Inverse problems of measurement with application on specification of surface profile

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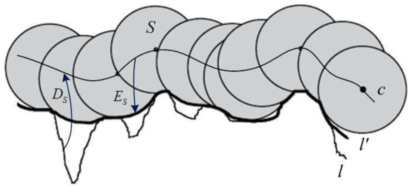
Introduction:

A contradiction of the specification of free-form surface profile is pointed out. The inverse problem of measurement (IPM) is defined base on the representational measurement theory. By using the concept of IPM, a desired property of specification limits is derived, and a correction for solving the contradiction is proposed.

Specification and measurement of surface profile

The upper and lower specification limits (USL and LSL) of a free-form surface profile defined in ISO 1101 are *two curves* enveloping circles of certain diameter *t*, the centers of which are situated on the nominal surface profile (see figure 2a). For an actual surface profile *l*, if all the points on *l* are within the tolerance zone, i.e. USL≤ *l* ≤ LSL, *l* is within the spec.

The canonical method of measuring surface profile is contact measurement by moving a tactile stylus along the surface to be measured to obtain the locus of the centre point of the stylus tip.



S: tactile measuring stylus;
l: actual surface profile;
c: locus of centre point of *s*;
l': estimated surface profile;
D_S: dilation filter
E_S: erosion filter

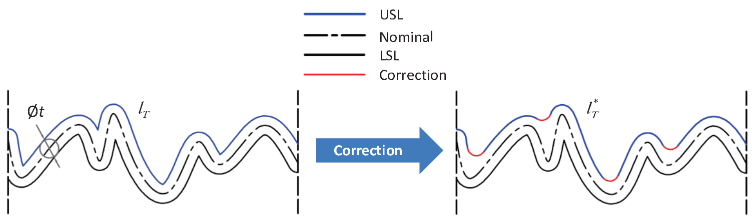
Figure 1. working principle of measuring surface profile with a tactile stylus

With *S* as the structuring element, the locus *c* is the dilation of *l*, and *l* can be estimated by the erosion of *c*, $l' = E_S D_S(l) = E_S(c)$. The combination of D_S followed by E_S is a *closing filter*, $C_S = D_S E_S$.

A Contradiction of the Specification of Free-form Surface

The contradiction

Due to the extensive property of closing filter, the estimated profile is always above the actual profile (see figure 1). Hence when an actual surface profile coincides with the USL (thus within spec.), the measurement result (without errors) would, however, be out of spec., which contradicts with the actual situation.



a. Before correction (ISO 1101)

b. After correction

Figure 2. A correction of the tolerance zone of surface profile

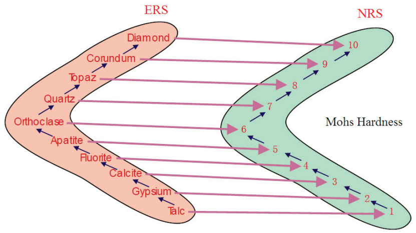
Representational model of measurement

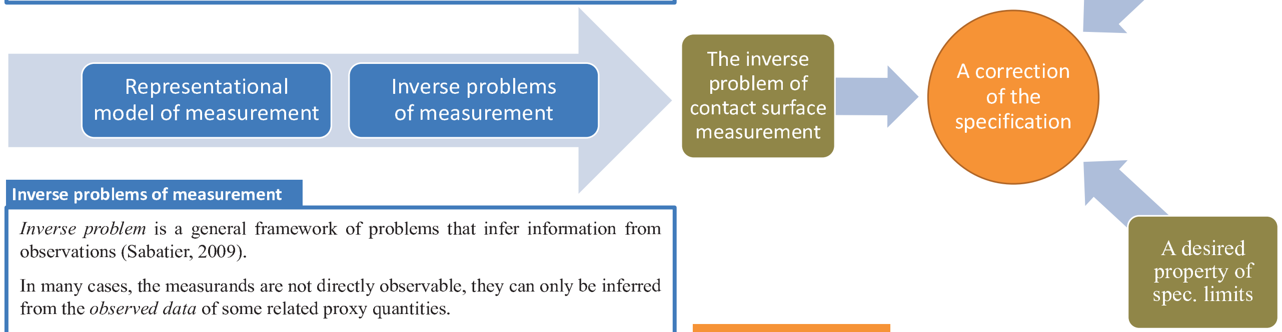
Representational measurement theory allows ***measurement*** to be defined as the assignment of numbers to attributes of objects in such a way as to describe them (Finkelstein, 1982). Hence measurement can be considered as a *mapping* from the measured objects to the measured values.

For the measurement of a attribute, one or more *empirical relations* would be defined between the measured objects. E.g. preorder ≾ is a very general empirical relation.

The set of the measured objects with the empirical relations, $R_1, R_2, \ldots, R_p$, can be taken as a mathematical object $\mathfrak{M}=\langle M, R_1, R_2, \ldots, R_p\rangle$, called an **empirical relational system** (**ERS**). E.g. the ERS of length (or mass, time) is $\langle M, \precsim, \circ\rangle$, where ≾ is a preorder, ∘ is a concatenation operation.

Measurability: a measurement is possible only if there exists a structure-preserving mapping from the ERS to a specified **numerical relational system** (**NRS**). E.g. the NRS representing the length is $\langle \mathbb{R}^+, \leq, +\rangle$. The numbers in the NRS are the values of the measurand (quantity to be measured).



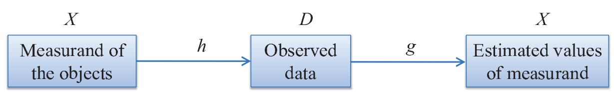


Inverse problems of measurement

Inverse problem is a general framework of problems that infer information from observations (Sabatier, 2009).

In many cases, the measurands are not directly observable, they can only be inferred from the *observed data* of some related proxy quantities.

Definition: Inferring the values of the measurands from the observed data is the **inverse problem of measurement (IPM)**. Its *forward mapping* is the characteristic function of the measurement process, $h: X \to D$.



The inverse or pseudo-inverse of *h*, denoted as $g: D \to X$, can be used to find the inverse solution. *g* is expected to satisfy the following equation,

$$hgh=h.$$

For any IPM, *X* and *D* are always determined by an measurable ERS.

Principle of correcting the contradiction

- To estimate the surface profile according to the observed locus is an **inverse problem**. D_S is the forward mapping and its pseudo-inverse is E_S, in the sense that $D_S E_S D_S = D_S$.
- **Essential reasons of the contradiction:**
  - the forward mapping D_S is not one-to-one;
  - the inverse solution *l'* is a maximal point of the possible inputs, i.e. $l \leq E_S D_S(l) = l'$.
- The spec. limits should reflect the required *measurement resolution*, e.g. 3.00 +/- 0.10mm. So the spec. limits given in ISO1101 should be amended.
- We expect that if the true value of a measured object is within spec., its measured value is also within spec. Hence the following **desired property of spec. limits** should be satisfied:

  Let *a* be a spec. limit, $a \in X$, then $gh(a)=a$.

A proposed correction

- Correcting the curve of USL from l_T to $l_T^* = C_S(l_T)$ (see figure 2b), where C_S is the *closing filter* with the structuring element *S*.

  The diameter of the stylus *S* is assumed to be smaller than the diameter *t*. It can be proved with the idempotent property of closing filter that the *desired property* is satisfied after the correction.

Acknowledgement: The authors gratefully acknowledge the UK's Engineering and Physical Sciences Research Council (EPSRC) funding of the EPSRC centre for Innovative Manufacturing in Advanced Metrology (Grant Ref: EP/I033424/1).

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