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Modelling and simulation of the dynamic behaviour of a ball screw system using transmission line modelling technique

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Abstract

This paper presents a new approach for the modelling of a screw shaft including the axial and torsional dynamics in the same model. The model includes the distributed parameter dynamics of the ball screw system and the effect of mass distribution. This is based on the flexibility of the Transmission Line Matrix Method (TLM) to develop lumped and distributed parameter systems. The procedure for the synchronisation of both axial and torsional effects is presented in detail.

1. Introduction

Various types of models for feed drives (lumped parameter models, modular approach, hybrid models) have been developed by industrial and academic researchers. However the simulated responses did not reflect entirely the overall dynamic behaviour of the machine tools because the stiffness calculations are made considering that the worktable is oscillating around one position.

A novel application of TLM for the modelling of the dynamic behaviour of Computer Numeric Controlled (CNC) machine tool feed drives for various running conditions was previously presented [1]. The considered feed drive was a non-linear hybrid system where the controller commanded the movement of a worktable linked to a motor through a ball-screw. This paper presents the improved TLM model of a ball-screw including the moving nut, the distributed inertia of the screw, the axial and torsional forces applied on the nut during its linear movement and the restraints applied by the bearings.

The application of TLM technique implies the division of the screw shaft into a large number of identical elements. This is necessary in order to achieve the synchronisation of events during simulation and produce acceptable resolution according to the maximum frequency of interest as presented by Beck et al [2]. This normally requires considerable computing effort when small time steps are used in the simulation process. This paper presents a solution to reduce the simulation time and calculation power and generate accurate and reliable results.

The TLM model is implemented in MATLAB and simulated values for different positions of the moving nut compare well with the measured data when same stimuli are applied to the model and the actual feed drive.

2. Transmission line matrix method

TLM is a numeric differential method usually used to solve problems of wave propagation through a medium. The system equations are made equivalent with the equations for voltages and currents for a mesh of transmission lines. TLM technique uses two circuits: The stub and the link.

Christopoulos [3] stated that any electrical circuit could be represented as a network of transmission sections by simply replacing the reactive components with corresponding stubs. Variables such as voltage and current are regarded as discrete pulses bouncing to and from the nodes of these stubs at each time step.

The voltage and current in each component (stub and link) is determined from the incident () and reflected () pulses in a port (Figure 1(a)). The TLM operation begins with the incident pulses representing the initial conditions being injected into the network. Incident pulses take a time () to travel between ports. When incident pulses reach a port (nodes), reflected pulses are generated according to boundary conditions. The reflected pulses thus become the incident pulses in the next time step. On incidence to the node, the pulse will interact with other parts of the circuit.

Figure 1: TLM Units

If $E^i(k)$ is known at time step k, the voltages and current in Figure 1(a) may be calculated. Taking $E^i(k)$ as the discrete stimulus applied to the stub, gives:

(1)

the reflected pulse will be (2)

the reflected pulse becomes the next incident pulse, hence

(3)

Now, with $E^i(k+1)$ obtained from eqn (4), i($k+1$) may be obtained from eqn (1). Then, the process (scattering algorithm) is repeated for as long as desired. The characteristic impedance Z and the reflection factor *(* are chosen accordingly to the nature of the element to be represented.

The application of the scattering algorithm to the TLM link gives:

(5)

(6)

the reflected pulses become the next incident pulses, hence

(7)

Eqns (5) to (7) represent the TLM link algorithm, which is a numerical method for the solution of the wave equation (Sadiku and Agba [4]) with the form:

(8)

Where the velocity of propagation (u) and the impedance (Z) are:

(9)

C and L are the capacitance and inductance per unit length of a transmission line. The function can represent either voltage or current on the transmission line.

Partidge et al [5] used this concept to model a shaft and turntable with linear and non-linear friction. TLM stubs represented lumped elements (turntable inertia), and distributed elements (shaft) were modelled by TLM links. Thus, for the equation for torsional vibration of a shaft

(10)

The function $y(x, t)$ signifies either torque or angle of twist. The velocity of propagation u_t and the impedance Z_t for the equivalent TLM link are:

(11)

(12)

G_m, J_m, $(_m$, E_m and A_m represent respectively: the material rigidity modulus, the polar moment of inertia, the material density, the material Young's modulus, and the cross sectional area of the shaft.

The same method can be applied for the equation of the longitudinal vibration of a bar - eqn (15), where the parameters of the equivalent TLM link are:

(13)

(14)

(15)

The function $y(x, t)$ represents either axial force or longitudinal displacement.

Figure 2: Ball screw arrangement

Figure 3: TLM model of a shaft divided into eight sections including supporting bearings friction and moving nut

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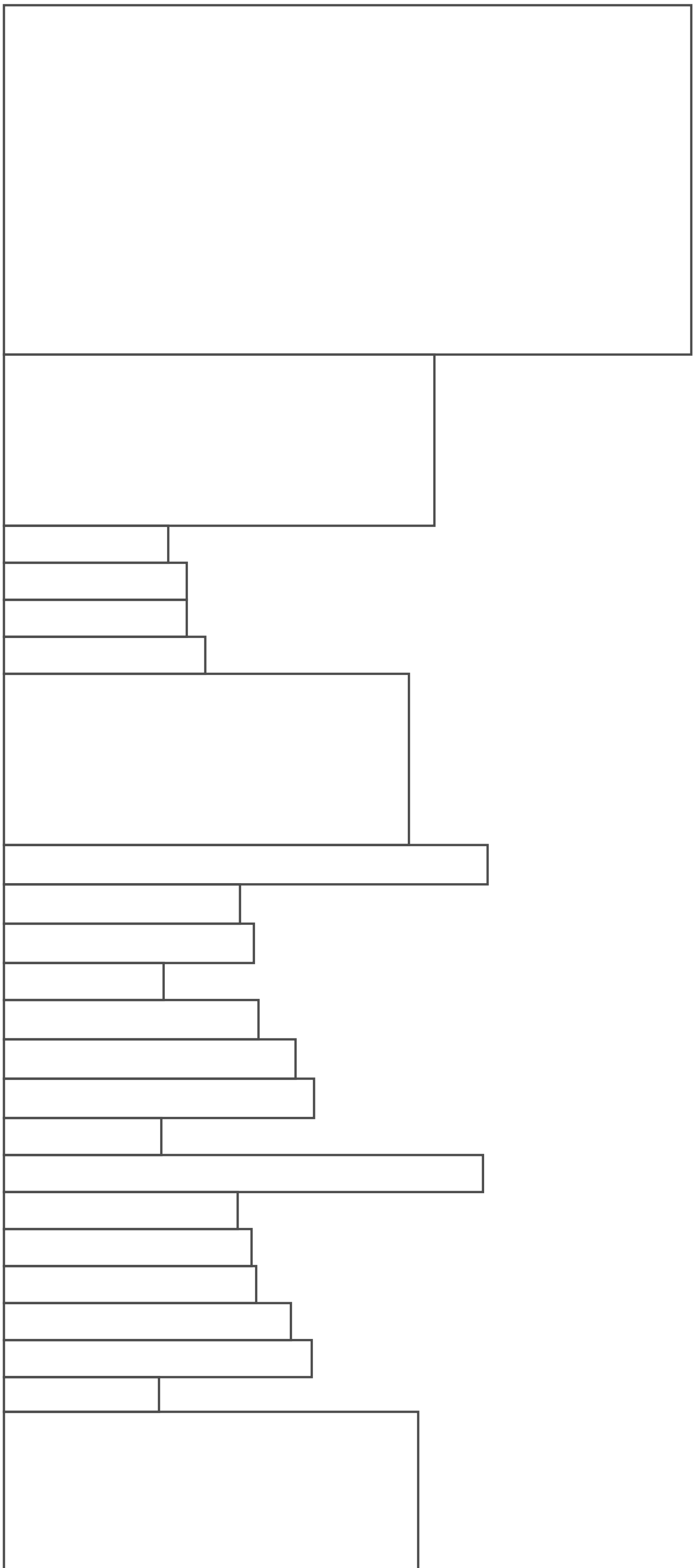

	$1.01*E_m$	$0.99*E_m$
$1.01*G_m$	1.6087	1.5927
$0.99*G_m$	1.6249	1.6087

Table 2: n_a/n_t ratio for variations of 1% in the values of G_m and E_m

It can be assumed from Table 2 that a ratio between 1.5929 and 1.6246 is valid taking into account the variations the screw shaft material may have due to the fabrication process. Therefore, the minimum rational number found into this interval (8/5) is selected for the modelling process (n_t = 5). In these conditions, E_m is approximated to 204.8x10^9 N/m^2 for a given value of G_m = 80x10^9 N/m^3.

The length of each section in the torsional model (l_{tor}) and m will be:

(44)

(45)

If m is not an integer number, it is rounded to the nearest integer. This implies to change the length of the screw shaft by certain quantity. Applying this procedure for l_{ss} = 1.346 m and t_t = 1 (s, gives:

[m/s] (46)

[m] (47)

sections (48)

This means, an increase in the length of the screw shaft (l_{ss}) of:

(m (49)

This error model could be present in the real system due to the tolerances in the machining process of the shaft and changes in the values of the physical properties of the material. For example, if the density value is changed by 0.63% to 7800, the number of sections will be 420 and the length of the screw shaft will be reduced by 92.35 *(m*.

An approach to cope with this limitation of the modelling technique is to assume that the density of the material could vary 1% its nominal value. In consequence, a valid number of sections can be defined (without altering the length of the shaft) by rounding the value of *m* towards minus infinity. Then:

sections (50)

The number of sections of the axial model will be

sections (51)

Rearranging eqn (44) gives:

[mm] (52)

[m/s] (53)

[kg/m³] (54)

The torsional impedance is calculated using eqn (12):

(55)

The propagation time for the axial model is

[s] (56)

u_a can be calculated from eqn (40) as:

[m/s] (57)

The axial impedance is calculated using eqn (14):

(58)

The length of each axial section is given by

(59)

5. Comparison between simulated and measured results

Figure 9 shows the measured and simulated frequency spectrum of the velocity control loop (impulse response). Measured results were obtained from the controller's oscilloscope through software provided by the manufacturer for the diagnosing of digital control loops. Simulations were conducted using specially written MATLAB code.

Analysing the measured and simulate results reveals that the TLM model is predicting the system's main oscillating frequency (at around 338 Hz) however the peaks in the simulated diagram are slightly displaced and are more difficult to distinguish.

Although the results demonstrate the accuracy of the TLM model in resembling the dynamic behaviour of the ball screw system, more research needs to be done in order to improve the model.

a) Measured b) Simulated

Figure 9: Frequency spectrum of the ball screw velocity control loop

6. Conclusions

The paper presents an approach for the TLM modelling of a screw shaft including the axial and torsional dynamics in the same model. This was accomplished by deriving a procedure for the synchronisation of both axial and torsional effects. In this regard, it was assumed that the physical properties of the screw shaft material (density) could vary with 1% its nominal value.

The simulated results compare well with the measured data when same stimuli are applied to the model and the actual feed drive.

The TLM method is faster and mathematically attractive avoiding becoming too involved in the boundary value problems associated with partial differential equations.

Future research will concentrate in using TLM for developing accurate feed drives models, which could become part of automatic tuning and condition-monitoring methods.

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