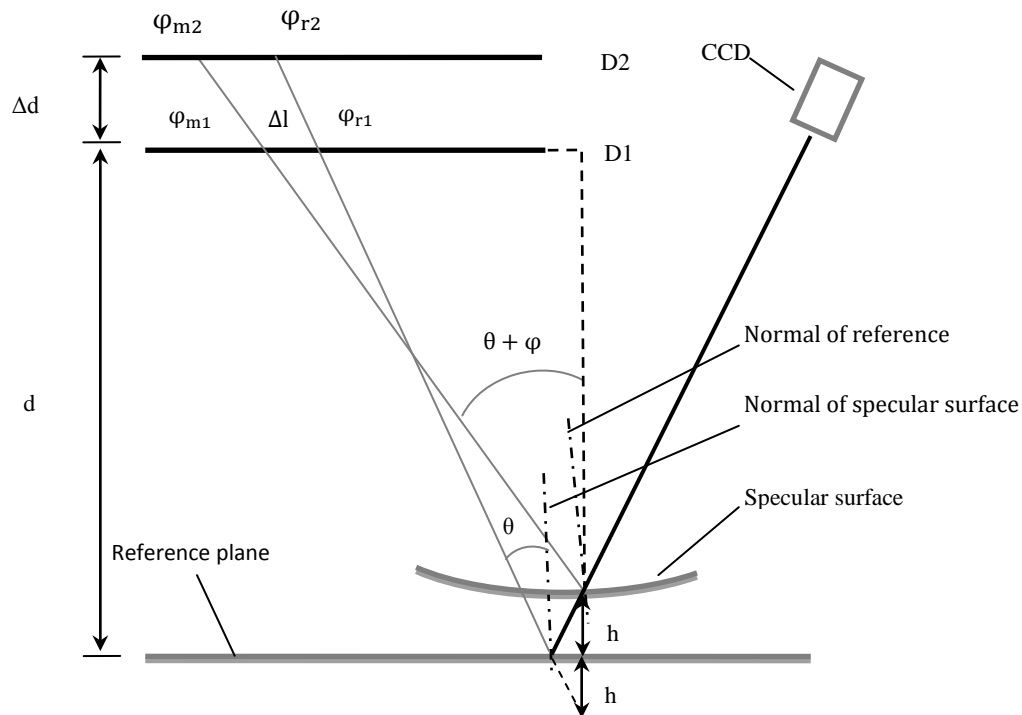


Figure 1 Schematic diagram of the measurement principle

specular surface with respect to the reference plane.

According the geometric relationship in Fig. 1, the following equations can be deduced.

$$(\varphi_{r1} - \varphi_{r2})q/2\pi = \Delta d \tan\theta \quad (1)$$

$$(\varphi_{m1} - \varphi_{m2})q/2\pi = \Delta d \tan(\theta + \varphi) \quad (2)$$

$$(d + h)\tan\theta + \Delta l = (d-h)\tan(\theta + \varphi) \quad (3)$$

$$(\varphi_{r1} - \varphi_{m1})q/2\pi = \Delta l \quad (4)$$

From the above four Eqs. (1)-(4), height of the measured specular surface is

$$h = \frac{\Delta d(\varphi_{r1} - \varphi_{m1}) - d[(\varphi_{r1} - \varphi_{r2}) - (\varphi_{m1} - \varphi_{m2})]}{(\varphi_{m1} - \varphi_{m2}) + (\varphi_{r1} - \varphi_{r2})} \quad (5)$$

This equation clearly shows that height information can be directly calculated from the captured fringe patterns only if two parameters  $d$  and  $\Delta d$ , and phase information on the reference plane are calibrated beforehand. Because the optimum three-fringe numbers selection method will be used to independantly calculate the absolute phase pixel by pixel, specular objects having isolated and/or discontinuous surfaces can be measured by the proposed method.

In order to capture the deformed fringe patterns on a specular surface, the diffuser film and projector are assembled together and then translated along a linear stage with known  $\Delta d$  distance in the range of depth of field of the CCD camera. At each position for the diffuser film, sinusoidal fringe patterns are deformed via the specular object under-test and captured by the CCD camera from a reflected viewpoint for post processing.

## 2.2 Four-Step Phase-Shifting Algorithm

For each fringe pattern set, there are four fringe patterns having 90 degrees shift in between, so the standard four-step phase-shifting algorithm is used to calculate the wrapped phase data. It is one of the most used phase calculation method in fringe pattern processing and has gained many achievements in research and industrial fields [15]. In order to make this paper self-contained, the mathematical model and distribution characteristics of fringe patterns are briefly described [18].

The intensity (grayscale) distribution of a fringe pattern can be expressed by the following mathematical expression:

$$I(x, y) = I_0(x, y) + I_1(x, y)\cos\varphi(x, y) + I_n(x, y) \quad (6)$$

where,  $I(x, y)$  is the only measurable parameter;  $I_0(x, y)$  is the intensity of background;  $I_1(x, y)$  is the fringe modulation;  $\cos\varphi(x, y)$  is the phase filed corresponding to objects shape;  $I_n(x, y)$  is additive random noise.

The four-step phase-shifting algorithm means four fringe patterns have  $\pi/2$  phase shift in between are successively displayed in the measuring volume from a displaying device. From another point of view, the CCD camera captures the deformed fringe patterns and saves them into a computer for post processing. One of the obtained fringe pattern can be expressed as

$$I_i(x, y) = I_0(x, y)\{1 + \gamma \cos[\varphi(x, y) + \alpha_i]\}, i = 1, 2, 3, 4. \quad \alpha_1 = 0, \alpha_2 = \pi/2, \alpha_3 = \pi, \alpha_4 = 3\pi/2 \quad (7)$$

where  $I_i(x, y)$  is the  $i^{th}$  captured gray value at one pixel position;  $I_0(x, y)$  is the intensity of background;  $\gamma(x, y)$  is the intensity modulation function;  $\varphi(x, y)$  is unknown phase filed corresponding to the measured object shape. If the four captured fringe patterns in the same optical field have the same gray value of background and modulation, the wrapped version of phase field  $\varphi(x, y)$  can be accurately calculated by the following triangle formula

$$\varphi_0(x, y) = \arctan \frac{I_4(x, y) - I_2(x, y)}{I_3(x, y) - I_1(x, y)} \quad (8)$$

The obtained phase is in the range of  $-\pi$  and  $\pi$ , which needs to be unwrapped for 3D shape measurement.

### 2.3 Optimum Three-Fringe Number Selection Method

The optimum multi-frequency selection process defines the numbers of projected fringes to be [17]:

$$N_{fi} = N_{f0} - N_{f0}^{(i-1)/(n-1)}, i = 1, \dots, n - 1 \quad (9)$$

where  $N_{f0}$  and  $N_{fi}$  are the maximum number of fringes and the number of fringes in the  $i_{th}$  fringe set, respectively, and  $n$  is the number of fringe sets used. With three fringe sets, the method is usually referred to as the optimum three-fringe number selection method. For example, if  $N_{f0} = 64$  and  $n = 3$ , the other two fringe sets have fringe numbers of  $N_{f1} = N_{f0} - 1 = 63$  and  $N_{f2} = N_{f0} - N_{f0}^{(i-1)/(n-1)} = 56$ . This method resolves fringe order ambiguity as the beat obtained between  $N_{f0}$  and  $N_{f1}$  is a single fringe over the full field of view and the reliability of the obtained fringe order is maximized as fringe order calculation is performed through a geometric series of beat fringes with 63, 63 and 56 fringes. The fringe order calculation for  $N_{f0} = 64$  is reliable to  $6\sigma$  (giving a probability of 99.73% of calculating the correct absolute fringe order) providing the phase noise to one standard deviation is better than  $1/59^{th}$  of a fringe [17]. The obtained absolute phase data are unwrapped pixel by pixel, so the method can measure objects having discontinuities and/or isolated surfaces.

## 3. PARAMETERS CALIBRATION

As shown in Eq. (5), in order to accurately measure depth data, two system parameters of  $d$  and  $\Delta d$  should be calibrated beforehand.

### 3.1 Calibration of $\Delta d$

A flat mirror will be originally placed in the reference position to calibrate  $\Delta d$  and the surface of the mirror is parallel to plane of the diffuser.

A high accurate translating stage will be used to locate the mirror at several known positions along the normal direction of the mirror. At each mirror position, fringe pattern sets having the optimum fringe numbers will be generated and projected onto the diffuser. The displayed fringe patterns are reflected by the plane mirror and captured by the CCD camera from another viewpoint for post processing. According to the geometric relationship of the parameters, the following equation can be obtained.

$$\Delta d = 2h(\Phi\varphi_{ri}' - \varphi_{ri}) / (\varphi_{ri}' - \varphi_{r0}') \quad (9)$$

In principle, one known translating depth  $h$  and the corresponding phase values can determine the parameter  $\Delta d$ . However, the axis of translation and surface normal of the reference plane are not parallel. In order to improve the accuracy of calibration, the translating stage should move to several known positions to build up an over-determined equation set.

### 3.2 Calibration of $d$

A machine vision method has been applied to calibrate  $d$  by using another flat mirror. There are markers with known distance on the mirror surface. The readers can refer to [19] for more details.

## 4. EXPERIMENTS AND RESULTS

A full-field 3D shape measurement system for specular objects has been setup to test the proposed method. A concave mirror and a monolithic multi-mirror array having multiple specular surfaces have been measured to show the feasibility of directly measuring depth information from the calculated absolute phase data.

### 4.1 Hardware system

A full-field 3D shape measurement system has been developed to obtain the 3D shape by projecting fringe patterns onto the diffuser film, as illustrated in Fig. 2. The hardware system consists of a computer, a CCD camera with lens, a DLP projector, a diffuser film and a mobile linear stage. The CCD is the latest industrial camera from Lumenera corporation with the model of Lw230 and the resolution of  $1616 \times 1216$  pixels. The camera supports external and internal trigger mode. The DLP projector is from Texas Instruments with the type of Pico-projector. The diffuser film and mobile linear stage are from Thorlabs Inc. The diffuser film has flatness of  $4\lambda$  ( $\lambda = 632 \text{ nm}$ ). The linear stage has a position accuracy of  $2\mu\text{m}$ . The projector and diffuser film are assembled together in the linear stage. During the procedure of calibration and measurement, they are translated to two different positions with distance of  $\Delta d$ . After the parameter of  $d$  has been calibrated, depth of the reflected object surface can be measured by the developed system.

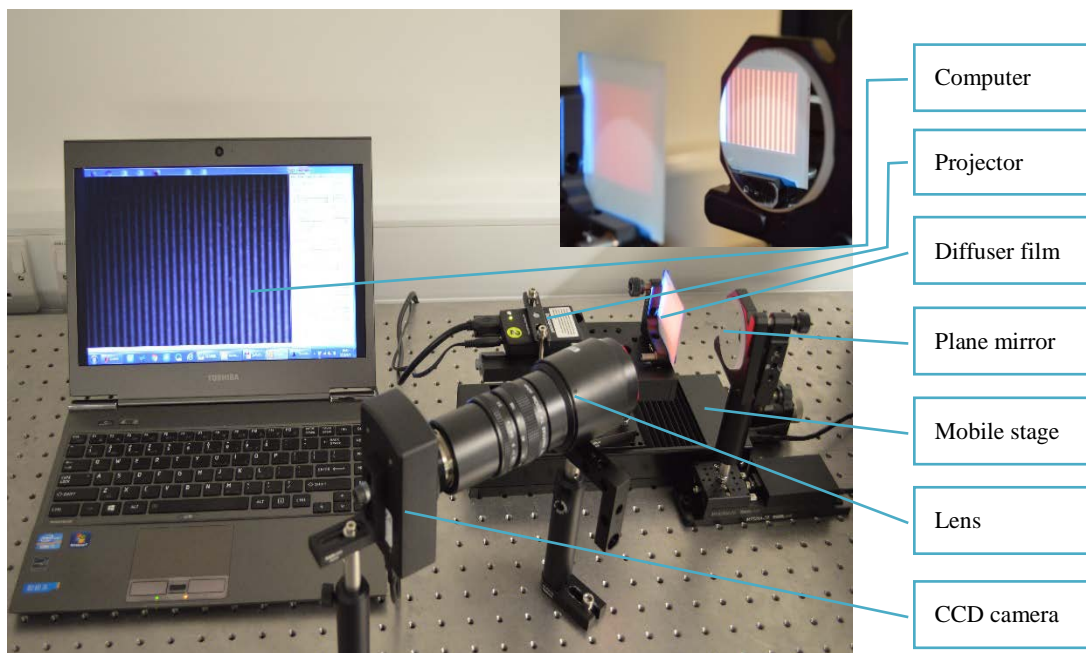


Figure 2: Hardware of the full-field 3D shape measurement.

## 4.2 Experimental results

Two reflected objects have been measured by the developed 3D system. One is a concave mirror having freeform shape; the other is a monolithic multi-mirror array on the Mid-Infrared Instrument (MIRI) Spectrometer Optics for the James Webb Space, which has multiple discontinuous specular surfaces, both of them are illustrated in Fig. 3.

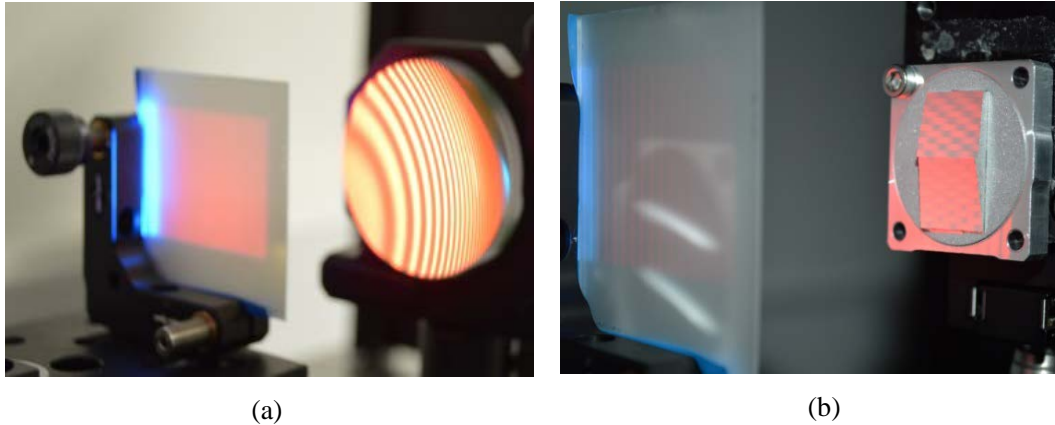


Figure 3: The tested specular objects with reflected fringe patterns. (a) concave mirror, and (b) monolithic multi-mirror arrays on the Mid-Infrared Instrument (MIRI) Spectrometer Optics for the James Webb Space

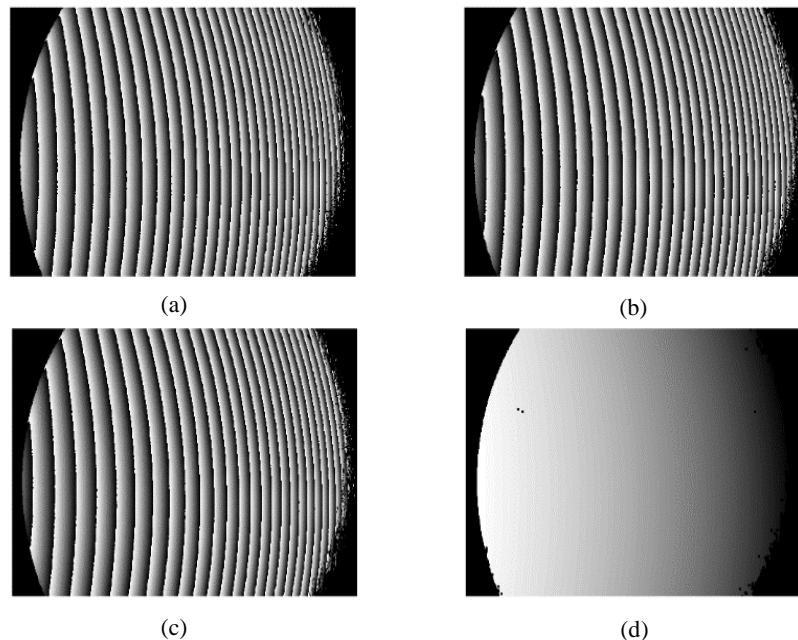


Figure 4: Phase maps of the measured concave mirror. (a), (b), (c) are the wrapped phase maps having the projected fringe numbers of 64, 63, 56, and (d) the absolute phase map.

Twelve fringe patterns having the optimum fringe numbers of 64, 63, and 56 were generated by software and sequentially projected onto the diffuser film. The reflected fringe patterns by the specular surface are deformed and captured by the triggered CCD camera. Using the four-step phase-shift algorithm, three wrapped phase maps are calculated, as shown in Fig. 4. The absolute phase of each pixel is determined by the optimum three-fringe selection method [16], as shown in Fig. 4(d). Using the calibrated parameters of  $\Delta d$  and  $d$ , depth data are obtained, as shown in Fig. 5.



Applying the same procedure to the monolithic multi-mirror array, phase maps and depth data are obtained, as shown in Fig. 6 and Fig. 7, respectively. The results show that the proposed method can directly measure specular object having isolated and/or discontinuous surfaces.

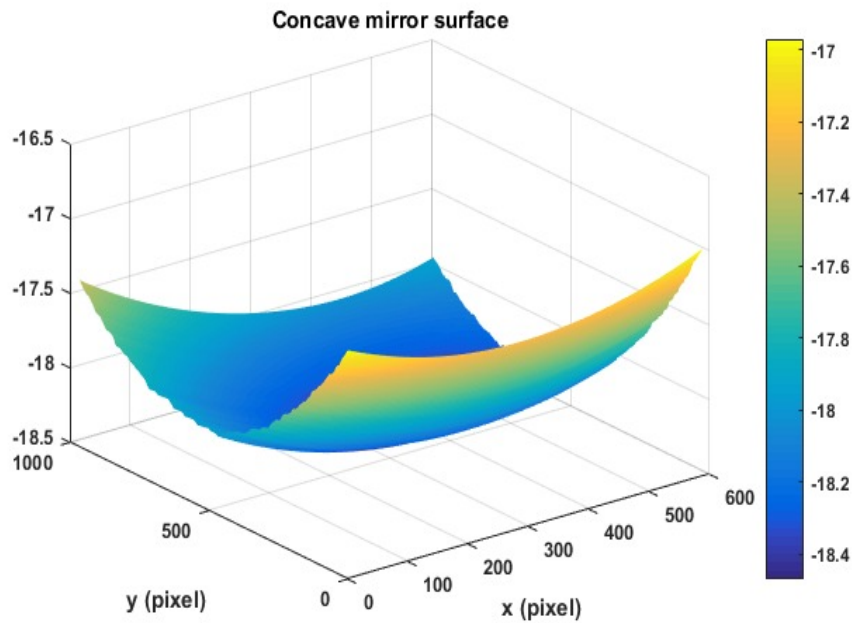


Figure 5: Depth of the concave mirror.

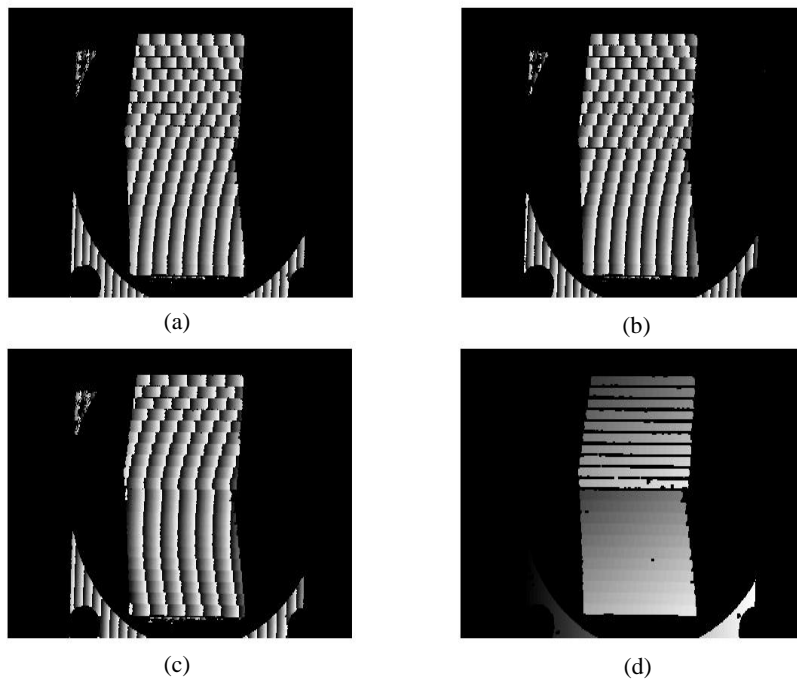


Figure 6 Phase maps of the monolithic multi-mirror arrays. (a), (b), (c) are three wrapped phase maps having the projected fringe numbers of 64, 63, 56, and (d) the absolute phase map.

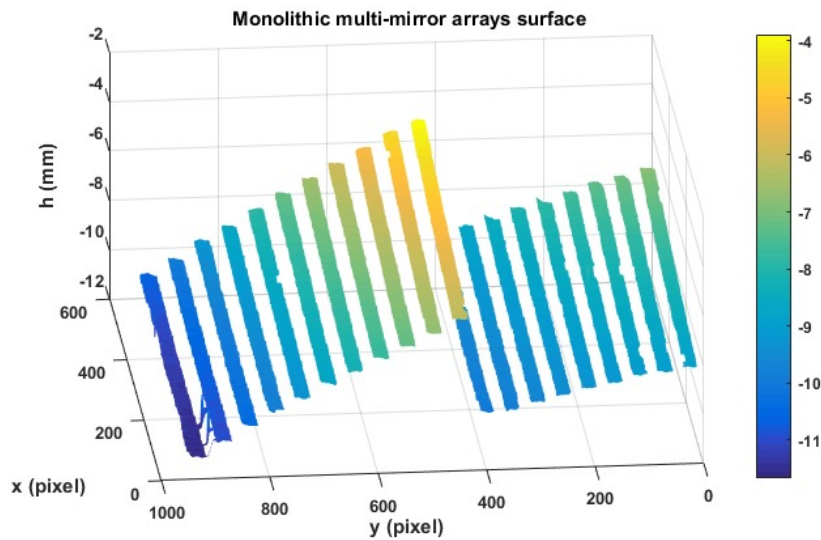


Figure 7 Depth of the monolithic multi-mirror arrays.

### 4.3 Performance Analysis

In order to evaluate the accuracy of the developed system, the plate mirror was placed on the accurate translating stage with resolution of  $1\mu\text{m}$ . The plate was positioned at  $-3.7\text{mm}$ ,  $-1.3\text{mm}$ ,  $1.3\text{mm}$  and  $3.7\text{mm}$  with respect to the reference plane. At each position, the depth data was calculated using the calibrated parameters. The profile along one row near the middle is illustrated in Fig. 8 for the position of  $-3.7\text{mm}$ . The measured average values for the four positions are  $-3.76\text{mm}$ ,  $-1.35\text{mm}$ ,  $1.38\text{mm}$  and  $3.74\text{mm}$  with respect to the reference plane. These results clear show that the proposed method can measure the depth data with high accuracy.

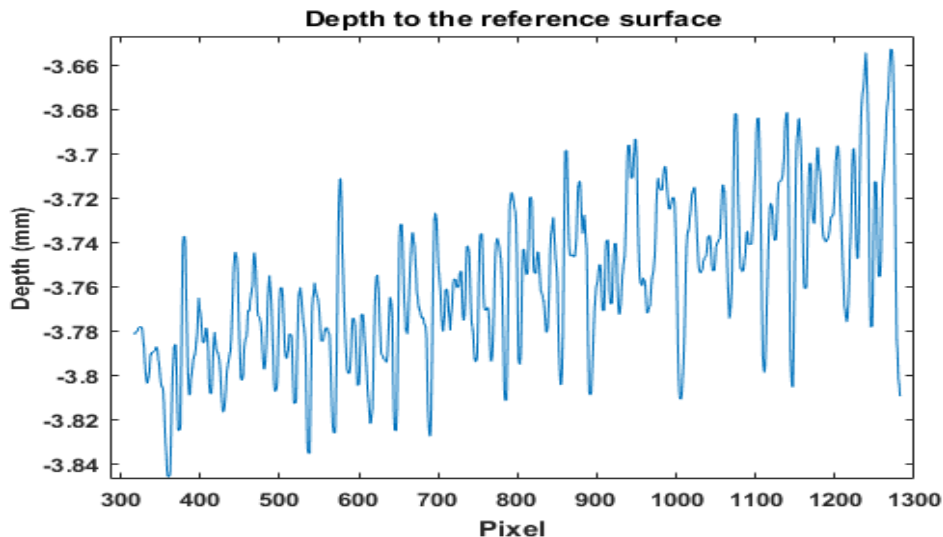


Figure 8 Measured depth along middle row direction for position of  $-3.7\text{mm}$ . X-axis represents the pixel positions along row direction, the vertical axis is the reconstructed depth to the reference surface (Unit mm).

## 5. CONCLUSION

This paper presents a novel full-field 3D shape measurement method of specular object having discontinuous surfaces by building the direct relationship between the absolute phase and depth data. A DLP projector and a diffuser are assembled together to replace a LCD display screen. Both of them are translated to two different positions during the procedure of calibration and measurement. Fringe pattern sets are generated by software and projected on the diffuser by the DLP projector. From a different viewpoint, the reflected fringe patterns are deformed with respect to slope of the specular surfaces and captured by a CCD camera. The absolute phase map is obtained from the captured fringe patterns by using the four-step phase-shifting algorithm and the optimum three-fringe numbers selection method. After two parameters of the system have been calibrated by using the machine vision method, depth data can be directly derived from the obtained absolute phase map. Because depth directly relates to the absolute phase without needing gradient integration, the proposed method can measure specular objects having isolated and/or discontinuous surfaces. Initial experimental results on measuring a concave mirror and a monolithic multi-mirror arrays with multiple specular surfaces show that the developed system effectively obtains full-field 3D shape measurement of specular object having discontinuous surfaces.

Next research directions of the proposed method are the following several parts. (1) Capturing speed: the generated fringe patterns will be coded into different color channels and then captured simultaneously by the corresponding color channels of the color CCD camera. (2) Accuracy: efficient calibration methods will be applied to obtain more accurate parameters and the performance will be evaluated by using an artificial standard step or other optical metrology. (3) Error analysis: effects of all kinds of error sources on the measurement results, such as the optimizing intensity and numbers of fringe patterns, inaccuracy of the calibrated parameters of  $\Delta d$  and  $d$ .

## ACKNOWLEDGEMENTS

The authors would like to thank the National Natural Science Foundation of China (under grant 51675160, 61171048), Key Basic Research Project of Applied Basic Research Programs Supported by Hebei Province (under grant 15961701D), Research Project for High-level Talents in Hebei University (under grant GCC2014049), Talents Project Training Funds in Hebei Province (NO: A201500503), Tianjin Science and Technology Project (under grant 15PTSYJC00260), Innovative and Entrepreneurial Talent Project Supported by Jiangsu Province. This project is also funded by European Horizon 2020 through Marie Skłodowska-Curie Individual Fellowship Scheme (under grant 767466-3DRM).

## REFERENCES

- [1] Y. Shimizu, S. Goto, J. Lee, S. Ito, W. Gao, S. Adachi, K. Omiya, H. Sato, T. Hisada, Y. Saito, and H. Kubota, "Fabrication of large-size SiC mirror with precision aspheric profile for artificial satellite," *Precis. Eng.*, 37, 640–649 (2013).
- [2] A. Miks, J. Novak, and P. Novak, "Method for reconstruction of shape of specular surfaces using scanning beam deflectometry," *Opt. Lasers Eng.*, 51, 867–872 (2013).
- [3] M. Petz, and R. Tutsch, "Reflection grating photogrammetry: a technique for absolute shape measurement of specular free-form surfaces," *Proc. of SPIE*, 58691:D1-D12 (2005).

- [4] C. F. Guo, X. Y. Lin, A. Hu, and J. Zou, "Improved phase-measuring deflectometry for aspheric surfaces test," *Appl. Opt.*, 55(8), 2059-2065(2016).
- [5] Y. Tang, X. Su, F. Wu, and Y. Liu, "A novel phase measuring deflectometry for aspheric mirror test," *Opt. Express*, 17(22), 19778–19784 (2009).
- [6] L. Huang, C. S. Ng, A. K. Asundi, "Dynamic three-dimensional sensing for specular surface with monoscopic fringe reflectometry," *Opt. Express*, 19(13), 12809-12814 (2011).
- [7] F. Chan, "Reflective fringe pattern technique for subsurface crack detection," *NDT&E International*, 41(8), 602-610 (2008).
- [8] G. Häusler, C. Richter, K. Leitz, and M. C. Knauer, "Microdeflectometry - a novel tool to acquire three-dimensional microtopography with nanometer height resolution," *Opt. Lett.*, 33(4), 396-398 (2008).
- [9] H. Zhang, S. Han, S. Liu, S. Li, L. Ji, and X. Zhang, "3D shape reconstruction of large specular surface," *Appl. Opt.*, 51(31), 7616-7625 (2012).
- [10] L. Huang, M. Idir, C. Zuo, K. Kaznatcheev, L. Zhou, and A. Asundi, "Shape reconstruction from gradient data in an arbitrarily-shaped aperture by iterative discrete cosine transforms in Southwell configuration," *Opt. Laser Eng.*, 67, 176-181 (2015).
- [11] M. C. Knauer, J. Kaminski, and G. Häusler, "Phase measuring deflectometry: a new approach to measure specular free-form surfaces," *Proc. SPIE*, 5457, 366-376 (2004).
- [12] P. Shore, P. Morantz, and D. Lee, "Manufacturing and measurement of the MIRI spectrometer optics for the James Webb space telescope," *Annals of the CIRP*, 55(1), 543-546 (2006).
- [13] Y. L. Xiao, X. Y. Su, W. J. Chen, and Y. K. Liu, "Three-dimensional shape measurement of aspheric mirrors with fringe reflection photogrammetry," *Appl. Opt.*, 51(4), 457-44 (2012).
- [14] H. Zhang, L. Ji, S. Liu, S. Li, S. Han, and X. Zhang, "Three-dimensional shape measurement of a highly reflected, specular surface with structured light method," *Appl. Opt.*, 51(31), 7724-7732 (2012).
- [15] K. Creath, "Phase measurement interferometry techniques," in *Progress in Optics XXVI*, E. Wolf, Ed. (North Holland Publ., Amsterdam, 1988).
- [16] Z. H. Zhang, C. E. Towers, D. P. Towers, "Time efficient colour fringe projection system for 3-D shape and colour using optimum 3-frequency interferometry," *Opt. Express*, 14(14), 6444-6455 (2006).
- [17] C. E. Towers, D. P. Towers, and J. D. Jones, "Optimum frequency selection in multifrequency interferometry," *Opt. Lett.*, 28(11), 887-889 (2003).
- [18] Z. H. Zhang, "Review of single-shot 3D shape measurement by phase calculation-based fringe projection techniques," *Opt. Laser Eng.*, 50(8), 1097-1106 (2012).
- [19] Z. H. Zhang, Y. Liu, S. J. Huang, Z. Q. Niu, J. Guo, N. Gao, F. Gao, and X. Jiang, "Full-field 3D shape measurement of specular surfaces by direct phase to depth relationship," *Proc. SPIE*, 10023, 100230X (2016).