A novel pitch evaluation method based on a cross-correlation filter

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Abstract

A cross correlation filter – a half period of sinusoidal waveform sequence (p_T period), is applied to filter the topographical signal (*p* period)of 1D arbitrary-form grating against positions. It, as a template, cross-correlate with the signal and the noise that coexists with signal is eliminated if $p_T \approx p$. After filtering, the distance between any two adjacent waveform peaks along the direction perpendicular to 1D grating lines is one pitch value, the peak-detection (PD) method. In this method, the pitch average and uniformity can be calculated.

1. If present pitch evaluation methods meet noise?

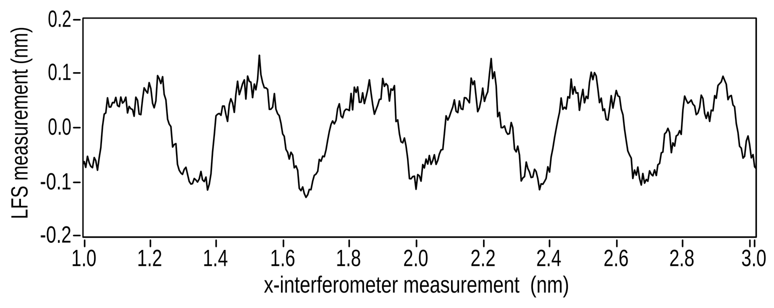


Figure 1. A grating position related topographical signal with noise and irregular waveforms. In such situation, centre-of-gravity (CG) evaluation method is difficult to be applied to calculate the pitches. Fourier-Transform-based (FT) method can be used, but it cannot decide the pitch uniformity.

2. 1D sinusoidal grating-probed signal filtering

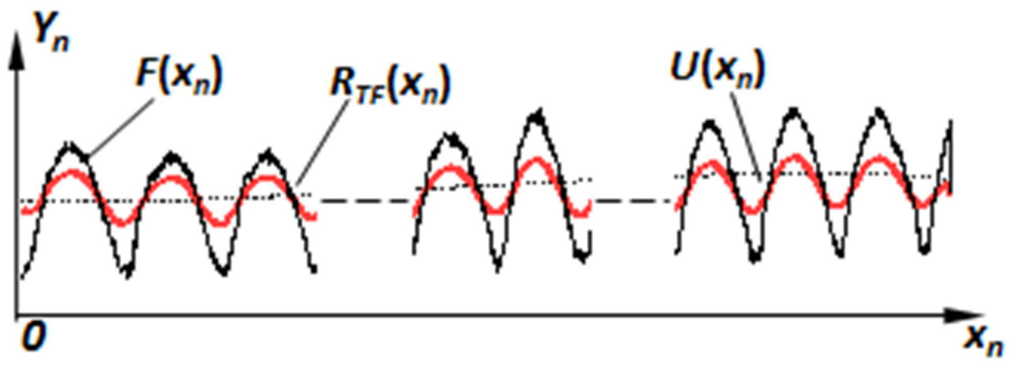


Figure 2. 1D sinusoidal grating position-related signal $F(x_n)$ probed by a AFM. It consists of sinusoidal f((x_n), noise W((x_n) and a tilt drafting U((x_n) signals:

$$F(x_n) = f(x_n) + U(x_n) + W(x_n) = A\sin(\frac{2\pi}{p}n) + \sum_{n=0}^{K} H \cdot n^K + \frac{1}{M}, \quad 0 \le n < M-1 \qquad (1)$$

A cross correlation filter:

$$T(x_n) = B\sin\frac{2\pi x_n}{p_T}, \quad 0 \le n < N-1 \qquad (2)$$

After cross-correlation filtering, the signal:

$$R_{TF}(x_m) = R_{Tf}(x_m) + R_{TU}(x_m) + R_{TW}(x_m) \qquad (3)$$

where $m = -(N-1), -(N-2), \cdots, -1, 0, 1, \cdots, M-2, M-1.$

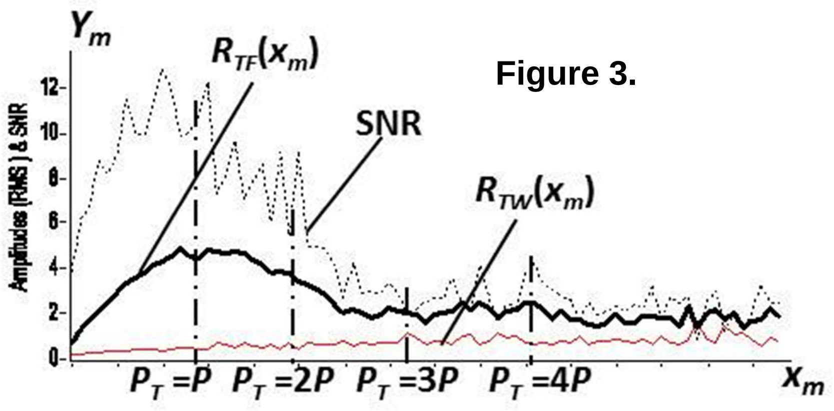


Figure 3.

Simulation experiment results [1] 1:

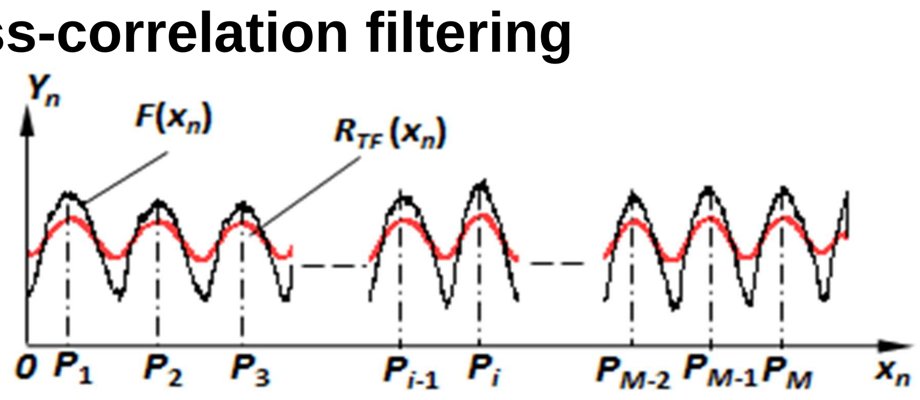
1. Cross correlation tilt signal $R_{TU}(x_m)$ is still a tilt component;
2. Cross correlation sinusoidal signal $R_{Tf}(x_m)$ is sinusoidal;
3. Signals are in same period of *p* , with phase shift *Φ*.

Illustrated by figure 3, simulation experiment results [1] 2:

$p_T = p$	$p_T << p$	$p_T >> p$
1.Cross correlation signal $R_{TF}(x_m)$ has max. amplitude; 2.Cross correlation noise $R_{TW}(x_m)$ is nearly to zero; 3.Signal to noise ratio (SNR) has maximum amplitude.	Noise can't be totally suppressed	$R_{TF}(x_m)$ drops as small as $R_{TW}(x_m)$ so that SNR is too low to distinguish periodical signal from noise.

3. Pitch evaluation after cross-correlation filtering

Figure 4. Signals before and after cross-correlation filtering, where low frequency tilt signals are eliminated in figure 2.



Individual pitch is calculated by:

$$\begin{cases} P_1 = (p_2 - p_1) \cdot \cos\alpha \\ P_2 = (p_3 - p_2) \cdot \cos\alpha \\ \cdots \\ P_{M-1} = (p_M - p_{M-1}) \cdot \cos\alpha \end{cases} \qquad (4)$$

Pitch Average: $$\bar{P} = \frac{1}{M-1}\sum_{i=1}^{M-1} P_i \qquad (5)$$

Pitch Uniformity: $$\delta = \frac{1}{M-1}\sqrt{\sum_{i=1}^{M-1}(P_i - \bar{P})^2} \qquad (6)$$

where, $p_1, p_2, p_3, \cdots, p_M$ – detected peaks; α – angle between *x*-axis and direction which is perpendicular to the 1D grating lines.

To decide the tilt angle α [2] :

$$\begin{cases} X = (fsx + Ix + fex) \cdot p / \sin\alpha \\ Y = (fsy + Iy + fey) \cdot p / \cos\alpha \end{cases} \qquad (7)$$

$$\alpha = \arctan(\frac{Y}{X} \cdot \frac{Ix + fsx + fex}{Iy + fsy + fey}) \qquad (8)$$

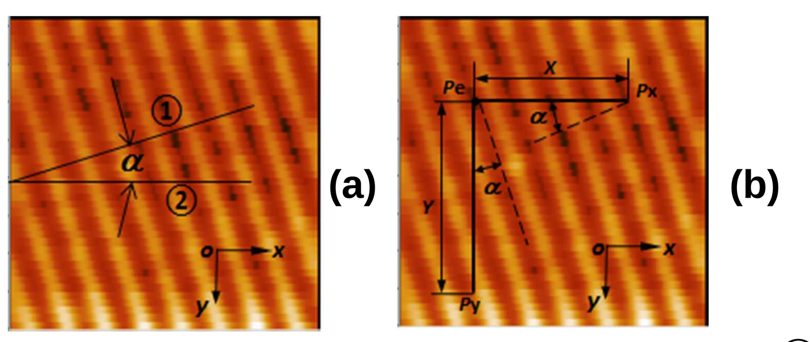


Figure 5. A tilt angle α between perpendicular line ① and displacement direct ② is shown in (a) and the diagram on evaluation of tilt angle α is shown in (b).

where, *Ix*, *fsx* and *fex* are integer periods, fractional parts at beginning and end in *x*-axis; *Iy*, *fsy* and *fey* are integer periods, fractional parts at beginning and end in *y*-axis.

4. Application of cross-correlation filter to signal of 1D, *p*-periodic and arbitrary-structured grating

It can be written as a Fourier sine series: $$f(x_n) = \sum_{k=1}^{\infty} A_k \cdot \sin\frac{2k\pi x_n}{p} \qquad (0 \le n \le N-1) \qquad (9)$$

where $$A_k = \frac{4}{p}\int_0^{p/2} f(x_n) \cdot \sin\frac{2k\pi x_n}{p} dx_n, (k = 1, 2, 3, \cdots) \qquad (10)$$

When it cross-correlated with a half sinusoidal waveform template of *p*-period,

$$R_{Tf}(x_m) = C_1 \cdot \sin(\frac{2\pi x_m}{p} + \phi_1) + C_2 \cdot \sin(\frac{2\pi x_m}{p_2} + \phi_2) + \cdots + C_k \cdot \sin(\frac{2\pi x_m}{p_k} + \phi_k) + \cdots \qquad (11)$$

where $p_k = p/k, (k = 2, 3, 4\cdots)$, **and C_k – proportional to A_k**

C_k decreases considerably with *k* increasing, 1st fundamental item in (11) nominates.

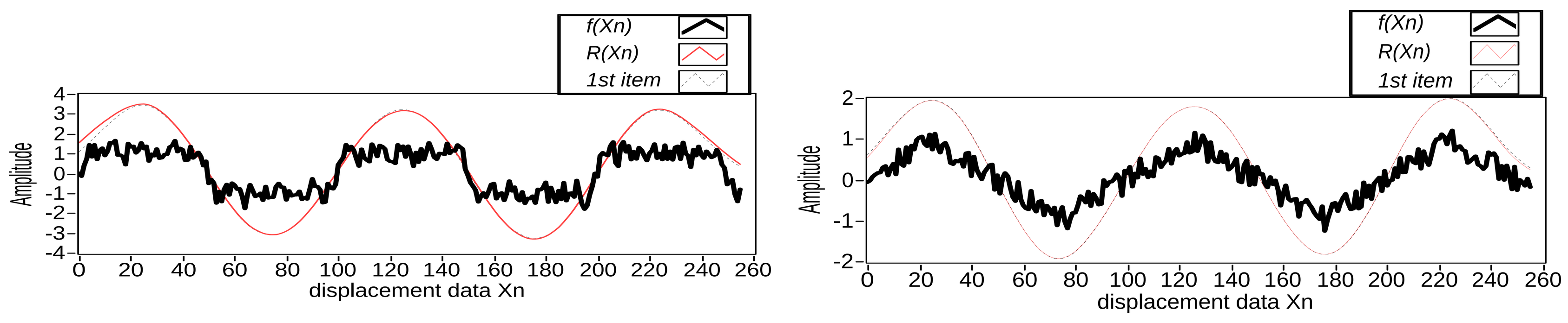


Figure 6. Simulations of 1D rectangular and 1D triangular grating position-related signals in legend $f(x_n)$ and their cross-correlation signals in legend $R(x_n)$ are shown in (a) and (b) respectively.

5. Experiments and results

5.1 Agreement between evaluated pitch and true pitch value

Table 1 Simulation results of average of pitch deviations and variations(arbitrary units)

Pitch evaluation method	Noise level	sinusoidal		rectangular		triangular	
		Deviation average	Variation (STD)	Deviation average	Repeat_ ability	Deviation average	Variation (STD)
PD	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.1	0.0	0.11	0.0	0.09	0.01	0.11
	0.2	-0.01	0.17	0.02	0.18	-0.01	0.20
	0.3	0.0	0.24	0.03	0.25	0.01	0.30
	0.4	-0.02	0.30	0.05	0.29	0.0	0.37
	0.5	0.0	0.38	0.09	0.39	-0.01	0.42
FT	0.0	-0.00	0.0	-0.01	0.0	0.01	0.0
	0.1	0.32	0.11	-0.09	0.10	-0.02	0.15
	0.2	-0.56	0.21	-0.07	0.17	-0.22	0.28
	0.3	-0.74	0.32	-0.49	0.23	-0.28	0.39
	0.4	-0.13	0.38	-0.11	0.35	-0.60	0.49
	0.5	0.09	0.49	-0.18	0.44	0.84	0.63

5.2 Cross-correlation filtering and pitch evaluation in PD method

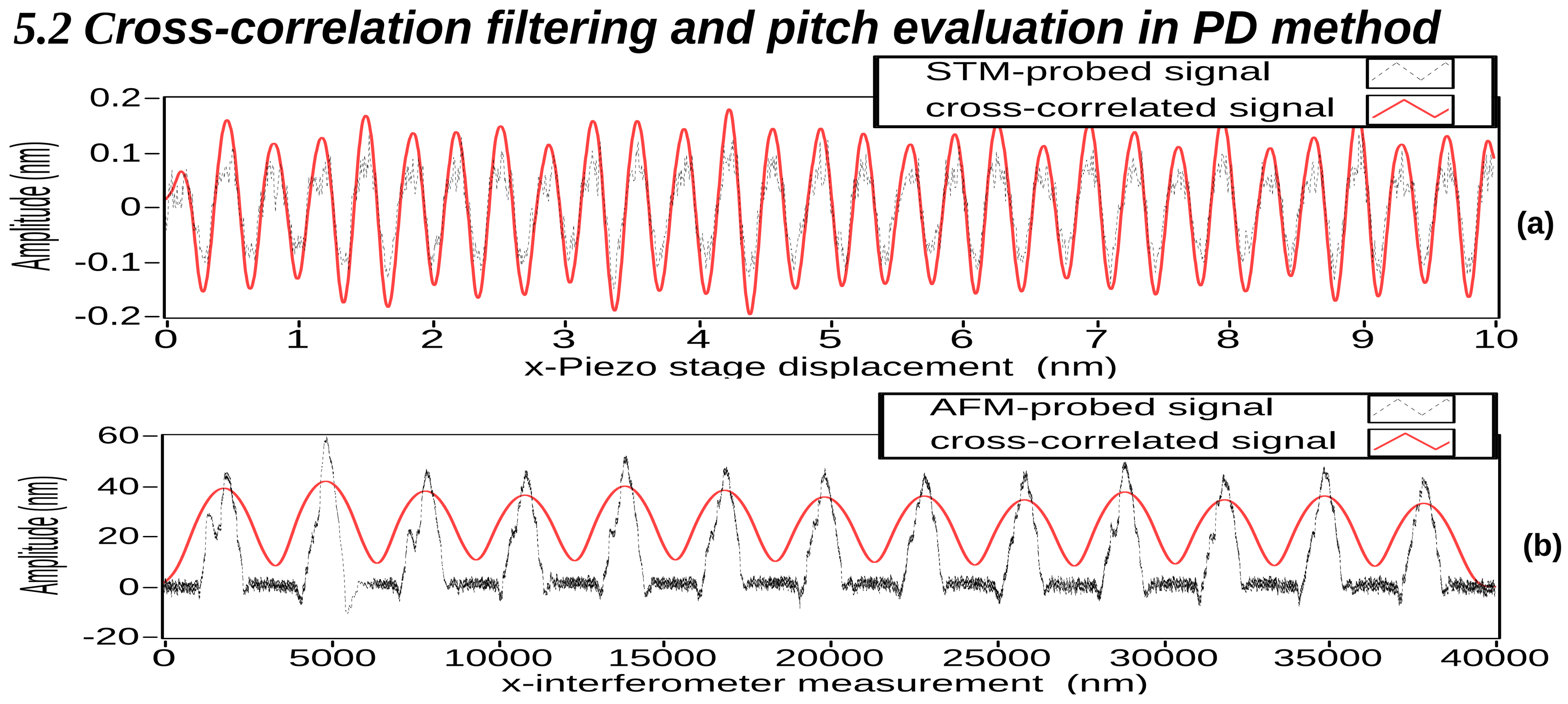


Figure 7. (a) STM- and (b) AFM-probed 1D grating position-related signal and its cross-correlation signal

Table 2 Average pitch and uniformity (STD) by PD method

Probe	PD (nm)	FT (nm)	STD in PD (nm)
STM	0.3392	0.3396	0.0142
AFM	3004.11	3003.34	19.45

5.3 In-plane tilt angles measurement

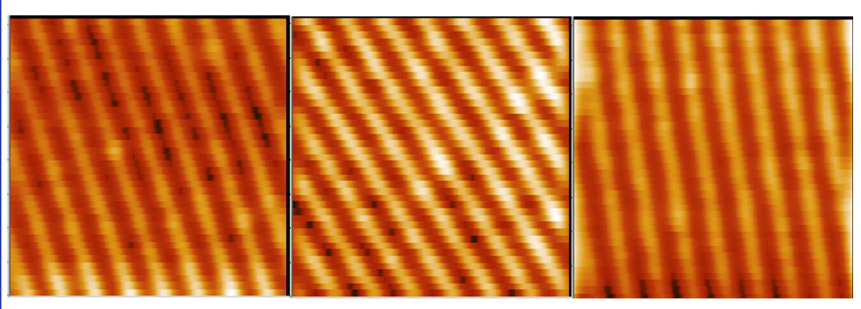


Figure 9. (a), (b) and (c) are the images of 1D sinusoidal grating with in-plane tilt angle I, II and III respectively. Angles are measured and calculated as 21.57°, 35.12° and 12.03° while measurement repeatability (STD) is 0.02°, 0.04° and 0.05° respectively.

Acknowledgements

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References

[1] Chen X, Koenders L, Haertig F 2011 Real time cross correlation process of one dimensional grating position encoded signal *Meas. Sci. Technol.* **22** 085105.

[2] Chen X, Koenders L, Wolff H and Härtig F **2011** Tuning fork atomic force microscope cantilever encoder and applications for displacement and in-plane rotation angle measurement, ***Procedia Engineering***, vol. 25, pp. 555 – 558.