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Extraction of Information from Vibration Data using Double Density Discrete Wavelet Analysis for Condition Monitoring

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Abstract

Many condition monitoring (CM) techniques have been investigated for the purpose of early fault detection and diagnosis in order to avoid unexpected machine breakdowns. However, non-stationary and non-linear characteristics of vibration data can make the signal analysis a challenging task. Multiresolution data analysis approaches have received significant attention in recent years and are widely applied to analyse non-stationary and non-linear data. Double-Density Discrete Wavelet Transform (DD-DWT), which was originally developed for image processing, is proposed and investigated in this paper for effectively extracting diagnostic features from the vibration measurements. DD-DWT has the merits of nearly shift-invariant and less frequency aliasing which and allows the effective extraction of non-stationary periodic peaks, compared with the undecimated DWT. Techniques based on thresholding of wavelet coefficients are gaining popularity for denoising data. The implementation of global, level-dependent, and subband-dependent thresholding based methods are investigated and implemented on the selected wavelet coefficients in order to denoise and enhance the periodic and impulsive fault features. The performance of the proposed method has been evaluated against DWT using both simulated data and experimental datasets from defective tapered roller bearings. Results, using the harmonic to signal ratio (HSR) as a measure, have demonstrated that DD-DWT outperforms conventional DWT in feature extraction and noise suppression. As a result, the proposed method is robust and effective in fault detection and diagnosis.

1 Introduction

The non-linear and non-stationary characteristics of vibration data make the extraction of fault features a challenging task in condition monitoring. Features extracted from time or frequency domains cannot include all useful information. Thus, time and frequency domain are combined as Time-Frequency domain methods such as Short-time Fourier Transform (STFT). However, STFT suffers from the limited and constant time-frequency resolution with the fixed window. For more details, the performances of the different time-frequency domain methods are reviewed and compared in[1]. To address this issue, several multiresolution time-frequency analysis methods have been developed, among them, Wavelet Transform methods are popular to process non-stationary data.

1.1 Wavelet Transform

Wavelet Transform (WT) is a multiresolution time-frequency analysis based on the idea of multiresolution analysis. Several WT methods are available such as Continues Wavelet Transform (CWT), Discrete Wavelet Transform (DWT), etc. In condition monitoring, there are different applications of wavelet transform such as the analysis of time-frequency domain, feature extraction, signal enhancement and denoising, signal compression etc. [2]. In terms of vibration signals, wavelet transform gives an excellent representation for nonstationary signals that containing jumps and spikes (singularities). It is able to provide optimal sparse representation for such signals because wavelets oscillate locally and only wavelets overlapping a singularity can have large coefficients [3]. WT has been successfully implemented for feature extraction tasks. The compact support property gives wavelets the feature of energy concentration, which results in yielding many coefficients with small energy. Consequently, small coefficients can be excluded without losing the important and informative components in analysed signals, and few coefficients can be used to represent the diagnostic features. Thresholding has been known as a promising solution and widely accepted to shrink the uninformative components from the analysed signal [1]. The key issue is to determine the best coefficients that can represent the diagnostic features.

1.2 Discrete Wavelet Transform (DWT)

In order to overcome the redundant transform produced form the Continues Wavelet Transform (CWT), a discretization method, DWT, is applied to dilate and translate parameters[4]. This can be done by changing the dilation parameter to a power formant[5]. The DWT was developed by Mallat [6]and it can be expressed as (1).

$$dwt(a,b) = \frac{1}{\sqrt{2^a}} \int_{-\infty}^{+\infty} x(t)\,\psi^*\left(\frac{t - b2^a\tau}{2^a}\right)dt \tag{1}$$

When the choice of scales and translations based on powers of two, the DWT analysis cab be more efficient and accurate. This transform can decompose the signals into orthogonal and non-redundant sets of wavelets[7]. DWT is critically sampled wavelet using perfect reconstruction FIR filter banks [8]. Wavelet and dilation functions at multi-scales are shown in equation (2) and (3).

$$\phi(t) = \sqrt{2}\sum_n h(n)\phi(2t-n) \tag{2}$$

$$\psi(t) = \sqrt{2}\sum_n g(n)\phi(2t-n) \tag{3}$$

where $h(n)$ represents the low pass filter; $g(n)$ is the high pass filter; $\phi(t)$ is the scaling function; and $\psi(t)$ is the mother wavelet function [9].

The interest in using DWT methods comes from the fact that signal impulses can be identified from the high frequencies of the wavelet with a good resolution [10]. Low time resolution and high-frequency resolution can be obtained at low frequencies, whereas, a high time resolution but a low-frequency resolution can be obtained at high frequencies [1]. DWT has been extensively used for rolling element bearings (REB) fault diagnosis in the past two decades[11]. For instance, Rubini et al.[12] used DWT to diagnose bearings with an incipient surface fault. Yan et al. in [13] presented a comprehensive review of the application of DWT for fault diagnosis of rotary machines. Moreover, Peng and Chu in [1] reviewed the application of DWT in machine condition monitoring and fault diagnosis.

2 Overview of Double Density DWT (DD-DWT)

Double Density DWT was introduced by [14]. It is grounded on over-sampled filter banks to reduce the shortage of translation sensitivity in the critically sampled DWT. The DD-DWT uses scaling function $\phi(t)$ and two distinct wavelets ψ_1 and ψ_2, where one wavelet is set to be offset by half from the other wavelet as shown in equation (4) [15].

$$\psi_2(t) \approx \psi_1(t-0.5) \tag{4}$$

The scaling function and two wavelet functions should satisfy equation (5) and (6), respectively[14]:

$$\phi(t) = \sqrt{2}\sum_n h_0(n)\phi(2t-n) \tag{5}$$

$$\psi_i(t) = \sqrt{2}\sum_n h_i(n)\phi(2t-n), \quad i = 1,2 \tag{6}$$

In the equations, $h_0(n)$ represents the low pass filter, while $h_i(n),\ i = 1,2$ are the high pass filters.

For satisfying perfect reconstruction condition, filters should satisfy the conditions expressed in equation (7) and (8) [14]:

$$H_0(z)H(1/z)+H_1(z)H_1(1/z)+H_2(z)H_2(1/z)=2 \tag{7}$$

$$H_0(z)H_o(-1/z)+H_1(z)H_1(-1/z)+H_2(z)H_2(-1/z)=0 \tag{8}$$

where, $H_i(z)$ is the Z transform of $h_i(n)$.

With the design of having more wavelets, a narrower spacing between adjacent wavelets within each scale will be obtained [16]. DD-DWT is constructed with decomposing and reconstructing three filter banks oversampled by 3/2. DD-DWT was proposed based on the Motivation of the success of improving shift sensitivity by adopting an overcomplete expansion in dual tree discrete wavelet transform DT-ℂWT[17]. It has several advantages that make it outperforms critically sampled DWT and undecimated DWT. The double density DWT is a less expansive version of the undecimated DWT. Also, DD-DWT has very smooth wavelets and it is nearly shift-invariant. This property is important for extracting periodical peaks. Another property is the reduced frequency aliasing effect which is claimed to be effective for detecting harmonic features. Consequently, the DD-DWT is well suited for analysing non-stationary signals. It has more wavelets than necessary, which gives a narrower spacing between adjacent wavelets within the same scale and is less redundant than undecimated wavelet [15].

DD-DWT has been implemented in image processing and denoising. Sveinsson. et al [18] applied DD-DWT to denoise Synthetic Aperture Radar (SAR) images by reducing the speckle of SAR images and claimed that the method was able to remove the speckles and enhance the performance of detection for SAR based recognition. In [19] and [20], DD-DWT was applied for image denoising in order to derive texture features of the images. The results showed the potential capacity of DD-DWT in performing the task. Comparative studies have been carried out in [21] and [22] between different discrete wavelet methods for image denoising and conclude that the DD-DWT outperforms the DWT with the same level of decomposing. However, to the best of authors' knowledge, DD-DWT has never been explored to the scenarios of detecting and diagnosing faults from machinery components such as bearings which is significantly different from the reported implementation cases of DD-DWT in the literature.

3 Data Denoising by Thresholding

Shrinkage denoising in the transformation domain is the process of removing the noise or unwanted components from a number of wavelet coefficients. The use of the shrinkage method has proven its ability as an effective way to suppress the noise with low computational complexity [23]. Nason [24] reviewed various thresholding methods including the selection and estimation. The thresholding rule decides the components of the coefficient that needs to be retained or eliminated. There are two main thresholding approaches, hard and soft thresholding, as expressed in equation (9) and (10) respectively.

$$T_{hard}(wi)= \begin{matrix} wi, |wi| \geq \mu \\ 0, |wi| < \mu \end{matrix} \tag{9}$$

$$T_{soft}(wi)= \begin{matrix} sign(wi)(|wi|-\mu)|wi| \geq \mu \\ 0, |wi| < \mu \end{matrix} \tag{10}$$

It has been reported that, in wavelet transformation domain, the energy of signal tends to be concentrated into a relatively few numbers of large coefficients, whilst the noise will be spread at a large number of the coefficients with relatively low energy [13, 25-27]. This increases the options to eliminate the noise while retaining the important information of the signal as much as possible. As a result, a signal can be enhanced by removing components smaller than an estimated threshold [28]. Based on this principle, wavelet coefficients thresholding for data denoising has been an extensive research domain since the first pioneering work by Donoho and Johnstone [29]. Several thresholding techniques and estimators have been developed, and this research focuses on evaluating four widely adopted techniques, including VisuShrink, SureShrink, HeurSure and Minimax.

SureShrink, also called **Rigrsure**, is generated under a risk rule by minimizing Stein's Unbiased Risk Estimate (SURE), which is represented by $\mu_l = \sigma_i \sqrt{w_m}$. For each detail level, a sub-band threshold is calculated based on SURE rule [30]. This technique is a subband-adaptive, level-dependant. With $((w_m))$ is the mth coefficient wavelet square at the lowest risk which selected from wavelet coefficient squares vector, sorted in ascending way [m1,m2,m3,…mn], and (σ_i) is the level-dependant standard deviation of a noisy signal [31].

VisuShrink, also considered as **Sqtwolog** threshold, which can be derived as $\mu = \sigma\sqrt{2\log(N)}$, where σ is the noise variance obtained by median absolute deviation (MAD) of the coefficients and (N) is the length of the observed signal.

HeurSure technique was developed as an automatic procedure and hybrid approach, which is combined the VisuShrink with SureShrink. It can be applied using one of two scenarios automatically, in the first scenario, it uses the SureShrink technique $\mu_1 = \sigma_i \sqrt{w_m}$. However, if a test of signal coefficients at a level l proved that the signal is deemed too small, the second scenario is then executed automatically by a fixed threshold will be applied instead based on $\mu_2 = \sigma\sqrt{2\log(N)}$ [32, 33].

Minimax technique is based on the statistical minimax principle for estimator designing. A constant threshold value is chosen to produce minimax performance for mean squared error (MSE) against an ideal procedure. [34]. More analytically, to recover the unknown function $s(\cdot)$ from $x(t) = s(t) + \sigma n(t)$, Minimaxi technique measures the performance of the estimation of $\hat{s} = (\hat{s}(t))$ from $s = (s(t))$, with regard to a quadratic loss at the sample points by minimizing the risk as small as possible as

$r(\hat{s}, s) = n^{-1} E \left\| \hat{s} - s \right\|_{2,n,}^{2}$, and $\mu_n^* = \inf_{\mu} \underset{\theta}{sub} \left\{ \frac{r_\mu(\theta)}{n^{-1} + (\theta^2 \mu 1)} \right\}$, where μ_n^* is the minimax risk bound and will be the value of μ. Generally, successful thresholding method in signal denoising highly relying on the accurate estimation of the noise level, thus noise level must be estimated correctly to obtain good performance denoising.

4 Implementation Algorithm

The implementation of the proposed methods starts by calculating the harmonic to signal ratio (HSR) of first three harmonics for the original signal and then the same procedure is used to calculate the HSR of denoised coefficients using DD-DWT based the adopted thresholding methods.

i. Calculate HSR ri_1 of the original signal. $ri_1 = rb / (ra - rb - rs)$
ii. Decompose the input signal into L levels $wli = wt(\mathrm{x})$, where L< log2(N)

iii. Calculate the energy of the coefficients using STD and the impulsiveness using Kurtosis
iv. Coefficients with similar high STD and Kurtosis values are considered to be selected and put the rest of the coefficients to zeros
v. Apply the selected threshold
vi. Reconstruct the decomposed signal $iwt = \tilde{w}^{-1}(a)$.
vii. calculating the ratio r of the first three harmonics of the demodulated signal as $r_i = rb_i / (ra - rb_i - rs_i)$

where ra is the average of the band range bd from 1Hz to 600Hz of the signal and obtained as

$$ra = \sum_{i=\min bd}^{\max bd} ahi ,$$

and rb is the average of the first three harmonics of the signal and obtained as $rb = \sum_{i=1}^{3} bhi$, and

$$rs = \sum_{i=1\times f_s}^{\max bd/f_s} shi$$ where f_S is the shaft frequency.

viii. Calculate the improvement of denoised signal with regard to the original signal as $hsr = (r_i / ri_1) - 1*100$, where ri_1 , ri are HSR of original and denoised signals respectively.

5 Experimental Setup and Data Collection

5.1 Test Rig Development

A test rig was developed for experimental studies. As shown in Figure 1 (a), it consists of a motor, a shaft, a coupling and bearings. This simple structure was adopted to avoid possible noise influences of additional components such as radial load devices. A TIMKEN 31308 tapered roller bearing is mounted in the SKF housing. In addition, a piezoelectric accelerometer (CA-YD-104T) is installed vertically on the top of the housing to acquire the vibration signal. A slip metric gauge box set, type Matrix Pitter 8075 C, was used to precisely measure the clearance.

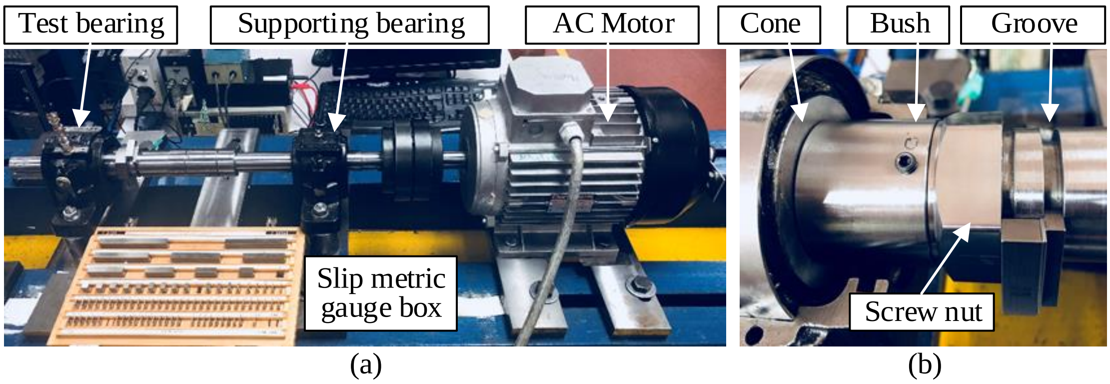

Figure 1. (a) Test rig, and (b) clearance measurement

5.2 Clearance Variance Mechanism

During the experiment, a mechanism for the preload adjustment was built by controlling the clearance between the bearing elements. This was executed by the movement of a precision positioning screw nut relative to the reference position. The schematic diagram of the test rig and the clearance measurement system are described in Figure 2 (a) and (b) respectively. Assume that the recommended installation condition is zero clearance, the measurement slips are used to measure the gap between the groove and the nut edge.

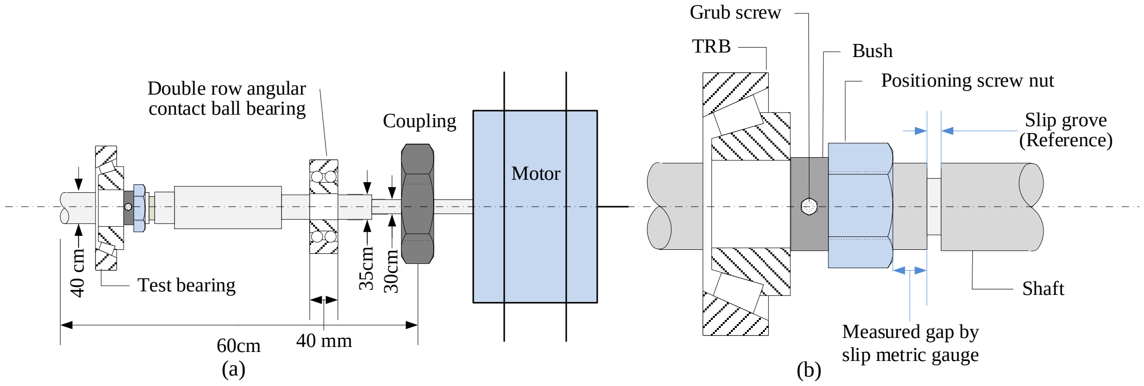

Figure 2. (a) Test rig sketch diagram, and (b) clearance measurement system

5.3 Fault Seeding and Experimental Procedure

In this experiment, one defective tapered roller bearing was used. The defect was artificially made using an electro-discharge machine (EDM). As it can be seen in Figure 3, the defective area has only a tiny size of 2.0mm length and 0.2mm depth, which was seeded to simulate the incipient faults. The data acquisition system is National Instruments PCI6221 and 5 channels were used for data recording at the sampling rate of 50kHz per channel. Five recordings were logged for 30 seconds when the bearing temperature was stable at around 30ºC, which can exclude the influence of the temperature on the bearing internal clearance. The vibration data with three different internal clearances (+0.02mm, 0.00mm, -0.20mm) are obtained for evaluating the developed approach.

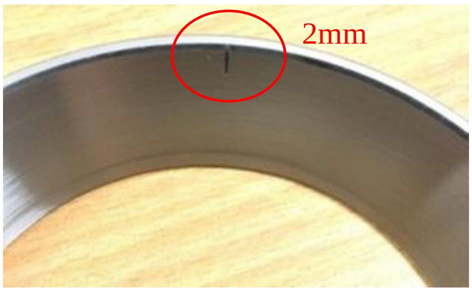

Figure 3. Bearings with seeded defects on the outer race

6 Results and Discussion

To accurately evaluate the performance of the proposed method, DD-DWT was applied to both simulated data and measured vibration data. The signals decomposed into 5 levels and the optimal decomposition levels are selected based on the energy distribution. Thus coefficients with the largest values have been selected[35]. The benchmark thresholding methods used for denoising the selected wavelet coefficients.

6.1 Performance Evaluation Based on Simulated Data

Several models have been developed to simulate the vibration signals since the first attempt to model a nonstationary vibration signal by McFadden in [36]. However, in reality, the combination of impacts produced by a defect on the bearing and stochastic components lead to periodically time-varying ensemble statistics. In this research, the model, described in equation (11), is that takes into account, the periodicity nature and the random variation in the spaces between adjacent pulses. Moreover, it takes into account the modulation due to the varying loads and transmission path influences. It was presented by Randall et al. in [37] as a statistical vibration model produced by a single defect. The model can be expressed as

$$x(t)=\sum_{j} A_j y(t-jT-\tau_j)+n(t) \tag{11}$$

where, A_j can be a possible modulator; T stands for average time between two adjacent impacts and it is derived by $T=1/f_r$; f_r is the fault frequency; jT is the jth time of single impact occurrence; τ_j is the randomness of the time between the impacts; and $n(t)$ is the additive white noise from other vibrations in the system. $y(t)$ is considered as the impulse response function, which can be simplified as an exponential damping cosinusoidal signal.

$$\mathrm{y}(n)=\begin{cases} e^{-\alpha n}\cos(2\pi f_d n) & ; n>0 \\ 0 & ; otherwise \end{cases} \tag{12}$$

where, f_d is the resonance frequencies and were set as (3kHz, 5kHz, and 8kHz), α is the damping ratio was set as (0.05). In order to simulate a vibration signal similar to the signal derived from experimental case studies, a signal was generated. Taking into account the outer race fault as well as the shaft imbalance, A_j can be simplified as

$$A_j = A_1\cos(2\pi f_s(jT+\tau_j)) \tag{13}$$

where, A_1 considers the amplitude of the modulator, f_s is the shaft frequency, $jT+\tau_j$ represents the specific time of the jth impact.

The simulated time signal of outer race fault with low SNR of -13.8dB is showed in Figure 4 (a). In Figure 4 (b), the envelope spectrum of the generated signal is displayed. Both DWT and DD-DWT were applied to the simulated signal and the results obtained are shown in Figure 5. Blue bars show a slight improvement achieved using DWT 5.8% without thresholding. Thresholding based DWT achieves the improvement from 3.2% to 12% when applying different threshold techniques. In contrast, Magenta bars show significant improvement by using DD-DWT. The reconstructed coefficients improved at least by 36%, which indicates that DD-DWT can effectively extract the diagnostic features even when applied without any thresholding methods. In addition, the thresholding methods of Sqtwolog, Heursure and Minimaxi can significantly improve the fault features.

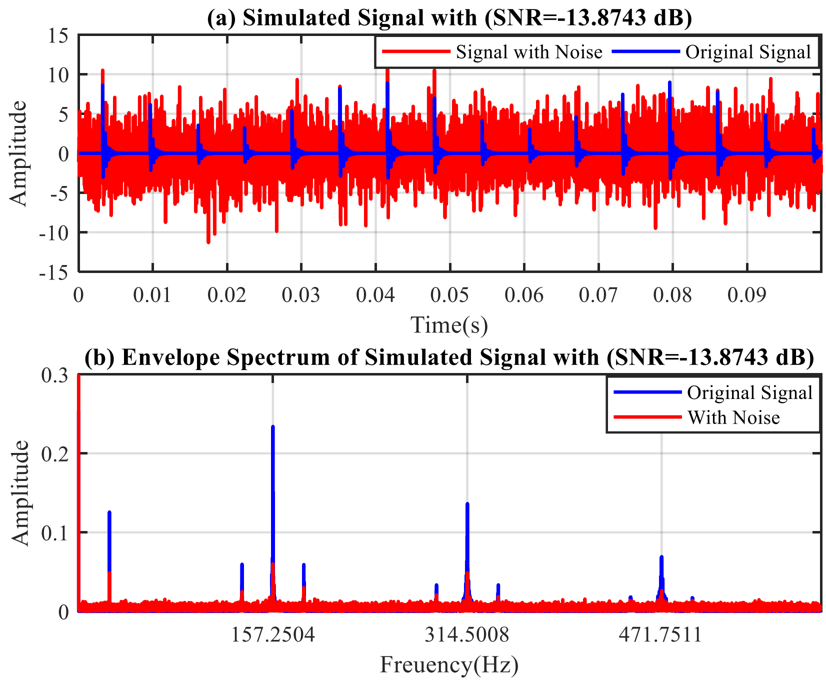

Figure 4 (a) Time domain, (b) envelope spectrum of the simulated vibration signal

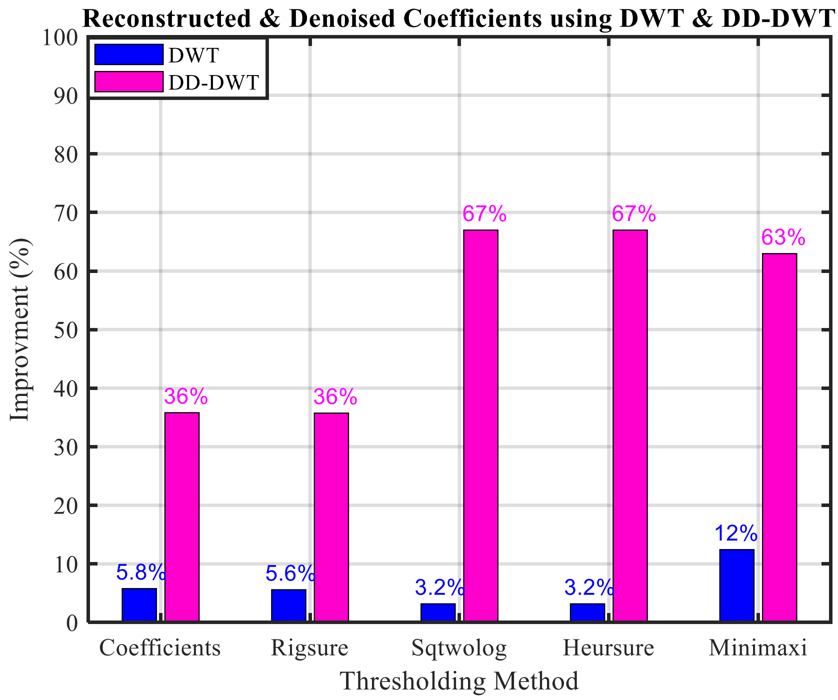

Figure 5 Improvement results of harmonic ratio using DWT and DD-DWT for simulated data

6.2 Performance Evaluation Based on Real Data

Three sets of data for a defective tapered roller bearing with small outer race fault are analysed using DD-DWT. The three sets are associated with three different clearances: +0.02 0.00 -0.02 which correspond to progressive wear levels. The envelope spectrum in Figure 6 shows the improvement of the first three harmonics to signal ratio of reconstructed coefficients in (b) compared to the original signal (a). It shows the effectiveness of DD-DWD in extracting and periodic fault features.

Figure 7 illustrates that the thresholding methods Sqtwolodt, Heusure and Minimaxi based DD-DWT can effectively suppress the noise and enhance the diagnostic features.

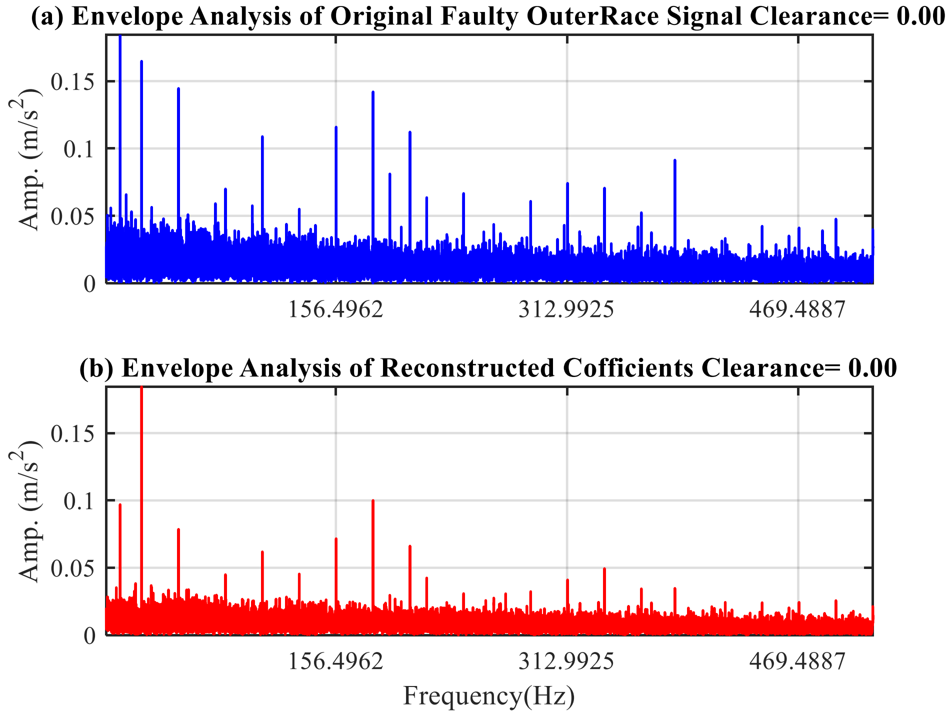

Figure 6 Improvement of the reconstructed coefficients compared to the original signal

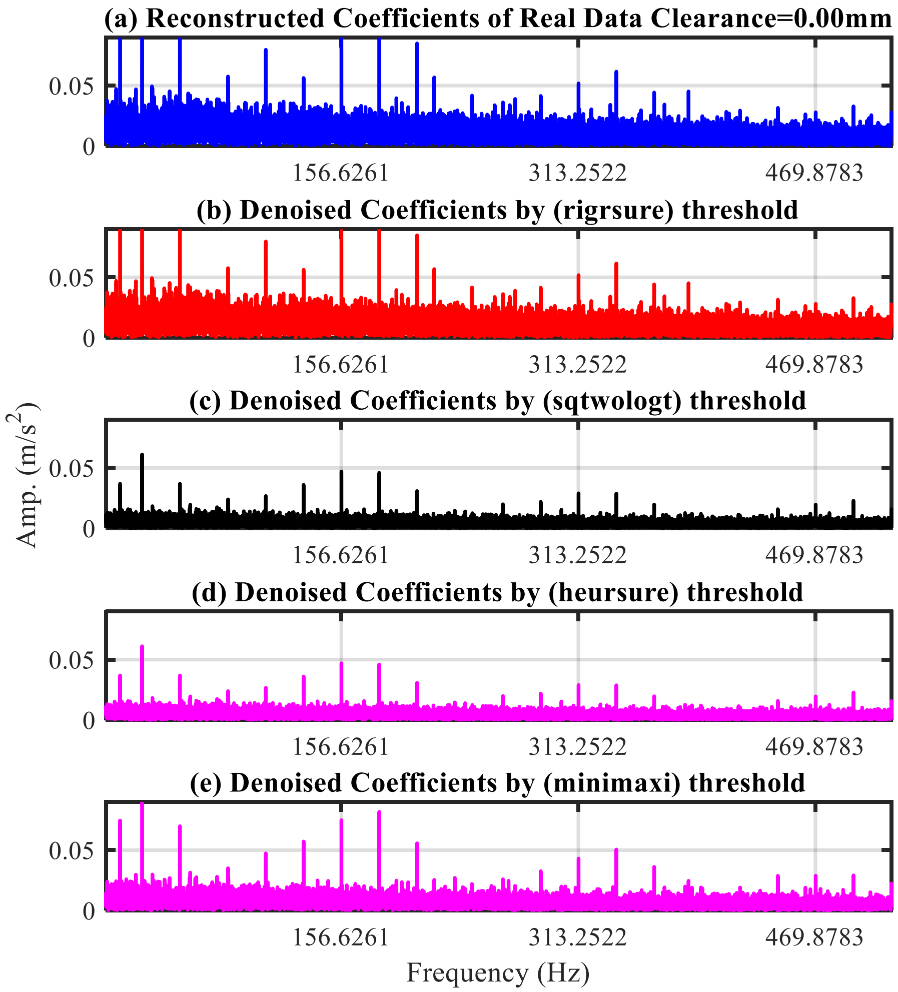

Figure 7 Envelope spectrum of the reconstructed and thresholded coefficients

Figure 8 presents a comparison made between the results obtained from applying both DWT and DD-DWT, and the results proved that DD-DWT outperforms DWT in all studied real cases. The harmonic ratio of outer race fault in reconstructed coefficients improved more than 30% by using the Sqtwolog and HeurSure thresholding based DD-DWT approach. The Rigsure thresholding method is not effective, which is identical with the simulation study. Both Sqtwolog, Heursure yielded the same improvement because Heursure is a hybrid approach and automatic procedure thus, it gives a fixed threshold value based on Sqtwolog as it explained above.

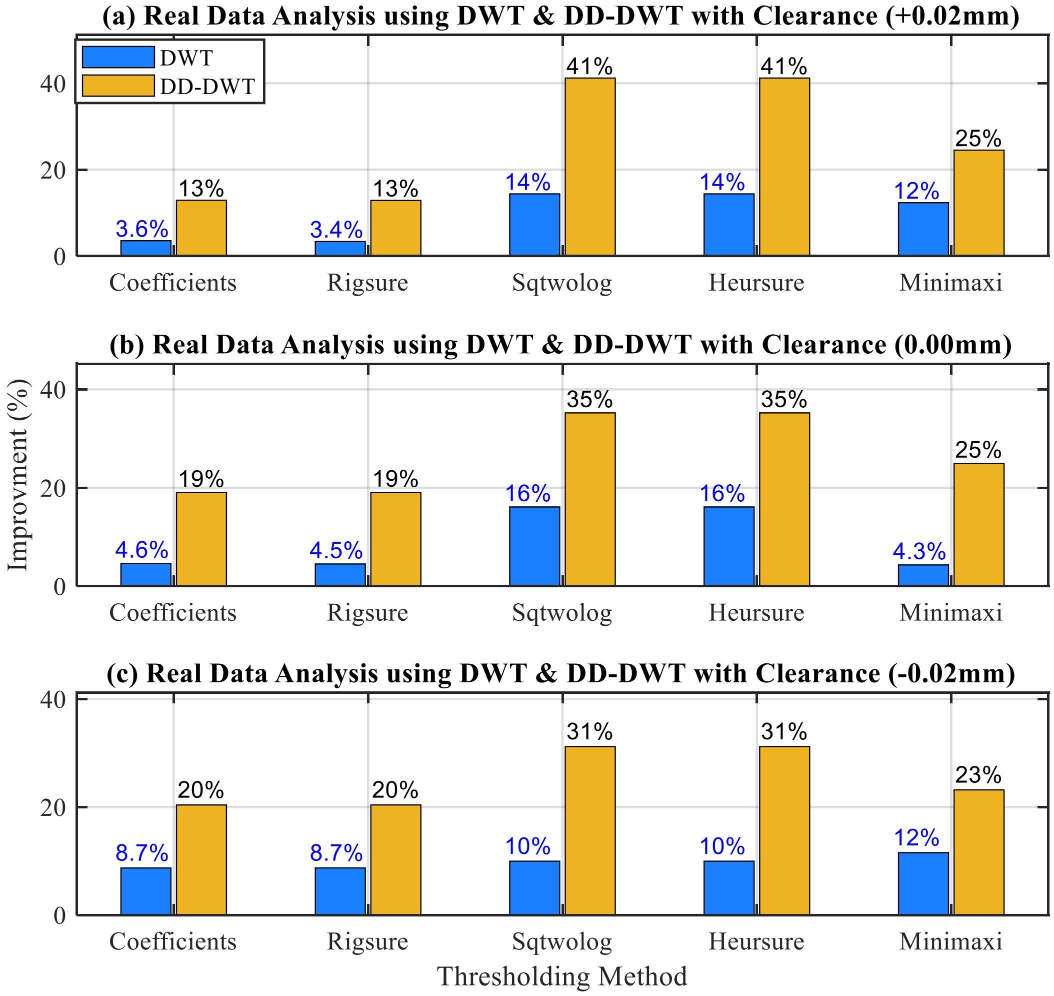

Figure 8 A comparison between DWT and DD-DWT results for all clearance cases

7 Conclusions

The study has shown the outstanding performance of the DD-DWT based method in extracting and enhancing the fault features. In addition, shrinkage in the transformation domain is found to be a very effective approach in suppressing the noise from the vibration data. Based on the evaluation of four thresholding methods, it can be concluded that Sqtwolog and Heursure consistently outperform the other studied wavelet-based techniques, which allows accurate early fault detection and diagnosis based on DD-DWT. Compared with about 10% improvement by conventional DWT, the thresholding based DD-DWT can produce more promising diagnostic results in that it achieves an improvement of more than 30% for enhancing the diagnostic features. In addition, the proposed method does not require any skilled labours or any advanced techniques to choose an optimal frequency band when applying demodulation analysis and the whole frequency band signals can be directly used to achieve the online detection and diagnosis of the rolling element bearings.

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