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Optimization rules for mill cutter and cutting parameters selection incorporating a control algorithm

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Abstract—In this paper an optimization procedure has been developed in order to select mill cutter and cutting parameters. For the purpose a cost function, which compares system performance, is contrived. An example is presented to illustrate the method.

I. INTRODUCTION

The selection of adequate tools and cutting parameters in machine tool environments has increased attention during the last years. The increased competence makes to plan thoroughly machining conditions in order to minimize costs and improve the final product [1].

This paper brings forward the concept of selecting an appropriate mill cutter, among a known set of candidate cutters, while obtaining the adequate cutting parameters for milling operations through an optimization algorithm.

The knowledge-base defines the allowable cutting parameters, given by the pairs spindle speed and axial depth of cut under the border line in the stability charts, and the feed velocity rate, which is used as control signal, is restricted by the control law. The triple axial depth of cut, spindle speed and feed velocity rate which fulfill control and stability restrictions conform the cutting space parameter, by taking into account the spindle power availability.

On the other hand, a novel cost function is designed, it hangs on spindle power consumption, material removing rate, a stability margin against possible perturbations in the spindle speed and the tracking error signal.

The Material Remove Rate (MRR) represents the amount of material to be removed from the work-piece, this parameter measures the efficiency of the removal process. It is required to be as large as possible, but the occurrence of machine tool vibrations limits this parameter. Another variable which limits the process effectiveness is the power available in the spindle motor. The third parameter taking part into the cost function is considered to ensure a well-posed behavior of the system if a perturbation in the spindle speed happened. Finally, the last parameter is an indicator of how well the milling forces track the reference one in order to avoid tool breakage and to achieve certain degree of quality in the finishing of the working-piece.

Note that, the proposed cost function is a measure of how the milling process is being carried out at certain operation conditions. The larger the cost function is, the worst operation conditions are. Thus, the cutter and cutting conditions which minimize the designed cost function are selected.

II. SYSTEM DESCRIPTION

The standard milling system is described as a second order differential equation such as typical mechanical systems. The system is excited by the cutting force \( F(t) \). Then, the equation of motion has the form [2,3]

\[
M \cdot r(t) + B \cdot r(t) + C \cdot r(t) = F(t)
\]

(1)

where \( r(t) = \begin{bmatrix} x(t) & y(t) \end{bmatrix}^T \), \( F(t) = [F_x(t) \ F_y(t)]^T \), \( M \), \( B \) and \( C \) are the modal mass, damping and stiffness matrices.

The milling cutting force is denoted by a tangential force proportional with the instantaneous chip thickness, and a radial force which is expressed in terms of the tangential force [2,3],

\[
F_x(t) = a_v \cdot K_t \cdot t_x(t); F_y(t) = K_t \cdot r \cdot F_x(t)
\]

(2)

\( K_t, K_r \) are specific cutting constants of the material, \( t_x(t) \) is the chip thickness and \( a_v \) is the axial depth of cut.

The chip thickness consists of a static part and a dynamic one. The static part is proportional to the feed rate, and it is attributed to the rigid body motion of the cutter. The dynamic part models two subsequent passes of the tool through the same part of the work-piece [2,3],

\[
t_x(t) = f_c \cdot \sin \phi + \left[x(t) - x(t - \tau)\right] \cdot \sin \phi + \left[y(t) - y(t - \tau)\right] \cdot \cos \phi
\]

(3)

where \( f_c \) is the feed rate and \( \tau \) is the delayed term defined as \( \tau = 60/N_t \), \( S_t \), \( N_t \) is the number of teeth and \( S_t \) the spindle speed. The resulted equation corresponds with a second order differential equation with time varying parameters. It can be solved numerically [2] or analytically, using some approximations [3]. The solution of the equation in the parameters space yields to the well-known stability lobes [2,3]. Uncertainties in the cutting process, the force-feed nonlinearity inherent in the metal cutting process, and fulfill machine requirements, such as avoid tool wear and tool breakage, achieve certain degree of quality in surface finish or save machining time and cost can be accommodated using a control strategy. Then, a continuous transfer function which relate both signals, measured resultant force and the actual feed delivered by the drive motors can be modelled as a first order dynamics [3], in chatter and resonant free zones,

\[
G_1(s) = \frac{F_x(s)}{f_c(s)} = K_p \cdot a_v \cdot r \cdot \left(\phi_\phi - N_t \cdot S_t \right) \cdot \frac{1}{r \cdot s + 1}
\]

(4)

where \( K_p \), \( (N/mm^2) \) is the cutting pressure constant, \( a_v \) (mm) is the axial depth of cut, \( r(\phi_\phi - N_t \cdot S_t) \) is a non-dimensional immersion function \( N_t \) is the number of teeth on the milling cutter and \( S_t \) (rev/s) is the spindle speed. Furthermore, it is desired to take into account the control dynamics in order to have a more accurate system. In this way, the machine tool control and drive system can be approximated as a first order system [3],

\[
G_1(s) = \frac{F_x(s)}{f_c(s)} = \frac{1}{r \cdot s + 1}
\]

(5)
where \( f_p \) and \( f_c \) are the actual and command velocity values of the table in \((mm/s)\) respectively and \( r \) is an average time constant, which depends on the type of the machine tool. In this study, it is assumed to be 0.1 ms.

From (5) and (6), the combined transfer function of the system,

\[
G_c(s) = \frac{F_c(s)}{F_r(s)} = \frac{K_r}{\tau s + 1}
\]

being the process gain \( K_r (N \cdot s/mm) = K_a r x f/N_S \).

### III. RULES

The following algorithmic methodologies are used as **preliminary rules** to have a more robust system.

**Rule 1**: Stability margin setting of the knowledge-base to ensure that the system plays in a stable region, despite the system model uncertainties. It is composed of two parts.

**Rule 1.1**: In order to calculate secure stability lobes \( char \), a small accurate stability margin is prescribed, i.e, it is supposed that the chatter vibrations happen at \( \delta + i \cdot \omega \), instead of at \( i \cdot \omega \). The reason is that the stability border line is calculated from a linear approximation [8] so that the nominal stability frontier and its neighborhood are inaccurate as stable regions of the real nonlinear problem. Then, \( i \cdot \omega \) is replace by \( \delta + i \cdot \omega \), \( \delta > 0 \), when the stability border line is calculated (see [6]).

**Rule 1.2**: For improving the robustness of the system, a margin at the final expression for chatter free axial depth of cut has been taken into account. Then, the limited axial depth of cut is programmed with a confine on this parameter; \( a_{k,lim} = \alpha \cdot a_{k,im} \) \( 0 < \alpha < 1 \). This rule lets a better control capacity in the spindle speed. On the other hand, a better MRR selection is lost.

**Rule 2**: For searching the allowable input parameter space, the set of spindle speed and axial depth of cut, the following rule methodology, consisting of two differentiated parts, is applied.

**Rule 2.1**: Calculate the boundary points, spindle speed and axial depth of cut pairs, which compose the border line between stable and unstable zones, satisfying rule 1. This rule is obtaining by plotting the stability lobes char, which gives the border line between stable and unstable zones.

**Rule 2.2**: Calculate the admissible input space, subjected to control and spindle motor power availability restrictions.

#### 2.2.1: Control restrictions: The main control restriction is to maintain the forces below a prescribed safety upper bound. This fact ensure avoiding or minimizing tool wear and tool breakage, save machining times and achieve certain degree of quality in finishing the surface on the work-piece. In this way, it is required that the force follows to a constant reference signal. Since the milling plant parameters are unknown and/or time varying, adaptive techniques are used in order to control milling forces. In this paper model reference adaptive control is used to retain the force signal at the reference signal. Then, the control scheme of the figure is scheduled in order to achieve force reference requirements. More detailed explanation of the discrete-time control scheme can be found in references [3].

The continuous model reference is chosen to be

\[
G_s(s) = \frac{\omega_t^2}{s^2 + 2 \xi \omega_t s + \omega_t^2}, \quad \xi = 0.75, \quad \omega_t = \frac{2.5}{T_r}, \quad \text{being} \ T_r, \ \text{the rise time, equal to four spindle periods is usually selected for practical applications [3]}
\]

The aim of the model-following control strategy is to force the closed-loop system to behave as a prescribed reference model. Recursive least-squares estimation algorithm is used in order to identify plant parameters. Perfect matching is achieved through the control signal:

\[
f_{rs} = \frac{T(z)}{R(z,k)} \quad \text{and} \quad H_{rs}(z,k) = \frac{S(z,k)}{R(z,k)}
\]

#### 2.2.2: Spindle Power availability constraints: The power draws from the spindle motor constraints the machining efficiency. It is found from, \( P_s = \pi \cdot D \cdot S \cdot F_p \), where \( D \) is the tool diameter, \( S \) is the spindle speed and \( F_p \) is the tangential force. Since \( F_p \) is the resultant cutting force, \( F_p = \sqrt{F^2_x + F^2_y} \) and \( F_p = K_a \cdot F_x \), and the power draws from spindle motor is calculated.

**A. Tool selection**

In this section, an approach for tool selection is suggested. For this purpose, a tool cost model function is designed. The designed tool cost model is used to select the appropriate tool among the candidates though the optimization rules, explained below. Then, the study requires a given set of candidates milling cutters characterized by the following set of parameters:

\[
R_t = \{a_{k}, \alpha_{r}, \alpha_{k}, \alpha_{e}, \alpha_{k}, k_{a}, k_{s}, N, D, \beta \}
\]

where the pair \((a_{k}, \alpha_{r})\) represents the tool natural frequencies, \((\xi, \phi)\) is the tool damping ratio, \((k_{a}, k_{s})\) is the tool static stiffness, and \( N, D \) and \( \beta \) characterize the number of teeth, the diameter and the tool helix angle, respectively. \( R_t, i = 1, 2, n \), where \( n \) is the number of tools and \( T \) the set of tools available to the designer. \( \alpha_{e} \) and \( \alpha_{k} \) represent the set of tools’ natural frequencies, damping ratio and static stiffness, respectively. Each one is conformed by the pairs \( (\alpha_{k}, \alpha_{r}), (\xi, \phi) \) and \((k_{a}, k_{s})\) for each tool.

1) **Tool cost model definition**

To carry out the selection of a suitable tool, and cutting parameters, a novel cost function has been conceived. The tool cost model for a single milling process can be calculated using the following equation:

\[
C(P_s, MRR, SS, c_i, R_t, c_1, c_2, c_3, c_4) = c_1 \cdot NF; P_s + c_2 \cdot NF + \frac{P_s}{MRR^\gamma}
\]

\[c_i \cdot NF + \frac{P_s}{SS} + c_2 \cdot NF \cdot c_3\]

with \( \sum c_i = 1 \), \( R_t \in T \).

The cost function is composed by the following terms:

- **Material Remove Rate**, \( MRR = a_{k} \cdot r_{c} \cdot f \cdot S \cdot N, \) where \( a_{k} \) is the axial depth of cut, \( r_{c} \) the radial depth of cut, \( f \) is the linear feed rate, \( S \) the spindle speed and \( N \) the number of teeth.
• Cutting Power draw from the spindle speed, \( P_r = V \cdot F_r \), where \( V = \pi \cdot a_d \cdot f_c \), where \( t_c \) is the chip thickness, and the tangential force, \( F_r \), is obtained from the measured cutting force, \( F_r \).

• Spindle Speed security change, \( \Delta S_r \), is an additional term added to the cost function model to be sure that chatter vibrations are avoided. The spindle speed security change measures the nearest spindle speed at which chatter vibrations happen. This fact allows to have an error margin due to possible perturbations in this variable. To calculate analytically \( \Delta S_r \), previous work of the authors is referred [4].

- Tracking error signal, \( e_r \), which gives measurements in the control constraints.

Standardizing factors are added to the cost function in order to equalize the magnitude of the various terms in the cost function. Standardizing factors, \( N_F \), are defined as follow, \( N_F = P_i^{\text{max}} \), where \( P_i \) is the available in the spindle motor, \( N_F = MRR_{\text{max}} \), where \( MRR_{\text{max}} \) is the maximum MRR with the chatter vibration and spindle power restrictions calculated among all the candidate cutters, \( N_F = e_{\text{max}} \), where \( e_{\text{max}} \) is the maximum measured value of this variable among the candidate cutters and \( N_F = e_{\text{max}} \), where \( e_{\text{max}} \) is the maximum error tracking assessed among the candidate cutters.

Finally, the \( e_i, i = 1, ..., 4 \), values are the weights of the cost function terms. They measure the importance of the cost function terms.

2) Optimization rules

The above defined tool cost function is used to select the appropriate tool and cutting parameters by selecting an optimal operation point, the selection of the pair, tool, operation point, is performed in a integrated way through the following optimization rules.

Rule 3: Weight factors selection

The weighting factors \( (c_i) \) are intended to be programmed by the machine operator and have the restriction to sum one. An extended explanation of their meaning and their adequate selection is given in this section. To select suitable values of \( c_i \), \( i = 1, ..., 4 \), their meaning has to be perceived. The \( c_i \) measures the importance of the spindle power consumption. The larger the \( c_i \) is, the more important to the spindle power consumption in the cost model function will be. The \( c_i \) measures the machine productivity, if the \( c_i \) is near to one high productivity is required, and if it is near to zero the productivity has no importance. The same reasoning can be applied to the \( c_i \), which measures the stability against possible perturbations in the spindle speed variable. Finally, the \( c_i \) -constant avoid tool breakage, achieve certain degree of quality on work-piece surface finishing and save machining production times and costs through the measurement of the tracking error signal in the adaptive control scheme of the milling process.

Rule 4: Tool selection criterion

A simple tool selection criterion has been developed. For a given values of \( c_i \), and a given tool characteristics, the cost function value is obtained for all the admissible input cutting parameter space. The minimum value of the cost function is saved. The procedure is repeated for all the available cutters, comparing the minimum value of the cost function for all the available or candidate cutters, the corresponding cutter to the minimum value of the minimum value of the cost function is the selected tool.

The selection criterion is, mathematically, expressed as,

- Compute

\[
C(P_i(q), MRR(q), \Delta S_r(q), e_r(q), R_i, c_i) \quad \text{for each} \quad R_i \in T, \quad k \in N, \quad N \text{is the given set of candidate tools and} \quad \forall q = \{S_i, a_d, f_c\} \in Q \quad \text{where} \quad j \in N = \{1, ..., N_p\}; \quad p \in N \text{(denoting that each machine can be potentially tested for distinct testing operating points) is a discrete subspace of the cutting parameters space where the cost function is calculated.}

- For obtaining the selected tool, \( ST \), compute

\[
ST = \arg \min_{q \in Q} \left[ C(P_i(q), MRR(q), \Delta S_r(q), e_r(q), R_i, c_i) \right] \quad (9)
\]

with \( ST \in T \), obtaining the appropriate tool according to the criterion. Following the rules, the expert system provides an appropriate tool among the candidates.

Rule 5: Cutting parameter selection

- Rule 5.1: Cutting selection

To select the cutting parameters, there are two possibilities. First of all, directly, calculate the cutting parameters, which correspond to the selected tool, which gives the minimum value of the cost function. It can be expressed mathematically as,

Compute the following equation (10),

\[
q^* = \{S_i, a_d, f_c\} = \arg \min_{q \in Q} \left[ C(P_i(q), MRR(q), \Delta S_r(q), e_r(q), ST, c_i) \right]
\]

obtaining an input cutting parameter for the selected tool. The cutting parameter space is obtained by checking all possible values of spindle speed and axial depth of cut which are below the stability line according to Rule 1. These values join to the allowable feed rates, given by spindle power consumption restriction and control requirements conform the cutting space parameter. Then, for the selected tool, the triple of cutting parameters which minimize the cost function, are selected.

- Rule 5.2: Refined cutting selection

In order to have a more accurate possibility, it has been taken into consideration that the cutting parameters can be searched with a more fine integration step around the point where the cost function gives its minimum value. Now, the cutting parameter space is given by a 3-tuple \( Q' = \{S_i^{(k)}, a_d^{(k)}, f_c^{(k)}\} \) around \( q^* \), for \( k = 1, ..., p \), where \( p \) is the number of points to be considered, according to rules 1 and 2. The procedure for obtaining the required cutting parameters is the same as used in Rule 5.1 through equation (10) for the above defined new cutting parameters.

Mathematically expressed, Compute,

\[
q^{**} = \arg \min_{q \in Q'} \left[ C(P_i(q^{**}), MRR(q^{**}), \Delta S_r(q^{**}), e_r(q^{**}), ST, c_i) \right] \quad (11)
\]

In this way, the refined cutting parameters are obtained.

Rule 6: Tuning \( c_i \) values to prevent process malfunctions

Nevertheless, in programming the selected tool and cutting parameters, malfunctions of the process may lead to poor behavior of the process. The most important are tool wear and burr formation. These phenomena, which are common in manufacturing processes, make that the analytical and experimental testes are not always in concordance. If it is
and 0.45, $c_1 = 0.05$ and $c_4 = 0.4$, the cutting parameters corresponding to the minimum of the tool cost function for the selected tool are obtained. In the studied case, these values are $q^* = (1000, 0.1641, 22.843)$. 

Figure 2 shows the situation in the stability lobes of the programmed point $q'^*$, the control signal or feed command and resultant force responses with control constraint.

IV. EXAMPLE

For the validation of this method, the above study has been applied for two practical straight cutters and full immersion up-milling operation. The example considers the tools to have the following characteristics, according with the section III.B notation, $R_1 = (603.666, 3.9, 3.5, 5.59, 5.715, 3.30, 0)$ and $R_2 = (900, 911, 1.39, 0.879, 0.971, 2, 12.7, 0)$.

The natural frequency is in hertz, the tool damping in %, the tool stiffness in KN/mm$^2$ and the diameter of the tool in mm. The work-piece is a rigid aluminum block whose specific cutting energy is chosen to be $K_{s2} = 600$ KN-mm$^2$ and the proportionally factor is taken to be $K_{s1} = 0.3$, for the tool one, and $K_{s2} = 0.07$ for the other one. Other parameters belong to the expert system take the following values, the stability margin factor, $\delta = 0.05$ and the stability margin factor for the axial depth of cut, $\alpha = 0.95$. The analytical tests for mill cutter selection were conducted using spindle speeds with increments of 1000 rpm, axial cutting depths started with its minimum value in the stability border line divided by ten, with increments of the same size. The spindle motor guarantees 2745.3 W of power.

The resulting tool is that leading to the minimum tool cost function value. A general analysis shows in figure 1, in which the minimum value of the tool cost function for all possible combinations of $c_1, c_2, c_3$ and $c_4 = 0.4$ with the restriction $c_1 + c_2 + c_3 + c_4 = 1$ is displayed. The analysis has revealed that the first tool has a better behavior than the second for some combinations of the $c_i$ parameters, and vice versa. For the cutting parameters selection, two steps have been done. First, once the tool has been selected, for example, for values of $c_1 = 0.1$, $c_2 = 0.45$, $c_3 = 0.05$ and $c_4 = 0.4$, the cutting parameters corresponding to the minimum of the tool cost function for the selected tool are obtained. In the studied case, these values are $q^* = (1000, 0.1641, 22.843)$. 

Figure 2 shows the situation in the stability lobes of the

![Figure 1: Minimum Cost function versus $c_i$. $c_j$ varies and $c_k = 0.4$](image1)

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