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Applications of Morphological Operations in Surface Metrology and Dimensional Metrology

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Abstract. In contrast to the widely used mean-line based evaluation techniques, the capabilities of morphological methods are not fully recognized in practice. Morphological operations, e.g. dilation, erosion, closing and opening, are useful tools in surface metrology and dimensional metrology. This paper presents a variety of novel applications of morphological operations in association with several of existing critical cases to demonstrate their usability and capability. These applications include scanning process analysis, real mechanical surface reconstruction, freeform surface deviation evaluation, open surface and roundness filtration, form approximation, contact phenomenon simulation, establishment of uncertainty zone for continuous surface reconstruction and stratified functional surface evaluation.

1. Introduction

Mathematical morphological is a mathematical discipline established in the 1960's [1]. It studies the geometrical structure of an image by matching it with the structuring element at various locations in the image. By varying the size and the shape of structuring elements, geometrical information of the different parts of the image and their interrelation can be extracted [2]. These useful tools were first widely used in image processing as nonlinear operations and then extended to other disciplines, including geometrical metrology.

In contrast to the widely used mean-line based evaluation techniques, e.g. the Gaussian filter, morphological methods are not universally adopted. Nonetheless they are of great value if not consciously recognized in practice. The initial application of morphological operations in geometrical metrology is the scanning of workpieces using tactile probes. The traversing process over the workpiece surface is in actual a hardware implementation of the envelope filter proposed by Von Weingraber [3], which is performed by rolling a ball over the workpiece surface. The envelope method, also known as the E-System, in comparison to the mean-line based filters (called as the M-System) is a function oriented approach for the functional prediction of workpieces in that it is related to geometrical properties of workpiece surfaces. Later morphological operations were introduced into the E-System, resulting morphological filters, which offer more tools and capabilities [4].

This paper proposes a variety of novel applications of morphological operations coupled with several of existing cases aiming to demonstrate their usability and capability for surface metrology and dimensional metrology.

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2. Mathematical morphology operations

Morphological filters are based on four basic morphological operations, namely dilation, erosion, opening and closing. They form the foundation of mathematical morphology [5].

Dilation combines two sets using the vector addition of set elements. The dilation of A by B is:

$$D(A,B) = A \oplus \check{B},$$

where $A \oplus B = \{c \mid c = a + b, a \subseteq A \ \& \ b \subseteq B\}$ and $\check{B}$ is the reflection of B through the origin of B. Figure 1 presents an example of dilating a square by a disk. The dilation of the light colour square by a disk results in the dark colour square with round corners.

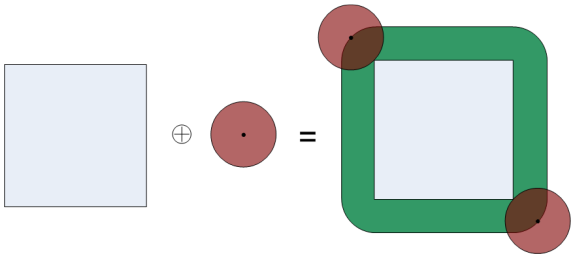

Figure 1. Dilation of a square by a disk

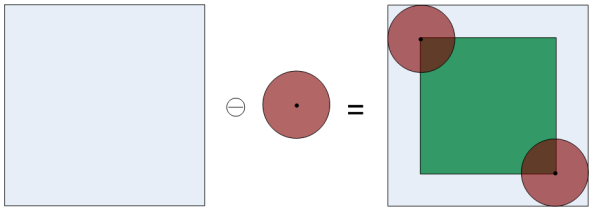

Figure 2. Erosion of a square by a disk

Erosion is the morphological dual to dilation. It combines two sets using the vector subtraction of set elements. The erosion of A by B is:

$$E(A,B) = A \ominus \check{B},$$

where $A \ominus B = \overline{\overline{A} + B}$. An example of erosion is illustrated in Figure 2. The erosion of the light colour square by a disk generates the dark colour square.

Opening and closing are dilation and erosion combined pairs in sequence. The opening of A by B is given by applying the erosion followed by the dilation,

$$O(A,B) = D(E(A,B),\check{B}) \ .$$

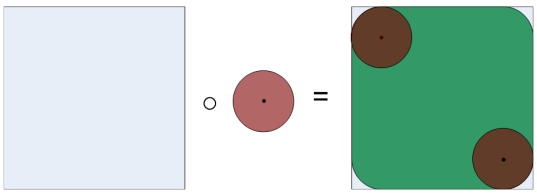

Figure 3. Opening of a square by a disk

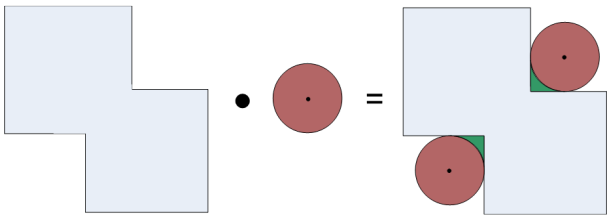

Figure 4. Closing of a union of two squares by a disk

In Figure 3, the opening of the light colour square by a disk generates the dark colour square with round corners.

Closing is the morphological dual to opening. The closing of A by B is given by applying the dilation followed by the erosion,

$$C(A,B) = E(D(A,B),\check{B}).$$

In Figure 4, the closing of the light colour shape (union of two squares) by a disk results in the union of the light colour shape and the dark colour areas.

3. Applications of morphological dilation and erosion operations

3.1. Surface scanning

The scanning of the workpiece surface using a tactile probe, e.g. the analog probe or the touch trigger probe, is a very common practice in geometrical measurement and a hardware implementation of morphological dilation operation [6]. The workpiece surface as the input set is dilated by the structuring element, the probe tip to generate the morphological output, the measured surface, which is also called the traced surface. Figure 5 illustrates the scanning process of a tactile probe. The scanning measurement is conducted by traversing the tip over the surface. The tip centre data are recorded at each sampling position and these sampled data form a discrete presentation of the measured surface. In ISO 3274 [7], the traced surface profile is defined as "*locus of the centre of a stylus tip which features an ideal geometrical form (conical with spherical tip) and nominal dimensions with nominal tracing force, as it traverses the surface within the intersection plane*".

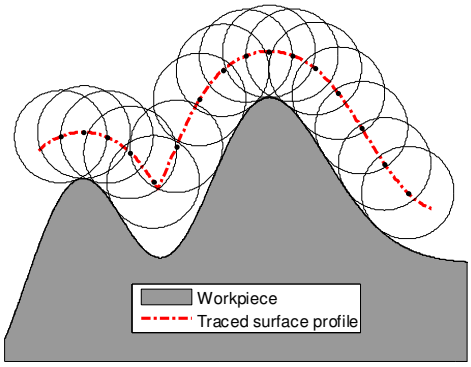

Figure 5. Scanning the probe tip over the workpiece surface

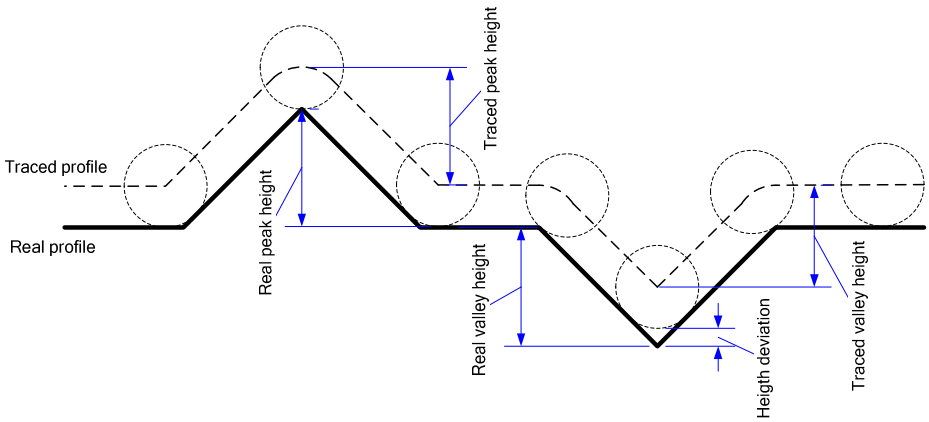

Figure 6. Tip mechanical filtering effects

In common practice, the probe tip employed for scanning are used to be small in size. However the tip size still influences the precision measurement of workpiece surfaces. Figure 6 demonstrates the effect of the probe tip traversing over the workpiece surface. By comparing the traced profile with the real workpiece profile in the figure, it is evident that the probe tip tends to round off peaks on the profile making it broader, nevertheless the peak height remains constant. The valleys on the profile are smoothed by the tip becoming narrow, meanwhile the valley height is reduced as well [8]. This effect introduces distortion into measurement of workpiece surfaces and is called as the mechanical filtration effect of tips. For the measurement of workpiece surfaces, especially for the freeform shaped workpieces, the distortions caused by the tip mechanical filtration effect appreciably influences the precision of measurement. Thus the correction to the traced surface is desired in order to restore to the

real workpiece surface. However the traced surface is unable to be perfectly reconstructed to the real surface, but only to an approximate one, i.e. the real mechanical surface.

3.2. Real mechanical surface reconstruction

ISO 14406 [9] presents the definition of mechanical surface: "*boundary of the erosion, by a sphere of radius r, of the locus of the centre of an ideal tactile sphere, also with radius r, rolled over the real surface of a workpiece.*" Figure 7 demonstrates the reconstruction process. Use an ideal sphere with the same size to the probe tip to roll over the traced profile, i.e. the dilated profile by the probe tip (which is already presented in Figure 5), the locus of the sphere centre is treated as the real mechanical surface. Rolling the ball from the below of the traced surface is in essence an erosion operation.

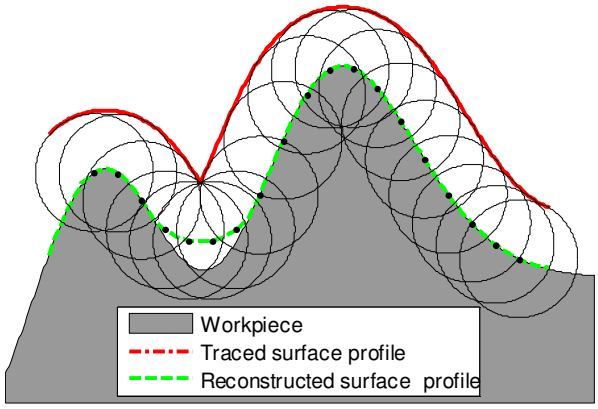

Figure 7. Reconstruction of mechanical surface

It is obvious that the morphological erosion operation is unable to perfectly reconstruct the original real surface of the workpiece. It was found that morphological operations can only reconstruct those portions of the surface where their local curvatures are larger than that of the probe tip [6, 10]. This indicates that the real mechanical surface differs from the real surface at the locations where the local surface curvature is small than the tips. Thus the reconstructed real mechanical surface varies with the probe tip size. Figure 8 presents such an example. Large probe tips tend to reduce and smooth the surface irregularities, while small tips enable the reconstructed surface more approximate to the real surface. The smaller the tip is, the closer the real mechanical surface approximates to the real surface.

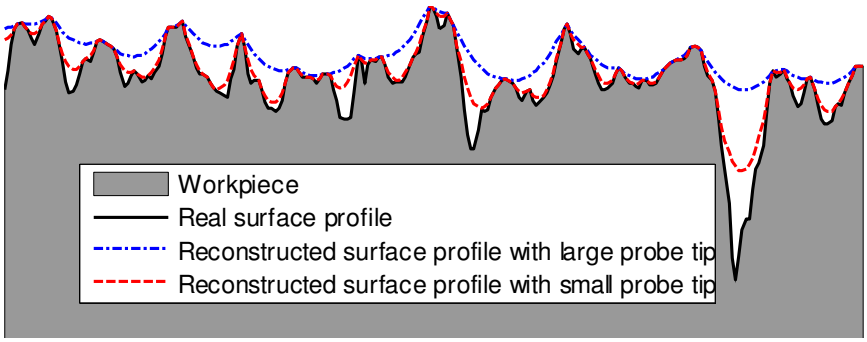

Figure 8. Reconstructed real mechanical surfaces vary with the tip size

In industry, the implementation of the reconstruction of the real mechanical surface varies from the application requirements. For surface texture instruments, for instance, the profilometer and the atomic force microscope, the reconstruction is usually performed by morphological image processing

techniques [11, 12]. Figure 9 presents a basic method to compute the dilation operation with the disk structuring element [13]. The disk ordinates are computed from the disk centre to the two ends. These ordinates are placed over the profile ordinates with the disk centre ordinate over the target profile point. The ordinate where the mapping pair of the profile ordinate and the disk ordinate gives the maximum value determines the height of the disk centre. This procedure is repeated for all the profile ordinates to obtain the whole envelope.

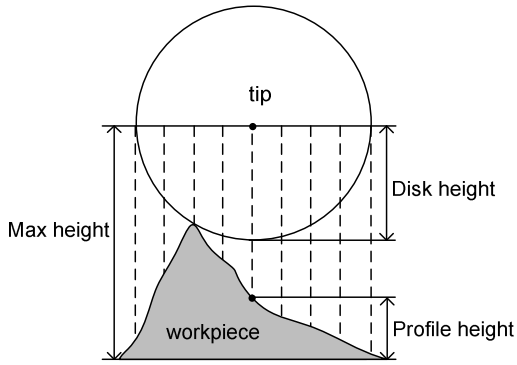

Figure 9. Computation of the dilation operation with the disk structuring element

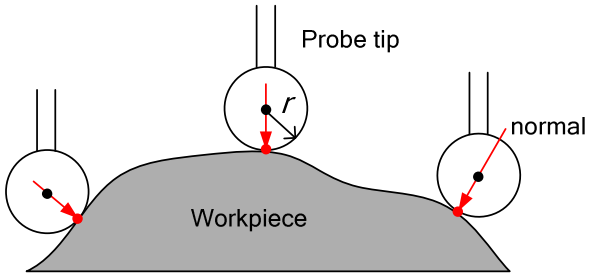

Figure 10. The radius compensation of CMM measurement

In dimensional metrology, for example, coordinate measurement, the reconstruction is usually implemented by the probe radius compensation. Compared to surface texture instruments, sampling of coordinate measurement machine (CMM) is usually less dense and the probe tip much bigger. As Figure 10 illustrates a couple of sampling positions, the contact points of the probe tip to the workpiece surface are obtained by compensating the tip radius in the direction of the surface normal at the contact point. The normal vectors for compensation are achieved either by guessing from the measured tip centre data in the case that the surface is densely scanned and nominal data is not available [14, 15], or by using the nominal vector at the matching point on the nominal surface model of the workpiece, e.g. the CAD model [16, 17]. Although dimensional metrology and surface metrology employ different routes, both of them are essentially morphological reconstruction to the real surface.

3.3. Freeform surface deviation evaluation

Usually the evaluation of freeform surfaces is challenging, especially for tactile measurement, due to their complex form. By convention, the evaluation routine for tactile measurement is constructed in the following sequence. The workpiece surface is measured using the scanning probe. The measured data are therefore compensated by tip radius along probing vectors, usually normal vectors extracted from the nominal surface model of the workpiece (e.g. CAD Model) at the corresponding point [18, 19]. The compensated points are then compared to the corresponding nominal points to capture the deviation. Although this method is reasonable and efficient, it is problematic in terms of two facts. For one thing, the extraction of the normal vectors are based on the nominal surface model and thus affected by the matching of measured data and nominal data. The misalignment between the coordinate frame of the nominal model and that of the real workpiece, which usually not precise in practice for freeform surface measurement because referencing datums are seldom available for establishing a fine alignment, will yield incorrect compensation vectors. For anther, this method assumes that the shape of the workpiece is same to that of the nominal surface model. Thus any deformation of the actual workpiece relative to its nominal shape will falsify the compensation vector [18].

Instead of compensating probe radius in the direction of the approaching vectors, an alternative method to evaluate the deviation of freeform surfaces is to dilate the nominal surface model of the

workpiece using the structuring element equivalent to the probe tip and compare the dilated data with the measured data. As presented in Section 3.1 that the traced surface is a dilation of the real workpiece surface by the probe tip, the differences between the actual dilated surface and the nominal dilated surface could thereafter be viewed as the deviation of the freeform surface. Refer to Figure 11 and Figure 12 as a demonstration of the profile example. In contrast to the traditional radius compensation approach, this method brings in a couple of merits. First, this method does not require calculating compensation vectors, which are sensitive to the misalignment of the real workpiece and its nominal model. Second, the dilation method is not affected by the deformation of the workpiece relative to the nominal model. Third, the mechanical filtration effect of the probe tip is not taken into account for the deviation evaluation, which is reasonable in that the deviation caused by the tip effect should not contribute to the deviation evaluation of the workpiece.

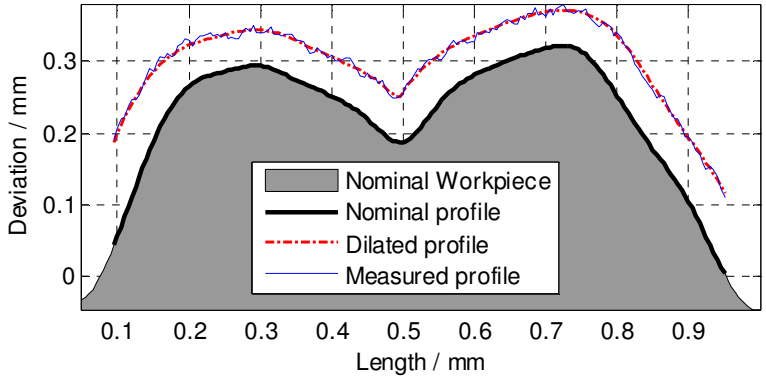

Figure 11. Evaluation of freeform surface profile using the dilation operation

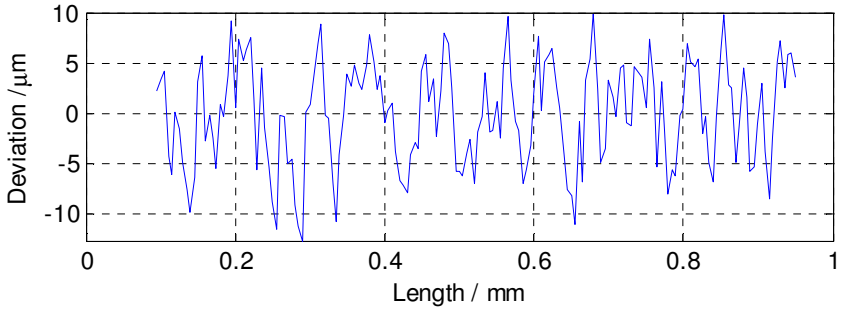

Figure 12. Deviation of freeform profile by subtracting the nominal dilated profile from the traced profile

4. Application morphological closing and opening operations

4.1. Closing and opening filters

Closing and opening filters are two envelope filter, whose output envelopes the input surface. They differ from the obsolete envelope filter proposed by Von Weingraber [3] in that the early envelope filter is essentially equivalent to the dilation operation, usually offset by ball radius.

Figure 13 and Figure 14 illustrate two examples of applying the closing operation and the opening operation on an open profile with the disk structuring element respectively. The closing filter is obtained by placing an infinite number of identical disks in contact with the profile from above along all the profile and taking the lower boundary of the disks [20]. On the contrary the opening filter is

archived by placing an infinite number of identical disks in contact with the profile from below along all the profile and taking the upper boundary of the disks.

It is obviously revealed in the figures that the closing filter suppresses the valleys on the profile which are smaller than the disk radius in size, meanwhile it remain the peaks unchanged. Conversely, the opening filter suppresses the peaks on the profile which are smaller than the disk radius in size, while it retains the valleys. The selection of the disk radius depends on the size of physical features on the surface of workpiece. Except circular structuring elements, other most commonly used structuring elements recommended by ISO 16610 are flat structuring elements, for instance, the horizontal line segment for profile data.

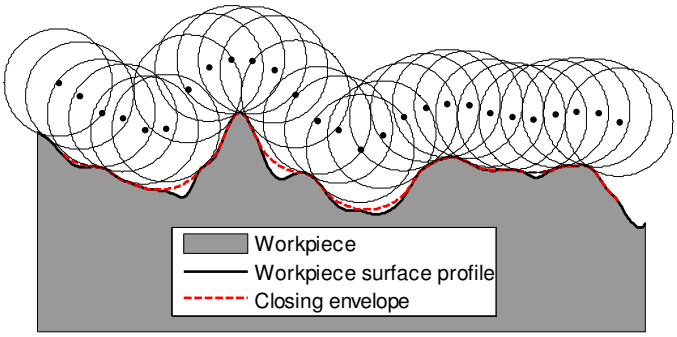

Figure 13. The closing envelope of an open profile by a disk

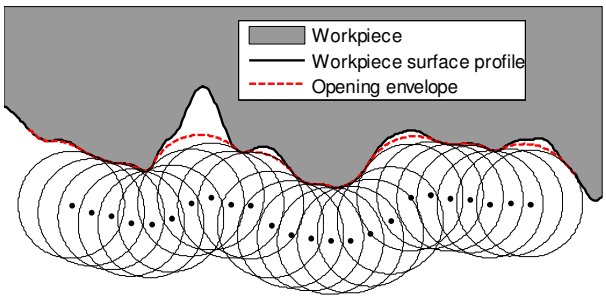

Figure 14. The opening envelope of an open profile by a disk

4.2. Form approximation

It has been illustrated that morphological envelopes could be utilized to approximate the form of functional surfaces for conformable interfaces [21], for instance a soft gasket in contact with a solid block in order to provide sealing function. The long wavelength component of the block surface could be tolerated by the compatibility of the gasket material while the middle wavelength components result in highly localized contacts. See Figure 15. The morphological closing envelope with the circular disk structuring element is used to approximate the conformable gasket surface such that the avoid areas between the conformable surface and the rigid surface can be obtained to characterize the sealing or load distribution. The radius of the circular structuring element should be chosen based on the compression and bending properties of the conformable component.

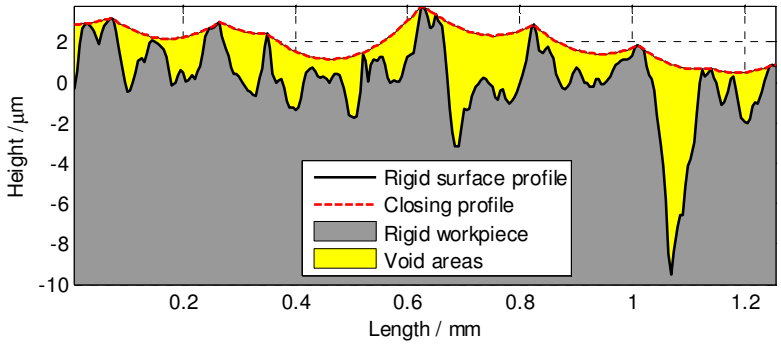

Figure 15. Approximation of the form of a conformable surface profile

4.3. Contact points

In many engineering applications, the contact between two surfaces is non-conforming, i.e., the contact area is very small when compared with the geometry of the bodies in contact. Even in situations between conforming contacts, the contact between the asperities that compose the surface topography is known to be non-conforming [22]. To investigate the contact phenomenon, the closing operation can be employed, whereby the interaction is simulated by rolling a ball with a given radius, which is sized to simulate the largest reasonable radius at a contact e.g. peak curvature, upon the underlying surface. The contact points of the rolling ball against the rolled surface are then captured. This serves as an indication of surface summits and surface portions which are in real contact.

In physics, the contact points are those points on the surface which are in contact with the rolling ball. From a point of view of mathematical morphology, the contact points are those points on the surface that remain constant with the morphological closing operation. Therefore these points might be captured by computing the closing envelope using appropriate algorithms [23]. Figure 16 illustrates such an example. The experimental profile is applied by the morphological closing filter with disk radius 5 mm. The contact points are those points on the profile remain constant before and after the closing operation, as illustrated by the dots in the figure.

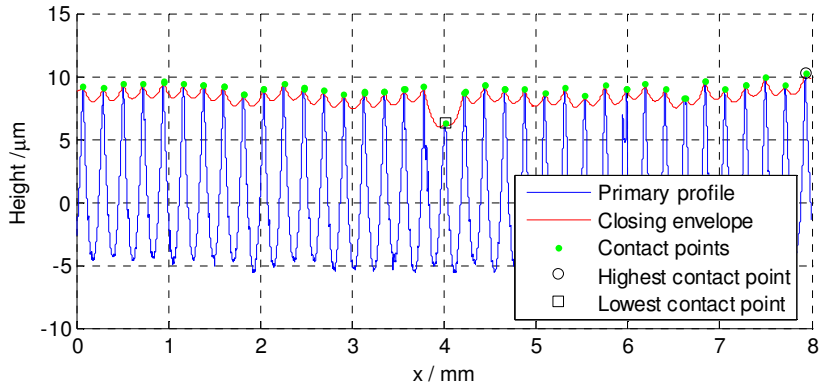

Figure 16. The contact points of the profile and the rolling ball with radius 5 mm.

4.4. Uncertainty zone for continuous surface reconstruction

The workpiece surface is the set of features that physically exist and separate the entire workpiece from the surrounding medium. The inspection of geometrical information of the workpiece surface is conducted by measuring the surface at certain sampling interval, using either the contact measurement instruments (e.g. CMM) or non-contact ones (e.g. interferometer). Either of them generates a series of sampled points, which form a discrete representation of the original surface. It should be noticed that

the sampled points in this scenario differ from those presented in the preceding cases in that they are supposed to be the contact points on the real workpiece surface, instead of the tip centre points for tactile measurement. For non-contact measurement, the sample data are all "contact points". It may be desired to reconstruct the original continuous workpiece surface from the discrete samples. In the theory of signal processing, the Nyquist theorem indicates that an infinitely long band-limited signal could be perfectly reconstructed without loss of information from the discrete data sampled at regularly spaced intervals if that interval is smaller than half of the minimal wavelength comprised by the original signal. In mathematical morphology, there is no theorem equivalent to the Nyquist theorem in that a universal equidistant sampling scheme can be found without loss of information, however there are a number of morphological sampling theorems to limit the amount of information lose [24].

Figure 18 illustrates an example of determining the uncertainty zone for the reconstruction of the original surface from a sequence of sampled points taken by a circular disk structuring element. The morphological sampling theorem takes the prerequisite that the surface profile Z under the examination remains unchanged after applying the opening and closing operation by a particular structuring element SE (e.g. a disk) of a given size (e.g. the disk radius), i.e. $C(Z,SE) = Z = O(Z,SE)$. If the original surface Z is sampled with a sampling interval strictly less than the size of SE, yielding a sampled surface Z_s, the original profile is supposed to lie in the region constructed by the opening envelope $O(Z_s)$ and the closing envelope $C(Z_s)$. This region defines the uncertainty zone in which the original profile lies, i.e. $C(Z_s,SE) \le Z \le O(Z_s,SE)$ [9].

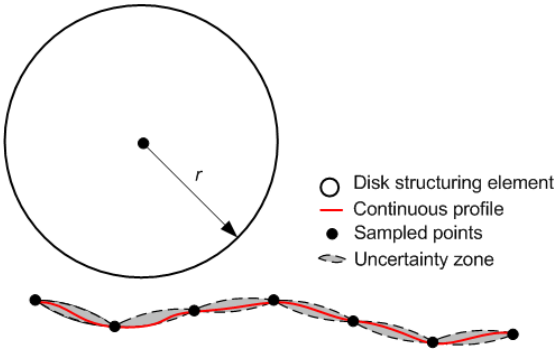

Figure 18. Uncertainty zone for continuous surface profile reconstruction with the disk

5. Applications of alternating symmetrical filters

5.1. Stratified functional surface evaluation

In engineering, surfaces with stratified functional properties are very common, for instance, the inner surface of cylinder liners for automotive engines. These kinds of surfaces are composed of deep valleys superimposed by plateaux. The plateaux support force, bearing and friction while the valleys serve as lubricant reservoirs and distribution circuits. The traditional method for the analysis of these surfaces is performed by applying the two-stage Gaussian filter, the so-called Rk filter. However there are several drawbacks of this method [25]. Firstly, it was derived from the empirical foundation with a significant assumption: surface contains a relative small amount of waviness. It is ambiguous and confusing. Secondly, running-in and running-out sections are generated from the Gaussian filter. These sections truncate the profile and only 20%-60% of the measurement data are used in evaluation. Thirdly, the form component needs to be removed from the profile before the Gaussian filter could apply to the data.

In contrast, morphological filters very suited for this kind of surfaces. Using morphological filters, the profile does not need to be pre-processed to remove the form. The roughness profile can be obtained over the complete measurement length if the end effects are cared for, therefore the roughness profile does not have running-in and running-out sections being "removed". Figure 19 presents such an example. The experimental profile was extracted from a plateau honed surface. The morphological alternating symmetrical filter, combination of first the closing filter and then the opening filter, with disk radius 5 mm, is employed to generate the reference line. As illustrated in Figure 19, the special alternating symmetrical filter reference line basically follows the form of the closing envelope, which is suitable for surfaces where valley features play a dominant role. The closing filter suppresses all the valleys on the original profile that are smaller than the disk radius and the opening filter removes all the peaks on the resulting closing envelope accordingly. The roughness profile is obtained by subtracting the reference line from the original profile.

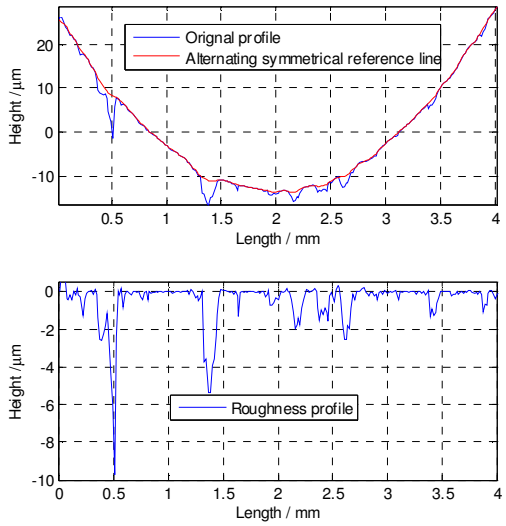

Figure 19. Roughness profile resulting from the alternating sequential filter with disk radius 5 mm

5.2. Roundness filtration

Unlike open profiles measured on planar surfaces, roundness profiles, such as those obtained using a rotating-spindle stationary probe roundness instrument, are closed profiles in shape. The roundness data should also be partitioned into different wavelength regimes to better understand process parameters and functional performance.

Conventionally, the data measured by roundness instruments are radius suppressed, which means the data only reflects the radial deviation while it is insensitive to the radius itself. In such a situation, the Gaussian filter and the spline filter have been successfully employed to decompose roundness data [26, 27]. Morphological filters can be applied for roundness filtration, however they should work on the unsuppressed roundness profiles, which means the component's radius and radius deviations are compounded together. Both the state of art roundness instruments and the CMM can provide such kind of data.

Figure 20 presents an example of applying the morphological alternating symmetrical filter (closing followed by opening) on a roundness profile measured from a cylinder part by a CMM. The cylinder is about 10 mm in radius and the roundness data is filtered with a disk of radius 1 mm. It should be noticed that for convenience of visualization both the roundness profile and the morphological envelope are radius suppressed (reduced by 9 mm).

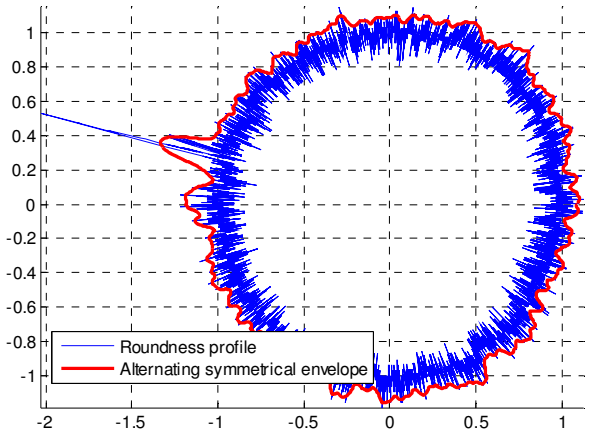

Figure 20. The morphological envelope obtained by the alternating symmetrical filter with disk radius 1 mm (Part radius suppressed)

6. Conclusion

Morphological operations, namely dilation, erosion, closing and opening, are useful tools in geometrical metrology. In this paper, a variety of novel applications of morphological operations in additional with several of existing cases are presented to demonstrate their usability and capability.

The traversing process using tactile probes and the reconstruction of real mechanical surfaces are morphological dilation and erosion respectively. The deviation evaluation of freeform surfaces could be conducted by dilating the nominal surface by tip radius and then compared to the measured data.

Closing and opening filters are superimposed results from combining dilation and closing operations in sequence. The closing filter could suppress the valleys on surfaces and retains the peaks, vice versa for the opening filter. Morphological envelopes could approximate the form of the functional surface, e.g. the conformable surface. The closing operation with the circular structuring element can be employed to simulate the contact phenomenon of two mating surfaces. To reconstruct the original continuous surface from the discrete samples, morphological closing and opening envelopes are able to provide an uncertainty zone for the reconstruction of original surfaces with limited amount of information loss.

The alternating symmetrical filter, as the combination of the closing filter and the opening filter, is an optimal alternative to the two-stage Gaussian filter for the evaluation of stratified functional surface. It can be also used in the roundness filtration.

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