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## Adaptive Control of Milling Forces under Fractional Order Holds.

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*Abstract*— This paper introduces a novel discrete-time modelreference based control of the tool-work-piece interaction force in a milling process. The novelty of the scheme relies on the use of a fractional order hold (FROH) instead of a traditional zero order hold (ZOH) used in the manufacturing literature to obtain a discrete-time model of the continuous system. The additional degree of freedom introduced by the FROH through its correcting gain allows the designer to improve the closed-loop behavior of the time-varying unknown system by an adequate choice of its value. Simulation examples showing the influence of the correcting gain in the closed-loop response are presented and compared.

## I. INTRODUCTION

Milling is a cutting process widely used in the manufacturing of mechanical components. It consists of the relative movement between feeding a work-piece clamped on a table and rotating multi-tooth cutter. In order to avoid machine malfunctions such as tool wear or breakage and to achieve a certain degree of quality in the finishing of the working-piece, the peak cutting force on the working piece has to be maintained below a prescribed safety upperbound. This fact implies that a control strategy has to be implemented on the system in order to fulfill such safety and performance requirements. Moreover, cutting parameters may be unknown or time-varying as a consequence of a complex milling geometry. Thus, the control law should be able to attain the desired objectives even in the presence of uncertainties or variations in the system parameters. In this way, the nature of the system suggests to use an adaptive controller to address the milling force control problem.

In this work, it is presented the design of an adaptive control law for milling processes which improves the behavior, specially the quality of the finishing of the working piece through a more precise tool-work-piece interaction force control, in comparison with previous approaches.

The key point to achieve such an improved behavior of the system is the use of fractional order holds (FROH) to obtain a discrete-time model of the system. The advantage of using a FROH instead of a traditional ZOH is that FROHs incorporate an additional degree of freedom, the gain of the FROH, which can be used to modify the overall closed-loop response of the system, improving, for instance, the stability of the discrete zeros or reducing the overshoot or bad transient responses which could lead, for example, to break the cutter shank, tool breakage or tool wear, [1, 2]. Hence, the model reference control is the designed from the so obtained FROH based discrete model.

The use of this kind of more complex hold devices is supported by the actual tendency in manufacturing environments consisting in optimizing the selection of machining parameters, through optimization algorithms, and in controlling the machining process on-line in contrast with the traditional CNC based systems, where the machining constant parameters are usually selected according to handbooks or operators' experience leading to an 'ad-hoc' tuning of the control system.

Thus, the influence of the FROH gain in the system's behavior is studied showing that an adequate tuning of it can lead to an improved closed-loop performance. The study is carried out by means of a cost function which compares the system transient responses when different gains of the FROH are used.

Previous works can be found in references [3-9]. In those papers, linear and time varying parameters models are widely used. Those models are cutting parameters dependent. Then, they will be time varying when complex parts are going to be milled. For this reason, the adaptive control techniques are mainly employed to control the milling process. A successful application of the adaptive control to milling process has potential machining-time savings, among other advantages.

### II. SYSTEM DESCRIPTION

### A. Continuous Model

The milling system can be modeled as the series decomposition of a Computerized Numerical Control (CNC), which includes all the circuitry involving in the table movement (amplifiers, motor drives), and the tool-work-piece interaction model itself. A feed rate command  $f_c$  (which plays the role of the control signal) is sent to the CNC unit. This feed rate represents the desired velocity for the table movement. Then, the CNC unit manages to make

the table move at an actual feed velocity of  $f_a$  according to the CNC dynamics. Even though the machine tool drive servos are typically modeled as high order transfer functions, they can usually be approximated as a second order transfer function within the range of working frequencies. Besides, they are tuned to be over-damped without overshoot, so that they can be modeled as the first order system [5]:

$$G_{s}(s) = \frac{f_{a}(s)}{f_{c}(s)} = \frac{1}{\tau_{s}s + 1}$$
(1)

where  $f_a$  and  $f_c$  are the actual and command velocity values of the table in (mm/s) respectively and  $\tau_s$  is an average time constant, which depends on the type of the machine tool. In this study, it is assumed to be 0.1 ms.

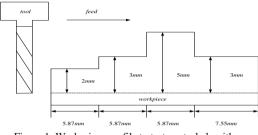


Figure 1: Work-piece profile to test control algorithms.

In addition, the chatter vibration and resonant free cutting process can be approximated as the first order system [5]:

$$G_{p}(s) = \frac{F_{p}(s)}{f_{a}(s)} = \frac{K_{c}ba(\phi_{st}, \phi_{ex}, N)}{N \cdot n} \frac{1}{\tau_{c}s + 1}$$
(2)

where  $K_c(N/mm^2)$  is the cutting pressure constant, b(mm) is the axial depth of cut,  $a(\phi_{st}, \phi_{ex}, N)$  is an adimensional immersion function, ranging between 0 and ~ N depending on the immersion angle and the number of teeth in cut, N is the number of teeth on the milling cutter and n(rev/s) is the spindle speed. The axial deep of cut function b in (2) may be time-varying leading to a potential time-varying system. In particular, the cutting process is assumed to be in this work piecewise constant, admitting sudden changes in the cutting parameters at certain time instants while remaining invariant between changes. This assumption allows us to consider the cutting process to be described by the transfer function (2) with the time interval between changes.

The combined transfer function of the system, obtained from (1) and (2) is

$$G_{c}(s) = \frac{F_{p}(s)}{f_{c}(s)} = \frac{B_{c}(s)}{A_{c}(s)} = \frac{1}{(\tau_{m}s+1)} \frac{K_{c}ab}{Nn(\tau_{c}s+1)} = \frac{K_{p}}{(\tau_{m}s+1)(\tau_{c}s+1)}$$
(3)

where the process gain is  $K_p (N \cdot s/mm) = K_c ab/Nn$ .

Figure 1 shows the sample work-piece depicting basic cutting geometry features with changes in the axial depth of cut used in the simulations. The spindle speed remains

constant, 715*rpm*; the work-piece is made of Aluminum 6067 whose specific cutting pressure is assumed to be  $K_c = 1200 \frac{N}{mm^2}$ . A 4-fluted carbide mill tool, full-immersed and rouging milling operation will be taken into consideration in the present paper.

Also, note that the desired final geometry of the piece to be milled involves changes in the axial deep of cut which implies suddenly changes in its value, according to the sudden changes assumption presented before. On the other hand, it has been taken into account that the control law computes new feed-rate command value at each sampling interval. Furthermore, it is worth to be mentioned that the CNC unit has its own digital position law executed at small time intervals in comparison with the sampled time of the control law, even though if high speed milling tool drives are used [5].

## B. Discrete model under $\beta$ – FROH

In this paper, the problem of controlling a continuous plant is addressed by using a discrete controller. The discrete controller is obtained applying a model-reference pole-placement based control design to a discrete model of the plant (3) obtained by means of a FROH with a certain correcting gain  $\beta$ . The additional "degree of freedom"  $\beta$  provided by the FROH can be used with a broad variety of objectives such as to improve the transient response behavior, to avoid the existence of oscillations in the continuous time output of the system or to improve the stability properties of the zeros of the discretized system. Hence, the discretization of (3) under a FROH is calculated as :

$$H_{\beta}(z) = Z \Big[ h_{\beta}(s) \cdot G_{\mathcal{C}}(s) \Big]$$
(4)

where 
$$h_{\beta}(s) = \left(1 - \beta e^{-sT} + \frac{\beta \left(1 - e^{-sT}\right)}{Ts}\right) \frac{1 - e^{-sT}}{s}$$
 is the transfer

function of a  $\beta$ -*FROH*, where *z* is the argument of the *Z*-*transforn*, being formally equivalent to the one step ahead operators, *q*, used in the time domain representation of difference equations. This allows us to keep a simple unambiguous notation for the whole paper content. The sampling time *T* has been chosen to be the spindle speed, *n*, as it is usual for this kind of systems [3-5]. Note that when  $\beta = 1$ , the *FROH* hold becomes a first order hold (*FOH*) and when  $\beta = 0$ , the zero order hold (*ZOH*) is obtained, being both particular cases of  $\beta \in [-1,1]$ . Furthermore,  $H_{\beta}(z)$  may be calculated using just *ZOH* devices in the following way:

$$H_{\beta}(z) = \frac{B_{\beta}(z)}{z^{\delta_{\beta}} \cdot A(z)} = \frac{z - \beta}{z} Z[h_{o}(s)G_{c}(s)] + \frac{\beta(z - 1)}{Tz} Z\Big[h_{o}(s)\frac{G_{c}(s)}{s}\Big] =$$
$$= \frac{B_{\beta}(z)}{z^{\delta_{\beta}} \cdot \left(z - e^{-T/\tau_{m}}\right) \cdot \left(z - e^{-T/\tau_{c}}\right)} = \frac{B_{\beta}(z)}{z^{\delta_{\beta}} \cdot \left(z^{2} + a_{1}z + a_{2}\right)}$$
(5)

where  $h_o(s) = \frac{1 - e^{-s_I}}{s}$  is the transfer function of a *ZOH* and  $\delta_\beta = 1$  if  $\beta \neq 0$  and  $\delta_\beta = 0$  if  $\beta = 0$ , which means that a fractional order hold with  $\beta \neq 0$  adds a pole at the origin.  $B_\beta(z) = b_0 z^2 + b_0 z + b_0$ , where

$$\begin{split} & b_{0} = \frac{\tau_{m} \Big( 1 - e^{-T/\tau_{m}} \Big) - \tau_{c} \Big( 1 - e^{-T/\tau_{c}} \Big)}{\tau_{m} - \tau_{c}} + \beta + \frac{\beta}{T} \frac{\tau_{c}^{2} \Big( 1 - e^{-T/\tau_{c}} \Big) - \tau_{m}^{2} \Big( 1 - e^{-T/\tau_{m}} \Big)}{\tau_{m} - \tau_{c}} \\ & b_{1} = \frac{\tau_{m} \Big( 1 - e^{-T/\tau_{m}} \Big) - \tau_{c} \Big( 1 - e^{-T/\tau_{c}} \Big)}{\tau_{m} - \tau_{c}} + \beta \frac{\tau_{c} \Big( 1 + e^{-T/\tau_{m}} \Big) - \tau_{m} \Big( 1 + e^{-T/\tau_{c}} \Big)}{\tau_{m} - \tau_{c}} + \\ & + \frac{\beta}{T} \begin{cases} \frac{\tau_{m}^{2} \Big( 1 + e^{-T/\tau_{c}} - e^{-T/\tau_{m}} - e^{-T/\tau_{c}} e^{-T/\tau_{m}} \Big)}{\tau_{m} - \tau_{c}} \\ & + \frac{\beta}{T} \begin{cases} \frac{\tau_{c}^{2} \Big( -1 + e^{-T/\tau_{c}} - e^{-T/\tau_{m}} + e^{-T/\tau_{c}} e^{-T/\tau_{m}} \Big)}{\tau_{m} - \tau_{c}} \end{cases} \end{cases} \\ & b_{2} = \frac{\beta}{\tau_{m} - \tau_{c}} \begin{cases} \tau_{c} e^{-T/\tau_{m}} \Big( \frac{\tau_{c}}{T} \Big( 1 - e^{-T/\tau_{c}} \Big) - 1 \Big) - \tau_{m} e^{-T/\tau_{c}} \Big( \frac{\tau_{m}}{T} \Big( 1 - e^{-T/\tau_{m}} \Big) - 1 \Big) \end{cases} \end{split}$$

## C. Desired response: model reference

A second order system 
$$G_m(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$
 (6) is

selected to represent the system model reference. This system is characterized by a desired damping ratio,  $\xi$  and a natural frequency,  $\omega_n$ . It is known that small  $\xi$  leads to a large overshoot and a large setting time. A general accepted range value for  $\xi$  to attain satisfactory performance is between 0.5 and 1, which corresponds to the so-called under-damped systems. In this way, a damping ratio of  $\xi = 0.75$  and a rise time,  $T_r$ , equal to four spindle periods is usually selected for practical applications. Furthermore, the natural frequency is then usually suggested to be  $\omega_n = \frac{2.5}{T_r} rad/s$ . This continuous-time reference model is then discretized with the same FROH as the real system was in order to obtain the corresponding discrete-time reference model for the controller. Thus, a number of different discrete models obtained from a unique continuous

reference model are considered depending on the value of

 $\beta$  used to obtain the discretization.

#### III. ADAPTIVE MODEL FOLLOWING CONTROLLER

The figure depicts a schematic representation of the model reference adaptive control algorithm:

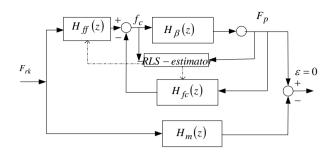


Figure 2: Adaptive model following control scheme.

where  $H_{ff}(z,k) = \frac{S(z,k)}{R(z,k)}$  is the feed-forward filter from

the reference signal,  $H_{fb}(z,k) = \frac{T(z,k)}{R(z,k)}$  is the feedback controller, H(z,k) is the discrete plant,  $H_m(z,k)$  is the model reference and  $F_{rk}$  is the reference force.

The adaptive control algorithm is obtained by adding a RLS estimation algorithm,

$$\hat{\theta}(k) = \hat{\theta}(k-1) + L(k) \left[ F_p(k) - \phi^T(k) \hat{\theta}(k-1) \right]$$

$$L(k) = P(k-1)\phi(k) \left( \lambda + \phi^T(k) P(k-1)\phi(k) \right)^{-1}$$

$$P(k) = \left( I - L(k)\phi^T(k) \right) P(k-1) \frac{1}{2}$$
(7)

simultaneously running in parallel with the control law at each sampling instant,  $k \cdot \hat{\theta}^T = \begin{pmatrix} \hat{a}_1, \hat{a}_2, \hat{b}_0, \hat{b}_1, \hat{b}_2 \end{pmatrix}$  is the

parameter vector and  $\hat{\phi}(k)$  is the regressor vector.

The transfer function of the reference model is,

$$H_m(z) = \frac{B^{-}(z)B_m(z)A_o(z)}{A_m(z)A_o(z)} = \frac{B_m(z)A_o(z)}{A_m(z)A_o(z)}$$
(8)

where  $B'_m(z)$  contains the free-design reference model zeros,  $B^-(z)$  is formed by the unstable (assumed known) plant zeros and  $A_o(z)$  is a polynomial including the eventual closed-loop stable pole-zero cancellations which are introduced when necessary to guarantee that the relative degree of the reference model is non less then that of the closed-loop system so that the synthesized controller is casual. A basic control scheme is displayed in figure 2. Then, it will be considered the polynomials  $R_k$ ,  $S_k$  and T(T depends only on the reference model zeros polynomial which is of constant coefficients) where  $T = B'_m A_o$  and  $R_k$  (monic),  $S_k$  are unique solutions with degrees fulfilling

$$\deg\left(R_{k}\right) = 2n - i, \deg\left(S_{k}\right) = i - 1, \deg\left(A_{m}A_{o}\right) = 2n$$

of the polynomial Diophantine equation

$$A_{k} R_{k} + B_{k} S_{k} = B_{k}^{+} A_{m} A_{o} \Leftrightarrow$$

$$A_{k} R_{1,k} + B^{-} S_{k} = A_{m} A_{o}$$

$$with R_{k} = B_{k} R_{1,k} .$$

$$(9)$$

From (8)-(9), perfect matching is achieved through the control signal:

$$f_{c,k} = \frac{\widehat{T}(z)}{\widehat{R}(z)} F_{r,k} - \frac{\widehat{S}(z)}{\widehat{R}(z)} F_{p,k}$$
(10)

Note that the zeros of the machine tool plant are always stable and within the unit circle. But since the RLS estimator does not predict accurately the parameters of the numerator of the plant, separate control system design are needed for cases when the zeros are stable or unstable.

An additional unstable zero can be introduced by the process discretization. In this paper, only stable discretization zero cases are taken into account.

## IV. EXPERIMENTAL RESULTS

There is an extensive literature which carefully explains the algorithms here developed, for example [10, 11], and show the robustness of the adaptive law [12]. The novelty of the control relies on the use of fractional order holds instead of the usual ZOH appearing in the manufacturing literature. In this paper, the correcting gain of  $\beta$ -FROH is handled to show that the system transient response can be enhanced respect to the use of ZOH. This can lead to avoid overloading of the insert, because the maximum removed chip-thickness would not increase the principal tensile stress in the cutting wedge beyond the ultimate tensile strength of the tool material, this can also lead to prevent fracture of the shank, and fulfill the machine tool requirements, such as power and torque availability [6]. Moreover, if the reference force is selected near the tool breakage limit, the large overshot lead to tool breakage [1, 2, 6]. Then, if the overshoot of the system response is reduced, the reference force can be increased, improving the time production requirements.

An adaptive model following controllers have been developed using different correcting gains of the fractional order hold. The milling system and the model reference are discretized via fractional order hold. The estimation vector has been initialized as the corresponding discretization from estimated continuous transfer function,

$$G(s) = \frac{-3s + 150}{s^2 + 20s + 550} \tag{11}$$

As example, the some representative cases are plotted in figures 3 and 4. The figures present the resultant force

keeping at the reference force, which is set to a constant value of 1.2KN. The system registers large overshoots in the transient responses, depending on the  $\beta$ -value and the initial values of the parameter vector.

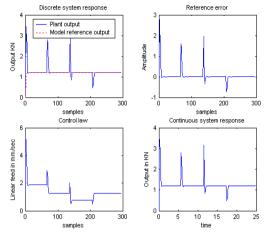


Figure 3: Relevant signals corresponding to  $\beta = 0$ .

The initial parameter vector has the ability that if it is near to the real values of the plant, the transient response of the system will be smooth and feasible. In contrast, if the initial value of the parameter vector has been selected in arbitrary manner the transient is normally oscillated with a great maximum overshoot and large setting time. In any case, fractional order holds can help to reduce large overshoots.

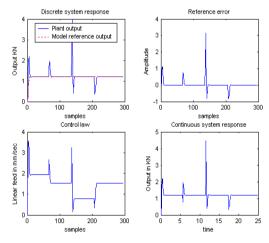


Figure 4: Relevant signals corresponding to  $\beta = -0.4$ .

On the other hand, there are abruptly overshoots in the output when the axial depth of cut changes suddenly. It is due to the intrinsic structure of the closed-loop output. It is not the main purpose of this paper reducing or avoiding these jumps. But, in that case, some 'a priori' information about the work-piece geometry is required to design a successful control, as in [6], where a CAD model of the work-piece is used to modify the control command when the axial depth of cut changes in order to minimize the overshoots due to abrupt changes in the transfer function.

#### V. TRANSIENT RESPONSES CHARACTERIZATION

In order to compare time domain transient behaviors when the designed control scheme respect to the use of traditional ZOHs, a cost function is defined:

$$J_{c} = \sum_{j=1}^{k} \int_{(j-1)T}^{jT} |F_{p}(\tau) - F_{p,m}(\tau)| d\tau$$
(12)

where  $F_p$  is the output signal and  $F_{p,m}$  is the model reference output signal, k is the number of periods which have been taken into account in the transient response characterization.

The cost function calculates a good approximation of the area between the continuous system output and the continuous model reference system response. The smaller this area is, the smaller cost function will be. It leads to choose an adequate value of  $\beta$  which achieves the best output transient response behavior.

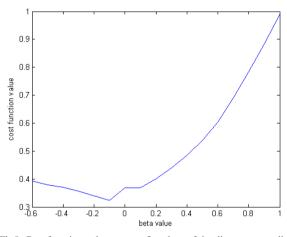


Fig5: Cost function value versus  $\beta$  values of the discrete controllers.

Figure 9 shows the cost function value when  $\beta$  value of the discrete controllers varies. In the figure it can be appreciated that, the use of  $\beta$ -value near to -0.2 leads to minimum values of the cost function. It concludes that better system transient responses will be achieved if the adaptive control algorithm is designed utilizing a FROH respect to the usual ZOH using in the manufacturing literature.

The cases when  $\beta < -0.6$  have not been taken into consideration because the plant is non-minimum phase. In those cases, 'a priori' knowledge about the system zero is needed to implement a successful control. Information about this case can be found in [11, 12].

## VI. CONCLUSION

In this paper an adaptive model following force control scheme has been proposed to deal with unknown timevarying milling systems. The novelty of the control scheme relies on the use of FROH instead of the usual ZOH appearing in the manufacturing literature. The FROH provides an "extra degree of freedom", which can be manipulated by the programmer to obtain a better transient response as the simulations have pointed out being then confirmed by the proposed cost functional. There is not a rule of thumb to select the adequate  $\beta$  value, only operators' experience can help to select a satisfying value of  $\beta$ , for a range of working cutting parameters.

On the other hand, the general FROH hold can be implemented by means of ZOH holds, which make this approach fairly feasible to be implemented in the manufacturing industry. Then, an easily implemented device can lead to save machining time in the production, avoid some process malfunctions or damage the tool less than if just a ZOH device is used.

#### ACKNOWLEDGMENT

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