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Cryan, R.A.

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# **SPECTRAL CHARACTERISATION OF THE DICODE PPM FORMAT**

R. A. Cryan

*Indexing terms:* Dicode, PSD, PPM

Dicode pulse position modulation (PPM) is a new modulation format that offers improved sensitivity over digital PPM but at a reduced line-rate. This Letter considers, for the first time, the spectral characterisation of the dicode PPM modulation format and presents original expressions for predicting both the continuous and discrete spectrum.

*Introduction:* Recent studies [1,2] have shown that dicode pulse position modulation (DiPPM) offers a sensitivity improvement of 7.5 dB and 2.4 dB over equivalent PCM and digital PPM systems respectively, but at a significantly reduced line rate in comparison to the latter. This makes the DiPPM modulation format particularly attractive for both optical fibre and optical wireless communication systems. Although a spectral characterisation has been presented for digital PPM [3], an evaluation of the power spectral density (PSD) for DiPPM has yet to be addressed. This issue is particularly important since the information is not only conveyed by the presence and absence of the pulse but also by its temporal position. In this Letter, a spectral characterisation of DiPPM is presented for the first time. Use is made of the cyclostationary properties of the pulse stream and original expressions derived for predicting both the continuous and discrete spectrum. It is shown that discrete components are available at the DiPPM frame repetition rate that can be used for

synchronisation purposes and that the continuous spectrum has a lower low-frequency content than PCM making it more attractive for optical wireless applications.

*Spectral Characterisation:* Several techniques are available for determining the PSD of a cyclostationary process including one previously reported by the author [3]. The approach adopted here is that outlined in [4] since it makes the cyclostationary nature of the sequence more explicit in the final solution of the PSD expressions.

The data pulse stream can be represented as

$$m(t) = \sum_{n=-\infty}^{\infty} a_n p(t - nT)$$

where  $\{a_n\}$  is the dicode sequence and  $p(t)$  is the pulse shape. To compute the

discrete PSD of  $m(t)$ , namely,  $S_m^d(f)$ , the statistical correlation function,

$R_m(t; \tau) \triangleq \overline{m(t)m(t+\tau)}$ , must first be averaged over  $t$  and then the Fourier transform

taken:

$$\begin{aligned} S_m^d(f) &= \mathcal{F}_\tau \left\{ \langle R_m(t; \tau) \rangle_t \right\} \\ &= \frac{1}{(4T_s)^2} \sum_{l=-\infty}^{\infty} \left| P\left(\frac{l}{4T_s}\right) \right|^2 \left| \sum_{n=1}^N \mathbb{E}\{a_n\} e^{+j\frac{\pi}{2}nl} \right|^2 \delta\left(f - \frac{l}{4T_s}\right) \end{aligned}$$

where  $T_s$  is the DiPPM slot-time. The term  $\sum_{n=1}^N \mathbb{E}\{a_n\} e^{+j\frac{\pi}{2}nl}$  represents the

characteristic function of the data distribution on the DiPPM frame and so makes the cyclostationary property explicit. Evaluating this and assuming a rectangular pulse of

height,  $A$ , and width,  $T_s$ , allows  $S_m^d(f)$  to be written as:

$$S_m^d(f) = \frac{A^2}{256} \sum_k \left| \frac{\sin\left(\pi \frac{k}{4}\right)}{\pi \frac{k}{4}} \right|^2 \left( 1 + \cos\left\{\frac{\pi}{2}k\right\} \right) \delta\left(f - \frac{k}{T_b}\right) \quad (1)$$

where  $T_b = 4T_s$  is the equivalent PCM bit-period.

The continuous PSD can be determined by evaluating the Fourier transform of the autocorrelation function of a zero-mean dicode sequence. The autocorrelation function is given by

$$\begin{aligned} R_M(t; \tau) &= \mathbb{E}\{M(t)M^*(t+\tau)\} \\ &= \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \int_y \int_z K_{a,\varepsilon}(n; m-n) \\ &\quad \times P(y)P^*(z)e^{-j2\pi y n T} e^{+j2\pi z m T} \\ &\quad \times e^{+j2\pi(y-z)t} e^{-j2\pi z \tau} dy dz \end{aligned}$$

where  $K_a(n; m-n) \triangleq \mathbb{E}\{a_n a_m^*\} - \mathbb{E}\{a_n\}\mathbb{E}\{a_m^*\}$ . Taking the Fourier transform of this gives

$$\begin{aligned} S_M^c(f) &= \mathcal{F}\left\{\langle R_M(t; \tau) \rangle_t\right\} \\ &= \frac{1}{T} |P(f)|^2 \sum_{l=-\infty}^{\infty} \left[ \frac{1}{N} \sum_{n=1}^N K_a(n; l) \right] e^{-j2\pi f l T} \end{aligned}$$

Which, through the contents of the square-brackets, makes the cyclostationary nature of the sequence explicit. Evaluating this for DiPPM gives

$$S_M^c(f) = A^2 T_s \left| \frac{\sin(\pi f T_s)}{\pi f T_s} \right|^2 \left\{ \begin{array}{l} \frac{3}{32} - \frac{1}{32} \cos(2\pi f T_s) + \frac{1}{32} \cos(6\pi f T_s) \\ -\frac{1}{16} \cos(8\pi f T_s) + \frac{1}{32} \cos(10\pi f T_s) \end{array} \right\} \quad (2)$$

*Results:* In order to validate the analytic results of (1) and (2), the PSD of DiPPM was evaluated numerically using the Fast Fourier Transform. Full width pulses were assumed and the equivalent PCM bit period set to 1 second. A sampling rate of 128 Hz was used and 100 FFT's were averaged in order to decrease the noise due to the randomness of the data sequence.

Fig. 1 compares the values of the DiPPM discrete spectrum, evaluated by Eqn. 1, to that determined using the FFT, at multiples of the DiPPM frame-rate. The results demonstrate that there is a strong discrete line at the DiPPM frame-rate and so this can be extracted for synchronisation purposes directly from the pulse stream. The numerical and analytical results are in excellent agreement so confirming the validity of Eqn. 1 for predicting the discrete spectrum of DiPPM.

Fig. 2 compares the DiPPM continuous spectrum, evaluated by Eqn. 2, to that determined using the FFT. Again, the numerical results are in excellent agreement with those determined analytically so validating the expression for predicting the DiPPM continuous spectrum. The results show that, unlike PCM, the spectrum is not concentrated at the near dc frequencies. This means that DiPPM may be a potential candidate for optical wireless systems since it offers increased immunity to low frequency noise sources such as that associated with fluorescent lights.

*Conclusions:* Original expressions, for both the discrete and continuous spectrum, have been presented that offer a full spectral characterisation of the dicode PPM modulation format. Using the FFT, the spectrum has been evaluated numerically and the results are in excellent agreement with those predicted using the new expressions presented in this Letter. They show that it is possible to extract the DiPPM frame rate component directly from the pulse stream and that, due to the low-frequency content of the continuous spectrum, DiPPM may be a suitable candidate for optical wireless communications.

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### Author affiliation:

R.A. Cryan (School of Engineering, Northumbria University, Ellison Building, Newcastle-upon-Tyne, NE1 8ST, United Kingdom)

Email: bob.cryan@northumbria.ac.uk

**Figure captions:**

Fig. 1 Discrete PSD of DiPPM

Fig. 2 Spectral characteristics of DiPPM



Figure 1

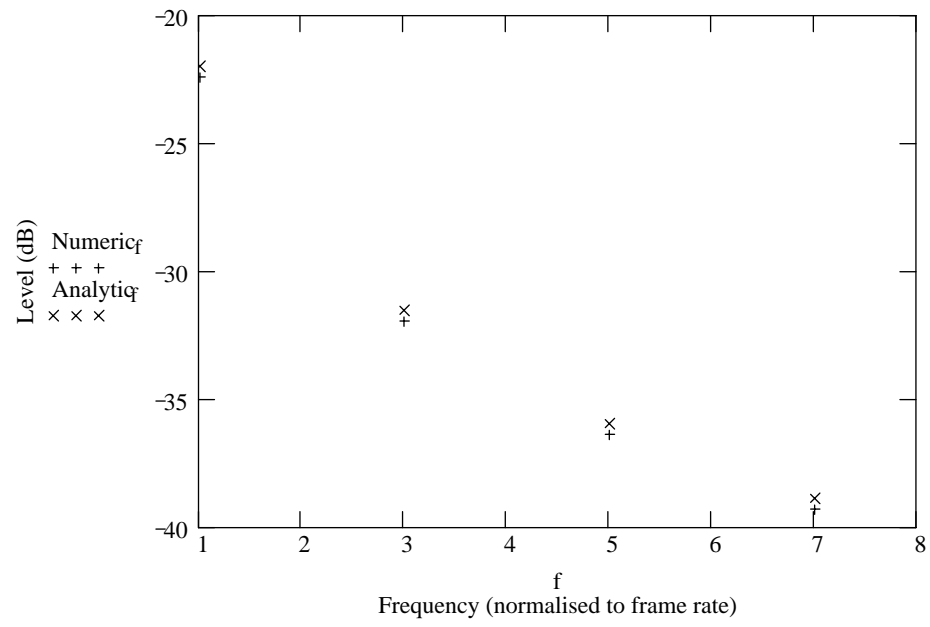


Figure 2

