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#### **Original Citation**

Chen, Xiaomei and Koenders, Ludger (2013) A novel pitch evaluation method based on a cross-correlation filter. In: Nanoscale 2013, 25-26, April 2013, Paris, France. (Unpublished)

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# A novel pitch evaluation method based on a cross-correlation filter

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## Abstract

A cross correlation filter – a half period of sinusoidal waveform sequence ( $p_{\tau}$  period), is applied to filter the topographical signal (p period) of 1D arbitrary-form grating against positions. It, as a template, cross-correlate with the signal and the noise that coexists with signal is eliminated if  $p_{\tau} \approx p$ . After filtering, the distance between any two adjacent waveform peaks along the direction perpendicular to 1D grating lines is one pitch value, the peak-detection (PD) method. In this method, the pitch average and uniformity can be calculated.

### **1. If present pitch evaluation methods meet noise?**



Figure 1. A grating position related topographical signal with noise and irregular waveforms. In such situation, centre-of-gravity (CG) evaluation method is difficult to be applied to calculate the pitches. Fourier-Transform-based (FT) method can be used, but it cannot decide the pitch uniformity.

4. Application of cross-correlation filter to signal of 1D, *p*periodic and arbitrary-structured grating

It can be written as a Fourier sine series:  $f(x) = \sum_{n=1}^{\infty} A \cdot \sin \frac{2k\pi x_n}{n}$   $(0 \le n \le N-1)$ (9)

#### 2.1D sinusoidal grating-probed signal filtering

 $F(x_n)$  $R_{TF}(x_n)$ Figure 2. 1D sinusoidal grating positionrelated signal  $F(x_n)$  probed by a AFM. It consists of sinusoidal  $f((x_n), noise W((x_n)))$ and a tilt drafting  $U((x_n)$  signals: 0  $F(x_n) = f(x_n) + U(x_n) + W(x_n) = A\sin(\frac{2\pi}{n}n) + \sum_{n=1}^{K} H \cdot n^{K} + \frac{1}{M}, \quad 0 \le n < M - 1$ (1) A cross correlation filter:  $T(x_n) = B\sin\frac{2\pi x_n}{p_T}, \quad 0 \le n < N-1$  $R_{TF}(x_m)$ (2) Figure 3. After cross-correlation filtering, the signal:  $R_{TW}(x_m)$ (3)  $R_{TF}(x_m) = R_{Tf}(x_m) + R_{TU}(x_m) + R_{TW}(x_m)$ where  $m = -(N-1), -(N-2), \dots, -1, 0, 1, \dots, M-2, M-1$ .

where 
$$A_k = \frac{4}{p} \int_{-\infty}^{p/2} f(x_n) \cdot \sin \frac{2k\pi x_n}{p} dx_n, (k = 1, 2, 3, \cdots)$$
 (0  $\le n \le n \le n \le 1$ ) (10)

When it cross-correlated with a half sinusoidal waveform template of *p*-period,

$$R_{Tf}(x_m) = C_1 \cdot \sin(\frac{2\pi x_m}{p} + \phi_1) + C_2 \cdot \sin(\frac{2\pi x_m}{p_2} + \phi_2) + \dots + C_k \cdot \sin(\frac{2\pi x_m}{p_k} + \phi_k) + \dots$$
(11)

where  $p_k = p/k, (k = 2, 3, 4 \cdots)$ , and  $C_k$  – proportional to  $A_k$ 

 $C_k$  decreases considerably with k increasing, 1<sup>st</sup> fundamental item in (11) nominates.



Figure 6. Simulations of 1D rectangular and 1D triangular grating position-related signals in legend  $f(x_n)$ and their cross-correlation signals in legend  $R(x_n)$  are shown in (a) and (b) respectively.

## **5. Experiments and results**

#### 5.1 Agreement between evaluated pitch and true pitch value

Table 1 Simulation results of average of pitch deviations and variations(arbitrary units)

	-										
Simulation experiment results [1] 1:	$P_T = P$	$P_{\tau} = P P_{\tau} = 2P P_{\tau} = 3P P_{\tau} = 4P X_{m}$		Noise	sinu	sinusoidal		rectangular		triangular	
<b>4</b> Cross correlation tilt signal $R_{(x)}$ is still		t -	evaluation	level	Deviation	Variation	Deviation	Repeat_	Deviation	Variation	
1. Cross correlation tilt signal $\Lambda_{TU}(x_m)$ is still 2. Cross correlation since side size $R$ (x)		ent;	method		average	(310)	average	ability	average	(310)	
2. Cross correlation sinusoidal signal $R_{Tf}(x_m)$	is sinusoidal	,		0.0	0.0	0.0	0.0	0.0	0.0	0.0	
3. Signals are in same period of $p$ , with phase	se shift <i>Ф.</i>			0.1	0.0	0.11	0.0	0.09	0.01	0.11	
Illustrated by figure 2, simulation experiment results [1] 2.			PD	0.2	-0.01	0.17	0.02	0.18	-0.01	0.20	
illustrated by figure 3, simulation experiment	results [1] 2:			0.3	0.0	0.24	0.03	0.25	0.01	0.30	
$p_{-} = p$	n // n	n > n		0.4	-0.02	0.30	0.05	0.29	0.0	0.37	
	$p_T \sim p$	$P_T \sim P$		0.5	-0.00	0.30		0.39	-0.01		
Cross correlation signal $R_{TF}(x_m)$ has max. amplitude;	Noise can't	$R_{TF}(x_m)$ drops as small as $R_{TW}(x_m)$ so		0.0	0.32	0.0	-0.01	0.0	-0.02	0.0	
Cross correlation noise $R_{TW}(x_m)$ is nearly to zero;	be totally	that SNR is too low to distinguish		0.1	-0.56	0.11	-0.07	0.10	-0.22	0.10	
Signal to noise ratio (SNR) has maximum amplitude.	suppressed	periodical signal from noise.	FT	0.3	-0.74	0.32	-0.49	0.23	-0.28	0.39	
				0.4	-0.13	0.38	-0.11	0.35	-0.60	0.49	
2 Ditch avaluation after croce	corrolatio	n filtoring		0.5	0.09	0.49	-0.18	0.44	0.84	0.63	
signals are eliminated in figure 2.	$P_1 P_2 P_3$	$\frac{1}{P_{i-1}} P_i P_{M-2} P_{M-1} P_M \overline{x_n}$	-0.0 -1.0 (Jun) -0 -1.0- -0.2-							(a)	
$\int P_1 = (p_2 - p_1) \cdot \cos \alpha$			$\dot{}$ $\phantom{$								
$P_{2} = (p_{2} - p_{2}) \cdot \cos \alpha \qquad \text{Pitch Average:} \qquad P = \frac{1}{M-1} \sum P_{i} \qquad (5)$											
$\begin{cases} 1_2 & (P_3 - P_2) & cos c \\ (4) \end{cases}$		$IVI - 1_{i=1}$	60- <sub>1</sub>					cross-corre	lated signa		
$\begin{bmatrix} \cdots \\ P_{M-1} = (p_M - p_{M-1}) \cdot \cos \alpha & \text{Pitch Un} \end{bmatrix}$	iformity: $\delta =$	$\frac{1}{M-1}\sqrt{\sum_{i=1}^{M-1} (P_i - \bar{P})^2}$ (6)	(junde 100- 20-								
where, $p_1$ , $p_2$ , $p_3$ ,, $p_M$ – detected peaks; $\alpha$ –	angle betwe	en <i>x</i> -axis and direction which is	Hunder - 0-	HANNING PARTICIPACITY	mining musically	Winning menunger	with high the	Vielescentrice Vielescentre	Mathematical Mathematical		
perpendicular to the 1D grating lines.			-20-	7	₩ <b>`</b>	. ,	1	Ţ Ţ	¥ ?	Ф	
			-20-4 C	) 50	00 1000	0 15000	2000	25000 3	0000 35	<u> </u>	
To decide the tilt angle $\alpha$ [2]:	S S A O S	Pe			X-			rement (nm			
			Figure 7. (a	) 51 WI- and	(D) AFM-prob	ed 1D grating	g position-rela	ated signal and	a its cross-co	rrelation signal	
$\int x = (fsx + Ix + fex) \cdot p / \sin \alpha $ (7)	10	(a) (b)				Probe	PD (nm)	FT (nm)	STD in	PD (nm)	
$Y = (fsy + Iy + fey) \cdot p / \cos \alpha$			Table 2 A	verage pi	tch and		, , , , , , , , , , , , , , , , , , ,		0	<u> </u>	
			uniformit	v (STD) by	v PD metho	d SIM	0.3392	0.339	0	0.0142	



Figure 5. A tilt angle  $\alpha$  between perpendicular line (1) and displacement direct (2) is shown in (a) and the diagram on evaluation of tilt angle  $\alpha$  is shown in (b).

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where, *Ix*, *fsx* and *fex* are integer periods, fractional parts at beginning and end in *x*-axis; Iy, fsy and fey are integer periods, fractional parts at beginning and end in y-axis.

(8)

#### References

[1] Chen X, Koenders L, Haertig F 2011 Real time cross correlation process of one dimensional grating position encoded signal *Meas. Sci. Technol.* 22 085105. [2] Chen X, Koenders L, Wolff H and Härtig F 2011 Tuning fork atomic force microscope cantilever encoder and applications for displacement and in-plane rotation angle measurement, *Procedia Engineering*, vol. 25, pp. 555 – 558.

AFM 3004.11 3003.34 19.45

#### 5.3 In-plane tilt angles measurement



Figure 9. (a), (b) and (c) are the images of 1D sinusoidal grating with in-plane tilt angle I, II and III respectively. Angles are measured and calculated as 21.57°, 35.12° and 12.03° while measurement repeatability (STD) is 0.02°, 0.04° and 0.05° respectively.

# Acknowledgements

Many thanks for the financial support s of University of Huddersfield and PTB.