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Spatiotemporal Growth of Laminar Boundary Layers in a Concentric Annulus

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ABSTRACT

Vortex rings are fascinating fluid flow phenomena occurring in nature. To accurately predict the dynamic behavior of such rings, researchers have been postulating analytical models and carrying out a huge number of experiments over the last couple of decades. It is an established fact that the most important feature of the flow that dictates the generation, and hence propagation, of vortex rings is in fact the growth of the boundary layer. As far as the generation process of these rings is concerned, Rayleigh's analytical solution to the Navier-Stokes equations for impulsive motion of fluid over a flat plate is considered as a benchmark for the development of dynamic models. The limitation of using Rayleigh's solution is that it is valid only at the edge of the boundary layer where the velocity of the boundary layer flow is nearly equal to the potential flow velocity. There is a need to develop a solution for the Navier-Stokes equations inside the boundary layer. Based on Computational Fluid Dynamics, an empirical model for the solution of the Navier-Stokes equations inside the boundary layer is presented here. The flow under consideration is Laminar in a concentric annular pipe. It has been concluded that the Rayleigh's solution is applicable to the flow in annuls as well.

Keywords boundary layer thickness, annular flow, CFD

1 INTRODUCTION

Rosenhead (1963) states the solution of the Navier-Stokes equations for an impulsive flow over a flat plate, given by Rayleigh, as:

$$u = U * \operatorname{erf}\left(\frac{y}{2\sqrt{\nu t}}\right) \tag{1}$$

Where *u* is the flow velocity inside the boundary layer, *U* is the velocity of the flow outside the boundary layer, *v* is the kinematic viscosity of the fluid and *t* corresponds to time. It can be seen that the Rayleigh's solution is independent of the axial distance *x*. The main limitation of this solution, as pointed out by Rosenhead, is its inability to predict the flow velocity inside the boundary layer. This means that this solution is applicable only at the edge of the boundary layer where *u = 0.99U*. Schlichting (2000) further simplified this equation and stated:

$$\eta = \operatorname{erf}\left(\frac{y}{2\sqrt{\nu t}}\right) \tag{2}$$

Where η is the non-dimensional boundary layer thickness. This solution has been used for the case of piston/cylinder geometry by Dabiri et al. (2004). The objectives of the work presented here are:

1. To analyze whether this solution can be applied to the flow in annular pipes
2. Development of an empirical model, based on CFD, which can predict the velocity of the flow inside the boundary layer

2 Numerical Modeling

The numerical modeling technique used by Shussher et al. (2002) for the case of piston/cylinder assembly has been applied for the case of annulus over here. 3D time-dependent incompressible Navier-Stokes equations are employed for simulating the unsteady flow. Concentric pipes of diameters 2.5cm and 1.5cm respectively having length of 20cm had been modeled as shown in figure (1).

Nomenclature

u	Axial velocity inside the boundary layer (m/s)
U	Axial velocity outside the boundary layer (m/s)
erf	Error function (-)
x	Axial distance (m)
y	Radial distance (m)
t	Time interval (s)
η	Dimensionless boundary layer thickness (-)
ν	Kinematic viscosity of the fluid (m^2/s)
L	Stroke length (m)

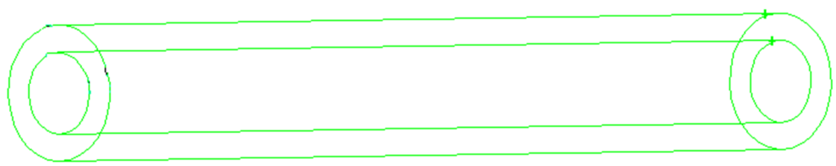

Figure 1: Modeling annular flow between two concentric pipes

At the inlet boundary, a uniform axial velocity of 8cm/sec had been specified which corresponds to a Reynolds number of 2000 for water flow. The pipes are stationary and the outflow is to the atmosphere. Zero velocity had been specified for initializing the solution. A commercial CFD solver (FLUENT 13) was employed to numerically solve the Navier-Stokes equations. Pressure implicit splitting of operators method was used for pressure – velocity coupling. Second order spatiotemporal schemes were used for obtaining accurate results. The mesh and time step size used by Shussher had been used.

3 RESULTS

The first objective of this study is to analyse whether Rayleigh's solution can be applied for the analysis of flows in annulus. If it is possible then the results should show that the velocity inside the boundary layer is:

i. Independent of axial distance x
ii. Proportional to radial distance y
iii. Proportional to 1/√t

In order to accurately analyse these points, it should be made sure that the analysis is done inside the boundary layer. Figure (2) shows the axial velocity profile of the flow between the pipes for the time interval 0.05sec and at an axial distance of 2cm from the inlet boundary. The boundary layer is in its initial stages of development in this case. The boundary layer increases in thickness at higher times and at larger axial distances. Hence, figure (2) suggests that if the analysis is carried out between y = 0.0122 and 0.0125m, it will ensure that the analysis region is deep inside the boundary layer where u is not equal to 0.99U.

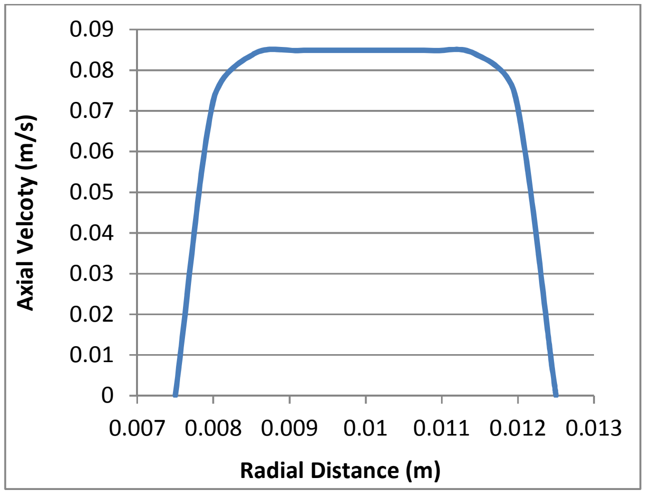

Figure 2: Velocity profile between the pipes

Figures (3), (4) and (5) confirm that the solution of Navier-Stokes equation given by Rayleigh is applicable to flows in annular pipes as well.

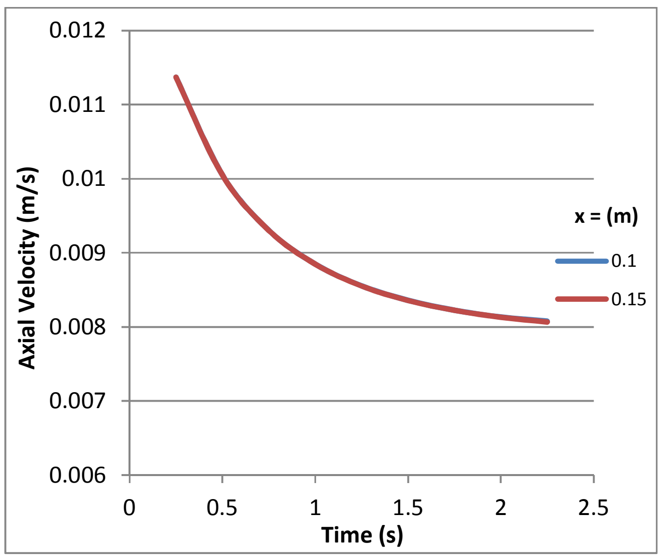

Figure 3: Dependence of the solution on axial distance

Figure (3) shows that the solution for boundary layer flow in concentric annulus is independent of the axial distance. Two different axial locations of x = 0.1m and 0.15m for a radial location of y = 0.0124m were chosen and axial velocity being plotted at different time intervals. The results are exactly similar to each other and hence only one curve can be seen in the figure as it has overshadowed the other curve. Figure (4) shows that the axial velocity inside the boundary layer of annulus is directly proportional to radial distance y. The analysis was carried out at an axial location of 0.1m and for different time intervals.

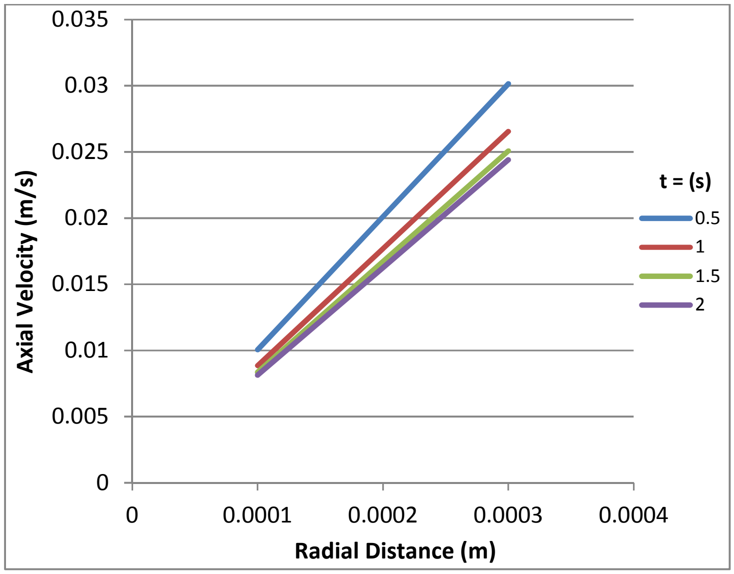

Figure 4: Dependence of the solution on radial distance

Figure (5) shows that the axial velocity inside the boundary layer of the annulus is inversely proportional to √t. The analysis was carried out at an axial location of x = 0.15m and radial location of y = 0.0124m.

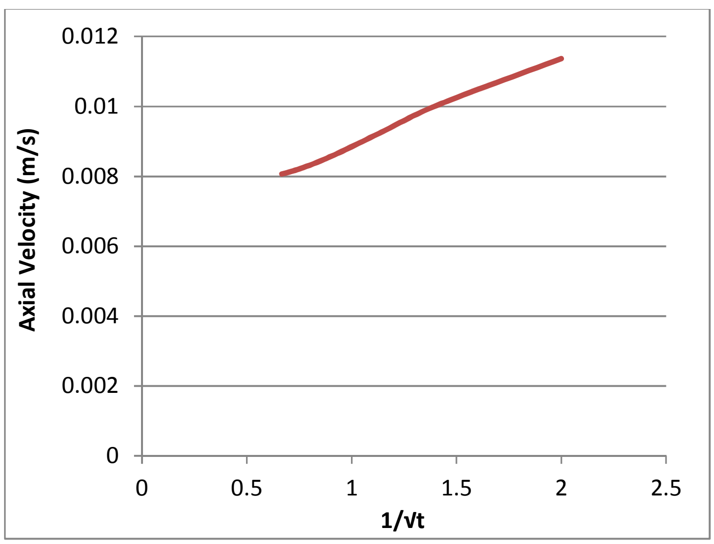

Figure 5: Dependence of the solution on time

Hence the above results confirm that the Rayleigh's solution, with some modifications, can be used for the analysis of boundary layer flows in concentric annulus.

4 EMPIRICAL MODEL

To predict a new model for the boundary layer growth in concentric annuls, CFD results were obtained for different time intervals and for different radial locations inside the boundary layer. The results were than compared with equation (1) and a correlation was found out. Table (1) shows some of the results that were used for the development of the empirical model.

TABLE 1: Comparison of analytical and empirical results

t	L	y	u from equation (1)	u from CFD
(sec)	(m)	(m)	(m/sec)	(m/sec)
0.5	0.04	0.0124	0.006357238	0.010047
		0.0122	0.018821966	0.030144
1	0.08	0.0124	0.004498972	0.008844
		0.0122	0.013407959	0.026535
1.5	0.12	0.0124	0.00367441	0.008355
		0.0122	0.010974689	0.025068
2	0.16	0.0124	0.003182573	0.008129
		0.0122	0.009516149	0.024389

Where *L* corresponds to different stroke lengths for the corresponding time intervals. The empirical model developed is:

$$u = 2.3 * U * \mathrm{erf}\left(\frac{y}{2\sqrt{\nu t}}\right) \quad (3)$$

Where the factor of 2.3 accounts for the axial velocity inside the laminar boundary layer of concentric annuls. Equation (3) predicts the axial velocity in the boundary layer with almost 90% accuracy and hence can be employed for further analysis.

5 CONCLUSIONS

An empirical model based on CFD techniques has been presented here which accounts for the limitations in Rayleigh's solution to the Navier-Stokes equations for impulsive flow over a flat plate. Laminar flow in concentric annulus was analyzed and transient results for boundary layer axial velocities were obtained. The results indicate towards a model similar to Rayleigh's solution. The results obtained from this model are more accurate for the spatiotemporal analysis of the laminar boundary layer growth in concentric annulus.

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