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Optimal Sizing of a Two Dimensional Duct for Transporting Solid-Liquid Mixtures

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***Abstract*— The transport of solid-liquid mixtures through pipelines is an advanced area of research and is being carried out to achieve energy efficiency and hence reduce transport cost of the material to be transported. The transport cost of the material in two dimensional ducts is considered to be less than that in a pipeline because of favourable concentration and velocity distributions within such ducts. In the present work, a methodology has been developed to design a two dimensional ducts for transporting solid-liquid mixtures.**

1. INTRODUCTION

The pipelines transporting solid-liquid mixtures have proven to be cost effective and have been instrumental in reducing the traffic and accidental hazards. This mode of transportation is environmentally friendly as compared to other modes of transportation of solids in the bulk form[1,2]. The cost of such piping systems depends on operational cost as well as capital costs. The operational costs depend on a number of parameters such as pipe size, solid fraction, solid particle size distribution, operating velocity, wear of the pipelines etc.

The design of pipelines transporting solid-liquid mixtures is reaching maturity with a number of excellent works published over the last 40 years [1-10]. There are some works that indicate that the flow through two dimensional ducts may offer significant advantages over a pipeline of same cross-section because the flow patterns in two dimensional ducts may be a lot more favorable than that seen in pipelines [11]. The investigations on flow pattern and pressured drop in a two dimensional duct have been reasonably well reported by various investigators [11] but detailed design process of such a mode of solid-liquid transport has rarely been reported in the literature. In this paper, an attempt has been made to develop a structured design methodology that may enable the pipeline designers to design a two dimensional duct system. For this purpose, various cost elements for a two dimensional duct system have been formulated which include cost of pumping power and capital cost of pipeline. Similar attempts have already been made for pipes with circular cross section [12-15]. A solution method has been proposed that gives the size of the duct for the least annual cost Finally, a design example is presented which demonstrates the application of the design process.

2. MODELLING AND ASSUMPTIONS:

The cost analysis of pipelines and ducts transporting solid-liquid mixtures includes determination of a size so that the annual total cost per unit length of the pipe is minimum. The following assumptions have been made when developing the methodology:

- The duct considered in the analysis is two-dimensional and that its width is minimum 5 times the depth.
- The flow through the duct is heterogeneous and hence the velocity of flow is higher than the deposition velocity.
- The major loss of energy is because of the frictional effects over the length of the duct and the minor head losses may be neglected.
- The particle size distribution is uniform and particles are represented by the weighted mean diameter in the analysis if the distribution is non-uniform.

The total cost of the duct transport system consists of the costs of pumping power, and the manufacturing costs. In the following, various cost models have been presented that satisfy various technical and operational constraints.

3. PUMPING ENERGY MODEL:

The pumping energy required to transport a given amount of solid-liquid mixture can be calculated if the head loss in a duct transporting such a mixture can be calculated. For the flow of water in a pipeline an equation of the following type is sufficient to give the information [16].

$$h_f = \frac{fLV^2}{2gD} \tag{1}$$

In the above equation 'f' is the friction factor [17], 'L' is the pipe length, 'V' is the flow velocity, 'g' is the acceleration due to gravity, and 'D' is the pipe diameter. For rectangular ducts D is taken as hydraulic diameter which is equal to the following expression

$$D = \frac{4\, d_1\, d_2}{2(d_1 + d_2)} \tag{2}$$

In the above equation d_1 and d_2 are the two sides of duct cross section.

The above equations work well for water flow through rectangular ducts but when the duct transports solid-liquid mixtures additional energy is needed to carry the solid particles in the fluid. The assumption of homogeneous flow of mixture with modified viscosity and density of the mixture has been shown to give inaccurate results for such flows especially for multi-sized solid-liquid mixtures even in ducts with circular cross section. [14]. For equi-sized solid-liquid flows, Durand [1,16] proposed a simple expression based on extensive experimentation to determine head loss in a unit length of pipe due to the mixture as a sum of two terms as given below:

$$\Delta h_S = \frac{fV^2}{2gD} + \frac{K\sqrt{gD}(S-1)^{1.5} C_v f}{VC_d^{0.75}} \quad (3)$$

The above equation has been empirically developed from the flow data on equi-sized slurries. The two terms on the right hand side represent head loss for water flow alone and the additional pressure drop over and above water flow respectively. K is a constant and depends on the flow situation. In the present work, this equation has been adapted to be used for two dimensional ducts and the flow difference effects between pipe and ducts have been included in K [1, 16]. It should be kept in view that while using this equation a suitable value of K should be chosen (either through experiments and/or analytical tools). The equation (3) can be used now to derive equation (4) to effectively compute the pressure drop in a two dimensional duct transporting solid liquid mixture.

$$\Delta h_S = \frac{fV^2(d_1+d_2)}{4gd_1\, d_2} + \frac{K2\sqrt{gd_1d_2}(S-1)^{1.5} C_v f}{(d_1+d_2)^{0.5}\, VC_d^{0.75}} \quad (4)$$

For the first term, the friction factor f can be calculated from the equation given below [16].

$$\frac{1}{f^{0.5}} = -2\log\left[\frac{\varepsilon / D}{3.7} + \left(\frac{7}{\mathrm{Re}}\right)^{0.9}\right] \quad (5)$$

In the above equation the Reynolds number can be expressed by:

$$\mathrm{Re} = \frac{\rho_L VD}{\mu} \quad (6)$$

The definition of the terms is provided in the nomenclature section. The flow velocity to be used should be such that it is slightly higher than the deposition velocity. It has been shown [11] that the concentration gradients in two dimensional ducts are fairly flatter as compared to circular pipelines. Keeping this in view

Wick's equation [18] has been used for determination of deposition velocity as this equation is expected to give conservative values for the two dimensional ducts. The diameter D in the equation is taken as hydraulic diameter

$$V_D = 1.87\left(\frac{d}{D}\right)^{1/6}\left[\frac{2gD(\rho_S-\rho_L)}{\rho_L}\right]^{0.5} \quad (7)$$

The operational flow velocity can now be chosen to be slightly higher than the deposition velocity.

$$V = V_D + 0.2 \quad (8)$$

The solid concentration 'C_v' can be expressed as given below and can be computed from known solid throughput 'Q_S':

$$Q_S = \frac{d_1 d_2 V C_v \rho_S}{} \quad (9)$$

The volumetric flow rate of the carrier fluid is expressed as:

$$Q_L = d_1 d_2 V(1-C_v) \quad (10)$$

Assuming no slip the total volumetric flow rate can be written as given below.:

$$Q = \frac{Q_S}{\rho_S} + Q_L \quad (11)$$

The drag coefficient and the settling velocity equations have been used as per Mishra [15].

4. **Cost Model**

To compute the total cost of the system the pumping energy cost and the manufacturing costs need to be combined. The power 'P' required per unit pipe length to transport a unit volume of mixture is obtained in terms of the total head loss in the pipeline as follows:

$$P = \frac{\rho_L g Q \Delta h_S / \eta}{Q} = \frac{\rho_L g \Delta h_S}{\eta} \quad (12)$$

The cost of duct can be calculated once its volume is known. As the dimensions of the duct are known, its volume can be calculated for a known thickness which is (5) operating pressure dependent. . The levelized net manufacturing cost of pipe per unit pipe length per unit volume of transported mixture is obtained from the levelized net cost of pipe per unit weight of pipe material 'C_2' [6]:

$$C_{manuf} = \frac{C_2 2(d_1+d_2)t\gamma_p}{Q} \quad (13) \quad (6)$$

The symbol 'γ_p' stands for specific weight for pipe material. Agarwal & Mishra [9] determined the pipe wall thickness in terms of pipe diameter 'D' and the coefficient 'C_c' for reasonable pressure range as:

$$t = C_c D \quad (14)$$

This expression has been modified to

t= Cc $(d_1+d_2)/2$ for ducts and this equation has been used for simplicity in its use. However more accurate equations can be used in its place if available.

Hence, the manufacturing cost of the pipe takes the form:

$$C_{manuf} = \frac{C_2 C_c (d1+d2)^2 \gamma_p}{Q} \quad (7) \quad (15)$$

Finally, the levelized total cost of the duct per unit length per unit volume of transported mixture throughout the lifetime of the pipeline is the sum of the cost of pumping energy and the cost of pipes:

$$C_{total} = C_{power} + C_{manuf} \quad (16)$$

Now putting a constraint of the duct being two dimensional and hence $d_1=5d_2$, the above cost equation can further be simplified. The equation 4 will now be

$$\Delta h_S = \frac{6fV^2}{20g\, d_2} + \frac{K2d_2\sqrt{gx5}(S-1)^{1.5} C_v f}{(6d_2)^{0.5}\, VC_d^{0.75}} \quad (17)$$

And hence the cost of pumping power can now be written as

$$Cpower = \frac{C_1 \rho_L g \Delta h_S}{\eta} \quad (18)$$

And the equation (15) will take the following shape,

$$C_{manuf} = \frac{C_2 C_c (6d_2)^2 \gamma_p}{Q} \quad (19)$$

In order to predict the optimal size of the duct, the total cost of the pipeline has now been represented in terms of the duct cross-section. The optimal duct dimensions can now be calculated by differentiating the total cost of the pipeline with respect to the duct dimension d_2. The results can then be summarised graphically where the relationship between total cost and duct dimensions can be depicted to find a local minima.

5. Design Example:

As a design example, the size of the duct will be determined, which can deliver 50 kg/s of Iron ore slimes for the lowest cost per unit pipe length per unit volumetric flow rate of mixture for the following data:

$S = 4.80$; $\mu = 1.003\times10^{-3}$ Pa·s; $\rho_L = 1000$ kg/m³; d = 93.4 □m

$\gamma_p = 78480$ N/m³;□□ = 0.9

C_{1,t_1} = 15 £/W/year; $C_2 = 0.1$ £/N; $C_c = 0.1$

The calculations for the dimensions are shown in Table 1. The computations have been shown in table 1 to demonstrate the design methodology developed. It can be seen that optimal size of the duct corresponds to a depth of 0.07 m and width of 0.35 m. Under such a condition the delivered concentration is 14.2% and the flow velocity is 2.2 m/s. The result shown in the table 1 will depend on the value of constants use and hence are subject to change depending on market conditions. The use of the design methodology should be carried out with caution as various parameters may change with respect to time and location. Furthermore, effects of simplifying assumptions may affect the total costs considerably and may invalidate the sue of this methodology for the cases where these assumptions are not fully applicable.

Table1: The details of the computations

d2 (m)	Pressure drop (m/m)	Pipe cost	Pumping cost	Total cost
0.04	0.784982331	64.19341	6978.828263	7043.022
0.05	0.640579618	167.5149	6733.198735	6900.714
0.06	0.446436931	366.9214	5632.219869	5999.141
0.07	0.334640754	712.1753	5097.690523	5809.866
0.08	0.264331139	1265.182	4860.68827	6125.87
0.09	0.217114394	2100.6	4799.294168	6899.894
0.1	0.183755048	3306.391	4851.747128	8158.138
0.11	0.159214325	4984.333	4983.979789	9968.313
0.12	0.140557116	7250.488	5175.881604	12426.37
0.13	0.125980639	10235.64	5414.901567	15650.55
0.14	0.114327986	14085.73	5692.831778	19778.56
0.15	0.104828839	18962.21	6004.085467	24966.3
0.16	0.096953893	25042.46	6344.725353	31387.19
0.17	0.09032943	32520.11	6711.890344	39232

6. Conclusions:

A design methodology has been developed to design a two dimensional duct for the transport of solid liquid mixtures. The developed design methodology, works on least cost principle and provides a framework for further development of such methodologies. A design example has demonstrated that such a design methodology can be used for such purposes if suitable simplifying assumptions are made.

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