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Inverse Design of Blade Shapes for Vertical Axis Wind Turbines

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Abstract

An inverse design process is applied to determine blade shapes used in vertical axis wind turbines (VAWTs). The method is based on the modified Garabedian-McFadden technique that uses the deviation of the pressure distribution or the velocity distribution on the surface from a target pressure or velocity distribution, and modifies the blade surface in order to reduce this deviation. The method was originally developed for wing design and applied for such aerofoil shapes as those of horizontal axis wind turbine blades. The procedure is employed here successfully to find a target shape that is used in a VAWT operating with blades of constant thickness rather than of aerofoil shape. The method is presently applicable to determine blade shapes in a two-dimensional section, and considers some performance criteria of VAWTs; thus, it contributes to finding blade shapes that most closely corresponds to the efficient operation of such turbines.

Keywords: *Inverse design, Numerical method, Velocity distribution, Vertical axis wind turbine*

I. INTRODUCTION

The rotor blade is a critical element of wind turbines, whose aerodynamic and dynamic properties influence the entire system. The main challenge during the design process of the rotor blade is to find the relationship between its shape and its aerodynamic properties. Although the direction of wind velocity is not constant, a prevailing direction can usually be determined in most of the geographical locations, and the variation from this prevailing direction does not affect the performance significantly. The design process is iterative in practice. A blade which has certain desired properties according to previous experience is tested using a computational or experimental model in order to find out if it satisfies the given performance criteria. Results are usually not satisfactory in the first approach, but the model provides an insight on how the actual blade design affected the results. Accordingly, the design is improved and the design loop is repeated until adequate blade design is obtained.

The design of rotor blades is based on concepts that are used in that of the aircraft wings in most of the cases. However, there exist some exceptions when the aerofoil shape is not optimal to satisfy the performance criteria. This is the case e.g. for small rotor blades and low-power vertical axis wind turbines (VAWTs) (Hau [1]). The present study considers a VAWT blade with constant thickness as constructed in the University of Huddersfield laboratory (Park [2]).

Inverse aerodynamic design methods have been proposed widely in the last three decades. Only a few of them are mentioned here, which are related to the technique used in the present paper. Garabedian & McFadden [3] developed an iterative procedure to design swept wings, which is referred to as the Garabedian-McFadden (GM) method. This original scheme was later modified by Malone *et al.* [4], which allowed design for a wide range of geometry with prescribed surface pressures. The authors referred to this technique as the modified Garabedian-McFadden (MGM) method. The MGM procedure was incorporated into a multigrid Navier-Stokes airfoil analysis method in Malone & Swanson [5]. Dulikravich & Baker [6] formulated a Fourier series solution of the MGM equation.

The applicability of the MGM technique for the design of aircraft wings and horizontal axis wind turbine blades has already been demonstrated [see e.g. 3-6]. The main goal of the present paper is to apply the MGM technique for the inverse design of VAWT blades with constant thickness, which represents a geometry substantially different from those described by aerofoils. First, the technique and the computational procedure is described. Then, the air flow field is determined around a VAWT blade with constant thickness, and the obtained velocity distribution is used as target velocities. Finally, it is shown that the process can result in the original blade shape with satisfactory accuracy.

II. INVERSE DESIGN PROCESS

A. Modified Garabedian-McFadden Technique

The positions of blade coordinates are modified so that they approach the positions along the surface that corresponds to the target velocity profile. The MGM technique (Malone *et al.* [4]) is used to achieve this goal. In this technique the deviation of pressure or velocity distribution near the surface from the target pressure or velocity distribution is expressed by the surface ordinate y, the slope of the surface dy/dx, and the second derivative of the surface, d^2y/dx^2. The change in the surface ordinate is denoted by Δy, which is positive when the surface has to be modified so that it moves away from the chord. The equation to be solved for the change in the position of the blade surface is the following

$$A\Delta y + B\frac{\mathrm{d}(\Delta y)}{\mathrm{d}x} - C\frac{\mathrm{d}^2(\Delta y)}{\mathrm{d}x^2} = V_{tar}^2 - V_{pr}^2 \qquad (2.1)$$

where V_{pr} and V_{tar} are the present and target velocities, respectively, and A, B and C are numerical constants that should be large enough to avoid the shape from changing too much in one step. The following values were applied in the present study: $A = 1000$; $B = C = 100$. Further refinement of these values is the subject of future study. When the present velocity approaches the target velocity, then the change in the ordinate y reduces, i.e. Δy tends to zero. If both of the top and bottom blade surfaces are divided into n subdomains, then the derivatives in Eq. (2.1) can be written in discrete form as follows

$$\frac{\mathrm{d}(\Delta y)}{\mathrm{d}x} = \frac{\Delta y_{i+1} - \Delta y_i}{x_{i+1} - x_i};$$

$$\frac{\mathrm{d}^2(\Delta y)}{\mathrm{d}x^2} = \frac{\dfrac{\Delta y_{i+1} - \Delta y_i}{x_{i+1} - x_i} - \dfrac{\Delta y_i - \Delta y_{i-1}}{x_i - x_{i-1}}}{\dfrac{x_{i+1} - x_{i-1}}{2}}; \qquad i = 2, ..., n-1 \qquad (2.2)$$

with i denoting the calculation point between two subdomains on the blade surface. Substituting Eq. (2.2) into Eq. (2.1), assuming constant step size $\Delta x = x_{i+1} - x_i$ for each $i = 2,\ldots,n\text{-}1$, and denoting the coefficients of Δy_{i-1}, Δy_i, Δy_{i+1} by k_1, k_2, k_3, respectively, then Eq. (2.1) can be organized in the following form

$$\begin{bmatrix} k_2 & k_3 & & & 0 \\ k_1 & k_2 & k_3 & & \\ & k_1 & k_2 & \ddots & \\ & & \ddots & \ddots & k_3 \\ 0 & & & k_1 & k_2 \end{bmatrix} \begin{bmatrix} \Delta y_2 \\ \Delta y_3 \\ \Delta y_4 \\ \vdots \\ \Delta y_{n-1} \end{bmatrix} = \begin{bmatrix} V_{tar,2}^2 - V_{pr,2}^2 \\ V_{tar,3}^2 - V_{pr,3}^2 \\ V_{tar,4}^2 - V_{pr,4}^2 \\ \vdots \\ V_{tar,n-1}^2 - V_{pr,n-1}^2 \end{bmatrix} \qquad (2.3)$$

The size of matrix in Eq. (2.3) is (n–2)x(n–2) since the change of ordinate at the leading edge Δy_1 and at the trailing edge Δy_n is zero. It is a tridiagonal matrix; thus, Eq. (2.3) can be solved by the tridiagonal matrix algorithm or Thomas algorithm (Conte & de Boor [7]).

B. Computational Procedure

In the computation, first the distribution of velocity or coefficient of pressure is determined, which represents flow conditions around the target blade profile. Then, these conditions are used in the inverse design process to obtain the blade profile that corresponds to the target conditions. Both parts of computations are implemented in Matlab.

A known blade shape is applied in order to create target conditions. This blade shape should later be obtained in the inverse design process, so it will also be used to validate the procedure. The geometry of one rotor blade of a VAWT is defined in the plane by the (x;y) coordinates of the surface points. The centre line of the blade is located on an arc, and the upper and lower profiles are determined so that the thickness of the blade be constant everywhere except at the first and last points. Then, the velocity field around this shape is determined using the panel method. The velocity distribution near the blade surface is used to calculate the coefficient of pressure at the same positions. The coefficient of pressure c_p at any position is obtained from Eq. (2.4)

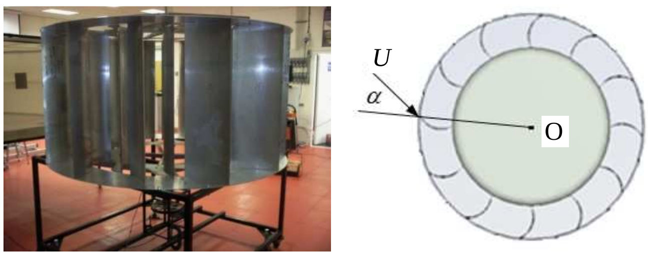
$$c_p = 1 - \frac{u^2 + v^2}{U^2} \qquad (2.4)$$

where u and v are x and y components, respectively, of the air velocity at the same position, and U is the free stream velocity.

In the inverse design process, the target velocities obtained as explained in the previous paragraph are used to reach the blade shape corresponding to the target conditions. Alternatively, if the coefficients of pressure represent the target conditions, then first the target velocities are calculated. The initial velocity distribution is determined by defining an arbitrary blade shape, and then calculating the velocity near this surface using the panel method. Then, Eq. (2.3) is solved and the blade shape is modified according to the obtained values Δy_2, …, Δy_{n-1}. This iteration continues until the change in the blade shape becomes smaller than a prescribed limit.

III. APPLICATION FOR A VAWT

A rotor blade of the VAWT located in the University of Huddersfield laboratory (Colley [8]; Park [2]) is chosen as subject of the present study (see Fig. 1a). The centre line of the blade is located on an arc with radius of 145 mm. The thickness of the blade is 2 mm. The chord length is 20 cm.



(a) (b)

Fig. 1: (a) VAWT as installed in University of Huddersfield laboratory (from Park [2]); (b) sketch of rotor blades showing the angle of attack α (O is axis of the turbine, and U is free stream velocity)

A. Pressure Distribution and Velocity Field around the Target Profile

The Matlab model of the blade defined above is shown in Fig. 2 by the dark area. The pressure distribution and air velocity field around this shape is determined in an air flow where the free stream velocity is 10 m/s, and the angle of attack is –15 degrees. More precisely, this angle is defined here by the angle of the direction of the line connecting the free end of the blade to the vertical axis of the turbine and the direction of free stream velocity (see Fig. 1b). The velocity field is determined by the panel method and is as shown in Fig. 2.

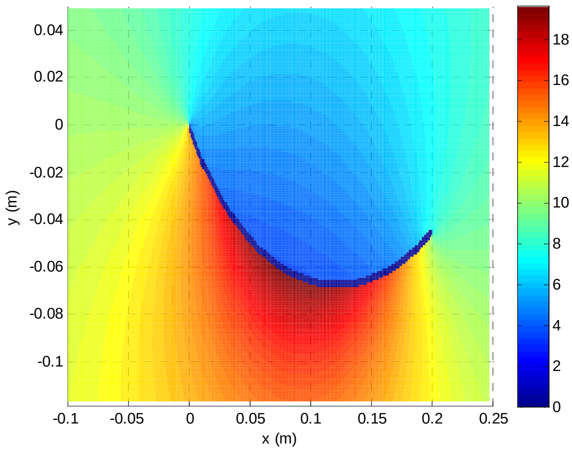


Fig. 2: Velocity field around a rotor blade of the VAWT of the University of Huddersfield laboratory

B. Blade Shape Obtained by the Inverse Design Process

The velocities near the surface of the blade defined earlier in this section are used as target velocity distribution for the inverse design process. An initial blade shape close to the optimal shape is usually known. However, in order to demonstrate the robustness of the method, the present computation began with a NACA0012 aerofoil shown by the blue curve in Fig. 3. This profile is symmetric and substantially different from the target shape that should be obtained (cf. the blue and black shapes in Fig. 3). Both sides of the blade are divided into 100 subdomains, and velocities were determined at the boundary of any two subdomains. The computation was terminated when the average of changes in the position of each point (i.e. 99 points at the boundaries of 100 subdomains) in one side of the blade, related to the blade thickness, became less than 0.0005. This accuracy was achieved in this particular example after 176 and 85 iterations on the top and bottom sides, respectively, of the blade. The calculated blade profile is shown in Fig. 3 by the green curve. The comparison of the target and calculated profiles in Fig. 3 shows that the calculation provides a smooth blade shape that closely approximates the target profile. This result is obtained for an initial profile that is substantially different from the target profile. Thus, the presented inverse design process was successfully applied to find the upper and lower profiles of a VAWT blade, satisfying given target conditions.

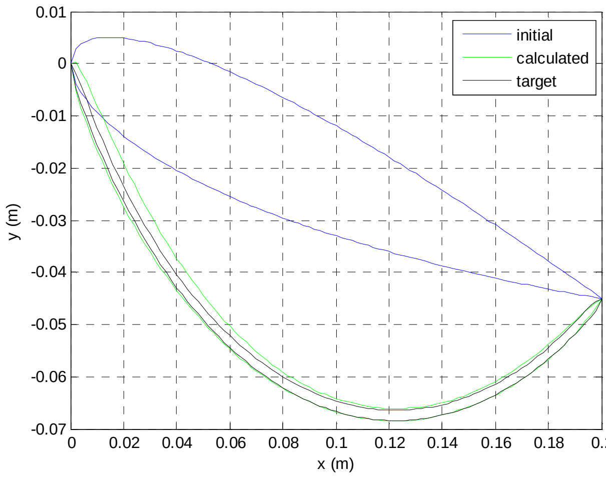


Fig. 3: Initial, target and calculated blade shapes

IV. SUMMARY AND CONCLUSIONS

An inverse design process based on the MGM technique is proposed to determine the shape of VAWT blades. The process uses a target velocity or pressure distribution as well as an initial blade profile and determines a blade profile that corresponds to the target velocity or pressure distribution. The velocity around any actual blade shape during the process is calculated by the panel method. The process was applied successfully with target conditions corresponding to a VAWT blade with constant thickness, which is used in the University of Huddersfield laboratory. First, the target velocity distribution around this blade was determined; and then the inverse design process provided a profile that closely approximates this blade shape. The process has already been applied for horizontal axis wind turbine blades and for aerofoil sections; however, the present application shows the reliability of the inverse design process for VAWT blades with constant thickness.

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