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SPECTRAL CHARACTERISATION OF THE SHORTENED PULSE POSITION MODULATION FORMAT

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Shortened pulse position modulation (SPPM) is a new modulation format that has recently been proposed for underwater wireless optical communication. This Letter considers, for the first time, a full spectral characterisation of SPPM and presents original expressions, which are validated numerically, for predicting both the continuous and discrete spectrum.

Introduction: Conventional n-ary pulse position modulation (PPM) has been proposed for the optical fibre [1], optical wireless [2], optical satellite [3] and optical underwater [4] channels due to its enhanced sensitivity performance. However, this is at the cost of significant bandwidth expansion and so alternative, more bandwidth efficient schemes have been proposed such as multiple PPM [5] and n^k-PPM [6].

In [7], a new format, termed as shortened PPM, was proposed for underwater wireless optical communication because of its bandwidth efficiency over n-ary PPM. The M bits of binary PCM, contained in a timeframe $T_f = MT_b$, are converted into the SPPM format by dividing T_f into $n = 1 + 2^{M-1}$ time slots. The first binary PCM bit is carried forward to the first SPPM slot and the remaining $(M-1)$ binary PCM bits are conveyed by positioning a single pulse in one of the remaining 2^{M-1} time slots. The bandwidth expansion of SPPM over binary PCM is $\left(2^{M-1}+1\right)/M$ which is more bandwidth efficient than the $2^M/M$ expansion required for n-ary PPM.

This Letter evaluates, for the first time, the power spectral density (PSD) of SPPM. By making use of the cyclostationary properties of the modulation format, original expressions are derived for predicting both the continuous and discrete spectrum and these are verified numerically by taking the Fast Fourier Transform of the SPPM pulse stream.

Spectral Characterisation: Following the approach outlined in [8] the data pulse stream can be represented as

$$m(t) = \sum_{q=-\infty}^{\infty} a_q p(t - qT)$$

where $\{a_q\}$ is the SPPM sequence and $p(t)$ is the pulse shape. To compute the discrete PSD of $m(t)$, namely, $S_m^d(f)$, the statistical correlation function, $R_m(t;\tau) = \overline{m(t)m(t+\tau)}$, must first be averaged over t and then the Fourier transform taken:

$$\begin{aligned} S_m^d(f) &= \mathrm{F}_\tau\left\{ \left\langle R_m(t;\tau) \right\rangle_t \right\} \\ &= \frac{1}{T_f^2} \sum_{l=-\infty}^{\infty} \left| P\left(\frac{l}{T_f}\right) \right|^2 \left| \sum_{q=1}^{n} \mathbf{E}\{a_q\} e^{+j\left(\frac{2\pi}{n} lq\right)} \right|^2 \delta\left(f - \frac{l}{T_f}\right) \end{aligned}$$

where T_f is the SPPM frame-time. The term $\sum_{q=1}^{n} \mathbf{E}\{a_q\} e^{+j\left(\frac{2\pi}{n} lq\right)}$ represents the characteristic function of the data distribution on the SPPM frame and so makes the cyclostationary property explicit. Evaluating this and assuming a rectangular pulse of height, A, and width, t_p, allows $S_m^d(f)$ to be written as:

$$S_m^d(f)=\sum_l \left|A\frac{t_p}{T_f}\operatorname{sinc}\left(l\frac{t_p}{T_f}\right)\right|^2 \left[\begin{array}{l}\left(\dfrac{(n-3)}{2(n-1)}\right)^2+\dfrac{(n-3)}{(n-1)^2}\dfrac{\sin(\pi l)\cos((n-1)\pi l/n)}{\sin(\pi l/n)}\\ +\dfrac{1}{(n-1)^2}\left|\dfrac{\sin(\pi l)}{\sin(\pi l/n)}\right|^2\end{array}\right]\delta\left(f-\frac{l}{T_f}\right) \tag{1}$$

where $\operatorname{sinc}(x)=\sin(\pi x)/(\pi x)$.

The continuous PSD can be determined by evaluating the Fourier transform of the autocorrelation function of a zero-mean SPPM sequence. The autocorrelation function is given by

$$\begin{aligned} R_M(t;\tau) &= \mathbf{E}\left\{M(t)M^*(t+\tau)\right\} \\ &= \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\int_y\int_z K_{a,\varepsilon}(n;m-n) \\ &\quad \times P(y)P^*(z)e^{-j2\pi ynT}e^{+j2\pi zmT} \\ &\quad \times e^{+j2\pi(y-z)t}e^{-j2\pi z\tau}dydz \end{aligned}$$

where $K_a(n;m-n)=\mathbf{E}\left\{a_n a_m^*\right\}-\mathbf{E}\left\{a_n\right\}\mathbf{E}\left\{a_m^*\right\}$. Taking the Fourier transform of this gives

$$\begin{aligned} S_M^c(f) &= \mathrm{F}\left\{\left\langle R_M(t;\tau)\right\rangle_t\right\} \\ &= \frac{1}{T}\left|P(f)\right|^2\sum_{l=-\infty}^{\infty}\left[\frac{1}{N}\sum_{n=1}^{N}K_a(n;l)\right]e^{-j2\pi flT} \end{aligned}$$

Evaluating this for SPPM gives

$$S_M^c(f)=\frac{\left|At_p\operatorname{sinc}(ft_p)\right|^2}{T_f}\left[\begin{array}{l}\dfrac{5n-1}{4(n-1)}-\dfrac{2}{(n-1)}\dfrac{\sin\left(\dfrac{(n-1)}{n}\pi fT_f\right)\cos\left(\dfrac{(n-2)}{n}\pi fT_f\right)}{\sin(\pi fT_f/n)}\\ +\dfrac{(n-2)}{(n-1)^2}\dfrac{\sin\left(\dfrac{(2n-3)}{n}\pi fT_f\right)}{\sin(\pi fT_f/n)}-\dfrac{1}{(n-1)^2}\left|\dfrac{\sin\left(\dfrac{(n-2)}{n}\pi fT_f\right)}{\sin(\pi fT_f/n)}\right|^2\end{array}\right] \tag{2}$$

Results: In order to validate the analytic results of (1) and (2), the PSD of SPPM was evaluated numerically using the Fast Fourier Transform. A sampling rate of 256 samples per SPPM slot duration was used and 50 FFT's were averaged in order to decrease the noise due to the randomness of the data sequence.

Fig. 1 shows the power spectral density, calculated both numerically and with the new analytic expressions of (1) and (2), for a 9-slot SPPM system with the pulse width set at the slot duration, $t_p = T_f / n$. Note that the frequency axis is normalised to the slot repetition frequency. As can be seen, unlike n-ary PPM, there are discrete lines at the frame repetition frequency that will facilitate frame synchronisation. However, for full-width pulses, the nulls of the sinc-function in the first term occur at the slot repetition frequency and so this masks the discrete spectrum and so there is no discrete spectrum at the slot repetition frequency, $f_s = n/T_f$. Fig. 1 also demonstrates that there is excellent agreement between the numerical and analytical results for predicting the continuous spectrum so validating the accuracy of expression (2).

Fig. 2 shows the PSD, calculated both numerically and analytically, for a 17-slot SPPM system with the pulse width set at half the slot duration ($t_p = T_f / (2n)$). Again, the numerical and analytical results are in excellent agreement so confirming the validity of (1) and (2) for predicting the PSD of SPPM. The results demonstrate that there is a strong discrete line at the SPPM slot-rate and so this can be extracted for synchronisation purposes directly from the pulse stream. Again, due to the non-uniform distribution of the data within the frame, there are components available at the frame repetition frequency.

Conclusions: Original expressions, for both the discrete and continuous spectrum, have been presented that offer a full spectral characterisation of the recently proposed shortened PPM modulation format. It is shown that, unlike conventional PPM, there are components at the frame repetition frequency that will facilitate frame clock extraction. Furthermore, when half-width pulses are used, there are components at both the slot and frame repetition frequency and so SPPM has the key advantages of both improved bandwidth efficiency and also being able extract synchronisation signals directly from the data stream.

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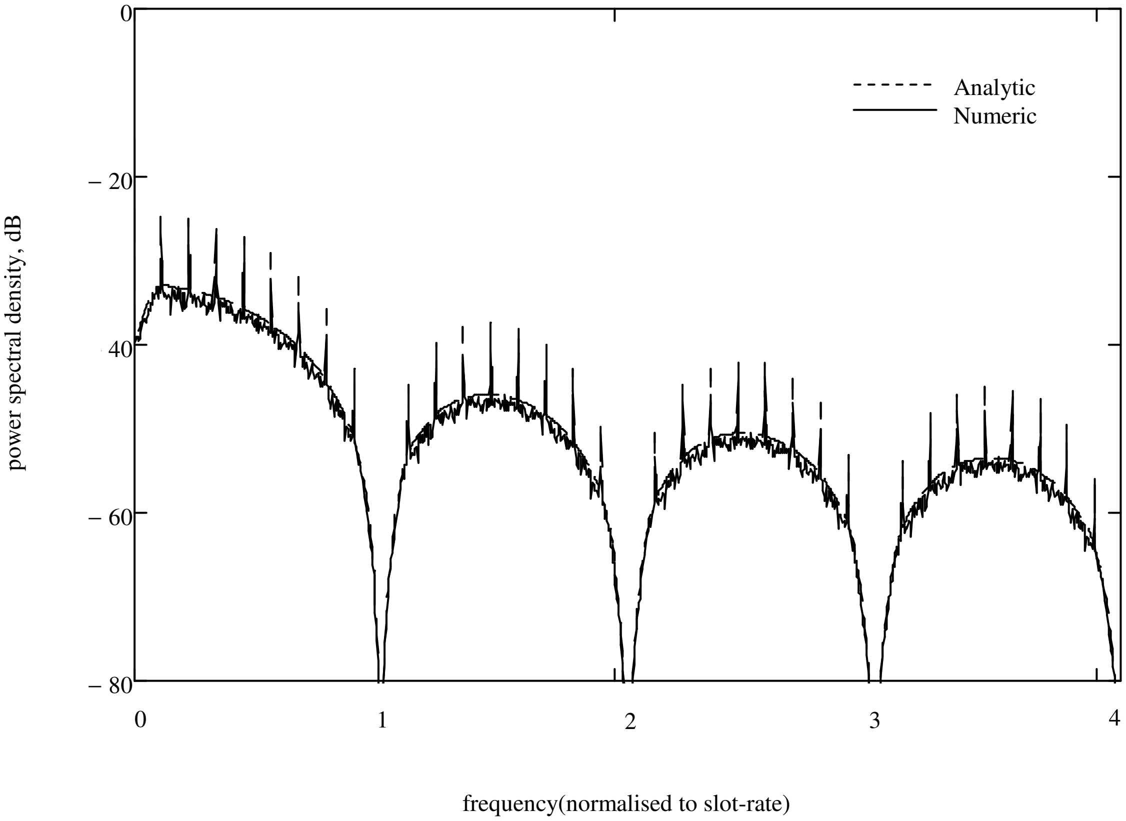

Fig. 1 Power spectral density of SPPM with $n = 9$ and $t_p = t_s$

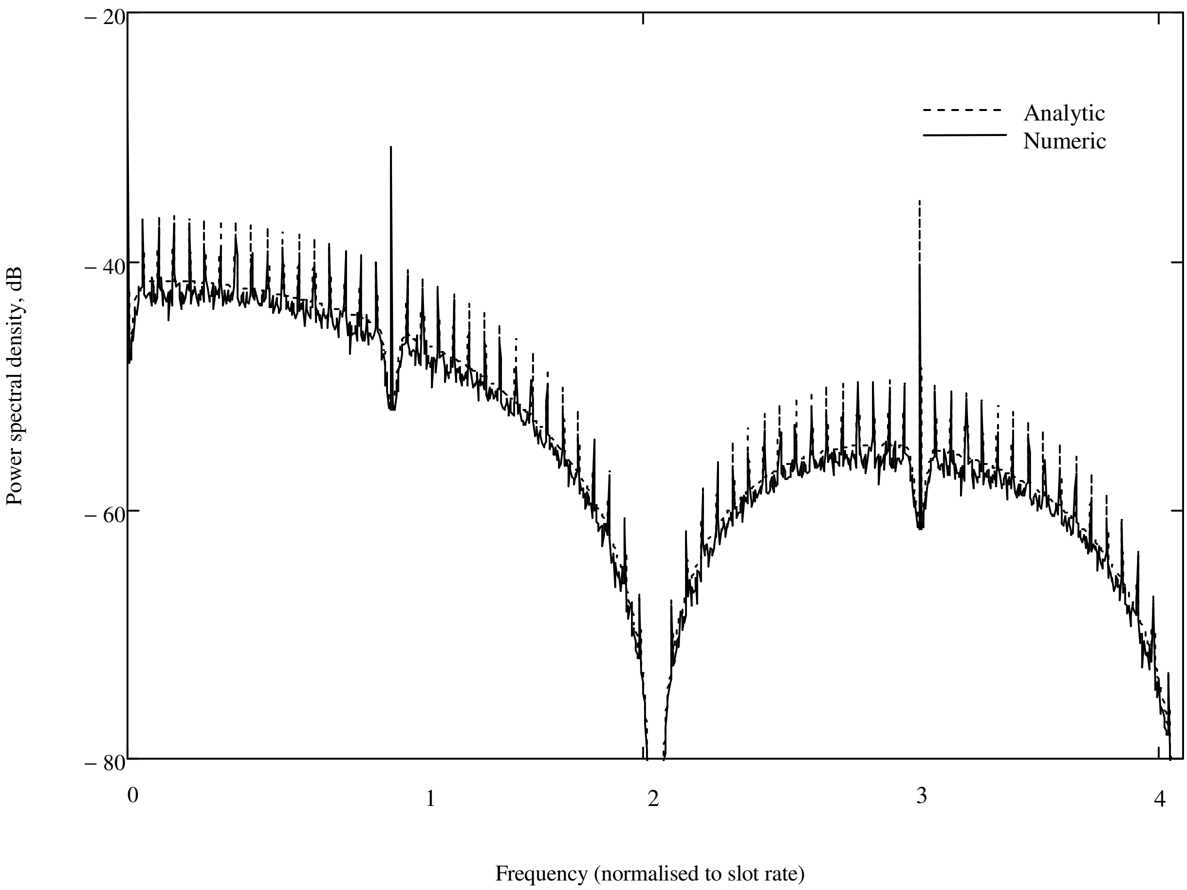

Fig. 2 Power spectral density of SPPM with $n = 17$ and $t_p = \frac{t_s}{2}$