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Original Citation

Zeng, Wenhan, Jiang, Xiang, Scott, Paul J., Xiao, Shaojun and Blunt, Liam (2009) A Fast Algorithm for the High Order Linear and Nonlinear Gaussian Regression filter. In: Proceedings of the 9 th international conference of the european society for precision engineering and nanotechnology. euspen, San Sebastian, Spain, pp. 356-359. ISBN 978-0-9553082-6-0

A Fast Algorithm for the High Order Linear and Nonlinear Gaussian Regression filter

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Abstract

In this paper, the general model of the Gaussian regression filter, including both the linear and nonlinear filter of zeroth, second order, has been reviewed. A fast algorithm based on the FFT algorithm has been proposed and tested for its speed and accuracy. Both simulated and practical engineering data have been used in the testing of the proposed algorithm. Results show that with the same accuracy, the processing times of the second order linear and nonlinear regression filters for a typical 40,000 points dataset have been reduced to under 0.5second from the several hours of the traditional convolution algorithm.

1 Introduction

The Gaussian filter has been defined as the standard filtering technique for surface roughness extraction [1]. However, a number of shortcomings have hampered its practical application in industry including: (1) the measured profile is truncated due to the boundary effect, especially when the measured data is not much long than the cutoff wavelength; (2) it is unsuitable for surface with relatively large form components; and (3) it is unable to handle measurement outliers or residual profiles with non-Gaussian distributions. To address the boundary effect and form removal issues with the traditional Gaussian filtering, Brinkman and Bodschwinna [2] have proposed a Gaussian regression filtering technique with the extension of up to second order polynomial form. Furthermore, by introduce a weighted iteration procedure, the Gaussian regression filter can obtain robust results against outliers and non-Gaussian distributions. Seewig [3] has given the discrete expression of the linear and nonlinear Gaussian regression filter.

However, the algorithm for the Gaussian regression filter is very slow if the convolution method is used directly, especially in the cases of the first or second order form removal, making the algorithm impractical for industrial application. In this paper, the general model of the Gaussian regression filter has been reviewed and a fast algorithm based on the FFT method has been proposed and tested.

2 General Gaussian Regression filter for profile analysis

Without considering the form removal, the general model of the zeroth order Gaussian regression filter can be described by the solution of the following minimisation problem:

$$\int_0^l \rho\left(z(\xi)-w(x)\right)s(\xi-x)d\xi \rightarrow \underset{w(x)}{Min} \qquad (1)$$

The resulted mean line $w(x)$ of the filtered profile is the deviations to the measured profile $z(\xi)$ weighted by the Gaussian function $s(\xi-x)$ over the whole measured length l. The function $\rho(\cdot)$ is the error metric function of the estimated residual, in the case of $\rho(x):=x^2$ is the least squares function and integration limits are expanded up to $-\infty<\xi<\infty$, the regression filter is then equal to the phase correct filtering according to ISO11562. In order to analysis profiles with significant form components, it is assumed that the form component $f(\xi)$ in the neighbourhood of a random point x can be adequately approximated by a second order polynomial curve within the measured length. Thus the equation (1) can be extended to include form removal as:

$$\int_0^l \rho\left(z(\xi)-w(x)-\beta_1(x)\left(\xi-x\right)-\beta_2(x)(\xi-x)^2\right)s(\xi-x)d\xi \rightarrow \underset{w(x),\beta_1(x),\beta_2(x)}{Min} \qquad (2)$$

By zeroing the partial derivations in directions of w, β_1 and β_2, the resulting mean profile with the form component can be obtained as:

$$\begin{bmatrix} A & B & C \\ B & C & D \\ C & D & E \end{bmatrix}\begin{bmatrix} w(x) \\ \beta_1(x) \\ \beta_2(x) \end{bmatrix}=\begin{bmatrix} F0 \\ F1 \\ F2 \end{bmatrix} \qquad (3)$$

Where , $A(x)=\int_0^l \delta(\xi)s(\xi-x)d\xi$, $B(x)=\int_0^l \delta(\xi)(\xi-x)s(\xi-x)d\xi$,

$C(x)=\int_0^l \delta(\xi)(\xi-x)^2 s(\xi-x)d\xi$, $D(x)=\int_0^l \delta(\xi)(\xi-x)^3 s(\xi-x)d\xi$,

$E(x)=\int_0^l \delta(\xi)(\xi-x)^4 s(\xi-x)d\xi$, $F0(x)=\int_0^l \delta(\xi)z(\xi)s(\xi-x)d\xi$,

$F1(x)=\int_0^l \delta(\xi)z(\xi)(\xi-x)s(\xi-x)d\xi$, $F2(x)=\int_0^l \delta(\xi)z(\xi)(\xi-x)^2 s(\xi-x)d\xi$.

with $\delta(x)=\rho'(x)/x$. When $\rho(x):=x^2$, $\delta(x)=2$ is a constant at all points, then the filter is the linear regression filter. When $\rho(x)$ is selected as other functions, such as the L1 norm, Huber, Cauchy, and Tukey functions, it's a nonlinear regression filter, where $\delta(\xi)$ has different values at different points.

3 Fast algorithm

Solving equation (3) directly for each individual point is extremely time-consuming and inefficient. A reasonable way is to pre-calculate all the A, B, C, D, E, F0, F1, F2 respectively from the whole length of profile, and then solve the linear equations point by point to get the mean profile. However, if convolution in the time domain is used directly to calculate the A, B, C, D, E, F0, F1, F2, the algorithm is still very slow. Thus, a method based on the FFT algorithm in the frequency domain is proposed by the authors to speed up the calculation of the above intermediate results as following:

$A(x)=IFT(FT(\delta(x))FT(s(x)))$, $B(x)=IFT(FT(\delta(x))FT(xs(x)))$

$C(x)=IFT(FT(\delta(x))FT(x^2 s(x)))$, $D(x)=IFT(FT(\delta(x))FT(x^3 s(x)))$,

$E(x)=IFT(FT(\delta(x))FT(x^4 s(x)))$, $F0(x)=IFT(FT(\delta(x)z(x))FT(s(x)))$,

$F1(x)=IFT(FT(\delta(x)z(x))FT(xs(x)))$, $F2(x)=IFT(FT(\delta(x)z(x))FT(x^2 s(x)))$,

Where, $\delta(x)=\rho'(x)/x, 0<x<l$ and $FT(.)\,\&\,IFT(.)$ are the forward and inverse Fourier transforms respectively.

4 Experiments

To evaluate the algorithm, simulated and measured profiles have been used to test the second order linear and nonlinear Gaussian regression filtering respectively. Both convolution and FFT algorithms have been conducted on those profiles with the aim of comparing speed and accuracy. Figure 1 is a simulated profile (43000 pts) with significant form component and a noticeable pip in the middle of the profile. Figure 2 is a measured honing profile (18,000 pts) with minor form components and many outlier spikes. The convolution algorithms need several hours to get the results, while the fast algorithms proposed only need several hundred milliseconds in all cases.

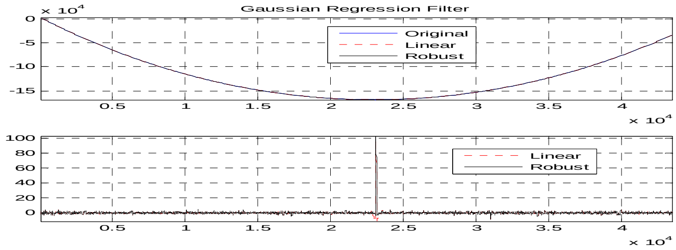

Figure 1: simulated data (up: original and mean profile; down: residual profile)

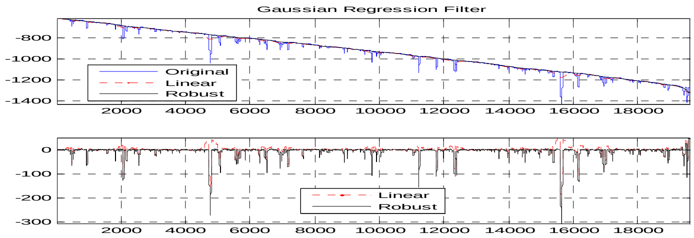

Figure 2: measured profile (up: original and mean profile; down: residual profile)

5 Conclusions

In this paper, the authors have reviewed the linear and nonlinear Gaussian regression filters of zeroth, second order. Fast algorithms based on the FFT method have also been proposed. Experiments have shown that the new algorithms have improved the speed significantly and achieved similar compatible accuracy.

6 Acknowledgement

The authors would like to thank the EPSRC in the UK for support fund to carry out this research work under its programme EP/F032242/1.

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