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Stress in V-section band clamps

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Abstract: This paper presents an analysis of the stresses in V-section band clamps by examining the correlation between experimental work and theoretical models. Theoretical models incorporating traditional beam-bending theories and allowing for friction were developed to calculate the stress distribution and displacements within the clamps. The theoretical models demonstrated that the normal manufacturing tolerances associated with this type of component, combined with the uncontrolled operating parameters, will produce a wide variation in working stresses.

These theoretical models were validated using strain and displacement measurements from a test with a V-section band clamp positioned around rigid flanges. The experimental results all fell within the range of stresses predicted by the theoretical models. The paper provides a knowledge base for the rational design of V-section band clamps.

Keywords: V-clamp, Marman clamp, clamp band, friction, T-bolt

NOTATION

A	area of the V-section (m)
d_p	pitch diameter (m)
d_1	inside diameter of the nut bearing surface (m)
d_2	outside diameter of the nut bearing surface (m)
E	elastic modulus (GPa)
f	flange edge thickness (m)
F_a	axial clamping load (N)
F_α	circumferential force at angle α (N)
F_β	force applied by the T-bolt (N)
h	clearance between the clamp and the flange (m)
I	second moment of the area through a section of the band (m^4)
l	arc length (m)
L	length of the moment arm (m)
m	bending moment per unit length (N)
M	bending moment (N m)
p	load per unit length in the radial direction (N/m)
q	load per unit length in the normal direction (N/m)
R_m	effective contact radius under the screw head (m)
R_1	initial/open radius of the band (m)
R_2	radius of the flange (m)
s	load per unit length in the axial direction (N/m)
S	tensile load (N)
t	thickness of the band (m)
T_W	wrench torque (N m)
y	distance from the neutral axis (used to determine the initial stage bending stress) (m)
z	distance from the neutral axis (used for determining the bending stress in the axial direction) (m)
α and θ	angular position around the band (rad)
α_h	angle of the thread (rad)
β	subtended angle of half the V-section band (rad)
δ_H	horizontal displacement
ζ	angle for half the open gap in the band (rad)
λ	thread friction angle (rad)
μ	coefficient of friction between the V-section band and rigid flanges
μ_h	coefficient of friction between the nut and the end cap
μ_t	coefficient of friction of thread
σ_b	axial bending stress (MPa)
σ_B	initial stage bending stress in the band (MPa)
σ_L	longitudinal stress (MPa)
σ_v	Von Mises stress (MPa)
σ_x	axial stress (MPa)

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σ_α hoop stress in the band at angle α (MPa)
ϕ angle of the V-section (rad)

1   INTRODUCTION

V-band clamps are widely used for connecting compo-
nents such as tubes, hose, duct, pipes, rigid flanges and
similar equipment. Apart from the aircraft industry,
turbochargers were one of the first applications where
V-section band clamps were used in high-volume
production. The working principle of a V-band clamp
is illustrated in Fig. 1. Tightening of the clamp nut
results in the application of a radial force due to the
increase in tension in the band. The wedging action of
the band on to the flanges then generates an axial load
that can seal the joint and allow it to sustain applied
rotational and bending moments.

Despite their wide use in industry, there is little or no
guidance given regarding the relationship between the
torque applied to the clamp nut and the resulting stress
distribution in the V-band. The majority of manufac-
turers base their designs on empirical equations of the
type presented by Hudson {1} and Mounthford {2}. The

Fig. 1   Working principle of the V-section band clamp

limitation in such equations is associated with obtaining
accurate experimental data over a wide range of design
variables.

V-band clamps are also frequently employed within
the aerospace industry to fasten satellites to the delivery
vehicle during launch and ascent. In this exacting
environment, the importance of the satellite–launch
vehicle interface on the dynamic behaviour of the
satellite has been recognized by Stavrinidis *et al.* {3}.
Lin and Cole {4} also identified the stiffness of the clamp
as a key variable in their dynamic analysis and stated
that stiffness values presented by the manufacturers
were inaccurate. The difficulty of using clamps of this
type has been recognized by NASA {5} who recommend
that when clamps are installed, they are loaded
incrementally and tapped around the periphery to help
produce a uniform internal circumferential force. It is
also recommended that this force be monitored using
a number of strain gauges. In order to avoid this
complicated and time-consuming procedure, Lancho *et
al.* {6} have proposed a clamp system using a much stiffer
band than normal, which is effectively shrunk on to the
interface flanges.

NASA acknowledges the complexity of V-band
behaviour and the poor knowledge about such joints is
the basis of their installation procedure. Within this
field, Takeuchi and Onoda {7} have recently proposed a
theoretical model that recognizes the variation of
circumferential force around such joints. This model is
based on the assumption that the clamp will be formed
from a separate band and relatively short, discrete V-
segments. This model is therefore very similar to the flat
band model recently demonstrated by Shoghi *et al.* {8}.
Within the Takeuchi–Onoda model, the effect of the size
of the V-section angle is absorbed into the coefficient of
friction. This parameter therefore has to be determined
by tests on clamps of the type to be used and is not
transferable to other clamps.

The theoretical equations presented in this paper
provide an understanding of V-band behaviour that
does not rely on experimental data.

2   THEORY

The theory presented here assumes that the operation of
the V-band can be considered in two discrete stages.
Initially there is little or no contact between the band
and the flanges and the main stresses are due to simple
bending. As the band closes, contact is established
around the full circumference and hoop stresses are
generated in the band. In addition to this, there will be
lateral stresses and bending action on the V-section of
the band as it is wedged on to the tapered flanges.

2.1 Initial stress

The stress and deformation analysis of the V-band for the initial stage of operation where the contact force between the band and the rigid component is negligible is similar to the case of flat band clamps as presented by Shoghi *et al.* {8} and will not be repeated here. With reference to Figs 2 and 3, the value of the bending stress, σ_B, can be determined in terms of displacement, δ_H, using Castigliano's theorem (see, for example, Hearn {9}) and is given by

$$\sigma_B = \pm \frac{yE\delta_H(\cos\varsigma + \cos\theta)}{R_1^2[\beta(\frac{1}{2} + \cos 2\beta) - \frac{3}{4}\sin 2\beta]} \quad (1)$$

The bending stress, σ_B, attains a maximum tensile value when $y = y_{max}$. However, because the V-section is not symmetrical about the neutral axis, the magnitude of the compressive stress could be larger than that of the tensile stress.

In order to avoid possible distortion and residual stresses during the installation of V-band clamps over rigid flanges, the relationship between the gap required to allow assembly and the bending stress thus generated should be assessed and this opening must be balanced properly with the aid of equation (1). When using equation (1), it must be assumed that the band material is homogeneous with the elastic modulus, with the same tension and compression, and that the yield stress is not exceeded. Shoghi *et al.* {10} have also demonstrated using finite element analysis where the radius of curvature must
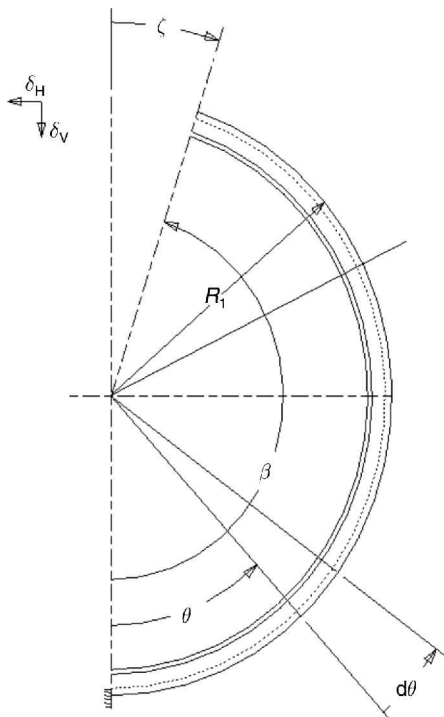

Fig. 3 Loading of the V-section

be at least 15 times the depth of the section if shear deformation of the section is to be ignored.

It is also important to make sure that the section is not disproportionately wide; otherwise the effect of warping has to be considered when using the theory of wide beams. Finally, the band section must be symmetrical to prevent twisting of the section as it is loaded.

2.2 Circumferential (hoop) stress due to friction

The mechanism of V-band clamps is similar to V-belt drives as described by Redford *et al.* {11}. The radial equilibrium of the small segment, $d\alpha$, subjected to the radial force per unit length, p, as shown in Fig. 4, is given by

Fig. 2 Deflection during the initial stage

Fig. 4 Loading with friction

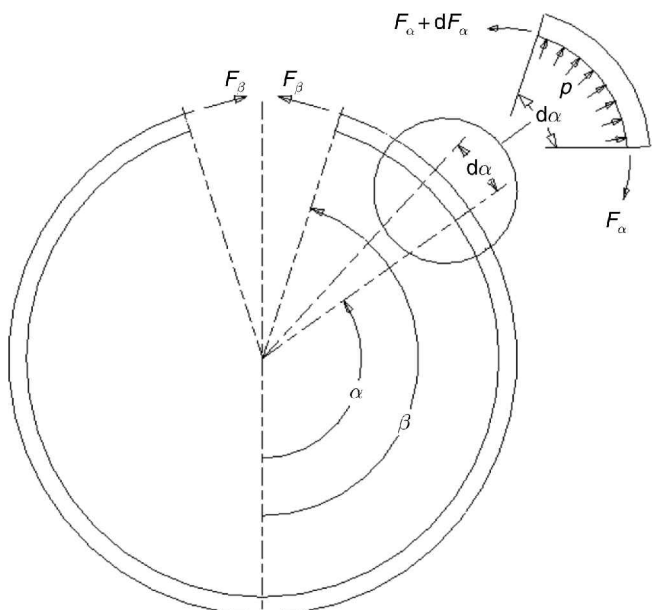$$pR_2 \, \mathrm{d}\alpha - 2F_\alpha \sin\left(\frac{\mathrm{d}\alpha}{2}\right) - \mathrm{d}F_\alpha \sin\left(\frac{\mathrm{d}\alpha}{2}\right) = 0$$

The above equation is based on the assumption that sliding frictional force acts only in the hoop direction. This is justified by the fact that there is negligible radial movement on the clamp in the second stage of loading. For small angles $\sin(\mathrm{d}\alpha/2) = \mathrm{d}\alpha/2$ and the term $(\mathrm{d}\alpha/2)\mathrm{d}F_\alpha$ can be neglected, being second order of small quantity. This gives

$$p = \frac{F_\alpha}{R_2} \tag{2}$$

Hence, with reference to Fig. 3, the contact force per unit length, q, is related to the internal circumferential force, F_α, by

$$q = \frac{p}{2\sin\phi} = \frac{F_\alpha}{2R_2 \sin\phi} \tag{3}$$

Using the relationship between the internal circumferential force, F_α, and the force applied by the T-bolt, F_β, derived for flat band clamps {8} and taking the radial component, p, of the contact force acting between the clamp and the flanges (see Fig. 3), it can therefore be shown that

$$\frac{F_\beta}{F_\alpha} = \exp\left[\frac{-\mu(\beta - \alpha)}{\sin\phi}\right] \tag{4}$$

Assuming that this circumferential force is uniformly distributed over the area, A, of the section, the value of the hoop stress, σ_α, at angle, α, is then given by

$$\sigma_\alpha = \left(\frac{F_\beta}{A}\right) \exp\left[\frac{-\mu(\beta - \alpha)}{\sin\phi}\right] \tag{5}$$

2.3 Axial load

The axial load imposed on the flanges of the component by the V-band clamp provides the clamping force required for the joint. This force is induced in the clamp due to the torque applied to the nut on the T-bolt.

Taking the axial component, s, of the contact load, q, per unit length of the contact line, as shown in Fig. 3, gives

$$s = q\cos\phi \tag{6}$$

The total axial force may then be determined from

$$F_a = 2\int_0^\beta s \, \mathrm{d}l \tag{7}$$

where $\mathrm{d}l$ is a small length of the contact line. This length

is a circular arc of radius R_2 and angle $\mathrm{d}\alpha$. Hence,

$$F_a = 2R_2 \int_0^\beta q\cos\phi \, \mathrm{d}\alpha \tag{8}$$

Combining equations (3), (4) and (8) then gives

$$
\begin{aligned}
F_a &= 2R_2 \int \frac{F_\alpha \cos\phi}{2R_2 \sin\phi} \mathrm{d}\alpha \\
&= \int_0^\beta \frac{F_\beta(\cos\phi)\exp[-\mu(\beta - \alpha)/\sin\phi]}{\sin\phi} \mathrm{d}\alpha \\
&= \frac{F_\beta \cos\phi}{\mu}\left[1 - \exp\left(\frac{-\mu\beta}{\sin\phi}\right)\right]
\end{aligned}
\tag{9}
$$

This force is independent of the radius, R_2, of the flanges on the component. Also, since axial load per unit length, s, is not uniform around the band, the sealing pressure will not be uniform. This is very important in some applications, where the main purpose of the V-section band is to seal the joint against high internal pressures.

2.4 Stress due to the bending of the V-section

The bending moment applied to the section due to the wedging action will subject the top of the band above the contact point to a bending stress (see Fig. 3). The effective section of the band subjected to the bending moment, M, is given by section A–A shown in Fig. 5. With reference to Fig. 6 the bending moment per unit length, m, is given by

$$m = q(h\cos\phi + f\sin\phi) \tag{10}$$

Over a small length, $\mathrm{d}l$, the total bending moment, M, is given by

$$M = q(h\cos\phi + f\sin\phi)R_2 \, \mathrm{d}\alpha \tag{11}$$

Ignoring any ring stiffening effect, the second moment of area, I, of this small length of the section is given by

$$I = t^3 \frac{(h + R_2)\,\mathrm{d}\alpha}{12} \tag{12}$$

and the maximum bending stress occurs when z, the distance from the neutral axis, is half the band thickness, $t/2$. Using an engineering bending theory with equations (8), (6) and (2), the value of the bending stress, σ_b, for the V-section is given by

$$
\begin{aligned}
\sigma_b &= \frac{Mz}{I} \\
&= \frac{3F_\beta(h\cos\phi + f\sin\phi)\exp[-\mu(\beta - \alpha)/\sin\phi]}{t^2 \sin\phi(R_2 + h)}
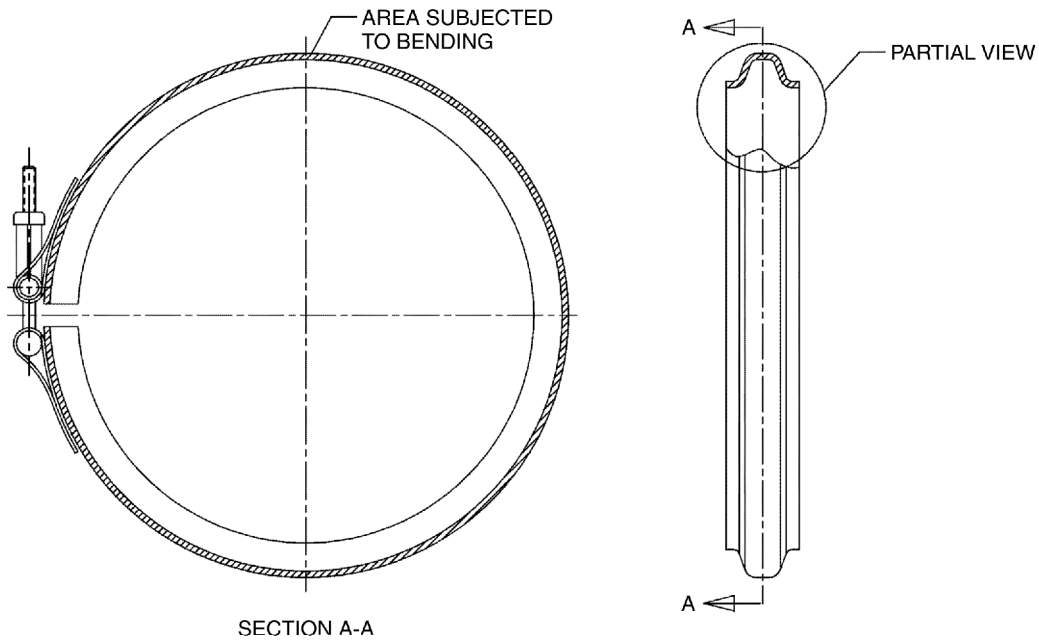\end{aligned}
$$

$$\tag{13}$$

Fig. 5 Section A–A of the band carries the bending moment

2.5 Longitudinal stress in V-section band clamps

The axial force per unit length, s, will also generate a simple tensile stress, σ_L, in the band perpendicular to the circumferential stress. This stress can again be determined by considering a small length of the band:

$$\sigma_L = \frac{s}{t} = \frac{F_\beta \exp[-\mu(\beta - \alpha)/\sin\phi]}{2tR_2 \tan\phi} \qquad (14)$$

Fig. 6 Bending moment generated by the normal force

2.6 Combined effect (V-section band clamps)

As full contact has been established between the V-section band and the rigid flanges of the components, no further bending will occur and the bending stress, σ_B, as given by Castigliano's theorem, will remain constant. Using the principle of superposition, the hoop stress, σ_α, due to friction can be added to the hoop stress due to initial bending, σ_B. As well as the above stresses, the stresses arising from the wedging action of the V-band (longitudinal stress, σ_L, and bending stress, σ_b) have to be considered. These act perpendicularly to the initial bending and hoop stresses and can therefore be taken independently. The bending stress, σ_b, is thus superimposed on the longitudinal stress; on the tensile surface it will be added to the longitudinal stress, σ_L, while on the compressive surface it will effectively reduce the value of σ_L.

For single-section continuous V-clamps all the stresses are taken up by the V-section. The two combined stresses described above are the principal stresses of the plane stress system present on the surface of the V-band. Hence, the combined effect of these stresses may be
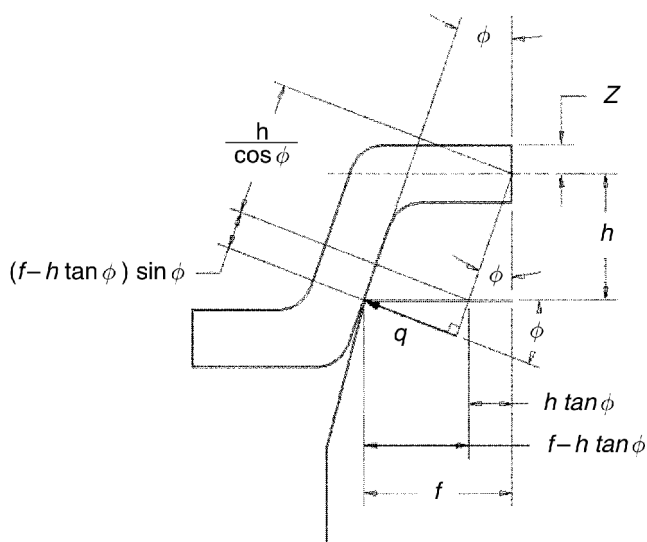
quantified by the von Mises equivalent stress, σ_v:

$$\sigma_v = \sqrt{(\sigma_b + \sigma_L)^2 + (\sigma_\alpha + \sigma_B)^2 - (\sigma_b + \sigma_L)(\sigma_\alpha + \sigma_B)}$$
(15)

3 EXPERIMENTAL VALIDATION

3.1 Strain gauge test arrangement

Strain gauges were installed on the V-band described in the Appendix at the positions shown in Fig. 7. To study the biaxial stress state of stress on the back of the band, a 45° three-element strain gauge rosette was installed at the 0° position with one of the grids aligned in the hoop direction and one linear gauge installed at the 70° position. Principal stresses were obtained from this rosette and linear gauge in the normal way (see, for example, Hearn {12}). The V-band clamp was then assembled on to a pair of solid flange plates of an appropriate size (see Fig. 12 in the Appendix). Using a digital calliper, the initial gap in the V-band was measured and torque was applied to the nut on the T-bolt. For every incremental value of gap the corresponding values of strain in the band were recorded. This was continued until the nut became finger-tight; then a torque wrench was used to apply incremental values of torque until failure occurred in the T-bolt.
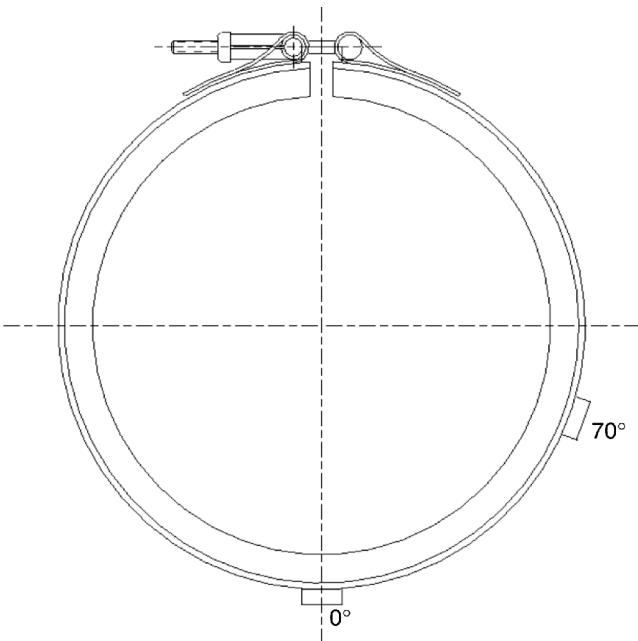
3.2 Relationship between the gap and the hoop stress in V-section bands

Although the theory developed gives the relationship between the gap and the stress for the initial stage, it does not cover this for contact with friction. However, for contact with friction the developed theory gives the relationship between the applied force and stress. The force applied was determined from the measured torque using the relationship given in the Appendix. In order to study the relationship between the gap and the corresponding stress in the V-band, for each value of applied torque the gap was measured. This enabled the calculation of stress in terms of displacement for the second stage.
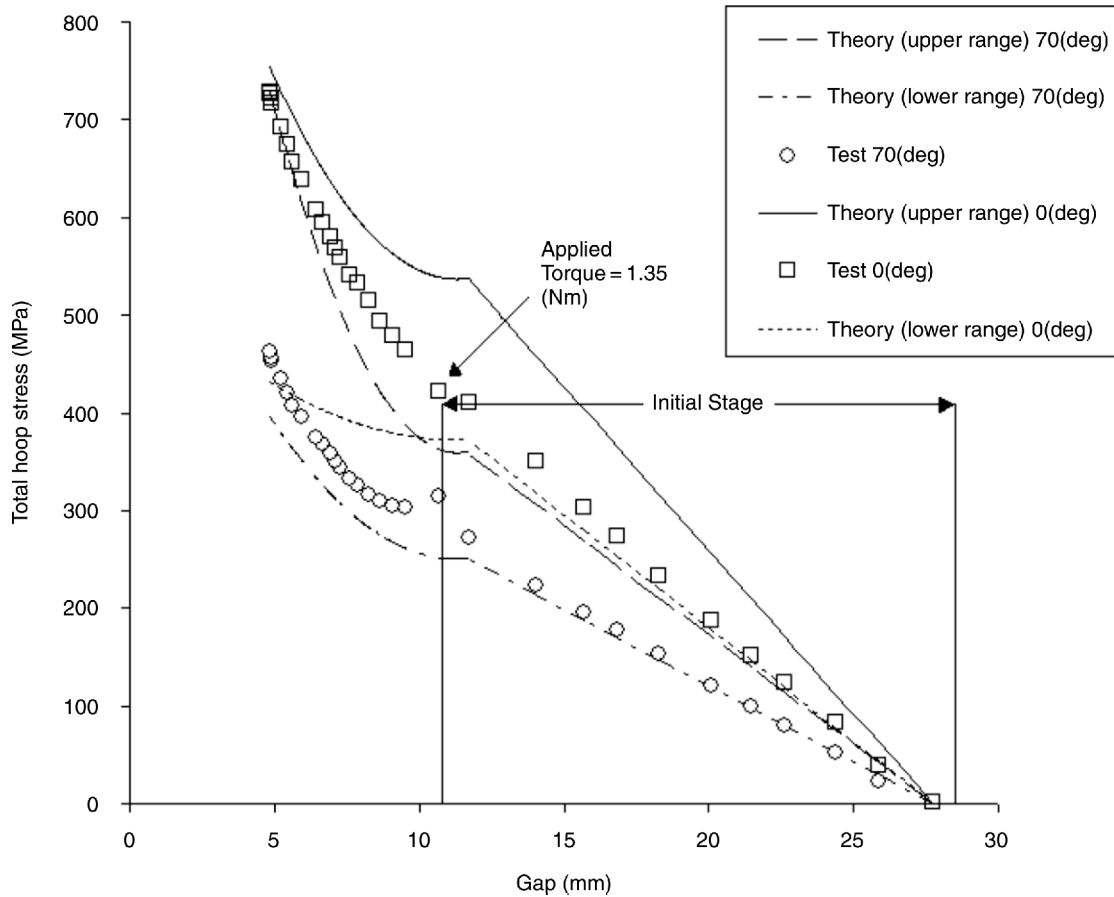
The properties of the V-section band clamp used in the theoretical prediction of the stress are shown in Table 1. All other properties are given in the Appendix.

Initially the stress in the V-band is due to bending. As the gap is reduced, the bending stress increases as shown in Fig. 8. The maximum stress occurs at the 0° gauge position, as predicted by the theory.

As shown in Fig. 8, the correlation between test and theory may deteriorate as the load is increased due to the fact that, unlike flat band clamps where during the initial stage the section remains constant, in V-section bands the section deforms due to the bending stress. This effect will be greatest at the 0° position. As a result of this deformation, the flanges of the V-section move outwards, resulting in a reduction of the distance from the neutral axis. This effect will contribute to the fact that the test results shown in Fig. 8 are towards the lower end of the predicted range. The very wide range of theoretical predictions for stress in the second stage has been produced with relatively small variations in the design parameters, certainly within the tolerance ranges normally applied to this type of product. Operating

Fig. 7 Position of the gauges on the V-band

Table 1 Properties of the V-section band clamp

Property	Lower range	Nominal	Upper range
t (mm)	1.2	1.25	1.3
A (mm^2)	20.6	25	29.4
β (deg)	161	167	174
ϕ (deg)	19	20	21
R_1 (mm)	62	63	64
R_2 (mm)	55.63	55.88	56.13
y (mm)	1.9	2.25	2.6
h (mm)		1.625	
f (mm)	3.68	3.73	3.78
μ	0.1	0.2	0.3
μ_t	0.08	0.18	0.28
μ_h	0.1	0.2	0.3
d_1 (mm)	6.3	6.35	6.4
d_2 (mm)	10.9	11	11.2
d_p (mm)	5.71	5.76	5.81
α_h		2.86	
E (GPa)		227	

Fig. 8   Relationship between the gap and the total hoop stress

parameters such as the coefficient of friction have also been kept within ranges that could be expected in practice. Given the sensitivity of this system to both design and operating parameters, it is encouraging to see such good agreement between the theoretical and experimental results.

An additional factor that has not been taken into account in this figure is the effect of the clamping load opening the angle of the V-section. Referring to equation (5), an increase in the section angle will tend to reduce the variation in stress around the band and hence, for a given T-bolt tension, increase the stress at the 0° position.

3.3   Relationship between the gap and the axial stress σ_x

The axial bending stress and the longitudinal stress cannot be measured individually, but the combined value, i.e. the axial stress σ_x, was measured. As it was not possible to fix a strain gauge on the inside of the V-band, gauges were fixed to the outer part. The theory developed predicts no axial stress during the initial stage of operation before the clamp comes into contact with

the flanges. However, as shown in Fig. 9, the test data for the 0° gauge position show that the axial stress increases linearly as the gap is reduced. This is due to the deformation of the V-section at this location. Since the theory does not allow for this deformation, the graph for the theoretical axial stress lags behind the test data by approximately this amount. If the value of the axial stress for the test data during the initial stage is added on to the theoretical values, the correlation between test and theory improves significantly. This further validates equations (10) and (11). The properties used in the prediction of the theoretical results shown in Fig. 9 were obtained from the nominal values presented in Table 1.

3.4   Relationship between the torque and the von Mises stress in V-section band clamps

Figure 10 shows the von Mises stress taken from the test data at the 0° gauge position along with results predicted using the range of design and operating parameters shown in Table 1. Initially the values of von Mises stress obtained from the test appear to be higher than the theoretical values. This is due to the fact that the theory
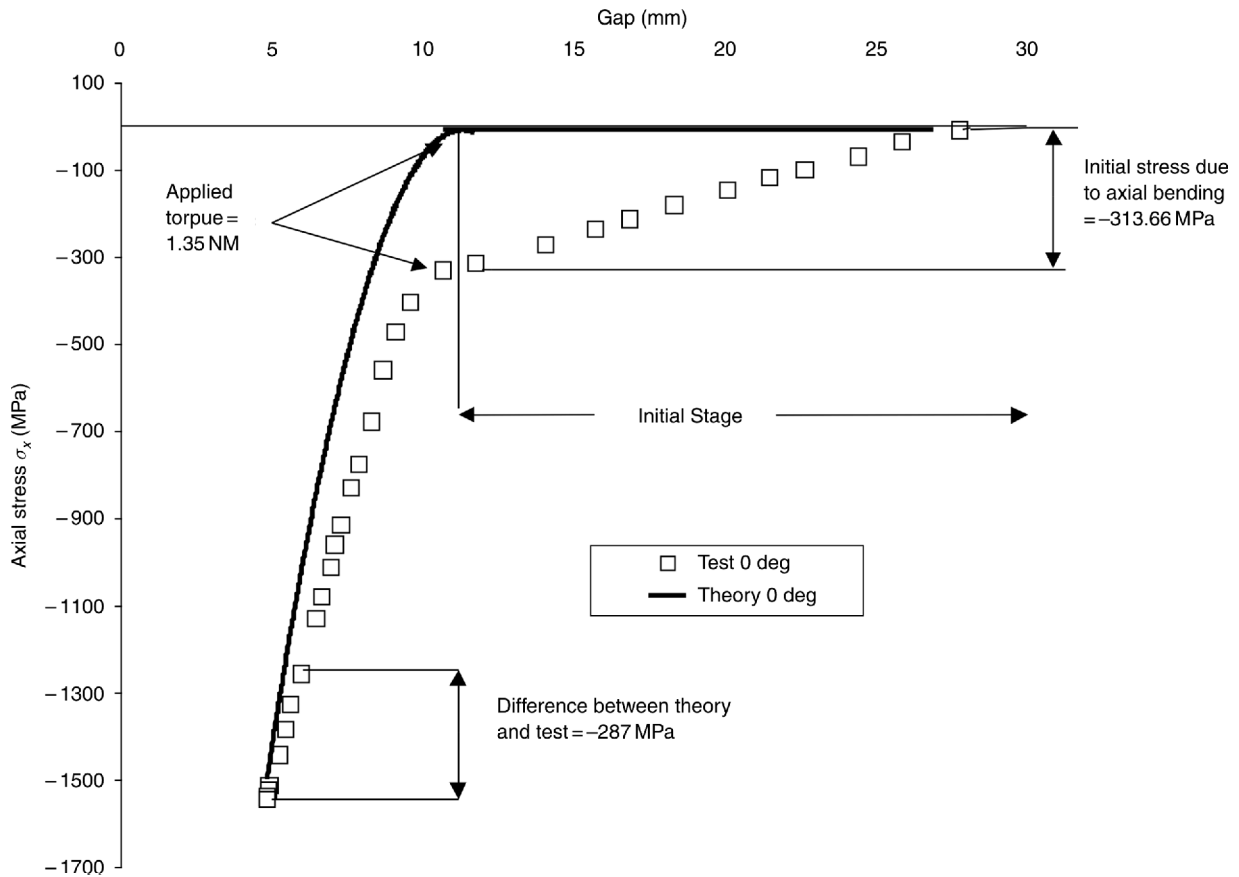
Fig. 9   Relationship between the axial stress and the gap

assumes that the total stress during stage 1 (and hence at the start of stage 2) is due only to bending, as given by equation (1). However, as explained in section 3.3, the theory does not account for the axial bending stress generated during the initial stage, whereas for the test data this value contributes to the von Mises stress at this location. Using the full range of operating and design parameters in the theoretical calculations results in a wide variation of the von Mises stress, as demonstrated in Fig. 10.
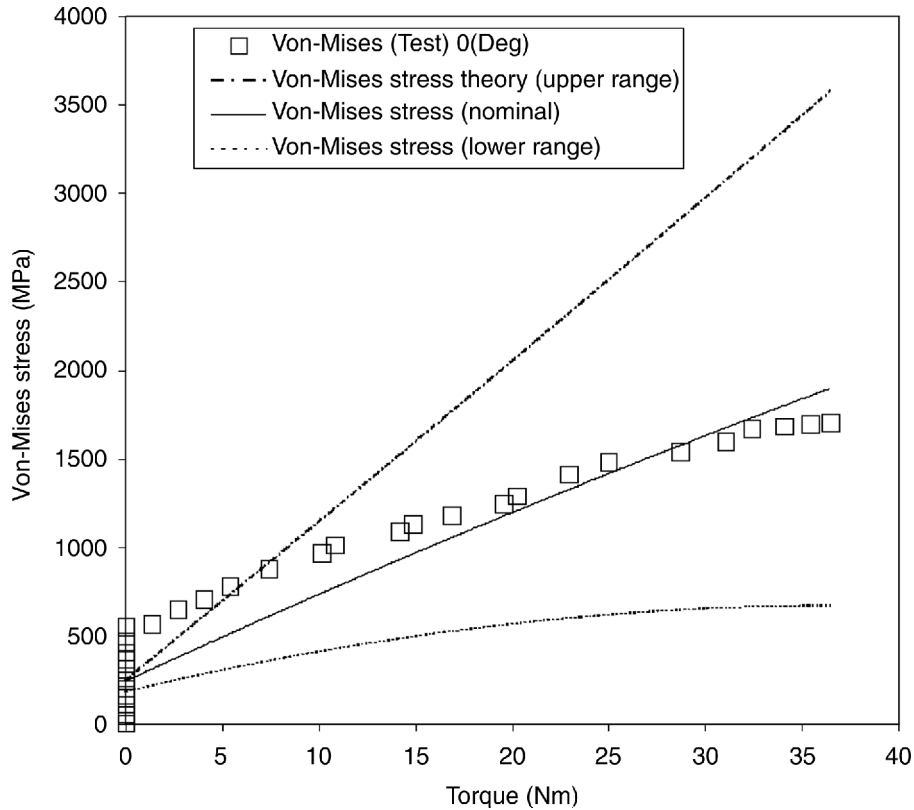
4   CONCLUSIONS

A theory has been developed for V-section band clamps, allowing the stress distribution to be calculated in terms of applied torque and displacement. This theory has been validated with experimental data for both non-contact and contact with friction stages of operation. It has been shown that for a given band section, the value of the axial clamping force is independent of the radius

of the flange plates. This is an important conclusion since V-bands from a given manufacturer tend to have identical sections, irrespective of their size.

In the non-contact condition the maximum stress in the V-band occurs at the furthest point from the open gap. During this stage of operation, transverse strains in the section result in axial stresses. These transverse strains also affect the stress distribution for contact with the friction stage of operation. The axial stress due to this deformation was determined experimentally. Allowing for this additional stress, the experimental results showed good agreement with the predicted axial stress.

The circumferential stresses generated during the contact stage appeared to be far more susceptible to the effects of operating conditions (e.g. the coefficient of friction) and manufacturing tolerances. However, it was demonstrated that the experimental results were well within the range of predicted results.

The theory developed in this paper will allow a more rational method of design and analysis of V-band clamps than has previously been possible.

Fig. 10 Relationship between the torque and the von Mises stress for the second stage at the $0°$ gauge position

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APPENDIX

(a) Torque conversion

The relationship between torque, T_W, applied to the bolt and the clamping load, F_β, is given by Monaghan {13} as

$$T_W = F_\beta \left[\frac{d_p}{2} \tan(\alpha_h + \lambda) + \mu_h R_m \right]$$

where

$$R_m = \left(\frac{d_1 + d_2}{4} \right) \quad \text{and} \quad \lambda = \tan^{-1} \mu_t$$

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Fig. 11 V-section band clamp used in the experimental work

In the equation relating the torque to the clamping load, variation in frictional values can have a considerable influence on results obtained for the clamping load. Hence it is recommended that these values are determined experimentally {13}.

For the T-bolt used in these tests the parameters used in the above equations were:

Size $= \frac{1}{4}''$ UNF, pitch diameter $d_p = 5.76$ mm, angle of the thread $\alpha_h = 2.86°$, effective contact radius under

Fig. 12 V-type flange plates used in the experimental work

the screw head $R_m = 4.4$ mm ($d_1 = 6.35$ mm, $d_2 = 11.25$ mm), coefficient of friction between the screw head and the end cap on the V-band clamp $\mu_h = 0.2$, coefficient of friction between the thread $\mu_t = 0.18$ and hence $\lambda = 10.20°$

Material = stainless steel—AISI 431, yield strength (0.2% offset) = 725 MPa, ultimate tensile strength = 860–1000 MPa (1029 MPa test)

(b) Properties of the V-band clamp

Material = stainless steel—AISI 304 quarter hard, yield strength (0.2% offset) = 515 MPa (648 MPa test), ultimate tensile strength = 860–1000 MPa (857 MPa test), $E = 190$ GPa (227 GPa stress–strain test)

The V-band clamp used in the experimental work had the cross-sectional area shown in Fig. 11, which was assembled on to a pair of solid flange plates as shown in Fig. 12.