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THE USE OF TUNED FRONT END OPTICAL RECEIVER AND PULSE POSITION MODULATION

KHALED TAHA

A thesis submitted to the University of Huddersfield in partial fulfilment of the requirements for the degree of Doctor of Philosophy

University of Huddersfield

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Abstract

The aim of this work is to investigate the use of tuned front-ends with OOK and PPM schemes, in addition to establish a theory for baseband tuned front end receivers. In this thesis, a background of baseband receivers, tuned receivers, and modulation schemes used in baseband optical communication is presented. Also, the noise theory of baseband receivers is reviewed which establishes a grounding for developing the theory relating to optical baseband tuned receivers.

This work presents novel analytical expressions for tuned transimpedance, tuned components, noise integrals and equivalent input and output noise densities of two tuned front-end receivers employing bi-polar junction transistors and field effect transistors as the input. It also presents novel expressions for optimising the collector current for tuned receivers. The noise modelling developed in this work overcomes some limitations of the conventional noise modelling and allows tuned receivers to be optimised and analysed.

This work also provides an in-depth investigation of optical baseband tuned receivers for on-off keying (OOK), Pulse position modulation (PPM), and Di-code pulse position modulation (Di-code PPM). This investigation aims to give quantitative predictions of the receiver performance for various types of receivers with different photodetectors (PIN photodetector and avalanche photodetector), different input transistors (bi-polar junction transistor BJT and field effect transistor FET), different pre-detection filters (1st order low pass filter and 3rd order Butterworth filter), different detection methods, and different tuned configurations (inductive shunt feedback front end tuned A and serial tuned front end tuned B). This investigation considers various optical links such as line of sight (LOS) optical link, non-line of sight (NLOS) link and optical fibre link.

All simulations, modelling, and calculations (including: channel modelling, receiver modelling, noise modelling, pulse shape and inter-symbol interference simulations, and error probability and receiver calculations) are performed by using a computer program (PTC Mathcad prime 4, version: M010/2017) which is used to evaluate and analyse the performance of these optical links.

As an outcome of this investigation, noise power in tuned receivers is significantly reduced for all examined configurations and under different conditions compared to non-tuned receivers. The overall receiver performance is improved by over 3dB in some cases. This investigation provides an overview and demonstration of cases where tuned receiver can be optimised for baseband transmission, offering a much better performance compared to non-tuned receivers. The performance improvement that tuned receivers offers can benefit a wide range of optical applications. This investigation also addresses some recommendations and suggestions for further work in some emerging applications such as underwater optical wireless communication (UOWC), visible light communication (VLC), and implantable medical devices (IMD).

Keyword: Optical communications, Baseband receivers, Noise modelling, tuned front end, pulse position modulation (PPM).
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Chapter 1: INTRODUCTION

This chapter describes the current context of optical communications, research question, research objectives, research methodology, thesis structure, and the original contribution.
1.1 Optical communications

Around 2035, the entire radio frequency (RF) spectrum will be fully used according to the predicted mobile data traffic, number of cell sites and achievable spectral efficiency for the US [1], [2]. Unlocking the visible light spectrum, which is 1000 times wider than the entire RF spectrum and deploying indoor/outdoor optical wireless systems would be a necessity for the next generation communication technologies in order to alleviate the spectrum crunch [1]-[4]. Beside the spectrum crunch problem, Optical wireless communications (OWC) has vastly attractive features include [5]-[8]:

- Cost-effective in terms of the price per bit and reduced time-to-market.
- Virtually unlimited bandwidth for providing near-optimum capacity.
- A high degree of security and privacy against eavesdropping.
- No licensing requirements or traffics for its utilisation.
- Low power consumption and reduced interference.
- A green technology with high energy efficiency.
- High scalability and re-configurability.
- Supporting high speed applications.
- Covering extensive link range.

OWC is covering ultra-violet (UV), infra-red (IR), visible light communication (VLC), free space optics (FSO) communications, which can be used in both the indoor and outdoor environments including underwater [9]. OWC systems are being developed and installed for several applications such as, but not limited to, urban networks [10], high speed ground to train networks [11]-[13], last-mile FSO links [14], high speed indoor links [15]-[16] and indoor VLC systems [17]-[22]. Most of OWC systems reported are based on the intensity modulation/direct detection (IM/DD) scheme. Coherent scheme in OWCs leads to improved channel usage but at the cost of increased system
complexity compared to IM/DD. The implementation of DD is simple and uses low-cost transceiver
devices without the need for complex high frequency circuit designs compared to coherent systems
[5], [23].

Moreover, Deep space exploration is also as important as other applications, it is a key means for
humans to investigate the Earth, solar system, and universe. This exploration may eventually reveal
the origins and evolution of the universe and enable exploration of inhabitable space. Deep space
exploration exhibits additional challenges, compared to near earth satellites, the longer distance and
transmission delays, further signal attenuation, and more complex environments. Optical
communications meet the requirements of the future planetary telemetry technologies such as
synthetic aperture radar, multispectral/hyperspectral imaging, and high definition video
communication. Although heterodyne detection offers a superior performance, IM/DD is preferred
due to its simplicity and low cost which makes it practically more favourable [24], [25].

Nevertheless, Optical transmission system became an essential in some modern applications, in brain
implantable neuroengineering applications, infrared light pulses provide a low interference solution
for implantable data transmission. The use of other communication links in such application is limited
to either the power consumption as in ultrasonic transmission, interference and high absorption ratio
as in radio frequency (RF) transmission, or the high possibility of infection as in wired transmission
[26].

In the following sections, some optical communication links are further discussed. The aim of
providing this background information is to demonstrate the emerging optical communication
applications and establish a ground for the research problem that this thesis addresses.

1.1.1 Free-space optical communications

Free space optical communication (FSO) refers to an optical communication system which utilises
modulated laser beams to transmit information through free space. Compared with the rapid
progression of optical fibre communication, FSO research and applications seemed to be declined for a long period of time during which the laser technologies have a long term of development. In recent years, the reckless advancement in the laser technique has decreased the financial cost, qualifying the FSO to become a suitable alternative for various applications [27]-[30]. Unlike the RF wireless communication, FSO has less restrictions for frequency license, electromagnetic interference (EMI), and bandwidth. It also has several advantages over optical fibre communication such as cheaper installations and flexible networking process.

Furthermore, it has the adaptability for unexpected high capacity communication situations where optical fibre system is not applicable; FSO system is an alternative in some specific scenarios such as outdoor military operations (known as tactical FSO) and short-haul communications between buildings, rivers, highways. [31]-[37]. There are other scenarios where FSO can be used such as [38].
• LAN to LAN connections on campuses or in cities at fast ethernet or gigabit ethernet speeds.
• Providing high speed connection in disaster recovery and emergency response.
• Ship to ship communications with high data rates and security.
• Satellite constellation.
• Space communications (near earth, interplanetary, and interstellar communication).

Figure 1-2 Deep-space optical communications

Deep space laser communication (lasercomm links) is a wireless communication method for transmitting images, video, and sound between deep space explorers and the Earth. These laser communication links provide various benefits, including high data rates, enhanced security, higher reliability, and more powerful networking flexibility. Their communication terminals are small, lightweight, and have low power consumption. Compared with microwave communication, laser communication operates on higher carrier frequencies and provides lower diffraction losses, better directivity, and greater transmission efficiency. It can thereby achieve high transmission rates and outstanding communication performances with lower transmitting power and smaller antenna sizes. [39]-[43]. However, Free space optical communication performance is limited by the constant presence of optical turbulence in the atmospheric channel. Absorption and scattering of the laser beam
are also atmospheric effect that degrade FSO. Also, a clear line of sight path between the laser transmitter and the receiver is one of this communication system requirements [44].

Research of FSO systems is focussed on channel model, forward error correction (FEC) and modulations. These three research areas are closely related when analysing the full system performance where the concern is given for energy efficiency, spectral efficiency and bit error rate. Channel model is the premier investigation as FEC and modulation schemes are selected based on the characteristics of channel model and system requirements. Modulation schemes can be described as baseband FEC encoded bits are converted into baseband symbols. As the main concern is the energy efficiency, Pulse position modulation (PPM) schemes take the dominant possession [45]-[55]. The energy efficiency refers to the ratio of the pulse energy and the bit energy. In traditional non-return-to-zero (NRZ) non-PPM, energy efficiency is defined as 1. While, energy efficiency of PPM schemes is commonly higher than 1 that makes PPM superior to the traditional intensity/phase modulation approaches [56].

1.1.2 Underwater optical wireless communication

Underwater wireless communication serves industry, military and the scientific community. It contributes in tactical surveillance, pollution monitoring, oil control and maintenance, offshore explorations, climate change monitoring, and oceanography research. Compared with RF and acoustic communications, optical communication provides much higher transmission bandwidth and date rate [57]. In recent years, Underwater optical wireless communication (UOWC) has been proposed for environmental monitoring, offshore exploration, disaster precaution, and military operations [58]. As laser sources enable highly efficient optical communications links due to their ability to be focused into very directive beam profiles, experience of developing FSO links demonstrate robust laser communications links at high rate with techniques that can be developed for the undersea environment [59], [60]. Although optical communication has several advantages over RF ad acoustic counterparts, UOWC is still a challenging task as it suffers from absorption and
scattering attenuate the transmitted light signal and cause multi-path fading. Also, Misalignment of optical transceivers and random movements of sea surface cause serious connectivity loss problem [61]-[63].

\[\text{Figure 1-3 Link configurations of UOWC (a) point to point LOS configuration. (b) Retroreflector-based LOS configuration. (c) Diffused LOS configuration. (d) NLOS configuration. [64]}\]

Based on underwater link, there are four link configurations [64]-[67]:

a) Point-to-point line-of-sight (LOS) configuration. This configuration is the most common in UOWC. It requires precise pointing between transmitter and receiver that limit the system performance in turbulent water environments.

b) Retroreflector-based LOS configuration, it is a variation of point to point LOS configuration. Since reflector has no laser, its power consumption, volume and weight are reduced. Therefore, it is used in limited power and weight budget scenario. As optical signal goes through water twice, it suffers from additional attenuation. Also, the backscatter of the transmitted optical signal interferes with the reflected signal which degrade the system performance.
c) Diffused LOS configuration, short communication distances and lower data rates are the two major limitations of this configuration. Since this link configuration employs a power LED with large divergence angle to broadcast from one node to multiple receivers, it relaxes the requirement of prise pointing. However, the link suffers from aquatic attenuation due to large interaction area with water.

d) Non-line-of-sight (NLOS) configuration overcomes the pointing restrictions of LOS configuration, however, it severs from huge signal dispersion due to the random sea surface slope caused by wind and other turbulence sources.

Figure 1-4 A typical laboratory LOS UOWC system based on (IM/DD) technique [58], [69].

Most of UOWC systems are experimental demonstrations and prototypes in a laboratory environment. The configuration of LOS UOWC link is used in most of UOWC experimental systems due its simplicity, a typical laboratory system shown in Figure 1-4 which is similar to The FSO communication setups (information source, light source, optics, photodetector, and optical receiver) [68]. However, a water tank or pipe is used to simulate the underwater transmission conditions [69].
UOWC is considered as an FSO communication system in underwater environment therefore the conventional intensity modulation techniques used in FSO are compatible with UOWC systems. On-off keying (OOK) modulation is used in UOWC in both forms: return to zero RZ-OOK and non-return to zero NRZ-OOK formats [7072].

Table 1-1 comparison of common UOWC modulation schemes [58]

<table>
<thead>
<tr>
<th>UOWC modulations</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>OOK</td>
<td>▪ Simplicity.</td>
<td>▪ Low energy efficiency.</td>
</tr>
<tr>
<td></td>
<td>▪ Low cost.</td>
<td></td>
</tr>
<tr>
<td>PPM</td>
<td>▪ High power efficiency.</td>
<td>▪ High timing requirements.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>▪ Low bandwidth utilisation rate.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>▪ Complex transceivers.</td>
</tr>
<tr>
<td>DPIM</td>
<td>▪ High bandwidth efficiency.</td>
<td>▪ Error spread in demodulation.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>▪ Complex demodulation devices.</td>
</tr>
<tr>
<td>PSK</td>
<td>▪ High receiver sensitivity.</td>
<td>▪ High implementation complexity.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>▪ High cost.</td>
</tr>
</tbody>
</table>

OOK modulation requires dynamic threshold (DT) techniques to be used underwater environment, DT is determined based on the estimation of channel fading [68]. The major drawbacks of OOK are the low power and bandwidth efficiency, however, it is still the most popular IM in UOWC due to its simplicity. Pulse position modulation (PPM) scheme is also used in UOWC due to its high energy efficiency and it does not require dynamic thresholding. However, this advantage comes at the expense of lower bandwidth utilisation rate and more complex transceivers. Also, PPM has strict timing synchronisation requirements. [73]-[81]. The performance of other modulation schemes, such as PSK [8284] and DPIM [85]-[87], has been evaluated for UOWC link. Performance comparison of these modulation schemes are shown in Table 1-1. Similar to any other communication link, the selection of light source (LED or LD) and optical detector (PIN and APD) depend on the link
requirements. Therefore, the performance of the communication link is determined by the optical
devices technologies and materials, and the link configuration [79], [8895].

1.1.3 Visible light communications

Visible light communication (VLC) is an emerging field of optical communications that focuses on
the visible part of the electromagnetic spectrum. VLC has gained interest due to its availability and
the ease at which it can be modulated using LEDs. LEDs are inexpensive, fast, and are widely adopted
in lighting. LED light sources appear in many applications in lighting and display including traffic
lights, flat panel displays, and instrumentation. VLC uses light that is intensity modulated to transmit
data. In this manner, LEDs are fast enough that can be intensity modulated in order of MHz.
Transmitted signals are detected as intensities via direct detection using a photodetector that can be
very cheap. Modulation formats in VLC vary extensively, and their optimisation under different
application scenarios is an active area of research [95100].

Applications of VLC include internet of things (IOT) [101]-[105], wireless Internet access [106107],
vehicle to vehicle communications (V2V) [107]-[111], machine to machine communications (M2M)
[112]-[114], indoor positioning systems (IPS), and navigation [115]-[120]. VLC systems demonstrate
great advantages over other communication techniques in various applications:

- VLC can be implemented as LOS configuration, and as NLOS configuration without
  interference to an existing RF network.
- VLC can be applied as a practical alternative in environments where RF signals are
  perceived as a hazard.
- VLC also provides an accurate and more easily deployed indoor positioning and localisation
  compared with RF and acoustic counterparts.
- Indoor Light-based systems provide improved security.

Outdoor VLC applications are also utilised in several areas such as traffic light signals, public
illumination systems, and underwater communication [121]-[125].
1.1.4 Optical communications in biomedical applications

Biomedical implants require communication links to communicate data and transmit power. Implantable medical devices (IMD) is vital tool that benefits medical doctors, researchers, life sciences and mankind as these devices are used to restore lost function, treat disorders, or monitor biological parameters. This area of research has a significant impact on human life, health, and body function studies [126]-[128]. There are different types of communication systems used in IMD, these systems range from wired baseband systems to wireless transceivers. Nowadays, most IMDs target wireless data transmission systems because of many reasons such as skin inflammation and risk of infections [129].

![Figure 1-5 Block diagram of a neural recording system with optical wireless telemetry link.](image)

The optical wireless link is a good candidate where data and power are transmitted by light [130][133]. A photovoltaic cell placed on implant side is used to capture external lights attenuated by the human
skin, which is a technique used to transmit power to internal device. For data transmission, a baseband modulated data is transmitted through human tissue [134136].

An example of IMD used in neuroscience applications is shown in Figure 1-5, this device is used to measure the extracellular electrical signal from the cortical area of the brain. These signals can be used to manipulate external life aid devices and develop brain machine interfaces (BMIs) which improve the quality of life of the patients with neurological diseases [130], [137]. Such a device should feature a small size, low interference, large bandwidth, and low power consumption. Optical communication link offers these features; however, it is very sensitive to misalignments between transmitter and receiver. Alignments requirements can be relaxed by increasing the tissue thickness at the cost of higher absorbed and scattered light in the thicker tissue. This technique increases the power consumption as more photons are required to maintain a valid communication link. The current research aims to develop modulation techniques, receiver configurations, and opto-electronic technologies that can enhance the link performance to allow higher bit rate transmissions, less pulse energy (photons per pulse) hence reduced power consumption [131138].

1.1.5 Optical fibre communications

Coherent detection of amplitude and phase enables multipoint modulations to be applied digitally. The current state of the art optical coherent transceivers uses phase and amplitude modulation, and polarization multiplexing on the transmitter side. At the receiver, it employs coherent detection, digital signal processing (DSP), and high performance forward error correction (FEC). Fibre capacity, network cost, network engineering simplicity, port density, and power consumption are all factors that influence technology selection for network operators building modern optical network. Binary phase shift keying (B-PSK), quadrature phase shift keying (Q-PSK), and 6-quadrature amplitude modulation (QAM) are the main commercial modulations, allowing 50 Gbit/s to 200 Gbit/s. The applications for these modulations and data rates include submarine links, terrestrial long-haul
systems and regional networks. As the number of transported bits increases, coherent transceivers are successfully implemented to lower the network cost per bit [139].

On the other hand, plastic optical fibre (POF) gained an increasing interest for use in short-range optical communications for in-home and automotive networks due to its low-cost, ease of installation and handling, and low weight. POF provides unique capabilities for these applications where long link lengths and high bit-rates are not required [140]-[145]. In automotive applications, optical solutions are used to solve complex issues such as integration of systems and avoiding electromagnetic compatibility (EMC) of the cables [146]. In addition, POF is classified as an appropriate optical cable to be used in harsh environments like automotive and factory industries due to its robustness and cost effectiveness [147]. Media oriented systems transport (MOST) is a high-speed multimedia network technology used by many car manufacturers. With a data rate of 150 Mbit/s over POF (released in 2012), MOST has been installed in more than 180 vehicle models during the period from 2012 to 2017. Currently, Gigabit Ethernet over plastic optical fibre (GEPOF) is also a fibre optic solution which is being developed for 1 Gbit/s communication in automotive [148].

Recently, there is an interest in a hybrid POF/VLC, which is a technology focuses on providing wired and wireless indoor optical communication. POF link is used to provide a backhaul link that connects VLC to the access network [149]-[153]. Also, Several modulation schemes have been investigated to optimise the performance and the reliability of the POF communication link in many applications [154]-[159].

1.2 Research problem

The emerging optical communication systems discussed in Section 1.1 share an important feature which is the sake of improving the communication link. The link performance of each system is critical in regard to its application. Some of which is concerned with the cost issue, link distance, bit rate, power efficiency, ease of handling and installation. Some other applications are focused to meet
exceptional link restrictions, installations in harsh or hazard environments. Research directions of these optical systems are not limited to one area; the areas of research include: photonic materials, modulation schemes, receiver design and detection techniques, analogue and digital signal processing, forward error correction, data rates, etc. Therefore, the selection of sub-system components (such as opto-electronic, modulation scheme, data rate) is subject to the application requirements.

For instance, Implantable biomedical systems need to transmit data through skin to achieve high accuracy measurements, high dimensionality and real-time control of complex prosthetic devices. These systems, as any other communication system, require a link with high data rate (in order of few hundred Mbit/s), low power consumption, low bit error rate, and good electromagnetic compliance.

In optical wireless biotelemetry links, a large area photodiode is required to collect more scattering photons and maximise the transmission efficiency through human tissue. As the junction capacitance of a photodiode affects the signal to noise ratio and the achievable bandwidth, increasing the size of the photodiode is critical in a way that the whole receiver input referred noise power is also increasing. For such a case, reducing the receiver noise is fundamental to achieve an overall performance improvement.

Also, in FSO systems, pulse energy efficiency is a main concern that modulations such as pulse position modulation is more compatible due to its power efficiency. Other systems (such as underwater unmanned units and offshore monitoring units) are battery operated in which the power consumption and pulse energy are also main concerns. Underwater optical wireless communication (UOWC) modulations such as on-off keying (OOK) and PPM are used due to their simplicity, cost effectiveness, and power efficiency.
1.2.1 Tuned front end

The use of tuned front end is reviewed in section 3.1. It is shown that the tuned front end is better than none-tuned in most optical communication systems where there are high frequency components associated with the detection of the signal in both coherent and direct detection receivers. The key role of the tuned front end is that the input stage of the optical receivers is tuned to a band of frequencies, same as the concept of the television and radio receivers, which contain the band of desired frequencies of the transmitted signal or carrier. This has an impact on the thermal noise at the front end prior to the pre-amplifier. It has been shown that the input noise at this stage has been significantly reduced. Another important property of the tuned front end is that the noise owing to photodetector is significantly reduced; especially avalanche photodetectors based front ends when the random fluctuation of the multiplication process generates an extra and unavoidable noise. It also reduces the high bias voltage required for APDs and lowers the optimum gain value.

This work proposes using a tuned resonant circuit in the front end of the ID/DD receivers that detect OOK or PPM coded signalling. The total receiver noise power will be reduced so increasing the signal to noise (SNR). One problem with a tuned circuit is that it rings for some time afterwards during which time it dissipates stored energy. This ringing causes Inter-Symbol-Interference (ISI) when detecting NRZ data and is the reason why the technique is not used.

With tuned front-end originally proposed for optical heterodyne receivers, the practical designs and theoretical analysis of tuned front end reported previously are not showing a theory that covers the operation of baseband transmission that can be implemented for either OOK or PPM. The existing baseband signalling detection techniques are based on the use of non-tuned front-end receivers. For example, the conventional baseband noise model (Personick integrals) defines the receiver transfer function in terms of the input and output pulse shape, independent of the preamplifier circuit. In this theory, the receiver is a first order approximation which makes this noise model not valid over other conditions. Although other baseband noise models offer noise expressions that take the preamplifier
in consideration, the preamplifier is expected to be non-tuned (single pole RC front end or second order GHz preamplifier). Therefore, noise modelling of baseband tuned front end receiver does not seem to be a straightforward task.

1.2.2 Research proposal

For tuned front end receiver to be used in baseband transmission, the receiver frequency response should be optimised in accordance to baseband transmission requirements. This will require an investigation of inter-symbol interference (ISI) to determine whether the use tuned front end degrades or improve the overall receiver performance. In addition, a valid noise model should be developed in order to estimate the difference in receive noise. The accurate modelling of tuned front-end receiver will help in predicting the overall receiver performance improvement. Any improvement in receiver performance will benefit a wide range of optical communication systems such as these mentioned in Section 1.1.

1.3 Research objectives and investigation overview

The aim is to investigate the use of tuned front-end with OOK and PPM schemes, in addition to establish a ground theory for baseband tuned front end receivers. The objectives of this investigation are as follows:

- Review the current context of optical communications.
- Review the theory of baseband receivers: photodetectors, pre-amplifier configurations, detection and filters, and receiver performance metrics (such as noise, noise equivalent bandwidth, bit error rate, and ISI).
- Review the related work of tuned receivers used in optical communications in order to find the suitable tuned circuit configurations.
• Review the basics of tuned circuits and tuned amplifiers. This will help to examine the frequency response of tuned front end and find a design approach that is suitable for baseband receivers.

• Review the conventional noise model of baseband receivers, and the input referred noise procedures of non-tuned receivers. Also, review the noise analysis of excising tuned receivers previously used in coherent systems. This will help in proposing a valid noise model for baseband tuned receivers.

• Review related work of pulse position modulation: theory, applications, detection, receiver design, receiver performance.

• Build a mathematical model that accurately simulate OOK and PPM systems: channel modelling, receiver modelling, noise modelling, input/output signal modelling, error probability and receiver sensitivity.

• Investigate the use of tuned front-end receivers with NRZ signalling, study the inter-symbol interference characteristics of output tuned pulse shapes and propose optimisation for tuned front end to overcome the ringing effects.

• Examine the tuned front-end response considering line of sight (LOS) and optical fibre links, assuming square input pulses for LOS and Gaussian input pulses for optical fibre.

• Examine the tuned receiver performance with different photodetectors (PIN and avalanche photodetectors) and different input transistors (Bi-polar junction transistors BJT and field effect transistor FET).

• Examine the effect of the pre-detection filter on tuned front end, considering 1st order low pass filter and 3rd order Butterworth filter. In addition to finding the optimum receiver bandwidth (noise/ISI trade off).

• Evaluate the tuned front-end receiver performance in term of noise, ISI and receiver sensitivity (for OOK and PPM).
1.3.1 Investigation overview

Investigation overview is illustrated in Figure 1-6.

There are three optical systems considered in this investigation: optical fibre communication, optical wireless communication with a line to sight characteristics (OWC-LOS), and optical wireless communication with non-line to sight characteristics (OWC-NLOS). The main reason behind
choosing these systems is that each system represents an optical channel. Accordingly, there will be three different optical input pulses: square pulse, Gaussian pulse and convolved pulse. These pulses are assumed to be the received optical pulse at the receiver input. Square pulse is a good approximation for an ideal channel. It is often used in line to sight systems, it may also be used in single mode optical fibre links. However, Gaussian pulse is preferable in optical fibre links due to dispersive nature of optical cables. Convoluted pulse is a more realistic pulse for diffusion links such as in VLC and NLOS optical wireless.

Figure 1-7 investigation stages
Figure 1-7 illustrates the investigation stages which are divided into system design, system modelling and system evaluation:

- System design involves designing different sub-system components such as a transmitter, receiver and an optical link.
- System modelling involves channel, received pulse shape, receiver electronics, output pulse shape, noise, error probability statistics, and receiver sensitivity.
- System evaluation involves receiver noise, inter-symbol interference, bit error rate, and receiver sensitivity.

1.3.2 System design

The contribution of this work is towards the receiver design of pulse position modulation system. Therefore, the system design will only involve receiver design since channel modelling provides the sufficient information about the received pulse. The design process involved in this study is illustrated in Figure 1-8.
Receiver design sub-components are illustrated in Figure 1-9. There are four different front-end input configurations: PIN-FET, PIN-BJT, APD-FET, and APD-BJT, in addition to the transimpedance amplifiers:

- Tuned A
- Tuned B
- Conventional TIA (non-tuned).

![Figure 1-9 Receiver design](image)

### 1.3.3 System modelling

The transfer function of the receiver is given in Section 3.4 for conventional front-end receiver and tuned front-end receiver. Noise expressions are explained and given in Section 2.5.2 and Section 3.5 for both receivers. For OOK receiver modelling, error probability expressions are given in Section 2.5.5. For pulse position modulation, error probability expressions are given in Section 4.3. Channel characteristics and optical link system will be discussed for each individual optical link in Chapter 6, 7. Since an analytical formulation can be derived for the systems under test, the performance of these systems can be evaluated using a mathematical modelling.
The TIA circuitry is altered, therefore, there will be an improvement to the conventional receiver noise model. This further explained in Section 5.4. Figure 1-10 indicates the contribution to the standard system modelling.

![Diagram of system modelling](image)

**Figure 1-10 System modelling (red indicates the contribution).**

System model presented in this work has the following features:

- It helps identify design trade-off. The model predicts the behaviour of a given receiver design under various operating conditions such as varying bit rate, modulation scheme, optical input power, input transistor technology, front-end configuration, photodetector size and type, pre-detection filter, and detection method.

- It can be used to study the impact of new technologies on system performance. The designer can explore the impact of the limits of current electronic and optoelectronic technologies on the link performance and suggests directions for technology and performance improvements.

- It can be integrated into a system level design tool that supports a multi-level and multi-technology simulation.

- The most important feature is that it takes into account the physical realisation of receiver components. PPM models do not take noise into account particularly well, which means that no real noise analysis can be carried out considering the physical realisation of the front end.
1.3.4 System evaluation

In order to be able to evaluate and optimise link performance criteria correctly, a clear definition of the performance metrics is required. The aim is to establish the overall power requirement for an optical link at a given data rate and BER. The calculation is conditioned by the receiver performance since the BER defines the lower limit for the received optical power.

![Evaluation Diagram]

**Figure 1-11 Evaluation stage**

Each receiver is evaluated in term of total noise performance, inter-symbol interference, bit error rate and receiver sensitivity. Digital system efficiency is specified in terms of the number of bits carried per Hz of occupied bandwidth. It could also be specified in terms of the number of photons required to transmit a bit of information at a specified bit error rate. Therefore, number of photon per pulse $b$ for a given bit error rate is a measure used to evaluate different receivers performance. Optical receiver sensitivity is then defined as the minimum optical power, averaged over time, necessary to achieve a specified BER.

1.4 Thesis structure

**Chapter 2.** Theory of optical receivers, introduces the conventional theory of baseband optical receivers. The structure of an optical receiver model used in baseband detection is presented in Section 2.1. In Section 2.2, the basics of two types of photodetectors, PIN photodetector and avalanche photodetector (APD) are presented. Section 2.3 introduces the common receiver front-end design configuration: low input impedance, high input impedance, and transimpedance amplifiers. In
Section 2.4, important pre-detection filter related topics are discussed and explained. In Section 2.5, other receiver fundamentals are discussed such as: noise in optical communications; noise equivalent bandwidth; input referred noise; the relation between signal; noise and bit error rate; and electrical and optical sensitivity.

Chapter 3, Optical tuned front-end receivers, starts with a review of the use of tuned front end in optical communication. In Section 3.2, the fundamentals of the tuned circuits are discussed. Topics included are basics of serial and parallel tuned amplifiers and pulsed tuned circuits. In Section 3.3, bandwidth extension in broadband receivers is discussed. A brief analysis of magnitude and phase response, components value, and bandwidth extension ratio (BWER) is provided in this section. In Section 3.4, original expressions are presented for two tuned front-end transimpedance amplifier (TIA) designs. In Section 3.5, an original noise analysis of tuned front-end is presented.

Chapter 4, Pulse position modulation, provides a brief discussion of pulse modulation., PCM and PPM theory is presented. Section 4.1 includes a literature review of the use of digital PPM in optical communications. In Section 4.2, a review of optimum and suboptimum detection of digital PPM is presented. In this section, there is a review of the theory of existing PPM optical receiver. The error probabilities analysis is explained in Section 4.3.

Chapter 5, System modelling, presents explanations of the mathematical models and calculations performed in this investigation.

Chapter 6, Results and discussion I (tuned front end optical receiver performance), presents the numerical results of an ideal optical channel. It gives an abstraction of tuned front-end performance away from channel effect. Results are obtained for tuned and non-tuned front end with different input configurations. The modulation scheme considered for this system is the on-off keying with non-return to zero signalling (OOK-NRZ). Section 6.1 provides a brief introduction to the full communication system. In this section, the simulation procedures are provided and explained in detail.
Chapter 7, Results and discussion II (tuned PPM receiver performance), presents the performance of tuned PPM receiver compared to non-tuned receiver in optical fibre and optical wireless links. The reason behind choosing these communication systems is to take into account the non-ideal optical channel effects. The modulation schemes considered are PPM and di-code PPM. The comparison is based on sub-optimal detection. In particular, the raised cosine filtering, since it is considered as a more practical and simple in term of receiver structure. In this chapter, the comparison criteria are limited to certain scenarios since tuned receivers are extensively investigated, and the performance advantages of tuned receivers compared to non-tuned are well established in Chapter 6.

1.5 Original contribution

1. Developed an algorithm for choosing the values of tuned front-end components based on central frequency, timed constant and transfer function optimisations. Transfer function approach is based on optimising the frequency response of the tuned receivers. Novel Expressions of two tuned transimpedance amplifiers are developed that allow tuned front end to be suitable for NRZ baseband transmission. (Chapter 3).

2. Developed expressions for noise bandwidth, noise transfer functions, and noise integrals; novel analytical expressions for noise integrals and equivalent input and output noise densities of two tuned front-end receivers employ Bi-polar junction input transistors and field effect input transistors; drive the optimum collector current expressions for tuned front end receivers employ Bi-polar junction input transistors. (Chapter 3).

3. Developed a noise analysis model for optical baseband receivers. The validity of this model is verified by comparing noise obtained from the conventional noise model and the developed model, the advantage of this model is that it can be used for non-tuned and tuned front end receivers, in addition to; novel noise modelling based on the novel expressions for optical baseband tuned receivers. A computer program (PTC Mathcad prime 4, version: M010/2017) is used to verify, validate and document calculations. (Chapter 5 and appendix A).
4. In-depth investigation of the performance of tuned receivers with on-off keying modulation (Chapter 6 and Appendix B):

- Investigation include different photodetectors (PIN photodetector and avalanche photodetector), different input transistors (Bi-polar junction transistor BJT and field effect transistor FET), different pre-detection filters (1st order low pass filter and 3rd order Butterworth filter), and different tuned configurations (inductive shunt feedback front end tuned A and serial tuned front end tuned B).

- The performance of tuned receivers is examined considering three different avalanche photodetector materials (Silicon APD, InGaAs APD, and Germanium APD) in order to evaluate the receiver performance for different avalanche gains and photodetector noise factors.

- Analysed inter-symbol interference of on-off keying modulation with tuned receivers.

- Optimised tuned receiver 3-dB bandwidth with 1st order low pass filter and 3rd order Butterworth filter.

- Original noise analysis of optical baseband tuned front-end receivers.

- All simulations and modelling are performed by using a computer program (PTC Mathcad prime 4, version: M010/2017) which is used to evaluate and analyse the performance of tuned receivers.

5. In-depth investigation (design, optimisation and evaluation) of the performance of tuned receivers with Pulse position modulation schemes (Chapter 7 and Appendix C):

- Investigation includes different modulation schemes (digital Pulse position modulation “DPPM” and di-code pulse position modulation “DiPPM”), different photodiodes (PIN photodetector and avalanche photodetector), different input transistors (bi-polar junction transistor “BJT” and field effect transistor “FET”), different pre-detection filters (1st order low pass filter and 3rd order Butterworth filter,
matched filter), and different tuned configurations (inductive shunt feedback front end “tuned A” and serial tuned front end “tuned B”).

- Investigated the performance of tuned PPM receivers in slightly and highly dispersive optical fibre channels (Gaussian input pulses for optical fibre links, considering different fibre bandwidth).

- Investigated the performance of tuned PPM receivers in diffuse optical wireless links (convoluted input pulses for non-line of sight optical wireless link).

- Investigated the performance of tuned PPM receivers in line of sight optical link (square input pulses for ideal optical channel).

- All simulations and modelling are performed by using a computer program (PTC Mathcad prime 4, version: M010/2017) which is used to evaluate and analyse the performance of optical links (Ideal channel, fibre channel and wireless channel).
Chapter 2: THEORY OF OPTICAL RECEIVERS

This chapter introduces the conventional theory of baseband optical receivers. The structure of an optical receiver model used in baseband detection is presented in Section 2.1. This model is used to explain relevant fundamentals of baseband receivers in optical communications. In Section 2.2, the basics of two types of photodetectors, PIN photodetector and avalanche photodetector (APD) are presented. Section 2.3 introduces the common receiver front-end design configuration: low input impedance, high input impedance, and transimpedance amplifiers. In Section 2.4, important pre-detection filter related topics are discussed and explained. In Section 2.5, other receiver fundamentals are discussed such as: noise in optical communications; noise equivalent bandwidth; input referred noise; the relation between signal; noise and bit error rate; and electrical and optical sensitivity.
2.1 Baseband Receiver Model

A basic receiver model used here is shown in Figure 2-1. This model is used to explain relevant receiver fundamentals. Later in other sections, this model is extended to include additional blocks. The receiver model consists of a photodetector (PD), a linear channel that includes the transimpedance amplifier (TIA), the main amplifier (MA), a low-pass filter and a binary decision circuit with a fixed threshold $V_{TH}$. The detector model includes a signal current source and a noise current source. The characteristics of these signal current sources are discussed in Section 2.2 for the PIN photo detector and the avalanche photo detector (APD).

![Figure 2-1 Basic receiver model.](image)

The signal current is linearly related to the received optical power and the noise current spectrum is approximately white and signal dependent. A complex transfer function can be used to model the linear channel. It represents the relation between the amplitude and phase of the output voltage to those of the input current. This transfer function can be decomposed into a product of three transfer functions - the transimpedance amplifier, filter and main amplifier. The main amplifier usually has a relatively large bandwidth that lets signal pass through, thus the response of this block is neglected. The noise characteristics of the linear channel are modelled by a single noise current source at the input of the pre-amplifier. Due to the nature of noise in optical receivers, receiver noise is mainly determined by the input-referred noise of the pre-amplifier so that noise from later stages can be neglected. The noise spectrum of the pre-amplifier is chosen such that after passing through the
noiseless amplifier, it produces the output noise spectrum of the actual noisy amplifier. The last block in this model, the decision circuit, compares the voltage at the output of receiver with a fixed threshold voltage. If the output voltage of the receiver is larger than the threshold, a one bit is detected. If it is smaller, a zero bit is detected. Although the receiver is modelled as a linear channel, this block is nonlinear. A clock signal is used as a trigger the decision circuit. This clock signal is provided by a clock recovery circuit.

2.2 Photodetectors

The main part of the optical receiver is the photodetector which acts as a demodulator converting the optical signal into an electrical signal. There are minimum performance requirements which the photodetector should have to perform this job. It should have a high sensitivity at the operating wavelength and a high fidelity that allows it to have a linear characteristic with the optical signal in the analogue systems. Also, high quantum efficiency is a necessary parameter to produce the maximum electric signal from the input optical power that enables the receiver to have a larger electric response to the input signal. Furthermore, as the optical bandwidth is increasing, it should have a short response time to obtain this bandwidth. The noise should be as low as possible in the photodetector and circuitry should have low noise. There are two main types of photodetectors that are commonly used, the PIN photodetector and the avalanche photodetector (APD) which are discussed in this section.

2.2.1 PIN photodiode

Figure 2-2 shows a reverse bias p-n photodiode. It also shows the depletion and diffusion region with the normal termination. The depletion region width depends on the doping concentration for a given reverse voltage. Thus, the width and absorption mechanism depends on the material of the photodiode where the photon may be absorbed by both the depletion and diffusion regions. It causes limitations for the response of photodiode due to electron generation from the two regions.
The output characteristics of the p-n photodiode are shown in Figure 2-3, in the case of no input till the high level of the input light.
PIN photodiodes have a different configuration whereby an n-type material is lightly doped as an intrinsic layer and a highly doped n-type layer is added to lower resistance. This modification allows the absorption to take place in the depletion region only. As shown in Figure 2-4, the absorption takes place in the depletion region P-I where the “n” is used as a low resistance contact without any absorption activity.

![Diagram of PIN photodiode with E field, depletion region, and absorption region labeled](image)

**Figure 2-4 Combined absorptions and depletion region of a PIN [160].**

The material used to fabricate a photodiode determines the amount of photocurrent $I_p$ which is produced because of the light incident signal into the semiconductor. As shown in Figure 2-5, each semiconductor has an absorption coefficient $\alpha_o$ which sets the relation between the optical power $P_0$ and the photocurrent $I_p$, this relation is given by [160]

$$I_p = \frac{P_0 q (1 - r)}{h \nu} [1 - e^{-\alpha_o d}]$$

(2-1)

where $q$ is the electron charge, $h\nu$ is the photon energy, $r$ is the Fresnel reflection coefficient, and $d$ is the width of the absorption region.
Figure 2-5 Optical absorption curves for common semiconductor photodiode material [160].

Figure 2-6 Responsivity $\mathcal{R}$ against wavelength $\lambda$ for the common materials of photodiodes [160].
As is clear from Figure 2-5, the relation between the material and the wavelength \( \lambda \) can affect the performance characteristics of the receiver in term of absorbing the optical signal and converting it into an electric signal. Therefore, it depends on the operating wavelength to choose the best material with an absorption coefficient which is suitable for a particular application. Quantum efficiency \( \eta \) is another parameter which depends on the absorption coefficient. It refers to the number of electrons delivered to the receiver circuitry divided by the number of the incident photons. Responsivity \( \mathcal{R} \) is another important parameter which is used to characterise the photodetector. Figure 2-6 shows the variation in \( \mathcal{R} \) for wavelength and common materials. \( \mathcal{R} \) is the relation between the input optical power \( P_o \) and the photocurrent \( I_p \). Since \( \mathcal{R} = \frac{I_p}{P_o} \), it is derived mathematically and given by \( \mathcal{R} = \frac{\eta e \lambda}{h c} \) which has a relation between Responsivity and Quantum efficiency at a particular wavelength [160].

### 2.2.2 Avalanche Photodiode

The second major type of optical communications detector is the avalanche photodiode (APD). As presented in [160], this has a more sophisticated structure than the PIN photodiode in order to create an extremely high electric field region (approximately \( 3 \times 10^5 \text{ V cm}^{-1} \)), as seen in Figure 2-7a. As the depletion region is where most of the photons are absorbed, and the primary carrier pairs generated, there is a high-field region in which holes and electrons can acquire sufficient energy to excite new electron-hole pairs. This process is known as impact ionization and is the phenomenon that leads to avalanche breakdown in ordinary reverse-biased diodes. It often requires high reverse bias voltages in order that the new carriers created by impact ionization can themselves produce additional carriers by the same mechanism as shown in Figure 2-7b.

At high speed, the operation of APD devices requires full depletion in absorption region. Carriers generated in un-depleted materials are collected slowly by the diffusion process. Therefore, it has an effect of introducing a long diffusion tail on the short optical pulse.

\( c \) is the velocity of light in a vacuum. \( h \) is Planck’s constant.
Once the APD is fully depleted by employing the electric field, all the carries drift at saturation limited velocities. In this event, there are three factors that limit the response time of the device [160]:

1. The transit time of the carriers across the absorption region.
2. The time taken by carriers to perform the avalanche multiplication process.
3. The RC time constant incurred by the junction capacitance of the diode and its load.
The transit time and RC effects dominate at low gain, giving a definite response time hence constant bandwidth for the device. However, at high gain the avalanche build-up time dominates and therefore the device bandwidth decreases proportionately with increasing gain. This APD operation is distinguished by a constant gain-bandwidth product. Fast rise time and slower fall time dictated by the transit time cause APD output pulse to be asymmetric. Although the use of suitable materials and structures may give rise times between 150 and 200 ps, fall times of 1 ns or more are quite common and limit the overall response of the device.

**Figure 2-8** Receiver sensitivity comparison of PIN photodiode and APD devices at a bit-error-rate of $10^{-9}$ using silicon detectors operating at a wavelength of 0.82 μm [160].

APDs have a distinct advantage over photodiodes without internal gain for the detection of the very low light levels often encountered in optical fibre communications. They generally provide an increase in sensitivity of between 5 and 15 dB over PIN photodiodes while often giving a wider dynamic range because of their gain variation with response time and reverse bias. The optimum sensitivity improvement of APD receivers over PIN photodiode devices is illustrated in the characteristics shown in Figure 2-8 and Figure 2-9.
The characteristics display the minimum detectable optical power for direct detection against the transmitted bit rate in order to maintain a bit-error-rate (BER) of $10^{-9}$ in the shorter and longer wavelength regions. Figure 2-8 compares silicon photodiodes operating at a wavelength of 0.82 $\mu$m where the APD is able to approach within 10 to 13 dB of the quantum limit. In addition, it may be observed that the PIN photodiode receiver has a sensitivity around 15 dB below this level. InGaAs photodiodes operating at a wavelength of 1.55 $\mu$m are compared in Figure 2-9. In this case, the APD requires around 20 dB more power than the quantum limit, whereas the PIN photodiode receiver is some 10 to 12 dB less sensitive than the APD. Avalanche photodiodes also have several drawbacks which include fabrication difficulties due to their more complex structure hence increased cost and the random nature of the gain mechanism which gives an additional noise contribution. The high bias voltages required particularly for silicon devices is also a disadvantage in addition to the variation of the gain (multiplication factor) with temperature which requires temperature compensation to stabilize the operation of the device. Figure 2-10 shows the variation of the gain for a silicon APD [160].
Figure 2-10 Current gain against reverse bias for a silicon RAPD operating at a wavelength of 0.825 μm [160].

The multiplication factor M is a measure of the internal gain provided by the APD. It is given by [161]

\[ M = \frac{I_s}{I_p} \] (2-2)

where \( I_s \) is the total output current at the operating voltage and \( I_p \) is the initial or primary photocurrent.

2.3 Pre-amplifiers

An optical receiver front-end design can usually be grouped into one of four basic configurations: resistor termination with a low-impedance voltage amplifier, high impedance amplifier, transimpedance amplifier, and noise-matched or resonant amplifier. Any of the configurations can be built using contemporary electronic devices such as operational amplifiers, bipolar junction transistors (BJT), field-effect transistors (FET), or high electron mobility transistors (HEMT). The receiver performance that is achieved will depend on the devices and design techniques used.
2.3.1 Low input impedance amplifiers

The primary purpose of using any preamplifier is to convert the photocurrent into a voltage signal which can then be amplified by a subsequent voltage amplifier. The preamplifier also should provide good low noise performance as the photocurrent is a relatively weak signal. This can be done by using a load resistor that also provides a bias for the photodetector.

The photodiode can be either AC coupled, or DC coupled to the amplifier. In the AC coupled case, a separate load resistor is used to derive a voltage proportional to the photocurrent and to provide a path for the DC photocurrent to flow. The low-frequency components of the photocurrent see a load resistor $R_l$ while the high-frequency components see a load resistance that is the parallel combination of $R_l$ and the amplifier input impedance $Z_{in}(\omega)$.

2.3.2 High input impedance amplifiers

The thermal-noise associated with the load resistor dominates the receiver noise in the resistor-terminated low-impedance voltage amplifier front-end. A high-impedance amplifier is an approach
that substantially reduces the effect of the thermal-noise of the load resistor, resulting in improved sensitivity. The high-impedance receiver is based on a technique that has been successfully used with other capacitive current sources such as vidicon tubes and is descended from vacuum tube amplifiers.

![Diagram of high-impedance front-end](image)

\[ Z_{th}(\omega) = \frac{1}{j\omega C_{in}} \]

\[ A_v(\omega) \]

\[ H_{eq}(\omega) \]

\[ V_{bias} \]

\[ V_{out} \]

\[ i_{photo} \]

\[ R_f \rightarrow \infty \]

(a)

(b)

Figure 2-12 high-impedance front-end a) Typical circuit configuration b) Example transfer functions [163].
It was one of the first low-noise front-ends used in optical receivers. The basic design principle is to load the current-source with as large an impedance as possible. This tends to maximize the amount of voltage developed at the input of the amplifier. Thus, the voltage is maximised, and the effects of any amplifier noise sources will be reduced. Therefore, it is the most sensitive preamplifiers due to the use of high input resistance which results in exceptionally low thermal noise. The high input resistance in combination with the receiver input capacitance results in a very low bandwidth which causes integration of the received signal hence it is not maintained for wideband operation. A differentiating network at the receiver output is used to correct this integration. The high-impedance (integrating) front-end structure gives a significant improvement in sensitivity over the low-impedance front-end design, but it creates a heavy demand for equalisation and has problems of limited dynamic range.

The limitations on the dynamic range are due to the attenuation of the low-frequency signal components by the equalisation process which causes the amplifier to saturate at high signal levels. When the amplifier saturates before equalisation has occurred, the signal is heavily distorted. Thus, the reduction in dynamic range is dependent upon the amount of integration and subsequent equalisation employed.

![Equaliser circuits](image)

**Figure 2-13 Equaliser circuits to the left a) Passive b) Active [163].**
As seen in Figure 2-12, if pole-zero cancellation is achieved by setting $\omega_z$ to $\omega_p$, the overall response will be flat out to $\omega_{eq}$. The equalisation process is graphically illustrated in Figure 2-12b. Equalisation can be performed using a simple passive RC circuit [164], shown in Figure 2-13a, or an active circuit [165], illustrated in Figure 2-13b. To obtain a wideband response with a high-impedance front-end, the ratio between $\omega_z$ and $\omega_{eq}$ must be as large as possible. Thus, the equaliser provides strong attenuation at low frequencies that cancels the high low-frequency gain of the high impedance amplifier. This introduces a dynamic range constraint on the high-impedance front-end.

The need to obtain accurate pole-zero cancellation to obtain a flat wideband response can also introduce some difficulties into the receiver design. The total amount of input capacitance can vary from device to device and can be a function of temperature and photodiode and transistor bias conditions. Accurate equalisation over time, temperature and production line variations may be difficult to obtain. Each individual equalised gain stage in the front-end could require independent manual adjustment to assure accurate pole-zero cancellation. Temperature compensation of the equalisation may be required. Line-coding may be needed to reduce the low-frequency content of the received signal. Low-noise is the high-impedance front-end principal benefit. The limited dynamic range is generally considered to be its principal drawback. Despite this limitation, high-impedance front-ends have been routinely used in many high sensitivity applications and are commercially available in the form of PIN-FET and APD-FET receivers [163].

2.3.3 Transimpedance Amplifiers

The transimpedance design relies on negative feedback to increase the bandwidth of the open loop preamplifier. This configuration largely overcomes the drawbacks of the high-impedance front end by utilizing a low-noise, high-input-impedance amplifier with negative feedback. The device, therefore, operates as a current mode amplifier where the high input impedance is reduced. An
An equivalent circuit for an optical fibre receiver incorporating a trans-impedance front-end structure is shown in Figure 2-14.

![Equivalent Circuit Diagram](image)

**Figure 2-14 An equivalent circuit for the optical fibre receiver incorporating a transimpedance preamplifier [160].**

In this equivalent circuit, the parallel resistances and capacitances are combined into $R_{TL}$ and $C_T$ respectively. The open loop current to voltage transfer function $H_{OL}(\omega)$ for this transimpedance configuration corresponds to the transfer function for the two structures described previously which do not employ feedback.

$$H_{OL}(\omega) = \frac{-A_v R_{TL}}{1 + j\omega R_{TL} C_T} \quad (2-3)$$

where $A_v$ is the open loop voltage gain of the amplifier and $\omega$ is the angular frequency of the input signal. In this case, the bandwidth (without equalisation) is constrained by RC time constant. When the feedback is applied, the closed loop current to voltage transfer function $H_{CL}(\omega)$ of the transimpedance configuration is given by
\[ H_{CL}(\omega) = \frac{-R_f}{1 + j\omega R_f C_T / A_v} \]  \hspace{1cm} (2-4)

Using a referred impedance noise analysis, it can be shown that to a good approximation the feedback resistance may be referred to the amplifier input to establish the noise performance of the configuration [178]. Thus when \( R_f \ll R_{TL} \), the major noise contribution is from thermal noise generated by \( R_f \). The noise performance of this configuration is therefore improved when \( R_f \) is large, and it approaches the noise performance of the high-impedance front end when \( R_f = R_{TL} \).

Unfortunately, the value of \( R_f \) cannot be increased indefinitely due to problems of stability with the closed loop design. Furthermore, increasing \( R_f \) reduces the bandwidth of the transimpedance configuration. This problem may be alleviated by making \( A_v \) as large as the stability of the closed loop will allow. Therefore, the noise in the trans-impedance amplifier will always exceed that incurred by the high-impedance front-end structure.

The other major advantage which the trans-impedance configuration has over the high impedance front end is a greater dynamic range. This improvement in the dynamic range obtained using the trans-impedance amplifier is a result of the different attenuation mechanism for the low-frequency components of the signal. The attenuation is accomplished in the trans-impedance amplifier through the negative feedback and therefore the low-frequency components are amplified by the closed loop rather than the open loop gain of the device. Hence for a particular amplifier, the improvement in dynamic range is approximately equal to the ratio of the open loop to the closed loop gains. The transimpedance structure overcomes some of the problems encountered with the other configurations and is often preferred for use in wideband optical fibre communication receivers. In Figure 2-15, another preamplifier modifies the trans-impedance design and uses a noiseless element as the feedback element.
This is accomplished by using a capacitor in feedback instead of a resistor. This results in integration of the received signal. This configuration can be intentionally designed to have an integrating response. However, instead of cancelling the pole at the origin with a corresponding zero (using a differentiator), a switch may be added across the amplifier to dump the integrator to zero. This reset switch is closed after every received bit and is then reopened to receive each subsequent bit.

### 2.3.4 Amplifier noise

At the front end of the non-tuned receiver, an ideal current source, shunted by the detector capacitance $C_d$, models the photodetector, which feeds the parallel combination input resistance and input capacitance, modelling the input impedance of the preamplifier $Z_{in}$. The voltage gain of the preamplifier and post amplifier is modelled as a voltage amplifier, with a transfer function $A(\omega)$ the output of which feeds the pre-detection filter with a transfer function of $H_f(\omega)$. In the case of using a matched filter, Butterworth or integrator as a pre-detection filter, the front end is assumed to have a large bandwidth as well as a large voltage gain in order to pass the signal without any distortion thus the front-end transfer function becomes negligible.
In the case of a FET input device, there will be various sources of noise. The first two are thermal noise from the bias resistor (feedback resistor if the front end is transimpedance design) and channel conductance of the input transistor. The other source is shot noise due to gate leakage current. Both thermal noise sources are connected to the input node of the amplifier, however, the shot noise may be referred to the input node in order to obtain the total equivalent input noise current. The shot noise generated produces a noise current spectral density in a short circuit placed across the drain and source. This current can be referred to the input of the FET by dividing by the transconductance. The transistor noise sources for a BJT input device are the base current shot noise, the base-spreading thermal noise and the collector current shot noise. The base noise and feedback resistor thermal noise are connected to the input node as current generators. The collector current can be referred in the same way as the channel noise in the FET [165]. The nature of noise in optical receivers, noise performance and noise referral for both FET and BJT input transistor TIAs are further discussed in Section 2.5 and 2.5.2.

### 2.4 Pre-detection filters

A practical receiver for on-off-keying (OOK) signalling is illustrated in Figure 2-17. The output of the photodetector passes through a low-noise front-end amplifier. The output of the front-end is split into two paths. One path is used to derive a clock signal that is aligned to the symbol transitions and...
the other is filtered and passed to a digital decision circuit. In this example, the falling edge of the recovered clock causes the decision circuit to sample the recovered waveform and compare the input voltage $V_{in}$ to the threshold voltage $V_{th}$. If the received waveforms voltage is below the threshold, the decision circuit produces a zero at its output. If the voltage exceeds the threshold, a one is produced.

![Diagram of Practical OOK receiver](image)

**Figure 2-17** Practical OOK receiver [161]

![Signals at the decision circuit input](image)

**Figure 2-18** Signals at the decision circuit input in a practical OOK receiver. (a) Example time-domain waveforms. (b) Example probability density functions
Examples of the signal that appears at the input to the decision circuit are illustrated in Figure 2-18. The recovered waveform is corrupted by noise, as illustrated in Figure 2-18a. There is a probability density associated with the reception of a zero and with a one. Both the shape of the pulse waveform and the amount of noise present at the input to the decision circuitry influence the receiver ability to correctly identify the symbol that has been received. The errors occurred due to noise is explained in Section 2.5.5 along with a further explaining of OOK detection and error bit rate statistics.

It is essential to select the waveform and noise level that maximise the ability to make the correct decision. The waveform that appears at the input to the decision circuit is designed such that any interference between adjacent received symbols is minimised. Inter-symbol interference (ISI) occurs when the detection of one symbol affects detection of the symbols that follow, and it leads to a basic source of degradation in digital systems. Also, signal to noise ratio at the decision time should be maximised in addition to using a waveform with minimal ISI. This trade-off is explained in detail in Section 2.4.1. In a digital receiver, pre-detection electrical filtering is commonly used to control the waveform shape and signal to noise ratio at the input to the decision circuit. The design of minimal ISI pulse waveforms and pulse shaping filters has become a common approach in digital communication since Nyquist’s original work on waveforms for use in telegraph transmission [166].

Nyquist theory identified criteria that a waveform must meet to avoid interactions between consecutive symbols. To avoid ISI, the minimum bandwidth that an ideal binary waveform could pass through is half the bit-rate (B). Otherwise, if a controlled amount of ISI is allowed within a system, it is possible to signal faster than that set by Nyquist criteria [168].

### 2.4.1 Bandwidth allocation

In communication systems, a bandwidth of at least half the bit rate B is needed for ISI free communications. This is also called the Nyquist bandwidth. Assuming that, the received pulse is a superposition of sinc pulses (Nyquist pulses). The spectrum of these pulses is rectangular which is flat up to B/2 and then drops to zero immediately. This signal is a desirable property in communication
systems, it is raised cosine spectrum with 0% excess bandwidth, therefore, the signal is free of ISI. If this signal passes through a brick-wall low pass of a bandwidth of B/2 with a linear phase, it will not incur any distortion. This happens as rectangular spectrum multiplied by the brick-wall low pass response yields the same rectangular spectrum. Therefore, to optimally receive a Nyquist pulse, the frequency response of the receiver should match the signal spectrum which is a brick-wall low pass response with a bandwidth of B/2 giving no ISI. The Nyquist bandwidth does not refer to the 3-dB bandwidth, but it rather refers to the absolute bandwidth (∞-dB bandwidth) where the signal is completely suppressed. Also, if the absolute bandwidth is less than B/2, the received bit stream could be error-free, but the signal will no longer be free of ISI [169], [179].

In the following, the primary question is how large the receiver bandwidth should be and, more generally, what is the optimum frequency response in case there is an amount of ISI accepted in the system. The problem divides between the noise and ISI as if the receiver bandwidth is made wide, the receiver passes the signal without distortion but picks up lots of noise which corrupts the signal. On the other hand, if the receiver bandwidth is lower than required, it will reduce the noise at the expense of producing ISI. It is concluded that there must be an optimum receiver bandwidth for which sensitivity is best. A rule of thumb for NRZ receivers says that this optimum 3-dB bandwidth is about 2/3 the bit-rate. Three cases are explained graphically in Figure 2-19. The output waveforms for three different receiver bandwidths are shown from right to left in the form of eye diagrams, 3-dB bandwidth = 4/3, 2/3 and 1/3 times the bit-rate [169].

Figure 2-19 illustrates the trade-off between ISI and noise for the example of a 10-Gb/s receiver with a second-order Butterworth response. It is assumed that the received input signal is an ideal NRZ waveform and that the input referred noise is white. Figure 2-19a shows the eye diagram of a wideband receiver with a 3-dB bandwidth of twice the optimum bandwidth leading to clean eye with almost no ISI. Figure 2-19b has the optimum bandwidth. Figure 2-19c shows the eye diagram of a narrowband receiver with only half the optimum bandwidth with severe ISI resulting in a closed eye.
The noise inside the eye is shown for ease. The eyes for the wide and narrowband receivers are completely closed by the noise, the eye for the optimum-bandwidth receiver is open at the centre which concludes that it is only possible to recover the received data at the desired BER for optimum bandwidth receiver [169].

Figure 2-19 Trade-off between ISI and noise in a receiver [169].

The receiver consists of a cascade of building blocks: photodetector, TIA, filter, MA and decision circuit. It is the combination of all these blocks that should have a bandwidth of about 2/3 B. The combined bandwidth can be approximated by adding the inverse-square bandwidths of the individual
blocks. Thus, each individual block must have a bandwidth that is larger than optimum bandwidth. There are several strategies for assigning bandwidths to the individual blocks to achieve the desired overall bandwidth. For a low-speed receiver, all receiver blocks are designed for a bandwidth much larger than the desired then one block is used to control the whole receiver bandwidth. This block could be a precise filter to be inserted after the TIA or at detection stage as a pre-detection filter. Alternatively, the TIA can be designed to have the desired receiver bandwidth and no filter is needed in this case. However, the frequency response, in this case, is less controlled to the former method but this approach provides higher trans-impedance and better noise performance. Another case where the filter is not needed is to design all blocks together to have the desired bandwidth, but this is a restricted strategy which is typically used for high-speed receivers [169].

2.4.2 Receiver Response

This topic might seem more related to the optimum frequency response for NRZ receivers in specific. However, it helps in the understanding of the optimum frequency response in general. The optimum frequency response for NRZ receivers depends on many factors such as the shape of the received pulses, the spectrum of the input-referred noise, the sampling jitter in the decision circuit and the bit estimation technique used. The most important cases are shown in Figure 2-20.

Each bit is estimated independently by comparing the sampled output voltage to a threshold voltage. The matched filter or modified matched filter is used in case of well-shaped pulses at the input of the receiver, typically when pulses are broadened by less than 14% of the pulse width. The matched-filter demodulator is equivalent to the maximum-likelihood demodulator. It allows optimum detection of a known waveform in the presence of additive noise. Hence, digital receiver optimisation criteria can be set to have a received pulse waveform that satisfies Nyquist's criteria for minimizing ISI and a matched filter to maximise the SNR at the decision time. The matched filter gives the best results in case of white input referred noise and absence of sampling jitter. Whitening noise filter is used in cascade with a matched filter to form the modified matched filter which is used in case of non-white
input referred noise. Maximum signal to noise ratio and lowest bit error rate is obtained for NRZ receivers employing matched filter when ISI is not presented. Integrate and dump filter is used as a matched filter where the input pulse shape is rectangular pulse starting at $t = 0$ and ending at $T = 1/B$, and hence this filter is also known as the rectangular filter [161], [166], [169], [178]-[179].

![Decision Tree Diagram]

**Figure 2-20 decision tree to determine the optimum receiver response [169].**

Practical optical communication receivers complicate this simple interpretation of receiver optimisation due to various reasons. Practical receivers combine bandlimited waveforms having signal dependent Poisson photodetection statistics with additive Gaussian noise due to electronic component. Although, the receiver electronic noise is Gaussian, the power spectral density of the electronic noise in wide bandwidth receivers is often frequency dependent instead of white. This may require an additional whitening filter to be included in the receiver design. The whitening filter can itself introduce unacceptable amounts of ISI. Avalanche multiplication-noise from an APD may also be present, and the waveforms may need to be designed to be tolerant of jitter in the receiver decision sampling or transmitter pulse timing. Other reasons are associated with the channel such as fibre-optic channel may be dispersive or suffer from optical nonlinearities, polarization dependent fading,
or channel-to-channel crosstalk. In case of free space, the channel may introduce fading due to atmospheric effects [161].

In long-haul transmission systems and because of fibre dispersion, the NRZ pulses at the input of the receiver usually are severely broadened. In this case, the matched filter would worsen ISI by further broadening the pulses leading to significant receiver sensitivity degradation. For broadened pulses, usually more than 20% of the bit interval, the raised cosine filter theoretically gives better results. This is done by transforming the broadened input pulses into pulses with a raised cosine spectrum, not necessarily by using a receiver that has a raised cosine response. Although raised-cosine filtering is common in the theoretical receiver literature, it is rarely used in practical optical receivers, meaning that such a receiver can only be realised as an approximation. This is due to the necessity of knowing the exact shape of the received pulses to design receiver transfer function [165], [169].

Filters that produce output waveforms that approximate members of the raised cosine family have been used in many digital communication systems [170]. These waveforms have also proven useful in optical communication systems, and practical direct detection receivers can be fabricated with filters that realize near-optimum raised cosine pulse waveforms at the input to the decision circuitry. This will be true whether the system uses NRZ pulses that occupy the full symbol time, RZ pulses that occupy only a portion of the symbol time, or Manchester coded signals [171].

### 2.4.3 Matched filter

The matched filter is defined by its impulse response \( h(t) \), which must be proportional to a time-reversed copy of the received pulses \( x(t) \), more precisely, \( h(t) \) is approximately equal to \( x(T - t) \), where \( T \) is the duration of the received pulses. This definition implies that the matched-filter frequency response matches the spectral shape of the input pulses. In the case of an undistorted NRZ signal, the matched filter is given by [169]
\[ h(t) = x(T - t) = x(t) \]  \hspace{1cm} (2-5)

where \( x(t) \) is a rectangular pulse starting at \( t = 0 \) and ending at \( T = 1/B \), hence this filter is known as the rectangular filter. In the frequency domain, the filter has a low-pass characteristic that can be calculated by taking the Fourier transform of a pulse of duration \( T = 1/B \). The normalized transfer function is [169]

\[
H(f) = \frac{\sin(\pi f/B)}{\pi f/B} \ e^{-j\pi f/B} \hspace{1cm} (2-6)
\]

Figure 2-21 shows the squared frequency response of the matched filter. The noise bandwidth of this response is half the bit rate and the 3-dB bandwidth is slightly less than half the bit rate. Even though in absence of ISI, the triangular eye shape implies that to avoid sensitivity degradation, it is required to sample exactly at the centre of the eye.

Figure 2-21 Rectangular filter to the left a) waveforms and b) frequency response [169]

For PPM receiver, the matched filter requires white noise at its input, so the preamplifier noise must be whitened first thus the frequency response of the noise whitening filter \( H_f(\omega) \) is given by [172]

\[
H_f(\omega) = \frac{1}{1 + \frac{\omega^2}{\omega_n^2}} H_p(\omega)^* \hspace{1cm} (2-7)
\]

The first term is the noise-whitening filter and the second is complex conjugate of the received signal.
The output voltage of whitening matched filter is given by [172]

\[ v_o(t) = \frac{b \eta q}{2\pi} \int_{-\infty}^{\infty} z_{\text{pre}}(\omega) \frac{H_p^2(\omega)}{1 + \left(\frac{\omega^2}{\omega_n^2}\right)} \exp(j\omega t) \, d\omega \]  
\[ (2-8) \]

\[ \langle n_o^2 \rangle = \frac{S_o}{2\pi} \int_{-\infty}^{\infty} \left[ \frac{H_p(\omega)}{1 + \left(\frac{\omega^2}{\omega_n^2}\right)} \right]^2 z_{\text{pre}}(\omega) \, d\omega \]  
\[ (2-9) \]

where \( S_o \) is the double-sided, equivalent input-noise current spectral density of the preamplifier, \( b \) is the number of photons per pulse, \( \eta \) is the quantum efficiency of the detector, \( q \) is the electronic charge.

For the Gaussian received pulse with a shape of \( h_p(t) \) and a Fourier transform of \( h(\omega) \), the output voltage and its derivative from matched filter (assuming white noise at the input) are given by [173]

\[ v_o(t) = \frac{\omega_n}{2} e^{\sigma^2 \omega_n^2} e^{-\omega_n t} \text{erfc}(\sigma \omega_n - \frac{t}{2\sigma}) \]  
\[ (2-10) \]

\[ \frac{d}{dt} v_o(t) = \frac{\omega_n}{2} e^{\sigma^2 \omega_n^2} e^{-\omega_n t} \left[ \frac{\exp\left( \frac{(\sigma \omega_n - t/2\sigma)}{\sigma \omega_n \sqrt{\pi}} \right)}{\sigma \omega_n \sqrt{\pi}} - \text{erfc}\left(\frac{\sigma \omega_n - t}{2\sigma}\right) \right] \]  
\[ (2-11) \]

The noise appearing on this signal is given by [172], [173]
\[ n_0^2 = S_o R_T^2 \frac{\omega_n}{2} e^{\sigma^2 \omega_n^2} \text{erfc}(\sigma \omega_n) \quad (2-12) \]

where \( R_T \) is the mid-band transimpedance of the receiver and \( \sigma \) is the variance of the received Gaussian pulse, which is linked to the fibre bandwidth and given by [172], [173]

\[ \sigma = \frac{\sqrt{2\ln2} T_b}{2\pi f_n} \quad (2-13) \]

### 2.4.4 Integrate and dump

Integrate and dump is another example of a matched filter which can be implemented as part of the decision circuit or embedded in TIA operation, however, implementation of the clock recovery might become an issue when choosing one of these two arrangements. The received signal is a square pulse and the pre-amp is an integrator that has a large bandwidth, the receiver will be modelled as shown in Figure 2-23.

![Integrate and dump receiver model](image)

**Figure 2-23 Integrate and dump receiver model**

The transfer function is given by

\[ H_f(\omega)_{I&D} = \frac{1}{j\omega \tau} (1 - e^{-j\omega T}) \quad (2-14) \]

where \( \tau \) is the RC time constant and \( T \) is the pulse-width. The noise at the output prior to detection circuit is given by

\[ n_o^2 = \int_{-\infty}^{\infty} n_i^2(f) |H_f(f)|^2 df \quad (2-15) \]

The maximum peak voltage is given by
\[ v_{pk} = b\eta q \frac{T}{T} \]  

(2-16)

Alternatively, the front end is assumed to be second order thus the transfer function becomes an integrator in combination with second order filter thus \( H_f(\omega)_{I&D} \) becomes

\[ H_f(\omega)_{I&D} = \frac{1}{\omega \tau} \frac{\omega_n^2}{(\omega^2 + \omega_n^2)} \]  

(2-17)

The noise at the output prior to detection circuit becomes

\[ n_o^2 = \int_{-\infty}^{+\infty} n_i^2(f) |H_f(f)_{I&D}|^2 d \]  

(2-18)

The output voltage due to a step input is given by

\[ v(t) = \frac{1}{\tau} \left( t - \frac{2}{\omega_n} \right) e^{-\omega_n t} \left( 1 + \frac{\omega_n t}{2} \right) \]  

(2-19)

Although the expression of matched filter suggests that the rectangular filter can be replaced by a circuit that integrates the received signal over the bit period, it is required to start the integration at the beginning of each bit period and thus the integrator must be reset immediately at the end of each bit period. Integrate and dump, similarly to the matched filter, is optimum only when receiving undistorted rectangular pulses with white noise which is rarely the case in practice [169], [174].

### 2.4.5 Butterworth filter

In the case of the receiver, which is accurately modelled as a signal in additive white or coloured Gaussian noise, a raised-cosine pulse is nearly optimum as in many broadband PIN-based receivers.

On the other hand, an APD or optically pre-amplified receiver, there is significant signal-dependent noise which is worse than optimum. Hence, it is possible to modify the pulse waveform and receiver filter to achieve performance near to the optimum. The time-domain waveform and the corresponding spectrum for the raised-cosine pulse family of symbol duration T are given by [161], [165]
\[ h_{out}(t) = \frac{\sin(\frac{\pi t}{T})}{\frac{\pi t}{T}} \times \cos\left(\frac{\alpha \pi t}{T}\right) x\left(1 - \left(\frac{2 \alpha t}{T}\right)^2\right) \] (2-20)

\[ H(f) = \begin{cases} T; & 0 \leq |f| \leq \left(\frac{1 - \alpha}{2T}\right) \\ \frac{T}{2} \left[1 - \sin\left(\pi T \left[\frac{|f|}{\alpha} - \frac{1}{2T\alpha}\right]\right)\right]; & \left(\frac{1 - \alpha}{2T}\right) \leq |f| \leq \left(\frac{1 + \alpha}{2T}\right) \\ 0; & |f| \geq \left(\frac{1 + \alpha}{2T}\right) \end{cases} \] (2-21)

Figure 2-24 Raised-cosine pulse waveform. (a) Time-domain (b) Frequency-domain [161].

Figure 2-24 illustrates examples of the raised cosine family. A raised-cosine pulse is described by the parameter \( \alpha \) and it is known as the roll-off factor. The value of \( \alpha \) is in the range of \( 0 < \alpha < -1 \). For \( \alpha = 0 \), the pulse has a perfectly rectangular spectrum with a single-sided bandwidth of \( 1/2T \) where \( T \) is the interval between symbol decisions and is equal to the time between bits for a binary system.

Figure 2-24a illustrates the pulse waveforms in the time-domain. Ideally, the decision for the current symbol occurs at \( t = 0 \). For next symbol and previous symbol, the decision is made at \( t = T \) and \( t = -T \).
T, respectively. The waveform always equals zero at all sample times $\pm nT$ except for $n = 0$, therefore, all waveforms have no ISI. Figure 2-24b illustrates the corresponding frequency-domain spectra. In case of $\alpha = 0$, the frequency response occupies the minimum bandwidth and would, therefore, have the smallest noise equivalent bandwidth. However, the tails of the minimum bandwidth pulse roll off as $1/t$ which gives rise to relatively large amplitude tails that result in ISI. The raised-cosine family allows trading an increase in bandwidth for a more rapid pulse roll-off. For $\alpha =1$, the tails of the pulse waveform roll-off as $1/t^3$ which leads to less significant errors in the decision circuit hence less ISI than for a pulse shape that rolls-off as $1/t$. This case is named the full raised-cosine waveform since there is 100% more bandwidth occupied than would be required by the minimum bandwidth pulse obtained when $\alpha = 0$. The term bandwidth is used here to describe the total width of the spectrum. This is not the receiver 3-dB bandwidth or the noise equivalent bandwidth [161].

Figure 2-25 shows a sequence of sin $x/x$ pulses. The main properties of these pulses are that the amplitude of the precursors and tails due to adjacent pulses is zero at the pulse centre. Also, the pulse spectrum is identical to the frequency response of an ideal low pass filter having a bandwidth of the half the bit rate. The characteristic of these pulses satisfies NRZ ISI requirements as this pulse shape is maximised at the sampling instant and results in zero amplitude at all other sampling points at multiples of the pulse width time. Raised cosine filter is rarely used in practice due to the necessity of knowing the exact shape of the received pulses to design receiver transfer function. This makes any receiver transfer function that results in such shape output pulses for a certain input pulse very intolerant of any changes in the input pulse shape, in addition to the importance of sampling at precisely the centre of the pulses. Although ideal sin $x/x$ pulse shape is impossible to achieve in practice, Butterworth filtering is used as a good approximation to convert NRZ pulses into raised cosine-like pulses.
As approximated in [165], the receiver frequency response is determined by dividing the output pulse shape by the input pulse shape. For full width rectangular input pulses and full raised cosine spectrum output pulses, the optimum transfer function is approximated by a single pole frequency response pre-amplifier with 3-dB cut off at 0.5 B feeding a third order Butterworth filter having a cut off frequency of 0.7 B. A transfer function of the Butterworth filter $H_f(\omega)_{BW}$ is given by [172]

$$H_f(\omega)_{BW} = \frac{1}{(j\omega)^3 + 2(j\omega)^2 \omega_B + 2j\omega\omega_B^2 + \omega_B^3} \quad (2-22)$$

### 2.5 Receiver Performance

In this section, the nature of noise in optical communications is discussed. In Section 2.5.2, there is a brief introduction of the TIA noise power spectrum followed by an introduction to feedback resistor noise and TIA noise sources. The characteristic of the high-pass transfer function for referred noise is discussed, explaining how TIA noise is referred in FET and BJT front-ends. This explains how white noise source such as channel noise is referred to the TIA input producing an equivalent coloured noise source. Noise equivalent bandwidth is briefly discussed in Section 2.5.3, distinguishing the difference between NEB and 3-dB bandwidth. The NEB of the low-pass filter, bandpass filter and filters with asymmetrical frequency response are briefly explained in this section. Personick integrals are presented in Section 2.5.4, with numerical values for some common receiver responses. In this
section, there is an interpretation for how noise bandwidths are obtained mathematically with an illustration of integrating the noise power spectrum. The relation between signal, noise and bit error rate is discussed in Section 2.5.5, explaining how the input noise power spectrum is shaped by the receiver transfer function producing the output noise power spectrum. In addition, there is an explanation of detection process leading to error probabilities, to briefly explain detection statistics and present bit-error rate mathematics relations. Electrical sensitivity and optical sensitivity are briefly discussed in Section 2.5.6, giving all related receiver sensitivity formulas used in evaluating and comparing the performance of different receivers.

2.5.1 Noise

Random fluctuations of electrons and photons are responsible for major performance impairments of optical transmission systems. Noise terms in optical communications arise, not only from random fluctuations of the electron density in the electronic amplifiers devoted to signal processing of the photodetected current but also from random photon fluctuation in the detected optical field. These optical density fluctuations are converted into the corresponding electrical noise term by the square law photon detection process and this electrical power is summed together with the other electrical power noise contribution resulting in the total noise power.

Thermal noise, this kind of noise is due to the thermal interaction between the electric charges and vibrating ions within a conductive medium and is governed by Gaussian statistics. Individual electron route is not predictable in any deterministic way owing to the larger number of collisions per unit time it suffers along a random path and at energy temperature above absolute zero. Among the fundamental concepts regarding noise theory, white noise refers to a stationary random process, which is completely uncorrelated. From the mathematical point of view, the autocorrelation function of white noise process coincides with the impulse function; meaning that every two indefinitely close events are still unpredictable. The corresponding frequency representation of the impulsive autocorrelation leads to a constant spectrum power density at which stems the term white noise. The
Indefinitely flat power spectrum of the white noise is a very useful mathematical abstraction, but it leads to an infinite energy paradox. The white noise process assumes physical meaning after it is integrated within a finite time window.

Figure 2-26 Block schematic of the front end of an optical receiver showing the various sources of noise.

Referring to electrical voltages or currents within the bandwidth capacities of either electrical or optical communication systems, the input equivalent thermal noise can be mathematically modulated as a zero-mean white Gaussian noise random process characterised by a Gaussian probability density function and flat power spectrum density. The important property of thermal noise is that it is independent of the signal level available in the same section. Thus, thermal noise is additive to the signal. If the signal is denoted by \( s(t) \) and noise thermal is \( n(t) \), the whole process can be described as \( x(0) = s(t) + n(t) \). Increases in the value of the signal power brings a corresponding increment in the signal to noise power ratio.
On the receiver side, only the input stage of the optical receiver contributes significantly to the thermal noise current. The design of the front-end amplifier is therefore fundamental for achieving low noise receiver performance. Therefore, the front-end architecture has a profound impact on the noise performance. For instance, in the well-known trans-impedance preamplifier structure, the feedback resistor has the dominant role in determining the input equivalent noise current, the power spectral density of noise current generated by the feedback resistor $R_f$ is white noise and is expressed as [160]

$$i = \frac{4KT}{R_f}$$

(2-23)

where $K$ is Boltzmann’s constant and $T$ is absolute temperature.

**Dark shot noise** is generated by leakages inside the photodetector without any incident light. It increases with diode defects and the photodetector reverse biasing. The nature of the leakage electrons generates the dark current shot noise. It is also independent of signal power. The dark current at the steady state is given by $I_D$ then the noise power spectral density given by $2qI_D$.

In PIN front ends, this kind of noise does not represent any limitation for the receiver performance, as it has very low values compared to thermal noise. In the low bit rate applications, it starts to affect significantly. In APD front end, the effect of shot noise is considered, as it follows the same multiplication as photocurrent.
Signal shot noise is another kind of shot noise in the front end. The photodetection process of the optical signal produces corresponding electron flux whose fluctuations are identified with the signal shot noise. It is assumed that the incoming photon flux is characterised by the constant rate photon/time without time variations or fluctuations. Thus, the corresponding photocurrent is instead a stationary random process characterised by a Poisson probability density function and white spectral density. Therefore, the linear relation between the power of shot noise and incident optical power shows the signal-dependent behaviour of the noise contribution. In digital systems, this kind of dependence will always affect level one bit whose incident optical power is always high, unlike level zero bit, where it is being almost negligible on the lower optical logic level.

In APD front ends, when a constant photocurrent is amplified by a current multiplication factor $M$, the DC signal power is equal to the square of the mean of the DC photocurrent times the square of the expected value of the multiplication [160]

$$P_s = I_{dc}^2 M^2$$  \hspace{1cm} (2-24)

The avalanche multiplication process in an APD is statistical in nature. The number of electrical carriers resulting from the absorption of a single photon is dependent upon where in the absorption region the photon was absorbed, the type of carrier (hole or electron), the magnitude of the local electric field, the local doping density of the semiconductor and the path the carriers travel through the semiconductor. This results in the mean of the square being larger than the square of the mean and the SNR after multiplication is lower than it was before multiplication [160].

$$P_n = I_{dc}^2 M^2 F(M)$$  \hspace{1cm} (2-25)

where $M$ is understood to be the expected value of the multiplication gain and $F(M)$ is an excess noise factor.
The value of the excess avalanche noise factor is dependent upon the detector material, the shape of the electric field profile within the device and whether the avalanche is initiated by holes or electrons. The excess noise factor is often represented as \( F(M) \) and one of the approximations is considered as

\[
F(M) = M^x
\]  

(2-26)

And the resulting noise is assumed to be white with a Gaussian distribution. The value of \( x \) is between 0.3 and 0.5 for silicon APDs and between 0.7 and 1.0 for germanium or III–V alloy APDs [160].

### 2.5.2 Input-Referred Noise Current

Figure 2-28a shows a noiseless TIA with an equivalent noise current source at the input. This current source combined with the noiseless TIA is equivalent to the actual noisy TIA, therefore, this combination reproduces the actual output noise of the noisy TIA. Thus, the current provided by this equivalent noise source is known as the input-referred noise current and it is one of the most critical TIA parameters. It determines the sensitivity of the receiver as it dominates all other noise sources. TIA noise becomes less critical in optically long-haul transmission systems as optical amplifiers are often used.

![Figure 2-28 (a) Input-referred noise current (b) Typical power spectrum [169].](image)

In Figure 2-28, the noise model consists of only a noise current source rather than a noise current and a noise voltage source, therefore, the value of the input referred noise current depends on the source
impedance which is determined mostly by the input capacitance. This capacitance must be specified when quoting the input referred noise current. Figure 2-28b illustrates schematically the power spectral density of the input referred noise current. It is measured in the square root of this spectrum and typically consists of a white part, at low frequencies and $f^2$ part at high frequencies. The root-mean-square rms noise value of the input referred noise current relates directly to the receiver sensitivity and it can be expressed by a single number measured in Ampere. It is also obtained by dividing the rms output noise voltage by the TIA mid-band transimpedance value. It is then called the total input referred noise current. For analytical calculations, the noise power spectrum is integrated up to infinity but for simulations and measurement, it usually enough to integrate up to twice the TIA bandwidth as rms output noise contribution become negligible. Hence, the input referred rms noise, also called the total input referred noise, directly determines the sensitivity of the TIA. Therefore, this noise measure is used to compare different TIAs designed for the same bit rate.

Figure 2-29 and Figure 2-31 show the familiar shunt-feedback TIA with the front-end implemented with a FET and a bipolar transistor, respectively. In an approximate noise analysis, it is sufficient to consider just the input transistor rather than the complete transistor level circuit. The FET front-end produces shot noise due to the gate current and channel noise. The bipolar front-end produces shot noise due to the base current, thermal noise due to the intrinsic base resistance, and shot noise due to the collector current. The thermal noise of the feedback resistor is present in both implementations. The effect of all these noise sources is presented by a single equivalent noise current source at the input of the TIA.

The input referred noise current spectrum of TIA is divided into two major components the noise from feedback resistor and the noise from amplifier front end. These two noise sources are uncorrelated and given by [169]

$$I_{n,TIA}^2(f) = I_{n,Rf}^2(f) + I_{n,front}^2(f) \quad (2-27)$$
The noise power spectrum of the feedback resistor $I_{n, Rf}^2(f)$ is frequency independent (white) and given by the thermal noise Eq. (2-23). It contributes directly to the input referred TIA noise as it has the same effect on TIA output noise. In receiver design, feedback resistor value is chosen as high as possible to optimise TIA noise performance.

![Figure 2-29 Noise sources in a TIA with FET front-end.](image)

For the noise contribution from the amplifier front-end $I_{n, \text{front}}^2(f)$, the major device noise sources in an FET common-source input stage are shown in Figure 2-29. The shot noise generated by the gate current $I_g$, is given by $I_{n,G}$, it contributes directly to the input-referred TIA noise same as feedback resistor noise. It is usually neglected for low gate-leakage current FETs. Channel noise is significant noise source in FET input stage, it is given as

$$I_{n,D}^2 = 4K_T\Gamma g_m$$  \hspace{1cm} (2-28)

where $g_m$ is the FET transconductance and $\Gamma$ is channel noise factor. This noise source is not located directly at the input of the TIA unlike the shot noise and feedback resistor noise. Therefore, it is referred to the input of TIA in order to obtain its contribution to the input referred TIA noise. The gain from the channel noise source to the output is found using nodal analysis [175]. As shown in Figure 2-30, at low frequencies, this gain is approximately given by $1/gm$ and at high frequencies, it
is as much as $1 + A_v$ times greater. Therefore, the high frequency output noise due to input transistor is much greater than the low frequency noise.

Figure 2-30 Noise spectrum components of a TIA FET front end.

The transfer functions from the input current to the output and from the channel noise to output are used to refer noise due to channel noise source of the input transistor to the input of TIA. The transfer function from the input current to the drain current has a low-pass characteristic thus the inverse function which refers the drain current back to the input has high-pass characteristics and is given by [175]

$$H(s) = \frac{1 + sR_f C_T}{g_m R_f}$$  \hspace{1cm} (2-29)

where $C_T$ is the total capacitance at the TIA input. If the gain of the first stage is sufficiently large other noise sources can be neglected. Using Eq. (2-29) to refer the white channel noise $I_{n,D}^2$ back to input yields

$$I_{n,D(\text{refered})}^2 = 4KT \Gamma g_m \cdot \frac{1 + (2\pi f R_f C_T)^2}{(g_m R_f)^2} = 4KT \Gamma \frac{1}{g_m R_f^2} + 4KT \Gamma \frac{(2\pi C_T)^2}{g_m} \cdot f^2$$  \hspace{1cm} (2-30)
The total input noise of FET input transistor is the sum of the channel noise referred to the input and shot noise generated by the gate current. Therefore, from Eq. (2-23), (2-27), (2-28), (2-30) the total input referred noise current spectrum of a FET front end TIA $I_{n,TIA,FET}^2(f)$ is written as

$$I_{n,TIA,FET}^2(f) = \frac{4kT}{R_f} + 2qI_g + 4kT\Gamma \frac{1}{g_mR_f^2} + 4kT\Gamma \frac{(2\pi C_T)^2}{g_m} f^2$$

(2-31)

Figure 2-31 Noise sources in a TIA with bipolar front-end

The situation for a BJT common-emitter front-end, as shown in Figure 2-31, is similar to that of the FET front-end. The shot noise generated by the base current $I_b$ is white noise current and it contributes directly to the input-referred TIA noise. The shot noise generated by the collector current must be transformed to find its contribution to the input-referred TIA noise current in the same way as the channel noise in FET. The total input referred noise current spectrum of BJT front end TIA is then given by

$$I_{n,TIA,BJT}^2(f) = \frac{4kT}{R_f} + 2qI_b + 2qI_c \frac{(2\pi C_T)^2}{g_m^2} f^2 + 2qI_c \frac{1}{g_m^2 R_f^2}$$

(2-32)

Equation (2-31) and (2-32) represent the power spectrum of the total input referred noise current in the TIA, the rms value (the total input referred noise current) will depend on noise equivalent bandwidth (NEB) and the actual bandwidth integrals which are explained in Section 2.5.3 and 0.
2.5.3 Noise equivalent bandwidth (NEB)

The noise equivalent bandwidth of a network is conceptually equivalent to the bandwidth that an ideal rectangular filter would have if it were to produce the same amount of rms noise power at its output as the network in question. The NEB of a network is generally not equal to the 3-dB bandwidth. The 3-dB bandwidth refers to the points in the frequency-domain at which the power gain transfer function falls to half its maximum value. Noise equivalent bandwidth is the area under the power gain transfer function normalized to the peak-power gain, as illustrated in Figure 2-32.

![Simple low-pass transfer function](image)

**Figure 2-32 Simple low-pass transfer function.**

Figure 2-32 illustrates the NEB for a low-pass transfer function. Note that the NEB is larger than the 3-dB bandwidth. For a simple single pole, roll-off low pass filter network with a 6-dB per octave roll-off, the equivalent noise bandwidth is nominally 1.57 times the 3-dB bandwidth. Figure 2-33 illustrates the NEB for a symmetrical band-pass transfer function. The noise bandwidth is again larger than the 3-dB bandwidth.

Additional care must be exercised whenever the transfer function is asymmetrical. As shown in Figure 2-34, two different noise-equivalent-bandwidths can be obtained, depending on the value used for peak power gain. If the true peak is used, an NEB corresponding to the solid rectangle is obtained. If the peak corresponding to the flat portion of the passband is used, a different larger NEB
corresponding to the dotted rectangle is obtained. Since both the solid and the dotted regions will have the same amount of noise power at their outputs, either NEB is considered correct. Either can be used if throughout the receiver design, the peak value of a transfer function is appropriately determined.

Figure 2-33 Symmetrical band-pass transfer function.

Figure 2-34 Asymmetrical band-pass transfer function.
2.5.4 Personick Integrals

The input-referred noise or the input referred rms noise plays a key role in determining the receiver sensitivity. As presented in [169], [176]. The input referred noise power can be written in terms of the input-referred power spectrum $I_n^2(f)$ as

$$i_n^2 = \frac{1}{H_o^2} \int_0^\infty |H(f)|^2 \cdot I_n^2(f) df$$

(2-33)

where $I_n^2(f)$ is the input-referred noise power spectrum of the combined detector and amplifier noise, $H_o$ is the mid-band value of receiver transfer function $H(f)$. This noise quantity is easy to use in numerical simulations, but it looks quite bulky for analytical hand calculations. It is a misconception to integrate the input-referred noise spectrum over all frequencies as the integral does not satisfy the noise spectrum that consists of white and coloured noise components leading to an unbounded noise current. It is not appropriate either to integrate up to the 3-dB bandwidth of the receiver frequency response. Figure 2-35 shows this quantity as the hatched area under the input referred noise spectrum.

Although, the current is bound to a finite value, the results will be very sensitive to the upper bound of the integration. The correct way to calculate the total input referred noise from the associated noise spectrum is to start out by writing the input noise spectrum in the general form

$$I_n^2(f) = \alpha_0 + \alpha_2 f^2$$

(2-34)

Parameter $\alpha_0$ describes the white part of the spectrum, and parameter $\alpha_2$ describes the squared-noise part, $1/f$ and $f$-noise terms are neglected here.

This spectrum is plugged into the general input-referred power spectrum leading to an exact expression of the referred noise

$$i_n^2 = \frac{1}{H_o^2} \int_0^\infty |H(f)|^2 \cdot (\alpha_0 + \alpha_2 f^2) df$$

(2-35)

Expanded as
\[ i_n^2 = \alpha_0 \cdot \frac{1}{H_0^2} \int_0^\infty |H(f)|^2 \, df + \alpha_2 \cdot \frac{1}{H_0^2} \int_0^\infty |H(f)|^2 \cdot f^2 \, df \]  

(2-36)

Integrals in first and second term of Eq. (2-36) represent the noise bandwidths \( \text{BW}_n \) and \( \text{BW}_{n_2} \)

\[ \text{BW}_n = \frac{1}{H_0^2} \int_0^\infty |H(f)|^2 \, df \]  

(2-37)

\[ \text{BW}_{n_2}^3 = \frac{1}{H_0^2} \int_0^\infty |H(f)|^2 \cdot f^2 \, df \]  

(2-38)

Now, rewriting \( i_n^2 \) in form of

\[ i_n^2 = \alpha_0 \cdot \text{BW}_n + \alpha_2/3 \cdot \text{BW}_{n_2}^3 \]  

(2-39)

**Figure 2-35 How to calculate referred noise [169]**
The bandwidths $BW_n$ and $BW_{n2}$ depend only on the receiver frequency response and the decision circuit bandwidth. The decision circuit bandwidth is uncritical if it is larger than the receiver bandwidth and the receiver has a steep roll-off. This is one of the assumptions used in tuned front end as when the front end employs a tuned circuit it becomes narrower in bandwidth compare to later stages and some tuned front ends experience steeper roll off. For ease of calculation, the decision-circuit bandwidth is assumed to be infinite. Numerical values for the two bandwidths of some simple receiver responses are listed in Table 2-1. As these bandwidths and the noise parameters $\alpha_0$ and $\alpha_2$ are known, it is handy to calculate the total input-referred noise.

### Table 2-1 Numerical values for $BW_n$ and $BW_{n2}$

<table>
<thead>
<tr>
<th>$H(f)$</th>
<th>$BW_n$</th>
<th>$BW_{n2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st order low pass</td>
<td>1.57 BW_{3dB}</td>
<td>$\infty$</td>
</tr>
<tr>
<td>2nd order low pass (Butterworth)</td>
<td>1.11 BW_{3dB}</td>
<td>1.49 BW_{3dB}</td>
</tr>
<tr>
<td>Brick wall low pass</td>
<td>1.00 BW_{3dB}</td>
<td>1.00 BW_{3dB}</td>
</tr>
<tr>
<td>Rectangular filter</td>
<td>0.500 B</td>
<td>$\infty$</td>
</tr>
<tr>
<td>NRZ to full raised cosine filter</td>
<td>0.564 B</td>
<td>0.639 B</td>
</tr>
</tbody>
</table>

By comparing this information in Table 2-1 with Eq. (2-39), this can be taken as a result of integrating the white noise component of the input referred spectrum up to $BW_n$ and the squared noise component up to $BW_{n2}$. This is explained graphically in Figure 2-36.

$BW_n$ is identical to noise equivalent bandwidth of the receiver frequency response. $BW_{n2}$ is called the second-order noise bandwidth, because it plays the same role as the zero-order noise bandwidth $BW_n$ but with the white noise replaced by squared noise.

In the optical receiver literature, however, the so-called Personick integrals are more widely used instead of the noise bandwidths. These integrals are usually designated with $I_2$ and $I_3$ and are defined such that the input-referred noise power can be written as
\[ i_n^2 = \alpha_0 \cdot I_2B + \alpha_2 \cdot I_3B^3 \]  \hspace{1cm} (2-40)

where B is the bit-rate. (The first Personick integral \( I_1 \) relates to the non-stationary detector noise case.) Thus, by comparing with Eq. (2-39), the second and third Personick integrals can be identified as

\[ I_2 = \frac{BW_n}{B} \]  \hspace{1cm} (2-41)

\[ I_3 = \frac{BW_{n2}^3}{3B^3} \]  \hspace{1cm} (2-42)

**Figure 2-36 Interpretation of BW\(_n\) and BW\(_{n2}\) integration bounds [169].**

In other words, the Personick integrals \( I_2 \) and \( I_3 \) are normalized noise bandwidths. It is clearly shown that these Personick integrals depend mainly on the noise components associated with the basic receiver model presented in Section 0.

### 2.5.5 Bit Error Rate

Bit errors mainly occur in the detection process due to noise and signal distortion. The voltage at the output of the linear channel is the desired signal voltage and the undesired noise voltage. The noise
voltage is caused by photodetector noise, amplifier noise and in some cases optical noise. Noise voltage may become so large that it corrupts the received signal, leading to a decision error causing a bit error. Noise power spectrum, at the output of the receiver prior to the decision circuit, is calculated by knowing the amplifier input-referred noise power spectrum and the receiver transfer function. The output noise power can be written as a sum of two components, one caused by the detector and one caused by the amplifier.

![Diagram of a receiver system showing noise components.](image)

**Figure 2-37 Illustration of the total output-referred noise [169].**

Total noise power due to the amplifier is obtained by integrating the output noise spectrum over the bandwidth of decision circuit. Figure 2-37 illustrates this integration, the input noise spectrum shown in Figure 2-37a increases with frequency because of the squared noise component, is then shaped by the frequency response of the receiver shown in Figure 2-37b, reducing an output spectrum which
rolls off rapidly at high frequencies Figure 2-37c. Because of the rapid roll-off, the precise value of the upper integration bound is uncritical and sometimes is set to infinity.

Figure 2-38 demonstrates the effect of the output noise power. It shows the relation between signal, noise and bit error rate. At the input of the decision circuit, the signal is assumed to be NRZ free of distortion and the noise is assumed as Gaussian and signal independent. NRZ pulses without ISI has a peak-to-peak value and noise has a root-mean-square rms value. The signal is sampled at the centre of each bit period producing the statistical distribution. Both distributions shown on the right-hand side of Figure 2-38 are Gaussian and have a standard deviation that is equal to the rms value of the total noise voltage at the receiver output.

Figure 2-38 Illustrations of NRZ signal + noise and noise statistics.

A threshold voltage located at the mid-point between the one and zero levels is used as a reference voltage to be compared to the sampled output voltage, determining whether one or zero was received. Then, the bit error rate can be defined as the probability that a zero is misinterpreted as a one or vice versa. Due to the limited bandwidth in the receiver, the signal has slow rising and falling edges at the input of the decision circuit thus this process also acts to clean-up the data. This model is used to derive bit error rate mathematically. The error probabilities are given by the shaded areas under the Gaussian tails in Figure 2-38. Assuming equal probabilities for one and zero each tail has a weight of half. Having two tails equal in area, one is used for calculation.
BER = \int_{Q_{BER}}^{\infty} Gauss(x) \, dx

With

\[ Q_{BER} = \frac{V_s}{2 \cdot \nu_{rms}} \]

The calculation involves normalised Gaussian distribution with an average of zero and standard deviation of one. The lower bound of the integral in this case is the difference between the one or zero level and the decision threshold level normalised to the standard derivation of the Gaussian distribution. Personick \( Q_{BER} \) is a measure of the ratio of signal and noise and is different to signal to noise ratio. The integral in the Eq. (2-43) can be expanded and approximated as follows

\[
\int_{Q_{BER}}^{\infty} Gauss(x) \, dx = \frac{1}{\sqrt{2\pi}} \int_{Q_{BER}}^{\infty} e^{-\frac{x^2}{2}} \, dx = \frac{1}{2} \text{erfc} \left( \frac{Q_{BER}}{\sqrt{2}} \right) \approx \frac{1}{\sqrt{2\pi}} e^{-\frac{Q_{BER}^2}{2}}
\]

The numerical relation between \( Q_{BER} \) and bit error rate is commonly used and shown in Table 2-2.

<table>
<thead>
<tr>
<th>( Q_{BER} )</th>
<th>BER</th>
<th>( Q_{BER} )</th>
<th>BER</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1/2</td>
<td>5.988</td>
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<tr>
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<td>10^{-5}</td>
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<td>10^{-12}</td>
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<tr>
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<tr>
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<td>7.651</td>
<td>10^{-14}</td>
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<tr>
<td>5.621</td>
<td>10^{-8}</td>
<td>7.942</td>
<td>10^{-15}</td>
</tr>
</tbody>
</table>

For an optically pre-amplified PIN detector or an APD, the assumption that the noise is signal independent is dropped as the noise on the ones is larger than the noise on the zeros due to the fact
that detector noise being significant compared with the amplifier noise. In this case, there are two Gaussian functions with two standard deviations because depending on whether the received bit is one or zero the rms noise alternates between two values. Calculating the crossover point for the optimum threshold voltage and integrating the error tails yields [169]

\[
BER = \int_{Q_{BER}}^{\infty} Gauss(x) \, dx \tag{2-45}
\]

with

\[
Q_{BER} = \frac{V_s}{v_{n,0}^{rms} + v_{n,1}^{rms}}
\]

where \(v_{n,0}^{rms}\) and \(v_{n,1}^{rms}\) are noises on zero and one. This equation can be simplified to the former one for the case of equal noise distributions.

**2.5.6 Sensitivity of OOK optical receiver**

In most optical applications, sensitivity is the primary measure of the receiver performance and it is used to compare different receiver designs. Sensitivity is explained here for a basic receiver model of NRZ signal, an extended analysis will be covered for more advanced coding schemes and receiver structures.

As a definition, the electrical receiver sensitivity is the minimum peak-to-peak signal current at the input of the receiver necessary to achieve a specified BER. The current swing at the input of the linear channel causes the output voltage swing. Thus, the electrical sensitivity can be derived by solving Eq. (2-46) for output voltage swing and dividing by mid-band \(H_o\) value of the transfer function, so that

\[
isp_{sens} = \frac{2 Q_{BER} v_n^{rms}}{H_o} \tag{2-46}
\]

The input-referred rms noise \(i_n^{rms}\) is the rms noise voltage at the output of the receiver \(v_n^{rms}\) divided by mid-band value of the transfer function [169]
The electrical sensitivity is written in more compact form as
\[ i_{sens}^{pp} = 2Q_{BER} i_n^{rms} \] (2-48)

Now, the optical receiver sensitivity is defined as the minimum optical power, averaged over time, necessary to achieve a specified BER. For a DC balanced signal with high extinction, considering the constant relation of the photo-detector current and optical power the optical sensitivity \( P_{sens} \) is given by [169]
\[ P_{sens} = Q_{BER} \frac{i_n^{rms}}{R} \] (2-49)

or more generally
\[ P_{sens} = Q_{BER} \frac{(i_{n,0}^{rms} + i_{n,1}^{rms})}{R} \] (2-50)

The optical sensitivity is based on the average signal value and the electrical sensitivity is based on the peak-to-peak signal value. Thus, the optical sensitivity depends on the pulse width of the optical signal. Due to the difference in average current in NRZ and RZ with 50% duty cycle, given an RZ and NRZ receiver with identical electrical sensitivities, the optical sensitivity of the RZ receiver will be 3 dB better than that of the NRZ receiver.

Sometimes, the optical sensitivity of a receiver with an ideal photodetector is given. This sensitivity is designated by \( \eta P_{sens} \), and is useful to compare the electrical performance of different receivers while excluding the quantum efficiency \( \eta \) of the photo-detector. With Eq. (2-51) and Eq. (2-52), this sensitivity is expressed as [169]
\[ \eta P_{sens} = \frac{hc}{\lambda q} Q_{BER} i_n^{rms} \] (2-51)

or more generally,
\[
\eta P_{sens} = \frac{hC}{\lambda q} \frac{Q_{BER}}{2} \left( i_{n,0}^{rms} + i_{n,1}^{rms} \right)
\]  
(2-52)

For very strong signals, effects such as pulse-width distortion and data-dependent jitter cause bit errors as well. Thus, besides the lower signal level, sensitivity or the sensitivity limit, there is an upper signal level, known as the overload limit, beyond which the required BER cannot be met. In similarity to the sensitivity, the input overload current is the maximum peak-to-peak signal current for which a specified BER can be achieved. Also, the optical overload power is the maximum time-averaged optical power for which a specified BER can be achieved. Thus, the dynamic range of a receiver is defined at its lower end by the sensitivity limit and at its upper end by the overload limit. The electrical dynamic range is electrical receiver sensitivity to input overload current whereas the optical dynamic range is optical receiver sensitivity to optical overload power.

### 2.6 Summary

This chapter explored some optical receiver related topics, which was important and useful to discuss. A basic receiver model in section 2.1 is used to explain relevant receiver fundamentals. As shown earlier, some of the receiver fundamentals are based on this basic receiver model. It is shown that Personick integrals depend mainly on the noise components associated with this basic model. Later in chapter 3, this model is replaced by the proposed tuned front end, hence noise sources, noise referral factions and noise bandwidths will be discussed for this extended model.

Another important issue was noise in the receiver as it is shown that the input stage of the optical receiver contributes significantly to the thermal noise current. The design of the front-end amplifier is therefore fundamental for achieving low noise receiver performance. Also, discussing noise referral is important as it will be used in chapter 3 to derive tuned front-end noise analytical expressions.
Chapter 3: OPTICAL TUNED FRONT END RECEIVERS

Section 3.1 provides a review of the use of tuned front end in optical communication. In Section 3.2, the fundamentals of the tuned circuits are discussed. Topics included are basics of serial and parallel tuned amplifiers and pulsed tuned circuits. In Section 3.3, bandwidth extension in broadband receivers is discussed. A brief analysis of magnitude and phase response, components value, and bandwidth extension ratio (BWDR) is provided in this section. In Section 3.4, original expressions are presented for two tuned front-end transimpedance amplifier (TIA) designs. In Section 3.5, an original noise analysis of tuned front-end is presented.
3.1 The use of tuned front end in optical communication

The fundamental problem this investigation is clearly addressing is the noise analysis of baseband tuned front end receivers. There is a lack of theory when it comes to designing a physical front end pre-amplifier which has different characteristics than that mentioned in the conventional noise analysis of baseband optical receivers. The baseband noise theory itself is built entirely on the fact that the receiver transfer function is independent of the preamplifier circuit and it is defined in terms of the input and output pulse shape. Despite the applicability of this definition, the system defines the receiver transfer function regardless of the preamplifier circuit components. This does not necessarily discount the validity of the model, however, with a first order system approximation this approach is not assumed to be valid over other conditions. Bearing this in mind, the use of tuned front end in baseband receiver, in particular, its noise modelling requires a critical determination. This chapter aims to develop an understanding of tuned circuits and its implementation in optical baseband communication.

Tuned front end is originally and only for heterodyning receivers according to the front-end classification in [180]. On some other optical literature, the optical receiver only must operate over a restricted bandwidth in certain modulation schemes that allows tuned front-end techniques to be employed [181]. The later claim excludes the use of tuned front end in baseband direct detection receivers.

3.1.1 Early use of tuned front end

The use of a Percival coil in television camera head amplifiers produces a significant improvement in the signal to noise ratio (SNR) [182]-[184]. This was an initiative for Hullett [185] to construct a front-end module for optical communication. A Percival coil connected between the photodetector and amplifier in the receiver of optical communication system offers the possibility of improved signal to noise ratio. Hullett’s analysis is based on a circuit module which has photodiode capacitance
with the input capacitance of the FET. The Percival coil with inductance $L$ and resistance $r$ connects the photodiode and the FET preamplifier. Also, it was assumed that all noise is coming from the first stage.

In a system employing an APD and binary amplitude shift keying (BASK), it is shown that the signal power requirements are improved for a given bit error rate due to Percival coil effect on the front end. This improvement is increasing along with the bit rate and avalanche gain. The use of a coil lowers the optimum avalanche gain results in eliminating those problems associated with high avalanche gain: high bias voltages, thermal instability, high cost, and reduced effective target area [186]. A 400 Mbit/s optical frequency shift keying (FSK) transmission experiment over 270 km of single-mode fibre showed the advantage of using an LC network in the receiver front end [187].

A resonant preamplifier (parallel inductance) is used to cancel the effect of the stray capacitance of the photodetector and reduce the intermediate frequency (IF) noise level. In a continuous phase frequency shift keying (CPFSK) transmission system, this resonant preamplifier is used to give a receiver sensitivity improvement of 1.3 dB over the conventional preamplifier [188]. Tuned front-end receiver allows the coherent system to be designed at high intermediate frequencies. A theoretical analysis in [189] shows how the tuned front end can be a solution for the frequency-squared noise spectral density components that ASK optical receivers suffer from, due to large IF bandwidth. Gimlett [190] reported an ultra-wide bandwidth and low noise optical receiver to be used in either multi-gigabit direct detection or coherent heterodyne systems. This tuned front end was used to reduce the noise at high frequencies.

A narrow-band resonant direct detection PIN-FET receiver is constructed for subcarrier multiple access networks in [191]. Theoretical and experimental analyses are given for serial and parallel peaking inductor in the receiver front end resulting in minimising the frequency dependent thermal noise and leaving shot noise as the ultimate limitation. The measured signal to noise ratio was in excellent agreement with that predicted by the noise analysis.
In multi Gbit/s applications, multi Gbit/s tuned optical receivers are constructed with high transimpedance gain and low averaged input equivalent noise current density. The results obtained in [192] and [193] were significantly improved compared to the existing optical receivers. A noise performance analysis of these Gbit/s tuned optical receivers is presented in [194].

Based on the theory presented in [185]-[195], more advanced tuned front-end techniques are demonstrated in [196]-[198]. The most effective technique is the so-called mixed tuning configuration which is a mixture between a low pass and a band pass filter where three inductances are used to tune out the influence of the parasitic capacitances in the broad frequency range of interest. The results show an improvement of 3dB in noise performance compared to what can be obtained using simple parallel or serial tuning schemes. T-equivalent circuit of transformer coupling is another technique used in tuned front-end receivers. In [198], a theoretical analysis of the noise performance of optical receivers with front-end tuning, suitable for wide-band coherent systems, is presented. It is proved that any tuning is better than none in the wide bandwidth designs considered. In this investigation, five different forms of tuned front-end receivers are compared, showing an improvement of up to 12 dB in thermal noise power compared to non-tuning front-end receiver. In [199] and [200], analytical expressions for calculating transimpedances and equivalent input noise densities of these five tuned optical receivers are presented.

The tuned front end is also used in optical receiver design for microwave subcarrier multiplexed lightwave systems (SCM). It is shown in [201]-[203] that the use of a tuned front end can improve the receiver performance in terms of noise, bandwidth and receiver sensitivity. A general optical receiver design method is presented in [205], It is based on the synthesis of optimum noise matching networks. Based on the noise figure concept in conjunction with the broadband matching theory, the design directly utilises the active device noise parameters such as minimum noise figure, noise resistance and optimum source impedance which are readily available at microwave frequencies. The analysis established the general noise-matching requirements of the tuning network that result in
getting the minimum obtainable equivalent input noise current and the fundamental noise limit in tuned receivers. An objective of obtaining both low noise and gain equalised characteristics simultaneously in tuned optical receivers is presented in [206]. Tuned front end is also used in monolithic microwave integrated circuit MMIC for optical communication systems [207]- [209].

3.1.2 The use of tuned amplifiers in modern applications

Bandwidth extension CMOS with optimized on-chip inductors [210] is a technique for enhancing the bandwidth of gigahertz broad-band circuitry by using optimized on-chip spiral inductors as shunt peaking elements. The main purpose of this technique is to enhance the bandwidth of broadband amplifiers and push the performance limits of low-cost CMOS implementations used in optical communication receivers. More detailed study for different inductive peaking is also presented in [211] with optimisation extended to cover more advanced circuit configurations to be used in broadband applications in both radio and optical communication. The basic theory behind the optimisation relies on the introduction of an inductance in series with the load resistance which alters the frequency response of the FET amplifier. This technique, called shunt peaking, enhances the bandwidth of the amplifier by transforming the frequency response from that of a single pole to one with two poles and a zero. The zero is determined solely by the time constant and is primarily responsible for the bandwidth enhancement. The frequency response of this shunt peaked amplifier is characterised by the ratio of the L/R and RC time constants. There are three interesting ratios for optimum group delay, maximally flat response and maximum bandwidth. This optimisation is based on work done for radio frequency (RF) circuits which is presented in [212], giving a maximum bandwidth extension of 1.8 while advanced techniques demonstrated total extended bandwidth of 4 times. Although the bandwidth extension technique is proposed for wideband amplifiers used in optical baseband receivers, there is no appropriate noise analysis for these front-end receivers.
3.1.3 Noise integrals misconception

There is a common misconception that in the case of tuned amplifiers, bandwidth extension technique or other tuned front receivers, the total input-referred noise can still be written in term of white and coloured noise terms using Personick integrals [213][215]. Similarly, a measured frequency response of optical receiver with a serial inductor in the front end is reported in [216]-[217]. The theoretical receiver model used in [216] and [217] is presented in [218], the expression given in this model for the input referred spectrum does not include the effect of the tuned front end on referred noise. This is somehow confusing because the response of the measured equivalent input noise current spectral density curve does not match what is theoretically assumed. This difference is because the measured spectrum indicates the effect of the tuned circuit on the input referred noise; however, this effect is not included in the theoretical analysis. The validity of this work is not discounted, but it shows a lack of understanding between the theoretical noise modelling and measured noise spectral curve.

3.1.4 Summary

Literature review on the use of tuned front end in optical communication can be concluded in the following key points:

- Hullett’s receiver model is one of the very early tuned front-end models in optical communications, showing that the signal power requirements are improved for a given bit error rate due to inductance effect on the front end, this improvement is increasing along with the bit rate.

- The tuned front end has been proposed across a wide range of optical communication applications. It allowed the coherent systems to be designed at high intermediate frequencies. It is also used in multi-gigabit direct detection systems and subcarrier multiple access networks.
For coherent systems, there is a theoretical analysis of the noise performance of optical receivers with front-end tuning. There are also several attempts to develop an algorithm choosing the values of the tuning components for noise optimisation in heterodyne receivers.

The most known tuned front-end circuits are serial tuning, parallel tuning, mixed tuning, T-equivalent circuit. At some point, serial tuning was the only suitable tuned front end for baseband transmission systems, any other tuning configuration was excluded for this type of communication.

Tuned amplifiers were proposed for bandwidth extension in CMOS. The main purpose of this technique is to enhance the bandwidth of broadband amplifiers.

It is proved that any tuning is better than none in the wide bandwidth designs. However, most of the noise optimisation was given for high-impedance FET receivers.

There is a clear deficiency in noise modelling of baseband direct detection receivers. In addition, there is a misuse of Personick integrals to obtain the total input referred noise power. Therefore, this investigation aims to:

- suggest expressions for transimpedance and optimised frequency response in accordance with baseband theory.
- perform noise referring to obtain exact expressions of referred noise, noise integrals and input referred noise power.

### 3.2 Tuned circuits

The fundamentals of the tuned amplifier are discussed in this section. A brief analysis of serial tuned and the parallel tuned amplifier is provided. This section also includes a brief explanation of frequency and phase response of tuned circuits. The signal shape and pulsed tuned circuit are discussed in this section to further understand tuned circuit performance. The analysis of tuned circuit in Section 3.2.1 is cited from [219].
3.2.1 Tuned circuit fundamentals

The basic principle of a tuned amplifier is that a load resistor of an amplifier is replaced by a tuned circuit. This arrangement can amplify a signal over a narrow band of frequencies centred at the central frequency $f_c$ of a tuned circuit, therefore, it is also called a narrow band amplifier. In tuned amplifier design, the central frequency is centred to a desire frequency and side frequencies are extended to a desired range for other amplifications. Figure 3-1 illustrates the frequency response of a tuned circuit resonating at a particular frequency. The response is maximum at the resonant frequency and it falls sharply for frequencies below and above the resonant frequency.

![Frequency response of the tuned amplifier.](image)

Inductive and capacitive effects of tuned circuit cancel each other at the resonance frequency. The circuit is purely resistive in that voltage and current are in phase, therefore, the circuit may be used as a load for an amplifier. At high frequencies above resonance, the circuit becomes capacitive while it becomes inductive at frequencies below resonance. Figure 3-2 shows the parallel resonant circuit, this circuit resonates at a resonant frequency $f_o$, the admittance of this circuit is given by

$$Y(\omega)_{\text{parallel}} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C \quad (3-1)$$

At resonance, the imaginary part of Eq. (3-1) is zero, equating this part to one yield
\[ \omega_0 C = \frac{1}{\omega_0 L} \quad (3-2) \]

Therefore, the resonance frequency is given by

\[ f_0 = \frac{1}{2\pi \sqrt{LC}} \quad (3-3) \]

![Parallel resonant LC circuit](image)

**Figure 3-2 Parallel resonant LC circuit.**

The quality factor (Q) is an important characteristic of the inductor, it is the ratio of reactance to resistance and it is unitless. The purity of an inductor is measured by its Q thus the higher Q of an inductor the fewer losses there in the inductor. Q-factor is also defined as the measure of the efficiency of how an inductor stores energy and is represented as

\[ Q = \frac{\text{Maximum energy stored per cycle}}{\text{Energy dissipated per cycle}} \quad (3-4) \]

![LC resonant circuit at FET output](image)

**Figure 3-3 LC resonant circuit at FET output.**

As the components in Figure 3-2 are in parallel, voltage is common thus the maximum energy of the circuit can be written in term of the capacitance. Equating the maximum energy of the circuit and energy loss per cycle, therefore, Q-factor can be re-written as
\[ Q = \omega_o C = \frac{R}{\omega_o L} = R \frac{L}{\sqrt{C}} \]  \hspace{1cm} (3-5)

For the circuit shown in Figure 3-3 LC resonant circuit, the current gain can be determined by the load resistor \( R_L \) since the circuit is resistive at resonance. Thus, current gain \( A_v \) is given by

\[ A_v = -g_m R_L \]  \hspace{1cm} (3-6)

In Figure 3-4a, R is in series with the inductor. At high Q-factor values, \( \omega^2 L^2 \gg R^2 \) thus, the admittance can be approximated as

\[ Y(\omega)_{series} = \frac{1}{R + j\omega L} \approx \frac{R}{\omega^2 L^2} + \frac{1}{j\omega L} \]  \hspace{1cm} (3-7)

Figure 3-4 Tuned circuit (a) R in series with L. (b) parallel tuned circuit with \( R' \) in parallel with L.

The approximation in Eq. (3-7) transforms this series circuit to a parallel form shown in Figure 3-4b, with the parallel resistor \( R' \) given as

\[ R' = \frac{\omega^2 L}{R} \]  \hspace{1cm} (3-8)
This relation in Eq. (3-8) is used to transform series circuit form to parallel form and vice versa. The only R is replaced by $R'$, and the inductor L does not change in this transformation. The new Q-factor for the parallel circuit in Figure 3-4a becomes

$$Q_{\text{series}} = \frac{\omega_0 L}{R} \quad (3-9)$$

Another important parameter is the frequency variation $\Delta f$. It indicates the frequency deviation from resonance frequency and it is given as

$$\Delta f = \frac{f - f_0}{f} \quad (3-10)$$

At resonance $\Delta f = 0$, the voltage of parallel resonant circuit is given by the product of current and circuit resistance. The impedance expression for parallel resonant circuit is given in terms of Q-factor and $\Delta f$ as

$$Z = \frac{R}{1 + j2Q\Delta f} \quad (3-11)$$

![Figure 3-5 An example of the parallel tuned circuit frequency and phase response.](image)
At a half power bandwidth limit the real and reactive terms of the circuit impedance are equal, equating expression in Eq. (3-11) with the below resonance band limit \( f_1 \) (lower frequency) and the above resonance band limit \( f_2 \) (upper frequency), the circuit bandwidth can be expressed as

\[
BW_{parallel} = f_2 - f_1 = \frac{f_o}{Q} \tag{3-12}
\]

Therefore, the bandwidth of a parallel tuned circuit is inversely proportional to the circuit Q-factor.

An illustration of parallel tuned circuit frequency response and phase response is shown in Figure 3-5.

This section gave a brief explanation of resonant circuits fundamentals, highlighting some important parameters of resonant circuits such as quality factor (Q-factor), resonant frequency \( f_o \), upper frequency \( f_1 \), lower frequency \( f_2 \), and the resonant circuit bandwidth \( BW_{parallel} \). In the following section, the response of a tuned circuit is briefly explained.

### 3.2.2 Pulse response of the tuned circuit

The signal shape and pulsed tuned circuit are discussed in this section to more understand tuned circuit performance in the relation to rectangular pulses. The rectangular pulse is the basic waveform of digital systems. Figure 3-6 shows the ideal form of a rectangular pulse.

![Figure 3-6: An ideal rectangular pulse.](image-url)
If such an ideal rectangular pulse is applied at the input of a tuned circuit, the output pulse will not be an ideal rectangular pulse. A tuned circuit in Figure 3-7 is considered to more understand the shape of the output pulse. It is assumed that photodiode capacitance is a part of the tuned circuit while the photocurrent is represented as the current source that brings into being the step input to the tuned filter. The refereed input resistor of the transimpedance $R_t = R_f/(A + 1)$ is taken as the tuned circuit resistor $R$. The pre-amplifier capacitance is neglected thus the photodiode capacitance $C_d$ is taken as the tuned circuit capacitance $C$.

![Figure 3-7 An example of a TIA with a tuned circuit at the front-end of optical receiver.](image)

The equivalent input circuit of this TIA front end is illustrated in Figure 3-8, assuming the TIA has an infinite bandwidth and large gain. The transfer function of this circuit $Z(\omega)$ when the output is taking across $R$ is given by
\[ Z(\omega) = \frac{v_o}{i} = \frac{R}{1 - \omega^2 LC + j\omega RC} \]  

(3-13)

It is assumed that \( R \) is significantly large that this circuit has a low Q-factor value, the natural resonance frequency \( \omega_{nat} \) when the circuit is stimulated by a sine wave is given by

\[ \omega_{nat} = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}} \]  

(3-14)

In case of the rectangular pulse, there will be forced oscillation \( \omega_f \) which is different to the natural resonant frequency and is given by

\[ \omega_f = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \]  

(3-15)

A step input \( I(s) \) is considered at the circuit input thus the response of the circuit \( v(s) \) is given as

\[ v(s) = \frac{1}{LC} \cdot \frac{R}{S^2 + \frac{R}{L} + \frac{1}{LC}} \cdot \frac{K}{s} e^{-xs} \]  

with

\[ I(s) = \frac{K}{s} e^{-xs} \]  

(3-16)

This gives the response of the circuit as a function of time \( v(t) \) that represents the output amplitude and its shape and given by

\[ v(t) = \frac{k}{C} \cdot \frac{2a}{a^2 + \omega_f^2} \left[ 1 - e^{-at} \cos \omega_f t - \frac{a}{\omega_f} e^{-at} \sin \omega_f t \right] \]  

(3-17)

with

\[ a = \frac{R}{2L} \]

The time of peak voltage \( t_{pk} \) is given as \( \frac{\pi}{\omega_f} \), so that
\[ v(t_{pk}) = \frac{k}{C} \cdot \frac{2a}{a^2 + \omega_f^2} \left[ 1 + e^{-\frac{a\pi}{\omega_f}} \right] \]  

(3-18)

For a pulsed tuned circuit, the major problem is that the output signal would have ringing. These ringings have characteristics that can be known thus the circuit components can be adjusted so that the output signal is valid for signal detection.

![Distorted pulse shape](image)

**Figure 3-9 Distorted pulse shape [219].**

Figure 3-9 illustrates the characteristics of a tuned circuit output pulse, it is used to explain related properties which helps developing an understanding for the pulsed tuned circuit:

- Overshoot is the amount by which the output pulse exceeds its peak amplitude. This is a result of the inclusion of inductive circuit elements, producing a second order response with conjugate poles.
- Rise time is the time taken by the output pulse to rise from 10% of peak pulse amplitude to 90%.
• Backswing is the portion of the trailing edge that extends below zero amplitude level, it is also called null.

Ringing (overshoots and backswing) on the output pulse will interfere with adjacent pulses, causing inter-symbol interference (ISI) and distortion unless the resonant frequency is set to minimise these effects. In some digital baseband modulation formats, where a separation between pulses or an acceptable amount of ISI is available within the system, ringing effect will become less significant.

As the central frequency and Q-factor determine the frequency response of the tuned circuit, therefore, the design of the tuned circuit should give a huge attention to the signal shape to satisfy ISI requirements. At the same time, the design should maintain the highest possible signal to noise ratio at receiver output prior to detection.

![Pulse width diagram](image.png)

**Figure 3-10 illustration of pulse time slot and pulse width.**

A frequency optimisation technique is suggested, this technique may be proposed to eliminate the negative effect of ringing hence determine the required time separation. The values of R and L in equation (3-13) are equated for different central frequencies and are given as in Table 3-1. Central frequency is equated to the time slot ($t_s = \frac{1}{B}$). Three cases are considered in Table 3-1, these cases are $\frac{2}{t_s}$, $\frac{1}{t_s}$, and $\frac{1}{2t_s}$. 

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Figure 3-11 illustrates the effect of different central frequency on the output pulse shape. The shape of the input pulse is assumed to have the characteristic of the ideal rectangular pulse in Figure 3-6). Figure 3-12 shows the step response of each central frequency.

**Figure 3-11** Output pulse shape (a) input pulse width equals the time slot ($f_0 = \frac{1}{2} t_s$, $Q = 1$). (b) input pulse width equals half the time slot, ($f_0 = \frac{1}{t_s}$, $Q = 1$). (c) input pulse width equals time slot, ($f_0 = \frac{2}{t_s}$, $Q = 1$).
Another suggested optimisation technique is to limit the time constant for a certain time duration to control the level of ISI, by making the resonant frequency and Q-factor variables. For zero ISI, the

Figure 3-12 Tuned front-end step response (a) $f_o = \frac{1}{2t_s}$, $Q = 1$. (b) $f_o = \frac{1}{t_s}$, $Q = 1$. (c) $f_o = \frac{2}{t_s}$, $Q = 1$. 
time constant of the tuned circuit is equated to the pulse width to accommodate ringing by considering the rise and falling time of the output pulse. The time constant may be evaluated to fit in a range of time between the time slot and pulse width, as in

\[ t_p \leq \tau_r \leq t_s \]  \hspace{1cm} (3-19)

In this situation, the design of the receiver front end response will give the attention to the availability of time separation. Therefore, the resonant frequency and tuned circuit components are obtained as a function of the time constant \( \tau_r \) and quality factor \( Q \), so that

\[ \tau_r = \frac{10 \omega_o}{Q} \]  \hspace{1cm} (3-20)

\[ R = \frac{10 L}{\tau_r} \]  \hspace{1cm} (3-21)

\[ L = \frac{1}{C} \left( \frac{\tau_r^2}{25^2 + (10 Q)^2} \right) \]  \hspace{1cm} (3-22)

![Figure 3-13 illustration of time definition in a digital codeword.](image)

In this case, the bandwidth of the tuned circuit is not controlled and the output pulse shape will vary depending on damping condition. Therefore, the values of Q-factor and central frequency will need to be adjusted to obtain an acceptable pulse shape. The disadvantage of this approach is that the 3-dB bandwidth is not controlled. Therefore, in some cases, the receiver bandwidth will be relatively large compared to the bit rate, causing a higher equivalent noise bandwidth.
Analysis and fundamentals of tuned circuits are presented in this section, frequency response and pulse shape for tuned front end designed depending on Q-factor, central frequency and time constant are briefly discussed in this section. As a conclusion, this method makes the frequency response of receiver less controlled, however, it is an aid to analyse the output pulse shape of tuned circuit (time domain analysis). In the following section, the bandwidth extension amplifiers are presented, with a brief analysis of the frequency response of these amplifiers.

### Table 3-1 R and L equated values

<table>
<thead>
<tr>
<th>( f_0 )</th>
<th>R</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2}{t_s} )</td>
<td>( \frac{4\pi}{t_s} \cdot \frac{L}{Q} )</td>
<td>( \frac{1}{C} \cdot \frac{t_s^2}{\pi^2} \left( \frac{Q^2}{4 + 16Q^2} \right) )</td>
</tr>
<tr>
<td>( \frac{1}{t_s} )</td>
<td>( \frac{2\pi}{t_s} \cdot \frac{L}{Q} )</td>
<td>( \frac{1}{C} \cdot \frac{t_s^2}{\pi^2} \left( \frac{Q^2}{1 + 4Q^2} \right) )</td>
</tr>
<tr>
<td>( \frac{1}{2t_s} )</td>
<td>( \frac{\pi}{t_s} \cdot \frac{L}{Q} )</td>
<td>( \frac{1}{C} \cdot \frac{t_s^2}{\pi^2} \left( \frac{4Q^2}{1 + 4Q^2} \right) )</td>
</tr>
</tbody>
</table>

### 3.3 Tuned amplifiers

Although shunt-peaking circuits are primarily designed for bandwidth extension in broadband receivers, it is discussed here to better understand the frequency response of these tuned circuits. A brief analysis of magnitude and phase response, components value, and bandwidth extension ratio (BWER) is provided in this section. This analysis is adapted in Section 3.4 to optimise the design of a tuned front-end TIA for baseband receiver.

#### 3.3.1 Shunt-peaking amplifier

A simple common-source (CS) amplifier illustrated in Figure 3-14. For simplicity, it is assumed that the small signal frequency response of this amplifier is determined by a single dominant pole, which
is determined by the output load resistance $R$ and the load capacitance $C$ shown in Figure 3-14. The transfer function of this arrangement is given by [210]

$$\frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{g_mR}{1 + j\omega RC} \quad (3-23)$$

![Figure 3-14](image)

**Figure 3-14 (a) Simple CS amplifier (b) equivalent small signal model [210].**

The introduction of an inductance in series with the load resistance alters the frequency response of the amplifier shown in Figure 3-15. This technique, called shunt peaking, enhances the bandwidth of the amplifier by transforming the frequency response from that of a single pole to one with two poles and a zero. The transfer function of this arrangement is given by [210]

$$\frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{g_m(R + j\omega L)}{1 + j\omega RC - \omega^2 LC} \quad (3-24)$$

Reduced rise time is another result of increasing the bandwidth, the inductor delays current flow to the resistive branch, therefore, more current initially charges $C$ which reduces rise time [210], [220].
Figure 3-15 (a) CS amplifier with shunt peaking. (b) equivalent small signal model [210].

The zero is determined by the L/R time constant and is primarily responsible for the bandwidth enhancement. The frequency response of this shunt peaked amplifier is characterised by the ratio of the L/R and RC time constants. This ratio is denoted by $m$ so that $L = \frac{R^2C}{m}$. Figure 3-16 and Figure 3-17 show the frequency response of the shunt-peaked amplifier for various values of $m$. The case with no shunt peaking ($m = 0$) is used as the reference so that its low-frequency gain and its (3-dB bandwidth) are equal to one. The frequency response is plotted for different values of $m$. The 3-dB bandwidth increases as $m$ increases. The maximum bandwidth is obtained when $m=1.41$ and yields an 85% improvement in bandwidth. However, this comes at the cost of significant gain peaking. A maximally flat response is obtained for $m=2.41$ with a still impressive bandwidth improvement of 72%. When $m=3.1$, there is an improvement of 60% with a linear phase response up to the 3-dB bandwidth. This case is called the optimum group delay case and is desirable for optimising pulse fidelity in broadband systems that transmit digital signals as mentioned in [210], [220].
Figure 3-16 Frequency response of shunt peaking.

Figure 3-17 Phase response of shunt peaking.*

* The values of m are presented in [51]. The data in fig. (3-16) and (3-17) is reproduced.
A compensation capacitance may be used to eliminate peaking with maximum bandwidth extenuation ratio. Figure 3-18 shows a CS amplifier incorporating bridged shunt network [211], [221]-[222].

![Diagram of CS amplifier with a bridged-shunt amplifier](image)

**Figure 3-18** A CS amplifier with a bridged-shunt amplifier [211].

The transfer function of compensation configuration is given as [211]

\[
\frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{1 + \frac{j\omega L}{R} - \omega^2 LC K_B}{1 + j\omega RC - (1 + K_B)\omega^2 LC - j\omega^3 LC^2 R K_B}
\]  

(3-25)

where \( K_B \) is compensation ratio \( (C_B = K_B C) \). Comparing to Eq. ((3-24), \( C_B \) introduces another pole and zero in the transfer function. Therefore, the value of \( C_B \) should be chosen that it is large enough to negate peaking but small enough to not significantly alter the gain response. Figure 3-19* shows magnitude responses for the bridged-shunt-peaked circuit for several practical values of along with the shunt-peaked and uncompensated case. For \( K_B = 0.3 \), there is approximately 84% improvement in bandwidth with a flat gain response, in contrast to the shunt-peaked design with a nearly identical bandwidth improvement but 1.5 dB of peaking. Figure 3-20 compares the magnitude response of compensated and uncompensated shunt peaking design. In Figure 3-20, the dotted line represents

* The data in fig. (3-19) is reproduced [210].
frequency response of the CS amplifier shown in Figure 3-14 (no shunt peaking). The black solid and dashed lines in the figure represent the frequency response of the shunt peaking design shown in Figure 3-15 (CS amplifier with shunt peaking, with maximum bandwidth and maximally flat response cases). The red solid line represents the frequency response of the compensated shunt peaking (bridged-shunt-peaked) design shown in Figure 3-18. The later response has the advantage that the maximum bandwidth is achieved for a larger value of m which translates to a smaller inductance value.

Figure 3-19 Frequency response of CS amplifier with a bridged-shunt amplifier $K_B =$

$0, 0.1, 0.2, 0.3$. 

$K_B = 0, m = 1.4$

$K_B = 0.1, m = 1.8$

$K_B = 0.2, m = 2.2$

$K_B = 0.3, m = 2.4$

Normalised frequency

Magnitude (dB)
Figure 3-20 Frequency response comparison (black solid: shunt peaking CC, black dashed: shunt peaking CC, black dotted: CC without shunt peaking and red solid CC with bridged shunt peaking).

3.3.2 Bridged-shunt-series peaking

If the drain parasitic $C_1$ is significant, better bandwidth extension ratio is achieved using capacitive splitting. This is done by inserting an inductor to separate the total load capacitance as shown in Figure 3-21.

This technique is called bridged-shunt-series peaking. For this configuration, the normalised transfer function is given by [211]

$$
\frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{1}{1 + j\omega RC - \omega^2 LC (1 - K_c) - j\omega^3 RLC^2 K_c (1 - K_c)}
$$

(3-26)
The separation of $C_1$ from C creates another pole which affects the frequency response. Bandwidth extension ratio would increase as the capacitance ratio increases, up to a maximum of 2.52 for splitting ratio of 0.3. For higher splitting ratio, the bandwidth extension continues to improve with a significant peaking [211]. Figure 3-22 shows 3-dB bandwidth improvements for different splitting ratio. Considering the step response of this amplifier, the transistor charges $C$ ($C_1+ C_2$), but with L inclusion, $C_1$ is only charged initially because L delays current flow to the rest of network. This reduces risetime at the drain and increases the bandwidth [211], [220]. It is important to note that
these amplifiers incorporating a tuned circuit at the output of CC stage which is then used as a broadband tuned amplifier.

However, the interpretation of the frequency response presented in this section provides an advantageous progression for optimising the tuned front-end receiver design with the 3-dB bandwidth more controlled compared to optimisation method suggested in Section 3.2. Therefore, in section 3.4, this theory is further developed to optimise the frequency response of tuned front-end receiver design. Then, in Section 3.5, the noise analysis is presented.

3.4 Tuned front-end transimpedance amplifiers

In this section, transimpedance expressions are suggested for two tuned front-end transimpedance amplifier (TIA) designs which are the tuned A TIA and tuned B TIA. These expressions are equated so that these two TIA designs have the same 3-dB bandwidth as an equivalent non-tuned front-end design. The analysis for non-tuned TIA design follows the theory presented in [160], [162], [165], [167]. The analysis and expressions for tuned front end TIA designs are built on the theory presented in [210]-[212], [221], [222].

3.4.1 Non-tuned front end

The circuit of a basic shunt-feedback TIA is shown in Figure 3-23. The photodetector is connected to the input of an inverting voltage amplifier, which has a feedback resistor $R_f$ leading from its output to the input. The current $I_p$ from the photo-detector flows into $R_f$, and the amplifier output responds in such a way that the input remains at virtual ground.

As a result, the output voltage $V_o$ equals to $-R_f I_p$, and thus the transimpedance is approximately $R_f$.

It is assumed that the amplifier transistors have an infinite bandwidth. The voltage gain of the inverting amplifier is $A$ and its output resistance is zero. The input resistance is taken to be infinite, a good assumption for an FET amplifier. For bipolar input amplifier, a transistor with larger input
impedance can be used. The referred input resistor $R_t$ is approximated by $R_f/(1 + A)$ and assumed to be the total input resistance of the TIA. The photodetector capacitance $C_d$ and the input amplifier capacitance $C_{in}$ appear in parallel from an AC point of view, both are combined into a single capacitance $C_T = C_d + C_{in}$.

![Diagram of a basic shunt-feedback TIA.](Image)

**Figure 3-23 basic shunt-feedback TIA.**

Given these assumptions, the frequency-dependent transimpedance is given by [169]

$$Z(\omega) = -R_T \frac{1}{1 + j\omega/\omega_p}$$

where $R_T$ and $\omega_p$ are given as

$$R_T = \frac{A}{A + 1} R_f$$  \hspace{1cm} (3-28)

$$\omega_p = \frac{A + 1}{R_f C_T}$$  \hspace{1cm} (3-29)

The closed-loop input resistance $R_t$ is (1+A) times smaller than $R_f$. This is one of shunt feedback TIA advantages that the bandwidth is A+1 times larger than high impedance design.
Figure 3-24 Open-loop frequency response of a TIA with a single-pole feedback amplifier.

In this analysis, it is assumed that the amplifier has infinite bandwidth thus, TIA bandwidth is determined by the low-frequency pole at \( 1/R_f C_T \) due to the low pass formed by feedback resistor \( R_f \) and total capacitance \( C_T \).

### 3.4.2 Tuned front end (A)

A TIA with a serial inductor introduced to the feedback loop is shown in Figure 3-25. The introduction of the inductor alters the frequency response of the TIA in the same manner as in tuned amplifier (cf. Section 3.3.1), provided that transistor load resistor \( R \) is replaced by the TIA feedback resistor \( R'_f \), the inductor is introduced to the feedback loop, and the tuned circuit capacitance is formed by the total capacitance at the TIA front end. Given these assumptions as for non-tuned front end, the frequency dependent transimpedance of the tuned front-end TIA \( Z_A(\omega) \) becomes

\[
Z_A(\omega) = -R_T \cdot \frac{1 + \frac{j\omega}{\omega_f}}{1 - \frac{\omega^2}{\omega_f \omega_T} + \frac{j\omega}{\omega_T}}
\]  

where

\[
R_T = \frac{A}{1 + A \cdot R'_f}
\]  

\( (3-30) \)

\( (3-31) \)
\[ \omega_T = \frac{(A + 1)}{R'_f C_T} \]  
(3-32)

\[ \omega_f = \frac{R'_f}{L'_f} \]  
(3-33)

**Figure 3-25 Feedback TIA with a serial \(L_f\) (Tuned A)**

If the feedback resistor in Eq. (3-29) kept the same, the tuned front-end bandwidth will further extend compared to a non-tuned front end due to the introduction of the feedback inductance. Therefore, with the same total capacitance in both TIA, the feedback resistor \(R'_f\) will be equated to have the same -3dB bandwidth. The new feedback resistor \(R'_f\) is given by

\[ R'_f = \Delta R_f \cdot R_f \]  
(3-34)

Where \(\Delta R_f\) is given by [212]

\[ \Delta R_f = \sqrt{\left(-\frac{m^2}{2} + m + 1\right) + \sqrt{\left(-\frac{m^2}{2} + m + 1\right)^2 + m^2}} \]  
(3-35)
\[ \Delta_{L_f} = \frac{R_f^2 C}{L_f} \] (3-36)

The first effect of tuned front end appears here as the value of transimpedance increases for the same receiver bandwidth. For the front end in Figure 3-25, \( R_f \) would increase by a factor of 1.8 for \( \Delta_{L_f} \) value of 1.8 without a significant peaking. The maximum flat response is at \( \Delta_{L_f} \) value of 2.41, best group delay at value of 3.1, \(|Z(\omega)| = R\) at value of 2 and the maximum increment for transimpedance is at \( \Delta_{L_f} = 1.41 \) with 1.5 dB peaking. These are the same ratios calculated for similar transfer function of tuned amplifier (not at front end) for bandwidth extension rather than feedback increment.

**Figure 3-26** The frequency response of Tuned A for different m ratios.

The frequency response in Figure 3-26 is not exactly the frequency response in Figure 3-16. The difference is that the frequency response plotted in Figure 3-16 is for \( m \) equated to obtain bandwidth extension (the tuned circuit is introduced at transistor output) as in FET load network. However, the
frequency response of all values of $m$ are equated here to have the same 3-dB bandwidth, hence higher $R_f$ is obtained (the tuned circuit is introduced at TIA input).

3.4.3 Tuned front-end (B)

A shunt-feedback TIA with a serial inductor connects the PIN photodetector and preamplifier input is shown in Figure 3-27. The introduction of the inductor alters the frequency response of the TIA in the same manner as in tuned amplifier (cf. Section 3.3.2), provided that transistor load resistor $R$ is replaced by the TIA feedback resistor $R''_f$, the inductor is introduced to TIA front end, and the tuned circuit capacitance is formed by the total capacitance at the TIA front end.

![Figure 3-27 Feedback TIA with a serial $L_B$ (Tuned B)](image)

The feedback resistor appears at the preamplifier input as Miller resistor in parallel with the input impedance, assuming finite amplifier bandwidth and the relation between $C_d$ and $C_{in}$ is defined such that

\[
C_d = (1 - a)C, \quad 0 \leq a \leq 1 \tag{3-37}
\]

\[
C_{in} = aC \tag{3-38}
\]
This configuration will result in a more complex frequency response \( Z_B(\omega) \), given as

\[
Z_B(\omega) = -R_T \frac{1}{1 - \frac{j\omega^2a(1-a)}{\omega_f\omega_T^3} - \frac{(1-a)\omega^2}{\omega_f\omega_T^2} + \frac{j\omega}{\omega_T}} \tag{3-39}
\]

where

\[
R_T = \frac{A}{A + 1} \cdot R''_f \tag{3-40}
\]

\[
\omega_T = \frac{(A + 1)}{R''_fC} \tag{3-41}
\]

\[
\omega_f = \frac{R''_f}{L_B(A + 1)} \tag{3-42}
\]

\[
R''_f = \Delta R_f R_f \tag{3-43}
\]

\[
L_B = \frac{R''_f^2 C}{(A + 1)\Delta L_B} \tag{3-44}
\]

The relations in Eq. (3-40) to Eq. (3-44) are equated such that the 3-dB bandwidth of Tuned B front-end receiver is equal to the same bandwidth of tuned-A and non-tuned front-end receivers. The values for \( \Delta L_B \) and \( \Delta R_f \) are previously discussed in the literature for similar transfer functions of bandwidth extension tuned amplifiers and are given in Table 3-2. It is clear that \( R_f \) would increase by a factor of 1.58 and 1.87 for \( \Delta L_B \) value of 1.8 if a splitting ratio achieved at 0.1 or 0.2 respectively. If the splitting ratio is 0.3, the transimpedance would increase by 2.52 if the inductor set to give a ratio \( \Delta L_B \) of 2.4.

The main result of having higher transimpedance for the same receiver bandwidth is the reduction in thermal noise by the same amount.

The transfer functions given in Eq. (3-30) and Eq. (3-39) are slightly different from the transfer function of similar tuned amplifier configurations used in bandwidth extension receiver designs. The main differences are that the introduction of an inductor results in increasing the feedback resistor.
rather than the amplifier bandwidth. The resistor in equations here is accounted for the feedback loop resistor rather than the FET bias and load resistors. As discussed earlier, these modifications will produce changes in receiver noise model.

Table 3-2 Total input capacitance splitting ratio $a$ and feedback increment $\Delta R_f$ [211].

<table>
<thead>
<tr>
<th>Splitting ratio ($a$)</th>
<th>$\Delta L_B$</th>
<th>$\Delta R_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>1.41</td>
</tr>
<tr>
<td>0.1</td>
<td>1.8</td>
<td>1.58</td>
</tr>
<tr>
<td>0.2</td>
<td>1.8</td>
<td>1.87</td>
</tr>
<tr>
<td>0.3</td>
<td>2.4</td>
<td>2.52</td>
</tr>
<tr>
<td>0.4</td>
<td>1.9</td>
<td>2.75</td>
</tr>
<tr>
<td>0.4</td>
<td>2.5</td>
<td>3.17</td>
</tr>
<tr>
<td>0.5</td>
<td>1.5</td>
<td>2.65</td>
</tr>
</tbody>
</table>

In case of a splitting ratio of $a = 0$, the frequency response can be approximated by

$$Z_B(\omega) = -R_f \frac{1}{1 - \frac{\omega^2}{\omega_f \omega_T} + \frac{j\omega}{\omega_T}} \quad (3-45)$$

This is a good approximation when large area photodetectors with large capacitance are used with such Gigahertz fast technology pre-amplifiers. Most of the optimisations previously made for such a frequency response in wideband designs might not be suitable for baseband operation in case the TIA is chosen to control the receiver bandwidth. Hence, the splitting capacitance coefficient $a$ should carefully be measured as pre-amplifier exhibits some capacitance at the input node. By examining both transfer functions in ((3-39) and the approximated in (3-45), it is more efficient to have an input capacitance as it produces a sharper roll-off hence reduce the noise bandwidth at the front end. Finally,

$\Delta R_f$ is represented here as the feedback increment rather than the bandwidth extension ration (BWER) previously presented in literature.
the absence of zero in Tuned B transfer function, compared to Tuned A transfer function, would produce a phase difference which should be accounted for during pulse detection. Such a problem can be easily dealt with in timing extraction.

Figure 3-28 The frequency response of Tuned B for different splitting ratios.

The expressions of tuned A and tuned B TIA designs are presented in this section. The frequency response of these two tuned front-end amplifiers is discussed in Section 3.3 and Section 3.4. The photodiode capacitance, pre-amplifier capacitance, and TIA feedback resistor are taken as part of the tuned circuit at the receiver front end. This arrangement alters the source impedance of the receiver front end thus the noise analysis of baseband receivers presented in Section 2.5.2, is no longer valid for this new configuration. The input referred noise power spectrum and noise bandwidth integrals will accordingly need to be re-equated. Therefore, the noise analysis of tuned A and tuned B TIA and noise expressions are presented in the following section.
3.5 Tuned front end noise analysis

In this section, an original noise analysis of tuned front-end is presented. Correspondingly, novel noise expressions for tuned A and tuned B are reproduced in this section. In the theoretical receiver literature [223]-[232], $I_2$ and $I_3$ are referred to as normalised noise bandwidths or Personick integrals, the total input-referred noise current is given in term of these noise integrals (cf. Eq. (2-40)). In this case, $I_2$ and $I_3$ are numerical values and are evaluated depending on the transfer function of the whole receiver regarding particular input and output pulse shape (cf. Table 2-1). Unfortunately, the latter numerical values often have been used inappropriately for noise calculations of practical receivers that do not have the mentioned transfer functions, typically resulting in overly optimistic noise numbers. In some other literature, $I_2$ and $I_3$ are presented as first and second noise bandwidths $BW_n$ and $BW_{n2}$ (cf. (2-39)). These noise integrals are the same and depend only on the mentioned receiver frequency response. These noise integrals must be used, and they are valid, only if, the receiver frequency response is the same as the one used in conventional analysis to produce these integrals.

3.5.1 Noise transferral methodology

It is common in the literature on optical receivers to express the total input referred noise spectrum of a FET front end TIA $I^2_{n,TIA,FET}(f)$ as in Eq. (2-31) [243]-[252]. The third and fourth terms in this expression arise from referring channel noise source back to TIA input. First term (feedback resistor) and second term (gate leakage current) are white noise sources at the TIA input. Comparing the input noise spectrum in Eq. (2-34) and noise expression in Eq. ((2-31), $\alpha_0$ and $\alpha_2$ can be expressed as

$$\alpha_0 = \frac{4KT}{R_f} + 2qI_g + 4KTT \left( \frac{1}{g_mR_f^2} + 2qI_g \right) \quad (3-46)$$

$$\alpha_2 = 4KTT \left( \frac{2\pi C_T}{g_m} \right)^2 \quad (3-47)$$
Γ is the excess noise factor [253] and it has a common use in optical literature, k is the Boltzmann constant, T is the absolute temperature, $C_T$ is the total capacitance at the TIA input, and $g_m$ is the FET transconductance. The relation between noise expression in Equations (2-31), (2-34), (2-39), and (2-40), leads to an understanding of the situations in which these noise expressions can and cannot be applied. These expressions arise from referring noise sources of a FET input transistor driven by a capacitance source. Since the source impedance in tuned front end is altered, therefore, the following methodology is used to reproduce noise expressions and noise bandwidth integrals for baseband tuned front end receiver:

- Review the assumptions and approximations used in the conventional noise model that result in non-tuned front-end baseband receiver noise expressions.
- Perform noise referring to re-produce noise expressions of input referred power spectrum for tuned front end baseband receiver, following the method explained in Section 2.5.2
- Compare the re-produced transfer functions used to refer noise under same circumstances in literature.
- Use the method explained in Section 2.5.4 to produce expressions for noise integrals and integrated input referred noise power.

The assumptions made for the tuned front-end noise model are as following:

- All reactive components forming the tuned circuit at the front end are ideal thus any noise contribution is neglected.
- The noise contribution of subsequent receiver blocks is insignificant and receiver noise is dominated by front-end input stage. Thus, the input referred noise current spectrum of TIA is divided into two major components the noise from feedback resistor and the noise from amplifier front end. These two noise sources are uncorrelated and given by (2-27).
- All receiver blocks either have infinite bandwidth or a relatively larger bandwidth compared to TIA. Thus, the receiver bandwidth is determined by TIA 3-dB bandwidth. The frequency
response of tuned front-end receiver is presented in Section 3.5, frequency response and 3-dB bandwidth are determined by Eq. (3-27) to Eq. (3-44).

- The frequency response of tuned front end, under some circumstances, is not symmetrical. Therefore, noise equivalent bandwidth may be expressed as discussed in Section 3.5.3 (Figure 2-34) for asymmetrical band-pass transfer functions. For tuned front end transfer function that experiences insignificant peaking, the usual NEB integral will be performed.

- Although, the investigation is not into the transistor design level, the excess noise factor $\Gamma$ approximation, as a single number, is appropriate for this noise model. However, for more accurate noise evaluation, it is suggested to consider the derivation of noise factor presented in [254].

- The contribution of noise sources due to gate leakage current noise in FET is not neglected in this noise model. Hence, its transferral functions will be considered.

- Noise sources are uncorrelated hence the total noise power will be proportional to the sum of the squared values of the individual noise processes.

**Noise terms used in this analysis are defined as follows:**

- Noise power spectral density (PSD) is a frequency domain characteristic of a noise process and it is defined as the Fourier transform of the time domain autocorrelation function, it has a unit of watts per Hz as it corresponds to the time-averaged noise power that is presented in a 1 Hz bandwidth.

- Therefore, in this analysis, the squared noise term (PSD) $I_n^2(f)$ refers to the power spectrum density (*watts per Hz*, $V^2_{Hz}$ or $A^2_{Hz}$) of a noise source, the squared root of PSD ($\frac{V}{\sqrt{Hz}}$ or $\frac{I}{\sqrt{Hz}}$) refers to the noise current density $I_n$ or voltage current density $V_n$ of the same source.

- For simplicity, power spectrum density $I_n^2(f)$ is referred to as $I_n^2$.

- Mean square noise power $i_n^2$ refers to the noise power spectrum density integrated up to the noise equivalent bandwidth. The squared root of this quantity refers to rms noise current $i_n$. 
• Noise equivalent bandwidth (NEB) is the noise bandwidth and it is not equal to the 3-dB bandwidth. It has a unit of Hz; the square root of NEB can be multiplied by voltage noise density or current noise density to obtain the current noise or voltage noise of an equivalent noise source.

3.5.2 Tuned A receiver noise

Noise sources of FET input TIA are illustrated in Figure 3-29. For non-tuned front-end receivers shown in Figure 3-29b, the characteristics of the noise sources and the way they are referred to TIA input is briefly discussed in Section 2.5.2. The same noise sources are considered for tuned front end TIA design as shown in Figure 3-29a.

![Figure 3-29 FET input TIA noise model. (a) Tuned front end. (b) Non-tuned front end.](image)

First noise source here is the one due to feedback resistor $R_f$. As discussed in chapter 2, the noise due to $R_f$ appears as a current generator connected to the TIA input as shown in Figure 3-30. one of the TIA advantages is that noise due to feedback is $(1+A)$ times less than the noise due to equivalent input refereed resistor ($R_t$) would produce, this case is illustrated in Figure 3-30.
In the non-tuned front end, this noise contributes directly to the total referred input current spectrum and it is frequency independent. Figure 3-31 illustrates the difference of noise situation after inductor is introduced. For the tuned front end, the noise source due to $R_f$ is not connected to the amplifier input hence it does not contribute directly to the total referred input noise.

![Figure 3-30 thermal noise due to the resistor at TIA input](image)

**Figure 3-30 thermal noise due to the resistor at TIA input**

(a) Noise due to input resistor.  
(b) Noise due feedback resistor.

![Figure 3-31 feedback resistor noise at the input of TIA](image)

**Figure 3-31 feedback resistor noise at the input of TIA**

(a) Noise in tuned front-end feedback TIA.  
(b) Noise in non-tuned front end.
Figure 3-32 Feedback resistor noise transferal (a) feedback resistor noise source. (b) impedance seen by the feedback resistor noise source. (c) impedance seen by photocurrent. (d) feedback resistor noise source transferred to the receiver input.

Therefore, this noise source may be referred to the input of the receiver to evaluate its contribution to the total input referred noise. Figure 3-32 illustrates the new noise source position regarding the front-end impedance. Figure 3-32b shows the impedance seen by \( I_{n,Rf} \). This impedance can be used to find the output noise voltage due to feedback resistor at the amplifier output. In this case, this transfer function is different compared to the TIA transfer function. Hence, the frequency response of this new transfer function (impedance see by noise source in tuned front end) can be represented as
$$Z(\omega)_{l_{n,rf}} = \frac{R_f}{1 - \frac{\omega^2}{\omega_f \omega_T} + \frac{j\omega}{\omega_T}}$$  \hspace{1cm} (3-48)

This transfer function can be used to directly obtain the noise contribution of $R_f$ to the total output noise voltage, not to the total input referred noise current. The transfer function used to refer the total input noise current to TIA output is the same as the transfer function used for the photocurrent and it is the transfer function of tuned front-end TIA given by Eq. (3-30) (i.e., Figure 3-32c). Figure 3-32(d) illustrates the position of noise source before and after referral, $I_{n,rf}$ is noise source and $I'_{n,rf}$ is referred noise source. The referred noise source $I'_{n,rf}$ can be written as

$$I'_{n,rf} = I_{n,rf} H(\omega)_{l_{n,rf}}$$ \hspace{1cm} (3-49)

where $H(\omega)_{l_{n,rf}}$ is used to transfer the noise source $I_{n,rf}$ to the input of the tuned circuit. It can be found by dividing the transfer function used to refer feedback resistor noise to the output $Z(\omega)_{l_{n,rf}}$ over the TIA transfer function $Z_A(\omega)$ and it is given by as

$$H(\omega)_{l_{n,rf}} = \frac{1}{1 + \frac{j\omega}{\omega_f}}$$ \hspace{1cm} (3-50)

The noise spectrum of the output noise voltage and the input-referred noise current of feedback resistor are plotted in Figure 3-33. The blue line represents the output voltage density which is obtained using Eq. (2-48). The red line represents the input current of the same noise source at the input of TIA which is obtained using Eq. (2-50).

So, the question is which transfer function can be used to obtain the NEB hence calculate the rms equivalent input current of the noise due to feedback resistor. The point is that if this referred noise $I'_{n,rf}$ is re-shaped by the frequency response of TIA, it should produce an equivalent noise voltage equal to the noise voltage obtained by referring the noise directly to the output. At the same time,
multiplying the NEB of the receiver by the white noise due to feedback resistor (without referring to input) will not produce the equivalent rms noise voltage at the receiver output.

![Graph showing noise density vs frequency](image)

**Figure 3-33 Output and input-referred noise density due to feedback resistor in tuned (A) TIA.**

This is understandable from the point of view discussed in Section 2.5.2, the input referred noise power spectrum is used in the general noise power spectrum of the receiver (cf. (2-35) to produce the input referred noise power. Therefore the actual input-referred noise power $i_{n,Rf}^2$ due to the feedback resistor $Rf$ for the tuned (A) TIA can be presented either as

$$i_{n,Rf}^2 = \frac{1}{2\pi Z_0^2} \int_0^\infty \left| Z(\omega)A H(\omega)_{\nu,Rf} \right|^2 d\omega$$  \hspace{1cm} (3-51)

or
\[ i_{n,Rf}^2 = I_{n,Rf} \frac{1}{2\pi Z_0^2} \int_0^\infty |Z(\omega)I_{n,Rf}|^2 d\omega \]  \hspace{1cm} (3-52)

The term \( I_{2A} \) can be defined now as the noise integral used to produce the equivalent referred input noise current at the tuned TIA due to the feedback resistor, given as

\[ I_{2A} = \frac{1}{2\pi Z_0^2} \int_0^\infty |Z(\omega)I_{n,Rf}|^2 d\omega \]  \hspace{1cm} (3-53)

This noise integral can be used to obtain the actual input referred noise power due to feedback resistor. NEB is the noise equivalent bandwidth of the whole receiver (first noise integral \( I_{1A} \)) and it is given for tuned (A) TIA as

\[ NEB = \frac{1}{2\pi Z_0^2} \int_0^\infty |Z(\omega)_A|^2 d\omega \]  \hspace{1cm} (3-54)

Figure 3-34 Gate leakage noise current in tuned and non-tuned front end. (a) Tuned A TIA, \( I_{n,G} \) contributes directly to input noise. (b) Non-tuned front end, \( I_{n,G} \) also contributes directly to input noise.
The first noise integral $I_{1A}$ can be used for the white noise sources appearing at the input of the tuned amplifier. As shown in Figure 3-34, this is the case for shot noise generated by the gate current $I_{n,G}$, as it contributes directly to the input-referred TIA noise same as in non-tuned front end.

![Figure 3-35 Channel noise transfer function](image)

Figure 3-35 Channel noise transfer function (a) non-tuned TIA. (b) Tuned A TIA.

The channel noise source will be referred using the same method as in non-tuned (cf. Section 2.5.2), however, the source impedance is different. The gain from the noise source $I_{n,D}$ to the output is found by using the same method is found to be

$$Z(\omega)_{I_{n,D}} = \frac{A}{gm} \left( \frac{1}{1 + A} - \frac{\omega^2}{\omega_f \omega_T} + \frac{j\omega}{\omega_T} \right)$$  \hspace{1cm} (3-55)$$

In order to refer noise due to $I_{n,D}$ back to the input, this transfer function is divided over the TIA transfer function $Z(\omega)_A$, therefore, the referred noise $I'_{n,D}$ can be written as

$$I'_{n,D} = I_{n,D} \cdot H(\omega)_{I_{n,D}}$$  \hspace{1cm} (3-56)$$

where $H(\omega)_{I_{n,D}}$ is used to transfer the noise source $I_{n,D}$ to the input of the tuned circuit and it is found to be
The term $I_{3A}$ can be defined now as the noise integral used to produce the equivalent refereed input noise power at the tuned TIA due to the channel noise, given as

$$I_{3A} = \frac{1}{2\pi Z_0^2} \int_0^{\infty} |Z(\omega)|_{n,D}^2 \, d\omega$$ \hspace{1cm} (3-58)

To summarise all this, the total referred noise spectral density of tuned A TIA is re-written as

$$I_{n,TIA,FET}^2(f) = \frac{4KT}{R_f} H(\omega)_{R_f}^2 + 2qI_g + 4KTT H(\omega)_{n,D}^2$$ \hspace{1cm} (3-59)

Now this noise spectral needs to be used in the general input-referred power spectrum to lead to an exact expression of the equivalent referred noise power $i_{n,TIA,FET}^2$, so that (cf. Eq. 2-35)

$$i_{n,TIA,FET}^2 = \frac{1}{2\pi Z_0^2} \int_0^{\infty} |I_{n,TIA,FET}^2(\omega)|^2 |Z(\omega)|_{A}^2 \, df$$ \hspace{1cm} (3-60)

expanded as

$$i_{n,TIA,FET}^2 = \frac{4KT}{R_f} \frac{1}{2\pi Z_0^2} \int_0^{\infty} |H(\omega)_{R_f}|^2 |Z(\omega)|_{A}^2 \, d\omega + 2qI_g \frac{1}{2\pi Z_0^2} \int_0^{\infty} |Z(\omega)|_{A}^2 \, df$$

$$+ 4KTT \frac{1}{2\pi Z_0^2} \int_0^{\infty} |H(\omega)_{n,D}|^2 |Z(\omega)|_{A}^2 \, df$$ \hspace{1cm} (3-61)

Therefore, using the noise integrals in Eq. (3-53), (3-54), and (3-58), the equivalent referred noise power $i_{n,TIA,FET}^2$ can be represented as

$$i_{n,TIA,FET}^2 = \frac{4KT}{R_f} I_{2A} + 2qI_g I_{NEBA} + 4KTI_{3A}$$ \hspace{1cm} (3-62)

which may be expanded as
\[
i_{n,TIA,FET}^2 = \frac{4KT}{R_f} \frac{1}{2\pi Z_0^2} \int_0^{\infty} |Z(\omega)|_{I_{n,rf}}^2 d\omega + 2qI_g \frac{1}{2\pi Z_0^2} \int_0^{\infty} |Z(\omega)|_A^2 d\omega + 4KT \frac{1}{2\pi Z_0^2} \int_0^{\infty} |Z(\omega)|_{I_{n,D}}^2 d\omega
\]  
\text{(3-63)}

The square root of this quantity is equivalent to the noise rms input noise current which is also equivalent to the output rms noise voltage divided by the mid-band of the TIA. Table 3-3 shows a comparison between noise terms for tuned A and non-tuned TIA in term of the noise power spectrum \(i_{n,TIA,FET}^2(f)\) and the noise power \(i_{n,TIA,FET}^2\).

**Table 3-3 FET input noise terms for tuned A and non-tuned TIA.**

<table>
<thead>
<tr>
<th>Noise expression</th>
<th>(i_{n,TIA,FET}^2(f))</th>
<th>(i_{n,TIA,FET}^2)</th>
<th>(i_{n,TIA,FET}^2(f)_{Tuned,A})</th>
<th>(i_{n,TIA,FET}^2_{Tuned,A})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I_{n,rf}) = (\frac{4KT}{R_f})</td>
<td>(4KT)</td>
<td>(I_{n,rf} I_2)</td>
<td>(4KT)</td>
<td>(I_{n,rf} I_{2A})</td>
</tr>
<tr>
<td>(I_{n,g} = 2qI_g)</td>
<td>(2qI_g)</td>
<td>(I_{n,g} I_2)</td>
<td>(2qI_g)</td>
<td>(I_{n,g} I_{NEBA})</td>
</tr>
<tr>
<td>(I_{n,D} = 4KT)</td>
<td>(4KT)</td>
<td>(H(f))</td>
<td>(4KT)</td>
<td>(H(f)<em>{I</em>{n,D}})</td>
</tr>
</tbody>
</table>

If the same noise analysis is applied to BJT input transistor TIA amplifier, the total noise power turns out to be

\[
i_{n,TIA,BJT}^2 = \frac{4KT}{R_f} I_{2A} + 2qI_B I_{NEBA} + 2qI_C I_{3A}
\]  
\text{(3-64)}

As in [165, 171, 255]-[Error! Reference source not found.], there must be an optimum collector current that minimises the total noise, it is obtained by equating the first derivative of Eq. (2-32) to zero. The optimum collector current can be written as

\[
i_{c,opt} = \sqrt{\frac{I_2}{I_3}} \beta
\]  
\text{(3-65)}

By doing the same for Eq. (3-3), The optimum collector current for tuned A TIA can be written as
\[ I_{c,\text{opt}} = V_T \sqrt{\frac{I_{2A}}{I_{3A}}} \beta \]

Figure 3-36 shows all noise sources referred to the input, in Figure 3-36a, a tuned TIA employing a FET input transistor and, in Figure 3-36b, a tuned TIA employing a BJT input transistor.

Figure 3-36 Noise in tuned A TIA (a) FET input TIA. (b) BJT input TIA.
As a summary;

- Noise due to feedback resistor is referred to receiver input using the transfer function in Eq. (3-50).
- Noise due to collector current in BJT and channel noise in FET are treated in the same way, and they are referred to the receiver input using the transfer function in Eq. (3-57).
- Noise due to gate current in FET and base current in BJT contribute directly to the total input noise.
- $I_{1A}$ is defined as the noise equivalent bandwidth of tuned A TIA receiver and it is used to obtain the input noise power due to gate current shot noise and base current shot noise.
- $I_{2A}$ is defined as the second noise integral for tuned A TIA receiver and it is used to obtain the equivalent input power noise due to feedback resistor.
- $I_{3A}$ is defined as the third noise integral for tuned A TIA receiver and it is used to obtain the input noise power due to the channel noise and collector shot noise.
- For FET input stage tuned A TIA, the total referred noise spectral density is given in Eq. (3-59) and the total input referred noise power is given in Eq. (3-62).
- For BJT input stage tuned A TIA, the total input referred noise power is given by Eq. (3-62).
- Collector current in BJT TIA receiver can be optimised to minimise the total noise by using Eq. (3-65)
- The input referred rms noise current for tuned front tuned A TIA receiver can be obtained as the square root of the noise power in Eq. (3-62) for FET and in Eq. (3-64) for BJT.

3.5.3 Tuned B receiver noise

Input-referred noise power and input-referred noise power spectrum of tuned B TIA front-end receiver are presented in this section. Illustration of tuned A and non-tuned TIA front end in the previous section is useful to develop the understanding of noise referred in tuned front-end receivers.
Figure 3.37 illustrates the noise model of the tuned front end, considering the same noise sources as in previous two designs.

The noise due to feedback resistor appears as current generator connected to the amplifier input, in the tuned front end, this referring does not yet obtain its contribution to the total input noise of the receiver. This noise source passes a network formed by the tuned circuit components at the receiver input. In a similar manner, as for tuned A noise model, $I_{n,Rf}$ can be referred to the receiver input in two ways. Firstly refer this noise source to the input of this network, passing noise current through the equivalent impedance seen by this source as shown in Figure 3-38. The other way is to find the transfer function that directly referring this source to the output and then divide it by the TIA transfer function. The transfer function that refers the noise source to the output can be written as

$$Z(\omega) I_{n,Rf} = R_T \frac{1 - (1 - a) \frac{\omega^2}{\omega_f \omega_T}}{1 - \frac{j\omega^3 a (1 - a)}{\omega_f \omega_T^2} - \frac{(1 - a)\omega^2}{\omega_f \omega_T} + \frac{j\omega}{\omega_T}}$$  (3-67)
By considering, both cases in Figure 3-38a and Figure 3-38b, the current spectrum of the referred noise can be represented as

\[ I'_{n,Rf} = I_{n,Rf} H(\omega)_{n,Rf} \]  

(3-68)

where

\[ H(\omega)_{n,Rf} = 1 - \frac{(1 - a)\omega^2}{\omega_f \omega_r} \]  

(3-69)

**Figure 3-38 Feedback resistor noise transferal**

* (a) Impedance seen by the feedback resistor noise source. (b) noise source appears as current in a short circuit at the TIA input. (c) The equivalent current produced in inductance due to referred noise.

It is important to realise that the shot noise generated by the gate current \( I_{n,G} \) contributes to the input-referred noise in the same feedback resistor noise does. Figure 3-39 illustrates the noise transferral

---

* Case one is when noise source is referred to output shown in (a). case two when noise is directly referred to input shown in (b) and (c).
for both noise sources. Therefore, the transfer function used to refer feedback resistor \( H(\omega)_{I_{n,Rf}} \) is the same as the transfer function used to refer gate current noise, so that \( H(\omega)_{I_{n,Rf}} = H(\omega)_{I_{n,G}} \).

![Figure 3-39 Noise transferral of \( I_{n,Rf} \) and \( I_{n,D} \)](image)

Although this transfer function is used to refer noise to the input, the referred noise source is plugged in the input power spectrum of the receiver (cf. Eq. 2-35) to produce the equivalent input-referred noise power due to feedback resistor and gate leakage noise sources. By doing so, the equivalent input-referred noise power due to feedback resistor can be written as

\[
i_{n,Rf}^2 = I_{n,Rf} \frac{1}{2\pi Z_0^2} \int_0^{\infty} \left| H(\omega)_{I_{n,Rf}} \right|^2 |Z_B(\omega)|^2 \, d\omega \quad (3-70)
\]

or

\[
i_{n,Rf}^2 = I_{n,Rf} \frac{1}{2\pi Z_0^2} \int_0^{\infty} \left| Z(\omega)_{I_{n,Rf}} \right|^2 \, d\omega \quad (3-71)
\]

Similarly, the equivalent input-referred noise power due to gate current can be written as

\[
i_{n,G}^2 = I_{n,G} \frac{1}{2\pi Z_0^2} \int_0^{\infty} \left| Z(\omega)_{I_{n,G}} \right|^2 \, d\omega \quad (3-72)
\]

Integrals in Eq. (3-70) and Eq. (3-72) lead to a conclusion that the noise integral for feedback resistor noise and gate current noise can be represented as
\[ I_{2B} = \frac{1}{2\pi Z_0^2} \int_0^\infty \left| Z(\omega)_{I_{n,rf}} \right|^2 d\omega \]  

(3-73)

NEB is the noise equivalent bandwidth of the tuned receiver and it is given for tuned (B) TIA as

\[ NEB = \frac{1}{2\pi} \int_0^\infty |Z(\omega)_B|^2 d\omega \]  

(3-74)

On the other hand, the gain from the noise source \( I_{n,D} \) to the output is found to be

\[ Z(\omega)_{I_{n,D}} = \frac{A}{g_m} \left[ \frac{1}{(A + 1)} - \frac{j\omega^3 a(1 - a)}{\omega_f \omega_T^2} - \frac{(1 - a)\omega^2}{(A + 1) \omega_f \omega_T} + \frac{j\omega}{\omega_T} \right] \]  

(3-75)

In order to refer noise due to \( I_{n,D} \) back to the input, this transfer function is divided over the TIA transfer function \( Z(\omega)_B \), therefore, the referred noise \( I'_{n,D} \) can be written as

\[ I'_{n,D} = I_{n,D} H(\omega)_{I_{n,D}} \]  

(3-76)

Where \( H(\omega)_{I_{n,D}} \) is used to transfer the noise source \( I_{n,D} \) to the input of the tuned circuit and it is found to be

\[ H(\omega)_{I_{n,D}} = \frac{(A + 1)}{g_m R} \left[ \frac{1}{(A + 1)} + \frac{j\omega^3 a(1 - a)}{\omega_f \omega_T^2} - \frac{(1 - a)\omega^2}{(A + 1) \omega_f \omega_T} + \frac{j\omega}{\omega_T} \right] \]  

(3-77)

The equivalent input-referred noise power due to channel noise can be written as

\[ i_{n,D}^2 = I_{n,D} \frac{1}{2\pi Z_0^2} \int_0^\infty \left| H(\omega)_{I_{n,D}} \right|^2 |Z_B(\omega)|^2 d\omega \]  

(3-78)

or

\[ i_{n,D}^2 = I_{n,D} \frac{1}{2\pi Z_0^2} \int_0^\infty \left| Z(\omega)_{I_{n,D}} \right|^2 d\omega \]  

(3-79)

With this in mind, the noise integral for channel noise can be represented as
\[ I_{2,\text{tuned}} = \frac{1}{2\pi Z_0} \int_0^\infty |Z(\omega)_{l,n,d}|^2 \, d\omega \]  \hspace{1cm} (3-80)

Therefore, the total referred noise power of tuned B TIA can be represented as

\[ i_{n,TIA,FET}^2 = \left( \frac{4KT}{R_f} + 2qI_g \right) I_B + 4KTTI_{3B} \]  \hspace{1cm} (3-81)

Figure 3-40 Noise in tuned B FET input stage.

Figure 3-41 Noise in tuned B BJT input stage.

If tuned TIA has a BJT input stage, then the noise characteristic will be the same as for FET input tuned TIA, except that the transistor noise sources for BJT are the base current shot noise and the
collector current shot noise. The expression of the equivalent referred noise power can be represented as

\[ i_{n,TIA,BJT}^2 = \left( \frac{4KT}{R_f} + 2qI_b \right) I_{2B} + 2qI_c I_{3B} \]  

(3-82)

The square root of the quantity in Eq. (3-82) (same for Eq. 3-81) is equivalent to the input rms noise current which is also equivalent to the output rms noise voltage divided by the mid-band of the TIA.

The optimum collector current for tuned B is equated as in Eq. (3-65) and it can be written as

\[ I_{c,\text{opt}} = V_T \sqrt{\frac{I_{2B}}{I_{3B}}} \beta \]  

(3-83)

As a summary;

- Feedback resistor noise is referred to the receiver input using Eq. (3-69). Gate current noise in FET and base current noise in BJT can be treated in the same way as the feedback resistor, they are referred to receiver input using same transfer function.

- Channel noise in FET and collector current in BJT are referred in the same way to the receiver input using Eq. (3-77).

- \( I_{2B} \) and \( I_{3B} \) in Eq. (3-73) and (3-80) are defined as noise integral used to produce the equivalent referred input noise power at the tuned TIA input.

- The total input referred noise power for tuned B TIA is given by Eq. (3-81) for FET and by Eq. (3-82) for BJT input stage.

- The input referred rms noise current for tuned B receiver is the square root of the input referred noise power.

- Collector current in BJT TIA receiver can be optimised to minimise the total noise by using Eq. (3-83).
3.6 Summary

Neither numerical Personick integrals nor conventional bandwidth integrals are valid for tuned front end receivers. This chapter gave an overview of tuned front-end receivers. The front-end classification in some optical literature exclude the tuned front end for heterodyne receivers, this is due to a lack of theory. The information presented in this chapter disputes this claim. It provides: expressions for transimpedance and optimised frequency response in accordance with baseband theory, and expressions of referred noise, noise integrals and input referred noise power for tuned front end baseband receivers.
Chapter 4: PULSE POSITION MODULATION

Section 4.1 provides a brief discussion of pulse modulation. In this section, PCM and PPM theory is presented. This section includes a literature review of the use of digital PPM in optical communications. In Section 4.2, a review of optimum and suboptimum detection of digital PPM is presented. In this section, there is a review of the theory of existing PPM optical receiver. The error probabilities analysis is explained in Section 4.3.
4.1 Digital transmission

The introduction of digital circuit techniques and integrated circuit technology made the transmission of discrete time signals both advantageous and economic. Digital transmission systems offer greater performance over their analogue counterparts. They also provide an ideal channel for data communications which is compatible with digital computing and storage techniques. Optical communication systems suit baseband digital transmission due to many factors. In some applications, optical digital transmission offers incredible advantage regarding the acceptable SNR at the optical receiver over analogue transmission. In addition, baseband digital transmission is often used as a solution to reduce problems with optical sources such as non-linearity and temperature dependence that analogue transmission may suffer from. Therefore, optical transmission systems convey digital information in the baseband using intensity modulation of the optical source [160]. In the subsequent sections, the basics of on-off keying OOK signalling and pulse position modulation signalling are presented along with a brief review of the recently reported applications.

4.1.1 Pulse code modulation

Pulse code modulation (PCM) may be used to digitise analogue signals for transmission. Figure 4-1 illustrates analogue to digital conversion. Analogue signals are sampled at a frequency in excess of the Nyquist rate to encode the original message into a digital pattern. Constant width sampling pulses are used in a way that their amplitude varies in proportion to the sample values of the analogue signal, resulting in a discrete signal known as pulse amplitude modulation (PAM). Figure 4-1 illustrates the quantising process in which PCM signal is provided. In this process, each PAM sample is encoded into three binary bits. In some applications, the bandwidth requirement for PCM transmission is greater than the corresponding baseband analogue transmission hence optical medium is used due to the wideband nature of optical channels. The analogue signal is thus digitised and may be transmitted as a baseband signal or, alternatively, be modulated by amplitude, frequency or phase shift keying.
Figure 4-1: The quantisation and encoding of an analogue signal into PCM using a linear quantizer with eight levels [160].
In optical communications, since a laser can be either amplitude, frequency, phase, or polarisation modulated, there is a wide variety of possible transmitter symbol sets. Some of which are associated with direct detection, while other formats are most often associated with coherent detection (Figure 4-2 and Figure 4-3). Different signal sets convey information with a higher efficiency than others. Thus, transmitted symbol set or modulation format are chosen depending on many factors such as optical source type, total cost, channel, performance criteria of the system. System efficiency is specified in terms of the number of bits carried per Hz of occupied bandwidth. It could also be specified in terms of the number of photons required to transmit a bit of information at a specified bit error rate, increasing the power efficiency is often conveyed by an increase in system complexity.

![Diagram showing modulation formats](image)

**Figure 4-2 Modulation formats for digital signalling, on-off keying (OOK) and Manchester coded OOK.**

A modulation scheme that is popular because of its simplicity is on-off keying (OOK), shown in Figure 4-2. The laser output is simply turned on and off in response to whether the data is a one or a zero. The OOK illustrated in Figure 4-2a is an example of non-return-to-zero signalling. The laser is turned on when transmitting a one and stays on for the entire bit time. A variation on OOK is return-to-zero signalling, shown in Figure 4-2b, where the laser is on during a one but for a time that is less
than the bit-time. This time is defined as a duty cycle which is a percentage of time occupied by the pulse-width.

Manchester coded OOK, shown in Figure 4-2c, is another variation of OOK signalling. For instance, a low-high transition can be sent to indicate a one and a high-low transition to indicate a zero. The Manchester coded waveform can be decoded by XOR'ing the data and clock waveforms. It has the benefit of producing a transition every bit and that aids clock acquisition. However, the penalty paid is that the bandwidth required is twice that of a simple OOK system. In some applications [259], Manchester coded OOK is implemented to eliminate the flickering of the light intensity caused by a long sequence of zero associated with other modulation formats.

![Modulation formats for digital signalling. (PSK, FSK, and ASK).](image)

Amplitude-shift-keying (ASK) format is similar to OOK except that the detected signal is obtained at an intermediate frequency (IF) instead of directly at baseband. Therefore, ASK receiver conveys a coherent detection which is also sensitive to the frequency of the received signal. The sensitivity is typically minimised by operating with a wide IF bandwidth or by precision frequency tracking. With
a coherent receiver, the spectral purity of the optical signals and the stability of the received polarization will strongly influence the receiver performance. As coherent receivers are made sensitive to frequency and phase of the optical carrier, frequency shift keying (FSK) and phase-shift-keying (PSK) are also alternatives to OOK signals. In an FSK system, the transmitter generates a specific frequency for a one and a different frequency for a zero, shown in Figure 4-3. In PSK, the transmitter sends an in-phase signal for a one and an out-of-phase signal for a zero, as shown in Figure 4-3. There are other variations of PSK such as differential phase-shift-keying (DPSK) and quadrature-phase-shift-keying (QPSK).

In this investigation, the interest is given for the use of tuned front end optical receivers with the on-off keying modulation schemes under non-coherent demodulation. In particular, non-return-to zero on-off keying (NRZ-OOK) and pulse position modulation (PPM) are considered as the main digital formats in this investigation. The receiver fundamentals and performance of OOK-NRZ are explained in greater detail in chapter 2. Therefore, the subsequent sections provide information on pulse position modulation (PPM) schemes, and receiver structure and performance.

4.1.2 Digital pulse position modulation (PPM)

Binary pulse position modulation (PPM) or digital PPM is another variation of OOK signalling. In the simplest form of PPM, shown in Figure 4-4, a bit time is divided into two slots. The pulse is sent in the first slot for one and in the second slot for zero.

PPM may be expanded to larger alphabet sizes, therefore, PPM formatting can then be described as M number of PCM bits is replaced by a highest optical power pulse in a unique position of the PPM frame of N number of slots according to N=2^M. PPM can be more efficient than simple OOK in terms of the number of photons required to transmit an individual bit of information since multiple bits of information can be transmitted using a single pulse [260]-[261].
In 4-PPM, the time required for two bits is divided into four time-slots. Each time slot has a corresponding two-bit address starting at 00 and counting through 01 and 10 up to 11. 4-PPM has been demonstrated in some moderate data rate free-space systems [262]-[263]. The signalling format is attractive because it becomes increasingly power efficient as the alphabet size increases. Large alphabets have been proposed for power efficient optical communications with space probes [264].

<table>
<thead>
<tr>
<th>PCM</th>
<th>4-PPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>0001</td>
</tr>
<tr>
<td>01</td>
<td>0010</td>
</tr>
<tr>
<td>10</td>
<td>0100</td>
</tr>
<tr>
<td>11</td>
<td>1000</td>
</tr>
</tbody>
</table>

Guard interval is a technique which is used in PPM frame in order to space the PPM words, reducing the effect of inter-symbol interference between PPM slots but at the cost of increased bandwidth. Figure 4-5 shows a 16 PPM with guard interval.

There have been three principal drawbacks to higher order PPM systems. The first is the need for accurate clock synchronisation across all of the slot-times. This becomes increasingly difficult as the number of slots increases [265]-[266]. However, PPM slot synchronisation has extensively been
researched, and several types of practical slot synchronisers have been proposed and implemented [267]-[272].

The second is that semiconductor lasers are typically peak power limited and as M increases, the pulse width decreases and the peak power increases. Ultimately a limit of a relatively modest M is reached [273]. However, Q-switched solid-state lasers may be used to advantage at the expense of a more complicated transmitter. In interplanetary missions, the transmission system requires lasers that are capable of emitting much more energetic pulses. Cavity-Dumped communication laser design is suggested by [274], this method can reduce or eliminate laser dead time hence could significantly enhance communication capacity. The third drawback with PPM is that the receiver bandwidth increases with increasing M. It is harder to achieve low-noise performance in a wider bandwidth, and PPM is typically restricted to moderate data-rates [275].

Di-code and digital PPM are combined in order to form di-code PPM scheme [276]. This scheme offers better sensitivity than PCM system as well as PPM system at high fibre bandwidth. At low fibre bandwidth, the sensitivity of di-code PPM is much higher than PPM. In di-code signalling data transitions from logic zero to logic one are coded as +V and transitions from logic one to logic zero are coded as -V as shown in Figure 4-2, a zero signal is transmitted if there is no change in the PCM

**Figure 4-5 PPM coded ward and equivalent PCM.**
signal. The positive pulse can be regarded as setting the data to logic one (pulse SET), whereas the negative pulse resets the data to logic zero (pulse RESET).

In di-code PPM, these SET and RESET signals are converted into two pulse positions in a data frame. Thus, a PCM transition from zero to one produces a pulse in slot S and a one to zero transition generates a pulse in slot R, as shown in Figure 4-6. If the PCM data is constant, no signal is

Table 4-2 4-PPM

<table>
<thead>
<tr>
<th>PCM</th>
<th>4-PPM</th>
<th>Di-code</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>0001</td>
<td>0000</td>
<td>N</td>
</tr>
<tr>
<td>01</td>
<td>0010</td>
<td>1000</td>
<td>S</td>
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<tr>
<td>10</td>
<td>0100</td>
<td>0100</td>
<td>R</td>
</tr>
<tr>
<td>11</td>
<td>1000</td>
<td>0000</td>
<td>N</td>
</tr>
</tbody>
</table>

Figure 4-6 Di-code PPM (from top to down: PCM, Di-code and Di-code PPM).
transmitted. Although two guard slots could be used in this system, to reduce the effects of inter-symbol interference (ISI), this depends on the channel characteristics. If there is minimal ISI, zero guard slots is used [172].

There are other different formats of digital PPM used in optical communications such as Offset-PPM, Duo-binary PPM, Multiple-PPM, Differential PPM, and \( n^k \) PPM [173], [277]-[282]. In recent years, PPM has become the most used digital scheme for low earth orbit satellites (LEO) due to its power efficiency which is a significant requirement for battery-powered LEO satellites [283]. In addition, the bit error rate performance of PPM is better than that of OOK modulation when a direct detection (DD) receiver is utilised in turbulent channels [284]-[287]. The performance of various PPM schemes in an atmospheric turbulence channel for ground to satellite and satellite to earth optical communication is evaluated in [288]-[289]. Among the intensity modulation (IM) schemes, with the same average received power, different PPM schemes perform better than conventional OOK in presence of turbulence and different weather conditions [290]-[295]. PPM schemes have also been developed for deep space optical communication links, and PPM signalling has been examined as a practical modulation of interplanetary optical communication [296]-[301]. This is again due to its power efficiency. This can be explained as if 10 photons are required to detect a pulse with \( 10^{-9} \) BER (which is same as detection sensitivity of IM/DD), therefore, a 1024-PPM can be used on the conventional 10 Gbps IM/DD transceivers with NRZ signalling, with 10 bits per pulse (10 photons to recover the bit correctly). In this system, sensitivity becomes 1 photon per bit. If higher PPM order is considered the receiver sensitivity becomes better than a single photon per bit. Based on this explanation, 0.3 photon per bit, assuming PPM signalling and high efficient error correction coding, has been shown to be feasible [301].

PPM schemes have also been proposed for underwater wireless optical communications (UWOC) [302]-[310]. Required transmission power and bandwidth of different modulation schemes have been evaluated in [309] for UWOC link. PPM outperform OOK in term of BER, achieving longer distance
communication compared to OOK. PPM power efficiency makes it a good choice for battery operated underwater sensors where low power consumption is a major concern. In addition, Alternative PPM schemes are also used to further improve the bandwidth efficiency of wireless communication system [310]. PPM has been shown to outperform OOK in a variety of application. For instance, in ultraviolet communication (UVC) system, Experiments are conducted with various key parameters and the system performances with OOK and 2-PPM were compared [311]. The results show that PPM offers better BER performance than OOK. In Passive optical networks (PON) based system presented in [312], among NRZ, RZ, Manchester coded and PPM, PPM with four pulse positions 4PPM shows the best overall system performance. PPM also demonstrates an efficient solution for short-range communications in distributed sensor networks as reported in [313]. The performance of PPM is also evaluated for optical fibre communication systems, showing that PPM outperforms OOK [314]-[317]. Different PPM schemes have been proposed recently for in-door visible light communication systems for the purpose of lighting and communication [319]-[322].

4.2 PPM detection and receiver design

In this section, PPM receiver structure is reviewed, considering the optimum (Section 4.2.1) and sub-optimum (Section 4.2.2) detection of PPM signalling.

One of the early designs of PPM optical communication system with a coherent source and direct photodetection is reported in [323]-[324]. Thermal noise due to receiver components was not considered in this model. Maximising the information rate and minimising the transmitted signal are the goals they targeted. They relied on the advantage of the photomultipliers of the signal rather than avoiding the noise by any sort of filtering. Thus, a poor level of error probability has been introduced.

In the system [325] shown in Figure 4-8, the performance of optical receivers with avalanche photodetection is evaluated for on-off keying and PPM signalling format. In this design, the signal power is kept at the minimum while the error probabilities had very high values between $10^{-1}$ and $10^{-4}$. As mentioned in [325], the range of error probabilities beyond this level will require a very large signal
power. This defines the trade-off between the signal power and error probability in the absence of the filtering.

![Diagram](image1)

**Figure 4-7 PPM detection in [323].**

![Diagram](image2)

**Figure 4-8 PPM detection in [325].**

A developed receiver is proposed in [326] for optical PPM systems. This receiver structure, shown in Figure 4-9, offers significant improvement signal to noise ratio compared to structures previously mentioned, this is due to using a pre-detection filter at the input of the threshold detector.

![Diagram](image3)

**Figure 4-9 Block diagram of receiver proposed in [326].**
In this model, the frequency response of the receiver is matched to that of the incident optical pulse, the avalanche gain and threshold detector are set at optimum levels. The frequency response matching is based on providing sufficient bandwidth to ensure that the received pulse is detected without further dispersion. However, in optical receivers, output noise is proportional to the second or third power of the bandwidth depending on whether a BJT or FET preamplifier is used [327]-[329]. A feedback amplifier, as shown in Figure 4-10, is also included in this model and it offers much higher bandwidth, a large signal dynamic range and reduced variation of avalanche gain.

![Figure 4-10 preamplifier used in [326].](image)

As in [326], receiver performance is evaluated for BJT and FET input amplifiers, with a complete analysis of noise in the receiver. As reported, the signal to noise ratio improvement for FET preamplifier is 10.8 dB with a PIN photodiode and 4.7 dB with an avalanche photodiode. For a BJT preamplifier, the SNR improvement is 3.1 dB and 2.3 dB with the PIN and avalanche diodes, respectively.

### 4.2.1 Optimum detection

In optical fibre PCM receiver, the matched filter produces a tail on the impulse response which makes the filter useless due to inter-symbol interference. However, in PPM system, the pulses are separated...
in time. There is also an emphasis on considering the modulation index and the number of time slots of PPM in the design of the filter.

![Parallel combination of whitened matched filters.](image)

**Figure 4-11 Parallel combination of whitened matched filters.**

Considering a matched filter for PPM detection, the filter proposed in [Error! Reference source not found.], is a parallel combination of whitened matched filter and proportional derivative (P/D) network. Figure 4-11 illustrates matched filters combination of PPM receiver for a slightly dispersive optical channel. Whitened filter 1 is matched to the input pulse and filter 2 is matched to the time derivative of the pulse. The filter can be mathematically derived to whitened matched filter and proportional derivative (P/D) network and shown in Figure 4-12. K is a factor to be determined in terms of the system parameters. The receiver used in [Error! Reference source not found.] is PIN-FET, shown in Figure 4-13.

![Optimal filter block diagram.](image)

**Figure 4-12 Optimal filter block diagram.**
Figure 4-13 Receiver model used in [Error! Reference source not found.], with input noise current and voltage of a FET front end.

The work presented in [Error! Reference source not found.] is extended and the use of PIN-FET and APD-FET front ends for PPM optical fibre receivers is compared in [330]. It is reported that the APD receiver can achieve sensitivity improvement of 2.3 dB over the PIN-FET while the PPM system initially archives 7.7 dB over the equivalent PCM system. Figure 4-14 illustrates the front-end pre-amplifiers used in this model.

It is important to note that digital PPM system has different format which depends on the number of slots in one frame, these time slots are determined by the number of coded PCM bits. Thus, the receiver sensitivity can be maximised upon the optimum number of time slots [332].

Figure 4-14 Pre-amplifier types used in [330]. High impedance amplifier and transimpedance amplifier.
A simplified block diagram of the optimum PPM receiver is shown in Figure 4-15. The first block is the receiver front end. The pre-detection filter is a pre-whitened matched filter (unless the front-end output noise spectrum is white) in cascade with a proportional derivative delay (PDD) network. PDD network is slightly different compared to (P/D) network in Figure 4-12, and $\lambda_r$, $\lambda_s$, $\lambda_t$ are the Lagrangian multipliers, which are to be determined in terms of the system parameters.

This receiver is implemented for digital PPM signalling over optical fibre channel [333]. Experimental results obtained at data rate of 8 Mbit/s show that the digital PPM system offers an improvement of 4.2 dB over equivalent binary PCM system. In the receiver design, the receiver front end employs a PIN-BJT transimpedance. The matched filter is designed using an approximated transfer function with 10 poles, which is physically implemented by a full pole removal operation to give the canonical ladder shown in Figure 4-16. For PDD network, the multiplier coefficients are realised using switched attenuators. Coaxial line is employed to obtain the delay. The differentiator branch is synthesised by a simple CR network.

![Figure 4-15 PPM receiver with optimal detection filter [333].](image-url)
There is a fact that the filter that optimises signal detection (optimal filter) is complex in that it comprises a pre-whitened matched filter in cascade with PDD network. Hence, the receiver would be unrealistic for a commercial system. There is a possibility of reducing the complexity of optimum receiver structure by using a sub-optimal receiver. However, the question is how far the implementation of sub-optimal filter would affect the sensitivity of the receiver.

### 4.2.2 Sub-optimum detection

Figure 4-17 illustrates sub-optimal pre-detection filters. In [335], it is shown that the noise whitening section of the optimum pre-detection filter has a continuous probability distribution known as Lorentz distribution and is unrealisable in practice. Thus, a double pole noise-whitening filter can be used with only 0.5dB degradation compared to idea detection. In [335], the filter complexity is reduced by replacing the 10th order matched filter and PDD network combination with a 3-pole filter. Due to this simplification, the receiver sensitivity is reduced by 1.6 dB. In [336], the performance of three suboptimum pre-detection filters (a matched filter, an optimised three pole filter, and a third order Butterworth filter) are examined. It is shown that the suboptimum pre-detection filters result in sensitivity degradations of 0.4 to 1.1 dB. This level of degradation can be considered as insignificant, relying on the fact that the PPM system was offering an advantage of 8.6dB of the receiver sensitivity over the typical PCM system.

The combination shown in Figure 4-18a (a noise whitening filter in cascade with a matched filter) is considered in [337]. The performance of Di-code PPM using PIN-FET receiver over slightly/highly
dispersive optical channel is evaluated. Although the PDD network can be removed, with a small loss in sensitivity, the practical implementation of the remaining matched filter is complex.

In [172], an alternatively sub-optimal filtering is proposed for a zero-guard interval di-code system. The performance of a third order Butterworth filter is evaluated for a dispersive optical channel. Gaussian-shape received pulses have been assumed and a bandwidth-limited PIN–bipolar receiver with both frequency invariant and variant noise was considered. This analysis shows that the bandwidth of the Butterworth filter is relatively independent from the channel. It is also shown that the performance of Butterworth filter is superior to that of a noise-whitened matched filter, reducing the complicity of the existed pre-detection filter.
In [338], the use of central detection combined with raised cosine filtering is discussed for di-code PPM optical fibre receiver. This leads to a more simplified receiver design, resulted in equivalent sensitivity performance at high fibre bandwidths and significantly improved performance at lower fibre bandwidths compared to the matched filter. Practical approximations to this approach are considered and design rules are given for determining the bandwidth of both the preamplifier and a simple 3rd order Butterworth equalisation filter. It is shown that setting the bandwidth of both, the preamplifier and equaliser, to 0.6 times the di-code slot rate, gives sensitivities within 0.2 dB of ideal.
di-code PPM raised cosine filtering and that such a receiver can operate over a wide range of fibre bandwidths, with minimal degradation in sensitivity.

As shown in Figure 4-18, the receiver structure for sub-optimum detection requires either: noise whitening filter in cascade with a matched filter or Butterworth filter. The Butterworth filter can be considered as a sub-optimal pre-detection filter, with minor degradation in receiver sensitivity.

Literature review on the receiver structure of digital PPM can be concluded in the following key points:

- The receiver front end is same as the conventional OOK receiver front end, it contains a photodetector (PIN or APD) with preamplifier with (BJT or FET) input stage.
- Thus, an additional pre-detection filter is used to maintain an accepted signal to noise ratio.
- There are two main receiver structures: optimum and sub-optimum detection.
- PPM receiver structure for optimum detection requires pre-whitened matched filter in cascade with PDD network.

**Figure 4-18 PPM sub-optimum receiver structure in [172].**
• PPM receiver structure for sub-optimum detection requires Butterworth filter or noise whitening filter in cascade with a matched filter.

• The implementation of the matched filter is a quit complex process.

• Butterworth filter can be considered as a sub-optimal pre-detection filter, with minor degradation in receiver sensitivity.

• There are two receiver bandwidth allocation strategies:
  - The bandwidth of the first stage is often large that the signal doesn’t incur any further dispersion (Matched filter case).
  - Both the preamplifier and equaliser can control the receiver bandwidth (Butterworth filter case).

4.3 Error probabilities

PPM systems are a digital system where the major measure of their performance depends mainly on the error probability associated with system’s receiver. The major cause of errors in a digital system is the amplitude of noise at the detector input that causes level one pulse to be detected as zero level pulse when the noise forces the amplitude of the signal to be below the threshold level at detection. On the other hand, if noise forces the amplitude of the pulse cross the threshold level then the pulse is detected as level one pulse. The same concept occurs in PPM systems but three different types of error rely on the fact that the detection of the PPM pulses depends mainly on the position of the pulse. The majority of the PPM coding schemes have only one-time slot transmitted in the frame. Thus, any contribution of noise causing the signal to cross the threshold level will result in having a false alarm in an unexpected position in the frame of PPM. Also, noise can affect the phase of the pulse so that it will be detected in the wrong position. These two sources of error are based on the fact that the noise is forcing the signal to be present where no signal has been transmitted. The noise can have a negative amplitude so that it forces an actual transmitted pulse to be below the threshold level, causing the pulse to be detected as zero. Accordingly, for PPM system which has only one pulse in the frame,
this type of error is referred to as an erasure error. Therefore, the three error probabilities in PPM systems are false alarm, wrong slot and erasure. These three types are discussed in detail. The total PCM error probability is found by adding together the probability of these three types for digital PPM or di-code PPM. The performance criterion is that these error probabilities should be the same as for the PCM. The total, equivalent PCM error probability due to is the sum of all error probabilities due to the three different types.

4.3.1 wrong slot

Figure 4-19 illustrates the generation of wrong slot error, with a pulse $V(t)$ and additive noise $n(t)$. The instant at which the receiver output voltage first crosses the threshold level $V_d$ is detected with an error $\tau$ which depends on the instantaneous noise voltage and the rate of rise of the pulse. The mean-square error is given by [Error! Reference source not found.]

$$\tau^2 = \frac{n^2}{|dV(t)/dt|_{t_d}}$$

(4-1)

where $n^2$ is the mean square noise voltage at the receiver output at threshold crossing instant. Errors occur if $\tau > \frac{t_s}{2}$ with probability $P_1$ which can be related to $n^2$ via the noise statistics. Noise is considered to have a gaussian statistics, with a good approximation $P_s$ is given by [Error! Reference source not found.]

$$P_s = \frac{2}{\tau \sqrt{2\pi}} \int_{\frac{t_s}{2\tau}}^{\infty} e^{-\frac{x^2}{2}} dx = erfc\left(\frac{Q_s}{\sqrt{2}}\right)$$

(4-2)

where
\[ Q_s = \frac{t_s}{2} \left( \frac{dV(t)}{dt} \right)^2 |_{t_d} \] 

(4-3)

**Figure 4-19 Generation of wrong slot errors** [Error! Reference source not found.].

In PPM system, noise can cause a pulse to appear either before or after the original slot. Thus, the probability of a wrong slot error for digital PPM is [336]

\[ P_{sPPM} = \text{erfc} \left( \frac{Q_s}{\sqrt{2}} \right) \] 

(4-4)

which gives rise to an equivalent PCM error probability \( P_{esPPM} \) given by [336]

\[ P_{esPPM} = \frac{2^M}{2(2^M - 1)} P_{wsPPM} \] 

(4-5)

In di-code, wrong slot event can cause four possible errors. Noise can cause the edge of the pulse in S slot to appear in the preceding guard slot or the following R slot. In the first case, no detection error occurs because the preceding slot is a guard and the decoder will not recognise the false threshold crossing. In addition, as the pulse is still present in the S slot, it will be detected correctly. In the second instance, the S pulse appears in R slot and this leads to detection errors. The probability of this happening is given by Eq. (4-4). This detection error causes an immediate PCM error, and all
following bits will be in error until an R pulse is received. (It is assumed that the probability of two errors occurring in a particular sequence is small.) Wrong-slot errors can cause an R pulse to appear in the preceding S slot or the following guard slot. In the first instance, the detection error gives the same number of errors as the S-R error. Loss of the R pulse due to noise shifting the rising edge into the following guard slot has a similar effect. The total, equivalent PCM error probability due to wrong slots is the sum of all the possible error probabilities cases [276]. However, using central detection in di-code PPM receiver shows that ISI due to noise and filter response can be eliminated. Therefore, wrong slot error becomes insignificant [338].

4.3.2 Erasure

![Diagram](image)

**Figure 4-20 Occurrence of errors [172].**

Erasure of a pulse occurs when the noise is large enough to reduce the peak signal voltage to below the threshold level. This error occurs with probability \( P_r \) given by [336]
\[ P_r = 0.5 \text{erfc} \left( \frac{Q_r}{\sqrt{2}} \right) \]  

(4-6)

with

\[ Q_r = \frac{V_{pk} - V_d}{\sqrt{n_0^2}} \]  

(4-7)

\( v_{pk} \) is the peak signal voltage of the receiver and \( v_d \) is the threshold crossing voltage. In digital PPM, erasure of a pulse gives an equivalent PCM error probability of \( P_{rDPMM} \), given by [336]

\[ P_{rDPMM} = \frac{n}{2(n - 1)} P_{er} \]  

(4-8)

In a di-code PPM system, erasure of a set or reset pulse generates the same number of PCM errors [276].

4.3.3 False-alarm

False-alarm errors are due to noise causing a threshold crossing event in any unoccupied data slot. The probability of False-alarm is given by [336]

\[ P_f = 0.5 \text{erfc} \left( \frac{Q_f}{\sqrt{2}} \right) \]  

(4-9)

with

\[ Q_f = \frac{V_d}{\sqrt{n_0^2}} \]  

(4-10)

The number of uncorrelated samples per time slot can be approximated to \( t_r \) which is the time at which the autocorrelation function of the receive filter has become small. The probability of a false alarm error then becomes [Error! Reference source not found.], [336]

\[ P_f = \frac{T_s}{t_r} 0.5 \text{erfc} \left( \frac{Q_f}{\sqrt{2}} \right) \]  

(2-4)
In a digital PPM system, this error source generates an equivalent PCM error probability of 
\( P_{f_{DPPM}} \) and is given by [336]

\[
P_{f_{PPM}} = \frac{2^M}{4} P_f
\]  

(2-5)

To cause PCM errors in a di-code PPM system, a false alarm error must be of the opposite type to the
symbol that started a sequence. With a pulse in slot S, a false alarm could occur in the following R
slot but, as the decoder stops when a pulse is received, no PCM errors will be generated. However,
an error will be generated if a false alarm occurs in the following string of N signals. The severity of
the error depends on where the false alarm occurs. The false-alarm error occurs on the \( k^{th} \) N symbol
in a run of \( xN \) symbols, and so the PCM error is \((x + 1 - k)\). In this instance, \( x \) must be greater than
zero because when S pulse is transmitted a false-alarm error in the R slot has no effect. A similar
situation applies to false-alarm errors with an R pulse. However, a false alarm could occur in the S
slot immediately before the pulse. The false alarm, therefore, is the sum of both false alarm of S and
R [276].

4.4 Summary

The literature review in this chapter has a sequential development of PPM receivers design which
tuned front end has not been part of it. The main intention of implementing PPM is to enhance the
PCM receiver sensitivity and make use of the large bandwidth available in optical communication.
PPM also offers higher transmission efficiency and less optical power required than the equivalent
PCM system in addition to a better error probability. Baseband receiver theory, tuned front-end
technique and pulse position modulation are combined to construct a complete receiver structure for
optical communication systems (optical fibre communication and optical wireless communication).
Chapter 5 provides an explanation of the methodology used to design, model and analyse these
systems.
Chapter 5: System modelling

This chapter presents explanations of the mathematical models and calculations performed in this investigation.
5.1 Mathematical model

In this section, the mathematical model used in this investigation is explained. Figure 5-1 illustrates the framework of the mathematical model.

Figure 5-1 Mathematical model framework (a) Channel modelling. (b) Receiver modelling. (c) Pulse shaping. (d) Noise modelling. (e) BER and receiver sensitivity calculations.
This mathematical model has a general structure that has the flexibility of modelling sub-component counterparts, ensuring the evaluation of different systems is carried out using the same method of calculations.

5.2 Channel modelling

There are three optical channels in this investigation. The first channel is an ideal channel that may represent a line of sight link or a non-dispersive optical link. In this case, the input received pulse is assumed to be an ideal rectangular pulse. This assumption is made to evaluate the receiver performance independently from the channel effect, however, the use of rectangular pulse still exists in some optical applications.

The input pulse shape is assumed as an ideal rectangular pulse $x(t)$ of unit height and width $T$. This pulse has a Fourier transform of

$$H_p(\omega) = \frac{\sin \left( \frac{\omega T}{2} \right)}{\left( \frac{\omega T}{2} \right)}$$

The second channel is assumed to be a dispersive optical channel that may represent optical fibre channel. Many optical fibres, and in particular jointed fibre links, exhibit pulse outputs with a temporal variation that is closely approximated by a Gaussian distribution. Hence the variation in the optical output power with time may be described as

$$H_p(t) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{t^2}{2\sigma^2}}$$

The pulse variance $\sigma$ is linked to the fibre bandwidth by

$$\sigma = \frac{\sqrt{2ln2T_b}}{2\pi f_n}$$

This pulse has a Fourier transform of
\[ H_p(\omega) = e^{-\omega^2 \sigma^2} \]

The third channel is assumed to be a diffusion channel that may represent a wireless optical link. In this case, the input pulse shape is affected by channel characteristics (dispersion and diffusion). The input pulse shape, therefore, will be more realistically affected by multipath nature of wireless channels resulting in a more dispersive pulse and ISI. In this case, the input pulse shape will be given by the convolution of transmitter pulse shape and channel impulse response.

### 5.3 Receiver modelling

Receiver modelling takes into account the photodiode characteristics and physical realisation of the transimpedance components and pre-detection filter. It is important to note that this approach is used to estimate receiver performance without making any assumptions on the input or output shapes. This will be further explained in pulse shaping and noise modelling.

The general framework in Figure 5-1 shows the receiver modelling block which is well applied to non-tuned front end (conventional model). A more detailed illustration of this block is shown in Figure 5-2.

![Figure 5-2 Conventional receiver model (non-tuned).](image-url)
These steps are taken to ensure a valid receiver model:

- TIA characteristics are modelled according to the theory presented in Chapter 2 and 3. All approximations and assumptions are made according to the theory presented in the optical literature previously discussed. This also applies to photodiode characteristics.
- For conventional receiver model, the 3-dB bandwidth of the TIA is calculated for a given bit-rate and total input capacitance, then the value of the feedback resistor is determined.
- The frequency response of the receiver is determined by the transfer function of TIA and pre-detection filter.
- Output pulse shape is obtained by convolving the received pulse and receiver frequency response.

For tuned A TIA receiver this model is extended as shown in Figure 5-3.

These steps are taken to ensure a valid receiver model:

- All approximations and assumptions of TIA are made similarly as for conventional receiver model.
- For tuned A TIA, the 3-dB bandwidth of the TIA can be calculated in two ways to ensure the same 3-dB bandwidth as a non-tuned front end.
- The 3-dB bandwidth of the TIA is calculated for a given bit-rate then feedback resistor and total capacitance are determined. This needs a further optimisation to include the inductor:
  - Using expressions of bandwidth extension to obtain inductor value.
  - Calculate the inductance value in the feedback loop.
  - Due to bandwidth extension ratio, the 3-dB bandwidth of the TIA will be altered.
  - Then, re-calculate 3-dB bandwidth of the TIA after inductor.
  - Total capacitance will be fixed however a new feedback resistor value is obtained.
- Alternatively, for a given bit-rate, feedback resistor and inductor are determined using expressions in Section 3.4.2.
- The frequency response of the receiver is determined by the transfer function of tuned A TIA and pre-detection filter.
- Output pulse shape is obtained by convolving the received pulse and receiver frequency response.

For tuned B receiver this model is similarly extended as shown in Figure 5-4. These steps are taken to ensure a valid receiver model:

- All approximations and assumptions of TIA are made similarly as for conventional receiver model.
- For tuned B TIA, the 3-dB bandwidth of the TIA can be calculated in two ways to ensure the same 3-dB bandwidth as a non-tuned front end.
- The 3-dB bandwidth of the TIA is calculated for a given bit-rate then feedback resistor and total capacitance are determined. This needs a further optimisation to include the inductor:
- Split the total capacitance.
- Using expressions of bandwidth extension to obtain inductor value.
- Due to bandwidth extension ratio, the 3-dB bandwidth of the TIA will be altered.
- Then, re-calculate 3-dB bandwidth of the TIA after inductor.
- Total capacitance will be fixed however a new feedback resistor value is obtained.

- Alternatively, for a given bit-rate, feedback resistor and inductor are determined using expressions in Section 3.4.3.
- The frequency response of the receiver is determined by the transfer function of tuned B TIA and pre-detection filter.
- Output pulse shape is obtained by convolving the received pulse and receiver frequency response.

**Figure 5-4 Tuned B receiver model.**
5.4 Noise modelling

Although the mathematical formulation of the conventional noise model (Figure 5-5) is correct, it has conditions that must be satisfied. Limitations of this approach can be summarised in these key points:

- The physical realisation of receiver components is not considered.
- Receiver transfer function is determined in terms of the input and output pulse shapes.
- Equalisation is assumed at the receiver output which is necessary to obtain the mentioned output pulse shape.
- Noise integrals are difficult to evaluate since the receiver transfer function is unknown.
- Input and output pulse shapes are normalised to the input pulse duration, and noise integrals are evaluated numerically by factoring out the bit rate dependency.
- Numerical values are given in Table 2-1. These numerical values are valid only if the receiver frequency response is the same as the one used to produce these integrals.
- In another word, this necessarily requires the same input and output pulse shapes associated with the assumed receiver frequency response.
- The bit rate dependency of noise sources leads to misinterpreted noise expressions and inappropriate use of noise integrals.

This approach can be modified in order to drop the input/output pulse shape condition [169]. This modified approach is explained in Section 0. A simplified model is shown in Figure 5-6.

The only limitation associated with this model is that the receiver frequency response is assumed to be non-tuned. This leads to the same conventional noise expressions in a more compact way, but it does not seem to be valid if the front-end circuitry is altered.
Figure 5-5 Conventional noise model (Personick approach).

Figure 5-6 Different representation of Personick integrals.

Figure 5-7 illustrates a more general noise model. This model takes into account:

- The bit-rate independency of noise sources, therefore, noise source has a frequency dependency of a realistic receiver frequency response.
• The transfer function of each noise source depends on the front-end circuitry.

• The equalisation independence of output pulse shape. A physical pre-detection filter design can be integrated into this model. Therefore, there the output pulse shape is obtained depending on a physical realisation of the receiver circuitry with no assumptions regarding input and output pulse shapes.

This model results in a flexibility of:

• Evaluating the receiver performance for any input pulse shape.

• Considering a wide range of physical implementation of the pre-detection filter and TIA configuration.

Figure 5-7a (blue line) illustrates the process of obtaining the total input noise power by dividing the output noise power over the mid-band transimpedance. Figure 5-7b (red line) illustrates the process of obtaining the total input noise power by using noise integrals.
Figure 5-7 Non-tuned receiver noise model used in this investigation $(H(\omega)^2)$ represents the noise transfer function.
5.5 The validity of noise model

Noise are calculated using both models to verify the proposed method. All calculations are attached in appendix A-5. TIA, pre-detection filter and the whole receiver frequency response is illustrated in Figure 5-8. This receiver has a 3-dB bandwidth of 59 MHz, with a feedback resistor of 15.56 kΩ and total input capacitance of 1.5 pF. If the receiver front end is PINFET, a FET transconductance is assumed to have a value of 30 mS and gate leakage current is assumed to have a value of 10 nA. Figure 5-9 illustrates noise power spectrum for each noise source. The total input noise power spectrum (red solid line) is white at low frequencies due to feedback resistor and gate leakage current, and it starts to increase rapidly at high frequencies due to channel noise. Total input noise power spectrum is shaped by the receiver frequency response, producing the output noise power spectrum as shown in Figure 5-10 Error! Reference source not found..

![Graph of receiver frequency response](image)

**Figure 5-8 Receiver frequency response (3rd order filter).**

Noise integrals $\text{BW}_n$ and $\text{BW}_{n^2}$ can be calculated as in Eq. (2-37) and (2-38) so that

$$\text{BW}_n = \frac{1}{H_0^2} \int_0^\infty |H(f)|^2 \, df = 59 \, \text{MHz}$$
\[ \text{BW}_{n2}^3 = \frac{3}{H_0} \int_0^\infty |H(f)|^2 f^2 \, df = 3.3 \times 10^{23} \text{ Hz}^3 \]

Hence, total input noise power \( i_n^2 \) can be calculated using these noise bandwidth values

\[ i_n^2 = \left[ \frac{4KT}{R_f} + 2ql_g + 4KTT \frac{1}{g_mR_f^2} \right] \text{BW}_n + 4KTT \frac{(2\pi C_T)^2}{g_m} \frac{\text{BW}_{n2}^3}{3} = 6.79 \times 10^{-17} \text{ A}^2 \]

Figure 5-9 Input noise power spectrum.

Figure 5-10 Output noise power spectrum (3rd order filter).
Figure 5.11 Receiver Frequency Response (1\textsuperscript{st} order filter).

Figure 5.12 Output noise power spectrum (1\textsuperscript{st} order filter).

Figure 5-11 illustrates receiver frequency response with a single pole front end and 1\textsuperscript{st} order low pass pre-detection filter. Input noise power spectrum still the same however, output noise power spectrum is different due to the roll-off of the 1\textsuperscript{st} low pass filter is less steppe than the 3\textsuperscript{rd} order filter.

In this case, $BW_n$ and $BW_{n2}$ becomes

$$BW_n = \frac{1}{H_0^2} \int_{0}^{\infty} |H(f)|^2 \, df = 59 \text{ MHZ}$$
The difference in total noise power is due to the channel noise at high frequencies. Input noise powers are calculated for the same receiver using the proposed noise model. For 1\textsuperscript{st} order pre-detection filter

\[ I_{2,\text{non-tuned}} = \frac{1}{Z_0} \int_0^\infty |H(f)|^2 \, df = 59 \times 10^6 \text{Hz} \]

\[ I_{3,\text{non-tuned}} = \frac{1}{Z_0} \int_0^\infty |Z(f) \cdot H(f)|^2 \, df = 11.17 \times 10^3 \text{Hz} \]

\[ H(f) = \frac{1 + 2\pi f R_f C_T}{g_m R_f} \]

\[ i_n^2 = \left[ \frac{4KT}{R_f} + 2ql_g \right] I_{2,\text{tuned}} + 2ql_c g_m I_{3,\text{non-tuned}} = 6.79 \times 10^{-17} A^2 \]

For 3\textsuperscript{rd} order pre-detection filter

\[ I_{2,\text{non-tuned}} = \frac{1}{Z_0} \int_0^\infty |H(f)|^2 \, df = 59 \times 10^6 \text{Hz} \]

\[ I_{3,\text{non-tuned}} = \frac{1}{Z_0} \int_0^\infty |Z(f) \cdot H(f)|^2 \, df = 32.95 \times 10^3 \text{Hz} \]
 Both methods have the same results, and calculations are attached in appendix A-4 and A-5.

5.6 Tuned receiver noise modelling

Now, it can be seen how this model can easily be modified to consider tuned TIA. Figure 5-13 and Figure 5-14 illustrate the noise models of tuned A and tuned B receivers, respectively. For noise optimisation in BJT, collector current can be calculated in two ways:

- Equate the total noise and collector current to find the minimum value that produces the minimum noise.
- Alternatively, use the expressions in Eq (3-65), (3-66), and (3-83).

Both should result in the same collector current value (this can be verified graphically).
Figure 5-14 Tuned B noise model, \((H(\omega))^2\) represents the noise transfer function for each noise source.

Figure 5-15 General noise model.
5.7 Output pulse shape

Output pulse shape is obtained by convolving the input pulse and whole receiver frequency response. Output pulse as a function of time is used to evaluate the peak voltage at decision time, pulse slope and ISI performance.

![Diagram](image)

Figure 5-16 Output pulse shape.

5.8 Bit error rate and receiver sensitivity

Each receiver will be evaluated based on the total receiver noise and ISI performance in addition to the bit error rate and the required photon per pulse. Expressions for BER are given for OOK and PPM in Section 2.5.5 and 4.3.

Figure 5-17 illustrates the general calculations of BER for OOK receiver. In addition to channel effect, TIA and filter characteristics are considered to evaluate the total ISI at the detection input, this is also followed for both non-tuned and tuned receivers. Since the BER expressions in Section 2.5.5 is for single pulse detection, the ISI contribution from isolated pulse to other symbols is evaluated. Since bits preceding is not predictable, ISI is evaluated for the worst case.
Depending on the input transistor, a noise optimisation routine is followed to obtain the minimum noise power at the receiver output. This routine is followed for both non-tuned and tuned front end.
BER calculations for OOK receiver, with optimised noise and ISI performance considered, is shown in Figure 5-18. For PPM receiver, error probability expressions are given in Section 4.3. Figure 5-19 illustrates the general model for PPM BER calculations.

![Diagram of Receiver Characteristics]

**Figure 5-19 PPM bit error rate calculations.**

For a given bit rate and bit error rate, the number of photons per pulse is calculated as shown in Figure 5-20. This model assumes a central detection with a fixed threshold level. Decision time $t_d$ and threshold level are to be evaluated for a given output pulse shape. Error probabilities are then evaluated to obtain the minimum required optical power. For slope detection, photons per pulse and threshold level are evaluated for a given bit error rate as shown in Figure 5-21. A threshold variable $v$ is defined as a ratio of output pulse peak voltage to voltage at decision time $t_d$. Then, for a given BER, decision time, optimum threshold level $v$, and the minimum number of photons per pulse are calculated.
Evaluating the required photon per pulse for a given bit error rate (central detection).

A diagnostic check is run to evaluate BER for the obtained number of photons and threshold level to ensure the validity of calculations. Minimum required optical power for a given bit error rate is then calculated for a given number of photons per pulse and bit rate.

All mathematical models and calculations are attached in Appendix A, Appendix B and Appendix C. Simulation procedures, system modelling (receiver modelling, noise modelling, optical link modelling), design optimisation, BER and sensitivity calculations, performance evaluation, and
modulations (OOK, PPM, di-code PPM) and signal modelling are developed based on the information presented in [339]-[358362].

Figure 5-21 Evaluating the required photon per pulse for a given bit error rate (slope detection).
Chapter 6: RESULTS AND DISCUSSION I (TUNED FRONT END OPTICAL RECEIVER PERFORMANCE)

This chapter presents the numerical results of an ideal optical channel. The received optical pulse is assumed to be a rectangular shaped. The reason behind choosing this communication system is to give an abstraction of tuned front-end performance away from channel effect. Results are obtained for tuned and non-tuned front end with different input configurations. The modulation scheme considered for this system is the on-off keying with non-return to zero signalling (OOK-NRZ). Section 6.1 provides a brief introduction to the full communication system, and the simulation procedures. Section presents a brief performance comparison between non-tuned and tuned receivers with a fixed receiver bandwidth. Section 6.3 discuss how non-tuned and tuned receivers bandwidth can be optimised to achieve best performance for both receivers. This optimisation leads to a fair comparison between non-tuned and tuned receivers. Optimised receivers are compared in detail in Section 6.4. In this section, input-referred noise power spectrums of both receivers are compared, showing the difference in noise spectrums due to noise referral of non-tuned and tuned front end.
6.1 System overview and simulation procedures

The performance of tuned front-end receiver is examined using NRZ-OOK modulation. The performance is evaluated based on numerical results obtained using the mathematical model explained in Chapter 5. Channel is assumed to be an ideal channel and the received pulse is taken as a pure full-width rectangular pulse. Based on numerical simulations of the bit error probability including ISI, results aim to give quantitative predictions of the OOK-NRZ sensitivity for various types of receivers (with different front-end configurations, different photodetectors, different input transistors and different pre-detection filters).

![Figure 6-1 Schematic diagram of the receiver front end.]

For comparison, the total input capacitance of 1.5 pF, operating wavelength of 650nm and quantum efficiency of 100% are assumed. APDs are practically limited by the slow response time thus it is only considered for a data rate of 100 Mbits/s. PIN photodetectors have large bandwidth and most likely to be implemented in high data rate applications thus it is considered for data rates of 100 Mbits/s, 1 Gbits/s and 10 Gbits/s. APD materials considered in this investigation are:

- Silicon APD with a gain of 10 and noise factor of 5.5.
- InGaAs APD with a gain of 100 and noise factor of 7.9.
- Germanium APD with a gain of 10 and noise factor of 9.2.

Transimpedance amplifier configuration is considered for tuned and non-tuned receivers with common source (CS) FET and common collector (CE) BJT input stages. In BJT input receiver, it is assumed to have a low re and high current gain which is enough to neglect noise coming from the second stage. The voltage gain of all pre-amplifiers is assumed to be 10 in order to maintain a stable feedback loop in practice. 1st order and 3rd order low pass filters are considered as pre-detection filters.

![Schematic diagram of a threshold-crossing detector and central decision gate.](image)

Figure 6-2 Schematic diagram of a threshold-crossing detector and central decision gate.

Bit error rate is taken as 1 error in $10^9$ pulses then the minimum number of photons are evaluated for this given BER. With central decision detection, assuming light source has an insignificant extinction ratio that received level zero pulses power is zero (no power). $V_{\text{max}}$ and $V_{\text{min}}$ represent the received signal levels at the output of the pre-detection filter, and $V_{\text{th}}$ is the threshold voltage with the decision made at the centre of each pulse at time $t_{\text{th}}$.

BER is evaluated in the term of input referred signal current ($V_{\text{max}}$ and $V_{\text{min}}$) and input-referred noise current. Input referred noise current is calculated as the output rms noise power divided over the mid-band transimpedance. Input referred signal current is the peak voltage at decision time divided over the mid-band transimpedance. $V_{\text{min}}$ is the voltage at the consequent decision time ($t_{\text{th}}+t_s$). For ISI consideration, $V_{\text{min}}$ is calculated at $t_{\text{th}}+t_s$, $t_{\text{th}}+2t_s$, and $t_{\text{th}}+3t_s$. 

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Numerical simulations and all related calculations are attached in Appendix B. Numerical simulation is performed for four different receiver configurations: PINFET, PINBJT, APDFET and APD BJT. Receiver performance is evaluated with two pre-detection filters: 1st order low pass filter and 3rd order Butterworth low pass filter. Calculations in Appendix B-1, for non-tuned front end, are as follow:
1. For a given bit-rate (B), TIA and filter 3-dB bandwidth are set to 0.7 times the bit rate.

2. The whole receiver 3-dB frequency response is then determined by the transfer functions of TIA and pre-detection filer.

3. The contribution of each noise source is calculated then total input noise power is calculated for both FET and BJT receivers.

4. For BJT receiver, collector current is optimised as in Eq (3-35) and it is verified graphically.

5. Total input noise power of BJT receiver is calculated for optimum collector current.

6. Output pulse shape is obtained by convoluting the input pulse shape and receiver frequency response (peak voltage and decision time are evaluated for the output pulse shape).

7. Error bit rate is evaluated across a range of number of photons (different input power levels) for all receiver configurations.

8. Error probability function (P_e(b)) is set to a given BER of 1 x 10^{-9}, number of photons (b) is obtained as the root of of P_e(b)), (the lowest number of photons b_{min}).

9. Receiver sensitivity is defined as the minimum required optical power per pulse averaged over pulse width.

10. For APD receivers, for a given avalanche gain and noise factor, total input noise is recalculated and P_e(b) is re-evaluated.

11. The same optimisation routine is performed as in step 8 thus receiver sensitivity for APD receiver is calculated for the new lowest number of photons.

Same procedures are followed for tuned receivers however, these modifications are performed for tuned A:

- Same TIA 3-dB is calculated as non-tuned receiver.
- for a given \(\Delta \ell_f\) and \(\Delta R_f\), values of feedback resistor and inductor are calculated.
- The frequency response of the whole receiver is determined by the tuned A transimpedance and pre-detection filter.
• Noise integrals are calculated for tuned A receiver (including the frequency response of the pre-detection filter).

• Total input noise power is calculated for tuned A receiver.

• Output pulse shape is obtained by convoluting the input pulse shape and receiver frequency response (tuned A TIA).

For tuned B:

• Same TIA 3-dB is calculated as non-tuned and tuned A receiver

• for a given splitting ratio \( a \), \( \Delta L_B \) and \( \Delta R_f \), values of feedback resistor and inductor are calculated.

• The frequency response of the whole receiver is determined by the tuned B transimpedance and pre-detection filter.

• Noise integrals are calculated for tuned B receiver.

For calculations in Appendix B-2 to B-5, receiver performance is simulated for a range of 3-dB bandwidth. TIA 3-dB bandwidth range is from 0.5 B to 1 B and pre-detection filter 3-dB bandwidth range is from 0.5 B to 2 B.

Non-tuned, tuned A and tuned B receivers are examined following these procedures:

• Examine noise integrals, each noise source power and total noise power for a range of bandwidth.

• Find the optimum bandwidth range for noise power.

• Examine the pulse shape for different receiver 3-dB bandwidth.

• Calculate the peak voltages at decision times.

• Plot all possible bit sequences and assume worst ISI case.

• Calculate Q for the worst ISI case.

• Evaluate BER for the worst case.
• Calculate receiver sensitivity.

Results obtained assuming fixed 3-dB bandwidth of 0.7 B is presented and discussed in Section 6.2. Of the variable 3-dB bandwidth, results are presented and summarised in Section 6.3. comparison of optimised receiver performance is presented in Section 6.4.

6.2 Tuned receiver performance

Receiver performance is evaluated assuming a fixed 3-dB bandwidth of 0.7 times the bit rate for both pre-amplifier and pre-detection filter.

6.2.1 Non-tuned front-end receiver

Non-tuned TIA frequency response is illustrated in Figure 6-5 (red trace). The whole receiver frequency response is shown in the blue trace for 1st order low pass filter and in the black trace for 3rd order Butterworth pre-detection filter.

![Figure 6-5 Receiver frequency response without a filter, with 1st order LPF, and with 3rd order Butterworth filter.](image)

Noise equivalent bandwidth of non-tuned front-end whose frequency response is approximated as a single pole equals to \( \frac{\pi}{2} \) times 3-dB bandwidth (NEB = 117 MHz for -3dB bandwidth of 75MHz, and
78 MHz for 3-dB of 50 MHz). For non-tuned receiver with 1\textsuperscript{st} order LPF pre-detection filter (filter 3-dB of 75 MHz), the whole receiver 3-dB bandwidth is 59 MHz with NEB of 58.9 MHz. For non-tuned receiver with 3\textsuperscript{rd} order Butterworth LPF pre-detection filter (filter 3-dB bandwidth of 75 MHz), the whole receiver bandwidth is 48.3 MHz with NEB of 58.9 MHz.

![Figure 6-6 Output noise power divided over mid-band transimpedance (frequency normalised to bit-rate).](image)

For TIA (amplifier 3-dB bandwidth of 75 MHz), feedback resistor of 15.5 kΩ produces an equivalent input noise power of $6.22 \times 10^{-17}$ A\textsuperscript{2} with both pre-detection filters. For FET input stage, assuming a gate leakage current of 10 nA, transconductance of 30 mS, the total input-referred noise power is $7.87 \times 10^{-17}$ A\textsuperscript{2} and $6.79 \times 10^{-17}$ A\textsuperscript{2} for 1\textsuperscript{st} order filter and 3\textsuperscript{rd} order Butterworth, respectively. The difference in total input referred noise power can be explained by examining the roll-off of both filters and input noise power spectrum at high frequencies. Output noise power spectrum divided over receiver mid-band transimpedance for both receivers with 1\textsuperscript{st} order LPF and 3\textsuperscript{rd} order Butterworth filter is illustrated in Figure 6-6. Since the roll-off of 3\textsuperscript{rd} order filter is steeper than that of the 1\textsuperscript{st} order,
a receiver with 3\textsuperscript{rd} order pre-detection filter produces less output noise power compared to the receiver with 1\textsuperscript{st} order pre-detection filter.

If FET input stage is replaced by a BJT transistor, assuming optimum collector current, total input noise power becomes $1.29 \times 10^{-16} \text{ A}^2$ (1\textsuperscript{st} order LPF), and $1.01 \times 10^{-16} \text{ A}^2$ (3\textsuperscript{rd} order Butterworth). These results demonstrate a difference of approximately 1.7 and 2.1 dB in noise power between FET and BJT receivers with 1\textsuperscript{st} order and 3\textsuperscript{rd} order pre-detection filters (operating bit-rate of 100 Mbits/s).

These results demonstrate a difference of approximately 1.7 and 2.1 dB in noise power between FET and BJT receivers with 1\textsuperscript{st} order and 3\textsuperscript{rd} order pre-detection filters (operating bit-rate of 100 Mbits/s).

![Figure 6-7 Optimum collector current for 100 Mbits/s PINBJT receiver.](image)

Total input noise power of both receivers is calculated over a range of operating bit-rates, it is found that FET receiver would have an advantage over BJT receiver at data rates up to 800 Mbits/s. however, BJT has an advantage for data rates over 1 Gbits/s. Figure 6-8 illustrates total receiver input-referred noise power for both input stages.
Figure 6-8 Noise performance of non-tuned BJT and FET receivers over range of bit-rates.

Output pulse shape obtained from each filter is illustrated in Figure 6-9 and Figure 6-10. 3\textsuperscript{rd} order Butterworth filter has less effect on ISI than 1\textsuperscript{st} order filter.

Figure 6-9 Output pulse shape (1\textsuperscript{st} order LPF receiver).
Error probability function is evaluated for different input optical power levels and is plotted against a range of number of photons per received pulse, shown in Figure 6-11, for PINFET and PINBJT receivers with 1\textsuperscript{st} order LPF and 3\textsuperscript{rd} order Butterworth pre-detection filters.

It can be concluded that for non-tuned front end, 3\textsuperscript{rd} order Butterworth has an advantage over 1\textsuperscript{st} order LPF pre-detection filter in terms of noise and ISI. Minimum optical power for each receiver configuration is calculated for a BER of 1 in $10^9$.

**Table 6-1 Receiver sensitivity (100 Mbits/s OOK-NRZ)**

<table>
<thead>
<tr>
<th>Receiver configuration</th>
<th>PINFET</th>
<th>PINBJT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-detection filter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1\textsuperscript{st} order LPF</td>
<td>-39.46 dBm</td>
<td>-38.39 dBm</td>
</tr>
<tr>
<td>3\textsuperscript{rd} order Butterworth</td>
<td>-40.21 dBm</td>
<td>-39.34 dBm</td>
</tr>
</tbody>
</table>
By examining Table 6-1, the use of 3rd order Butterworth pre-detection filter has an advantage over the 1st order LPF within 1 dB in receiver sensitivity, while PINFET has also an advantage of 1 dB over PINBJT receiver. If the PIN detector is replaced by Avalanche detector (APD), assuming multiplication gain of 10 and noise factor of 5.5 (typical values for silicon APD), receiver sensitivity can be improved within 6 dB.

Table 6-2 Receiver sensitivity with APD (100 Mbits/s OOK-NRZ)

<table>
<thead>
<tr>
<th>Pre-detection filter</th>
<th>PINFET</th>
<th>PINBJT</th>
<th>APDFET</th>
<th>APDBJT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st order LPF</td>
<td>-39.8 dBm</td>
<td>-38.39 dBm</td>
<td>-45.61 dBm</td>
<td>-45.30 dBm</td>
</tr>
<tr>
<td>3rd order Butterworth</td>
<td>-39.47 dBm</td>
<td>-39.28 dBm</td>
<td>-46.28 dBm</td>
<td>-46.05 dBm</td>
</tr>
</tbody>
</table>
Since a FET input receiver would produce less noise power than BJT receivers, APD will have higher improvement in receiver sensitivity for BJT than FET, compared with that obtained with a PIN detector. APDBJT with 1st order pre-detection receiver has a sensitivity improvement of 6.9 dB over PINBJT. This improvement is reduced to 6 dB for PINFET compared to APDFET.

Although PINBJT receiver has less sensitivity among other configurations, it has the best performance in high data rates due to two reasons:

- BJT input stage will have less noise power at high data rates (with optimised collector current).
- Avalanche photodetectors have low bandwidth compared to PIN photodetectors, and the use of APD is limited to low data rates.

### 6.2.2 Tuned A front-end receiver

The performance of the same receiver configurations (PINFET, PINBJT, APDFET, and APDBJT) is simulated, providing that the TIA is replaced by a tuned A TIA. Same receiver model is used with modified TIA transfer function and noise integrals. Noise, output pulse shape, ISI, error probability, and receiver sensitivity are evaluated in the same way as for non-tuned front end.

Tuned A TIA frequency response is illustrated in Figure 6-12 (red line). The variation of frequency response due to pre-detection filter is shown in Figure 6-12 (1st order filter blue line and 3rd order Butterworth black line). Noise equivalent bandwidth of tuned A receiver is 100 MHz (17 MHz less than non-tuned receiver), 64.9 MHz with 1st order pre-detection filter (6 MHz higher than non-tuned), and 69.8 MHz with 3rd order Butterworth pre-detection filter (10 MHz higher than non-tuned).
Tuned A TIA frequency response.

Table 6-3 Comparison of non-tuned and tuned A receiver noise power

<table>
<thead>
<tr>
<th>Noise power (A^2)</th>
<th>Non-tuned</th>
<th>Tuned A</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PINFET</td>
<td>PINFET</td>
</tr>
<tr>
<td>Pre-detection filter</td>
<td>6.79 10^{17}</td>
<td>1.29 10^{16}</td>
</tr>
<tr>
<td>1^{st} order LPF</td>
<td>7.87 10^{17}</td>
<td>1.01 10^{16}</td>
</tr>
</tbody>
</table>

Total noise power in tuned A receiver is significantly reduced compared to non-tuned front end. Tuned A front end with 1^{st} order low pass filter has an advantage over non-tuned front end with 1^{st} order and 3^{rd} order pre-detection filters. Main reasons for noise power reduction of tuned A receiver are:

- Feedback resistor in tuned A front end is 1.8 times higher than non-tuned front end.
- Feedback resistor in tuned A receiver produces 3 dB less input-referred noise power compared to non-tuned front end.
- Tuned A transimpedance has two poles and a zero with either 1^{st} order LPF or 3^{rd} order Butterworth output noise power spectrum is sharper than that of the non-tuned receiver.
Table 6.4 Comparison of non-tuned and tuned A receiver sensitivity (PIN)

<table>
<thead>
<tr>
<th>Sensitivity</th>
<th>Non-tuned</th>
<th>Tuned A</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PINFET</td>
<td>PINBJT</td>
</tr>
<tr>
<td>Pre-detection</td>
<td>-39.8 dBm</td>
<td>-38.39 dBm</td>
</tr>
<tr>
<td>1st order LPF</td>
<td>-39.8 dBm</td>
<td>-38.39 dBm</td>
</tr>
<tr>
<td>3rd order Butterworth</td>
<td>-39.47 dBm</td>
<td>-39.28 dBm</td>
</tr>
</tbody>
</table>

Pre-detection filter has no significant impact on tuned A receiver sensitivity. However, Tuned A receiver with 1st order LPF pre-detection filter has an advantage over non-tuned receiver with either 1st order or 3rd order pre-detection filter. Tuned A has a sensitivity improvement within 2 dB over non-tuned receiver.

Table 6.5 Comparison of non-tuned and tuned A receiver sensitivity (APD)

<table>
<thead>
<tr>
<th>Sensitivity</th>
<th>Non-tuned</th>
<th>Tuned A</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>APDFET</td>
<td>APDBJT</td>
</tr>
<tr>
<td>Pre-detection</td>
<td>-45.61 dBm</td>
<td>-45.30 dBm</td>
</tr>
<tr>
<td>1st order LPF</td>
<td>-45.61 dBm</td>
<td>-45.30 dBm</td>
</tr>
<tr>
<td>3rd order Butterworth</td>
<td>-46.28 dBm</td>
<td>-46.05 dBm</td>
</tr>
</tbody>
</table>

For APD receivers, tuned A has no significant improvement over non-tuned since NEB of tuned A receiver is larger than non-tuned receiver. These results indicate that the pre-detection filter cut-off frequency may be optimum for non-tuned receiver, however, it is not optimum for tuned A receiver.

6.2.3 Tuned B front-end receiver

The performance of the same receiver configurations (PINFET, PINBJT, APDFET, and APDBJT) is simulated, providing that the TIA is replaced by a tuned B TIA. Same receiver model is used with
modified TIA transfer function and noise integrals. Noise, output pulse shape, ISI, error probability, and receiver sensitivity are evaluated in the same way as for non-tuned and tuned A receivers.

Table 6-6 comparison of non-tuned, tuned A and tuned B receivers (tuned B TIA with splitting ratio of 0.2).

<table>
<thead>
<tr>
<th></th>
<th>PINFET</th>
<th>PINBJT</th>
<th>APDFET</th>
<th>APDBJT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-tuned</td>
<td>Tuned A</td>
<td>Tuned B</td>
<td>Non-tuned</td>
</tr>
<tr>
<td>1st order LPF</td>
<td>-45.61</td>
<td>-46.57</td>
<td>-46.28</td>
<td>-45.30</td>
</tr>
<tr>
<td>3rd order Butterworth</td>
<td>-46.19</td>
<td>-46.29</td>
<td>-46.47</td>
<td>-45.97</td>
</tr>
</tbody>
</table>

Table 6-6 summarise the results obtained for non-tuned and tuned receivers. Although tuned receivers have higher sensitivity compared to non-tuned receiver, the pre-detection filter seems to degrade the performance of tuned receivers. In the next section, the effect of bandwidth variation of TIA and pre-detection filter is examined in order to optimise the performance of tuned front end.

6.3 Tuned receiver optimisation

The performance of each receiver is examined across a range of 3-dB bandwidth from 0.5 B to 1 B for the TIA and from 0.5 B to 2 B for the pre-detection filter.

6.3.1 Non-tuned front-end receiver

Figure 6-13 and Figure 6-14 show the variation of the total input referred noise power with the variation of receiver bandwidth. The horizontal axis is the 3-dB bandwidth of the pre-detection. The vertical axis is the 3-dB bandwidth of the TIA amplifier. Noise power increases more rapidly along the vertical axis due to the increase of feedback resistor noise, this can be noticed as in Figure 6-15.
Feedback resistor value and noise equivalent bandwidth dominate the total noise power. Therefore, TIA bandwidth should be at the minimum to obtain the lowest feedback resistor value. The variation of pre-detection 3-dB bandwidth only affects the noise bandwidths. It always limits the total noise power at high frequencies. Since 3rd order filter frequency response would have a steeper roll-off than 1st order filter, 3rd order filter produces less noise power than 1st order filter.

The same pre-detection bandwidth effect can be noticed in BJT receiver, however, total noise power of a BJT receiver is higher than a FET receiver because base and collector shot noise has higher noise power than gate leakage and channel noise*.

Figure 6-13 PINFET non-tuned receiver noise power with 1st order low pass pre-detection filter.

* The simulations of non-tuned receiver with BJT input stage can be found in Appendix B.3.1 (page: B-197, for 1st order pre-detection filter) and B.5.1 (Page: B-270, for 3rd order Butterworth filter).
Figure 6-14 PINFET non-tuned receiver noise power with 3\textsuperscript{rd} order Butterworth filter pre-detection filter.

Figure 6-15 Feedback resistor input noise power (1\textsuperscript{st} order LPF).
For a single pulse, the maximum peak voltage is calculated at central of the pulse and minimum peak voltage is calculated at subsequent decision instants. For inter-symbol interference, the minimum peak to peak voltage is considered for error probability calculations. It is important to note that ISI performance of non-tuned receiver doesn’t degrade the total receiver performance as it does in the case of tuned receivers.

**Figure 6-16 Output pulse shape (1st order low pass filter).**

**Figure 6-17 Output pulse shape (3rd order Butterworth).**
Optimum receiver bandwidth is determined by trading off the lowest noise power and lowest ISI. Compared with optimum 1st order pre-detection filter, for FET receiver, the use of 3rd order predetection improves the ISI performance within 1 dB and reduces noise power by 0.4 dB. For BJT receiver, noise power is reduced by 0.87 dB.

As a summary, the use of 3rd order predetection filter with an optimum receiver bandwidth has a sensitivity improvement of 0.7 dB (FET) and 1 dB (BJT) over 1st order low pass filter with an optimum receiver bandwidth.

6.3.2 Tuned A front-end receiver

Tuned A receiver produces less noise power for the same receiver bandwidth. It is noticeable that the variation in noise power is less sensitive to pre-detection 3-dB bandwidth variation compared to non-tuned front end.

![Figure 6-18 PINFET tuned A receiver noise power (1st order LPF).](image-url)
The introduction of inductance in the feedback loop would have a benefit of increasing the receiver transimpedance for the same 3-dB cut-off frequency; the amount of increment would have a direct benefit of reducing the thermal noise at the receiver front end as the noise due to the feedback resistor can drop by 3dB. In comparison to 1\textsuperscript{st} order LPF, the use of 3\textsuperscript{rd} order pre-detection has an insignificant effect on improving the ISI performance, however, it reduces the total noise power by 0.89 dB for FET, and 1.85 dB for BJT. The optimum 3-dB bandwidth of the 1\textsuperscript{st} order LPF is 0.8 times the bit-rate, and 0.7 times the bit-rate for 3\textsuperscript{rd} order Butterworth.

![Filter 3-dB bandwidth Hz](image)

**Figure 6-19 PINFET tuned A receiver (3\textsuperscript{rd} order Butterworth).**

Although noise performance is improved by replacing 1\textsuperscript{st} order LPF by a 3\textsuperscript{rd} order Butterworth filter, ISI performance of tuned A receiver is limited by the frequency response of TIA. This behaviour results in less variation of the overall system performance compared to non-tuned receiver. However, tuned A receiver has a better overall performance with both pre-detection filter compared to non-tuned receiver.
6.3.3 Tuned B front-end receiver

The performance of tuned B receiver is examined for splitting ratio of 0 to 0.5. The optimum splitting ratio is 0.4 in terms of overall receiver performance. The optimum pre-detection 3-dB bandwidth is 0.5 times the bit rate for all splitting ratio.
Figure 6-22 PINFET tuned B receiver noise power (1st order LPF/a=0.2).

Figure 6-23 PINFET tuned B receiver (3rd order Butterworth/a=0.2).
Figure 6-24 PINFET tuned B receiver noise power (1st order LPF/a=0.4).

Figure 6-25 PINFET tuned B receiver (3rd order Butterworth/a=0.4).
By comparing Figure 6-23 and Figure 6-23, a=0.2, the use of 3rd order filter demonstrates a significant reduction in noise power. In Figure 6-24 and Figure 6-25, a=0.4, the effect of the pre-detection filter becomes less significant since the roll-off of the TIA frequency response becomes sharper.

In FET receiver, a=0.2, the use of 3rd order filter reduces the total noise power by 2 dB compared to the 1st order filter. While for a=0.4, it reduces the noise power by 0.84 dB.

In BJT receiver, for a=0.2, the use of 3rd order filter reduces the total noise power by 1.86 dB compared to the 1st order filter. while for a=0.4, it reduces the noise power by 1.16 dB. This behaviour is further explained when it comes to comparing the output noise power spectrum of different receivers in Section 6.4.

6.3.4 Receiver optimisation summary

In this section, the optimisation of non-tuned and tuned receivers is briefly discussed, highlighting some important points regards the effect of the 3-dB bandwidth of the whole receiver. In the following section, non-tuned and tuned receivers (with optimum receiver bandwidth) are compared in more detail.

6.4 Performance comparison

Non-tuned and tuned receivers are compared in terms of the minimum number of photons required to achieve BER of 1 in $10^9$. Performance comparison when input FET stage is employed is presented in Section 6.4.1, and when input BJT stage is employed is presented in Section 6.4.2. Then, performance comparison when PIN PD is replaced by an APD is presented in Section 6.4.3.
6.4.1 FET receivers

The performance of Non-tuned and tuned receivers that employ PINFET front end is evaluated (with different pre-detection filters).

![Diagram of receiver components]

**Figure 6-26 illustration of receiver components (FET comparison).**

Figure 6-27 and Figure 6-28 illustrate the equivalent input noise current spectral density and noise spectrum components of tuned A TIA and tuned B TIA in comparison to non-tuned TIA.

Gate leakage current referred noise in non-tuned and tuned A have the same characteristics. The curve of this noise source is constant at all frequencies (white noise), this is because noise is referred directly to the input of the receiver. Channel noise curves of non-tuned and tuned A have the same characteristics at high frequency, however, at low frequencies, tuned A channel noise curve has a lower equivalent value due to feedback resistor being less than that of non-tuned TIA.

Feedback resistor curve of tuned A TIA has lower equivalent value, but only at low frequencies. At high frequencies, the curve is decaying due to referring this noise source through RL branch. Feedback resistor curve in non-tuned TIA is white as the gate leakage current since they are referred in the same way.

By comparing the total input noise current spectral density of both non-tuned and tuned A, both have the same characteristic at high frequencies and are constant at low frequencies. The noticeable difference is that non-tuned curve starts to rise at the frequency given by the zero in the channel noise transfer function, however, tuned A curve has a smooth notch due to the pole in feedback resistor noise transfer function.
Figure 6-27 Equivalent input noise current spectral density of a PINFET TIA (noise spectrum components are shown in dotted lines).

Tuned B TIA has totally different characteristic for all noise spectrum component that results in the spectral curve in Figure 6-28 (red curve). Tuned B input noise spectrum has a sharp notch, this characteristic can be explained by examining individual noise spectrum components as shown in Figure 6-29. Since feedback resistor noise and gate noise are referred through the same impedance

\[
\left[1 - \frac{(1-\alpha)\omega^2}{\omega_f\omega_T}\right]^2,
\]

their spectrums have a notch at the resonant frequency formed by \(L_B\) and \(C_{in}\). At high frequencies, both spectrums are proportional to \(\omega^4\) term of this transfer function.

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Figure 6-28 Input noise current spectral density of tuned B TIA.

Figure 6-29 Input noise power spectral curves of individual noise sources (Tuned B TIA).
Figure 6-30 input noise spectral density of channel noise in tuned B TIA verses non-tuned TIA.

Channel noise has a notch at the frequency formed by \( L_B, C_{in} \) and \( C_d \). At high frequencies, noise spectrum is proportional to \( \omega^6 \) term. Figure 6-30 illustrates the characteristic of channel noise spectrum of tuned B TIA in comparison to non-tuned TIA.
As a result, the output noise spectrum of tuned B TIA has a notch (due to the resonance of L and C) and hump (due to $\omega^6$ term) as shown in Figure 6-31 and Figure 6-32. The notch is less noticed in tuned A noise spectrum. The hump is not noticeable in non-tuned and tuned A as high-frequency part of the input noise spectrums is proportional to $\omega^2$ compared to the higher order term in tuned B TIA
input noise spectrum ($\omega^6$). Figure 6-31 and Figure 6-32 illustrate the total output noise spectrums (input noise spectrum shaped by the receiver frequency response) of non-tuned, tuned A and tuned B receivers with 1\textsuperscript{st} order and 3\textsuperscript{rd} order pre-detection filters. Integrated noise value for each receiver is shown in Table 6-7.

Table 6-7 PINFET receiver performance comparison (100 Mbits/s). Each TIA has a 3-dB bandwidth of 50 MHz and optimum filter 3-dB bandwidth is used for each receiver.

<table>
<thead>
<tr>
<th>1\textsuperscript{st} order LPF</th>
<th>Non-tuned</th>
<th>Tuned A</th>
<th>Tuned B (a=0.4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rf</td>
<td>23.3 kΩ</td>
<td>42.6 kΩ</td>
<td>64.2 kΩ</td>
</tr>
<tr>
<td>BW\textsubscript{3dB,filter}</td>
<td>70 MHz</td>
<td>80 MHz</td>
<td>50 MHz</td>
</tr>
<tr>
<td>$i^2_{n,Rf}$</td>
<td>$3.23 \times 10^{-17}$ A$^2$</td>
<td>$1.47 \times 10^{-17}$ A$^2$</td>
<td>$0.78 \times 10^{-17}$ A$^2$</td>
</tr>
<tr>
<td>$i^2_{n,G}$</td>
<td>$1.47 \times 10^{-19}$ A$^2$</td>
<td>$1.63 \times 10^{-19}$ A$^2$</td>
<td>$0.97 \times 10^{-19}$ A$^2$</td>
</tr>
<tr>
<td>$i^2_{n,D}$</td>
<td>$7.85 \times 10^{-18}$ A$^2$</td>
<td>$4.78 \times 10^{-18}$ A$^2$</td>
<td>$0.78 \times 10^{-18}$ A$^2$</td>
</tr>
<tr>
<td>$i^2_{n,TIA,FET}$</td>
<td>$4.02 \times 10^{-17}$ A$^2$</td>
<td>$1.96 \times 10^{-17}$ A$^2$</td>
<td>$0.87 \times 10^{-17}$ A$^2$</td>
</tr>
<tr>
<td>b</td>
<td>$6.12 \times 10^3$ A$^2$</td>
<td>$3.78 \times 10^3$ A$^2$</td>
<td>$2.95 \times 10^3$ A$^2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3\textsuperscript{rd} order Butterworth</th>
<th>Non-tuned</th>
<th>Tuned A</th>
<th>Tuned B (a=0.4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BW\textsubscript{3dB,filter}</td>
<td>70 MHz</td>
<td>70 MHz</td>
<td>50 MHz</td>
</tr>
<tr>
<td>$i^2_{n,Rf}$</td>
<td>$3.32 \times 10^{-17}$ A$^2$</td>
<td>$1.56 \times 10^{-17}$ A$^2$</td>
<td>$0.66 \times 10^{-17}$ A$^2$</td>
</tr>
<tr>
<td>$i^2_{n,G}$</td>
<td>$1.51 \times 10^{-19}$ A$^2$</td>
<td>$1.70 \times 10^{-19}$ A$^2$</td>
<td>$0.83 \times 10^{-19}$ A$^2$</td>
</tr>
<tr>
<td>$i^2_{n,D}$</td>
<td>$3.22 \times 10^{-18}$ A$^2$</td>
<td>$2.774 \times 10^{-18}$ A$^2$</td>
<td>$0.43 \times 10^{-18}$ A$^2$</td>
</tr>
<tr>
<td>$i^2_{n,TIA,FET}$</td>
<td>$3.66 \times 10^{-17}$ A$^2$</td>
<td>$1.86 \times 10^{-17}$ A$^2$</td>
<td>$0.71 \times 10^{-17}$ A$^2$</td>
</tr>
<tr>
<td>b</td>
<td>$5.19 \times 10^3$ A$^2$</td>
<td>$3.36 \times 10^3$ A$^2$</td>
<td>$2.58 \times 10^3$ A$^2$</td>
</tr>
</tbody>
</table>

Tuned A receiver has 3.12 dB (1\textsuperscript{st} order pre-detection filter) and 2.95 dB (3\textsuperscript{rd} order pre-detection filter) less noise power compared to non-tuned with the same pre-detection filters. While tuned B receiver has 6.67 dB -7.1 dB less noise power compared to the non-tuned receiver. With ISI
considered, tuned A and tuned B receivers have 3.16 dB and 2.09 dB higher sensitivity compared to the non-tuned receiver when 1\textsuperscript{st} order filter is used, and -3.03 dB and 1.88 dB higher sensitivity when 3\textsuperscript{rd} order filter is used.

As a summary, Tuned B receiver requires approximately half of the number of photons per pulse \( b \) to achieve the specified BER compared to the non-tuned receiver. Tuned receivers (with 1\textsuperscript{st} order pre-detection filter) has an overall performance improvement compared to the non-tuned receiver with either 1\textsuperscript{st} or 3\textsuperscript{rd} order pre-detection filter.

### 6.4.2 BJT receivers

The same simulation is performed for optimised non-tuned and tuned receivers that employ PINBJT front end, with 1\textsuperscript{st} order and 3\textsuperscript{rd} order pre-detection filters.

![Figure 6-33 illustration of receiver components (BJT comparison).](image)

The situation for a BJT front-end noise is similar to that of the FET front end. Therefore, noise spectrum components of a BJT front end has the same characteristics as of the FET noise spectrum components, for each receiver respectively.

The difference in total noise power of non-tuned and tuned B receivers is significant with low front-end bias current compared to high bias current values. However, for a fair comparison, each receiver front-end is assumed to be optimally biased.
Figure 6-34 Total input noise power verses front-end bias current (1st order LPF).

Table 6-8 PINFET receiver performance comparison (100 Mbits/s). non-tuned and tuned B amplifiers have a 3-dB bandwidth of 50 MHz, tuned A amplifier has a 3-dB bandwidth of 60 MHz.

<table>
<thead>
<tr>
<th>1st order LPF</th>
<th>Non-tuned</th>
<th>Tuned A</th>
<th>Tuned B (a=0.4)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rf</strong></td>
<td>23.3 kΩ</td>
<td>32.3 kΩ</td>
<td>64.2 kΩ</td>
</tr>
<tr>
<td><strong>BW_{3dB,filter}</strong></td>
<td>70 MHz</td>
<td>70 MHz</td>
<td>50 MHz</td>
</tr>
<tr>
<td><strong>i_n^2,\text{Rf}</strong></td>
<td>3.23 10^{-17} A^2</td>
<td>1.97 10^{-17} A^2</td>
<td>0.78 10^{-17} A^2</td>
</tr>
<tr>
<td><strong>i_n^2,\text{TIA,BJT}</strong></td>
<td>7.33 10^{-17} A^2</td>
<td>5.83 10^{-17} A^2</td>
<td>1.83 10^{-17} A^2</td>
</tr>
<tr>
<td><strong>b</strong></td>
<td>8.25 10^3</td>
<td>5.92 10^3</td>
<td>4.36 10^3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3rd order Butterworth</th>
<th>Non-tuned</th>
<th>Tuned A</th>
<th>Tuned B (a=0.4)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rf</strong></td>
<td>23.3 kΩ</td>
<td>42.6 kΩ</td>
<td>64.2 kΩ</td>
</tr>
<tr>
<td><strong>BW_{3dB,filter}</strong></td>
<td>60 MHz</td>
<td>60 MHz</td>
<td>50 MHz</td>
</tr>
<tr>
<td><strong>i_n^2,\text{Rf}</strong></td>
<td>3.07 10^{-17} A^2</td>
<td>1.52 10^{-17} A^2</td>
<td>0.66 10^{-17} A^2</td>
</tr>
<tr>
<td><strong>i_n^2,\text{TIA,BJT}</strong></td>
<td>5.28 10^{-17} A^2</td>
<td>3.80 10^{-17} A^2</td>
<td>1.38 10^{-17} A^2</td>
</tr>
<tr>
<td><strong>b</strong></td>
<td>6.56 10^3</td>
<td>5.03 10^3</td>
<td>3.59 10^3</td>
</tr>
</tbody>
</table>
It is also important to note that tuned A receiver has NEB higher than the non-tuned receiver. As a result, the total noise power of tuned A receiver becomes higher than that of the non-tuned receiver when both front ends have a high bias current as shown in Figure 6-34, due to BJT transistor noise being dominant when front-end is overly biased.

Tuned A receiver has 0.99 dB (1st order) and 1.42 dB (3rd order) less noise power compared to non-tuned with the same pre-detection filters. While tuned B receiver has 6.03 dB -5.81 dB less noise power compared to non-tuned receiver. With ISI considered, tuned A and tuned B receivers have 1.44 dB and 2.78 dB higher sensitivity compared to non-tuned receiver when 1st order filter is used, and 1.15 dB and 2.62 dB higher sensitivity when 3rd order filter is used. Similar to FET receivers, BJT tuned receivers (with 1st order pre-detection filter) has an overall performance improvement compared to non-tuned receiver with either 1st or 3rd order pre-detection filter.

6.4.3 APD receivers

There APD materials are selected to compare the performance of non-tuned and tuned receivers; Silicon, InGaAs, and Germanium. Figure 6-36 summarises the overall performance of APD-based receivers for non-tuned and tuned front ends. The performance of each receiver is evaluated BJT and FET transistors. Each APD has a gain and noise factor, and the reason for choosing different material is to compare tuned front end and non-tuned over different conditions.

<table>
<thead>
<tr>
<th>Photodetector type/material</th>
<th>Pre-amp input transistor</th>
<th>Pre-amp type</th>
<th>Pre-detection filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>APD</td>
<td>Silicon</td>
<td>FET</td>
<td>1st order LPF</td>
</tr>
<tr>
<td></td>
<td>InGaAs</td>
<td>BJT</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Germanium</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non-tuned</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Tuned A</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Tuned B</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3rd order Butterworth</td>
</tr>
</tbody>
</table>

Figure 6-35 illustration of receiver components (APD comparison).
In general, Germanium has the lowest receiver sensitivity because it has avalanche gain of 10 and a noise factor of 9.2. On the other hand, silicon based APD has the same avalanche gain with less noise factor of 5.5 which makes silicon far superior to Germanium. InGaAs APD has avalanche gain of 100 and noise factor of 7.9, and its performance is at the midway between silicon and Germanium.

![Comparison of non-tuned and tuned receivers, with data rate of 100 Mbits/s](image)

**Figure 6-36 Comparison of non-tuned and tuned receivers, with data rate of 100 Mbits/s**

(Silicon APD M= 10 F(M)= 5.5, InGaAs APD M= 100 F(M)= 7.9, and Germanium APD M= 10 F(M)= 9.2). Results are presented in tabular form in Appendix B (B.6).

If silicon is considered as the best performance (square grey scatters), it would be noticed that tuned receivers (tuned A and tuned B) have significant difference in receiver sensitivity. Although, the use
of 3rd order filter significantly improves non-tuned receiver sensitivity, tuned receivers with a 1st order filter is still offers a better sensitivity than non-tuned receiver with a 3rd order filter.

6.5 Summary

Results can be concluded in these key points:

- The overall tuned receiver’s performance is better than non-tuned receivers.
- Noise power in tuned receivers is significantly reduced for all examined receiver configurations.
- Tuned receivers are less sensitive to the variation of receiver bandwidth than non-tuned receiver.
- For tuned B receiver, although the capacitance splitting ratio depends on the size of the photodetector and the input transistor, the overall performance of tuned B receiver overcome non-tuned for all splitting rations.
- Tuned B receiver overs a significant reduction in noise power, but the overall receiver performance is still limited due to the additional ISI. However, the overall performance of tuned receiver is better than non-tuned receiver. Therefore, it is anticipated that tuned receiver would have improved overall receiver performance over non-tuned in case of other modulation schemes where ISI is less significant.
- The significance of performance improvement can be well noticed in high data rates due to these reasons:
  - Feedback resistors become too small due to the large bandwidth hence the reduction of thermal noise benefits the overall noise power.
  - Since BJT's are preferable in high data rates receiver design, tuned receivers would offer less optimum collector current values hence less total noise power.
- Tuned receiver offer sensitivity improvement within 1 dB over non-tuned receiver when APDs are used, however, APDs are less attractive to high data rates.
Chapter 7: RESULTS AND DISCUSSION II (TUNED PULSE POSITION MODULATION RECEIVER PERFORMANCE)

This chapter presents the performance of tuned PPM receiver compared to non-tuned receiver. The received optical pulse is assumed to be Gaussian shape for optical fibre systems. For wireless optical communication system, channel model is explained in Section 7.1. In this section, system overview and simulation procedures are explained. The reason behind choosing these communication systems is to take into account the non-ideal optical channel effects. The modulation schemes considered are PPM and di-code PPM. Optimum and sub-optimum detection of PPM are previously discussed, however, in this investigation, the comparison is based on sub-optimal detection. In particular, the raised cosine filtering, since it is considered as more practical and simple in term of receiver structure. In this chapter, the comparison criteria are limited to certain scenarios since tuned receivers are extensively investigated, and the performance advantages of tuned receivers compared to non-tuned are well established in Chapter 6.
7.1 System overview and simulation procedures

The performance of tuned front-end receiver is examined using digital pulse position modulation and Di-code PPM. The performance is evaluated based on simulations and numerical results obtained using the mathematical model explained in Chapter 5. For optical fibre channel, the received pulse is assumed to be Gaussian shape. A brief explanation of the pulse characteristics is presented in Section 7.1.

Based on numerical simulations of the bit error probability including ISI, results aim to give quantitative predictions of the PPM and di-code PPM tuned receivers sensitivity in comparison to non-tuned receivers. In Section 7.2, the performance of tuned PPM receiver is compared to non-tuned receiver using two different pre-detection filters (Matched filter and 3rd Butterworth filter), assuming slope detection. In Section 7.2, PPM is replaced by a di-code signalling. The performance of a tuned receiver is then compared to a non-tuned receiver with 3rd order Butterworth filter, assuming central detection. In the same section, the performance of tuned receiver is briefly compared to non-tuned receiver when 3rd order Butterworth is replaced by a 1st order low pass filter.

The performance evaluation is performed for different channel bandwidths in order to examine the advantage of tuned receiver in high and low dispersive optical fibre channels. All simulations and calculations of tuned receivers and non-tuned receivers are included in Appendix C. System modelling and calculations of PPM and di-code are performed in the same way as for OOK, providing that some sections of the model are developed to include channel effect and PPM modulation terms, PPM error probability functions, and PPM receiver sensitivity. The investigation in Chapter 6 is used as a guideline to pick the simulation scenarios. For instance, the bit-rate of original data is taken as 1 Gbit/s. Since BJT receivers perform better than FET receivers in higher bit-rates, the comparison between tuned and non-tuned PPM receivers are based on a BJT input front end. Also, the photodetector is taken as a PIN photodetector with a total capacitance of 1.5 pf. A different operating
wavelength of 1550 nm (which is commonly used in optical fibre links) is used to evaluate the receiver sensitivity.

Calculations in Appendix C.2, for non-tuned front end, are as follow:

1. For a given PCM bit-rate (B) and PPM modulation order, PPM terms such as (PPM line rate, slot time, modulation index) are calculated.
2. TIA 3-dB bandwidth is set to 2 times the bit rate, and the pre-detection filter is designed to match the amplitude and phase of the received optical pulse.
3. The whole receiver frequency response is then determined by the transfer functions of TIA and pre-detection filer.
4. The contribution of each noise source is calculated then total input noise power is calculated for both FET and BJT receivers.
5. For BJT receiver, collector current is optimised as in Eq (3-35).
6. Total input noise power of BJT receiver is calculated for optimum collector current.
7. Output pulse shape is obtained by convoluting the input pulse shape and receiver frequency response (peak voltage and decision time are evaluated for the output pulse shape).
8. False alarm, erasure, wrong slot and the total error functions are evaluated in terms of the number of photons and threshold voltage.
9. Error probability function (P_e(b)) is set to a given BER of $1 \times 10^{-9}$, number of photons (b) is obtained as the root of of $P_e(b)$, generating the lowest number of photons $b_{\text{min}}$ per PPM pulse required to achieve an overall BER of $1 \times 10^{-9}$. The same evaluation algorithm scans for the optimum threshold level and decision time by evaluating the slope of the received pulse.
10. Receiver sensitivity is defined as the minimum required optical power per pulse averaged over original PCM pulse width.
11. The delay at which the autocorrelation function of each filter becomes small is considered for each filter.
12. The total error probability function is examined graphically in terms of the number of photons and threshold level in order to confirm the validity of algorithm calculations used to obtain the minimum number of photons.

Same procedures are followed for tuned receivers however, these modifications are performed for tuned B:

- Same TIA 3-dB is calculated as non-tuned.
- For a given splitting ratio \( a \), \( \Delta L_B \) and \( \Delta R_f \), values of feedback resistor and inductor are calculated.
- The frequency response of the whole receiver is determined by the tuned B transimpedance and pre-detection filter.
- Noise integrals are calculated for tuned B receiver.
- Calculations in Appendix C.3, for non-tuned and tuned receivers, are performed following the same procedures, providing that an additional block is added to the receiver model. This additional block is used to modify the matched filter response so that pre-detection filter matches the shape of TIA output pulse rather than the received optical pulse. Models in C.3.1 to C.3.8 are used to evaluate the overall performance of PPM optical link assuming two channel bandwidths, BJT and FET front ends, and non-tuned and tuned amplifiers. Additionally, the noise performance of the tuned receiver is evaluated for a range of front end capacitive ratios in order to optimise the receiver performance and determine the optimum tuned components. This evaluation is performed considering BJT and FET front ends, and different optical channel bandwidths. Calculations in Appendix C.4, for non-tuned and tuned receivers, are also performed following the same procedures, providing that matched filter is replaced by a 3\textsuperscript{rd} order Butterworth filter. A modulation index of 0.8 is used to provide addition guards to the PPM frame in order to minimise the inter-symbol interference in non-tuned and tuned receivers. Calculations in Appendix C.4, for non-tuned and tuned receivers,
are also performed following the same procedures, providing that PPM is replaced by di-code modulation. Calculations in C.4.1 to C.4.4 are as follows

1. For a given PCM bit-rate \( B \), di-code PPM terms such as (di-code PPM line rate, slot time, and, number of like symbols in PCM) are calculated.

2. TIA 3-dB bandwidth is set to 0.5 times the bit rate, and the pre-detection filter 3-dB bandwidth is set to 0.7 for non-tuned receiver and 0.5 for tuned receiver.

3. The whole receiver frequency response is then determined by the transfer functions of TIA and pre-detection filer (3\textsuperscript{rd} order Butterworth and 1\textsuperscript{st} order low pass filter).

4. Total input noise power of BJT receiver is calculated for optimum collector current so that tune and non-tuned receiver are optimally biased.

5. Output pulse shape is obtained by convolving the input pulse shape and receiver frequency response (peak voltage and decision time are evaluated for the output pulse shape).

6. Since central decision detection is assumed, False alarm, erasure, and the total error functions are evaluated in terms of the number of photons and optimum threshold voltage.

7. Error probability function \( P_e(b) \) is set to a given BER of \( 1 \times 10^{-9} \), number of photons \( b \) is obtained as the root of of \( P_e(b) \), providing the lowest number of photons \( b_{\text{min}} \) per PPM pulse required to acheive an overall BER of \( 1 \times 10^{-9} \).

8. Receiver sensitivity is defined as the minimum required optical power per pulse averaged over original PCM pulse width.

9. The total error probability function is examined graphically in terms of the number of photons and threshold level in order to confirm the validity of algorithm calculations used to obtain the minimum number of photons.

The performance of a tuned receiver is evaluated for all splitting ratios, and the overall performance of a tuned receiver with an optimum capacitive ratio is compared to non-tuned receiver in Section 7.2.2.
7.1.1 Gaussian pulse (Optical fibre)

Many optical fibres, and in particular jointed fibre links, exhibit pulse outputs with a temporal variation that is closely approximated by a Gaussian distribution. Hence the variation in the optical output power with time may be described as

\[ H_p(t) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{t^2}{2\sigma^2}} \]

With a Fourier transform of

\[ H_p(\omega) = e^{-\frac{\omega^2\sigma^2}{2}} \]

The pulse variance \( \sigma \) is linked to the fibre bandwidth by

\[ \sigma = \frac{\sqrt{2\ln2}T_b}{2\pi f_n} \]

Where \( T_b \) is the PCM bit-time and \( f_n \) is the channel bandwidth normalised to the PCM data rate. a Gaussian pulse with theses parameters is assumed at the receiver input.

Figure 7-1 Received pulses assumed to be gaussian in shape. times is normalised to slot time.
In the low-bandwidth region, the pulse is dispersed and so the rise time is large in comparison to the slot width, this causes the wrong slot error to increase. As channel bandwidth increase the input pulse becomes less dispersed hence wrong slot errors become less significant.

![Figure 7-2 Ideal matched filter output (TIA bandwidth is neglected).](image)

7.1.2
7.2 Tuned pulse position modulation for optical fibre

In this section, the performance of the tuned receiver is compared with a non-tuned receiver. Normalised channel bandwidth ($f_n$) of 2 and 5 are used to examine the difference of both receivers while channel effect is considered. $f_n$ of 2 will represent a highly dispersive optical channel and $f_n$ of 5 represents a less dispersive channel.

PPM receiver has improved performance compared to equivalent PCM. This improvement increases with modulation order of PPM. This fact is true when photon counting is implemented as in free space optical transmission systems. This restricts the implementation of PPM when the system runs over high bit rates. The use of a conventional baseband receiver such as a receiver with a preamplifier is more used with high data rates applications. The problem is this type of receiver will suffer from high noise; increasing the PPM modulation order will not further improve the system performance due to high noise associated with receiver.

Therefore, in this case, there are two ways to examine the use of a matched filter. Theoretically, a preamplifier can be assumed to have an infinite bandwidth. Although the theoretical simulation is valid in this case, the physical implementation of such a matched filter becomes highly complex or unrealistic. The other way to examine the use of a matched filter is to consider a preamplifier with limited bandwidth. This requires additional equalisation before the detection circuit since the filter response must match the pre-amplifier output pulse shape rather than the received pulse shape. This process also results in a complicated practical implementation of the matched filter. This is why sub-optimum filtering is considered to be a simpler and more practical compared to optimum filtering.

However, in this section, an example of tuned receiver with a matched filter is briefly explained before considering the sub-optimum filter.
7.2.1 Tuned receiver performance (Matched filter/Gaussian Pulse)

For a comparison, a low PPM order modulation of 8 is used. The corresponding PCM codeword of 3 bits are encoded in 8 bits PPM codeword that line-rate is 8/3 times the original PCM bit-rate. Therefore, PPM slot width with zero guards (modulation index of 1) equals to 0.375 ns. For PPM line rate of 2.667 GHz, the 3-dB bandwidth of the TIA is wide as twice this line rate and independent from fibre bandwidth.

![Graph showing pulse positions](image)

**Figure 7-3 Output pulse shape of tuned receiver with a matched filter (PCM bit rate = 1 Gbit/s and \( f_n = 5 \)).**

Figure 7-3 illustrates the output pulse of a tuned receiver with a filter whose response is matched to received pulse shape. Receiver front end bandwidth is limited to twice the bit rate which affects the output pulse shape. Although the matched filter limits the ringing, peaking in amplifier frequency response can still be noticed (approximately below 10% of the peak voltage). Output pulse shape due to an equalised response is illustrated in Figure 7-4 (non-tuned receiver with a matched filter and equaliser has the same output pulse shape as tuned receiver).
Figure 7-4 Output pulse shape of tuned receiver with a matched filter and equaliser (PCM bit rate = 1 Gbit/s $f_n = 5$).

Figure 7-5 Output noise power spectrum due to channel noise (non-tuned receiver).

Equalisation will further worsen the receiver noise, the noise performance of the receiver with a matched filter is poor since the input noise spectrum is not white. Figure 7-5 and Figure 7-6 illustrate the output noise spectrum of non-tuned and tuned receiver. Higher noise order components in both
cases affect the output noise spectrum. For tuned receiver, coloured noise causes more serious problems along with higher capacitor splitting ratios as shown in Figure 7-7.

As a conclusion, matched filter is optimum for signal detection, however, it is not the optimum for noise filtering since the preamplifier noise is not white. Although this fact applies for both receivers (non-tuned and tuned receiver), tuned receiver demonstrates a sensitivity improvement of 0.58 to 2
dB for all splitting ratios. The optimum input capacitor splitting ratio is 0.3, with the minimum output noise power among other splitting ratios.

**Figure 7-8** Total output noise power spectrum (PINFET tuned receiver/$f_n=5$).

**Figure 7-9** Total output noise power spectrum (PINFET tuned receiver/$f_n=2$).
If a lower channel bandwidth is considered \( (f_n = 2) \), the received pulse is wider in time domain and the frequency response of the matched filter tends to further limit the noise performance of both receivers. Figure 7-9 illustrates the matched filter effect in higher dispersive channel \( (f_n = 2) \). Tuned receiver has a sensitivity improvement from 0.7 to 1.7 for all splitting ratios. The optimum input capacitor splitting ratio is 0.4, with the minimum output noise power among other splitting ratios. Splitting ratio of 0.4 is the optimum because matched filter bandwidth is smaller than that of higher channel bandwidth. This results in noise equivalent bandwidth being smaller. In addition, splitting ratio of 0.4 has the highest feedback resistor value which in return reduces the total input noise power. Based on these results, a tuned receiver with splitting ratio of 0.4 (PINFET) requires 4051 less photons per PPM pulse to achieve a bit error rate of \( 10^{-9} \) compared to the same receiver whose front end is not tuned. Error! Not a valid bookmark self-reference. shows performance comparison of PINBJT non-tuned receiver and PINBJT tuned receiver.

**Table 7-1 Summary of tuned receiver performance (Matched filter with equaliser). Receiver sensitivity per PCM bit. \( b_{\text{min}} \) is the minimum number of photons.**

<table>
<thead>
<tr>
<th>PINBJT PPM</th>
<th>( f_n = 5 )</th>
<th>( f_n = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( b_{\text{min}} )</td>
<td>Sensitivity</td>
</tr>
<tr>
<td>Non-tuned</td>
<td>7059</td>
<td>-35.20 dBm</td>
</tr>
<tr>
<td>Tuned B (( a = 0.1 ))</td>
<td>5192</td>
<td>-36.53 dBm (1.33 dB)</td>
</tr>
<tr>
<td>Tuned B (( a = 0.2 ))</td>
<td>4931</td>
<td>-36.76 dBm (1.56 dB)</td>
</tr>
<tr>
<td>Tuned B (( a = 0.3 ))</td>
<td>4654</td>
<td>-37.01 dBm (1.81 dB)</td>
</tr>
<tr>
<td>Tuned B (( a = 0.4 ))</td>
<td>5377</td>
<td>-36.38 dBm (1.18 dB)</td>
</tr>
<tr>
<td>Tuned B (( a = 0.5 ))</td>
<td>5668</td>
<td>-36.15 dBm (0.95 dB)</td>
</tr>
</tbody>
</table>
By examining If a lower channel bandwidth is considered ($f_n = 2$), the received pulse is wider in time domain and the frequency response of the matched filter tends to further limit the noise performance of both receivers. Figure 7-9 illustrates the matched filter effect in higher dispersive channel ($f_n = 2$). Tuned receiver has a sensitivity improvement from 0.7 to 1.7 for all splitting ratios. The optimum input capacitor splitting ratio is 0.4, with the minimum output noise power among other splitting ratios. Splitting ratio of 0.4 is the optimum because matched filter bandwidth is smaller than that of higher channel bandwidth. This results in noise equivalent bandwidth being smaller. In addition, splitting ratio of 0.4 has the highest feedback resistor value which in return reduces the total input noise power. Based on these results, a tuned receiver with splitting ratio of 0.4 (PINFET) requires 4051 less photons per PPM pulse to achieve a bit error rate of $10^{-9}$ compared to the same receiver whose front end is not tuned. **Error! Not a valid bookmark self-reference.** shows performance comparison of PINBJT non-tuned receiver and PINBJT tuned receiver.

Table 7-1, tuned PINBJT receiver has a much better overall performance compared to non-tuned receiver. For $f_n = 5$, tuned receiver requires a minimum of 4654 phonons per PPM pulse compared to 7059 for non-tuned receiver. While for $f_n = 2$, tuned receiver requires a minimum of 5771 phonons per PPM pulse compared to 9767 for non-tuned receiver. It is important to note that both receivers, in both cases, are assumed to be optimally biased and the pre-detection filter matches the pre-amplifier output pulse.

### 7.2.2 Tuned receiver performance (Butterworth filter/ Gaussian Pulse)

The use of a Butterworth filter is a more realistic scenario (sub-optimum detection). A physical realisation of a 3rd Butterworth filter can practically be implemented; compared to a matched filter. The same line rate is used to examine the difference between non-tuned and tuned front end receivers when the matched filter is replaced with a Butterworth filter.
System simulations of non-tuned and tuned receivers with a Butterworth pre-detection filter are performed following the same procedures as for matched filter. The surface plot in Figure 7-10 illustrates the error probability function in term of threshold voltage and number of photons per pulse. This plot is used to confirm the results obtained from the mathematical models in Appendix C which are used to evaluate the optimum threshold level and the minimum number of photons per pulse required to achieve error rate of 1 in $10^9$.

The overall performance of non-tuned and tuned receivers is evaluated assuming a normalised channel bandwidth of 5. The 3-dB bandwidth of TIA is set to 0.5 times the line rate and the 3-dB bandwidth of the pre-detection filter is set to 0.7 times the line rate (line rate is 1/PPM time slot, with a modulation index of 0.8). Receiver front end is a PINBJT and both receivers are assumed to be optimally biased. The performance of tuned receiver is evaluated assuming all capacitive splitting ratio (0.1 to 0.5).

Non-tuned receiver has a total noise power of $6.6 \times 10^{-14}$ A$^2$. The same receiver with a tuned front end and splitting ratio of 0.3 has 4.5 dB less noise power. By evaluating both receivers’ performance, tuned receiver requires 1562 less photons per pulse compared to non-tuned receiver. By examining all splitting ratios, tuned receiver has an overall sensitivity improvement of 1.22 dB to 1.96 dB compared to non-tuned receiver.
Figure 7-10 An example of error probability evaluation in terms of threshold voltage and number of photos per pulse (Non-tuned receiver).
Figure 7-11 An example of error probability evaluation in terms of threshold voltage and number of photos per pulse (Tuned B receiver).

Furthermore, the variation of pre-detection filter bandwidth significantly affects the non-tuned performance. This effect is explained in detail in Chapter 6. Therefore, the performance of both receivers is examined with a filter 3-dB bandwidth of 0.5 times the PPM line rate. Results show that non-tuned receiver sensitivity degrades by 0.22 dB due to filter bandwidth variation which seems insignificant. However, compared to tuned receiver that will gain a sensitivity improvement of 0.31 dB due to the same filter bandwidth variation, the sensitivity difference between non-tuned and tuned receiver \((a = 0.3)\) increases from 1.66 dB to 2.2 dB. The same simulation is performed for different splitting ratios. Tuned receiver has sensitivity improvement of 1.47 dB to 2.5 dB.
7.3 Tuned Di-code PPM optical fibre receiver

After examining the performance of tuned receivers with OOK and PPM, same figures of improvement can be expected for tuned receiver compared to non-tuned receiver for other PPM schemes in various scenarios and under different conditions. In this section, the performance of tuned receiver is compared to non-tuned receiver when Butterworth filter is replaced with a 1st order low pass filter. The examination of both receivers is performed with a di-code PPM in order to further expand the comparison between both receivers, covering additional sub-system components.

7.3.1 Tuned receiver performance (Butterworth filter/ Gaussian Pulse)

The simulation is performed assuming a 3rd order Butterworth filter following the first order response of the preamplifier. Both receivers have PINBJT front end and are assumed to be optimally biased. A di-code PPM signalling with 2 guards is used to evaluated receiver performance, assuming central decision detection. PCM bit rate is 1 Gbit/s so that di-code line rate is 4 times the original bit rate. The performance of both receivers is evaluated for low dispersive optical channel ($f_n = 5$) and in highly dispersive channel ($f_n = 1$ and $f_n = 0.7$). Pre-detection 3-dB bandwidth of non-tuned receiver is set to 0.7 times the di-code line rate while it is set to 0.5 times the line rate for tuned receiver.

For a low dispersive channel ($f_n = 5$), tuned receiver ($a = 0.5$) has sensitivity improvement of 2.01 dB compared to non-tuned receiver which is 1927 less photons per di-code pulse. The difference in number of photons per pulse decreases to 1210 for tuned receiver with $a = 0.1$. for a higher dispersive channel ($f_n = 1$), tuned receiver ($a = 0.5$) has 2.95 dB higher sensitivity than non-tuned receiver. For a highly dispersive channel ($f_n = 0.7$), tuned receiver would have 3.29 dB higher sensitivity than non-tuned receiver which is approximately $1.57 \times 10^4$ less photons per di-code pulse.

7.3.2 Tuned receiver performance (1st order low pass filter)

The same simulation is performed for both receivers, providing that 3rd order Butterworth filter is replaced by a 1st order low pass filter.
Tuned receiver with a splitting ratio of 0.5 and 1st order pre-detection filter would have sensitivity improvement of 3.18 dB ($f_n = 0.7$) and 2.2 dB ($f_n = 5$) compared to non-tuned receiver with the same pre-detection filter.

These results show a significant improvement of the overall system when a non-tuned receiver is replaced with a tuned receiver. In addition to PPM and OOK, the examination of tuned receiver with di-code scheme can also confirm the advantage of this receiver technique.

7.4 Tuned Di-code PPM optical wireless receiver

In this section, the performance of tuned receiver is compared to non-tuned receiver considering a diffuse optical channel. The transmitted pulse shape is assumed to be rectangle shape. Figure 7-12 illustrates the output pulse shape of a non-tuned receiver assuming an ideal channel. Channel effect on the output pulse shape is shown in Figure 7-13. The same channel effect is simulated with the tuned receiver and the output pulse shape is shown in Figure 7-14.

![Figure 7-12 Output pulse shape due to receiver response, assuming an ideal optical channel (non-tuned receiver).](image-url)
The performance of both receivers is evaluated following the same procedures as in Section 7.3. Tuned receiver would have up to 3.4 dB higher receiver sensitivity than non-tuned receiver, assuming both receivers have the same pre-detection filter (Butterworth) and are optimally biased. More results are
presented in Appendix C.5. The overall performance of tuned receivers indicates that a tuned di-code link can operate with approximately 6800 less photons per di-code pulse compared to non-tuned di-code optical link.

7.5 Summary

This Chapter provides a further demonstration of cases where tuned receiver can be optimised for baseband transmission, offering a much better performance compared to non-tuned receivers. The performance of both receivers is examined using different modulation schemes, different detection methods, and different pre-detection filters. Besides, results show that a tuned receiver can further enhance the receiver performance when optical channel effects are presented. Optimally biased tuned BJT receivers are also shown to optimise the receiver performance, enhancing the overall performance of PPM and di-code optical links.
Conclusions and further work
8.1 Conclusions

Some optical communication links were introduced in Chapter 1. The aim of the background information in this chapter was to demonstrate some emerging optical communication applications such as underwater wireless optical communications, visible light communications, and wireless optical links used in medical devices. The significance of these optical links and their benefits in various fields motivated this project which examined the use of tuned front-end receivers with baseband signalling. Research proposal, research problem and methodology used in this investigation were discussed in detail in Chapter 1. All this background information in line with the research motivations helped to set effective objectives in order to respond to the research problems.

Objectives of this investigation included reviews of several topics such as the theory of baseband receivers, the use of tuned receivers in optical communication, and baseband modulation schemes. Therefore, these topics were divided into three chapters (Chapter 1, Chapter 2 and Chapter 3) in order to provide a joined link between them. As a result

- Chapter 2 explored important optical receiver related topics which are important and useful to discuss. Topic such as Personick integrals, noise equivalent bandwidth, and the input referred noise in non-tuned receivers contributed to develop the background theory all of which used later in Chapter 3 to establish a ground for tuned receiver theory development. It has also shown that the input stage of the optical receiver contributes significantly to the total receiver noise, and the design of the front-end amplifier is therefore fundamental to achieve a low noise receiver performance hence an overall link performance improvement.

- Chapter 3 provides an overview of optical tuned front-end receivers. The literature review in this chapter addresses the evaluation of tuned receivers in optical communication. It also discusses some research problems, (such as tuned receiver classification in some literature and noise methods previously used to refer noise in optical receivers) in further detail.
• Since neither numerical Personick integrals nor conventional bandwidth integrals are valid for tuned front end receivers, novel analytical expressions for noise integrals and equivalent input and output noise densities of two tuned front-end receivers employ Bi-polar junction input transistors and field effect input transistors are presented in this work (Chapter 3).

• Another significant contribution of this work was to drive the optimum collector current expressions for tuned front end receivers employ Bi-polar junction input transistors.

The literature review in Chapter 2 and Chapter 3 as well as the theory developed in these two chapters result in producing a novel noise model for baseband tuned receivers. This developed model overcomes some limitations of the conventional noise model as it takes into account

• The bit-rate independency of noise sources, therefore, noise source has a frequency dependency of a realistic receiver frequency response.

• The transfer function of each noise source depends on the front-end circuitry.

• The equalisation independence of output pulse shape. A physical pre-detection filter design can be integrated into this model. Therefore, the output pulse shape is obtained depending on a physical realisation of the receiver circuitry with no assumptions regarding input and output pulse shapes.

This model also has a flexibility of

• Evaluating the receiver performance for any input pulse shape.

• Considering a wide range of physical implementation of the pre-detection filter and TIA configuration.

The developed noise model is explained in detail in Chapter 5, showing how this model is integrated with the system modelling. In this Chapter, the mathematical models and calculations performed in this investigation are presented and explained in order to provide sufficient information that allow
this work to be reliably reproduced. This will also allow examining the use of tuned front end in other application as the system modelling in this work

- Can help to identify design trade-off. The model predicts the behaviour of a given receiver design under various operating conditions such as varying bit rate, modulation scheme, optical input power, input transistor technology, front-end configuration, photodetector size and type, pre-detection filter, and detection method.
- It can also be used to study the impact of new technologies on system performance. The designer can explore the impact of the limits of current electronic and optoelectronic technologies on the link performance and suggest directions for technology and performance improvements.
- It can be integrated into a system level design tool that supports a multi-level and multi-technology simulation.
- The most important feature is that it takes into account the physical realisation of receiver components. PPM models do not take noise into account particularly well, which means that no real noise analysis can be carried out considering the physical realisation of the front end.

PCM and PPM theory is reviewed in Chapter 4 in addition to reviewing the sequential development of PPM receivers design in optical communication. This background is used to simulate PPM links and evaluate their performance based on related work in the literature.

In addition to the theoretical contribution, this work provides an in-depth investigation of the performance of tuned receivers with on-off keying modulation (Chapter 6 and Appendix B). this investigation includes

- Different photodetectors (PIN photodetector and avalanche photodetector), different input transistors (Bi-polar junction transistor BJT and field effect transistor FET), different pre-detection filters (1\textsuperscript{st} order low pass filter and 3\textsuperscript{rd} order Butterworth filter),
and different tuned configurations (inductive shunt feedback front end tuned A and serial tuned front end tuned B).

- An examination of the performance of tuned receivers, considering three different avalanche photodetector materials (Silicon APD, InGaAs APD, and Germanium APD) which helps to evaluate the receiver performance for different avalanche gains and photodetector noise factors.
- An analysis of the inter-symbol interference of on-off keying modulation with tuned receivers.
- An Optimisation tuned receiver 3-dB bandwidth with 1st order low pass filter and 3rd order Butterworth filter.
- Original noise analysis of optical baseband tuned front-end receivers.

The key points of this investigation can be summarised as follows

- The overall tuned receiver’s performance is better than non-tuned receivers.
- Noise power in tuned receivers is significantly reduced for all examined receiver configurations.
- Tuned receivers are less sensitive to the variation of receiver bandwidth than non-tuned receiver.
- For tuned B receiver, although the capacitance splitting ratio depends on the size of the photodetector and the input transistor, the overall performance of tuned B receiver overcome non-tuned for all splitting ratios.
- Feedback resistors become too small due to the large bandwidth required for high data rates hence the reduction of thermal noise of tuned receivers benefits the overall noise power.
- Tuned receivers also operate with less optimum collector current values hence less total noise power compared to non-tuned receiver.
This investigation is extended to cover higher modulation formats such as PPM and di-code PPM (Chapter 7 and appendix C), in this investigation

- The performance of tuned PPM receivers in slightly and highly dispersive optical fibre channels (Gaussian input pulses for optical fibre links, considering different fibre bandwidth) is presented.
- The performance of tuned PPM receivers in diffuse optical wireless links (convolved input pulses for non-line of sight optical wireless link) is also presented.

Chapter 7 provides a further demonstration of cases where tuned receiver can be optimised for baseband transmission, offering a much better performance compared to non-tuned receivers. The performance of both receivers is examined using different modulation schemes, different detection methods, and different pre-detection filters. Besides, results show that tuned receiver can further enhance the receiver performance when optical channel effects are presented. Optimally biased tuned BJT receivers are also showed to optimise the receiver performance, enhancing the overall performance of PPM and di-code optical links.

Finally, the use of tuned receiver with either OOK, PPM or di-code PPM offers a significant reduction in total receiver noise. In addition, the overall receiver performance is improved by over 3dB in some cases which less than half the transmitted optical power compared to non-tuned receiver.

### 8.2 Further work

The recommendations of this investigation suggest:

- To facilitate an underwater simulated environment to further examine tuned pulse position modulation systems. As tuned receiver technique further enhances the performance of PPM optical links (in terms of less photons per pulse), this will benefit the underwater operations in term of power consumption, link distance, and maintenance cost of communication devices.
(battery operated based). The research might take into account different link configurations such as LOS and NLOS.

- To facilitate a biological research environment to examine the performance of tuned receivers with baseband modulation such as PPM and OOK. Since tuned receiver technique further enhances the performance of PPM and OOK optical links (in terms of less photons per pulse and higher signal to noise ratio), this will benefit battery operated implantable medical devices allowing it to be implemented with a larger area photodetector without degrading the link performance. Also, it might increase the battery changing period.

- To facilitate an indoor optical wireless link (such as VLC or IR) to examine the performance of tuned baseband receivers in these links. As IR wireless links have a restricted optical power, tuned receivers will further improve the bit error rate of such link. This also applies to VLC links as one of current challenge is the high bit error rate due to light dispersion.

- The current trend in automotive industry is to switch to eco technologies. Most of commercial car manufacturers replace current car models with electric engines, the use of tuned receiver might offer a better power consumption of in-car optical systems. In-car optical systems employ plastic optical fibre (such as MOST standard links). Usually in such environments, cables must be bent to fit the space and connection requirements. POF cable bent introduces additional loss (bend loss), in short-range link, tuned receiver is a solution to compensate this optical loss.
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Appendix A: Tuned circuit and Tuned amplifiers (General)

A.1 Tuned circuit .............................................................................................................A-2
A.2 Bandwidth extension.................................................................................................A-17
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A.1 Tuned circuit

Step response, output pulse shape and frequency response of tuned front end for different central frequency.

\[ C := 1 \cdot 10^{-12} \quad \text{Total capacitance} \]
\[ ts := 2 \cdot 10^{-8} \quad \text{Time slot} \]
\[ Ql := 1 \quad \text{Q-factor} \]

\[ Ll := \frac{1}{C} \cdot \frac{ts^2}{\pi^2} \cdot \frac{4 \cdot Ql^2}{1 + 4 \cdot Ql^2} \quad \text{Inductance value} \]

\[ Rl := \frac{\pi}{ts} \cdot \frac{Ll}{Ql} \quad \text{Resistor value} \]

\[ Zl(\omega) := \frac{Rl}{1 + \frac{Rl \cdot \omega \cdot C \cdot (1 - \omega^2 \cdot Ll \cdot C)}{1 + Rl \cdot \omega \cdot C \cdot (1 - \omega^2 \cdot Ll \cdot C)} \quad \text{Transfer function} \]

\[ al := \frac{Rl}{2 \cdot Ll} \]
\[ \omega_0 l := \sqrt{\frac{1}{Ll \cdot C} - \frac{Rl^2}{4 \cdot Ll^2}} = 1.571 \cdot 10^8 \quad \text{Resonance} \]
\[ fo l := \frac{\omega_0 l}{2 \cdot \pi} = 2.5 \cdot 10^7 \]

\[ \text{step\_response } \text{fo}_{0,5o}(t) := Rl \cdot \left( 1 - \exp(-al \cdot t) \cdot \cos(\omega_0 l \cdot t) - \frac{al}{\omega_0 l} \cdot \exp(-al \cdot t) \cdot \sin(\omega_0 l \cdot t) \right) \]

\[ t := 0, 0.1 \cdot 10^{-9}, \ldots, 200 \cdot 10^{-9} \quad \text{Step response, tpk = ts} \]
Different input pulse widths (tp = ts, tp = 0.5 ts, tp = 0.25 ts)
Output pulse shape/ input pulse width equals the time slot

\[ v_{out}(t) \]

\[ time \_slot(t) \]

\[ \frac{t}{ts} \]

\[ v_{out,0.25s}(t) = \frac{ts}{4 \pi Rl} \cdot \int_0^{1/100} \frac{\sin \left( \frac{\omega \cdot ts}{8} \right)}{\left( \frac{\omega \cdot ts}{8} \right)} \cdot \Re \left( Z(\omega) \cdot \exp \left( i \omega \cdot \left( t - \frac{ts}{8} \right) \right) \right) d\omega \]

\[ v_{out,0.5s}(t) = \frac{ts}{2 \pi Rl} \cdot \int_0^{1/100} \frac{\sin \left( \frac{\omega \cdot ts}{4} \right)}{\left( \frac{\omega \cdot ts}{4} \right)} \cdot \Re \left( Z(\omega) \cdot \exp \left( i \omega \cdot \left( t - \frac{ts}{4} \right) \right) \right) d\omega \]

Output pulse shape/ input pulse width equals 0.25 times slot

\[ v_{out,0.25s}(t) \]

\[ input\_pulse_{0.25s}(t) \]

\[ \frac{t}{ts} \]
Output pulse shape/ input pulse width equals half the time slot

\[ \frac{t}{ts} \]

\[ Q2 := 1 \]

\[ f_o = \frac{1}{T_s} \]

\[ L2 := \frac{1}{C} \cdot \frac{ts^2}{\pi^2} \cdot \frac{Q2^2}{1 + 4 \cdot Q2^2} \]

\[ R2 := \frac{2 \cdot \pi}{ts} \cdot \frac{L2}{Q2} \]

\[ a2 := \frac{R2}{2 \cdot L2} \quad \omega_0^2 := \sqrt{\frac{1}{L2 \cdot C} - \frac{R2^2}{4 \cdot L2^2}} \quad f_o^2 := \frac{\omega_0^2}{(2 \cdot \pi)} \]

\[ \text{step_response}_f(t) := R2 \cdot \left(1 - \exp(-a2 \cdot t) \cdot \cos(\omega_0^2 \cdot t) - \frac{a2}{\omega_0^2} \cdot \exp(-a2 \cdot t) \cdot \sin(\omega_0^2 \cdot t)\right) \]

Step response, tpk = 0.5 ts
\[ Z_2(\omega) := \frac{R_2}{1 + R_2 \cdot \omega \cdot C \cdot (1 - \omega^2) \cdot L_2 \cdot C} \]

\[ v_{out_{fo}}(t) := \frac{ts}{2} \cdot \frac{1}{\pi \cdot R_1} \cdot \int_0^{\frac{1}{n} \cdot 10^{-3}} \sin \left( \frac{\omega \cdot ts}{4} \right) \cdot \Re \left( Z_2(\omega) \cdot \exp \left( i \cdot \omega \cdot \left( t - \frac{ts}{4} \right) \right) \right) d\omega \]

\[ \text{inputpulse}(t) := \frac{ts}{2} \cdot \frac{1}{\pi} \cdot \int_0^{\frac{1}{n} \cdot 10^{-3}} \sin \left( \frac{\omega \cdot ts}{4} \right) \cdot \Re \left( \exp \left( i \cdot \omega \cdot \left( t - \frac{ts}{4} \right) \right) \right) d\omega \]

Output pulse shape/ input pulse width equals half the time slot.

Output pulse shape/ input pulse width equals half the time slot.

\[ v_{out_{fo}}(t) \]

\[ \text{inputpulse}(t) \]

\[ \text{time_slot}(t) \]
Output pulse shape/ input pulse width equals 0.25 times the time slot

\[
\frac{t}{ts}
\]

Output pulse shape/ input pulse width equals the time slot

\[
\frac{t}{ts}
\]
\[ f_0 = \frac{2}{T_s} \]

\[ Q^3 = 1 \]
\[ L^3 = \frac{1}{C} \cdot \frac{t^2}{\pi^2} \cdot \frac{1}{4 + 16 \cdot Q^2} \]
\[ R^3 = \frac{4 \cdot \pi \cdot L^3}{Q^3} \]
\[ a_3 = \frac{R^3}{2 \cdot L^3} \]
\[ \omega_0^3 = \sqrt{1 - \frac{R^3}{4 \cdot L^3}} \]
\[ f_{04} = \frac{\omega_0^3}{(2 \cdot \pi)} \]

\[ \text{step response } f_{o2n} (t) = R^3 \cdot \left( 1 - \exp(-a_3 \cdot t) \cdot \cos(\omega_0^3 \cdot t) - \frac{a_3}{\omega_0^3} \cdot \exp(-a_3 \cdot t) \cdot \sin(\omega_0^3 \cdot t) \right) \]

Step response, \( tpk = 0.25 \) ts

\[ Z^3(\omega) = \frac{R^3}{1 + R^3 \cdot \omega \cdot C \cdot (1 - \omega^2 \cdot L^3 \cdot C)} \]

\[ \text{inputpulse } f_{o2n} (t) = \frac{4}{\pi} \cdot \frac{1}{t_s} \cdot 10^4 \cdot \sin \left( \frac{\omega \cdot t_s}{8} \right) \cdot \text{Re} \left( \exp \left( 1i \cdot \omega \left( t - \frac{t_s}{8} \right) \right) \right) \, d\omega \]

\[ \text{vout } f_{o2n} (t) = \frac{4}{\pi \cdot R^3} \cdot \frac{1}{t_s} \cdot 10^4 \cdot \sin \left( \frac{\omega \cdot t_s}{8} \right) \cdot \text{Re} \left( Z^3(\omega) \cdot \exp \left( 1i \cdot \omega \left( t - \frac{t_s}{8} \right) \right) \right) \, d\omega \]
Output pulse shape/ input pulse width equals 0.25 times the time slot

\[ \text{Output pulse shape/ input pulse width equals the time slot} \]

\[ \frac{v_{\text{out, } f_{2s}}(t)}{v_{\text{in, } f_{2s}}(t)} = \frac{1}{n} \cdot 10^{3} \cdot \frac{\sin(\omega \cdot ts)}{\omega \cdot ts} \cdot \text{Re}\left( Z(\omega) \cdot \exp\left( i \cdot \omega \cdot \left( t - \frac{ts}{8} \right) \right) \right) \, d\omega \]

\[ \frac{v_{\text{out, } f_{2s}}(t)}{v_{\text{in, } f_{2s}}(t)} = \frac{1}{n} \cdot 10^{3} \cdot \frac{\sin(\omega \cdot ts)}{\omega \cdot ts} \cdot \text{Re}\left( Z(\omega) \cdot \exp\left( i \cdot \omega \cdot \left( t - \frac{ts}{2} \right) \right) \right) \, d\omega \]

Output pulse shape/ input pulse width equals the time slot

\[ \frac{v_{\text{out, } f_{2s}}(t)}{v_{\text{in, } f_{2s}}(t)} = \frac{1}{n} \cdot 10^{3} \cdot \frac{\sin(\omega \cdot ts)}{\omega \cdot ts} \cdot \text{Re}\left( Z(\omega) \cdot \exp\left( i \cdot \omega \cdot \left( t - \frac{ts}{2} \right) \right) \right) \, d\omega \]

A-9
Output pulse shape/ input pulse width equals half the time slot

\[ v_{0.5t_f}^{0.5t_s}(t) \]

\[ \text{inputpulse}_{0.5t_s}(t) \]
\[ ZRL(\omega) := \frac{RI + \omega \cdot LI \cdot 1i}{1 + RI \cdot \omega \cdot C \cdot L1 \cdot L1 \cdot C} \]

\[ \text{vouRL}_{ts}(t) := \frac{ts}{\pi} \cdot \int_{0}^{\frac{1}{\pi} \cdot 10^4} \frac{\sin \left( \frac{\omega \cdot ts}{2} \right)}{\left( \frac{\omega \cdot ts}{2} \right)} \cdot \text{Re} \left( ZRL(\omega) \cdot \exp \left( 1i \cdot \omega \cdot \left( t - \frac{ts}{2} \right) \right) \right) \, d\omega \]

\[ \text{vouRL}_{0.5ts}(t) := \frac{ts}{\pi} \cdot \int_{0}^{\frac{1}{\pi} \cdot 10^4} \frac{\sin \left( \frac{\omega \cdot ts}{4} \right)}{\left( \frac{\omega \cdot ts}{4} \right)} \cdot \text{Re} \left( ZRL(\omega) \cdot \exp \left( 1i \cdot \omega \cdot \left( t - \frac{ts}{4} \right) \right) \right) \, d\omega \]

\[ \text{vouRL}_{0.25ts}(t) := \frac{4ts}{\pi} \cdot \int_{0}^{\frac{1}{\pi} \cdot 10^4} \frac{\sin \left( \frac{\omega \cdot ts}{8} \right)}{\left( \frac{\omega \cdot ts}{8} \right)} \cdot \text{Re} \left( ZRL(\omega) \cdot \exp \left( 1i \cdot \omega \cdot \left( t - \frac{ts}{8} \right) \right) \right) \, d\omega \]
\[ ZRL2(\omega) := \frac{R2 + \omega \cdot L2 \cdot i}{1 + R2 \cdot \omega \cdot C \cdot i - \omega^2 \cdot L2 \cdot C} \]

\[
vouRL2_{1s}(t) := \frac{ts}{\pi} \cdot \int_{0}^{\frac{1}{\pi} \cdot 10^3} \frac{\sin\left(\frac{\omega \cdot ts}{2}\right)}{\left(\frac{\omega \cdot ts}{2}\right)} \cdot \text{Re}\left(ZRL2(\omega) \cdot \exp\left(\frac{i \cdot \omega \cdot \left(t - \frac{ts}{2}\right)}{2}\right)\right) d\omega
\]

\[
vouRL2_{0.5s}(t) := \frac{ts}{\pi} \cdot \int_{0}^{\frac{1}{\pi} \cdot 10^3} \frac{\sin\left(\frac{\omega \cdot ts}{4}\right)}{\left(\frac{\omega \cdot ts}{4}\right)} \cdot \text{Re}\left(ZRL2(\omega) \cdot \exp\left(\frac{i \cdot \omega \cdot \left(t - \frac{ts}{4}\right)}{4}\right)\right) d\omega
\]

\[
vouRL2_{0.25s}(t) := \frac{ts}{\pi} \cdot \int_{0}^{\frac{1}{\pi} \cdot 10^3} \frac{\sin\left(\frac{\omega \cdot ts}{8}\right)}{\left(\frac{\omega \cdot ts}{8}\right)} \cdot \text{Re}\left(ZRL2(\omega) \cdot \exp\left(\frac{i \cdot \omega \cdot \left(t - \frac{ts}{8}\right)}{8}\right)\right) d\omega
\]
\[ ZRL3(\omega) := \frac{R3 + \omega \cdot L3 \cdot 1i}{1 + R3 \cdot \omega \cdot C \cdot 1i - \omega^2 \cdot L3 \cdot C} \]

\[ voulR3(t) = \frac{ts}{\pi} \cdot \frac{1}{1 \cdot 10^3} \sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \text{Re} \left( ZRL3(\omega) \cdot \exp \left( 1i \cdot \omega \cdot \left( t - \frac{ts}{2} \right) \right) \right) d\omega \]

\[ voulR3_{0.5s}(t) = \frac{ts}{\pi} \cdot \frac{1}{1 \cdot 10^3} \sin \left( \frac{\omega \cdot ts}{4} \right) \cdot \text{Re} \left( ZRL3(\omega) \cdot \exp \left( 1i \cdot \omega \cdot \left( t - \frac{ts}{4} \right) \right) \right) d\omega \]

\[ voulR3_{0.25s}(t) = \frac{ts}{\pi} \cdot \frac{1}{1 \cdot 10^3} \sin \left( \frac{\omega \cdot ts}{8} \right) \cdot \text{Re} \left( ZRL3(\omega) \cdot \exp \left( 1i \cdot \omega \cdot \left( t - \frac{ts}{8} \right) \right) \right) d\omega \]

\[ voulR3_{ts}(t) \]
\[ voulR3_{0.5s}(t) \]
\[ voulR3_{0.25s}(t) \]
Frequency response

\[ Z_1(\omega) := 20 \log \left( \frac{R_1}{1 + R_1 \cdot \omega \cdot C \cdot 1i - \omega^2 \cdot L_1 \cdot C} \right) \]

\[ Z_2(\omega) := 20 \log \left( \frac{R_2}{1 + R_2 \cdot \omega \cdot C \cdot 1i - \omega^2 \cdot L_2 \cdot C} \right) \]

\[ Z_3(\omega) := 20 \log \left( \frac{R_3}{1 + R_3 \cdot \omega \cdot C \cdot 1i - \omega^2 \cdot L_3 \cdot C} \right) \]

\[ Z_{RL}(\omega) := 20 \log \left( \frac{R_1 + \omega \cdot L_1 \cdot 1i}{1 + R_1 \cdot \omega \cdot C \cdot 1i - \omega^2 \cdot L_1 \cdot C} \right) \]

\[ Z_{RL2}(\omega) := 20 \log \left( \frac{R_2 + \omega \cdot L_2 \cdot 1i}{1 + R_2 \cdot \omega \cdot C \cdot 1i - \omega^2 \cdot L_2 \cdot C} \right) \]

\[ Z_{RL3}(\omega) := 20 \log \left( \frac{R_3 + \omega \cdot L_3 \cdot 1i}{1 + R_3 \cdot \omega \cdot C \cdot 1i - \omega^2 \cdot L_3 \cdot C} \right) \]

\[ \omega = 1 \cdot 10^6, 1.01 \cdot 10^6 \ldots 1 \cdot 10^{10} \]
\[
Z_1\phi(\omega) := \frac{180}{\pi} \cdot \arg \left( \frac{R1}{1 + R1 \cdot \omega \cdot C \cdot \frac{1}{i} \cdot \omega^2 \cdot L1 \cdot C} \right)
\]

\[
Z_2\phi(\omega) := \frac{180}{\pi} \cdot \arg \left( \frac{R2}{1 + R2 \cdot \omega \cdot C \cdot \frac{1}{i} \cdot \omega^2 \cdot L2 \cdot C} \right)
\]

\[
Z_3\phi(\omega) := \frac{180}{\pi} \cdot \arg \left( \frac{R3}{1 + R3 \cdot \omega \cdot C \cdot \frac{1}{i} \cdot \omega^2 \cdot L3 \cdot C} \right)
\]

\[
Z_RL\phi(\omega) := \frac{180}{\pi} \cdot \arg \left( \frac{R1 + \omega \cdot L1 \cdot \frac{1}{i}}{1 + R1 \cdot \omega \cdot C \cdot \frac{1}{i} \cdot \omega^2 \cdot L1 \cdot C} \right)
\]

\[
Z_RL2\phi(\omega) := \frac{180}{\pi} \cdot \arg \left( \frac{R2 + \omega \cdot L2 \cdot \frac{1}{i}}{1 + R2 \cdot \omega \cdot C \cdot \frac{1}{i} \cdot \omega^2 \cdot L2 \cdot C} \right)
\]

\[
Z_RL3\phi(\omega) := \frac{180}{\pi} \cdot \arg \left( \frac{R3 + \omega \cdot L3 \cdot \frac{1}{i}}{1 + R3 \cdot \omega \cdot C \cdot \frac{1}{i} \cdot \omega^2 \cdot L3 \cdot C} \right)
\]

\[
\omega := 1 \cdot 10^6, 2 \cdot 10^6, 10 \cdot 10^9
\]
\[ \tau_r := 2 \, ts \]

\[ Q_r := 0.5 \]

\[ L_r := \frac{1}{C} \cdot \frac{\tau_r^2}{25^2 + \left(10 \, Q_r\right)^2} \]

\[ R_r := 10 \cdot \frac{L_r}{\tau_r} \]

\[ Z_r(\omega) := \frac{R_r}{1 + R_r \cdot \omega \cdot C \cdot \left(1 - \omega^2 \cdot L_r \cdot C\right)} \]

\[ Z_{\text{Response}}(t) = \frac{ts}{\pi} \cdot \left[ 1 \cdot \text{Re} \left( \frac{R_r}{1 + R_r \cdot \omega \cdot C \cdot \left(1 - \omega^2 \cdot L_r \cdot C\right)} \right) \cdot \exp \left( i \cdot \omega \cdot \left( t - \frac{ts}{2} \right) \right) \right] \text{d}\omega \]
A.2 Bandwidth extension

Bandwidth extension for CMOS

\[ R = 1 \quad C = 1 \quad \text{RC} = 1, \text{ for normalised response} \]

\[ Z_{\text{basis}}(\omega) := \frac{1}{1 + R \omega C i} \]

Initial transfer function

\[ Z(\omega) := 1 + i \omega \left[ \frac{1}{1.4} R C - \omega^2 R^2 C^2 \frac{0}{1.4} - \frac{(1 + 0)}{1.4} \omega^2 \left( R^2 C^2 \right) - i \frac{0}{1.4} \omega^3 \left( R^3 C^3 \right) \right] \]

\[ Z(x)(\omega) := 1 + i \omega \left[ \frac{1}{1.8} R C - \omega^2 R^2 C^2 \frac{0.1}{1.8} - \frac{(1 + 0.1)}{1.8} \omega^2 \left( R^2 C^2 \right) - i \frac{0.1}{1.8} \omega^3 \left( R^3 C^3 \right) \right] \]

\[ Z_{2x}(\omega) := 1 + i \omega \left[ \frac{1}{2.2} R C - \omega^2 R^2 C^2 \frac{0.2}{2.2} - \frac{(1 + 0.2)}{2.2} \omega^2 \left( R^2 C^2 \right) - i \frac{0.2}{2.2} \omega^3 \left( R^3 C^3 \right) \right] \]

\[ Z_{3x}(\omega) := 1 + i \omega \left[ \frac{1}{2.4} R C - \omega^2 R^2 C^2 \frac{0.3}{2.4} - \frac{(1 + 0.3)}{2.4} \omega^2 \left( R^2 C^2 \right) - i \frac{0.3}{2.4} \omega^3 \left( R^3 C^3 \right) \right] \]

\[ Z_{2z}(\omega) := \frac{R + i \omega R^2 C}{1 + \omega^2 R^2 C^2 + R \omega C i} \]

\[ Z_{3z}(\omega) := \frac{R + i \omega R^2 C}{1 + \omega^2 R^2 C^2 + R \omega C i} \]
\[ Z_1(\omega) := \frac{R}{1 + R\omega Ci} \]

\[ Z_2(\omega) := \frac{R + i\omega R^2 \frac{C}{1.41}}{1 - \omega^2 R^2 \frac{C}{1.41} + R\omega Ci} \]

\[ Z_3(\omega) := \frac{R + i\omega R^2 \frac{C}{2.41}}{1 - \omega^2 R^2 \frac{C}{2.41} + R\omega Ci} \]

\[ Z_4(\omega) := \frac{R + i\omega R^2 \frac{C}{3.1}}{1 - \omega^2 R^2 \frac{C}{3.1} + R\omega Ci} \]

---

**Graphical Representation**

- **No peaking**
- **Maximum bandwidth**
- **Flat response**
- **Best group delay**

**Axes:**
- **Normalised frequency**
- **Magnitude (dB)**
\[ P_1(\omega) := \frac{180}{3.14} \arg(Z_1(\omega)) \]

\[ P_2(\omega) := \frac{180}{3.14} \arg(Z_2(\omega)) \]

\[ P_3(\omega) := \frac{180}{3.14} \arg(Z_3(\omega)) \]

\[ P_4(\omega) := \frac{180}{3.14} \arg(Z_4(\omega)) \]
Normalised frequency

--- uncompensated (Max bandwidth)
--- compensated
--- uncompensated (Max flat response)
--- No peaking

\[ Z_{b1}(\omega) := \frac{1}{1 + R\omega C - \frac{(1-0.1)}{1.8} \cdot \omega^2 (R^2 C^2) - \frac{0.11 - 0.1}{1.8} \cdot \omega^3 (R^3 C^3)} \]

\[ Z_{b2}(\omega) := \frac{1}{1 + R\omega C - \frac{(1-0.3)}{2.4} \cdot \omega^2 (R^2 C^2) - \frac{0.31 - 0.3}{2.4} \cdot \omega^3 (R^3 C^3)} \]

\[ Z_{b3}(\omega) := \frac{1}{1 + R\omega C - \frac{(1-0.5)}{1.5} \cdot \omega^2 (R^2 C^2) - \frac{0.51 - 0.5}{1.5} \cdot \omega^3 (R^3 C^3)} \]
Normalised frequency

Magnitude (dB)

- KC = 0.1
- KC = 0.3
- KC = 0.5
- No peaking

KC = 0.1
KC = 0.3
KC = 0.5
No peaking
A.3 Noise transfer functions

Input/output noise transfer functions (without filter)

\[ Av := 10 \quad \text{Gain} \]

\[ C := 1.5 \cdot 10^{-12} \text{ total C} \]

\[ m := 1.8 \quad \text{time constant ratio of } L/R \text{ and } RC \]

\[ y = \sqrt{\left( \frac{-m^2}{2} + m + 1 \right)} + \sqrt{\left( \frac{-m^2}{2} + m + 1 \right)^2 + m^2} = 1.825 \]

\[ d := \frac{1}{y} \cdot 100 \cdot 10^6 \]

\[ f_c := 0.5 \cdot d = 2.739 \cdot 10^7 \quad \text{Cut-off of half the bit rate, before BWER} \]

\[ R := \frac{1}{(2 \cdot \pi \cdot C \cdot f_c)} = 3.874 \cdot 10^3 \]

\[ L := R^2 \cdot \frac{C}{m} = 1.25 \cdot 10^{-5} \]

\[ t_s := \frac{1}{y \cdot d} \quad B := \frac{1}{t_s} = 1 \cdot 10^8 \]

\[ f_c := 0.5 \cdot B = 5 \cdot 10^7 \quad \text{Actual Bit rate(cut-off)} \]

TUNED A

\[ R_f := R \cdot (1 + Av) = 4.261 \cdot 10^4 \]

\[ L_f := L \cdot (1 + Av) = 1.376 \cdot 10^{-4} \]

\[ Z_x(\omega) := \frac{R_f + 1i \cdot \omega \cdot L_f}{\left(\left(1 - \omega^2 \cdot L_f \cdot \frac{C}{1 + Av}\right) + R_f \cdot \omega \cdot \frac{C}{(1 + Av) \cdot 1i}\right)} \]

\[ H_{\text{not}}(\omega) := 1 \]

\[ Z_{RL}(\omega) := H_{\text{not}}(\omega) \cdot \frac{Z_x(\omega)}{R_f} \]
Non-tuned

\[ \omega_c := 2 \cdot \pi \cdot f_c = 3.142 \cdot 10^8 \]

\[ \text{Rt} := \frac{Av + 1}{\omega_c \cdot C} = 2.334 \cdot 10^4 \quad \text{non-tuned Rf} \]

\[ Z(\omega) := H_{nt}(\omega) \cdot \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_c}} \]

Tuned B

\[ x := 2.75 \quad Mc := 1.9 \]

\[ Rf2 := x \cdot Rt = 6.419 \cdot 10^4 \]

\[ a := 0.4 \]

\[ Lc := \frac{\left( \frac{Rf2}{1 + Av} \right)^2 \cdot C}{Mc} \]

\[ C1 := (1 - a) \cdot (C) = 9 \cdot 10^{-13} \]

\[ C2 := a \cdot (C) = 6 \cdot 10^{-13} \]

\[ Z2R(\omega) := \frac{Rf2}{\left( \left(1 - \omega^2 \cdot Lc \cdot C1\right) + \frac{Rf2}{(1 + Av)} \cdot (\omega) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \cdot 1i \right)} \]

\[ ZR(\omega) := H_{nt}(\omega) \cdot \frac{Z2R(\omega)}{Rf2} \]

\[ \omega := 1 \cdot 10^6, 10 \cdot 10^6 \ldots 1 \cdot 10^{11} \]
Noise transfer function to output/input for Ic or Id

**non-tuned**

Noise transfer function to output (output power)

\[ Z_{IC\_out}(\omega) := \left( \frac{1 + j \cdot \omega \cdot R_t \cdot C}{1 + j \cdot \omega \cdot \frac{R_t}{1 + A_v} \cdot C} \right)^2 \]

Noise transfer function to input (input power)

\[ H_{IC\_in}(\omega) := \left( \frac{1 + j \cdot \omega \cdot R_t \cdot C}{1} \right)^2 \]

Output power (input x TF)

\[ Z_{IC\_out,2}(\omega) := \left( \frac{1 + j \cdot \omega \cdot R_t \cdot C}{1} \right)^2 \cdot \left( |Z(\omega)| \right)^2 \]

![Graph showing the comparison of transfer functions](image)
Tuned A

Noise transfer function to output (output power)

\[ Z_{IC\_out\_A}(\omega) := \left( \left( \frac{(1 - \omega^2 \cdot Lf \cdot C) + Rf \cdot \omega \cdot C \cdot 1i}{1 + Av} \right) \cdot \left( \frac{1}{1 + Av} \right) + Rf \cdot \omega \cdot \frac{C}{(1 + Av)} \cdot 1i \right)^2 \]

Noise transfer function to input (input power)

\[ H_{IC\_in\_A}(\omega) := \left( \left( \frac{(1 - \omega^2 \cdot Lf \cdot C) + Rf \cdot \omega \cdot C \cdot 1i}{1 + 1i \cdot \omega \cdot \frac{Lf}{Rf}} \right)^2 \right) \]

Output power (input x TF)

\[ Z_{out\_A.2}(\omega) := \left( \left( \frac{(1 - \omega^2 \cdot Lf \cdot C) + Rf \cdot \omega \cdot C \cdot 1i}{1 + 1i \cdot \omega \cdot \frac{Lf}{Rf}} \right)^2 \right) \cdot \left( |ZRL(\omega)| \right)^2 \]
**Tuned B**

Noise transfer function to output (output power)

\[
Z_{IC_{out_B}}(\omega) := \left( \frac{1}{\left(1 - \omega^2 \cdot Lc \cdot C1\right) + Rf2 \cdot (\omega) \cdot \left(C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2\right) \cdot 1i} \right)^2
\]

Noise transfer function to input (input power)

\[
H_{IC_{in_B}}(\omega) := \left( \frac{1}{\left(1 - \omega^2 \cdot Lc \cdot C1\right) + \frac{Rf2}{1 + Av} \cdot (\omega) \cdot \left(C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2\right) \cdot 1i} \right)^2
\]

Output power (input x TF)

\[
Z_{IC_{out_B_2}}(\omega) := \left( \frac{1}{\left(1 - \omega^2 \cdot Lc \cdot C1\right) + Rf2 \cdot (\omega) \cdot \left(C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2\right) \cdot 1i} \right)^2 \cdot \left(\left|ZR(\omega)\right|\right)^2
\]
Rf noise

non-tuned

\( H_{Rf\_in} := 1 \)

\( Z_{Rf\_out}(\omega) := H_{Rf\_in} \cdot |Z(\omega)|^2 \)
Tuned A

Noise transfer function to output (output power)

\[ Z_{RF \text{ out } A}(\omega) := \left( \left( 1 - \omega^2 \cdot L_f \cdot \frac{C}{1 + A v} \right) + R_f \cdot \omega \cdot \frac{C}{1 + A v} \cdot 1i \right)^2 \]

Noise transfer function to input (input power)

\[ H_{RF \text{ in } A}(\omega) := \left( \left( \frac{1}{1 + 1i \cdot \omega \cdot \frac{L_f}{R_f}} \right) \right)^2 \]

Output power (input x TF)

\[ Z_{RF \text{ out } A \cdot 2}(\omega) := \left( \left( \frac{1}{1 + 1i \cdot \omega \cdot \frac{L_f}{R_f}} \right) \right)^2 \cdot \left( |ZRL(\omega)| \right)^2 \]
Tuned B
Noise transfer function to output (output power)

\[ Z_{RF\_out\_B}(\omega) := \left( \frac{\left| 1 - \omega^2 \cdot Lc \cdot C1 \right|}{\left( \left( 1 - \omega^2 \cdot Lc \cdot C1 \right) + \frac{Rf^2}{Av + 1} \cdot (\omega) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \cdot 1i \right)} \right)^2 \]

Noise transfer function to input (input power)

\[ H_{RF\_in\_B}(\omega) := \left( \left| 1 - \omega^2 \cdot Lc \cdot C1 \right| \right)^2 \]

output power (input x TF)

\[ Z_{RF\_out\_B\_2}(\omega) := \left( \left( \left| 1 - \omega^2 \cdot Lc \cdot C1 \right| \right)^2 \cdot \left( \left| ZR(\omega) \right|^2 \right) \right) \]
noise for Ib and Ig

non-tuned

\[ H_{IB\text{ in}} := 1 \]

\[ Z_{IB\text{ out}}(\omega) := H_{IB\text{ in}} \cdot (|Z(\omega)|)^2 \]
Noise transfer function to input (input power)

\[ H_{IB\text{--}in\text{--}A}(\omega) := 1 \]

Output power (input x TF)

\[ Z_{IB\text{--}out\text{--}A}(\omega) := \left( H_{IB\text{--}in\text{--}A}(\omega) \cdot |ZRL(\omega)|^2 \right) \]
total power spectrum
non-tuned
Input

\[ I_{n_{\text{in}}} (\omega) := H_{IB_{\text{in}}} + H_{Rf_{\text{in}}} + H_{IC_{\text{in}}} (\omega) \]

\[ I_{n_{\text{out}}} (\omega) := I_{n_{\text{in}}} (\omega) \cdot (|Z(\omega)|)^2 \]
Total power spectrum

Tuned A

\[ I_{n\_in\_A}(\omega) := H_{IB\_in\_A}(\omega) + H_{RF\_in\_A}(\omega) + H_{IC\_in\_A}(\omega) \]

\[ I_{n\_out\_A}(\omega) := I_{n\_in\_A}(\omega) \cdot (|ZRL(\omega)|)^2 \]
Total power spectrum

Tuned B

\[ H_{IB\ in\ B}(\omega) := H_{RF\ in\ B}(\omega) \]

\[ I_{in\ B}(\omega) := H_{IB\ in\ B}(\omega) + H_{RF\ in\ B}(\omega) + H_{IC\ in\ B}(\omega) \]

\[ I_{n\ out\ B}(\omega) := I_{n\ in\ B}(\omega) \cdot (|ZR(\omega)|)^2 \]
Total power spectrum

Non-tuned vs Tuned A vs Tuned B

\[ H_{IB \text{ in } B}(\omega) := H_{RF \text{ in } B}(\omega) \]

\[ I_{n \text{ in }}(\omega) := H_{IB \text{ in } B}(\omega) + H_{RF \text{ in } B}(\omega) + H_{IC \text{ in }}(\omega) \]

\[ I_{n \text{ in } A}(\omega) := H_{IB \text{ in } A}(\omega) + H_{RF \text{ in } A}(\omega) + H_{IC \text{ in } A}(\omega) \]

\[ I_{n \text{ in } B}(\omega) := H_{IB \text{ in } B}(\omega) + H_{RF \text{ in } B}(\omega) + H_{IC \text{ in } B}(\omega) \]
A.4 Noise modelling (1st order low-pass pre-detection filter)

Noise model verification (non-tuned Front-End receiver/1st order LPF)

**Bit-rate, pulse duration**

\[ C_T := 1.5 \times 10^{-12} \quad \text{Total capacitance} \]
\[ B := 100 \times 10^6 \quad \text{Bit-rate} \]
\[ f_c := 0.75 \times B = 7.5 \times 10^7 \quad \text{cut-off frequency TIA} \]
\[ t_s := \frac{1}{B} \]
\[ f_B := 0.75 \times B \quad \text{cut-off frequency LPF} \]
\[ \omega_B := 2 \pi f_B = 4.712 \times 10^8 \]

**Pre-dec filter**

\[ H_{but}(\omega) := \frac{1}{1 + j \omega \frac{\omega}{\omega_B}} \]

**TIA**

\[ A_v := 10 \quad \text{gain} \]
\[ \omega_c := 2 \pi f_c = 4.712 \times 10^8 \quad \text{cut-off} \]
\[ R_f := \frac{A_v + 1}{\omega_c \cdot C_T} = 1.556 \times 10^4 \quad \text{Feedback} \]
\[ Z_{TIA}(\omega) := \frac{1}{1 + j \omega \frac{\omega}{\omega_c}} \]

**Receiver freq-response**

\[ Z(\omega) := H_{but}(\omega) \cdot Z_{TIA}(\omega) \quad \text{Frequency dependence of NON-TUNED TI} \]
\[ \omega := 0.1 \times 10^6, 1 \times 10^6, \ldots, 1 \times 10^1 \]
\[ |H_{but}(\omega_B)| = 0.707 \]
\[ |Z_{TIA}(\omega_c)| = 0.707 \]
\[ |Z(2 \pi 0.483 B)| = 0.707 \]
\[ \omega_f := \sqrt{\frac{1}{\omega_B^2 + \omega_c^2}} \]
\[ \omega_{3dB} := 0.483 \]
FET parameters

\[ Ig := 10 \times 10^{-9} \quad gm_l := 30 \times 10^{-3} = 0.03 \quad noise\_factor := 1 \]

\[ q := 1.6 \times 10^{-19} \quad k := 1.38 \times 10^{-23} \quad T := 298 \]

Noise equivalent bandwidth

\[ Beq = \frac{1}{2 \pi} \int_{0}^{10^{12}} \left( \frac{1 + i \omega \cdot R_T \cdot C_T}{gm_l \cdot R_t} \cdot |Z(\omega)| \right)^2 d\omega = 5.89 \times 10^7 \]

\[ \frac{Beq}{B_{\omega dB}} = 0.589 \quad \text{noise calculations (proposed method)} \]

\[ Beqc := \frac{1}{2 \pi} \int_{0}^{10^{12}} \left( \frac{1 + i \omega \cdot (q \cdot Ig + \frac{4 \cdot k \cdot T}{R_t}) \cdot (Beq) + 4 \cdot k \cdot T \cdot gm_l \cdot Beqc}{gm_l \cdot R_t} \cdot |Z(\omega)| \right)^2 d\omega = 3.295 \times 10^4 \]

\[ noise := \left( 2 \cdot q \cdot Ig + \frac{4 \cdot k \cdot T}{R_t} \right) \cdot (Beqc) + 4 \cdot k \cdot T \cdot gm_l \cdot Beqc = 7.87155075362924 \times 10^{-17} \]

Noise calculations Conventional + check

\[ NEB := \frac{1}{2 \pi} \int_{0}^{10^{12}} \left( \frac{1 + i \omega \cdot (q \cdot Ig)}{gm_l \cdot R_t} \cdot |Z(\omega)| \right)^2 d\omega = 5.89 \times 10^7 \]

\[ Test\_BW_n := \frac{1}{2 \pi} \int_{0}^{10^{12}} \left( \frac{1 + i \omega \cdot 0.7 \cdot 0.7}{gm_l \cdot R_t} \cdot |Z(\omega)| \right)^2 d\omega = 4.687 \times 10^7 \]

\[ BW := \frac{1}{2 \pi} \int_{0.7 \pi}^{10^{12}} \left( \frac{1 + i \omega \cdot 0.7}{gm_l \cdot R_t} \cdot |Z(\omega)| \right)^2 d\omega = 5.89 \times 10^7 \quad \text{This should be the right integration bound} \]

\[ Test\_BW_{n2} := 3 \cdot \int_{0}^{10^{12}} \left( \frac{1 + i \omega \cdot 0.7 \cdot 0.7}{gm_l \cdot R_t} \cdot |Z(\omega)| \right)^2 \cdot |f|^2 df = 1.595 \times 10^{-23} \]

\[ BW_{n2} := 3 \cdot \int_{0}^{10^{11}} \left( \frac{1 + i \omega \cdot 0.7 \cdot 0.7}{gm_l \cdot R_t} \cdot |Z(\omega)| \right)^2 \cdot |f|^2 df = 9.931 \times 10^{-23} \]

\[ Test\_I_{nd} := Test\_BW_n \cdot \frac{4 \cdot k \cdot T}{R_t} = 4.95389134541763 \times 10^{-17} \]

\[ Test\_I_{ndg} := Test\_BW_n \cdot (2 \cdot q \cdot Ig) = 1.49968969673026 \times 10^{-19} \]

\[ Test\_I_{channel} := Test\_BW_n \cdot \left( \frac{1}{R_t^2} \cdot \frac{4 \cdot k \cdot T}{gm_l} \right) = 1.0611210448387 \times 10^{-19} \]

\[ Test\_I_{channel2} := Test\_BW_{n2} \cdot \left( \frac{4 \cdot k \cdot T}{3 \cdot gm_l} \cdot \left( \frac{2 \cdot \pi \cdot C_T}{2} \right)^2 \right) = 2.59019087494843 \times 10^{-18} \]

A-39
Correct noise calculations input

\[
\text{Test noise}_2 = 5.23851854032804 \cdot 10^{-17}
\]

\[
I_{\text{bg}} = BW_n \cdot \frac{4 \cdot k \cdot T}{Rt} = 6.22653126183735 \cdot 10^{-17}
\]

\[
I_{\text{lg}} = BW_n \cdot (2 \cdot q \cdot \text{Ig}) = 1.884955084376 \cdot 10^{-19}
\]

\[
I_{\text{channel}} = BW_n \cdot \left( \frac{1}{Rt^2} \cdot \frac{4 \cdot k \cdot T}{\text{gml}} \right) = 1.33371987746832 \cdot 10^{-19}
\]

\[
I_{\text{channel2}} = BW_{n2} \cdot \left( \frac{4 \cdot k \cdot T}{3 \cdot \text{gml}} \cdot (2 \cdot \pi \cdot C_T)^2 \right) = 1.61226005714715 \cdot 10^{-17}
\]

\[
I_{\text{channel}} + I_{\text{channel2}} = 1.62559725592183 \cdot 10^{-17}
\]

\[
\frac{I_{\text{channel}} + I_{\text{channel2}}}{4 \cdot k \cdot T \cdot \text{gml} \cdot \text{Beq}_c} = 1
\]

\[
\text{noise}_2 = \left( \frac{1}{Rt^2} \cdot \frac{4 \cdot k \cdot T}{\text{gml}} + 2 \cdot q \cdot \text{Ig} + \frac{4 \cdot k \cdot T}{Rt} \right) \cdot BW_n + \frac{4 \cdot k \cdot T}{\text{gml}} \cdot (2 \cdot \pi \cdot C_T)^2 \cdot \frac{BW_{n2}}{3} = 7.87098 \cdot 10^{-17}
\]

\[
\text{noise}_2 = 7.87 \cdot 10^{-17}
\]

Noise calculations double check (output noise power, mid-band TI)

\[
V_{\text{bg}} = \frac{4 \cdot k \cdot T}{Rt} \cdot \left( \frac{1}{2 \cdot \pi} \right) \cdot \int_0^{10^1} \left( |Z(\omega)| \right)^2 \text{d}\omega = 6.22653126183735 \cdot 10^{-17}
\]

\[
V_{\text{lg}} = 2 \cdot q \cdot \text{Ig} \cdot \left( \frac{1}{2 \cdot \pi} \right) \cdot \int_0^{10^1} \left( |Z(\omega)| \right)^2 \text{d}\omega = 1.884955084376 \cdot 10^{-19}
\]

\[
I_{\text{channel11}} = \left( \frac{1}{Rt^2} \cdot \frac{4 \cdot k \cdot T}{\text{gml}} \right) \cdot \left( \frac{1}{2 \cdot \pi} \right) \cdot \int_0^{10^1} \left( |Z(\omega)| \right)^2 \text{d}\omega
\]

\[
I_{\text{channel12}} = \frac{4 \cdot k \cdot T}{\text{gml}} \cdot (2 \cdot \pi \cdot C_T)^2 \cdot \int_0^{10^1} \left( |Z(2 \cdot \pi \cdot f)| \right)^2 \text{d}f
\]

\[
V_{\text{channel}} = I_{\text{channel11}} + I_{\text{channel12}}
\]
\[ I_{\text{rms}} := V_{\text{RF}} + V_{n} + V_{\text{channel}} = 7.870978 \times 10^{-17} \]

\[ \text{noise}_2 = 7.870978 \times 10^{-17} \]

\[ \text{noise} = 7.871551 \times 10^{-17} \]

**Rx response**

\[ I_n(\omega) := \left( \frac{1}{Rt^2} \cdot \frac{4 \cdot k \cdot T}{gm1} + 2 \cdot q \cdot Ig + 4 \cdot k \cdot T \right) + 4 \cdot k \cdot T \cdot \left(2 \cdot \pi \cdot C_T \right)^2 \cdot \omega^2 \]

\[ I_{nRF}(\omega) := \frac{4 \cdot k \cdot T}{Rt} \quad I_{nBG}(\omega) := 2 \cdot q \cdot Ig \]

\[ I_{nChannel}(\omega) := \left( \frac{1}{Rt^2} \cdot \frac{4 \cdot k \cdot T}{gm1} \right) + 4 \cdot k \cdot T \cdot \left(2 \cdot \pi \cdot C_T \right)^2 \cdot \omega^2 \]
\[ BW1(\omega) = Z(\omega) \]
A.5 Noise modelling (3rd order Butterworth pre-detection filter)

Noise model verification (non-tuned Front-End receiver/3rd order Butterworth)

**Bit-rate, pulse duration**

\[ C_T := 1.5 \cdot 10^{-12} \]  
Total capacitance

\[ B := 100 \cdot 10^6 \]  
Bit-rate

\[ f_c := 0.75 \cdot B = 7.5 \cdot 10^7 \]  
cut-off frequency TIA

\[ t_s := \frac{1}{B} \]

\[ f_B := 0.75 \cdot B \]  
cut-off frequency LPF

\[ \omega_B := 2 \cdot \pi \cdot f_B = 4.712 \cdot 10^8 \]

**pre-dec filter**

\[
H_{\text{but}}(\omega) := \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3}
\]

**TIA**

\[ Av := 10 \]  
gain

\[ \omega_c := 2 \cdot \pi \cdot f_c = 4.712 \cdot 10^8 \]  
cut-off

\[
R_f := \frac{Av + 1}{\omega_c \cdot C_T} = 1.556 \cdot 10^4
\]
Feedback

\[ Z_{\text{TIA}}(\omega) := \frac{1}{1 + j \frac{\omega}{\omega_c}} \]

**Receiver freq-response**

\[ Z(\omega) := H_{\text{but}}(\omega) \cdot Z_{\text{TIA}}(\omega) \]  
Frequency dependence of NON-TUNED TI

\[ \omega := 0.1 \cdot 10^6, 1 \cdot 10^6 .. 1 \cdot 10^{11} \]

\[ |H_{\text{but}}(\omega)| = 0.707 \]

\[ |Z_{\text{TIA}}(\omega_c)| = 0.707 \]

\[ |Z(2 \cdot \pi \cdot 0.483 \cdot B)| = 0.812 \]

\[ \omega_T := \sqrt{\frac{1}{\omega_B^2 + \frac{1}{\omega_c^2}}} \]

\[ \omega_{3\text{dB}} := 0.483 \]
FET parameters

\[ Ig := 10 \cdot 10^{-9} \quad gm := 30 \cdot 10^{-3} = 0.03 \quad noise \_ factor := 1 \]

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \]

Noise equivalent bandwidth

\[ Beq := \frac{1}{2 \cdot \pi} \int_0^{10^{12}} \left( \frac{1}{2} \left( 1 + 1 \cdot \frac{T \cdot g m \cdot \omega}{R t} \right) \cdot \left| Z(\omega) \right| \right)^2 d \omega = 5.89 \cdot 10^7 \]

\[ \frac{Beq}{B} = 0.589 \quad \frac{0.589}{\omega_{id}} = 1.219 \]

Noise calculations (proposed method)

\[ Beqc := \frac{1}{2 \cdot \pi} \int_0^{10^{12}} \left( \frac{1}{2} \left( 1 + 1 \cdot \frac{T \cdot g m \cdot \omega}{R t} \right) \cdot \left| Z(\omega) \right| \right)^2 d \omega = 1.117 \cdot 10^4 \]

\[ noise := \left( 2 \cdot q \cdot Ig + 4 \cdot k \cdot T \cdot \frac{Beq}{R t} \right) + 4 \cdot k \cdot T \cdot g m \cdot Beq = 6.79665200566981 \cdot 10^{-17} \]

Noise calculations Conventional + check

\[ NEB := \frac{1}{2 \cdot \pi} \int_0^{10^{12}} \left( \left| Z(\omega) \right| \right)^2 d \omega = 5.89 \cdot 10^7 \]

\[ Test \_ BW_ n := \frac{1}{2 \cdot \pi} \int_0^{2 \cdot \pi \cdot 0.7 \cdot B} \left( \left| Z(\omega) \right| \right)^2 d \omega = 5.329 \cdot 10^7 \]

\[ BW_{n2} := \frac{1}{2 \cdot \pi} \int_0^{10^{11}} \left( \left| Z(\omega) \right| \right)^2 d \omega = 5.89 \cdot 10^7 \quad \text{This should be the right integration bound} \]

\[ Test \_ BW_{n2} := 3 \cdot \int_0^{10^{11}} \left( \left| Z(\omega) \right| \right)^2 (\omega)^2 d \omega = 1.982 \cdot 10^{23} \]

\[ BW_{n2} := 3 \cdot \int_0^{10^{11}} \left( \left| Z(\omega) \right| \right)^2 (\omega)^2 d \omega = 3.313 \cdot 10^{23} \]

\[ Test \_ I_{nag} := Test \_ BW_ n \cdot \frac{4 \cdot k \cdot T}{R t} = 5.63256044883055 \cdot 10^{-17} \]

\[ Test \_ I_{ng} := Test \_ BW_ n \cdot (2 \cdot q \cdot Ig) = 1.70514294366491 \cdot 10^{-19} \]

\[ Test \_ I_{nchannel} := Test \_ BW_ n \cdot \left( \frac{1}{R t^2} \cdot \frac{4 \cdot k \cdot T}{g m} \right) = 1.20649162683726 \cdot 10^{-19} \]

\[ Test \_ I_{nchannels} := Test \_ BW_{n2} \cdot \left( \frac{4 \cdot k \cdot T}{3 \cdot g m} \cdot \left( \frac{2 \cdot \pi \cdot C T}{g m} \right)^2 \right) = 3.21725558861275 \cdot 10^{-18} \]

A-44
Test I_{channel} + Test I_{channel2} = 3.33790475129648 \times 10^{-18}

Test I_{nbg} + Test I_{nlg} + Test I_{nchannel2} + Test I_{channel} = 5.98340235339685 \times 10^{-17}

Test noise2 = \left( \frac{1}{R^2} \cdot \frac{4 \cdot k \cdot T}{gm1} + 2 \cdot q \cdot Ig + \frac{4 \cdot k \cdot T}{Rt} \right) \cdot \text{Test BW}_{n} \cdot \frac{\text{Test BW}_{n2}}{3} = 59.834\text{E-018}

\text{Test noise2} = 5.98340235339685 \times 10^{-17}

\text{Correct noise calculations input}

I_{nbg} = \text{BW}_{n} \cdot \frac{4 \cdot k \cdot T}{Rt} = 6.22653153883211 \times 10^{-17}

I_{nlg} = \text{BW}_{n} \cdot (2 \cdot q \cdot Ig) = 1.88495559229213 \times 10^{-19}

I_{nchannel} = \text{BW}_{n} \cdot \left( \frac{1}{R^2} \cdot \frac{4 \cdot k \cdot T}{gm1} \right) = 1.33371993680046 \times 10^{-19}

I_{nchannel2} = \text{BW}_{n} \cdot \left( \frac{4 \cdot k \cdot T}{3 \cdot gm1} \cdot (2 \cdot \pi \cdot C_T)^2 \right) = 5.37933707803375 \times 10^{-18}

\frac{I_{nchannel} + I_{nchannel2}}{4 \cdot k \cdot T \cdot gm1 \cdot Beqc} = 1

noise2 = \left( \frac{1}{R^2} \cdot \frac{4 \cdot k \cdot T}{gm1} + 2 \cdot q \cdot Ig + \frac{4 \cdot k \cdot T}{Rt} \right) \cdot \text{BW}_{n} + \frac{4 \cdot k \cdot T}{gm1} \cdot (2 \cdot \pi \cdot C_T)^2 \cdot \frac{\text{BW}_{n2}}{3} = 6.79665 \times 10^{-12}

\text{noise2} = 6.8 \times 10^{-17}

\text{noise} = 6.8 \times 10^{-17}

\text{noise calculations double check (output noise power, mid-band TI)}

V_{nbg} = \frac{4 \cdot k \cdot T}{Rt} \cdot \frac{1}{2 \cdot \pi} \int_{0}^{10^{11}} \left( \frac{\left| Z(\omega) \right|}{Z(\omega)} \right)^2 d\omega = 6.22653153883211 \times 10^{-17}

V_{nlg} = 2 \cdot q \cdot Ig \cdot \frac{1}{2 \cdot \pi} \int_{0}^{10^{11}} \left( \frac{\left| Z(\omega) \right|}{Z(\omega)} \right)^2 d\omega = 1.88495559229213 \times 10^{-19}

I_{nChannel11} = \left( \frac{1}{R^2} \cdot \frac{4 \cdot k \cdot T}{gm1} \right) \cdot \frac{1}{2 \cdot \pi} \int_{0}^{10^{11}} \left( \frac{\left| Z(\omega) \right|}{Z(\omega)} \right)^2 d\omega

I_{nChannel22} = \frac{4 \cdot k \cdot T}{gm1} \cdot (2 \cdot \pi \cdot C_T)^2 \cdot \int_{0}^{10^{11}} \left( \frac{\left| Z(\omega) \right|}{Z(\omega)} \right)^2 d\omega

V_{nchannel} = I_{nChannel11} + I_{nChannel22}
\[ I_{\text{rms}} := V_{\text{alf}} + V_{\text{nlg}} + V_{\text{channel}} = 6.796652 \times 10^{-17} \]

\[ \text{noise}_2 = 6.796652 \times 10^{-17} \]

\[ \text{noise} = 6.796652 \times 10^{-17} \]

**Rx response**

**NOISE PSD**

\[ I_n(\omega) := \left( \frac{1}{Rt^2} \cdot \frac{4 \cdot k \cdot T}{gm^l} + 2 \cdot q \cdot Ig + \frac{4 \cdot k \cdot T}{Rt} \right) + \frac{4 \cdot k \cdot T}{gm^l} \cdot (2 \cdot \pi \cdot C_T)^2 \cdot \omega^2 \]

\[ I_{\text{nalg}}(\omega) := \frac{4 \cdot k \cdot T}{Rt} \]

\[ I_{\text{nalg}}(\omega) := 2 \cdot q \cdot Ig \]

\[ I_{\text{channel}}(\omega) := \left( \frac{1}{Rt^2} \cdot \frac{4 \cdot k \cdot T}{gm^l} + \frac{4 \cdot k \cdot T}{gm^l} \cdot (2 \cdot \pi \cdot C_T)^2 \cdot \omega^2 \right) \]

**Diagram**
\( BW1(\omega) = Z(\omega) \)

\[ \begin{align*}
I_n(\omega) \\
I_n(\omega) \cdot (|BW1(\omega)|)^2 \\
Rt^2 \cdot I_n(\omega) \cdot (|BW1(\omega)|)^2 \\
\left( (|BW1(\omega)|)^2 \right)
\end{align*} \]
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B.1 Receiver performance (General)

B.1.1 Non-tuned front-end receiver

(Non-tuned front-end receiver/1st and 3rd order LPF- 100 Mbit/s)

Non-tuned front-end Rx performance with PIN-BJT PIN-FET APD-BJT APD-FET input configurations. 1st order LPF and 3rd order LPF pre-detection filters. Calculations are as follow (variable bit-rate)

- Frequency response
- Noise integrals
- Total noise
- Pulse shaping, peak voltage, ISI
- Error bit rate, minimum number of photons, receiver sensitivity
- APD noise, APD BER, APD receiver sensitivity

**Bit-rate, pulse duration**

\( C_T = 1.5 \cdot 10^{-12} \)

**Pre-dec filter**

\[ f_B(B) := 0.75 \cdot B \]

\[ \omega_B(B) := 2 \cdot \pi \cdot f_B(B) \]

\[ H_{bat}(\omega, B) := \frac{1}{1 + 1j \cdot \omega \overline{\omega_B(B)}} \]

\[ H_{bat3}(\omega, B) := \frac{\omega_B(B)^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B(B) + 2 \cdot (1j \cdot \omega) \cdot \omega_B(B)^2 + \omega_B(B)^3} \]

**TIA**

\[ f_c(B) := 0.75 \cdot B \]

\[ A_v := 10 \]

\[ \omega_c(B) := 2 \cdot \pi \cdot f_c(B) \]

\[ R_t(B) := \frac{A_v + 1}{2 \cdot \pi \cdot f_c(B) \cdot C_T} \]

\[ Z_{TIA}(\omega, B) := \frac{1}{1 + 1j \cdot \omega \overline{\omega_c(B)}} \]

**Receiver freq-response**

\[ Z1(\omega, B) := H_{bat}(\omega, B) \cdot Z_{TIA}(\omega, B) \]  
Frequency dependence of NON-TUNED TI

\[ Z3(\omega, B) := H_{bat3}(\omega, B) \cdot Z_{TIA}(\omega, B) \]

\[ \omega = 1 \cdot 10^6, 10 \cdot 10^6, \ldots, 1 \cdot 10^{11} \]

\[ B = 10 \cdot 10^6, 20 \cdot 10^6, \ldots, 10000 \cdot 10^6 \]
Noise equivalent bandwidth

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \]

\[ NEB1(B) := \frac{1}{2 \pi} \int_0^{10^{12}} \left( \left| Z1(\omega, B) \right|^2 \right) d\omega \]

\[ NEB3(B) := \frac{1}{2 \pi} \int_0^{10^{11}} \left( \left| Z3(\omega, B) \right|^2 \right) d\omega \]

Rx 3-dB check (for all bit rates)

\[
\left| Z1 \left( 2 \pi \cdot 0.483 \cdot B, B \right) \right| = \begin{bmatrix}
0.707 \\
0.707 \\
0.707 \\
0.707 \\
0.707 \\
\vdots
\end{bmatrix}
\]

\[
\left| Z3 \left( 2 \pi \cdot 0.59 \cdot B, B \right) \right| = \begin{bmatrix}
0.707 \\
0.707 \\
0.707 \\
0.707 \\
0.707 \\
\vdots
\end{bmatrix}
\]
Rf noise

\[ I_{nRf}(B) = \frac{4 \cdot k \cdot T}{R_t(B)} \]

\[ I_{nRf}(B) = I_{nRf}(B) \cdot NEB1(B) \]

\[ I_{nRf}(B) = I_{nRf}(B) \cdot NEB3(B) \]

FET-input stage

\[ Ig := 10 \cdot 10^{-9} \quad \text{gm}l := 30 \cdot 10^{-3} = 0.03 \quad \text{noise factor} := 1 \]

Ig noise \( (\text{gate current}) \)

\[ I_{nG}(B) := 2 \cdot q \cdot Ig \]

\[ I_{nG1}(B) := I_{nG}(B) \cdot NEB1(B) \]

\[ I_{nG3}(B) := I_{nG}(B) \cdot NEB3(B) \]

\[ Beq1(B) := \frac{1}{2 \cdot \pi} \int_{0}^{10^{11}} \left( \left| 1 + j \cdot (\omega) \cdot R_t(B) \cdot C_t \cdot |Z1(\omega, B)| \right| \right)^2 d\omega \]

\[ Beq3(B) := \frac{1}{2 \cdot \pi} \int_{0}^{10^{11}} \left( \left| 1 + j \cdot (\omega) \cdot R_t(B) \cdot C_t \cdot |Z3(\omega, B)| \right| \right)^2 d\omega \]

Channel noise \( (\text{Gate-source}) \)

\[ I_{nC}(B) = 4 \cdot k \cdot T \cdot \frac{1}{\text{gm}l} \cdot Beq1(B) \]
Total noise - FET

\[ I_{no3}(B) := 4 \cdot k \cdot T \cdot \frac{1}{gml} \cdot Beqc3(B) \]

\[ I_{no1}(B) \]

\[ I_{no3}(B) \]

\[ noise1(B) := \left( 2 \cdot q \cdot Ig + \frac{4 \cdot k \cdot T}{Rt(B)} \right) \cdot NEB1(B) + 4 \cdot k \cdot T \cdot \frac{1}{gml} \cdot Beqc1(B) \]

\[ noise3(B) := \left( 2 \cdot q \cdot Ig + \frac{4 \cdot k \cdot T}{Rt(B)} \right) \cdot NEB3(B) + 4 \cdot k \cdot T \cdot \frac{1}{gml} \cdot Beqc3(B) \]
**BJT input stage**

\[ I_c := 2 \cdot 10^{-3} \quad hfe := 100 \quad I_b := \frac{I_c}{hfe} \quad gm_{BJT} := \frac{I_c}{(25 \cdot 10^{-3})} = 0.08 \]

**Ib noise (base)**

\[ I_{nib} (B) := 2 \cdot q \cdot I_b \]

\[ I_{nib1} (B) := I_{nib} (B) \cdot NEB1 (B) \]

\[ I_{nib3} (B) := I_{nib} (B) \cdot NEB3 (B) \]

**Ic noise (collector)**

\[ I_{nc1} (B) := \frac{2 \cdot q \cdot I_c}{gm_{BJT}^2} \cdot Beqc1 (B) \]

\[ I_{nc3} (B) := \frac{2 \cdot q \cdot I_c}{gm_{BJT}^2} \cdot Beqc3 (B) \]

**Total noise-BJT**

\[ noise_{1st\_BJT} (B) := \left( 2 \cdot q \cdot I_b + \frac{4 \cdot k \cdot T}{R_t (B)} \right) \cdot NEB1 (B) + 2 \cdot q \cdot \frac{I_c}{gm_{BJT}^2} \cdot Beqc1 (B) \]

\[ noise_{3rd\_BJT} (B) := \left( 2 \cdot q \cdot I_b + \frac{4 \cdot k \cdot T}{R_t (B)} \right) \cdot NEB3 (B) + 2 \cdot q \cdot \frac{I_c}{gm_{BJT}^2} \cdot Beqc3 (B) \]

**FET vs BJT**
noise-opt BJT (Ic re-calculations)

\[
\text{noise\_1st\_BJT\_opt}(B,x) = \left(2 \cdot q \cdot \frac{x}{hfe} + \frac{4 \cdot k \cdot T}{R_t(B)}\right) \cdot \text{NEB1}(B) + 2 \cdot q \cdot \frac{x}{\left(25 \cdot 10^{-3}\right)^2} \cdot \text{Beqc1}(B)
\]

\[
\text{noise\_3rd\_BJT\_opt}(B,x) = \left(2 \cdot q \cdot \frac{x}{hfe} + \frac{4 \cdot k \cdot T}{R_t(B)}\right) \cdot \text{NEB3}(B) + 2 \cdot q \cdot \frac{x}{\left(25 \cdot 10^{-3}\right)^2} \cdot \text{Beqc3}(B)
\]

\[
x := 0.00001, 0.00002 \ldots 0.01
\]

\[
\text{noise\_1st\_BJT\_opt}(100 \cdot 10^6, 2 \cdot 10^{-3}) = 4.422 \cdot 10^{-16} \quad \text{noise\_1st\_BJT}(100 \cdot 10^6) = 4.422 \cdot 10^{-16}
\]

\[
\text{noise\_3rd\_BJT\_opt}(100 \cdot 10^6, 2 \cdot 10^{-3}) = 4.403 \cdot 10^{-16}
\]

\[
I_{\text{opt1}}(B) := 25 \cdot 10^{-3} \cdot \sqrt{\frac{hfe}{\text{NEB1}(B)}}
\]

\[
I_{\text{opt3}}(B) := 25 \cdot 10^{-3} \cdot \sqrt{\frac{hfe}{\text{NEB3}(B)}}
\]

\[
I_{\text{opt1}}(100 \cdot 10^6) = 1.774 \cdot 10^{-4}
\]

\[
I_{\text{opt3}}(100 \cdot 10^6) = 1.033 \cdot 10^{-4}
\]

\[
\text{noise\_1st\_BJT\_opt}(100 \cdot 10^6, 1.774 \cdot 10^{-4}) = 1.291 \cdot 10^{-16}
\]

\[
\text{noise\_3rd\_BJT\_opt}(100 \cdot 10^6, 1.033 \cdot 10^{-4}) = 1.012 \cdot 10^{-16}
\]

\[
\text{test1}(x) := \text{noise\_1st\_BJT\_opt}(100 \cdot 10^6, x)
\]

\[
\text{test2}(x) := \text{noise\_3rd\_BJT\_opt}(100 \cdot 10^6, x)
\]
FET vs BJT-opt

\[
\text{noise}_1\text{st\_BJT\_optx}(B) = \left\{ \begin{array}{l}
2 \cdot q \cdot \frac{25 \cdot 10^{-3} \cdot \sqrt{hfe \cdot \sqrt{\frac{\text{Beqc1}(B)}{\text{NEB1}(B)}} + 4 \cdot k \cdot T}}{\frac{\text{Rt}(B)}{\text{R}_t(B)}} \cdot \text{NEB1}(B) \\
+ 2 \cdot q \cdot \frac{25 \cdot 10^{-3} \cdot \sqrt{hfe \cdot \sqrt{\frac{\text{Beqc1}(B)}{\text{NEB1}(B)}}}}{\frac{25 \cdot 10^{-3}}{}} \cdot \text{Beqc1}(B) \\
\end{array} \right\
\]

\[
\text{noise}_3\text{rd\_BJT\_optx}(B) = \left\{ \begin{array}{l}
2 \cdot q \cdot \frac{25 \cdot 10^{-3} \cdot \sqrt{hfe \cdot \sqrt{\frac{\text{Beqc3}(B)}{\text{NEB3}(B)}} + 4 \cdot k \cdot T}}{\frac{\text{Rt}(B)}{\text{R}_t(B)}} \cdot \text{NEB3}(B) \\
+ 2 \cdot q \cdot \frac{25 \cdot 10^{-3} \cdot \sqrt{hfe \cdot \sqrt{\frac{\text{Beqc3}(B)}{\text{NEB3}(B)}}}}{\frac{25 \cdot 10^{-3}}{}} \cdot \text{Beqc3}(B) \\
\end{array} \right\
\]

Testnoise1(B) := noise_1st_BJT_optx(B)
Testnoise2(B) := noise_3rd_BJT_optx(B)
output pulse shape

\[ t := -6 \cdot t_s, -5.99 \cdot t_s, \ldots, 6 \cdot t_s \]

\[ vout1(t, B) := \frac{t_s}{\pi} \cdot \frac{1}{n} \cdot 10^7 \cdot \frac{1}{n} \cdot 10^7 \cdot \sin \left( \frac{\omega \cdot t_s}{2} \right) \cdot \text{Re} \left( \frac{1}{1 + l_j \cdot \frac{\omega}{\omega_c(B)}} \cdot \frac{1}{1 + l_j \cdot \frac{\omega}{\omega_c(B)}} \cdot \exp \left( i \cdot \omega \cdot \left( t - \frac{t_s}{2} \right) \right) \right) \cdot d\omega \]

\[ vout3(t, B) := \frac{t_s}{\pi} \cdot \frac{1}{n} \cdot 10^7 \cdot \sin \left( \frac{\omega \cdot t_s}{2} \right) \cdot \text{Re} \left( H_{bus1}(\omega, B) \cdot Z_{TLA}(\omega, B) \cdot \exp \left( i \cdot \omega \cdot \left( t - \frac{t_s}{2} \right) \right) \right) \cdot d\omega \]

peak voltage and time

\[ xvout1(t) := \frac{t_s^2}{\pi} \cdot \frac{1}{n} \cdot 10^7 \cdot 0 \cdot 10^7 \cdot \sin \left( \frac{\omega \cdot t_s}{2} \right) \cdot \text{Re} \left( i \cdot \omega \cdot H_{bus1}(\omega, \frac{1}{t_s}) \cdot Z_{TLA}(\omega, \frac{1}{t_s}) \cdot \exp \left( i \cdot \omega \cdot \left( t - \frac{t_s}{2} \right) \right) \right) \cdot d\omega \]

\[ xvout3(t) := \frac{t_s^2}{\pi} \cdot \frac{1}{n} \cdot 10^7 \cdot 0 \cdot 10^7 \cdot \sin \left( \frac{\omega \cdot t_s}{2} \right) \cdot \text{Re} \left( i \cdot \omega \cdot H_{bus3}(\omega, \frac{1}{t_s}) \cdot Z_{TLA}(\omega, \frac{1}{t_s}) \cdot \exp \left( i \cdot \omega \cdot \left( t - \frac{t_s}{2} \right) \right) \right) \cdot d\omega \]
\( t \equiv 0.4 \cdot ts \)
\( t_{pk1} := \text{root}(xvout1(t), t) \)
\( \frac{t_{pk1}}{ts} = 1.009 \)
\( v_{max1} := xvout1\left(t_{pk1}, \frac{1}{ts}\right) = 0.949 \)

\( v_{max2} := xvout1\left(t_{pk1} + 1 \cdot ts, \frac{1}{ts}\right) = 0.049 \)
\( v_{max3} := xvout1\left(t_{pk1} + 2 \cdot ts, \frac{1}{ts}\right) = 7.965 \cdot 10^{-4} \)
\( v_{min1} := xvout1\left(t_{pk1} + 3 \cdot ts, \frac{1}{ts}\right) = 9.234 \cdot 10^{-6} \)

\( t := -6 \cdot ts, -5.99 \cdot ts, \ldots, 6 \cdot ts \)

\( t := 0.9 \cdot ts \)
\( t_{pk3} := \text{root}(xvout3(t), t) \)
\( \frac{t_{pk3}}{ts} = 1.182 \)
\( v_{max3} := xvout3\left(t_{pk3}, \frac{1}{ts}\right) = 1 \)
\( v_{min1} := xvout3\left(t_{pk3} + 2 \cdot ts, \frac{1}{ts}\right) = 0.008 \)
\( v_{min2} := xvout3\left(t_{pk3} + 2 \cdot ts, \frac{1}{ts}\right) = 0.008 \)
\( v_{min3} := xvout3\left(t_{pk3} + 3 \cdot ts, \frac{1}{ts}\right) = -3.531 \cdot 10^{-4} \)
BER

\[ nq \approx 1.6 \cdot 10^{-10} \quad Test_{b_{\text{photons}}} = 1100, 1200, \ldots, 12000 \]

\[ test_{Q1}(Test_{b_{\text{photons}}}, B) := \frac{1}{ts} \cdot Test_{b_{\text{photons}}} \cdot nq \cdot \left( \frac{v_{\text{max1}} - v_{\text{min1}}}{2} \right) \]

\[ \sqrt{\text{noise}_{1st\_BJT\\_optx}(B)} \]

\[ test_{Q2}(Test_{b_{\text{photons}}}, B) := \frac{1}{ts} \cdot Test_{b_{\text{photons}}} \cdot nq \cdot \left( \frac{v_{\text{max3}} - v_{\text{min3}}}{2} \right) \]

\[ \sqrt{\text{noise}_{3rd\_BJT\\_optx}(B)} \]

\[ test_{Q3}(Test_{b_{\text{photons}}}, B) := \frac{1}{ts} \cdot Test_{b_{\text{photons}}} \cdot nq \cdot \left( \frac{v_{\text{max1}} - v_{\text{min1}}}{2} \right) \]

\[ \sqrt{\text{noise1}(B)} \]

\[ test_{Q4}(Test_{b_{\text{photons}}}, B) := \frac{1}{ts} \cdot Test_{b_{\text{photons}}} \cdot nq \cdot \left( \frac{v_{\text{max3}} - v_{\text{min3}}}{2} \right) \]

\[ \sqrt{\text{noise3}(B)} \]

\[ Test1(Test_{b_{\text{photons}}}, B) := \frac{1}{2} \cdot \text{erfc} \left( \frac{test_{Q1}(Test_{b_{\text{photons}}}, \frac{1}{ts})}{\sqrt{2}} \right) \]

\[ Test2(Test_{b_{\text{photons}}}, B) := \frac{1}{2} \cdot \text{erfc} \left( \frac{test_{Q2}(Test_{b_{\text{photons}}}, \frac{1}{ts})}{\sqrt{2}} \right) \]

\[ Test3(Test_{b_{\text{photons}}}, B) := \frac{1}{2} \cdot \text{erfc} \left( \frac{test_{Q3}(Test_{b_{\text{photons}}}, \frac{1}{ts})}{\sqrt{2}} \right) \]

\[ Test4(Test_{b_{\text{photons}}}, B) := \frac{1}{2} \cdot \text{erfc} \left( \frac{test_{Q4}(Test_{b_{\text{photons}}}, \frac{1}{ts})}{\sqrt{2}} \right) \]
BER PIN-BJT and PIN-FET (PIN noise is neglected)

\[ xQ_{1\text{BJT}_{\text{1st}}}(b, B) = \frac{1}{ts} \cdot b \cdot nq \cdot \frac{v_{\text{max}1} - v_{\text{min}1}}{2} \sqrt{\text{noise}_{\text{1st BJTOptx}}(B)} \]

\[ xQ_{2\text{BJT}_{\text{3rd}}}(b, B) = \frac{1}{ts} \cdot b \cdot nq \cdot \frac{v_{\text{max}3} - v_{\text{min}3}}{2} \sqrt{\text{noise}_{\text{3rd BJTOptx}}(B)} \]

\[ xQ_{1\text{FET}_{\text{1st}}}(b, B) = \frac{1}{ts} \cdot b \cdot nq \cdot \frac{v_{\text{max}1} - v_{\text{min}1}}{2} \sqrt{\text{noise1}(B)} \]

\[ xQ_{3\text{FET}_{\text{3rd}}}(b, B) = \frac{1}{ts} \cdot b \cdot nq \cdot \frac{v_{\text{max}3} - v_{\text{min}3}}{2} \sqrt{\text{noise3}(B)} \]

\[ xP_{\text{eb}_{\text{BJT}_{\text{1st}}}}(b) = \frac{1}{2} \cdot \text{erfc} \left( \frac{xQ_{1\text{BJT}_{\text{1st}}}(b, 1/\text{ts})}{\sqrt{2}} \right) \]

\[ xP_{\text{eb}_{\text{BJT}_{\text{3rd}}}}(b) = \frac{1}{2} \cdot \text{erfc} \left( \frac{xQ_{2\text{BJT}_{\text{3rd}}}(b, 1/\text{ts})}{\sqrt{2}} \right) \]

\[ xP_{\text{eb}_{\text{FET}_{\text{1st}}}}(b) = \frac{1}{2} \cdot \text{erfc} \left( \frac{xQ_{1\text{FET}_{\text{1st}}}(b, 1/\text{ts})}{\sqrt{2}} \right) \]

\[ xP_{\text{eb}_{\text{FET}_{\text{3rd}}}}(b) = \frac{1}{2} \cdot \text{erfc} \left( \frac{xQ_{3\text{FET}_{\text{3rd}}}(b, 1/\text{ts})}{\sqrt{2}} \right) \]
\[ p_{\text{B}15} = \log (xP_{eb,\text{B15}}(b)) + 9 \]
\[ p_{\text{B}3} = \log (xP_{eb,\text{B3}}(b)) + 9 \]
\[ p_{\text{F}1} = \log (xP_{eb,\text{F1}}(b)) + 9 \]
\[ p_{\text{F}3} = \log (xP_{eb,\text{F3}}(b)) + 9 \]

\[ b \equiv 3 \cdot 10^3 \]
\[ \text{BJT}_{-1} := \text{root} \left( p_{\text{BJT1}}(b), b \right) \]
\[ \text{BJT}_{-3} := \text{root} \left( p_{\text{BJT3}}(b), b \right) \]
\[ \text{FET}_{-1} := \text{root} \left( p_{\text{FET1}}(b), b \right) \]
\[ \text{FET}_{-3} := \text{root} \left( p_{\text{FET3}}(b), b \right) \]

\[ \text{minimum}_{\text{BJT1}} := \min \left( \text{BJT}_{-1} \right) = 9.46 \cdot 10^3 \]
\[ \text{minimum}_{\text{BJT3}} := \min \left( \text{BJT}_{-3} \right) = 7.599 \cdot 10^3 \]
\[ \text{minimum}_{\text{FET1}} := \min \left( \text{FET}_{-1} \right) = 7.386 \cdot 10^3 \]
\[ \text{minimum}_{\text{FET3}} := \min \left( \text{FET}_{-3} \right) = 6.227 \cdot 10^3 \]

\[ xQ_{\text{BJT1}} \left( \frac{1}{ts} \right) = 5.998 \]
\[ xQ_{\text{BJT3}} \left( \frac{1}{ts} \right) = 5.998 \]
\[ xQ_{\text{FET1}} \left( \frac{1}{ts} \right) = 5.998 \]
\[ xQ_{\text{FET3}} \left( \frac{1}{ts} \right) = 5.998 \]

**Sens PIN-BJT and PIN-FET**

\[ \lambda := 650 \cdot 10^{-9} \]

\[ \text{photon \_energy} = \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\( ts \) is defined as a global value (bit-rate, cut-off, NEB and noise will be controlled by \( ts \))

\[ ts \equiv \frac{1}{100 \cdot 10^6} \]
PIN_{BJT\_1\_dBm} := 10 \cdot \log \left( \frac{\text{minimum}_{BJT\_1}}{2} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \frac{1}{\text{ts}} \right) = -38.394

PIN_{BJT\_3\_dBm} := 10 \cdot \log \left( \frac{\text{minimum}_{BJT\_3}}{2} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \frac{1}{\text{ts}} \right) = -39.346

PIN_{FET\_1\_dBm} := 10 \cdot \log \left( \frac{\text{minimum}_{FET\_1}}{2} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \frac{1}{\text{ts}} \right) = -39.469

PIN_{FET\_3\_dBm} := 10 \cdot \log \left( \frac{\text{minimum}_{FET\_3}}{2} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \frac{1}{\text{ts}} \right) = -40.21

**APD noise**

\[ M_{APD} := 10 \]

\[ F_M = 5.5 \]

\[ APD\_1 (b) := \frac{1}{\text{ts}} \cdot b \cdot nq \cdot (v_{\text{max}1}) \]

\[ APD\_3 (b) := \frac{1}{\text{ts}} \cdot b \cdot nq \cdot (v_{\text{max}3}) \]

\[ I_{APD\_d} := 10 \cdot 10^{-9} \]

\[ \text{Noise}_{APD\_1} (b, B) := \left[ 2 \cdot q \cdot APD\_1 (b) \cdot M_{APD} \cdot F_M \cdot \frac{1}{\text{ts}} \right] \cdot \text{NEB1} \left( \frac{1}{\text{ts}} \right) \]

\[ \text{Noise}_{APD\_3} (b, B) := \left[ 2 \cdot q \cdot APD\_3 (b) \cdot M_{APD} \cdot F_M \cdot \frac{1}{\text{ts}} \right] \cdot \text{NEB3} \left( \frac{1}{\text{ts}} \right) \]

**BER APD-BJT and APD-FET**

\[ Q_{APD\_BJT\_1} (b, B) := \frac{M_{APD} \cdot b \cdot nq \cdot \left( v_{\text{max}1} - v_{\text{min}1} \right)}{\sqrt{\text{noise\_1st\_BJT\_optx}(B) + \text{Noise}_{APD\_1} (b, B)}} \]

\[ Q_{APD\_BJT\_3} (b, B) := \frac{M_{APD} \cdot b \cdot nq \cdot \left( v_{\text{max}3} - v_{\text{min}3} \right)}{\sqrt{\text{noise\_3rd\_BJT\_optx}(B) + \text{Noise}_{APD\_3} (b, B)}} \]

\[ Q_{APD\_FET\_1} (b, B) := \frac{M_{APD} \cdot b \cdot nq \cdot \left( v_{\text{max}1} - v_{\text{min}1} \right)}{\sqrt{\text{noise1}(B) + \text{Noise}_{APD\_1} (b, B)}} \]

\[ Q_{APD\_FET\_3} (b, B) := \frac{M_{APD} \cdot b \cdot nq \cdot \left( v_{\text{max}3} - v_{\text{min}3} \right)}{\sqrt{\text{noise3}(B) + \text{Noise}_{APD\_3} (b, B)}} \]
\[ P_{e,\text{APD}_{\text{BJT}_1}}(b) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD}_{\text{BJT}_1}} \left( b, \frac{1}{ts} \right)}{\sqrt{2}} \right) \]

\[ P_{e,\text{APD}_{\text{BJT}_3}}(b) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD}_{\text{BJT}_3}} \left( b, \frac{1}{ts} \right)}{\sqrt{2}} \right) \]

\[ P_{e,\text{APD}_{\text{FET}_1}}(b) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD}_{\text{FET}_1}} \left( b, \frac{1}{ts} \right)}{\sqrt{2}} \right) \]

\[ P_{e,\text{APD}_{\text{FET}_3}}(b) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD}_{\text{FET}_3}} \left( b, \frac{1}{ts} \right)}{\sqrt{2}} \right) \]

\[ P_{c,\text{APD}_{\text{BJT}_1}}(b) := (\log(P_{e,\text{APD}_{\text{BJT}_1}}(b)) + 9) \]

\[ P_{c,\text{APD}_{\text{BJT}_3}}(b) := (\log(P_{e,\text{APD}_{\text{BJT}_3}}(b)) + 9) \]

\[ P_{c,\text{APD}_{\text{FET}_1}}(b) := (\log(P_{e,\text{APD}_{\text{FET}_1}}(b)) + 9) \]

\[ P_{c,\text{APD}_{\text{FET}_3}}(b) := (\log(P_{e,\text{APD}_{\text{FET}_3}}(b)) + 9) \]

\[ \text{APD}_{\text{BJT}_1}(b) = \text{root}(P_{c,\text{APD}_{\text{BJT}_1}}(b), b) \]

\[ \text{APD}_{\text{BJT}_3}(b) = \text{root}(P_{c,\text{APD}_{\text{BJT}_3}}(b), b) \]

\[ \text{APD}_{\text{FET}_1}(b) = \text{root}(P_{c,\text{APD}_{\text{FET}_1}}(b), b) \]

\[ \text{APD}_{\text{FET}_3}(b) = \text{root}(P_{c,\text{APD}_{\text{FET}_3}}(b), b) \]

\[ \text{minimum}_{\text{APD}_{\text{BJT}_1}} := \min(\text{APD}_{\text{BJT}_1}, b) = 1.928 \cdot 10^3 \]

\[ \text{minimum}_{\text{APD}_{\text{BJT}_3}} := \min(\text{APD}_{\text{BJT}_3}, b) = 1.653 \cdot 10^3 \]

\[ \text{minimum}_{\text{APD}_{\text{FET}_1}} := \min(\text{APD}_{\text{FET}_1}, b) = 1.795 \cdot 10^3 \]

\[ \text{minimum}_{\text{APD}_{\text{FET}_3}} := \min(\text{APD}_{\text{FET}_3}, b) = 1.57 \cdot 10^3 \]

\[ Q_{\text{APD}_{\text{BJT}_1}} \left( \text{minimum}_{\text{APD}_{\text{BJT}_1}}, \frac{1}{ts} \right) = 5.998 \]

\[ Q_{\text{APD}_{\text{BJT}_3}} \left( \text{minimum}_{\text{APD}_{\text{BJT}_3}}, \frac{1}{ts} \right) = 5.998 \]

\[ Q_{\text{APD}_{\text{FET}_1}} \left( \text{minimum}_{\text{APD}_{\text{FET}_1}}, \frac{1}{ts} \right) = 5.998 \]

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\[ Q_{\text{APD\_FET\_3}}(\text{minimum\_APD\_FET\_3}, \frac{1}{ts}) = 5.998 \]

**Sens APD-BJT and APD-FET**

\[ APD\_BJT\_dBm\_1 = 10 \cdot \log \left( \frac{\text{minimum\_APD\_BJT\_1}}{2}, \frac{\text{photon\_energy}}{10^{-3}}, \frac{1}{ts} \right) = -45.302 \]

\[ APD\_BJT\_dBm\_3 = 10 \cdot \log \left( \frac{\text{minimum\_APD\_BJT\_3}}{2}, \frac{\text{photon\_energy}}{10^{-3}}, \frac{1}{ts} \right) = -45.969 \]

\[ APD\_FET\_dBm\_1 = 10 \cdot \log \left( \frac{\text{minimum\_APD\_FET\_1}}{2}, \frac{\text{photon\_energy}}{10^{-3}}, \frac{1}{ts} \right) = -45.612 \]

\[ APD\_FET\_dBm\_3 = 10 \cdot \log \left( \frac{\text{minimum\_APD\_FET\_3}}{2}, \frac{\text{photon\_energy}}{10^{-3}}, \frac{1}{ts} \right) = -46.194 \]

\[ PIN\_BJT\_1\_dBm = -38.394 \quad APD\_BJT\_dBm\_1 = -45.302 \]

\[ PIN\_BJT\_3\_dBm = -39.346 \quad APD\_BJT\_dBm\_3 = -45.969 \]

\[ PIN\_FET\_1\_dBm = -39.469 \quad APD\_FET\_dBm\_1 = -45.612 \]

\[ PIN\_FET\_3\_dBm = -40.21 \quad APD\_FET\_dBm\_3 = -46.194 \]

**Sens APD vs PIN**

\[ PIN\_BJT\_1\_dBm - APD\_BJT\_dBm\_1 = 6.908 \]

\[ PIN\_BJT\_3\_dBm - APD\_BJT\_dBm\_3 = 6.623 \]

\[ PIN\_FET\_1\_dBm - APD\_FET\_dBm\_1 = 6.143 \]

\[ PIN\_FET\_3\_dBm - APD\_FET\_dBm\_3 = 5.983 \]

**Noise BJT**

\[ \text{noise}\_1st\_BJT\_optx\left( \frac{1}{ts} \right) + \text{Noise}\_APD\_1\left( \text{minimum\_APD\_BJT\_1}, \frac{1}{ts} \right) = 5.364 \cdot 10^{-16} \]

\[ \text{noise}\_3rd\_BJT\_optx\left( \frac{1}{ts} \right) + \text{Noise}\_APD\_3\left( \text{minimum\_APD\_BJT\_3}, \frac{1}{ts} \right) = 4.792 \cdot 10^{-16} \]

\[ \text{noise}\_1\left( \frac{1}{ts} \right) + \text{Noise}\_APD\_1\left( \text{minimum\_APD\_FET\_1}, \frac{1}{ts} \right) = 4.651 \cdot 10^{-16} \]

\[ \text{noise}\_3\left( \frac{1}{ts} \right) + \text{Noise}\_APD\_3\left( \text{minimum\_APD\_FET\_3}, \frac{1}{ts} \right) = 4.321 \cdot 10^{-16} \]

\[ \text{noise}\_1st\_BJT\_optx\left( \frac{1}{ts} \right) = 1.291 \cdot 10^{-16} \]

\[ \text{noise}\_3rd\_BJT\_optx\left( \frac{1}{ts} \right) = 1.012 \cdot 10^{-16} \]

\[ \text{noise}\_1\left( \frac{1}{ts} \right) = 7.872 \cdot 10^{-17} \]
\[ \text{noise3} \left( \frac{1}{t_s} \right) = 6.797 \cdot 10^{-17} \]

\[ 10 \log \left( \frac{\text{noise}_3 \text{rd BJT optx} \left( \frac{1}{t_s} \right)}{\text{noise}_1 \text{st BJT optx} \left( \frac{1}{t_s} \right)} \right) = -1.059 \]

\[ 10 \log \left( \frac{\text{noise}_1 \text{st BJT optx} \left( \frac{1}{t_s} \right) + \text{Noise APD}_1 \left( \text{minimum APD BJT}_1, \frac{1}{t_s} \right)}{\text{noise}_1 \text{st BJT optx} \left( \frac{1}{t_s} \right)} \right) = 6.184 \]

\[ 10 \log \left( \frac{\text{noise}_3 \text{rd BJT optx} \left( \frac{1}{t_s} \right) + \text{Noise APD}_3 \left( \text{minimum APD BJT}_3, \frac{1}{t_s} \right)}{\text{noise}_3 \text{rd BJT optx} \left( \frac{1}{t_s} \right)} \right) = 6.753 \]

\[ 10 \log \left( \frac{\text{noise}_3 \text{rd BJT optx} \left( \frac{1}{t_s} \right)}{\text{noise}_1 \left( \frac{1}{t_s} \right)} \right) = 1.091 \]

\[ 10 \log \left( \frac{\text{noise}_3 \text{rd BJT optx} \left( \frac{1}{t_s} \right)}{\text{noise}_3 \left( \frac{1}{t_s} \right)} \right) = 1.729 \]

\[ 10 \log \left( \frac{\text{noise}_1 \text{st BJT optx} \left( \frac{1}{t_s} \right)}{\text{noise}_1 \left( \frac{1}{t_s} \right)} \right) = 2.15 \]

\[ 10 \log \left( \frac{\text{noise}_1 \text{st BJT optx} \left( \frac{1}{t_s} \right)}{\text{noise}_2 \left( \frac{1}{t_s} \right)} \right) = 2.788 \]

\[ 10 \log \left( \frac{\text{noise}_3 \left( \frac{1}{t_s} \right)}{\text{noise}_1 \left( \frac{1}{t_s} \right)} \right) = -0.638 \]

\[ 10 \log \left( \frac{\text{noise}_1 \left( \frac{1}{t_s} \right) + \text{Noise APD}_1 \left( \text{minimum APD FET}_1, \frac{1}{t_s} \right)}{\text{noise}_1 \left( \frac{1}{t_s} \right)} \right) = 7.715 \]

\[ 10 \log \left( \frac{\text{noise}_3 \left( \frac{1}{t_s} \right) + \text{Noise APD}_3 \left( \text{minimum APD FET}_3, \frac{1}{t_s} \right)}{\text{noise}_3 \left( \frac{1}{t_s} \right)} \right) = 8.033 \]
B.1.2 Tuned A receiver

(Tuned A front-end receiver/1st and 3rd order LPF-100 Mbit/s)

Tuned-A front-end Rx performance with PIN-BJT PIN-FET APD-BJT APD-FET input configurations. 1st order LPF and 3rd LPF pre-detection filters. calculations are as follow

- Frequency response
- Noise integrals
- Total noise
- Pulse shaping, peak voltage, ISI
- Error bit rate, minimum number of photons, receiver sensitivity
- APD noise, APD BER, APD receiver sensitivity

**Bit-rate, pulse duration**

\[ C_T := 1.5 \cdot 10^{-12} \quad B := 100 \cdot 10^6 \]

**pre-dec filter**

\[
f_B := 0.75 \cdot B \quad A_v := 10
\]

\[ \omega_B := 2 \cdot \pi \cdot f_B \]

\[
| H_{bt} (\omega) | := \frac{1}{1 + j \frac{\omega}{\omega_B}}
\]

\[
| H_{btt} (\omega) | := \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3}
\]

**TIA**

\[ m := 1.8 \quad \text{time constant ratio of L/R and RC} \]

\[
y := \sqrt{\left( \frac{-m^2}{2} + m + 1 \right) + \sqrt{\left( \frac{-m^2}{2} + m + 1 \right)^2 + m^2}} = 1.825
\]

\[ d := \frac{1}{y} \cdot 100 \cdot 10^6 \]

\[ f_c := 0.75 \cdot d = 4.109 \cdot 10^7 \quad \text{cut-off of half the bit rate, before BWER} \]

\[ R := \frac{1}{(2 \cdot \pi \cdot C_T \cdot f_c)} = 2.582 \cdot 10^3 \]

\[ L := R^2 \cdot \frac{C_T}{m} = 5.558 \cdot 10^{-6} \]

\[ t_s := \frac{1}{y \cdot d} \quad B := \frac{1}{t_s} = 1 \cdot 10^8 \]
\[ f_c := 0.75 \times B = 7.5 \times 10^7 \]
\[ Rf := R \times (1 + Av) = 2.841 \times 10^4 \]
\[ Lf := L \times (1 + Av) = 6.114 \times 10^{-5} \]
\[ Z_{TLA}(\omega) := \left( \frac{1 + 1i \omega \times \frac{Lf}{Rf}}{\left( \left( 1 - \omega^2 \times \frac{C_T}{1 + Av} \right) + Rf \times \omega \times \frac{C_T}{1 + Av} \times 1i \right)} \right) \]

**Receiver freq-response**

\[ ZI(\omega) := H_{bat}(\omega) \times Z_{TLA}(\omega) \quad \text{Frequency dependence of NON-TUNED TI} \]

\[ Z3(\omega) := H_{bat3}(\omega) \times Z_{TLA}(\omega) \]
\[ \omega := 1 \times 10^6, 10 \times 10^6 \ldots 1 \times 10^{11} \]
\[ |Z_{TLA}(2 \times 10^6)\times 0.75 \times B)| = 0.707 \]
Noise equivalent bandwidth

\[ q = 1.6 \cdot 10^{-19} \quad k = 1.38 \cdot 10^{-23} \quad T = 298 \]

\[ NEB1 = \frac{1}{2 \cdot \pi} \int_0^{10^{12}} \left( \frac{1}{|Z1(\omega)|^2} \right) d\omega = 6.493 \cdot 10^7 \]

\[ NEB3 = \frac{1}{2 \cdot \pi} \int_0^{10^{11}} \left( \frac{1}{|Z3(\omega)|^2} \right) d\omega = 6.986 \cdot 10^7 \]

Rf noise

\[ I_{\text{Rf}} = \frac{4 \cdot k \cdot T}{Rf} \]

\[ I_{2-1} = \frac{1}{2 \cdot \pi} \int_0^{10^{12}} \left( \frac{1}{|1 + \frac{1}{\omega \cdot Z1(\omega) Rf}|^2} \right) d\omega = 5.074 \cdot 10^7 \]

\[ I_{2-3} = \frac{1}{2 \cdot \pi} \int_0^{10^{11}} \left( \frac{1}{|1 + \frac{1}{\omega \cdot Z3(\omega) Rf}|^2} \right) d\omega = 5.612 \cdot 10^7 \]

FET-input stage

\[ Ig = 10 \cdot 10^{-5} \quad gmI = 30 \cdot 10^{-3} = 0.03 \quad \text{noise_factor} := 1 \]

Ig noise (gate current)

\[ I_{\text{Ig}} := 2 \cdot q \cdot Ig \]

\[ I_{\text{Ig1}} := I_{\text{Ig}} \cdot NEB1 = 2.078 \cdot 10^{-19} \]

\[ I_{\text{Ig3}} := I_{\text{Ig}} \cdot NEB3 = 2.236 \cdot 10^{-19} \]
Channel noise (Gate-source)

\[
Beqc1 = \frac{1}{2 \cdot \pi} \int_0^{10^7} \left( \frac{\left( \left( 1 - \omega^2 \cdot Lf \cdot C_T \right) + Rf \cdot \omega \cdot C_T \cdot 1 i \right)}{(Rf + 1 i \cdot \omega \cdot Lf)} \cdot |Z1(\omega)| \right)^2 \, d\omega = 17.079
\]

\[
Beqc3 = \frac{1}{2 \cdot \pi} \int_0^{10^7} \left( \frac{\left( \left( 1 - \omega^2 \cdot Lf \cdot C_T \right) + Rf \cdot \omega \cdot C_T \cdot 1 i \right)}{(Rf + 1 i \cdot \omega \cdot Lf)} \cdot |Z3(\omega)| \right)^2 \, d\omega = 10.177
\]

\[
I_{nc1} = 4 \cdot k \cdot T \cdot \frac{1}{gm1} \cdot Beqc1 = 9.365 \cdot 10^{-18}
\]

\[
I_{nc3} = 4 \cdot k \cdot T \cdot \frac{1}{gm1} \cdot Beqc3 = 5.58 \cdot 10^{-18}
\]

Total noise - FET

\[
noise1 = \left( 2 \cdot q \cdot Iq \cdot NEB1 + \frac{4 \cdot k \cdot T}{Rf} \cdot I_{z - 1} + 4 \cdot k \cdot T \cdot \frac{1}{gm1} \cdot Beqc1 \right) = 3.8925479665541 \cdot 10^{-17}
\]

\[
noise3 = \left( 2 \cdot q \cdot Iq \cdot NEB3 + \frac{4 \cdot k \cdot T}{Rf} \cdot I_{z - 3} + 4 \cdot k \cdot T \cdot \frac{1}{gm1} \cdot Beqc3 \right) = 3.82981391358371 \cdot 10^{-17}
\]

BJT input stage

\[
Ic = 2 \cdot 10^{-3}
\]

\[
hfe := 100 \quad \frac{Ic}{hfe} = \frac{Ic}{hfe} = 0.08
\]

\[
Ib := \frac{Ic}{hfe} = \frac{Ic}{hfe} = 0.08
\]

\[
gm_{BJT} := \frac{Ic}{(25 \cdot 10^{-3})} = 0.08
\]

Ib noise (base)

\[
I_{nb1} := I_{nbx} \cdot NEB1 = 4.156 \cdot 10^{-16}
\]

\[
I_{nb3} := I_{nbx} \cdot NEB3 = 4.471 \cdot 10^{-16}
\]

Ic noise (collector)

\[
I_{nce1} := \frac{2 \cdot q \cdot Ic}{gm_{BJT}^2} \cdot Beqc1 = 1.708 \cdot 10^{-18}
\]

\[
I_{nce3} := \frac{2 \cdot q \cdot Ic}{gm_{BJT}^2} \cdot Beqc3 = 1.018 \cdot 10^{-18}
\]

Total noise - BJT

\[
noise_{1st\_BJT} := \left( \frac{4 \cdot k \cdot T}{Rf} \cdot I_{z - 1} + \left( 2 \cdot q \cdot I_b \right) \cdot NEB1 + 2 \cdot q \cdot \frac{Ic}{gm_{BJT}^2} \cdot Beqc1 \right) = 4.46640911886504 \cdot 10^{-16}
\]

\[
noise_{3rd\_BJT} := \left( \frac{4 \cdot k \cdot T}{Rf} \cdot I_{z - 3} + \left( 2 \cdot q \cdot I_b \right) \cdot NEB3 + 2 \cdot q \cdot \frac{Ic}{gm_{BJT}^2} \cdot Beqc3 \right) = 4.80624830641738 \cdot 10^{-16}
\]
noise-opt BJT (Ic re-calculations)

\[\text{noise}_1\text{st}_\text{BJT opt}\,(x) := \left(\frac{4 \cdot k \cdot T}{R_f} \cdot I_{2,1} + \left(2 \cdot q \cdot \frac{x}{hfe}\right) \cdot \text{NEB1} + 2 \cdot q \cdot \frac{x}{\left(25 \cdot 10^{-3}\right)^2} \cdot \text{Beq1}\right)\]

\[\text{noise}_3\text{rd}_\text{BJT opt}\,(x) := \left(\frac{4 \cdot k \cdot T}{R_f} \cdot I_{2,3} + \left(2 \cdot q \cdot \frac{x}{hfe}\right) \cdot \text{NEB3} + 2 \cdot q \cdot \frac{x}{\left(25 \cdot 10^{-3}\right)^2} \cdot \text{Beq3}\right)\]

\[x := 0.00001, 0.00002 \ldots 0.01\]

\[\text{noise}_1\text{st}_\text{BJT opt}\,(2 \cdot 10^{-3}) = 4.466 \cdot 10^{-16}\]

\[\text{noise}_3\text{rd}_\text{BJT opt}\,(2 \cdot 10^{-3}) = 4.806 \cdot 10^{-16}\]

\[I_{c\text{opt}1} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \frac{\text{Beq1}}{\text{NEB1}}\]

\[I_{c\text{opt}3} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \frac{\text{Beq3}}{\text{NEB3}}\]

\[I_{c\text{opt}1} = 1.282 \cdot 10^{-4}\]

\[I_{c\text{opt}3} = 9.542 \cdot 10^{-5}\]

\[\text{noise}_1\text{st}_\text{BJT opt}\,(I_{c\text{opt}1}) = 8.266 \cdot 10^{-17}\]

\[\text{noise}_3\text{rd}_\text{BJT opt}\,(I_{c\text{opt}3}) = 7.516 \cdot 10^{-17}\]

\[\text{test1}\,(x) := \text{noise}_1\text{st}_\text{BJT opt}\,(x)\]

\[\text{test2}\,(x) := \text{noise}_3\text{rd}_\text{BJT opt}\,(x)\]

**FET vs BJT-opt**

\[\text{noise}_1\text{st}_\text{BJT opt}\,(I_{c\text{opt}1}) = 8.266 \cdot 10^{-17}\]

\[\text{noise}_3\text{rd}_\text{BJT opt}\,(I_{c\text{opt}3}) = 7.516 \cdot 10^{-17}\]

\[\text{noise}_1 = 3.895 \cdot 10^{-17}\]

\[\text{noise}_3 = 3.83 \cdot 10^{-17}\]
output pulse shape

\[
t_{s} = 1 \cdot 10^{-8} \quad B = 1 \cdot 10^{8}
\]

\[
t := -6 \cdot ts, -5.99 \cdot ts \ldots 6 \cdot ts
\]

\[
vout1 \left( t \right) = \frac{ts}{\pi} \int_{0}^{\frac{1}{n} \cdot 10^{7}} \left[ \sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \text{Re} \left( Z1 \left( \omega \right) \cdot \exp \left( i \cdot \omega \cdot \left( t - \frac{ts}{2} \right) \right) \right) \right] d\omega
\]

\[
vout3 \left( t \right) = \frac{ts}{\pi} \int_{0}^{\frac{1}{n} \cdot 10^{7}} \left[ \sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \text{Re} \left( Z3 \left( \omega \right) \cdot \exp \left( i \cdot \omega \cdot \left( t - \frac{ts}{2} \right) \right) \right) \right] d\omega
\]

peak voltage and time

\[
xvout1 \left( i \right) = \frac{ts^{2}}{\pi} \int_{0}^{\frac{1}{ts} \cdot 10^{7}} \left[ \sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \text{Re} \left( i \cdot \omega \cdot Z1 \left( \omega \right) \cdot \exp \left( i \cdot \omega \cdot \left( t - \frac{ts}{2} \right) \right) \right) \right] d\omega
\]

\[
xvout3 \left( i \right) = \frac{ts^{2}}{\pi} \int_{0}^{\frac{1}{ts} \cdot 10^{7}} \left[ \sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \text{Re} \left( i \cdot \omega \cdot Z3 \left( \omega \right) \cdot \exp \left( i \cdot \omega \cdot \left( t - \frac{ts}{2} \right) \right) \right) \right] d\omega
\]

\[
tt := 0.5 \cdot ts
\]

\[
t_{pk1} := \text{root} \left( xvout1 \left( tt, t \right) \right)
\]

\[
\frac{t_{pk1}}{ts} = 1.019
\]

\[
v_{max1} := vout1 \left( t_{pk1} \right) = 1.038
\]

\[
v_{min1} := vout1 \left( t_{pk1} + 1 \cdot ts \right) = -0.038
\]

\[
v_{min2t_{s}} := vout1 \left( t_{pk1} + 2 \cdot ts \right) = -0.004
\]

\[
v_{min3t_{s}} := vout1 \left( t_{pk1} + 3 \cdot ts \right) = 0.001
\]

\[
t := -6 \cdot ts, -5.99 \cdot ts \ldots 6 \cdot ts
\]

\[
t := 0.9 \cdot ts
\]

\[
t_{pk3} := \text{root} \left( xvout3 \left( t, i \right) \right)
\]

\[
\frac{t_{pk3}}{ts} = 1.236
\]

\[
v_{max3} := vout3 \left( t_{pk3} \right) = 1.092
\]

\[
v_{min3t_{s}} := vout3 \left( t_{pk3} + 2 \cdot ts \right) = 0.01
\]

\[
v_{min2t_{s}} := vout3 \left( t_{pk3} + 2 \cdot ts \right) = 0.01
\]

\[
v_{min3t_{s}} := vout3 \left( t_{pk3} + 3 \cdot ts \right) = -1.396 \cdot 10^{-4}
\]
\[
\text{BER} = 1.6 \cdot 10^{-10} \quad \text{Test}_{\text{b_{photons}}} = 1100, 1200 \ldots 12000
\]

\[
\text{test}_1\left(\text{Test}_{\text{b_{photons}}}\right) := \frac{1}{I_s} \cdot \text{Test}_{\text{b_{photons}}} \cdot n_g \cdot \frac{\left(\frac{v_{\text{max}} - v_{\text{min}}}{2}\right)}{\sqrt{\text{noise}_{\text{1st BJT opt}}} (I_{\text{opt}})}
\]

\[
\text{test}_2\left(\text{Test}_{\text{b_{photons}}}\right) := \frac{1}{I_s} \cdot \text{Test}_{\text{b_{photons}}} \cdot n_g \cdot \frac{\left(\frac{v_{\text{max}} - v_{\text{min}}}{2}\right)}{\sqrt{\text{noise}_{\text{3rd BJT opt}}} (I_{\text{opt}})}
\]

\[
\text{test}_3\left(\text{Test}_{\text{b_{photons}}}\right) := \frac{1}{I_s} \cdot \text{Test}_{\text{b_{photons}}} \cdot n_g \cdot \frac{\left(\frac{v_{\text{max}} - v_{\text{min}}}{2}\right)}{\sqrt{\text{noise}}} \sqrt{2}
\]

\[
\text{test}_4\left(\text{Test}_{\text{b_{photons}}}\right) := \frac{1}{I_s} \cdot \text{Test}_{\text{b_{photons}}} \cdot n_g \cdot \frac{\left(\frac{v_{\text{max}} - v_{\text{min}}}{2}\right)}{\sqrt{\text{noise}^3}} \sqrt{2}
\]

\[
\text{Test}_1\left(\text{Test}_{\text{b_{photons}}}\right) := \frac{1}{2} \cdot \text{erfc}\left(\frac{\text{test}_1\left(\text{Test}_{\text{b_{photons}}}\right)}{\sqrt{2}}\right)
\]

\[
\text{Test}_2\left(\text{Test}_{\text{b_{photons}}}\right) := \frac{1}{2} \cdot \text{erfc}\left(\frac{\text{test}_2\left(\text{Test}_{\text{b_{photons}}}\right)}{\sqrt{2}}\right)
\]

\[
\text{Test}_3\left(\text{Test}_{\text{b_{photons}}}\right) := \frac{1}{2} \cdot \text{erfc}\left(\frac{\text{test}_3\left(\text{Test}_{\text{b_{photons}}}\right)}{\sqrt{2}}\right)
\]

\[
\text{Test}_4\left(\text{Test}_{\text{b_{photons}}}\right) := \frac{1}{2} \cdot \text{erfc}\left(\frac{\text{test}_4\left(\text{Test}_{\text{b_{photons}}}\right)}{\sqrt{2}}\right)
\]
BER PIN-BJT and PIN-FET (PIN noise is neglected)

\[ xQ_{l_{BJT,1st}}(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot \frac{v_{max1} - v_{min1}}{2} \sqrt{\text{noise}_{1st\_BJT\_opt}(I_{opt1})} \]

\[ xQ_{l_{BJT,3rd}}(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot \frac{v_{max3} - v_{min3}}{2} \sqrt{\text{noise}_{3rd\_BJT\_opt}(I_{opt3})} \]

\[ xQ_{l_{FET,1st}}(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot \frac{v_{max1} - v_{min1}}{2} \sqrt{\text{noise1}} \]

\[ xQ_{l_{FET,3rd}}(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot \frac{v_{max3} - v_{min3}}{2} \sqrt{\text{noise3}} \]

\[ xP_{eb\_BJT,1st}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{xQ_{l_{BJT,1st}}(b)}{\sqrt{2}} \right) \]

\[ xP_{eb\_BJT,3rd}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{xQ_{l_{BJT,3rd}}(b)}{\sqrt{2}} \right) \]

\[ xP_{eb\_FET,1st}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{xQ_{l_{FET,1st}}(b)}{\sqrt{2}} \right) \]
\[
x P_{eb,FET_{ind}}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{x Q t_3 FET_{ind}(b)}{\sqrt{2}} \right)
\]

\[
\begin{align*}
pc_{BJT_1}(b) &:= \log \left( x P_{eb,BJT_{ind}}(b) \right) + 9 \\
pc_{BJT_3}(b) &:= \log \left( x P_{eb,BJT_{ind}}(b) \right) + 9 \\
pc_{FET_1}(b) &:= \log \left( x P_{eb,FET_{ind}}(b) \right) + 9 \\
pc_{FET_3}(b) &:= \log \left( x P_{eb,FET_{ind}}(b) \right) + 9
\end{align*}
\]

\[b \equiv 3 \cdot 10^3\]

\[
	ext{BJT}_{b_1} := \text{root} \left( pc_{BJT_1}(b), b \right)
\]

\[
	ext{BJT}_{b_3} := \text{root} \left( pc_{BJT_3}(b), b \right)
\]

\[
	ext{FET}_{b_1} := \text{root} \left( pc_{FET_1}(b), b \right)
\]

\[
	ext{FET}_{b_3} := \text{root} \left( pc_{FET_3}(b), b \right)
\]

\[
\begin{align*}
\text{minimum } &\text{BJT}_1 := \min \left( \text{BJT}_{b_1} \right) = 6.335 \cdot 10^3 \\
\text{minimum } &\text{BJT}_3 := \min \left( \text{BJT}_{b_3} \right) = 6.008 \cdot 10^3 \\
\text{minimum } &\text{FET}_1 := \min \left( \text{FET}_{b_1} \right) = 4.349 \cdot 10^3 \\
\text{minimum } &\text{FET}_3 := \min \left( \text{FET}_{b_3} \right) = 4.289 \cdot 10^3
\end{align*}
\]

\[
x Q t_{1,\text{BJT}_{1w}}(\text{minimum } \text{BJT}_1) = 5.998
\]

\[
x Q t_{3,\text{BJT}_{3w}}(\text{minimum } \text{BJT}_3) = 5.998
\]

\[
x Q t_{1,\text{FET}_{1w}}(\text{minimum } \text{FET}_1) = 5.998
\]

\[
x Q t_{3,\text{FET}_{3w}}(\text{minimum } \text{FET}_3) = 5.998
\]

**Sens PIN-BJT and PIN-FET**

\[
\lambda := 650 \cdot 10^{-9}
\]

\[
\text{photon energy} = \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda}
\]
\[
\begin{align*}
PIN\_BJT\_1\_dBm &= 10 \cdot \log\left( \frac{\text{minimum\_BJT\_1} \cdot \text{photon\_energy} \cdot 1}{2 \cdot 10^{-3} \cdot ts} \right) = -40.14 \\
PIN\_BJT\_3\_dBm &= 10 \cdot \log\left( \frac{\text{minimum\_BJT\_3} \cdot \text{photon\_energy} \cdot 1}{2 \cdot 10^{-3} \cdot ts} \right) = -40.37 \\
PIN\_FET\_1\_dBm &= 10 \cdot \log\left( \frac{\text{minimum\_FET\_1} \cdot \text{photon\_energy} \cdot 1}{2 \cdot 10^{-3} \cdot ts} \right) = -41.77 \\
PIN\_FET\_3\_dBm &= 10 \cdot \log\left( \frac{\text{minimum\_FET\_3} \cdot \text{photon\_energy} \cdot 1}{2 \cdot 10^{-3} \cdot ts} \right) = -41.83
\end{align*}
\]

**APD noise**

\[ M\_APD = 10 \]
\[ F\_M = 5.5 \]
\[ APD\_1\_b = \frac{1}{ts} \cdot b \cdot nq \cdot (v\_max1) \]
\[ APD\_3\_b = \frac{1}{ts} \cdot b \cdot nq \cdot (v\_max3) \]
\[ I\_APD\_d = 10 \cdot 10^{-9} \]
\[ \text{Noise\_APD1\_b} = \left( 2 \cdot q \cdot APD\_1\_b \cdot M\_APD^2 \cdot F\_M \right) + \left( 2 \cdot q \cdot I\_APD\_d \cdot M\_APD^2 \cdot F\_M \right) \]
\[ \text{Noise\_APD3\_b} = \left( 2 \cdot q \cdot APD\_3\_b \cdot M\_APD^2 \cdot F\_M \right) + \left( 2 \cdot q \cdot I\_APD\_d \cdot M\_APD^2 \cdot F\_M \right) \]

**BER APD-BJT and APD-FET**

\[ Q\_APD\_BJT\_1\_b = \frac{M\_APD}{ts} \cdot b \cdot nq \cdot \left( \frac{v\_max1 - v\_min1}{2} \right) \sqrt{\text{noise\_1st\_BJT\_opt} + \text{Noise\_APD1\_b}} \]
\[ Q\_APD\_BJT\_3\_b = \frac{M\_APD}{ts} \cdot b \cdot nq \cdot \left( \frac{v\_max3 - v\_min3}{2} \right) \sqrt{\text{noise\_3rd\_BJT\_opt} + \text{Noise\_APD3\_b}} \]
\[ Q\_APD\_FET\_1\_b = \frac{M\_APD}{ts} \cdot b \cdot nq \cdot \left( \frac{v\_max1 - v\_min1}{2} \right) \sqrt{\text{noise1} + \text{Noise\_APD1\_b}} \]
\[ Q\_APD\_FET\_3\_b = \frac{M\_APD}{ts} \cdot b \cdot nq \cdot \left( \frac{v\_max3 - v\_min3}{2} \right) \sqrt{\text{noise3} + \text{Noise\_APD3\_b}} \]
\[ P_{e, \text{APD BJT 1}}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD BJT 1}}(b)}{\sqrt{2}} \right) \]

\[ P_{e, \text{APD BJT 3}}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD BJT 3}}(b)}{\sqrt{2}} \right) \]

\[ P_{e, \text{APD FET 1}}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD FET 1}}(b)}{\sqrt{2}} \right) \]

\[ P_{e, \text{APD FET 3}}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD FET 3}}(b)}{\sqrt{2}} \right) \]

\[ pc_{\text{APD BJT 1}}(b) := (\log(P_{e, \text{APD BJT 1}}(b)) + 9) \]

\[ pc_{\text{APD BJT 3}}(b) := (\log(P_{e, \text{APD BJT 3}}(b)) + 9) \]

\[ pc_{\text{APD FET 1}}(b) := (\log(P_{e, \text{APD FET 1}}(b)) + 9) \]

\[ pc_{\text{APD FET 3}}(b) := (\log(P_{e, \text{APD FET 3}}(b)) + 9) \]

\[ \text{APD BJT 1} := \text{root}(pc_{\text{APD BJT 1}}(b), b) \]

\[ \text{APD BJT 3} := \text{root}(pc_{\text{APD BJT 3}}(b), b) \]

\[ \text{APD FET 1} := \text{root}(pc_{\text{APD FET 1}}(b), b) \]

\[ \text{APD FET 3} := \text{root}(pc_{\text{APD FET 3}}(b), b) \]

\[ \text{minimum APD BJT 1} := \min(\text{APD BJT 1}) = 1.542 \times 10^3 \]

\[ \text{minimum APD BJT 3} := \min(\text{APD BJT 3}) = 1.619 \times 10^3 \]

\[ \text{minimum APD FET 1} := \min(\text{APD FET 1}) = 1.439 \times 10^3 \]

\[ \text{minimum APD FET 3} := \min(\text{APD FET 3}) = 1.536 \times 10^3 \]

\[ Q_{\text{APD BJT 1}}(\text{minimum APD BJT 1}) = 5.998 \]

\[ Q_{\text{APD BJT 3}}(\text{minimum APD BJT 3}) = 5.998 \]

\[ Q_{\text{APD FET 1}}(\text{minimum APD FET 1}) = 5.998 \]

\[ Q_{\text{APD FET 3}}(\text{minimum APD FET 3}) = 5.998 \]
Sens APD-BJT and APD-FET

\[ APD_{BJT\ dBm\_1} = 10 \cdot \log \left( \frac{\text{minimum } APD_{BJT\_1}}{2} \cdot \frac{\text{photon\ energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -46.272 \]

\[ APD_{BJT\ dBm\_3} = 10 \cdot \log \left( \frac{\text{minimum } APD_{BJT\_3}}{2} \cdot \frac{\text{photon\ energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -46.06 \]

\[ APD_{FET\ dBm\_1} = 10 \cdot \log \left( \frac{\text{minimum } APD_{FET\_1}}{2} \cdot \frac{\text{photon\ energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -46.573 \]

\[ APD_{FET\ dBm\_3} = 10 \cdot \log \left( \frac{\text{minimum } APD_{FET\_3}}{2} \cdot \frac{\text{photon\ energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -46.29 \]

PIN \_1 \ dBm = -40.136 \quad APD_{BJT\ dBm\_1} = -46.272

PIN \_3 \ dBm = -40.365 \quad APD_{BJT\ dBm\_3} = -46.06

PIN \_1 \ dBm = -41.769 \quad APD_{FET\ dBm\_1} = -46.573

PIN \_3 \ dBm = -41.829 \quad APD_{FET\ dBm\_3} = -46.29

Sens APD vs PIN

PIN \_1 \ dBm - APD_{BJT\ dBm\_1} = 6.137

PIN \_3 \ dBm - APD_{BJT\ dBm\_3} = 5.695

PIN \_1 \ dBm - APD_{FET\ dBm\_1} = 4.804

PIN \_3 \ dBm - APD_{FET\ dBm\_3} = 4.46

Noise BJT

\[ \text{noise\_1st\_BJT\_opt}(I_{opt1}) + \text{Noise\ APD1}(\text{minimum } APD_{BJT\_1}) = 4.897 \cdot 10^{-16} \]

\[ \text{noise\_3rd\_BJT\_opt}(I_{opt3}) + \text{Noise\ APD3}(\text{minimum } APD_{BJT\_3}) = 5.458 \cdot 10^{-16} \]

\[ \text{noise1} + \text{Noise\ APD1}(\text{minimum } APD_{FET\_1}) = 4.264 \cdot 10^{-16} \]

\[ \text{noise3} + \text{Noise\ APD3}(\text{minimum } APD_{FET\_3}) = 4.911 \cdot 10^{-16} \]

\[ \text{noise\_1st\_BJT\_opt}(I_{opt1}) = 8.266 \cdot 10^{-17} \]

\[ \text{noise\_3rd\_BJT\_opt}(I_{opt3}) = 7.516 \cdot 10^{-17} \]

\[ \text{noise1} = 3.895 \cdot 10^{-17} \]

\[ \text{noise3} = 3.83 \cdot 10^{-17} \]

\[ 10 \log \left( \frac{\text{noise\_1st\_BJT\_opt}(I_{opt1})}{\text{noise\_3rd\_BJT\_opt}(I_{opt3})} \right) = 0.413 \]
$$10 \log \left( \frac{noise_{1st_{BJT}_{\text{opt}(I_{c}opt)}} + \text{Noise}_APD1(minimum_{APD_{BJT}})}{noise_{1st_{BJT}_{\text{opt}(I_{c}opt)}}} \right) = 7.726$$

$$10 \log \left( \frac{noise_{3rd_{BJT}_{\text{opt}(I_{c}opt)}} + \text{Noise}_APD3(minimum_{APD_{BJT}})}{noise_{3rd_{BJT}_{\text{opt}(I_{c}opt)}}} \right) = 8.611$$

$$10 \log \left( \frac{noise_{1st_{BJT}_{\text{opt}(I_{c}opt)}}}{noise} \right) = 3.268$$

$$10 \log \left( \frac{noise_{1st_{BJT}_{\text{opt}(I_{c}opt)}}}{noise3} \right) = 3.341$$

$$10 \log \left( \frac{noise_{3rd_{BJT}_{\text{opt}(I_{c}opt)}}}{noise1} \right) = 2.854$$

$$10 \log \left( \frac{noise_{3rd_{BJT}_{\text{opt}(I_{c}opt)}}}{noise1} \right) = 2.928$$

$$10 \log \left( \frac{noise3}{noise1} \right) = -0.074$$

$$10 \log \left( \frac{noise1 + \text{Noise}_APD1(minimum_{APD_{FET}})}{noise1} \right) = 10.393$$

$$10 \log \left( \frac{noise3 + \text{Noise}_APD3(minimum_{APD_{FET}})}{noise3} \right) = 11.079$$
B.1.3 Tuned B receiver

(Tuned B front end receiver/1st and 3rd order LPF- 100 Mbit/s)

Tuned-B front-end Rx performance with PIN-BJT PIN-FET APD-BJT APD-FET input configurations. 1st order LPF and 3rd LPF predetection filters. calculations are as follow

- Frequency response
- Noise integrals
- Total noise
- Pulse shaping, peak voltage, ISI
- Error bit rate, minimum number of photons, receiver sensitivity
- APD noise, APD BER, APD receiver sensitivity

**Bit-rate, pulse duration**

\[ C_T := 1.5 \times 10^{-12} \quad B := 100 \times 10^6 \]

**pre-dec filter**

\[ f_B := 0.75 \times B \quad Av := 10 \]

\[ \omega_B := 2 \times \pi \times f_B \]

\[ H_{bat}(\omega) := \frac{1}{1 + 1j \times \frac{\omega}{\omega_B}} \]

\[ H_{bat3}(\omega) := \frac{\omega_B^3}{(1j \times \omega)^3 + 2 \times (1j \times \omega)^2 \times \omega_B + 2 \times (1j \times \omega) \times \omega_B^2 + \omega_B^3} \]

**TIA**

Feedback value for (R) Tuned B

\[ \Delta_L := 1.8 \quad \Delta_R := 1.87 \quad 	ext{feedback “\Delta R”, time constant ratio “\Delta L”} \]

\[ R_x := \frac{1 + Av}{\omega_B \times C_T} = 1.556 \times 10^4 \quad \text{Test } R := \Delta_R \times R_x = 2.91 \times 10^4 \]

\[ R_f := \Delta_R \times \frac{1 + Av}{2 \times \pi \times 0.75 \times B \times C_T} = 2.91 \times 10^4 \quad \text{feedback for Tuned B} \]

\[ \alpha := 0.2 \quad \text{splitting ratio} \]

\[ L_c := \left( \frac{R_f}{1 + Av} \right)^2 \times C_T = 5.832 \times 10^{-6} \]

\[ C_1 := (1 - \alpha) \times (C_T) = 1.2 \times 10^{-12} \]

\[ C_2 := \alpha \times (C_T) = 3 \times 10^{-13} \]
TUNED B (R) Transimpedance

\[ ZI(\omega) := H_{\text{inter}}(\omega) \cdot Z_{\text{TLA}}(\omega) \]

\[ Z_{\text{TLA}}(\omega) := \frac{Z2R(\omega)}{R_f} \]

\[ |Z_{\text{TLA}}(2 \cdot \pi \cdot 0.1 \cdot B)| = 0.997 \]

**Receiver freq-response**

**Frequency dependence of NON-TUNED TI**

\[ Z3(\omega) := H_{\text{crosstalk}}(\omega) \cdot Z_{\text{TLA}}(\omega) \]

\[ \omega := 1 \cdot 10^6, 10 \cdot 10^6 \ldots 1 \cdot 10^{11} \]

\[ |Z_{\text{TLA}}(2 \cdot \pi \cdot 0.75 \cdot B)| = 0.713 \]

\[ |ZI(\omega)| \]

\[ |Z_{\text{TLA}}(\omega)| \]
Noise equivalent bandwidth

\[ q = 1.6 \cdot 10^{-19} \quad k = 1.38 \cdot 10^{-23} \quad T = 298 \]

\[ NEB1 := \frac{1}{2 \cdot \pi} \left( \int_{0}^{10^{12}} \left( |Z1(\omega)| \right)^2 \, d\omega \right) = 5.713 \cdot 10^7 \]

\[ NEB3 := \frac{1}{2 \cdot \pi} \left( \int_{0}^{10^{11}} \left( |Z3(\omega)| \right)^2 \, d\omega \right) = 6.208 \cdot 10^7 \]

Rf noise

\[ I_{Rf} := \frac{4 \cdot k \cdot T}{Rf} \]

\[ I_{2-1} := \frac{1}{2 \cdot \pi} \left( \int_{0}^{10^{12}} \left( \left| 1 - \omega^2 \cdot Lc \cdot C1 \cdot |Z1(\omega)| \right| \right)^2 \, d\omega \right) = 6.012 \cdot 10^7 \]

\[ I_{2-3} := \frac{1}{2 \cdot \pi} \left( \int_{0}^{10^{12}} \left( \left| (1 - \omega^2 \cdot Lc \cdot C1) \cdot Z3(\omega) \right| \right)^2 \, d\omega \right) = 3.691 \cdot 10^7 \]

\[ I_{Rf} = I_{Rf} \cdot I_{2-1} = 3.398 \cdot 10^{-17} \]

\[ I_{Rf} = I_{Rf} \cdot I_{2-3} = 2.086 \cdot 10^{-17} \]
FET-input stage

\[ I_g := 10 \cdot 10^{-9} \quad \text{gm}I := 30 \cdot 10^{-3} = 0.03 \quad \text{noise_factor} := 1 \]

**Ig noise (gate current)**

\[ I_{ng} := 2 \cdot q \cdot I_g = 3.2 \cdot 10^{-27} \]

\[ I_{ng1} := I_{ng} \cdot I_{2\cdot1} = 1.924 \cdot 10^{-19} \]

\[ I_{ng3} := I_{ng} \cdot I_{2\cdot3} = 1.181 \cdot 10^{-19} \]

**Channel noise (Gate-source)**

\[ Beqc1 := \frac{1}{2 \cdot \pi} \int_{0}^{10^9} \left( \frac{\left(\left(\left(1 - \omega^2 \cdot Lc \cdot Cl\right) + Rf \cdot (\omega) \cdot \left(C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2\right) \cdot 1i\right)}{Rf} \cdot |Z1(\omega)| \right)^2 d\omega = 8.309 \]

\[ Beqc3 := \frac{1}{2 \cdot \pi} \int_{0}^{10^9} \left( \frac{\left(\left(\left(1 - \omega^2 \cdot Lc \cdot Cl\right) + Rf \cdot (\omega) \cdot \left(C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2\right) \cdot 1i\right)}{Rf} \cdot |Z3(\omega)| \right)^2 d\omega = 5.992 \]

\[ I_{nc1} := 4 \cdot k \cdot T \cdot \frac{1}{\text{gm}I} \cdot \text{Beqc1} = 4.556 \cdot 10^{-18} \]

\[ I_{nc3} := 4 \cdot k \cdot T \cdot \frac{1}{\text{gm}I} \cdot \text{Beqc3} = 3.286 \cdot 10^{-18} \]

**Total noise - FET**

\[ \text{noise1} := \left(2 \cdot q \cdot I_g \cdot I_{2\cdot1} + \frac{4 \cdot k \cdot T}{Rf} \cdot I_{2\cdot1} \cdot 4 \cdot k \cdot T \cdot \frac{1}{\text{gm}I} \cdot \text{Beqc1} \right) = 3.87301562727899 \cdot 10^{-17} \]

\[ \text{noise3} := \left(2 \cdot q \cdot I_g \cdot I_{2\cdot3} + \frac{4 \cdot k \cdot T}{Rf} \cdot I_{2\cdot3} \cdot 4 \cdot k \cdot T \cdot \frac{1}{\text{gm}I} \cdot \text{Beqc3} \right) = 2.42667086023124 \cdot 10^{-17} \]

**BJT input stage**

\[ I_c := 2 \cdot 10^{-3} \quad \text{hfe} := 100 \quad I_b := \frac{I_c}{\text{hfe}} \quad \text{gm}_\text{BJT} := \frac{I_c}{(25 \cdot 10^{-3})} = 0.08 \]

**Ib noise (base)**

\[ I_{nb} := 2 \cdot q \cdot I_b \]

\[ I_{nb1} := I_{nb} \cdot I_{2\cdot1} = 3.847 \cdot 10^{-16} \]

\[ I_{nb3} := I_{nb} \cdot I_{2\cdot3} = 2.362 \cdot 10^{-16} \]

**Ic noise (collector)**

\[ I_{nc} := \frac{2 \cdot q \cdot I_c}{\text{gm}_\text{BJT}^2} \cdot \text{Beqc1} = 8.309 \cdot 10^{-19} \]
\[ I_{n3c} = \frac{2 \cdot q \cdot I_c}{g_{m\_BJT}} \cdot B_{eq3} = 5.992 \cdot 10^{-19} \]

**Total noise-\text{BJT}**

\[\text{noise\_1st\_BJT} := \left( \frac{4 \cdot k \cdot T}{R_f} \cdot I_{Z\_1} + \left(2 \cdot q \cdot I_b\right) \cdot I_{Z\_1} + 2 \cdot q \cdot \frac{2 \cdot q \cdot I_c}{g_{m\_BJT}} \cdot B_{eq1}\right) = 4.18723883020899 \cdot 10^{-16} \]

\[\text{noise\_3rd\_BJT} := \left( \frac{4 \cdot k \cdot T}{R_f} \cdot I_{Z\_3} + \left(2 \cdot q \cdot I_b\right) \cdot I_{Z\_2} + 2 \cdot q \cdot \frac{2 \cdot q \cdot I_c}{g_{m\_BJT}} \cdot B_{eq3}\right) = 2.57076376873765 \cdot 10^{-16} \]

**\text{noise\_opt\_BJT} (Ic re-calculations)**

\[\text{noise\_1st\_BJT\_opt}(x) := \left( \frac{4 \cdot k \cdot T}{R_f} \cdot I_{Z\_1} + \left(2 \cdot q \cdot \frac{x}{hfe}\right) \cdot I_{Z\_1} + 2 \cdot q \cdot \frac{x}{\left(25 \cdot 10^{-3}\right)} \right) \cdot B_{eq1} \]

\[\text{noise\_3rd\_BJT\_opt}(x) := \left( \frac{4 \cdot k \cdot T}{R_f} \cdot I_{Z\_3} + \left(2 \cdot q \cdot \frac{x}{hfe}\right) \cdot I_{Z\_2} + 2 \cdot q \cdot \frac{x}{\left(25 \cdot 10^{-3}\right)} \right) \cdot B_{eq3} \]

\[x := 0.00001, 0.00002 \ldots 0.01 \]

\[\text{noise\_1st\_BJT\_opt} \left(2 \cdot 10^{-3}\right) = 4.196 \cdot 10^{-16} \]

\[\text{noise\_3rd\_BJT\_opt} \left(2 \cdot 10^{-3}\right) = 2.577 \cdot 10^{-16} \]

\[I_{c\_opt1} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{B_{eq1}}{I_{Z\_1}}} \]

\[I_{c\_opt3} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{B_{eq3}}{I_{Z\_2}}} \]

\[I_{c\_opt1} = 9.295 \cdot 10^{-5} \]

\[I_{c\_opt3} = 1.007 \cdot 10^{-4} \]

\[\text{noise\_1st\_BJT\_opt}(I_{c\_opt1}) = 6.974 \cdot 10^{-17} \]

\[\text{noise\_3rd\_BJT\_opt}(I_{c\_opt3}) = 4.466 \cdot 10^{-17} \]

\[\text{test1}(x) := \text{noise\_1st\_BJT\_opt}(x) \]

\[\text{test2}(x) := \text{noise\_3rd\_BJT\_opt}(x) \]
noise\_1st\_BJT\_opt(I_{opt1}) = 6.974 \cdot 10^{-17}

noise\_3rd\_BJT\_opt(I_{opt3}) = 4.466 \cdot 10^{-17}

noise_I = 3.873 \cdot 10^{-17}

noise_3 = 2.427 \cdot 10^{-17}

output pulse shape

\[ ts := \frac{1}{B} \quad B = 1 \cdot 10^8 \]

\[ t := -6 \cdot ts, -5.99 \cdot ts, \ldots 6 \cdot ts \]

\[ vout1(t) = \frac{ts}{\pi} \left[ \sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \text{Re} \left( Z1(\omega) \cdot \exp \left( i \cdot \omega \cdot \left( t - \frac{ts}{2} \right) \right) \right) \right] \mathrm{d}\omega \]

\[ vout3(t) = \frac{ts}{\pi} \left[ \sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \text{Re} \left( Z3(\omega) \cdot \exp \left( i \cdot \omega \cdot \left( t - \frac{ts}{2} \right) \right) \right) \right] \mathrm{d}\omega \]

peak voltage and time

\[ xvout1(t) := \frac{ts^2}{\pi} \left[ \sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \text{Re} \left( L_1 \cdot Z1(\omega) \cdot \exp \left( i \cdot \omega \cdot \left( t - \frac{ts}{2} \right) \right) \right) \right] \mathrm{d}\omega \]

\[ xvout3(t) := \frac{ts^2}{\pi} \left[ \sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \text{Re} \left( L_1 \cdot Z3(\omega) \cdot \exp \left( i \cdot \omega \cdot \left( t - \frac{ts}{2} \right) \right) \right) \right] \mathrm{d}\omega \]

\[ tt := 1.1 \cdot ts \]

\[ t_{pk1} := \text{root}(xvout1(tt), tt) \]

\[ \frac{t_{pk1}}{ts} = 1.141 \]

\[ v_{max1} := vout1(t_{pk1}) = 0.97 \]

\[ t := 1.1 \cdot ts \]

\[ t_{pk3} := \text{root}(xvout3(t), t) \]

\[ \frac{t_{pk3}}{ts} = 1.391 \]

\[ v_{max3} := vout3(t_{pk3}) = 1.031 \]

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\[ v_{\text{min}1} := v_{\text{out}1}(t_{p1} + 1 \cdot ts) = 0.02 \]

\[ v_{\text{min}2} := v_{\text{out}1}(t_{p1} + 2 \cdot ts) = 0.001 \]

\[ v_{\text{min}3} := v_{\text{out}1}(t_{p1} + 3 \cdot ts) = 2.268 \cdot 10^{-6} \]

\[ t := -6 \cdot ts, -5.99 \cdot ts, 6 \cdot ts \]

\[ v_{\text{out}1}(t) \quad v_{\text{out}3}(t) \]

\[ v_{\text{out}3}(t) + v_{\text{out}3}(t - 2 \cdot ts) \]
BER

\[ n_q = 1.6 \cdot 10^{-19} \quad \text{Test}_{\text{b photons}} = 1100, 1200, \ldots, 12000 \]

\[
test_{Q1}(Test_{\text{b photons}}) = \frac{1}{ts} \cdot Test_{\text{b photons}} \cdot n_q \cdot \left( \frac{V_{\text{max}} - V_{\text{min}}}{2} \right)
\]

\[
test_{Q2}(Test_{\text{b photons}}) = \frac{1}{ts} \cdot Test_{\text{b photons}} \cdot n_q \cdot \left( \frac{V_{\text{max}} - V_{\text{min}}}{2} \right)
\]

\[
test_{Q3}(Test_{\text{b photons}}) = \frac{1}{ts} \cdot Test_{\text{b photons}} \cdot n_q \cdot \left( \frac{V_{\text{max}} - V_{\text{min}}}{2} \right)
\]

\[
test_{Q4}(Test_{\text{b photons}}) = \frac{1}{ts} \cdot Test_{\text{b photons}} \cdot n_q \cdot \left( \frac{V_{\text{max}} - V_{\text{min}}}{2} \right)
\]

\[
Test1(Test_{\text{b photons}}) = \frac{1}{2} \cdot \text{erfc}\left( \frac{test_{Q1}(Test_{\text{b photons}})}{\sqrt{2}} \right)
\]

\[
Test2(Test_{\text{b photons}}) = \frac{1}{2} \cdot \text{erfc}\left( \frac{test_{Q2}(Test_{\text{b photons}})}{\sqrt{2}} \right)
\]

\[
Test3(Test_{\text{b photons}}) = \frac{1}{2} \cdot \text{erfc}\left( \frac{test_{Q3}(Test_{\text{b photons}})}{\sqrt{2}} \right)
\]

\[
Test4(Test_{\text{b photons}}) = \frac{1}{2} \cdot \text{erfc}\left( \frac{test_{Q4}(Test_{\text{b photons}})}{\sqrt{2}} \right)
\]
BER PIN-BJT and PIN-FET (PIN noise is neglected)

\[ x_{Q1_{BJT\_1st}}(b) := \frac{\frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{max1} - v_{min1}}{2} \right)}{\sqrt{\text{noise\_1st\_BJT\_opt}(I_{opt1})}} \]

\[ x_{Q1_{BJT\_3rd}}(b) := \frac{\frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{max3} - v_{min3}}{2} \right)}{\sqrt{\text{noise\_3rd\_BJT\_opt}(I_{opt3})}} \]

\[ x_{Q1_{FET\_1st}}(b) := \frac{\frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{max1} - v_{min1}}{2} \right)}{\sqrt{\text{noise1}}} \]

\[ x_{Q1_{FET\_3rd}}(b) := \frac{\frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{max3} - v_{min3}}{2} \right)}{\sqrt{\text{noise3}}} \]

\[ x_{P_{e\_BJT\_1st}}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{x_{Q1_{BJT\_1st}}(b)}{\sqrt{2}} \right) \]

\[ x_{P_{e\_BJT\_3rd}}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{x_{Q1_{BJT\_3rd}}(b)}{\sqrt{2}} \right) \]

\[ x_{P_{e\_FET\_1st}}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{x_{Q1_{FET\_1st}}(b)}{\sqrt{2}} \right) \]

\[ x_{P_{e\_FET\_3rd}}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{x_{Q1_{FET\_3rd}}(b)}{\sqrt{2}} \right) \]
\[ p_{c_{\text{BJT},1}}(b) = (\log (x_{P_{e_b,\text{BJT},1}}(b)) + 9) \]
\[ p_{c_{\text{BJT},3}}(b) = (\log (x_{P_{e_b,\text{BJT,3rd}}}(b)) + 9) \]
\[ p_{c_{\text{FET},1}}(b) = (\log (x_{P_{e_b,\text{FET},1}}(b)) + 9) \]
\[ p_{c_{\text{FET},3}}(b) = (\log (x_{P_{e_b,\text{FET,3rd}}}(b)) + 9) \]

\[ b \equiv 3 \cdot 10^3 \]

\[ BJT_{-}b_{-}1 := \text{root} (p_{c_{\text{BJT},1}}(b), b) \]
\[ BJT_{-}b_{-}3 := \text{root} (p_{c_{\text{BJT},3}}(b), b) \]
\[ FET_{-}b_{-}1 := \text{root} (p_{c_{\text{FET},1}}(b), b) \]
\[ FET_{-}b_{-}3 := \text{root} (p_{c_{\text{FET},3}}(b), b) \]

\[ \text{minimum}_{\text{BJT},1} := \min (BJT_{-}b_{-}1) = 6.595 \cdot 10^3 \]
\[ \text{minimum}_{\text{BJT},3} := \min (BJT_{-}b_{-}3) = 4.925 \cdot 10^3 \]
\[ \text{minimum}_{\text{FET},1} := \min (FET_{-}b_{-}1) = 4.915 \cdot 10^3 \]
\[ \text{minimum}_{\text{FET},3} := \min (FET_{-}b_{-}3) = 3.631 \cdot 10^3 \]

\[ xQ1_{\text{BJT,1}}(\text{minimum}_{\text{BJT},1}) = 5.998 \]
\[ xQ1_{\text{BJT,3}}(\text{minimum}_{\text{BJT},3}) = 5.998 \]
\[ xQ1_{\text{FET,1}}(\text{minimum}_{\text{FET},1}) = 5.998 \]
\[ xQ1_{\text{FET,3}}(\text{minimum}_{\text{FET},3}) = 5.998 \]

**Sens PIN-BJT and PIN-FET**

\[ \lambda := 650 \cdot 10^{-9} \]

\[ \text{photon\_energy} = \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]
\[
\begin{align*}
\text{PIN}_{\text{BJT}}_{\text{1 dBm}} &= 10 \cdot \log \left( \frac{\text{minimum}_{\text{BJT}}_{\text{1}}, \text{photon energy}}{2 \cdot 10^{-3}} \frac{1}{ts} \right) = -39.961 \\
\text{PIN}_{\text{BJT}}_{\text{3 dBm}} &= 10 \cdot \log \left( \frac{\text{minimum}_{\text{BJT}}_{\text{3}}, \text{photon energy}}{2 \cdot 10^{-3}} \frac{1}{ts} \right) = -41.229 \\
\text{PIN}_{\text{FET}}_{\text{1 dBm}} &= 10 \cdot \log \left( \frac{\text{minimum}_{\text{FET}}_{\text{1}}, \text{photon energy}}{2 \cdot 10^{-3}} \frac{1}{ts} \right) = -41.238 \\
\text{PIN}_{\text{FET}}_{\text{3 dBm}} &= 10 \cdot \log \left( \frac{\text{minimum}_{\text{FET}}_{\text{3}}, \text{photon energy}}{2 \cdot 10^{-3}} \frac{1}{ts} \right) = -42.553
\end{align*}
\]

**APD noise**

\[M_{\text{APD}} = 10\]
\[F_{M} = 5.5\]
\[
\text{APD}_{\text{1}}(b) := \frac{1}{ts} \cdot b \cdot nq \cdot (v_{\text{max}1})
\]
\[
\text{APD}_{\text{3}}(b) := \frac{1}{ts} \cdot b \cdot nq \cdot (v_{\text{max}3})
\]
\[
I_{\text{APD}_{\text{d}}} = 10 \cdot 10^{-9}
\]
\[
\text{Noise}_{\text{APD1}}(b) := \left( 2 \cdot q \cdot \text{APD}_{\text{1}}(b) \cdot M_{\text{APD}} \cdot F_{M} + I_{\text{APD}_{\text{d}}} \cdot M_{\text{APD}} \cdot F_{M} \right) \cdot \text{NEB1}
\]
\[
\text{Noise}_{\text{APD3}}(b) := \left( 2 \cdot q \cdot \text{APD}_{\text{3}}(b) \cdot M_{\text{APD}} \cdot F_{M} + I_{\text{APD}_{\text{d}}} \cdot M_{\text{APD}} \cdot F_{M} \right) \cdot \text{NEB3}
\]

**BER APD-BJT and APD-FET**

\[
\text{Q}_{\text{APD}_{\text{BJT}}_{\text{1}}}(b) := \frac{M_{\text{APD}}}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max}1} - v_{\text{mints}1}}{2} \right) \sqrt{\text{noise}_{1st} \cdot \text{Bjt opt}(I_{\text{opt1}}) + \text{Noise}_{\text{APD1}}(b)}
\]
\[
\text{Q}_{\text{APD}_{\text{BJT}}_{\text{3}}}(b) := \frac{M_{\text{APD}}}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max}3} - v_{\text{mints}3}}{2} \right) \sqrt{\text{noise}_{3rd} \cdot \text{Bjt opt}(I_{\text{opt3}}) + \text{Noise}_{\text{APD3}}(b)}
\]
\[
\text{Q}_{\text{APD}_{\text{FET}}_{\text{1}}}(b) := \frac{M_{\text{APD}}}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max}1} - v_{\text{mints}1}}{2} \right) \sqrt{\text{noise}1 + \text{Noise}_{\text{APD1}}(b)}
\]
\[
\text{Q}_{\text{APD}_{\text{FET}}_{\text{3}}}(b) := \frac{M_{\text{APD}}}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max}3} - v_{\text{mints}3}}{2} \right) \sqrt{\text{noise}3 + \text{Noise}_{\text{APD3}}(b)}
\]
\[
P_{\text{APD}_{\text{BJT}}_{\text{1}}}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{\text{Q}_{\text{APD}_{\text{BJT}}_{\text{1}}}(b)}{} \right)
\]
\[ P_{e,\text{APD_{BJT}_3}}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD_{BJT}_3}}(b)}{\sqrt{2}} \right) \]

\[ P_{e,\text{APD_{FET}_1}}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD_{FET}_1}}(b)}{\sqrt{2}} \right) \]

\[ P_{e,\text{APD_{FET}_3}}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD_{FET}_3}}(b)}{\sqrt{2}} \right) \]

\[ pc_{\text{APD_{BJT}_1}}(b) := (\log (P_{e,\text{APD_{BJT}_1}}(b))) + 9 \]

\[ pc_{\text{APD_{BJT}_3}}(b) := (\log (P_{e,\text{APD_{BJT}_3}}(b))) + 9 \]

\[ pc_{\text{APD_{FET}_1}}(b) := (\log (P_{e,\text{APD_{FET}_1}}(b))) + 9 \]

\[ pc_{\text{APD_{FET}_3}}(b) := (\log (P_{e,\text{APD_{FET}_3}}(b))) + 9 \]

\[ \text{APD_{BJT}_0}_1 := \text{root} \left( pc_{\text{APD_{BJT}_1}}(b), b \right) \]

\[ \text{APD_{BJT}_0}_3 := \text{root} \left( pc_{\text{APD_{BJT}_3}}(b), b \right) \]

\[ \text{APD_{FET}_0}_1 := \text{root} \left( pc_{\text{APD_{FET}_1}}(b), b \right) \]

\[ \text{APD_{FET}_0}_3 := \text{root} \left( pc_{\text{APD_{FET}_3}}(b), b \right) \]

\[ \text{minimum_{APD_{BJT}_1}} := \min (\text{APD_{BJT}_0}_1) = 1.626 \cdot 10^3 \]

\[ \text{minimum_{APD_{BJT}_3}} := \min (\text{APD_{BJT}_0}_3) = 1.527 \cdot 10^3 \]

\[ \text{minimum_{APD_{FET}_1}} := \min (\text{APD_{FET}_0}_1) = 1.538 \cdot 10^3 \]

\[ \text{minimum_{APD_{FET}_3}} := \min (\text{APD_{FET}_0}_3) = 1.472 \cdot 10^3 \]

\[ Q_{\text{APD_{BJT}_1}}(\text{minimum_{APD_{BJT}_1}}) = 5.998 \]

\[ Q_{\text{APD_{BJT}_3}}(\text{minimum_{APD_{BJT}_3}}) = 5.998 \]

\[ Q_{\text{APD_{FET}_1}}(\text{minimum_{APD_{FET}_1}}) = 5.998 \]

\[ Q_{\text{APD_{FET}_3}}(\text{minimum_{APD_{FET}_3}}) = 5.998 \]
**Sens APD-BJT and APD-FET**

\[
APD_{\text{BJT}}_{\text{dBm}_1} := 10 \cdot \log \left( \frac{\text{minimum}_{\text{APD-BJT}_1}}{2} \cdot \frac{\text{photon energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -46.041
\]

\[
APD_{\text{BJT}}_{\text{dBm}_3} := 10 \cdot \log \left( \frac{\text{minimum}_{\text{APD-BJT}_3}}{2} \cdot \frac{\text{photon energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -46.315
\]

\[
APD_{\text{FET}}_{\text{dBm}_1} := 10 \cdot \log \left( \frac{\text{minimum}_{\text{APD-FET}_1}}{2} \cdot \frac{\text{photon energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -46.284
\]

\[
APD_{\text{FET}}_{\text{dBm}_3} := 10 \cdot \log \left( \frac{\text{minimum}_{\text{APD-FET}_3}}{2} \cdot \frac{\text{photon energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -46.474
\]

**PIN_BJT_1_{dBm} = -39.961**  
**APD_BJT_{dBm}_1 = -46.041**

**PIN_BJT_3_{dBm} = -41.229**  
**APD_BJT_{dBm}_3 = -46.315**

**PIN_FET_1_{dBm} = -41.238**  
**APD_FET_{dBm}_1 = -46.284**

**PIN_FET_3_{dBm} = -42.553**  
**APD_FET_{dBm}_3 = -46.474**

**Sens APD vs PIN**

**PIN_BJT_1_{dBm} − APD_BJT_{dBm}_1 = 6.081**

**PIN_BJT_3_{dBm} − APD_BJT_{dBm}_3 = 5.087**

**PIN_FET_1_{dBm} − APD_FET_{dBm}_1 = 5.046**

**PIN_FET_3_{dBm} − APD_FET_{dBm}_3 = 3.921**

**Noise BJT**

\[\text{noise} \_1st_{\text{BJT opt}}(I_{\text{copt1}}) + \text{Noise}_{\text{APD1}}(\text{minimum}_{\text{APD-BJT}_1}) = 4.24 \cdot 10^{-16}\]

\[\text{noise} \_3rd_{\text{BJT opt}}(I_{\text{copt3}}) + \text{Noise}_{\text{APD3}}(\text{minimum}_{\text{APD-BJT}_3}) = 4.291 \cdot 10^{-16}\]

\[\text{noise1} + \text{Noise}_{\text{APD1}}(\text{minimum}_{\text{APD-FET}_1}) = 3.792 \cdot 10^{-16}\]

\[\text{noise3} + \text{Noise}_{\text{APD3}}(\text{minimum}_{\text{APD-FET}_3}) = 3.988 \cdot 10^{-16}\]

\[\text{noise} \_1st_{\text{BJT opt}}(I_{\text{copt1}}) = 6.974 \cdot 10^{-17}\]

\[\text{noise} \_3rd_{\text{BJT opt}}(I_{\text{copt3}}) = 4.466 \cdot 10^{-17}\]

\[\text{noise1} = 3.873 \cdot 10^{-17}\]

\[\text{noise3} = 2.427 \cdot 10^{-17}\]
\[
10 \log \left( \frac{\text{noise}_{\text{1st BJT opt}}(I_{\text{opt}1})}{\text{noise}_{\text{3rd BJT opt}}(I_{\text{opt}3})} \right) = 1.936
\]

\[
10 \log \left( \frac{\text{noise}_{\text{1st BJT opt}}(I_{\text{opt}1}) + \text{Noise}_{\text{APD1}}(\text{minimum APD BJT 1})}{\text{noise}_{\text{1st BJT opt}}(I_{\text{opt}1})} \right) = 7.839
\]

\[
10 \log \left( \frac{\text{noise}_{\text{3rd BJT opt}}(I_{\text{opt}3}) + \text{Noise}_{\text{APD3}}(\text{minimum APD BJT 3})}{\text{noise}_{\text{3rd BJT opt}}(I_{\text{opt}3})} \right) = 9.827
\]

\[
10 \log \left( \frac{\text{noise}_{\text{1st BJT opt}}(I_{\text{opt}1})}{\text{noise}_{\text{1}}} \right) = 2.554
\]

\[
10 \log \left( \frac{\text{noise}_{\text{1st BJT opt}}(I_{\text{opt}1})}{\text{noise}_{\text{3}}} \right) = 4.585
\]

\[
10 \log \left( \frac{\text{noise}_{\text{3rd BJT opt}}(I_{\text{opt}3})}{\text{noise}_{\text{1}}} \right) = 0.618
\]

\[
10 \log \left( \frac{\text{noise}_{\text{3rd BJT opt}}(I_{\text{opt}3})}{\text{noise}_{\text{3}}} \right) = 2.649
\]

\[
10 \log \left( \frac{\text{noise}_{\text{3}}}{\text{noise}_{\text{1}}} \right) = -2.03
\]

\[
10 \log \left( \frac{\text{noise}_{\text{1}} + \text{Noise}_{\text{APD1}}(\text{minimum APD FET 1})}{\text{noise}_{\text{1}}} \right) = 9.909
\]

\[
10 \log \left( \frac{\text{noise}_{\text{3}} + \text{Noise}_{\text{APD3}}(\text{minimum APD FET 3})}{\text{noise}_{\text{3}}} \right) = 12.158
\]
## B.1.4 Summary

Table A-1 Performance summary

<table>
<thead>
<tr>
<th>Sensitivity (dBm)</th>
<th>PINFET</th>
<th>PINBJT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-tuned</td>
<td>Tuned A</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st order LPF</td>
<td>-45.61</td>
<td>-46.57</td>
</tr>
<tr>
<td>3rd order Butterworth</td>
<td>-46.19</td>
<td>-46.29</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
B.2 Receiver optimisation (FET/ 1st order LPF pre-detection filter)

B.2.1 Non-tuned

(Non-tuned optimum Performance (PIN-FET APD-FET 100 Mbit/s)

Rx performance opt PIN-FET and APD-FET input configurations, 1st order LPF pre-detection filter. opt as follows

- TIA 3-db bandwidth rt/Filter 3-dB bandwidth rd
- range cut-off
- minimum noise
- examine ISI
- highest SNR
- BER
- APD

**Bit-rate, pulse duration**

\[ C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]

\[ B := 100 \cdot 10^6 \quad \text{Bit-rate} \]

**pre-dec filter**

\[ \omega_B(rd) := 2 \cdot \pi \cdot rd \]

\[ H_{\text{but}}(\omega, rd) := \frac{1}{1 + j \cdot \frac{\omega}{2 \cdot \pi \cdot rd}} \]

**TIA(Non-tuned)**

\[ \omega_c(rt) := 2 \cdot \pi \cdot rt \]

\[ Av := 10 \]

\[ Rf_N(rt) := \frac{Av + 1}{2 \cdot \pi \cdot rt \cdot C_T} \]

\[ Z_{TIA,N}(\omega, rt) := \frac{1}{1 + j \cdot \frac{\omega}{2 \cdot \pi \cdot rt}} \]

\[ Rf_N(0.5 \cdot B) = 2.334 \cdot 10^4 \]

**Receiver freq-response**

\[ Z_N(\omega, rd, rt) := H_{\text{but}}(\omega, rd) \cdot Z_{TIA,N}(\omega, rt) \]

\[ \omega := 1 \cdot 10^6, 10 \cdot 10^6 \ldots 1 \cdot 10^{11} \]

\[ rd := 0.7 \cdot B, 0.75 \cdot B \ldots 2 \cdot B \]

\[ rt := 0.5 \cdot B, 0.55 \cdot B \ldots B \]
Noise equivalent bandwidth (NEB and I2)

\[ \text{NEB}_N(\text{rd}, \text{rt}) := \frac{1}{2 \cdot \pi} \int_{0}^{10^{12}} \left( \left| \mathcal{Z}_N(\omega, \text{rd}, \text{rt}) \right| \right) d\omega \]

\[ \text{NEB}_N(0.7 \, B, 0.5 \, B) = 4.581 \cdot 10^7 \quad \text{NEB}_B(\text{rd}, \text{rt}) := \text{NEB}_N(\text{rd}, \text{rt}) \]

---

Noise equivalent bandwidth (I3)

\[ I_3\, N(\text{rd}, \text{rt}) := \frac{1}{2 \cdot \pi} \int_{0}^{10^{12}} \left( \left( \frac{1 + j \cdot (\omega) \cdot R_f(\text{rt}) \cdot C_T \cdot \mathcal{Z}_N(\omega, \text{rd}, \text{rt})}{1 \cdot R_f(\text{rt})} \right)^2 \right) d\omega \]

---

Feedback noise

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \]
\[ I_{n_{RF,N}}(rd, rt) := \frac{4 \cdot k \cdot T}{R_{f,N}(rt)} \cdot NEB_N(rd, rt) \]

\[ I_{n_{RF,N}}(0.7 B, 0.5 B) = 3.229 \cdot 10^{-17} \]

**Ig noise (gate current)**

\[ Ig := 10 \cdot 10^{-9} \]

\[ I_{nG,N}(rd, rt) := 2 \cdot q \cdot Ig \cdot NEB_N(rd, rt) \]

\[ I_{nG,N}(0.7 B, 0.5 B) = 1.466 \cdot 10^{-19} \]

**Channel noise (Gate-source)**

\[ gm_1 := 30 \cdot 10^{-3} = 0.03 \quad noise\_factor := 1 \]

\[ I_{nD} := 4 \cdot k \cdot T \cdot \frac{1}{gm_1} \]

\[ I_{nD,N}(rd, rt) := I_{nD} \cdot I_{3,N}(rd, rt) \]

\[ I_{nD,N}(0.7 B, 0.5 B) = 7.852 \cdot 10^{-18} \]

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Total noise - PIN-FET

\[ I_{n,B}(r_d, r_t) = I_{nRF,N}(r_d, r_t) + I_{nG,N}(r_d, r_t) + I_{nD,N}(r_d, r_t) \]

\[ I_{n,B}(0.7 B, 0.5 B) = 4.028 \cdot 10^{-17} \]
\[ I_{nB}(0.7 \, B, \, 0.5 \, B) = 4.03 \cdot 10^{-17} \]
\[ I_{nB}(0.8 \, B, \, 0.5 \, B) = 4.37 \cdot 10^{-17} \]
\[ I_{nB}(0.9 \, B, \, 0.5 \, B) = 4.69 \cdot 10^{-17} \]
\[ I_{nB}(1 \, B, \, 0.5 \, B) = 4.99 \cdot 10^{-17} \]

\[ I_{nB}(0.7 \, B, \, 0.55 \, B) = 4.68 \cdot 10^{-17} \]
\[ I_{nB}(0.8 \, B, \, 0.55 \, B) = 5.09 \cdot 10^{-17} \]
\[ I_{nB}(0.9 \, B, \, 0.55 \, B) = 5.47 \cdot 10^{-17} \]
\[ I_{nB}(1 \, B, \, 0.55 \, B) = 5.84 \cdot 10^{-17} \]

\[ I_{nB}(0.7 \, B, \, 0.7 \, B) = 6.76 \cdot 10^{-17} \]
\[ I_{nB}(0.8 \, B, \, 0.7 \, B) = 7.41 \cdot 10^{-17} \]
\[ I_{nB}(0.9 \, B, \, 0.7 \, B) = 8.03 \cdot 10^{-17} \]
\[ I_{nB}(1 \, B, \, 0.7 \, B) = 8.62 \cdot 10^{-17} \]

\[ I_{nB}(0.7 \, B, \, 0.75 \, B) = 7.5 \cdot 10^{-17} \]
\[ I_{nB}(0.8 \, B, \, 0.75 \, B) = 8.24 \cdot 10^{-17} \]
\[ I_{nB}(0.9 \, B, \, 0.75 \, B) = 8.94 \cdot 10^{-17} \]
\[ I_{nB}(1 \, B, \, 0.75 \, B) = 9.61 \cdot 10^{-17} \]

\[ I_{nB}(0.5 \, B, \, 0.6 \, B) = 4.27 \cdot 10^{-17} \]
\[ I_{nB}(0.5 \, B, \, 0.7 \, B) = 5.32 \cdot 10^{-17} \]
\[ I_{nB}(0.5 \, B, \, 0.8 \, B) = 6.42 \cdot 10^{-17} \]
\[ I_{nB}(0.5 \, B, \, 0.9 \, B) = 7.54 \cdot 10^{-17} \]

\[ I_{nB}(0.7 \, B, \, 0.6 \, B) = 5.35 \cdot 10^{-17} \]
\[ I_{nB}(0.8 \, B, \, 0.6 \, B) = 5.84 \cdot 10^{-17} \]
\[ I_{nB}(0.9 \, B, \, 0.6 \, B) = 6.29 \cdot 10^{-17} \]
\[ I_{nB}(1 \, B, \, 0.6 \, B) = 6.73 \cdot 10^{-17} \]

\[ I_{nB}(0.7 \, B, \, 0.65 \, B) = 6.05 \cdot 10^{-17} \]
\[ I_{nB}(0.8 \, B, \, 0.65 \, B) = 6.61 \cdot 10^{-17} \]
\[ I_{nB}(0.9 \, B, \, 0.65 \, B) = 7.15 \cdot 10^{-17} \]
\[ I_{nB}(1 \, B, \, 0.65 \, B) = 7.66 \cdot 10^{-17} \]

\[ I_{nB}(0.7 \, B, \, 0.8 \, B) = 8.24 \cdot 10^{-17} \]
\[ I_{nB}(0.8 \, B, \, 0.8 \, B) = 9.08 \cdot 10^{-17} \]
\[ I_{nB}(0.9 \, B, \, 0.8 \, B) = 9.87 \cdot 10^{-17} \]
\[ I_{nB}(1 \, B, \, 0.8 \, B) = 1.06 \cdot 10^{-16} \]

\[ I_{nB}(0.7 \, B, \, 0.85 \, B) = 9.01 \cdot 10^{-17} \]
\[ I_{nB}(0.8 \, B, \, 0.85 \, B) = 9.94 \cdot 10^{-17} \]
\[ I_{nB}(0.9 \, B, \, 0.85 \, B) = 1.08 \cdot 10^{-16} \]
\[ I_{nB}(1 \, B, \, 0.85 \, B) = 1.17 \cdot 10^{-16} \]

0.5/0.7 > rd, range rt 0.5-0.7
Output pulse shape

\[ t_s := \frac{1}{B}, \quad B = 1 \cdot 10^8 \]

\[ \text{vout}(t, rd, rt) := t_s \cdot \frac{1}{\pi} \left[ \sin \left( \frac{\omega \cdot t_s}{2} \right) \cdot \text{Re} \left( Z_N(\omega, rd, rt) \cdot \exp(1i \cdot \omega \cdot t) \right) \right] d\omega \]

\[ x_{Bt}(t) := t_s \cdot \frac{1}{\pi} \left[ \sin \left( \frac{\omega \cdot t_s}{2} \right) \cdot \text{Re} \left( \exp(1i \cdot \omega \cdot t) \right) \right] d\omega \]

\[ x_{vout}(t, rd, rt) := t_s^2 \cdot \frac{1}{\pi} \left[ \sin \left( \frac{\omega \cdot t_s}{2} \right) \cdot \text{Re} \left( 1i \cdot \omega \cdot Z_N(\omega, rd, rt) \cdot \exp(1i \cdot \omega \cdot t) \right) \right] d\omega \]

\[ t := -1 \cdot t_s, -0.99 \cdot t_s, \ldots, 4 \cdot t_s \]
\( t := -1 \, ts, -0.99 \, ts, 6 \, ts \)

\[ V_1(t, rd, rt) = vout(t, rd, rt) \]
\[ V_{101}(t, rd, rt) = vout(t, rd, rt) + vout(t - 2 ts, rd, rt) \]
\( t_{B1} := 0.1 \cdot ts \)
\( t_{pk1} := \text{root}(x\text{vout}(t_{B1}, 0.7 B, 0.5 B), t_{B1}) \)
\( \frac{t_{pk1}}{ts} = 0.526 \)

\( v_{\text{max}_{B1}} := v\text{out}(t_{pk1}, 0.7 B, 0.5 B) = 0.884 \)
\( v_{\text{min}_{B1}} := v\text{out}(t_{pk1} + 1 \cdot ts, 0.7 B, 0.5 B) = 0.106 \)
\( v_{\text{max}_{B1}} - v_{\text{min}_{B1}} = 0.777 \)
\( t_{B2} := 0.2 \cdot ts \)
\( t_{pk2} := \text{root}(x\text{vout}(t_{B2}, 0.7 B, 0.6 B), t_{B2}) \)
\( \frac{t_{pk2}}{ts} = 0.519 \)

\( v_{\text{max}_{B2}} := v\text{out}(t_{pk2}, 0.7 B, 0.6 B) = 0.915 \)
\( v_{\text{min}_{B2}} := v\text{out}(t_{pk2} + 1 \cdot ts, 0.7 B, 0.6 B) = 0.08 \)
\( v_{\text{max}_{B2}} - v_{\text{min}_{B2}} = 0.835 \)
\( t_{B3} := 0.2 \cdot ts \)
\( t_{pk3} := \text{root}(x\text{vout}(t_{B3}, 0.7 B, 0.7 B), t_{B3}) \)
\( \frac{t_{pk3}}{ts} = 0.513 \)

\( v_{\text{max}_{B3}} := v\text{out}(t_{pk3}, 0.7 B, 0.7 B) = 0.935 \)
\( v_{\text{min}_{B3}} := v\text{out}(t_{pk3} + 1 \cdot ts, 0.7 B, 0.7 B) = 0.062 \)
\( v_{\text{max}_{B3}} - v_{\text{min}_{B3}} = 0.873 \)
\( t_{B4} := 0.01 \cdot ts \)
\( t_{pk4} := \text{root}(x\text{vout}(t_{B4}, 0.8 B, 0.5 B), t_{B4}) \)
\( \frac{t_{pk4}}{ts} = 0.521 \)

\( v_{\text{max}_{B4}} := v\text{out}(t_{pk4}, 0.8 B, 0.5 B) = 0.899 \)
\( v_{\text{min}_{B4}} := v\text{out}(t_{pk4} + 1 \cdot ts, 0.8 B, 0.5 B) = 0.093 \)
\( v_{\text{max}_{B4}} - v_{\text{min}_{B4}} = 0.805 \)
\( t_{B5} := 0.1 \cdot ts \)
\( t_{pk5} := \text{root}(x\text{vout}(t_{B5}, 0.8 B, 0.6 B), t_{B5}) \)
\( \frac{t_{pk5}}{ts} = 0.514 \)

\( v_{\text{max}_{B5}} := v\text{out}(t_{pk5}, 0.8 B, 0.6 B) = 0.929 \)
\[ v_{\text{min, B5}} := v_{\text{o}} (t_{\text{pk5}} + 1 \cdot t_s, 0.8 \cdot B, 0.6 \cdot B) = 0.067 \]

\[ v_{\text{max, B5}} - v_{\text{min, B5}} = 0.862 \]

\[ v_{\text{max, B5}} - v_{\text{min, B5}} = 0.862 \]
\[ v_{\text{max, B4}} - v_{\text{min, B4}} = 0.805 \]
\[ v_{\text{max, B3}} - v_{\text{min, B3}} = 0.873 \]
\[ v_{\text{max, B2}} - v_{\text{min, B2}} = 0.835 \]
\[ v_{\text{max, B1}} - v_{\text{min, B1}} = 0.777 \]

\[ \frac{v_{\text{max, B1}} - v_{\text{min, B1}}}{\sqrt{10^{17} \cdot I_{\text{n, B}}(0.7 \cdot B, 0.5 \cdot B)}} = 0.387 \]
\[ v_{\text{max, B1}} - v_{\text{min, B1}} = 0.777 \]

\[ \frac{v_{\text{max, B2}} - v_{\text{min, B2}}}{\sqrt{10^{17} \cdot I_{\text{n, B}}(0.7 \cdot B, 0.6 \cdot B)}} = 0.361 \]

\[ \frac{v_{\text{max, B3}} - v_{\text{min, B3}}}{\sqrt{10^{17} \cdot I_{\text{n, B}}(0.7 \cdot B, 0.7 \cdot B)}} = 0.336 \]

\[ \frac{v_{\text{max, B4}} - v_{\text{min, B4}}}{\sqrt{10^{17} \cdot I_{\text{n, B}}(0.8 \cdot B, 0.5 \cdot B)}} = 0.385 \]

\[ \frac{v_{\text{max, B5}} - v_{\text{min, B5}}}{\sqrt{10^{17} \cdot I_{\text{n, B}}(0.8 \cdot B, 0.6 \cdot B)}} = 0.357 \]

**Highest SNR @ rd=0.7 rt=0.5**

\[ v_{\text{max, B}} := v_{\text{max, B1}} \quad v_{\text{min, B}} := v_{\text{min, B1}} \]

**Eye-diagram check**

\[ x_{111}(t, rd, rt) := v_{\text{o}}(t, rd, rt) + v_{\text{o}}(t - 2 \cdot ts, rd, rt) + v_{\text{o}}(t - ts, rd, rt) \]
\[ x_{110}(t, rd, rt) := v_{\text{o}}(t, rd, rt) + v_{\text{o}}(t - ts, rd, rt) \]
\[ x_{100}(t, rd, rt) := v_{\text{o}}(t, rd, rt) \]
\[ x_{001}(t, rd, rt) := v_{\text{o}}(t - 2 \cdot ts, rd, rt) \]
\[ x_{010}(t, rd, rt) := v_{\text{o}}(t - ts, rd, rt) \]
\[ x_{011}(t, rd, rt) := v_{\text{o}}(t - ts, rd, rt) + v_{\text{o}}(t - 2 \cdot ts, rd, rt) \]
\[ x_{101}(t, rd, rt) := v_{\text{o}}(t, rd, rt) + v_{\text{o}}(t - 2 \cdot ts, rd, rt) \]
\[ x_{101}(t_{\text{pk1}} + ts, 0.7 \, B, 0.5 \, B) = 0.111 \]
\[ x_{101}(t_{\text{pk1}}, 0.7 \, B, 0.5 \, B) = 0.884 \]
\[ x_{101}(t_{\text{pk1}} + 2 \, ts, 0.7 \, B, 0.5 \, B) = 0.889 \]
\[ x_{100}(t_{\text{pk1}}, 0.7 \, B, 0.5 \, B) = 0.884 \]
\[ x_{100}(t_{\text{pk1}} + ts, 0.7 \, B, 0.5 \, B) = 0.106 \]
\[ x_{100}(t_{\text{pk1}} + 2 \, ts, 0.7 \, B, 0.5 \, B) = 0.005 \]

Check ratio >1 is ok 100, <1 re-opt

\[ \frac{x_{100}(t_{\text{pk1}}, 0.7 \, B, 0.5 \, B) - x_{100}(t_{\text{pk1}} + 2 \, ts, 0.7 \, B, 0.5 \, B)}{v_{\text{max}_{B1}} - v_{\text{min}_{B1}}} = 1.13 \]

\textbf{Sens PINFET}

\[ \lambda := 650 \cdot 10^{-9} \]
\[ \text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]
\[ nq := 1.6 \cdot 10^{-19} \]
\[ Q_N(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max},B} - v_{\text{min},B}}{2} \right) \sqrt{I_{n,B}(0.7 B, 0.5 B)} \]
\[ P_{e,N}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right) \]
\[ P_{c,N}(b) := (\log(P_{e,N}(b)) + 9) \]

\[ b \equiv 3 \cdot 10^3 \]
\[ b_N := \text{root} (P_{c,N}(b), b) \]

**minimum_B := min(b_N) = 6.12 \cdot 10^3**

**APD-1FET**

\[ APD_N(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot (v_{\text{max},B}) \]
\[ APD_B(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot (v_{\text{max},B}) \]
\[ I_{APD,d} := 10 \cdot 10^{-9} \]

\[ M_{APD1} := 100 \]
\[ F_{M1} := 7.9 \]

\[ \text{Noise}_{APD1,B}(b, rd, rt) := \left( 2 \cdot q \cdot APD_B(b) \cdot M_{APD1}^2 \cdot F_{M1} \right) \cdot \text{NEB}_B(rd, rt) \]
\[ + 2 \cdot q \cdot I_{APD,d} \cdot M_{APD1}^2 \cdot F_{M1} \]

\[ Q_{APD1,B}(b) := \frac{M_{APD1}}{t_s} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max},B} - v_{\text{min},B}}{2} \right) \]
\[ \sqrt{I_{n,B}(0.7 B, 0.6 B) + \text{Noise}_{APD1,B}(b, 0.7 B, 0.6 B)} \]

\[ P_{e,APD1,B}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD1,B}(b)}{\sqrt{2}} \right) \]
\[ p_{c,APD1,B}(b) := (\log(P_{e,APD1,B}(b)) + 9) \]

\[ APD1_b_B := \text{root} (p_{c,APD1,B}(b), b) \]

**minimum_{APD1,B} := min(APD1_b_B) = 2.225 \cdot 10^3**

**APD-2FET**

\[ M_{APD2} := 10 \quad F_{M2} := 5.5 \]

\[ \text{Noise}_{APD2,B}(b, rd, rt) := \left( 2 \cdot q \cdot APD_B(b) \cdot M_{APD2}^2 \cdot F_{M2} \right) \cdot \text{NEB}_B(rd, rt) \]
\[ + 2 \cdot q \cdot I_{APD,d} \cdot M_{APD2}^2 \cdot F_{M2} \]

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\[ Q_{\text{APD}2\_B}(b) := \frac{M_{\text{APD}2}}{t_s} \cdot b \cdot n_q \cdot \left( \frac{v_{\text{max\_B}} - v_{\text{min\_B}}}{2} \right) \sqrt{I_{\text{n\_B}}(0.7 \ B, 0.6 \ B) + \text{Noise} \_{\text{APD}2\_B}(b, 0.7 \ B, 0.6 \ B)} \]

\[ P_{\epsilon \_{\text{APD}2\_B}}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD}2\_B}(b)}{\sqrt{2}} \right) \]

\[ p_{\text{APD}2\_B}(b) := \log \left( P_{\epsilon \_{\text{APD}2\_B}}(b) \right) + 9 \]

\[ \text{APD}2\_b\_B := \text{root} \left( p_{\text{APD}2\_B}(b), b \right) \]

\[ \text{minimum} \_{\text{APD}2\_B} := \text{min} \left( \text{APD}2\_b\_B \right) = 1.881 \cdot 10^3 \]

### APD-3FET

\[ M_{\text{APD}3} := 10 \quad F_{\text{M3}} := 9.2 \]

\[ \text{Noise} \_{\text{APD}3\_B}(b, rd, rt) := \left( 2 \cdot q \cdot \text{APD}3\_B(b) \cdot M_{\text{APD}3}^2 \cdot F_{\text{M3}} \right) \cdot \text{NEB}(rd, rt) + \left( 2 \cdot q \cdot I_{\text{APD}3\_d} \cdot M_{\text{APD}3}^2 \cdot F_{\text{M3}} \right) \]

\[ Q_{\text{APD}3\_B}(b) := \frac{M_{\text{APD}3}}{t_s} \cdot b \cdot n_q \cdot \left( \frac{v_{\text{max\_B}} - v_{\text{min\_B}}}{2} \right) \sqrt{I_{\text{n\_B}}(0.7 \ B, 0.6 \ B) + \text{Noise} \_{\text{APD}3\_B}(b, 0.7 \ B, 0.6 \ B)} \]

\[ P_{\epsilon \_{\text{APD}3\_B}}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD}3\_B}(b)}{\sqrt{2}} \right) \]

\[ p_{\text{APD}3\_B}(b) := \log \left( P_{\epsilon \_{\text{APD}3\_B}}(b) \right) + 9 \]

\[ \text{APD}3\_b\_B := \text{root} \left( p_{\text{APD}3\_B}(b), b \right) \]

\[ \text{minimum} \_{\text{APD}3\_B} := \text{min} \left( \text{APD}3\_b\_B \right) = 2.671 \cdot 10^3 \]

\[ \text{Sens}_N_{\text{PIN}} := 10 \cdot \log \left( \frac{\text{minimum} \_B \cdot \text{photon\_energy}}{2 \cdot 10^{-3}} \cdot \frac{1}{t_s} \right) = -40.285 \]

\[ \text{Sens}_N_{\text{APD\_silicon}} := 10 \cdot \log \left( \frac{\text{minimum} \_{\text{APD1\_B}} \cdot \text{photon\_energy}}{2 \cdot 10^{-3}} \cdot \frac{1}{t_s} \right) = -44.68 \]

\[ \text{Sens}_N_{\text{APD\_InGaAs}} := 10 \cdot \log \left( \frac{\text{minimum} \_{\text{APD2\_B}} \cdot \text{photon\_energy}}{2 \cdot 10^{-3}} \cdot \frac{1}{t_s} \right) = -45.41 \]

\[ \text{Sens}_N_{\text{APD\_germanium}} := 10 \cdot \log \left( \frac{\text{minimum} \_{\text{APD3\_B}} \cdot \text{photon\_energy}}{2 \cdot 10^{-3}} \cdot \frac{1}{t_s} \right) = -43.887 \]

"
B.2.2 Tuned A

**Tuned A Optimum Performance (PIN-FET APD-FET 100 Mbit/s)**

Rx performance opt PIN-FET and APD-FET input configurations, 1st order LPF pre-detection filter. opt as follows

- TIA 3-db bandwidth rt/Filter 3-db bandwidth rd
- range cut-off
- minimum noise
- examine ISI
- highest SNR
- BER
- APD

**Bit-rate, pulse duration**

\[ C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]

\[ B := 100 \cdot 10^6 \quad \text{Bit-rate} \]

**pre-dec filter**

\[ \omega_B (rd) := 2 \cdot \pi \cdot rd \]

\[ H_{but}(\omega, rd) := \frac{1}{1 + 1j \cdot \omega \cdot 2 \cdot \pi \cdot rd} \]

**TIA(Non-tuned)**

\[ Av := 10 \]

\[ m := 1.8 \quad \text{time constant ratio of L/R and RC} \]

\[ y := \sqrt{\left(\frac{-m^2}{2} + m + 1\right) + \sqrt{\left(\frac{-m^2}{2} + m + 1\right)^2 + m^2}} = 1.825 \]

\[ Rf_{-N}(rt) := y \cdot \frac{Av + 1}{2 \cdot \pi \cdot rt \cdot C_T} \]

\[ Rf_{-N}(0.6 \ B) = 3.551 \cdot 10^4 \]

\[ L(rt) := Rf_{-N}(rt)^2 \cdot \frac{C_T}{m} \]

\[ L(0.6 \cdot B) = 0.001 \]

\[ Lf(rt) := \left( \frac{Rf_{-N}(rt)}{Av + 1} \right)^2 \cdot \frac{(Av + 1) \cdot C_T}{m} \]

\[ Lf(0.6 \cdot B) = 9.552 \cdot 10^{-5} \]

\[ Z_{TIA\_N}(\omega, rt) := \frac{1 + 1i \cdot \omega \cdot \frac{Lf(rt)}{Rf_{-N}(rt)}}{\left(1 - \omega^2 \cdot Lf(rt) \cdot \frac{C_T}{1 + Av}\right) + Rf_{-N}(rt) \cdot \omega \cdot \frac{C_T}{(1 + Av) \cdot 1i}} \]
**Receiver freq-response**

\[ Z_N(\omega, rd, rt) := H_{\text{btl}}(\omega, rd) \cdot Z_{\text{TIA}}(\omega, rt) \]

\[ \omega := 1 \cdot 10^6, 10 \cdot 10^6, \ldots 1 \cdot 10^{11} \]

\[ rd := 0.7 \cdot B, 0.75 \cdot B, 2 \cdot B \quad rt := 0.5 \cdot B, 0.55 \cdot B, 1 \cdot B \]

**Noise equivalent bandwidth (NEB and I2)**

\[ NEB_N(rd, rt) := \frac{1}{2 \pi} \cdot \left( \int_0^{10^3} \left( |Z_N(\omega, rd, rt)|^2 \right) d\omega \right) \]

\[ NEB_B(rd, rt) := NEB_N(rd, rt) \]

**Noise equivalent bandwidth (I2)**

\[ I2_A(rd, rt) := \frac{1}{2 \pi} \cdot \left( \int_0^{10^3} \left( \left( \frac{1}{1 + i \cdot \omega \cdot \frac{L_f(rt)}{R_f N(rt)}} \right) \cdot Z_N(\omega, rd, rt) \right)^2 d\omega \right) \]
Noise equivalent bandwidth (13)

\[
I_{3\cdot N}(rd, rt) := \frac{1}{2 \cdot \pi} \int_0^{10^9} \left( \left( \frac{1 - \omega^2 \cdot LF(rt) \cdot CT}{Rf_N(rt) \cdot \omega \cdot CT \cdot Li} \right) \cdot Z_N(\omega, rd, rt) \right)^2 d\omega
\]

Feedback noise

\[
q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298
\]

\[
I_{nRf_N}(rd, rt) := \frac{4 \cdot k \cdot T \cdot J_{2\cdot A}(rd, rt)}{Rf_N(rt)}
\]

Ig noise (gate current)

\[
Ig := 10 \cdot 10^{-9}
\]

\[
I_{nG_N}(rd, rt) := 2 \cdot q \cdot Ig \cdot NEB_N(rd, rt)
\]
Channel noise (Gate-source)

\[ \text{gm1} = 30 \cdot 10^{-3} = 0.03 \quad \text{noise}_{\text{factor}} = 1 \]

\[ I_{nD} = 4 \cdot k \cdot T \cdot \frac{1}{\text{gm1}} \]

\[ I_{nD,N}(rd, rt) = I_{nD} \cdot I_{3 \cdot N}(rd, rt) \]

\[ I_{nD,N}(0.7 B, 0.5 B) = 4.118 \cdot 10^{-18} \]

Total noise - PIN-FET

\[ I_{n_B}(rd, rt) = I_{nRF,N}(rd, rt) + I_{nG,N}(rd, rt) + I_{nD,N}(rd, rt) \]
\[ I_{n_B}(0.7 B, 0.5 B) = 1.86 \times 10^{-17} \]
\[ I_{n_B}(0.8 B, 0.5 B) = 1.96 \times 10^{-17} \]
\[ I_{n_B}(0.9 B, 0.5 B) = 2.06 \times 10^{-17} \]
\[ I_{n_B}(1 B, 0.5 B) = 2.15 \times 10^{-17} \]

\[ I_{n_B}(0.7 B, 0.55 B) = 2.2 \times 10^{-17} \]
\[ I_{n_B}(0.8 B, 0.55 B) = 2.33 \times 10^{-17} \]
\[ I_{n_B}(0.9 B, 0.55 B) = 2.45 \times 10^{-17} \]
\[ I_{n_B}(1 B, 0.55 B) = 2.57 \times 10^{-17} \]

\[ I_{n_B}(0.7 B, 0.7 B) = 3.34 \times 10^{-17} \]
\[ I_{n_B}(0.8 B, 0.7 B) = 3.58 \times 10^{-17} \]
\[ I_{n_B}(0.9 B, 0.7 B) = 3.79 \times 10^{-17} \]
\[ I_{n_B}(1 B, 0.7 B) = 3.99 \times 10^{-17} \]

\[ I_{n_B}(0.7 B, 0.75 B) = 3.75 \times 10^{-17} \]
\[ I_{n_B}(0.8 B, 0.75 B) = 4.03 \times 10^{-17} \]
\[ I_{n_B}(0.9 B, 0.75 B) = 4.29 \times 10^{-17} \]
\[ I_{n_B}(1 B, 0.75 B) = 4.52 \times 10^{-17} \]
Output pulse shape

\[ t_s := \frac{1}{B} \quad B = 1 \cdot 10^8 \]

\[ v_{out}(t, rd, rt) := \frac{t_s}{\pi} \cdot \frac{1}{t_s \cdot 10^3} \left\{ \sin \left( \omega \cdot \frac{t_s}{2} \right) \cdot \text{Re} \left( Z_N(\omega, rd, rt) \cdot \exp \left( i \cdot \omega \cdot (t) \right) \right) \right\} \text{d}\omega \]

\[ x_{Bt}(t) := \frac{t_s}{\pi} \cdot \frac{1}{t_s \cdot 10^3} \left\{ \sin \left( \omega \cdot \frac{t_s}{2} \right) \cdot \text{Re} \left( \exp \left( i \cdot \omega \cdot (t) \right) \right) \right\} \text{d}\omega \]

\[ x_{vout}(t, rd, rt) := \frac{t_s^2}{\pi} \cdot \frac{1}{t_s \cdot 10^3} \left\{ \sin \left( \omega \cdot \frac{t_s}{2} \right) \cdot \text{Re} \left( 1 \cdot \omega \cdot Z_N(\omega, rd, rt) \cdot \exp \left( i \cdot \omega \cdot (t) \right) \right) \right\} \text{d}\omega \]

\[ t := -1 \cdot t_s, -0.99 \cdot t_s, .4 \cdot t_s \]
\[ t := -1 \, ts, -0.99 \, ts..6 \, ts \]

\[ V_1(t, rd, rt) := vout(t, rd, rt) \]
\[ V_{101}(t, rd, rt) = v_{out}(t, rd, rt) + v_{out}(t - 2 \cdot ts, rd, rt) \]
\( t_{B1} := 0.5 \cdot ts \)
\( t_{pk1} := \text{root}(xvout(t_{B1}, 0.7 B, 0.5 B), t_{B1}) \)
\[ t_{pk1} = 0.576 \]
\( v_{max_{B1}} := vout(t_{pk1}, 0.7 B, 0.5 B) = 0.94 \]
\( v_{min_{B1}} := vout(t_{pk1} + 1 \cdot ts, 0.7 B, 0.5 B) = 0.094 \]
\( v_{min1_{B1}} := vout(t_{pk1} + 2 \cdot ts, 0.7 B, 0.5 B) = -0.055 \]
\( v_{min2_{B1}} := vout(t_{pk1} + 3 \cdot ts, 0.7 B, 0.5 B) = -0.001 \]
\[ v_{max_{B1}} - v_{min_{B1}} = 0.846 \quad v_{max_{B1}} - v_{min1_{B1}} = 0.995 \quad v_{max_{B1}} - v_{min2_{B1}} = 0.941 \]
\[
\frac{v_{max_{B1}} - v_{min_{B1}}}{\sqrt{10^{17} \cdot I_{n,B}(0.7 B, 0.5 B)}} = 0.62 \quad \text{1st pulse (worst ISI)}
\]
\[
\frac{v_{max_{B1}} - v_{min1_{B1}}}{\sqrt{10^{17} \cdot I_{n,B}(0.7 B, 0.5 B)}} = 0.73 \quad \text{2nd pulse}
\]
\[
\frac{v_{max_{B1}} - v_{min2_{B1}}}{\sqrt{10^{17} \cdot I_{n,B}(0.7 B, 0.5 B)}} = 0.69 \quad \text{3rd pulse}
\]
\( t_{B2} := 0.5 \cdot ts \)
\( t_{pk2} := \text{root}(xvout(t_{B2}, 0.8 B, 0.5 B), t_{B2}) \)
\[ t_{pk2} = 0.563 \]
\( v_{max_{B2}} := vout(t_{pk2}, 0.8 B, 0.5 B) = 0.959 \]
\( v_{min_{B2}} := vout(t_{pk2} + 1 \cdot ts, 0.8 B, 0.5 B) = 0.08 \]
\( v_{min1_{B2}} := vout(t_{pk2} + 2 \cdot ts, 0.8 B, 0.5 B) = -0.055 \]
\( v_{min2_{B2}} := vout(t_{pk2} + 3 \cdot ts, 0.8 B, 0.5 B) = -3.193 \cdot 10^{-4} \]
\[ v_{max_{B2}} - v_{min_{B2}} = 0.879 \quad v_{max_{B2}} - v_{min1_{B2}} = 1.014 \quad v_{max_{B2}} - v_{min2_{B2}} = 0.959 \]
\[
\frac{v_{max_{B2}} - v_{min_{B2}}}{\sqrt{10^{17} \cdot I_{n,B}(0.8 B, 0.5 B)}} = 0.63 \quad \text{1st pulse (worst ISI)}
\]
\[
\frac{v_{max_{B2}} - v_{min1_{B2}}}{\sqrt{10^{17} \cdot I_{n,B}(0.8 B, 0.5 B)}} = 0.72 \quad \text{2nd pulse}
\]
\[
\frac{v_{max_{B2}} - v_{min2_{B2}}}{\sqrt{10^{17} \cdot I_{n,B}(0.8 B, 0.5 B)}} = 0.68 \quad \text{3rd pulse}
\]
\( t_{B3} := 0.5 \cdot ts \)

\[ t_{pk3} := \text{root} \left( xvout \left(t_{B3}, 0.9 \ B, 0.5 \ B\right), t_{B3}\right) \]

\[ \frac{t_{pk3}}{ts} = 0.553 \]

\[ v_{max\_B3} := \text{vout} \left(t_{pk3}, 0.9 \ B, 0.5 \ B\right) = 0.973 \]

\[ v_{min\_B3} := \text{vout} \left(t_{pk3} + 1 \cdot ts, 0.9 \ B, 0.5 \ B\right) = 0.068 \]

\[ v_{min1\_B3} := \text{vout} \left(t_{pk3} + 2 \cdot ts, 0.9 \ B, 0.5 \ B\right) = -0.055 \]

\[ v_{min2\_B3} := \text{vout} \left(t_{pk3} + 3 \cdot ts, 0.9 \ B, 0.5 \ B\right) = 2.237 \cdot 10^{-4} \]

\[ v_{max\_B3} - v_{min\_B3} = 0.904 \quad v_{max\_B3} - v_{min1\_B3} = 1.028 \quad v_{max\_B3} - v_{min2\_B3} = 0.972 \]

\[
\frac{v_{max\_B3} - v_{min\_B3}}{\sqrt{10^{17} \cdot I_{n\_B} (0.9 \ B, 0.5 \ B)}} = 0.63 \\
\text{1st pulse} \quad \text{(worst ISI)}
\]

\[
\frac{v_{max\_B3} - v_{min1\_B3}}{\sqrt{10^{17} \cdot I_{n\_B} (0.9 \ B, 0.5 \ B)}} = 0.72 \\
\text{2nd pulse}
\]

\[
\frac{v_{max\_B3} - v_{min2\_B3}}{\sqrt{10^{17} \cdot I_{n\_B} (0.9 \ B, 0.5 \ B)}} = 0.68 \\
\text{3rd pulse}
\]

\( (rt = 0.6) \)

\( t_{B4} := 0.5 \cdot ts \)

\[ t_{pk4} := \text{root} \left( xvout \left(t_{B4}, 0.7 \ B, 0.6 \ B\right), t_{B4}\right) \]

\[ \frac{t_{pk4}}{ts} = 0.547 \]

\[ v_{max\_B4} := \text{vout} \left(t_{pk4}, 0.7 \ B, 0.6 \ B\right) = 0.992 \]

\[ v_{min\_B4} := \text{vout} \left(t_{pk4} + 1 \cdot ts, 0.7 \ B, 0.6 \ B\right) = 0.025 \]

\[ v_{min1\_B4} := \text{vout} \left(t_{pk4} + 2 \cdot ts, 0.7 \ B, 0.6 \ B\right) = -0.031 \]

\[ v_{min2\_B4} := \text{vout} \left(t_{pk4} + 3 \cdot ts, 0.7 \ B, 0.6 \ B\right) = 0.004 \]

\[ v_{max\_B4} - v_{min\_B4} = 0.967 \quad v_{max\_B4} - v_{min1\_B4} = 1.023 \quad v_{max\_B4} - v_{min2\_B4} = 0.988 \]

\[
\frac{v_{max\_B4} - v_{min\_B4}}{\sqrt{10^{17} \cdot I_{n\_B} (0.7 \ B, 0.6 \ B)}} = 0.6 \\
\text{1st pulse} \quad \text{(worst ISI)}
\]

\[
\frac{v_{max\_B4} - v_{min1\_B4}}{\sqrt{10^{17} \cdot I_{n\_B} (0.7 \ B, 0.6 \ B)}} = 0.64 \\
\text{2nd pulse}
\]

\[
\frac{v_{max\_B4} - v_{min2\_B4}}{\sqrt{10^{17} \cdot I_{n\_B} (0.7 \ B, 0.6 \ B)}} = 0.62 \\
\text{3rd pulse}
\]

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\[ t_{B5} := 0.5 \cdot ts \]

\[ t_{pk5} := \text{root} \left( xvout \left( t_{B5}, 0.8 \, B, 0.6 \, B \right), t_{B5} \right) \]

\[ \frac{t_{pk5}}{ts} = 0.538 \]

\[ v_{max_{B5}} := \text{vout} \left( t_{pk5}, 0.8 \, B, 0.6 \, B \right) = 1.011 \]

\[ v_{min_{B5}} := \text{vout} \left( t_{pk5} + 1 \cdot ts, 0.8 \, B, 0.6 \, B \right) = 0.008 \]

\[ v_{min1_{B5}} := \text{vout} \left( t_{pk5} + 2 \cdot ts, 0.8 \, B, 0.6 \, B \right) = -0.03 \]

\[ v_{min2_{B5}} := \text{vout} \left( t_{pk5} + 3 \cdot ts, 0.8 \, B, 0.6 \, B \right) = 0.004 \]

\[ v_{max_{B5}} - v_{min_{B5}} = 1.003 \quad v_{max_{B5}} - v_{min1_{B5}} = 1.041 \quad v_{max_{B5}} - v_{min2_{B5}} = 1.007 \]

\[ \frac{v_{max_{B5}} - v_{min_{B5}}}{\sqrt{10^{17} \cdot I_{n_{B}} (0.8 \, B, 0.6 \, B)}} = 0.61 \quad \text{1st pulse} \]

\[ \frac{v_{max_{B5}} - v_{min1_{B5}}}{\sqrt{10^{17} \cdot I_{n_{B}} (0.8 \, B, 0.6 \, B)}} = 0.63 \quad \text{2nd pulse} \]

\[ \frac{v_{max_{B5}} - v_{min2_{B5}}}{\sqrt{10^{17} \cdot I_{n_{B}} (0.8 \, B, 0.6 \, B)}} = 0.61 \quad \text{3rd pulse (worst ISI)} \]

\[ t_{B6} := 0.5 \cdot ts \]

\[ t_{pk6} := \text{root} \left( xvout \left( t_{B4}, 0.9 \, B, 0.6 \, B \right), t_{B4} \right) \]

\[ \frac{t_{pk6}}{ts} = 0.532 \]

\[ v_{max_{B6}} := \text{vout} \left( t_{pk6}, 0.9 \, B, 0.6 \, B \right) = 1.025 \]

\[ v_{min_{B6}} := \text{vout} \left( t_{pk6} + 1 \cdot ts, 0.9 \, B, 0.6 \, B \right) = -0.006 \]

\[ v_{min1_{B6}} := \text{vout} \left( t_{pk6} + 2 \cdot ts, 0.9 \, B, 0.6 \, B \right) = -0.028 \]

\[ v_{min2_{B6}} := \text{vout} \left( t_{pk6} + 3 \cdot ts, 0.9 \, B, 0.6 \, B \right) = 0.004 \]

\[ v_{max_{B6}} - v_{min_{B6}} = 1.031 \quad v_{max_{B6}} - v_{min1_{B6}} = 1.053 \quad v_{max_{B6}} - v_{min2_{B6}} = 1.02 \]

\[ \frac{v_{max_{B6}} - v_{min_{B6}}}{\sqrt{10^{17} \cdot I_{n_{B}} (0.9 \, B, 0.6 \, B)}} = 0.61 \quad \text{1st pulse} \]

\[ \frac{v_{max_{B6}} - v_{min1_{B6}}}{\sqrt{10^{17} \cdot I_{n_{B}} (0.9 \, B, 0.6 \, B)}} = 0.62 \quad \text{2nd pulse} \]

\[ \frac{v_{max_{B6}} - v_{min2_{B6}}}{\sqrt{10^{17} \cdot I_{n_{B}} (0.9 \, B, 0.6 \, B)}} = 0.6 \quad \text{3rd pulse (worst ISI)} \]
\( t_{B7} := 0.5 \cdot ts \)
\( t_{pk7} := \sqrt{v_{out}(t_{B7}, 0.7 B, 0.7 B)} \)
\( \frac{t_{pk7}}{ts} = 0.529 \)

\( v_{\text{max}_{B7}} := v_{out}(t_{pk7}, 0.7 B, 0.7 B) = 1.021 \)
\( v_{\text{min}_{B7}} := v_{out}(t_{pk7} + 1 \cdot ts, 0.7 B, 0.7 B) = -0.017 \)
\( v_{\text{min1}_{B7}} := v_{out}(t_{pk7} + 2 \cdot ts, 0.7 B, 0.7 B) = -0.011 \)
\( v_{\text{min2}_{B7}} := v_{out}(t_{pk7} + 3 \cdot ts, 0.7 B, 0.7 B) = 0.002 \)

\[
\frac{v_{\text{max}_{B7}} - v_{\text{min}_{B7}}}{\sqrt{10^{17} \cdot I_n(B, 0.7 B)}} = 0.57 \\
\frac{v_{\text{max}_{B7}} - v_{\text{min1}_{B7}}}{\sqrt{10^{17} \cdot I_n(B, 0.7 B)}} = 0.56 \\
\frac{v_{\text{max}_{B7}} - v_{\text{min2}_{B7}}}{\sqrt{10^{17} \cdot I_n(B, 0.7 B)}} = 0.56
\]

1st pulse

\( t_{B8} := 0.5 \cdot ts \)
\( t_{pk8} := \sqrt{v_{out}(t_{B8}, 0.8 B, 0.7 B)} \)
\( \frac{t_{pk8}}{ts} = 0.523 \)

\( v_{\text{max}_{B8}} := v_{out}(t_{pk8}, 0.8 B, 0.7 B) = 1.039 \)
\( v_{\text{min}_{B8}} := v_{out}(t_{pk8} + 1 \cdot ts, 0.8 B, 0.7 B) = -0.034 \)
\( v_{\text{min1}_{B8}} := v_{out}(t_{pk8} + 2 \cdot ts, 0.8 B, 0.7 B) = -0.009 \)
\( v_{\text{min2}_{B8}} := v_{out}(t_{pk8} + 3 \cdot ts, 0.8 B, 0.7 B) = 0.002 \)

\[
\frac{v_{\text{max}_{B8}} - v_{\text{min}_{B8}}}{\sqrt{10^{17} \cdot I_n(B, 0.8 B)}} = 0.57 \\
\frac{v_{\text{max}_{B8}} - v_{\text{min1}_{B8}}}{\sqrt{10^{17} \cdot I_n(B, 0.8 B)}} = 0.55 \\
\frac{v_{\text{max}_{B8}} - v_{\text{min2}_{B8}}}{\sqrt{10^{17} \cdot I_n(B, 0.8 B)}} = 0.55
\]

1st pulse

\( \text{worst ISI} \)
\[ t_{B9} := 0.5 \cdot ts \]
\[ t_{pk9} := \text{root}(xvout(t_{B9}, 0.9 \ B, 0.7 \ B), t_{B9}) \]
\[ t_{pk9} = 0.518 \]
\[ ts \]
\[ v_{\max \_B9} := xvout(t_{pk9}, 0.9 \ B, 0.7 \ B) = 1.051 \]
\[ v_{\min \_B9} := xvout(t_{pk9} + 1 \cdot ts, 0.9 \ B, 0.7 \ B) = -0.047 \]
\[ v_{\min1 \_B9} := xvout(t_{pk9} + 2 \cdot ts, 0.9 \ B, 0.7 \ B) = -0.008 \]
\[ v_{\min2 \_B9} := xvout(t_{pk9} + 3 \cdot ts, 0.9 \ B, 0.7 \ B) = 0.002 \]
\[ v_{\max \_B9} - v_{\min \_B9} = 1.099 \quad v_{\max \_B9} - v_{\min1 \_B9} = 1.059 \quad v_{\max \_B9} - v_{\min2 \_B9} = 1.049 \]
\[ \frac{v_{\max \_B9} - v_{\min \_B9}}{\sqrt{10^{17} \cdot I_{n \_B}(0.9 \ B, 0.7 \ B)}} = 0.56 \quad \text{1st pulse} \]
\[ \frac{v_{\max \_B9} - v_{\min1 \_B9}}{\sqrt{10^{17} \cdot I_{n \_B}(0.9 \ B, 0.7 \ B)}} = 0.54 \quad \text{2nd pulse} \]
\[ \frac{v_{\max \_B9} - v_{\min2 \_B9}}{\sqrt{10^{17} \cdot I_{n \_B}(0.9 \ B, 0.7 \ B)}} = 0.54 \quad \text{3rd pulse \ worst ISI} \]

\[ \text{opt (3-dB cut-off)} \]
\[ rt \text{ opt } > 0.6 \]
\[ rd \text{ opt } > 0.5:0.7 \]

\[ v_{\max \_B} := v_{\max \_B2} \quad v_{\min \_B} := v_{\min \_B2} \]

**Eye-diagram check**

\[ x_{111}(t, rd, rt) := xvout(t, rd, rt) + xvout(t - 2 \ ts, rd, rt) + xvout(t - ts, rd, rt) \]
\[ x_{110}(t, rd, rt) := xvout(t, rd, rt) + xvout(t - ts, rd, rt) \]
\[ x_{100}(t, rd, rt) := xvout(t, rd, rt) \]
\[ x_{001}(t, rd, rt) := xvout(t - 2 \ ts, rd, rt) \]
\[ x_{010}(t, rd, rt) := xvout(t - ts, rd, rt) \]
\[ x_{011}(t, rd, rt) := xvout(t - ts, rd, rt) + xvout(t - 2 \ ts, rd, rt) \]
\[ x_{101}(t, rd, rt) := xvout(t, rd, rt) + xvout(t - 2 \ ts, rd, rt) \]
Check ratio >1 is ok 100, <1 re-opt

\[
x_{101}(t_{pk4} + ts, 0.7 B, 0.6 B) = 0.035
\]

\[
x_{101}(t_{pk4}, 0.7 B, 0.6 B) = 0.992
\]

\[
x_{101}(t_{pk4} + 2 ts, 0.7 B, 0.6 B) = 0.962
\]

\[
x_{100}(t_{pk4}, 0.7 B, 0.6 B) = 0.992
\]

\[
x_{100}(t_{pk4} + ts, 0.7 B, 0.6 B) = 0.025
\]

\[
x_{100}(t_{pk4} + 2 ts, 0.7 B, 0.6 B) = -0.031
\]

Sens PINFET

\[
\lambda := 650 \cdot 10^{-9}
\]

\[
\text{photon}_\text{energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda}
\]

\[
nq := 1.6 \cdot 10^{-19}
\]

\[
Q_N(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{max_B} - v_{min_B}}{2} \right)
\]

\[
P\epsilon_N(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right)
\]
\[ P_{c,N}(b) := (\log(P_{e,N}(b)) + 9) \]

\[ b := 3 \times 10^3 \]

\[ b_N := \text{root}(P_{c,N}(b), b) \]

**minimum \_B := min\{ b_N \} = 3.779 \times 10^3**

**APD-1FET**

\[ APD_N(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot (v_{\text{max}_B}) \]

\[ APD_B(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot (v_{\text{max}_B}) \]

\[ I_{APD} := 10 \times 10^{-9} \]

\[ M_{APD} := 100 \]

\[ F_{M1} := 7.9 \]

\[ \text{Noise}_{APD1\_B}(b, rd, rt) := \left( 2 \cdot q \cdot APD_B(b) \cdot M_{APD} \cdot F_{M1} \right) \cdot \text{NEB}_B(rd, rt) \]

\[ + 2 \cdot q \cdot I_{APD} \cdot M_{APD} \cdot F_{M1} \]

\[ Q_{APD1\_B}(b) := \frac{M_{APD}}{t_s} \cdot b \cdot nq \cdot \frac{v_{\text{max}_B} - v_{\text{min}_B}}{2} \]

\[ \sqrt{I_{n\_B}(0.7 \ B \ , \ 0.6 \ B) + \text{Noise}_{APD1\_B}(b, 0.7 \ B \ , \ 0.6 \ B)} \]

\[ P_{c,APD1\_B}(b) := \frac{\text{erfc}\left( \frac{Q_{APD1\_B}(b)}{2} \right)}{2} \]

\[ p_{APD1\_B}(b) := (\log(P_{c,APD1\_B}(b)) + 9) \]

\[ APD1\_b\_B := \text{root}(p_{APD1\_B}(b), b) \]

\[ \text{NEB}_B(0.7 \ B \ , \ 0.6 \ B) = 5.528 \times 10^7 \]

**minimum \_APD1\_B := min\{ APD1\_b\_B \} = 2.055 \times 10^3**

**APD-2FET**

\[ M_{APD} := 10 \quad F_{M2} := 5.5 \]

\[ \text{Noise}_{APD2\_B}(b, rd, rt) := \left( 2 \cdot q \cdot APD_B(b) \cdot M_{APD}^2 \cdot F_{M2} \right) \cdot \text{NEB}_B(rd, rt) \]

\[ + 2 \cdot q \cdot I_{APD} \cdot M_{APD}^2 \cdot F_{M2} \]

\[ Q_{APD2\_B}(b) := \frac{M_{APD}}{t_s} \cdot b \cdot nq \cdot \frac{v_{\text{max}_B} - v_{\text{min}_B}}{2} \]

\[ \sqrt{I_{n\_B}(0.7 \ B \ , \ 0.6 \ B) + \text{Noise}_{APD2\_B}(b, 0.7 \ B \ , \ 0.6 \ B)} \]

\[ P_{c,APD2\_B}(b) := \frac{1}{2} \cdot \text{erfc}\left( \frac{Q_{APD2\_B}(b)}{2} \right) \]

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\[ p_{\text{APD2}_B}(b) := (\log(P_{e-\text{APD2}_B}(b)) + 9) \]

\[ \text{APD2}_B := \text{root}(p_{\text{APD2}_B}(b), b) \]

\[ \text{minimum}_{\text{APD2}_B} := \min(\text{APD2}_B, b) = 1.633 \cdot 10^3 \]

**APD-3FET**

\[ M_{\text{APD3}} := 10 \quad F_{M3} := 9.2 \]

\[ \text{Noise}_{\text{APD3}_B}(b, rd, rt) := \left( 2 \cdot q \cdot \text{APD}_B(b) \cdot M_{\text{APD3}}^2 \cdot F_{M3} \right) \cdot \text{NEB}_B(rd, rt) + 2 \cdot q \cdot I_{\text{APD}_d} \cdot M_{\text{APD3}}^2 \cdot F_{M3} \]

\[ Q_{\text{APD3}_B}(b) := \frac{M_{\text{APD3}}}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max}_B} - v_{\text{min}_B}}{2} \right) \]

\[ \text{APD3}_B := \text{root}(p_{\text{APD3}_B}(b), b) \]

\[ \text{minimum}_{\text{APD3}_B} := \min(\text{APD3}_B, b) = 2.389 \cdot 10^3 \]

\[ \text{Sens}_{\text{N PIN}} := 10 \cdot \log \left( \frac{\text{minimum}_B \cdot \text{photon}_{\text{energy}} \cdot 1}{ts} \right) = -42.379 \]

\[ \text{Sens}_{\text{N APD silicon}} := 10 \cdot \log \left( \frac{\text{minimum}_{\text{APD1}_B} \cdot \text{photon}_{\text{energy}} \cdot 1}{ts} \right) = -45.025 \]

\[ \text{Sens}_{\text{N APD InGaAs}} := 10 \cdot \log \left( \frac{\text{minimum}_{\text{APD2}_B} \cdot \text{photon}_{\text{energy}} \cdot 1}{ts} \right) = -46.023 \]

\[ \text{Sens}_{\text{N APD germanium}} := 10 \cdot \log \left( \frac{\text{minimum}_{\text{APD3}_B} \cdot \text{photon}_{\text{energy}} \cdot 1}{ts} \right) = -44.37 \]

#
B.2.3 Tuned B a=0

Tuned B Optimum Performance (PIN-FET APD-FET 100 Mbit/s) a=0

Rx performance opt PIN-FET and APD-FET input configurations, 1st order LPF pre-detection filter. opt as follows
- TIA 3-dB bandwidth rt/Filter 3-dB bandwidth rd
- range cut-off
- minimum noise
- examine ISI
- highest SNR
- BER
- APD

**Bit-rate, pulse duration**

\[ C_T := 1.5 \cdot 10^{-12} \text{ total C} \]

\[ B := 100 \cdot 10^6 \text{ Bit-rate} \]

**pre-dec filter**

\[ \omega_B (rd) := 2 \cdot \pi \cdot rd \]

\[ H_{\text{bnt}} (\omega, rd) := \frac{1}{1 + j \cdot \frac{\omega}{2 \cdot \pi \cdot rd}} \]

**TIA(Non-tuned)**

\[ \omega_c (rt) := 2 \cdot \pi \cdot rt \]

\[ A_v := 10 \]

\[ Rf_N (rt) := \frac{A_v + 1}{2 \cdot \pi \cdot rt \cdot C_T} \]

\[ Z_{TIA,N} (\omega, rt) := \frac{1}{1 + j \cdot \frac{\omega}{2 \cdot \pi \cdot rt}} \]

**TIA**

Feedback value for (R) Tuned B

\[ \Delta_L := 2 \quad \Delta_R := 1.41 \quad \text{feedback } \Delta R \text{, time constant ratio } \Delta L \]

\[ Rf_B (rt) := \Delta_R \cdot \frac{A_v + 1}{2 \cdot \pi \cdot rt \cdot C_T} \quad \text{feedback for Tuned B} \]

\[ \alpha := 0 \quad \text{splitting ratio} \]

\[ \begin{bmatrix} \alpha & \Delta_L & \Delta_R \\ 0 & 2 & 1.41 \\ 0.1 & 1.8 & 1.58 \\ 0.2 & 1.8 & 1.87 \\ 0.3 & 2.4 & 2.52 \\ 0.4 & 1.9 & 2.75 \\ 0.4 & 2.5 & 3.17 \\ 0.5 & 1.5 & 2.65 \end{bmatrix} \]
\[
Lc(rt) := \frac{\left( \frac{Rf_B(rt)}{1+Av} \right)^2 \cdot C_T}{\Delta_L}
\]

\[
C1 := (1 - \alpha) \cdot (C_T) = 1.5 \cdot 10^{-12}
\]

\[
C2 := \alpha \cdot (C_T) = 0
\]

**TUNED B (R) Transimpedance**

\[
Z_{TIA,B}(\omega, rt) = \frac{1}{\left( (1 - \omega^2 \cdot Lc(rt) \cdot C1) + \frac{Rf_B(rt)}{(1+Av)} \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc(rt) \cdot C1 \cdot C2) \right)}
\]

**Receiver freq-response**

\[
Z_N(\omega, rd, rt) := H_{in}(\omega, rd) \cdot Z_{TIA,N}(\omega, rt)
\]

\[
Z_B(\omega, rd, rt) := H_{in}(\omega, rd) \cdot Z_{TIA,B}(\omega, rt)
\]

\[
\omega := 1 \cdot 10^6, 10 \cdot 10^6 .. 1 \cdot 10^{11}
\]

\[
rd := 0.7 \cdot B, 0.75 \cdot B, .2 \cdot B
\]

\[
rt := 0.5 \cdot B, 0.55 \cdot B, .1 \cdot B
\]

**Noise equivalent bandwidth (NEB and I2)**

\[
NEB_N(rd, rt) := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{15}} \left( \left| Z_N(\omega, rd, rt) \right|^2 \right) d\omega \right)
\]

\[
NEB_B(rd, rt) := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{15}} \left( \left| Z_B(\omega, rd, rt) \right|^2 \right) d\omega \right)
\]

range >NEB >APD

---

**NEB_N(rd, rt)**

NEB >APD#0.55-0.65 rt/ ++>2 rd

---

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\[ I_{2B}(rd, rt) := \frac{1}{2 \cdot \pi} \left[ \frac{1}{\left( \left( 1 - \omega^2 \cdot Lc(rt) \cdot C1 \right) \cdot \left( \frac{d}{Z_B(\omega, rd, rt)} \right) \right)^2} \right] d\omega \]
Noise equivalent bandwidth (13)

\[ I_{3\_N}(rd, rt) = \frac{1}{2 \cdot \pi} \int_{0}^{10^{12}} \left( \frac{(1 + 1j \cdot \omega \cdot C_{T} \cdot Rf_{N}(rt))}{Rf_{N}(rt)} \cdot Z_{N}(\omega, rd, rt) \right)^{2} \, d\omega \]

\[ I_{3\_B}(rd, rt) = \frac{1}{2 \cdot \pi} \int_{0}^{10^{12}} \left( \frac{(1 - \omega^{2} \cdot Lc(rt) \cdot C1)^{\downarrow}}{Rf_{B}(rt) \cdot (\omega)^{\downarrow}} + \frac{Rf_{B}(rt) \cdot (\omega)^{\downarrow}}{(C1 + C2 - (\omega)^{2} \cdot Lc(rt) \cdot C1 \cdot C2) \cdot 1i} \cdot Z_{B}(\omega, rd, rt) \right)^{2} \, d\omega \]

range >FET,D rt 0.7, 1.1 rd
Feedback noise

\[ q := 1.6 \cdot 10^{-10} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \]

\[ I_{n_{RF,N}}(rd, rt) := \frac{4 \cdot k \cdot T}{Rf_N(rt)} \cdot NEB_N(rd, rt) \]

\[ I_{n_{RF,B}}(rd, rt) := \frac{4 \cdot k \cdot T}{Rf_B(rt)} \cdot I_{2_B}(rd, rt) \]
**Ig noise (gate current)**

\[ I_g = 10 \cdot 10^{-9} \]

\[ I_{nG,N}(rd,rt) = 2 \cdot q \cdot I_g \cdot NEB_N(rd,rt) \]

\[ I_{nG,B}(rd,rt) = 2 \cdot q \cdot I_g \cdot I_{2\_B}(rd,rt) \]

**Channel noise (Gate-source)**

\[ g_m1 = 30 \cdot 10^{-3} = 0.03 \quad \text{noise\_factor} = 1 \]

\[ I_{nD} = 4 \cdot k \cdot T \cdot \frac{1}{g_m1} \]

\[ I_{nD,N}(rd,rt) = I_{nD} \cdot I_{3\_N}(rd,rt) \]

\[ I_{nD,B}(rd,rt) = I_{nD} \cdot I_{3\_B}(rd,rt) \]
$$I_{n,N}(rd, rt) := I_{nRF,N}(rd, rt) + I_{nG,N}(rd, rt) + I_{nD,N}(rd, rt)$$

$$I_{n,B}(rd, rt) := I_{nRF,B}(rd, rt) + I_{nG,B}(rd, rt) + I_{nD,B}(rd, rt)$$
\[ I_{n,B}(0.7 \, B, \, 0.5 \, B) = 3.58 \cdot 10^{-17} \]
\[ I_{n,B}(0.8 \, B, \, 0.5 \, B) = 4.16 \cdot 10^{-17} \]
\[ I_{n,B}(0.9 \, B, \, 0.5 \, B) = 4.77 \cdot 10^{-17} \]
\[ I_{n,B}(1 \, B, \, 0.5 \, B) = 5.39 \cdot 10^{-17} \]

\[ I_{n,B}(0.7 \, B, \, 0.55 \, B) = 3.94 \cdot 10^{-17} \]
\[ I_{n,B}(0.8 \, B, \, 0.55 \, B) = 4.56 \cdot 10^{-17} \]
\[ I_{n,B}(0.9 \, B, \, 0.55 \, B) = 5.2 \cdot 10^{-17} \]
\[ I_{n,B}(1 \, B, \, 0.55 \, B) = 5.87 \cdot 10^{-17} \]

\[ I_{n,B}(0.7 \, B, \, 0.7 \, B) = 5.08 \cdot 10^{-17} \]
\[ I_{n,B}(0.8 \, B, \, 0.7 \, B) = 5.82 \cdot 10^{-17} \]
\[ I_{n,B}(0.9 \, B, \, 0.7 \, B) = 6.59 \cdot 10^{-17} \]
\[ I_{n,B}(1 \, B, \, 0.7 \, B) = 7.39 \cdot 10^{-17} \]

0.8/0.7>rd, range rt 0.55-0.75

\[ I_{n,B}(0.7 \, B, \, 0.6 \, B) = 4.3 \cdot 10^{-17} \]
\[ I_{n,B}(0.8 \, B, \, 0.6 \, B) = 4.96 \cdot 10^{-17} \]
\[ I_{n,B}(0.9 \, B, \, 0.6 \, B) = 5.65 \cdot 10^{-17} \]
\[ I_{n,B}(1 \, B, \, 0.6 \, B) = 6.37 \cdot 10^{-17} \]

\[ I_{n,B}(0.7 \, B, \, 0.65 \, B) = 4.68 \cdot 10^{-17} \]
\[ I_{n,B}(0.8 \, B, \, 0.65 \, B) = 5.38 \cdot 10^{-17} \]
\[ I_{n,B}(0.9 \, B, \, 0.65 \, B) = 6.11 \cdot 10^{-17} \]
\[ I_{n,B}(1 \, B, \, 0.65 \, B) = 6.87 \cdot 10^{-17} \]

\[ I_{n,B}(0.7 \, B, \, 0.75 \, B) = 5.49 \cdot 10^{-17} \]
\[ I_{n,B}(0.8 \, B, \, 0.75 \, B) = 6.27 \cdot 10^{-17} \]
\[ I_{n,B}(0.9 \, B, \, 0.75 \, B) = 7.08 \cdot 10^{-17} \]
\[ I_{n,B}(1 \, B, \, 0.75 \, B) = 7.92 \cdot 10^{-17} \]
Output pulse shape

\[ ts := \frac{1}{B} \quad B = 1 \cdot 10^8 \]

\[ vout(t, rd, rt) := ts \cdot \frac{1}{\pi} \int_{0}^{\frac{1}{ts} \cdot 10^5} \frac{\sin \left( \frac{\omega \cdot ts}{2} \right)}{\left( \frac{\omega \cdot ts}{2} \right)} \cdot \text{Re} \left( Z_B(\omega, rd, rt) \cdot \exp \left( 1i \cdot \omega \cdot t(\omega) \right) \right) d\omega \]

\[ x_{Bt}(t) := \frac{ts}{\pi} \int_{0}^{\frac{1}{ts} \cdot 10^5} \frac{\sin \left( \frac{\omega \cdot ts}{2} \right)}{\left( \frac{\omega \cdot ts}{2} \right)} \cdot \text{Re} \left( \exp \left( 1i \cdot \omega \cdot t(\omega) \right) \right) d\omega \]

\[ xvout(t, rd, rt) := \frac{ts^2}{\pi} \int_{0}^{\frac{1}{ts} \cdot 10^5} \frac{\sin \left( \frac{\omega \cdot ts}{2} \right)}{\left( \frac{\omega \cdot ts}{2} \right)} \cdot \text{Re} \left( 1i \cdot \omega \cdot Z_B(\omega, rd, rt) \cdot \exp \left( 1i \cdot \omega \cdot t(\omega) \right) \right) d\omega \]

\[ t := -1 \, ts, -0.99 \, ts, 0.4 \, ts \]

\[ vout(t, 0.7 \, B, 0.6 \, B) \]

\[ x_{Bt}(t) \]

\[ xvout(t, 0.7 \, B, 0.6 \, B) \]

\[ \frac{t}{ts} \]

ISI

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\[ t := -1 \, ts, -0.99 \, ts, 6 \, ts \]

\[ V_1(t, rd, rt) = vout(t, rd, rt) \]
ISI-range rd-rt
0.7/0.7>0.7/0.8>0.7/.09>0.7/1>0.8/.08>0.8/0.6>0.9>0.6

\[ V_{101}(t, rd, rt) := vout(t, rd, rt) + vout(t - 2 \, ts, rd, rt) \]
\( t_{B1} := 0.5 \cdot t_s \)

\( t_{pk1} := \text{root}\left( x\text{vout}(t_{B1}, 0.7 B, 0.5 B), t_{B1} \right) \)

\[ \frac{t_{pk1}}{t_s} = 0.691 \]

\( v_{\text{max}_1} := v\text{out}(t_{pk1}, 0.7 B, 0.5 B) = 0.894 \)

\( v_{\text{min}_1} := v\text{out}(t_{pk1} + 1 \cdot t_s, 0.7 B, 0.5 B) = 0.088 \)

\( v_{\text{min}_1} := v\text{out}(t_{pk1} + 2 \cdot t_s, 0.7 B, 0.5 B) = -0.016 \)

\( v_{\text{min}_2} := v\text{out}(t_{pk1} + 3 \cdot t_s, 0.7 B, 0.5 B) = 0.001 \)

\( v_{\text{max}_1} - v_{\text{min}_1} = 0.806 \quad v_{\text{max}_1} - v_{\text{min}_1} = 0.91 \quad v_{\text{max}_1} - v_{\text{min}_2} = 0.893 \)

\[
\frac{v_{\text{max}_1} - v_{\text{min}_1}}{\sqrt{10^{17} \cdot I_{n,B}(0.7 B, 0.5 B)}} = 0.43 \quad \text{1st pulse \ worst ISI} \]

\[
\frac{v_{\text{max}_1} - v_{\text{min}_1}}{\sqrt{10^{17} \cdot I_{n,B}(0.7 B, 0.5 B)}} = 0.481 \quad \text{2nd pulse} \]

\[
\frac{v_{\text{max}_1} - v_{\text{min}_2}}{\sqrt{10^{17} \cdot I_{n,B}(0.7 B, 0.5 B)}} = 0.471 \quad \text{3rd pulse} \]

\( t_{B2} := 0.5 \cdot t_s \)

\( t_{pk2} := \text{root}\left( x\text{vout}(t_{B2}, 0.8 B, 0.5 B), t_{B2} \right) \)

\[ \frac{t_{pk2}}{t_s} = 0.675 \]

\( v_{\text{max}_2} := v\text{out}(t_{pk2}, 0.8 B, 0.5 B) = 0.911 \)

\( v_{\text{min}_2} := v\text{out}(t_{pk2} + 1 \cdot t_s, 0.8 B, 0.5 B) = 0.075 \)

\( v_{\text{min}_2} := v\text{out}(t_{pk2} + 2 \cdot t_s, 0.8 B, 0.5 B) = -0.016 \)

\( v_{\text{min}_2} := v\text{out}(t_{pk2} + 3 \cdot t_s, 0.8 B, 0.5 B) = 0.001 \)

\( v_{\text{max}_2} - v_{\text{min}_2} = 0.835 \quad v_{\text{max}_2} - v_{\text{min}_1} = 0.927 \quad v_{\text{max}_2} - v_{\text{min}_2} = 0.909 \)

\[
\frac{v_{\text{max}_2} - v_{\text{min}_2}}{\sqrt{10^{17} \cdot I_{n,B}(0.8 B, 0.5 B)}} = 0.41 \quad \text{1st pulse \ worst ISI} \]

\[
\frac{v_{\text{max}_2} - v_{\text{min}_2}}{\sqrt{10^{17} \cdot I_{n,B}(0.8 B, 0.5 B)}} = 0.45 \quad \text{2nd pulse} \]

\[
\frac{v_{\text{max}_2} - v_{\text{min}_2}}{\sqrt{10^{17} \cdot I_{n,B}(0.8 B, 0.5 B)}} = 0.45 \quad \text{3rd pulse} \]
\[ t_{B3} := 0.5 \cdot t_s \]
\[ t_{pk3} := \text{root} \left( xvout \left( t_{B3}, 0.9 \ B, 0.5 \ B \right), t_{B3} \right) \]
\[ \frac{t_{pk3}}{t_s} = 0.662 \]
\[ v_{max\_B3} := vout \left( t_{pk3}, 0.9 \ B, 0.5 \ B \right) = 0.923 \]
\[ v_{min\_B3} := vout \left( t_{pk3} + 1 \cdot t_s, 0.9 \ B, 0.5 \ B \right) = 0.066 \]
\[ v_{min1\_B3} := vout \left( t_{pk3} + 2 \cdot t_s, 0.9 \ B, 0.5 \ B \right) = -0.016 \]
\[ v_{min2\_B3} := vout \left( t_{pk3} + 3 \cdot t_s, 0.9 \ B, 0.5 \ B \right) = 0.001 \]
\[ v_{max\_B3} - v_{min\_B3} = 0.858 \quad v_{max\_B3} - v_{min1\_B3} = 0.94 \quad v_{max\_B3} - v_{min2\_B3} = 0.922 \]

\[
\frac{v_{max\_B3} - v_{min\_B3}}{\sqrt{10^{17} \cdot I_{n,B}(0.9 \ B, 0.5 \ B)}} = 0.39
\]
\[
\frac{v_{max\_B3} - v_{min1\_B3}}{\sqrt{10^{17} \cdot I_{n,B}(0.9 \ B, 0.5 \ B)}} = 0.43
\]
\[
\frac{v_{max\_B3} - v_{min2\_B3}}{\sqrt{10^{17} \cdot I_{n,B}(0.9 \ B, 0.5 \ B)}} = 0.42
\]

\((rt = 0.6)\)

\[ t_{B4} := 0.8 \cdot t_s \]
\[ t_{pk4} := \text{root} \left( xvout \left( t_{B4}, 0.7 \ B, 0.6 \ B \right), t_{B4} \right) \]
\[ \frac{t_{pk4}}{t_s} = 0.634 \]
\[ v_{max\_B4} := vout \left( t_{pk4}, 0.7 \ B, 0.6 \ B \right) = 0.942 \]
\[ v_{min\_B4} := vout \left( t_{pk4} + 1 \cdot t_s, 0.7 \ B, 0.6 \ B \right) = 0.044 \]
\[ v_{min1\_B4} := vout \left( t_{pk4} + 2 \cdot t_s, 0.7 \ B, 0.6 \ B \right) = -0.005 \]
\[ v_{min2\_B4} := vout \left( t_{pk4} + 3 \cdot t_s, 0.7 \ B, 0.6 \ B \right) = 4.592 \cdot 10^{-4} \]
\[ v_{max\_B4} - v_{min\_B4} = 0.898 \quad v_{max\_B4} - v_{min1\_B4} = 0.947 \quad v_{max\_B4} - v_{min2\_B4} = 0.942 \]

\[
\frac{v_{max\_B4} - v_{min\_B4}}{\sqrt{10^{17} \cdot I_{n,B}(0.7 \ B, 0.6 \ B)}} = 0.43
\]
\[
\frac{v_{max\_B4} - v_{min1\_B4}}{\sqrt{10^{17} \cdot I_{n,B}(0.7 \ B, 0.6 \ B)}} = 0.46
\]
\[
\frac{v_{max\_B4} - v_{min2\_B4}}{\sqrt{10^{17} \cdot I_{n,B}(0.7 \ B, 0.6 \ B)}} = 0.45
\]
\[ t_{B5} := 0.8 \cdot ts \]
\[ t_{pk5} := \text{root} \left( xvout \left( t_{B5}, 0.8 B, 0.6 B \right), t_{B5} \right) \]
\[ \frac{t_{pk5}}{ts} = 0.62 \]
\[ v_{\text{max}, B5} := \text{vout} \left( t_{pk5}, 0.8 B, 0.6 B \right) = 0.96 \]
\[ v_{\text{min}, B5} := \text{vout} \left( t_{pk5} + 1 \cdot ts, 0.8 B, 0.6 B \right) = 0.029 \]
\[ v_{\text{min1}, B5} := \text{vout} \left( t_{pk5} + 2 \cdot ts, 0.8 B, 0.6 B \right) = -0.005 \]
\[ v_{\text{min2}, B5} := \text{vout} \left( t_{pk5} + 3 \cdot ts, 0.8 B, 0.6 B \right) = 4.671 \cdot 10^{-4} \]
\[ v_{\text{max}, B5} - v_{\text{min}, B5} = 0.931 \quad v_{\text{max}, B5} - v_{\text{min1}, B5} = 0.965 \quad v_{\text{max}, B5} - v_{\text{min2}, B5} = 0.959 \]

\[ \frac{v_{\text{max}, B5} - v_{\text{min}, B5}}{\sqrt{10^{17} \cdot I_{n, B}(0.8 B, 0.6 B)}} = 0.42 \] [1st pulse] [worst ISI]
\[ \frac{v_{\text{max}, B5} - v_{\text{min1}, B5}}{\sqrt{10^{17} \cdot I_{n, B}(0.8 B, 0.6 B)}} = 0.43 \] [2nd pulse]
\[ \frac{v_{\text{max}, B5} - v_{\text{min2}, B5}}{\sqrt{10^{17} \cdot I_{n, B}(0.8 B, 0.6 B)}} = 0.43 \] [3rd pulse]

\[ t_{B6} := 0.5 \cdot ts \]
\[ t_{pk6} := \text{root} \left( xvout \left( t_{B4}, 0.9 B, 0.6 B \right), t_{B4} \right) \]
\[ \frac{t_{pk6}}{ts} = 0.608 \]
\[ v_{\text{max}, B6} := \text{vout} \left( t_{pk6}, 0.9 B, 0.6 B \right) = 0.973 \]
\[ v_{\text{min}, B6} := \text{vout} \left( t_{pk6} + 1 \cdot ts, 0.9 B, 0.6 B \right) = 0.018 \]
\[ v_{\text{min1}, B6} := \text{vout} \left( t_{pk6} + 2 \cdot ts, 0.9 B, 0.6 B \right) = -0.004 \]
\[ v_{\text{min2}, B6} := \text{vout} \left( t_{pk6} + 3 \cdot ts, 0.9 B, 0.6 B \right) = 4.622 \cdot 10^{-4} \]
\[ v_{\text{max}, B6} - v_{\text{min}, B6} = 0.954 \quad v_{\text{max}, B6} - v_{\text{min1}, B6} = 0.977 \quad v_{\text{max}, B6} - v_{\text{min2}, B6} = 0.972 \]

\[ \frac{v_{\text{max}, B6} - v_{\text{min}, B6}}{\sqrt{10^{17} \cdot I_{n, B}(0.9 B, 0.6 B)}} = 0.4 \] [1st pulse] [worst ISI]
\[ \frac{v_{\text{max}, B6} - v_{\text{min1}, B6}}{\sqrt{10^{17} \cdot I_{n, B}(0.9 B, 0.6 B)}} = 0.41 \] [2nd pulse]
\[ \frac{v_{\text{max}, B6} - v_{\text{min2}, B6}}{\sqrt{10^{17} \cdot I_{n, B}(0.9 B, 0.6 B)}} = 0.41 \] [3rd pulse]
$$t_{B7} = 0.5 \cdot t_s$$

$$t_{pk7} := \text{root}(xvout(t_{B7}, 0.7 B, 0.7 B), t_{B7})$$

$$\frac{t_{pk7}}{t_s} = 0.595$$

$$v_{max_{B7}} := vout(t_{pk7}, 0.7 B, 0.7 B) = 0.969$$

$$v_{min_{B7}} := vout(t_{pk7} + 1 \cdot t_s, 0.7 B, 0.7 B) = 0.022$$

$$v_{min1_{B7}} := vout(t_{pk7} + 2 \cdot t_s, 0.7 B, 0.7 B) = -2.398 \cdot 10^{-4}$$

$$v_{min2_{B7}} := vout(t_{pk7} + 3 \cdot t_s, 0.7 B, 0.7 B) = 2.282 \cdot 10^{-5}$$

$$v_{max_{B7}} - v_{min_{B7}} = 0.948 \hspace{1cm} v_{max_{B7}} - v_{min1_{B7}} = 0.969 \hspace{1cm} v_{max_{B7}} - v_{min2_{B7}} = 0.969$$

$$\frac{v_{max_{B7}} - v_{min_{B7}}}{\sqrt{10^{17} \cdot I_{n_{B}}(0.7 B, 0.7 B)}} = 0.42$$ \hspace{1cm} 1st pulse \hspace{1cm} (worst ISI)

$$\frac{v_{max_{B7}} - v_{min1_{B7}}}{\sqrt{10^{17} \cdot I_{n_{B}}(0.7 B, 0.7 B)}} = 0.43$$ \hspace{1cm} 2nd pulse

$$\frac{v_{max_{B7}} - v_{min2_{B7}}}{\sqrt{10^{17} \cdot I_{n_{B}}(0.7 B, 0.7 B)}} = 0.43$$ \hspace{1cm} 3rd pulse

$$t_{B8} = 0.5 \cdot t_s$$

$$t_{pk8} := \text{root}(xvout(t_{B8}, 0.8 B, 0.7 B), t_{B8})$$

$$\frac{t_{pk8}}{t_s} = 0.582$$

$$v_{max_{B8}} := vout(t_{pk8}, 0.8 B, 0.7 B) = 0.986$$

$$v_{min_{B8}} := vout(t_{pk8} + 1 \cdot t_s, 0.8 B, 0.7 B) = 0.007$$

$$v_{min1_{B8}} := vout(t_{pk8} + 2 \cdot t_s, 0.8 B, 0.7 B) = -4.049 \cdot 10^{-5}$$

$$v_{min2_{B8}} := vout(t_{pk8} + 3 \cdot t_s, 0.8 B, 0.7 B) = 6.85 \cdot 10^{-6}$$

$$v_{max_{B8}} - v_{min_{B8}} = 0.98 \hspace{1cm} v_{max_{B8}} - v_{min1_{B8}} = 0.986 \hspace{1cm} v_{max_{B8}} - v_{min2_{B8}} = 0.986$$

$$\frac{v_{max_{B8}} - v_{min_{B8}}}{\sqrt{10^{17} \cdot I_{n_{B}}(0.8 B, 0.7 B)}} = 0.41$$ \hspace{1cm} 1st pulse

$$\frac{v_{max_{B8}} - v_{min1_{B8}}}{\sqrt{10^{17} \cdot I_{n_{B}}(0.8 B, 0.7 B)}} = 0.41$$ \hspace{1cm} 2nd pulse

$$\frac{v_{max_{B8}} - v_{min2_{B8}}}{\sqrt{10^{17} \cdot I_{n_{B}}(0.8 B, 0.7 B)}} = 0.41$$ \hspace{1cm} 3rd pulse \hspace{1cm} (worst ISI)
\[ t_{B9} := 0.8 \cdot ts \]
\[ t_{pk9} := \text{root}(x\text{vout}(t_{B9}, 0.6 B, 0.6 B), t_{B9}) \]
\[ \frac{t_{pk9}}{ts} = 0.652 \]
\[ v_{\text{max}_{B9}} := v\text{out}(t_{pk9}, 0.6 B, 0.6 B) = 0.918 \]
\[ v_{\text{min}_{B9}} := v\text{out}(t_{pk9} + 1 \cdot ts, 0.6 B, 0.6 B) = 0.064 \]
\[ v_{\text{min1}_{B9}} := v\text{out}(t_{pk9} + 2 \cdot ts, 0.6 B, 0.6 B) = -0.005 \]
\[ v_{\text{min2}_{B9}} := v\text{out}(t_{pk9} + 3 \cdot ts, 0.6 B, 0.6 B) = 4.354 \cdot 10^{-4} \]
\[ v_{\text{max}_{B9}} - v_{\text{min}_{B9}} = 0.854 \quad v_{\text{max}_{B9}} - v_{\text{min1}_{B9}} = 0.923 \quad v_{\text{max}_{B9}} - v_{\text{min2}_{B9}} = 0.918 \]

\[
\begin{align*}
\frac{v_{\text{max}_{B9}} - v_{\text{min}_{B9}}}{\sqrt{10^{17} \cdot I_{n_{B}}(0.6 B, 0.6 B)}} &= 0.45 \quad \text{1st pulse} \\
\frac{v_{\text{max}_{B9}} - v_{\text{min1}_{B9}}}{\sqrt{10^{17} \cdot I_{n_{B}}(0.6 B, 0.6 B)}} &= 0.48 \quad \text{2nd pulse} \\
\frac{v_{\text{max}_{B9}} - v_{\text{min2}_{B9}}}{\sqrt{10^{17} \cdot I_{n_{B}}(0.6 B, 0.6 B)}} &= 0.48 \quad \text{3rd pulse \ worst ISI} 
\end{align*}
\]

\[ v_{\text{max}_{B}} := v_{\text{max}_{B9}} \quad v_{\text{min}_{B}} := v_{\text{min}_{B9}} \]

\[ x_{111}(t, rd, rt) := v\text{out}(t, rd, rt) + v\text{out}(t - 2 \cdot ts, rd, rt) + v\text{out}(t - ts, rd, rt) \]
\[ x_{110}(t, rd, rt) := v\text{out}(t, rd, rt) + v\text{out}(t - ts, rd, rt) \]
\[ x_{100}(t, rd, rt) := v\text{out}(t, rd, rt) \]
\[ x_{001}(t, rd, rt) := v\text{out}(t - 2 \cdot ts, rd, rt) \]
\[ x_{010}(t, rd, rt) := v\text{out}(t - ts, rd, rt) \]
\[ x_{011}(t, rd, rt) := v\text{out}(t - ts, rd, rt) + v\text{out}(t - 2 \cdot ts, rd, rt) \]
\[ x_{101}(t, rd, rt) := v\text{out}(t, rd, rt) + v\text{out}(t - 2 \cdot ts, rd, rt) \]
\[
x_{101}(t_{pk1} + ts, 0.7 B, 0.5 B) = 0.121 \\
x_{101}(t_{pk1}, 0.7 B, 0.5 B) = 0.894 \\
x_{101}(t_{pk1} + 2 ts, 0.7 B, 0.5 B) = 0.878 \\
x_{100}(t_{pk1}, 0.7 B, 0.5 B) = 0.894 \\
x_{100}(t_{pk1} + ts, 0.7 B, 0.5 B) = 0.088 \\
x_{100}(t_{pk1} + 2 ts, 0.7 B, 0.5 B) = -0.016
\]

Check ratio<1 is ok 100, >1 re-opt

\[
\frac{x_{100}(t_{pk1}, 0.7 B, 0.5 B) - x_{100}(t_{pk1} + 2 ts, 0.7 B, 0.5 B)}{v_{\text{max}_{B1}} - v_{\text{min}_{B1}}} = 1.129
\]

**Sens PINFET**

\[\lambda := 650 \cdot 10^{-9}\]

\[\text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda}\]

\[nq := 1.6 \cdot 10^{-19}\]

\[Q_N(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot \left(\frac{v_{\text{max}} - v_{\text{min}}}{2}\right)
\]

\[\sqrt{I_{n,B}(0.6 B, 0.6 B)}\]
\[ P_{e\_N}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\_N}(b)}{\sqrt{2}} \right) \]

\[ P_{c\_N}(b) := (\log(P_{e\_N}(b)) + 9) \]

\[ b \equiv 3 \cdot 10^3 \]

\[ b_{\_N} := \text{root} \left( P_{c\_N}(b), b \right) \]

minimum \_B := min(b_{\_N}) = 5.321 \cdot 10^3

**APD\_1FET**

\[ APD\_N(b) := \frac{1}{t_{\text{s}}} \cdot b \cdot nq \cdot (v_{\text{max} \_B}) \]

\[ APD\_B(b) := \frac{1}{t_{\text{s}}} \cdot b \cdot nq \cdot (v_{\text{max} \_B}) \]

\[ I_{\_APD \_d} := 10 \cdot 10^{-9} \]

\[ M_{\_APD1} := 100 \]

\[ F_{\_M1} := 7.9 \]

\[ \text{Noise\_APD1\_B}(b, rd, rt) := \left( 2 \cdot q \cdot APD\_B(b) \cdot M_{\_APD1}^2 \cdot F_{\_M1} \right) \cdot \text{NEB\_B}(rd, rt) \]

\[ + 2 \cdot q \cdot I_{\_APD \_d} \cdot M_{\_APD1}^2 \cdot F_{\_M1} \]

\[ Q_{\_APD1\_B}(b) := \frac{M_{\_APD1}}{t_{\text{s}}} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max} \_B} - v_{\text{min} \_B}}{2} \right) \]

\[ \sqrt{I_{n\_B}(0.6 B, 0.6 B) + \text{Noise\_APD1\_B}(b, 0.6 B, 0.6 B)} \]

\[ P_{e\_APD1\_B}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\_APD1\_B}(b)}{\sqrt{2}} \right) \]

\[ p_{\text{APD1\_B}}(b) := (\log(P_{e\_APD1\_B}(b)) + 9) \]

\[ APD1\_b\_B := \text{root} \left( p_{\text{APD1\_B}}(b) \right) \]

\[ \text{NEB\_B}(0.7 B, 0.6 B) = 5.012 \cdot 10^7 \]

minimum \_APD1\_B := min(APD1\_b\_B) = 1.849 \cdot 10^3

**APD\_2FET**

\[ M_{\_APD2} := 10 \]

\[ F_{\_M2} := 5.5 \]

\[ \text{Noise\_APD2\_B}(b, rd, rt) := \left( 2 \cdot q \cdot APD\_B(b) \cdot M_{\_APD2}^2 \cdot F_{\_M2} \right) \cdot \text{NEB\_B}(rd, rt) \]

\[ + 2 \cdot q \cdot I_{\_APD \_d} \cdot M_{\_APD2}^2 \cdot F_{\_M2} \]

\[ Q_{\_APD2\_B}(b) := \frac{M_{\_APD2}}{t_{\text{s}}} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max} \_B} - v_{\text{min} \_B}}{2} \right) \]

\[ \sqrt{I_{n\_B}(0.6 B, 0.6 B) + \text{Noise\_APD2\_B}(b, 0.6 B, 0.6 B)} \]

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\[ P_{e\_APD2\_B}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\_APD2\_B}(b)}{\sqrt{2}} \right) \]

\[ P_{c\_APD2\_B}(b) := (\log(P_{e\_APD2\_B}(b)) + 9) \]

\[ \text{APD2\_b\_B} := \text{root}(P_{c\_APD2\_B}(b), b) \]

\[ \text{minimum\_APD2\_B} := \text{min}(\text{APD2\_b\_B}) = 1.54 \cdot 10^3 \]

**APD-3FET**

\[ M_{\_APD3} := 10 \quad F_{\_M3} := 9.2 \]

\[ \text{Noise\_APD3\_B}(b, rd, rt) := \left( 2 \cdot q \cdot \text{APD\_B}(b) \cdot M_{\_APD3}^2 \cdot F_{\_M3} \right) \cdot \text{NEB\_B}(rd, rt) \]

\[ + 2 \cdot q \cdot I_{\_APD\_d} \cdot M_{\_APD3}^2 \cdot F_{\_M3} \]

\[ Q_{\_APD3\_B}(b) := \frac{M_{\_APD3} \cdot b \cdot nq \cdot (v_{\text{max\_B}} - v_{\text{min\_B}})}{2} \]

\[ \sqrt{I_{\_n\_B}(0.6 \text{ B}, 0.6 \text{ B}) + \text{Noise\_APD3\_B}(b, 0.6 \text{ B}, 0.6 \text{ B})} \]

\[ P_{e\_APD3\_B}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\_APD3\_B}(b)}{\sqrt{2}} \right) \]

\[ P_{c\_APD3\_B}(b) := (\log(P_{e\_APD3\_B}(b)) + 9) \]

\[ \text{APD3\_b\_B} := \text{root}(P_{c\_APD3\_B}(b), b) \]

\[ \text{minimum\_APD3\_B} := \text{min}(\text{APD3\_b\_B}) = 2.191 \cdot 10^3 \]

\[ \text{Sens\_B\_PIN} := 10 \cdot \log \left( \frac{\text{minimum\_B}}{2}, \frac{\text{photon\_energy}}{10^{-3}}, \frac{1}{ts} \right) = -40.893 \]

\[ \text{Sens\_B\_APD\_silicon} := 10 \cdot \log \left( \frac{\text{minimum\_APD1\_B}}{2}, \frac{\text{photon\_energy}}{10^{-3}}, \frac{1}{ts} \right) = -45.483 \]

\[ \text{Sens\_B\_APD\_InGaAs} := 10 \cdot \log \left( \frac{\text{minimum\_APD2\_B}}{2}, \frac{\text{photon\_energy}}{10^{-3}}, \frac{1}{ts} \right) = -46.279 \]

\[ \text{Sens\_B\_APD\_germanium} := 10 \cdot \log \left( \frac{\text{minimum\_APD3\_B}}{2}, \frac{\text{photon\_energy}}{10^{-3}}, \frac{1}{ts} \right) = -44.747 \]
B.2.4 Tuned B $a=0.1$

**Tuned B optimum Performance (PIN-FET APD-FET 100 Mbit/s) $a=0.1$**

Rx performance opt PIN-FET and APD-FET input configurations, 1st order LPF pre-detection filter. opt as follows

- TIA 3-db bandwidth rt/Filter 3-db bandwidth rd
- range cut-off
- minimum noise
- examine ISI
- highest SNR
- BER
- APD

<table>
<thead>
<tr>
<th>$\text{opt } \text{rd} = 0.5, \text{rt} = 0.5&gt;0.6$</th>
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**Bit-rate, pulse duration**

\[
C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \\
B := 100 \cdot 10^6 \quad \text{Bit-rate}
\]

**pre-dec filter**

\[
\omega_B(\text{rd}) := 2 \cdot \pi \cdot \text{rd} \\
H_{\text{int}}(\omega, \text{rd}) := \frac{1}{1 + j \cdot \frac{\omega}{2 \cdot \pi \cdot \text{rd}}}
\]

**TIA**

Feedback value for (R) Tuned B

\[
\Delta_L := 1.8 \\
\Delta_R := 1.58 \\
\alpha := 0.1
\]

feedback $\Delta R$, time constant ratio $\Delta L$  

\[
Av := 10 \\
\begin{bmatrix}
\alpha & \Delta_L & \Delta_R \\
0 & 2 & 1.41 \\
0.1 & 1.8 & 1.58 \\
0.2 & 1.8 & 1.87 \\
0.3 & 2.4 & 2.52 \\
0.4 & 1.9 & 2.75 \\
0.4 & 2.5 & 3.17 \\
0.5 & 1.5 & 2.65
\end{bmatrix}
\]

Feedback for Tuned B

\[
Rf_B(rt) := \Delta_R \cdot \frac{Av + 1}{2 \cdot \pi \cdot rt \cdot C_T}
\]

splitting ratio

\[
\alpha := 0.1
\]

**feedback**

\[
\Delta_L := 1.8 \\
\Delta_R := 1.58 \\
Av := 10 \\
\begin{bmatrix}
\alpha & \Delta_L & \Delta_R \\
0 & 2 & 1.41 \\
0.1 & 1.8 & 1.58 \\
0.2 & 1.8 & 1.87 \\
0.3 & 2.4 & 2.52 \\
0.4 & 1.9 & 2.75 \\
0.4 & 2.5 & 3.17 \\
0.5 & 1.5 & 2.65
\end{bmatrix}
\]

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\]
TUNED B (R) Transimpedance

\[ Z_{TIA,B}(\omega, rt) := \frac{1}{\left(1 - \omega^2 \cdot Lc(rt) \cdot C1\right) + \frac{Rf_B(rt)}{1 + Av} \cdot (\omega \cdot i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc(rt) \cdot C1 \cdot C2)} \]

**Receiver freq-response**

\[ Z_B(\omega, rd, rt) := H_{but}(\omega, rd) \cdot Z_{TIA,B}(\omega, rt) \]

\( \omega := 1 \cdot 10^6, 10 \cdot 10^6 .. 1 \cdot 10^{11} \)

\( rd := 0.5 \cdot B, 0.75 \cdot B .. 2 \cdot B \)

\( rt := 0.5 \cdot B, 0.55 \cdot B .. 1 \cdot B \)

**Noise equivalent bandwidth (NEB and I2)**

\[ NEB_B(rd, rt) := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{11}} \left( Z_B(\omega, rd, rt) \right)^2 d\omega \right) \]

\[ I_{2,B}(rd, rt) := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{11}} \left( \frac{\left(1 - \omega^2 \cdot Lc(rt) \cdot C1\right)}{\left| Z_B(\omega, rd, rt) \right|} \right)^2 d\omega \right) \]
Noise equivalent bandwidth (13)

\[ I_{3B}(rd, rt) = \frac{1}{2\pi} \cdot \int_{0}^{10^9} \left( \frac{\left( 1 - \omega^2 \cdot Lc(rt) \cdot C1 \right)}{Rf_B(rt) \cdot \left( \omega \right)} + Rf_B(rt) \cdot \left( \omega \right) \cdot \left( C1 + C2 - (\omega)^2 \cdot Lc(rt) \cdot C1 \cdot C2 \cdot 1_1 \right) \cdot Z_B(\omega, rd, rt) \right)^2 \, \mathrm{d}\omega \]

Feedback noise

\[ q = 1.6 \cdot 10^{-19} \quad k = 1.38 \cdot 10^{-23} \quad T = 298 \]

\[ I_{nRf_B}(rd, rt) = \frac{4 \cdot k \cdot T}{Rf_B(rt)} \cdot I_{2B}(rd, rt) \]
Ig noise (gate current)

\[ Ig := 10 \cdot 10^{-9} \]

\[ I_{nG,B}(rd, rt) := 2 \cdot q \cdot Ig \cdot I_2B(rd, rt) \]

Channel noise (Gate-source)

\[ gm1 := 30 \cdot 10^{-3} = 0.03 \quad noise\_factor := 1 \]

\[ I_{nD} := 4 \cdot k \cdot T \cdot \frac{1}{gm1} \]

\[ I_{nD,B}(rd, rt) := I_{nD} \cdot I_3B(rd, rt) \]

Total noise- PIN-FET

\[ I_n(rd, rt) := I_{nRF,B}(rd, rt) + I_{nG,B}(rd, rt) + I_{nD,B}(rd, rt) \]
\[
\frac{I_{n,B}(rd, rt)}{10^{-17}}
\]
$I_{n_B}(0.7\ B, 0.5\ B) = 2.99 \cdot 10^{-17}$
$I_{n_B}(0.8\ B, 0.5\ B) = 3.42 \cdot 10^{-17}$
$I_{n_B}(0.9\ B, 0.5\ B) = 3.85 \cdot 10^{-17}$
$I_{n_B}(1\ B, 0.5\ B) = 4.28 \cdot 10^{-17}$

$\text{Output pulse shape}$

\[ ts := \frac{1}{B} \quad B = 1 \cdot 10^8 \]

\[
vout(t, rd, rt) := ts \cdot \frac{1}{\pi} \int_{0}^{\frac{1}{ts} \cdot 10^{3}} \sin\left(\frac{\omega \cdot ts}{2}\right) \cdot \text{Re}\left(Z_B(\omega, rd, rt) \cdot \exp\left(1i \cdot \omega \cdot t\right)\right) d\omega
\]

\[
x_{Bt}(t) := ts \cdot \frac{1}{\pi} \int_{0}^{\frac{1}{ts} \cdot 10^{4}} \sin\left(\frac{\omega \cdot ts}{2}\right) \cdot \text{Re}\left(\exp\left(1i \cdot \omega \cdot t\right)\right) d\omega
\]
\[ x_{\text{vout}}(t, rd, rt) = \frac{ts^2}{\pi} \cdot \frac{1}{ts} \cdot \left\{ \sin \left( \frac{\omega \cdot ts}{2} \right) \right\} \frac{d\omega}{\omega \cdot ts} \cdot \Re \left( 1i \cdot \omega \cdot Z_B(\omega, rd, rt) \cdot \exp(1i \cdot \omega \cdot t) \right) \]

\[ t := -1 \ ts, -0.99 \ ts, 4 \ ts \]

\[ t \]

\[ \frac{t}{ts} \]

\[ |S| \]

\[ t := -1 \ ts, -0.99 \ ts, 6 \ ts \]

\[ V_1(t, rd, rt) := vout(t, rd, rt) \]

\[ V_1(t, 0.7 B, 0.5 B) \]

\[ V_1(t, 0.7 B, 0.6 B) \]

\[ V_1(t, 0.7 B, 0.7 B) \]
\[
\begin{align*}
V_1(t, 0.8 B, 0.5 B) \\
V_1(t, 0.8 B, 0.6 B) \\
V_1(t, 0.8 B, 0.7 B) \\
V_1(t, 0.8 B, 0.8 B) \\
V_1(t, 0.8 B, 0.9 B) \\
V_1(t, 0.8 B, 1 B)
\end{align*}
\]

\[
\begin{align*}
V_1(t, 0.9 B, 0.5 B) \\
V_1(t, 0.9 B, 0.6 B) \\
V_1(t, 0.9 B, 0.7 B) \\
V_1(t, 0.9 B, 0.8 B) \\
V_1(t, 1 B, 0.5 B)
\end{align*}
\]

\[
V_{101}(t, rd, rt) := \text{vout}(t, rd, rt) + \text{vout}(t - 2 \, ts, rd, rt)
\]

\[
\begin{align*}
V_{101}(t, 0.7 B, 0.7 B) \\
V_{101}(t, 0.7 B, 0.9 B) \\
V_{101}(t, 0.9 B, 0.6 B) \\
V_{101}(t, 0.9 B, 0.8 B)
\end{align*}
\]
(rt = 0.5)

$t_{B1} := 0.5 \cdot ts$

$t_{pk1} := \text{root}(xvout(t_{B1}, 0.7 B, 0.5 B), t_{B1})$

\[
\frac{t_{pk1}}{ts} = 0.749
\]

$v_{\text{max\_B1}} := xvout(t_{pk1}, 0.7 B, 0.5 B) = 0.893$

$v_{\text{min\_B1}} := xvout(t_{pk1} + 1 \cdot ts, 0.7 B, 0.5 B) = 0.084$

$v_{\text{min1\_B1}} := xvout(t_{pk1} + 2 \cdot ts, 0.7 B, 0.5 B) = -0.016$

$v_{\text{min2\_B1}} := xvout(t_{pk1} + 3 \cdot ts, 0.7 B, 0.5 B) = 0.001$

$v_{\text{max\_B1}} - v_{\text{min\_B1}} = 0.809 \quad v_{\text{max\_B1}} - v_{\text{min1\_B1}} = 0.909 \quad v_{\text{max\_B1}} - v_{\text{min2\_B1}} = 0.892$

\[
\frac{v_{\text{max\_B1}} - v_{\text{min\_B1}}}{\sqrt{10^{17} \cdot I_{n\_B}(0.7 B, 0.5 B)}} = 0.47 \quad \text{(1st pulse worst ISI)}
\]

\[
\frac{v_{\text{max\_B1}} - v_{\text{min1\_B1}}}{\sqrt{10^{17} \cdot I_{n\_B}(0.7 B, 0.5 B)}} = 0.526 \quad \text{(2nd pulse)}
\]

\[
\frac{v_{\text{max\_B1}} - v_{\text{min2\_B1}}}{\sqrt{10^{17} \cdot I_{n\_B}(0.7 B, 0.5 B)}} = 0.516 \quad \text{(3rd pulse)}
\]

$t_{B2} := 0.7 \cdot ts$

$t_{pk2} := \text{root}(xvout(t_{B2}, 0.8 B, 0.5 B), t_{B2})$

\[
\frac{t_{pk2}}{ts} = 0.732
\]

$v_{\text{max\_B2}} := xvout(t_{pk2}, 0.8 B, 0.5 B) = 0.91$

$v_{\text{min\_B2}} := xvout(t_{pk2} + 1 \cdot ts, 0.8 B, 0.5 B) = 0.072$

$v_{\text{min1\_B2}} := xvout(t_{pk2} + 2 \cdot ts, 0.8 B, 0.5 B) = -0.016$

$v_{\text{min2\_B2}} := xvout(t_{pk2} + 3 \cdot ts, 0.8 B, 0.5 B) = 0.001$

$v_{\text{max\_B2}} - v_{\text{min\_B2}} = 0.838 \quad v_{\text{max\_B2}} - v_{\text{min1\_B2}} = 0.926 \quad v_{\text{max\_B2}} - v_{\text{min2\_B2}} = 0.908$

\[
\frac{v_{\text{max\_B2}} - v_{\text{min\_B2}}}{\sqrt{10^{17} \cdot I_{n\_B}(0.8 B, 0.5 B)}} = 0.45 \quad \text{(1st pulse worst ISI)}
\]

\[
\frac{v_{\text{max\_B2}} - v_{\text{min1\_B2}}}{\sqrt{10^{17} \cdot I_{n\_B\_B}(0.8 B, 0.5 B)}} = 0.5 \quad \text{(2nd pulse)}
\]

\[
\frac{v_{\text{max\_B2}} - v_{\text{min2\_B2}}}{\sqrt{10^{17} \cdot I_{n\_B}(0.8 B, 0.5 B)}} = 0.49 \quad \text{(3rd pulse)}
\]
\[ t_{B3} := 0.5 \cdot ts \]
\[ t_{pk3} := \text{root}(xvout(t_{B3}, 0.9 B, 0.5 B), t_{B3}) \]
\[ \frac{t_{pk3}}{ts} = 0.718 \]
\[ v_{\text{max}_B3} := vout(t_{pk3}, 0.9 B, 0.5 B) = 0.922 \]
\[ v_{\text{min}_B3} := vout(t_{pk3} + 1 \cdot ts, 0.9 B, 0.5 B) = 0.062 \]
\[ v_{\text{min1}_B3} := vout(t_{pk3} + 2 \cdot ts, 0.9 B, 0.5 B) = -0.016 \]
\[ v_{\text{min2}_B3} := vout(t_{pk3} + 3 \cdot ts, 0.9 B, 0.5 B) = 0.002 \]
\[ v_{\text{max}_B3} - v_{\text{min}_B3} = 0.86 \quad v_{\text{max}_B3} - v_{\text{min1}_B3} = 0.938 \quad v_{\text{max}_B3} - v_{\text{min2}_B3} = 0.921 \]

\[
\begin{align*}
\frac{v_{\text{max}_B3} - v_{\text{min}_B3}}{\sqrt{10^{17} \cdot I_{n, B}(0.9 B, 0.5 B)}} &= 0.44 \quad \text{(1st pulse, worst ISI)} \\
\frac{v_{\text{max}_B3} - v_{\text{min1}_B3}}{\sqrt{10^{17} \cdot I_{n, B}(0.9 B, 0.5 B)}} &= 0.48 \quad \text{(2nd pulse)} \\
\frac{v_{\text{max}_B3} - v_{\text{min2}_B3}}{\sqrt{10^{17} \cdot I_{n, B}(0.9 B, 0.5 B)}} &= 0.47 \quad \text{(3rd pulse)}
\end{align*}
\]

\[ (rt = 0.6) \]

\[ t_{B4} := 0.8 \cdot ts \]
\[ t_{pk4} := \text{root}(xvout(t_{B4}, 0.7 B, 0.6 B), t_{B4}) \]
\[ \frac{t_{pk4}}{ts} = 0.68 \]
\[ v_{\text{max}_B4} := vout(t_{pk4}, 0.7 B, 0.6 B) = 0.942 \]
\[ v_{\text{min}_B4} := vout(t_{pk4} + 1 \cdot ts, 0.7 B, 0.6 B) = 0.041 \]
\[ v_{\text{min1}_B4} := vout(t_{pk4} + 2 \cdot ts, 0.7 B, 0.6 B) = -0.004 \]
\[ v_{\text{min2}_B4} := vout(t_{pk4} + 3 \cdot ts, 0.7 B, 0.6 B) = 4.205 \cdot 10^{-4} \]
\[ v_{\text{max}_B4} - v_{\text{min}_B4} = 0.901 \quad v_{\text{max}_B4} - v_{\text{min1}_B4} = 0.946 \quad v_{\text{max}_B4} - v_{\text{min2}_B4} = 0.942 \]

\[
\begin{align*}
\frac{v_{\text{max}_B4} - v_{\text{min}_B4}}{\sqrt{10^{17} \cdot I_{n, B}(0.7 B, 0.6 B)}} &= 0.47 \quad \text{(1st pulse, worst ISI)} \\
\frac{v_{\text{max}_B4} - v_{\text{min1}_B4}}{\sqrt{10^{17} \cdot I_{n, B}(0.7 B, 0.6 B)}} &= 0.5 \quad \text{(2nd pulse)} \\
\frac{v_{\text{max}_B4} - v_{\text{min2}_B4}}{\sqrt{10^{17} \cdot I_{n, B}(0.7 B, 0.6 B)}} &= 0.49 \quad \text{(3rd pulse)}
\end{align*}
\]
\[ t_{B5} := 0.8 \cdot ts \]
\[ t_{pk5} := \text{root} \left( v_{\text{out}} \left( t_{B5}, 0.8 \ B, 0.6 \ B \right), t_{B5} \right) \]
\[ \frac{t_{pk5}}{ts} = 0.665 \]
\[ v_{\text{max} \ B5} := v_{\text{out}} \left( t_{pk5}, 0.8 \ B, 0.6 \ B \right) = 0.959 \]
\[ v_{\text{min} \ B5} := v_{\text{out}} \left( t_{pk5} + 1 \cdot ts, 0.8 \ B, 0.6 \ B \right) = 0.027 \]
\[ v_{\text{min1} \ B5} := v_{\text{out}} \left( t_{pk5} + 2 \cdot ts, 0.8 \ B, 0.6 \ B \right) = -0.004 \]
\[ v_{\text{min2} \ B5} := v_{\text{out}} \left( t_{pk5} + 3 \cdot ts, 0.8 \ B, 0.6 \ B \right) = 4.217 \times 10^{-4} \]
\[ v_{\text{max} \ B5} - v_{\text{min} \ B5} = 0.933 \quad v_{\text{max} \ B5} - v_{\text{min1} \ B5} = 0.964 \quad v_{\text{max} \ B5} - v_{\text{min2} \ B5} = 0.959 \]

\[
\frac{\sqrt{10^{17} \cdot I_{n,B}(0.8 \ B, 0.6 \ B)}}{v_{\text{max} \ B5} - v_{\text{min} \ B5}} = 0.46 \quad \text{1st pulse} \quad \text{(worst ISI)}
\]
\[
\frac{\sqrt{10^{17} \cdot I_{n,B}(0.8 \ B, 0.6 \ B)}}{v_{\text{max} \ B5} - v_{\text{min1} \ B5}} = 0.47 \quad \text{2nd pulse}
\]
\[
\frac{\sqrt{10^{17} \cdot I_{n,B}(0.8 \ B, 0.6 \ B)}}{v_{\text{max} \ B5} - v_{\text{min2} \ B5}} = 0.47 \quad \text{3rd pulse}
\]

\[ t_{B6} := 0.5 \cdot ts \]
\[ t_{pk6} := \text{root} \left( v_{\text{out}} \left( t_{B4}, 0.9 \ B, 0.6 \ B \right), t_{B4} \right) \]
\[ \frac{t_{pk6}}{ts} = 0.653 \]
\[ v_{\text{max} \ B6} := v_{\text{out}} \left( t_{pk6}, 0.9 \ B, 0.6 \ B \right) = 0.972 \]
\[ v_{\text{min} \ B6} := v_{\text{out}} \left( t_{pk6} + 1 \cdot ts, 0.9 \ B, 0.6 \ B \right) = 0.016 \]
\[ v_{\text{min1} \ B6} := v_{\text{out}} \left( t_{pk6} + 2 \cdot ts, 0.9 \ B, 0.6 \ B \right) = -0.004 \]
\[ v_{\text{min2} \ B6} := v_{\text{out}} \left( t_{pk6} + 3 \cdot ts, 0.9 \ B, 0.6 \ B \right) = 4.125 \times 10^{-4} \]
\[ v_{\text{max} \ B6} - v_{\text{min} \ B6} = 0.956 \quad v_{\text{max} \ B6} - v_{\text{min1} \ B6} = 0.976 \quad v_{\text{max} \ B6} - v_{\text{min2} \ B6} = 0.972 \]

\[
\frac{\sqrt{10^{17} \cdot I_{n,B}(0.9 \ B, 0.6 \ B)}}{v_{\text{max} \ B6} - v_{\text{min} \ B6}} = 0.44 \quad \text{1st pulse} \quad \text{(worst ISI)}
\]
\[
\frac{\sqrt{10^{17} \cdot I_{n,B}(0.9 \ B, 0.6 \ B)}}{v_{\text{max} \ B6} - v_{\text{min1} \ B6}} = 0.45 \quad \text{2nd pulse}
\]
\[
\frac{\sqrt{10^{17} \cdot I_{n,B}(0.9 \ B, 0.6 \ B)}}{v_{\text{max} \ B6} - v_{\text{min2} \ B6}} = 0.45 \quad \text{3rd pulse}
\]
$$t_{B7} := 0.5 \cdot ts$$

$$t_{pk7} := \text{root}(xvout(t_{B7}, 0.7 \ B, 0.7 \ B), t_{B7})$$

$$\frac{t_{pk7}}{ts} = 0.632$$

$$v_{\max B7} := vout(t_{pk7}, 0.7 \ B, 0.7 \ B) = 0.969$$

$$v_{\min B7} := vout(t_{pk7} + 1 \cdot ts, 0.7 \ B, 0.7 \ B) = 0.02$$

$$v_{\min1 B7} := vout(t_{pk7} + 2 \cdot ts, 0.7 \ B, 0.7 \ B) = 7.674 \cdot 10^{-5}$$

$$v_{\min2 B7} := vout(t_{pk7} + 3 \cdot ts, 0.7 \ B, 0.7 \ B) = 1.874 \cdot 10^{-7}$$

$$v_{\max B7} - v_{\min B7} = 0.949 \quad v_{\max B7} - v_{\min1 B7} = 0.969 \quad v_{\max B7} - v_{\min2 B7} = 0.969$$

$$\frac{v_{\max B7} - v_{\min B7}}{\sqrt{10^{17} \cdot I_{n,B}(0.7 \ B, 0.7 \ B)}} = 0.46$$

$$\frac{v_{\max B7} - v_{\min1 B7}}{\sqrt{10^{17} \cdot I_{n,B}(0.7 \ B, 0.7 \ B)}} = 0.47$$

$$\frac{v_{\max B7} - v_{\min2 B7}}{\sqrt{10^{17} \cdot I_{n,B}(0.7 \ B, 0.7 \ B)}} = 0.47$$

$$t_{B8} := 0.7 \cdot ts$$

$$t_{pk8} := \text{root}(xvout(t_{B8}, 0.8 \ B, 0.7 \ B), t_{B8})$$

$$\frac{t_{pk8}}{ts} = 0.618$$

$$v_{\max B8} := vout(t_{pk8}, 0.8 \ B, 0.7 \ B) = 0.986$$

$$v_{\min B8} := vout(t_{pk8} + 1 \cdot ts, 0.8 \ B, 0.7 \ B) = 0.005$$

$$v_{\min1 B8} := vout(t_{pk8} + 2 \cdot ts, 0.8 \ B, 0.7 \ B) = 2.769 \cdot 10^{-4}$$

$$v_{\min2 B8} := vout(t_{pk8} + 3 \cdot ts, 0.8 \ B, 0.7 \ B) = -1.695 \cdot 10^{-5}$$

$$v_{\max B8} - v_{\min B8} = 0.981 \quad v_{\max B8} - v_{\min1 B8} = 0.986 \quad v_{\max B8} - v_{\min2 B8} = 0.986$$

$$\frac{v_{\max B8} - v_{\min B8}}{\sqrt{10^{17} \cdot I_{n,B}(0.8 \ B, 0.7 \ B)}} = 0.44$$

$$\frac{v_{\max B8} - v_{\min1 B8}}{\sqrt{10^{17} \cdot I_{n,B}(0.8 \ B, 0.7 \ B)}} = 0.44$$

$$\frac{v_{\max B8} - v_{\min2 B8}}{\sqrt{10^{17} \cdot I_{n,B}(0.8 \ B, 0.7 \ B)}} = 0.44$$
\[ t_{B9} := 0.7 \cdot ts \]
\[ t_{pk9} := \text{root}(x\text{vout}(t_{B9}, 0.5B, 0.6B), t_{B9}) \]
\[ \frac{t_{pk9}}{ts} = 0.723 \]
\[ v_{\text{max}_{B9}} := v\text{out}(t_{pk9}, 0.5B, 0.6B) = 0.883 \]
\[ v_{\text{min}_{B9}} := v\text{out}(t_{pk9} + 1 \cdot ts, 0.5 \cdot B, 0.6B) = 0.088 \]
\[ v_{\text{min1}_{B9}} := v\text{out}(t_{pk9} + 2 \cdot ts, 0.5B, 0.6B) = -0.002 \]
\[ v_{\text{min2}_{B9}} := v\text{out}(t_{pk9} + 3 \cdot ts, 0.5B, 0.6B) = 4.263 \cdot 10^{-4} \]
\[ v_{\text{max}_{B9}} - v_{\text{min}_{B9}} = 0.795 \quad v_{\text{max}_{B9}} - v_{\text{min1}_{B9}} = 0.886 \quad v_{\text{max}_{B9}} - v_{\text{min2}_{B9}} = 0.883 \]
\[
\begin{align*}
\frac{v_{\text{max}_{B9}} - v_{\text{min}_{B9}}}{\sqrt{10^{17} \cdot I_{n_{B}}(0.5B, 0.6B)}} &= 0.49 \quad \text{1st pulse} \\
\frac{v_{\text{max}_{B9}} - v_{\text{min1}_{B9}}}{\sqrt{10^{17} \cdot I_{n_{B}}(0.5B, 0.6B)}} &= 0.55 \quad \text{2nd pulse} \\
\frac{v_{\text{max}_{B9}} - v_{\text{min2}_{B9}}}{\sqrt{10^{17} \cdot I_{n_{B}}(0.5B, 0.6B)}} &= 0.54 \quad \text{3rd pulse worst ISI} \\
\end{align*}
\]
\[ v_{\text{max}_{B}} := v_{\text{max}_{B9}} \quad v_{\text{min}_{B}} := v_{\text{min}_{B9}} \]

\text{opt} \quad rd = 0.5, \quad rt = 0.5 > 0.6

\[ x_{111}(t, rd, rt) := v\text{out}(t, rd, rt) + v\text{out}(t - 2 \cdot ts, rd, rt) + v\text{out}(t - ts, rd, rt) \]
\[ x_{110}(t, rd, rt) := v\text{out}(t, rd, rt) + v\text{out}(t - ts, rd, rt) \]
\[ x_{100}(t, rd, rt) := v\text{out}(t, rd, rt) \]
\[ x_{001}(t, rd, rt) := v\text{out}(t - 2 \cdot ts, rd, rt) \]
\[ x_{010}(t, rd, rt) := v\text{out}(t - ts, rd, rt) \]
\[ x_{011}(t, rd, rt) := v\text{out}(t - ts, rd, rt) + v\text{out}(t - 2 \cdot ts, rd, rt) \]
\[ x_{101}(t, rd, rt) := v\text{out}(t, rd, rt) + v\text{out}(t - 2 \cdot ts, rd, rt) \]
\[ x_{101}(t_{pk1} + ts, 0.5 B, 0.5 B) = 0.178 \]
\[ x_{101}(t_{pk1}, 0.5 B, 0.5 B) = 0.834 \]
\[ x_{101}(t_{pk1} + 2 ts, 0.5 B, 0.5 B) = 0.821 \]
\[ x_{100}(t_{pk1}, 0.5 B, 0.5 B) = 0.834 \]
\[ x_{100}(t_{pk1} + ts, 0.5 B, 0.5 B) = 0.15 \]
\[ x_{100}(t_{pk1} + 2 ts, 0.5 B, 0.5 B) = -0.012 \]

Check ratio < 1 is ok 100, > 1 re-opt

\[
\frac{x_{100}(t_{pk1}, 0.5 B, 0.5 B) - x_{100}(t_{pk1} + 2 ts, 0.5 B, 0.5 B)}{v_{max,B1} - v_{min,B1}} = 1.046
\]

**Sens PINFET**

\( \lambda := 650 \cdot 10^{-9} \)

\( photon\_energy := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \)

\( nq := 1.6 \cdot 10^{-19} \)

\[
Q_N(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{max,B} - v_{min,B}}{2} \right) \sqrt{I_{n,B}(0.5 B, 0.5 B)}
\]
\[ P_{e_N}(b) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{e_N}(b)}{\sqrt{2}} \right) \]

\[ P_{c_N}(b) = (\log(P_{e_N}(b)) + 9) \]

\[ b \equiv 3 \cdot 10^3 \]

\[ b_{N} := \text{root}\left(P_{c_N}(b), b\right) \]

\[ \text{minimum}_B := \min(b_{N}) = 4.363 \cdot 10^3 \]

**APD-1FET**

\[ P_{D_N}(b) := \frac{1}{b_{N}} \cdot b \cdot nq \cdot (v_{\text{max}_B}) \]

\[ P_{D_B}(b) := \frac{1}{b_{N}} \cdot b \cdot nq \cdot (v_{\text{max}_B}) \]

\[ I_{\text{APD},d} := 10 \cdot 10^{-9} \]

\[ M_{\text{APD}} := 100 \]

\[ F_{M} := 7.9 \]

\[ \text{Noise}_{\text{APD},1}(b, rd, rt) := \left( 2 \cdot q \cdot P_{D_B}(b) \cdot M_{\text{APD}} \cdot F_{M} \right) + 2 \cdot q \cdot I_{\text{APD},d} \cdot M_{\text{APD}} \cdot F_{M} \]

\[ Q_{\text{APD},1}(b) := \frac{M_{\text{APD}}}{b_{N}} \cdot \frac{nq \cdot (v_{\text{max}_B} - v_{\text{min}_B})}{2} \]

\[ \text{Noise}_{\text{APD},1}(b, 0.5 B, 0.5 B) + \text{Noise}_{\text{APD},1}(b, 0.5 B, 0.5 B) \]

\[ P_{c\text{APD},1}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD},1}(b)}{\sqrt{2}} \right) \]

\[ b_{\text{APD},1} := \text{root}(P_{c\text{APD},1}(b), b) \]

\[ \text{minimum}_{\text{APD},1}(b) := \min(b_{\text{APD},1}) = 1.754 \cdot 10^3 \]

**APD-2FET**

\[ M_{\text{APD}} := 10 \quad F_{M} := 5.5 \]

\[ \text{Noise}_{\text{APD},2}(b, rd, rt) := \left( 2 \cdot q \cdot P_{D_B}(b) \cdot M_{\text{APD}} \cdot F_{M} \right) + 2 \cdot q \cdot I_{\text{APD},d} \cdot M_{\text{APD}} \cdot F_{M} \]

\[ Q_{\text{APD},2}(b) := \frac{M_{\text{APD}}}{b_{N}} \cdot \frac{nq \cdot (v_{\text{max}_B} - v_{\text{min}_B})}{2} \]

\[ \text{Noise}_{\text{APD},2}(b, 0.5 B, 0.5 B) + \text{Noise}_{\text{APD},2}(b, 0.5 B, 0.5 B) \]

B-111
\[ P_{e\_APD2\_B}(b) = \frac{1}{2} \cdot \text{erfc}\left( \frac{Q_{APD2\_B}(b)}{\sqrt{2}} \right) \]

\[ p_{APD2\_B}(b) := (\log(P_{e\_APD2\_B}(b)) + 9) \]

\[ APD2\_B := \text{root}(p_{APD2\_B}(b), b) \]

\[ \text{minimum}_{\ APD2\_B} := \min(\ APD2\_B) = 1.432 \cdot 10^3 \]

**APD-3FET**

\[ M_{\ APD3} := 10 \quad F_{\ M3} := 9.2 \]

\[ \text{Noise}_{\ APD3\_B}(b, rd, rt) := \left( 2 \cdot q \cdot \text{APD\_B}(b) \cdot M_{\ APD3}^2 \cdot F_{\ M3} \right) \cdot \text{NEB\_B}(rd, rt) \]

\[ + 2 \cdot q \cdot I_{\ APD\_d} \cdot M_{\ APD3}^2 \cdot F_{\ M3} \]

\[ Q_{\ APD3\_B}(b) := \frac{M_{\ APD3}}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{\max\_B} - v_{\min\_B}}{2} \right) \]

\[ \sqrt{I_{\ n\_B}(0.5 B, 0.5 B) + \text{Noise}_{\ APD3\_B}(b, 0.5 B, 0.5 B)} \]

\[ P_{e\_APD3\_B}(b) = \frac{1}{2} \cdot \text{erfc}\left( \frac{Q_{\ APD3\_B}(b)}{\sqrt{2}} \right) \]

\[ p_{APD3\_B}(b) := (\log(P_{e\_APD3\_B}(b)) + 9) \]

\[ APD3\_B := \text{root}(p_{APD3\_B}(b), b) \]

\[ \text{minimum}_{\ APD3\_B} := \min(\ APD3\_B) = 2.05 \cdot 10^3 \]

**Sens\_B\_PIN**

\[ 10 \cdot \log\left( \frac{\text{minimum}\_B \cdot \text{photon\_energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -41.756 \]

**Sens\_B\_APD\_silicon**

\[ 10 \cdot \log\left( \frac{\text{minimum}\_APD1\_B \cdot \text{photon\_energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -45.712 \]

**Sens\_B\_APD\_InGaAs**

\[ 10 \cdot \log\left( \frac{\text{minimum}\_APD2\_B \cdot \text{photon\_energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -46.592 \]

**Sens\_B\_APD\_germanium**

\[ 10 \cdot \log\left( \frac{\text{minimum}\_APD3\_B}{\frac{\text{photon\_energy}}{10^{-3}}} \cdot \frac{1}{ts} \right) = -45.035 \]

#
B.2.5 Tuned B $a=0.2$

Tuned B optimum Performance (PIN-FET APD-FET 100 Mbit/s) $a=0.2$

Rx performance opt PIN-FET and APD-FET input configurations, 1st order LPF pre-detection filter. opt as follows

- TIA 3-db bandwidth rt/Filter 3-db bandwidth rd
- range cut-off
- minimum noise
- examine ISI
- highest SNR
- BER
- APD

opt rd= 0.5, rt = 0.5

Bit-rate, pulse duration

$C_T := 1.5 \cdot 10^{-12}$ total C

$B := 100 \cdot 10^6$ Bit-rate

pre-dec filter

$\omega_B(\text{rd}) := 2 \cdot \pi \cdot \text{rd}$

$H_{\text{bnt}}(\omega, \text{rd}) := \frac{1}{1 + j \frac{\omega}{2 \cdot \pi \cdot \text{rd}}}$

TIA Feedback value for (R) Tuned B

$\Delta_L := 1.8$ $\Delta_R := 1.87$ feedback $\Delta R$, time constant ratio $\Delta L$

$Av := 10$

\[ \begin{bmatrix} \alpha & \Delta_L & \Delta_R \\ 0.1 & 1.8 & 1.58 \\ 0.2 & 1.8 & 1.87 \\ 0.3 & 2.4 & 2.52 \\ 0.4 & 1.9 & 2.75 \\ 0.4 & 2.5 & 3.17 \\ 0.5 & 1.5 & 2.65 \end{bmatrix} \]

$Rf_B(\text{rt}) := \Delta_R \cdot \frac{Av + 1}{2 \pi \cdot \text{rt} \cdot C_T}$ feedback for Tuned B

$\alpha := 0.2$ splitting ratio

$LC(\text{rt}) := \frac{(Rf_B(\text{rt}))^2}{1 + Av} \cdot C_T$

$C1 := (1 - \alpha) \cdot (C_T) = 1.2 \cdot 10^{-12}$

$C2 := \alpha \cdot (C_T) = 3 \cdot 10^{-13}$
TUNED B (R) Transimpedance

\[ Z_{TIA,B}(\omega, rt) := \frac{1}{\left((1-\omega^2 \cdot Lc(\omega) \cdot C1) + \frac{Rf_B(rt)}{1+Av} \cdot (\omega \cdot 1i) \cdot (C1+C2-(\omega)^2 \cdot Lc(\omega) \cdot C1 \cdot C2) \right)} \]

Receiver freq-response

\[ Z_B(\omega, rd, rt) := H_{but}(\omega, rd) \cdot Z_{TIA,B}(\omega, rt) \]

\[ \omega := 1 \cdot 10^6, 10 \cdot 10^6 .. 1 \cdot 10^{11} \]

\[ rd := 0.5 \cdot B, 0.75 \cdot B, 2 \cdot B \]

\[ rt := 0.5 \cdot B, 0.55 \cdot B, 1 \cdot B \]

Noise equivalent bandwidth (NEB and I2)

\[ NEB_B(rd, rt) := \frac{1}{2\pi} \int_0^{10^{11}} \left( \left| Z_B(\omega, rd, rt) \right|^2 \right) d\omega \]

\[ I_{2,B}(rd, rt) := \frac{1}{2\pi} \int_0^{10^{11}} \left( \left(\left(1-\omega^2 \cdot Lc(\omega) \cdot C1\right)^2 \cdot Z_B(\omega, rd, rt) \right)^2 \right) d\omega \]
Noise equivalent bandwidth \( I_{3B}(rd, rt) \)

\[
I_{3B}(rd, rt) = \frac{1}{2\pi} \int_{0}^{10^{12}} \left( \left( \frac{1 - \omega^2 \cdot Lc(rt) \cdot C1}{Rf_B(rt) \cdot \omega} \right) + \left( \frac{C1 + C2 - \left( \omega \right)^2 \cdot Lc(rt) \cdot C1 \cdot C2 \cdot 1i}{Rf_B(rt)} \right) \right)^2 d\omega
\]

Feedback noise

\[
q = 1.6 \cdot 10^{-19} \quad k = 1.38 \cdot 10^{-23} \quad T = 298
\]

\[
I_{nRf_B}(rd, rt) = \frac{4 \cdot k \cdot T}{Rf_B(rt)} \cdot I_{2B}(rd, rt)
\]
Ig noise (gate current)

\[ Ig = 10 \cdot 10^{-5} \]

\[ \text{In}_{\text{G,B}}(rd, rt) = 2 \cdot q \cdot Ig \cdot I_{\text{-2_B}}(rd, rt) \]

Channel noise (Gate-source)

\[ gm1 = 30 \cdot 10^{-2} = 0.3 \]

\[ \text{noise factor} = 1 \]

\[ \text{In}_D = 4 \cdot k \cdot T \cdot \frac{1}{gm1} \]

\[ \text{In}_{\text{D,B}}(rd, rt) = \text{In}_D \cdot I_{\text{-3_B}}(rd, rt) \]

Total noise - PIN-FET

\[ \text{In}_B(rd, rt) = \text{In}_{\text{RF,B}}(rd, rt) + \text{In}_{\text{G,B}}(rd, rt) + \text{In}_{\text{D,B}}(rd, rt) \]
$I_{\text{n,B}(rd, rt)} \times 10^{-17}$

opt-total noise- PIN-FET
\[
I_{n,B}(0.7 \, B, 0.5 \, B) = 2.24 \cdot 10^{-17}
\]
\[
I_{n,B}(0.8 \, B, 0.5 \, B) = 2.53 \cdot 10^{-17}
\]
\[
I_{n,B}(0.9 \, B, 0.5 \, B) = 2.8 \cdot 10^{-17}
\]
\[
I_{n,B}(1 \, B, 0.5 \, B) = 3.06 \cdot 10^{-17}
\]
\[
I_{n,B}(0.7 \, B, 0.55 \, B) = 2.51 \cdot 10^{-17}
\]
\[
I_{n,B}(0.8 \, B, 0.55 \, B) = 2.83 \cdot 10^{-17}
\]
\[
I_{n,B}(0.9 \, B, 0.55 \, B) = 3.14 \cdot 10^{-17}
\]
\[
I_{n,B}(1 \, B, 0.55 \, B) = 3.44 \cdot 10^{-17}
\]
\[
I_{n,B}(0.7 \, B, 0.7 \, B) = 3.35 \cdot 10^{-17}
\]
\[
I_{n,B}(0.8 \, B, 0.7 \, B) = 3.78 \cdot 10^{-17}
\]
\[
I_{n,B}(0.9 \, B, 0.7 \, B) = 4.21 \cdot 10^{-17}
\]
\[
I_{n,B}(1 \, B, 0.7 \, B) = 4.63 \cdot 10^{-17}
\]
\[
0.8/0.7 > rd, \text{ range rt } 0.55-0.75
\]

Output pulse shape

\[
Ts := \frac{1}{B} \quad B = 1 \cdot 10^8
\]

\[
vout(t, rd, rt) := \frac{ts}{\pi} \cdot \frac{1}{ts \cdot 10^3} \cdot \sin\left(\omega \cdot \frac{ts}{2}\right) \cdot \Re\left(Z_B(\omega, rd, rt) \cdot \exp\left(i \omega \cdot t\right)\right) d\omega
\]

\[
x_{Bt}(t) := \frac{ts}{\pi} \cdot \frac{1}{ts \cdot 10^4} \cdot \sin\left(\omega \cdot \frac{ts}{2}\right) \cdot \Re\left(\exp\left(i \omega \cdot t\right)\right) d\omega
\]
\[
x_{\text{vout}}(t, rD, rT) := ts^2 \cdot \frac{1}{\pi} \int_{0}^{\frac{1}{ts}} \frac{\sin\left(\frac{\omega \cdot ts}{2}\right)}{\left(\frac{\omega \cdot ts}{2}\right)} \cdot \text{Re}(1i \cdot \omega \cdot Z_B(\omega, rD, rT) \cdot \exp(1i \cdot \omega \cdot t)) \, d\omega
\]

\[
t := -1 \, ts, -0.99 \, ts, 0.4 \, ts
\]

\[
v_{\text{out}}(t, 0.7 \, B, 0.6 \, B)
\]

\[
x_{\text{Bt}}(t)
\]

\[
x_{\text{vout}}(t, 0.7 \, B, 0.6 \, B)
\]

\[
\frac{1}{2}
\]

\[
t := -1 \, ts, -0.99 \, ts, 0.6 \, ts
\]

\[
V_1(t, rD, rT) := v_{\text{out}}(t, rD, rT)
\]

\[
V_1(t, 0.7 \, B, 0.5 \, B)
\]

\[
V_1(t, 0.7 \, B, 0.6 \, B)
\]

\[
V_1(t, 0.7 \, B, 0.7 \, B)
\]
\[ V_1(t, 0.8 B, 0.5 B) \]
\[ V_1(t, 0.8 B, 0.6 B) \]
\[ V_1(t, 0.8 B, 0.7 B) \]
\[ V_1(t, 0.8 B, 0.8 B) \]
\[ V_1(t, 0.8 B, 0.9 B) \]
\[ V_1(t, 0.8 B, 1 B) \]

\[ V_1(t, 0.9 B, 0.5 B) \]
\[ V_1(t, 0.9 B, 0.6 B) \]
\[ V_1(t, 0.9 B, 0.7 B) \]
\[ V_1(t, 0.9 B, 0.8 B) \]
\[ V_1(t, 1 B, 0.5 B) \]

\[ V_{101}(t, rd, rt) := vout(t, rd, rt) + vout(t - 2 \cdot ts, rd, rt) \]

\[ V_{101}(t, 0.7 B, 0.7 B) \]
\[ V_{101}(t, 0.7 B, 0.9 B) \]
\[ V_{101}(t, 0.9 B, 0.6 B) \]
\[ V_{101}(t, 0.9 B, 0.8 B) \]
\( t_{B1} := 0.6 \cdot ts \)
\( t_{pk1} := \text{root}(xvout(t_{B1}, 0.7 \cdot B, 0.5 \cdot B), t_{B1}) \)
\( \frac{t_{pk1}}{ts} = 0.822 \)
\( v_{max_{B1}} := xvout(t_{pk1}, 0.7 \cdot B, 0.5 \cdot B) = 0.878 \)
\( v_{min_{B1}} := xvout(t_{pk1} + 1 \cdot ts, 0.7 \cdot B, 0.5 \cdot B) = 0.079 \)
\( v_{min1_{B1}} := xvout(t_{pk1} + 2 \cdot ts, 0.7 \cdot B, 0.5 \cdot B) = 0.003 \)
\( v_{min2_{B1}} := xvout(t_{pk1} + 3 \cdot ts, 0.7 \cdot B, 0.5 \cdot B) = 5.81 \cdot 10^{-4} \)
\( v_{max_{B1}} - v_{min_{B1}} = 0.799 \quad v_{max_{B1}} - v_{min1_{B1}} = 0.881 \quad v_{max_{B1}} - v_{min2_{B1}} = 0.877 \)
\( \frac{v_{max_{B1}} - v_{min_{B1}}}{\sqrt{10^{17} \cdot I_{n_{B}}(0.7 \cdot B, 0.5 \cdot B)}} = 0.53 \quad \text{1st pulse worst ISI} \)
\( \frac{v_{max_{B1}} - v_{min1_{B1}}}{\sqrt{10^{17} \cdot I_{n_{B}}(0.7 \cdot B, 0.5 \cdot B)}} = 0.588 \quad \text{2nd pulse} \)
\( \frac{v_{max_{B1}} - v_{min2_{B1}}}{\sqrt{10^{17} \cdot I_{n_{B}}(0.7 \cdot B, 0.5 \cdot B)}} = 0.586 \quad \text{3rd pulse} \)

\( t_{B2} := 0.7 \cdot ts \)
\( t_{pk2} := \text{root}(xvout(t_{B2}, 0.8 \cdot B, 0.5 \cdot B), t_{B2}) \)
\( \frac{t_{pk2}}{ts} = 0.804 \)
\( v_{max_{B2}} := xvout(t_{pk2}, 0.8 \cdot B, 0.5 \cdot B) = 0.894 \)
\( v_{min_{B2}} := xvout(t_{pk2} + 1 \cdot ts, 0.8 \cdot B, 0.5 \cdot B) = 0.067 \)
\( v_{min1_{B2}} := xvout(t_{pk2} + 2 \cdot ts, 0.8 \cdot B, 0.5 \cdot B) = 0.003 \)
\( v_{min2_{B2}} := xvout(t_{pk2} + 3 \cdot ts, 0.8 \cdot B, 0.5 \cdot B) = 5.629 \cdot 10^{-4} \)
\( v_{max_{B2}} - v_{min_{B2}} = 0.827 \quad v_{max_{B2}} - v_{min1_{B2}} = 0.897 \quad v_{max_{B2}} - v_{min2_{B2}} = 0.893 \)
\( \frac{v_{max_{B2}} - v_{min_{B2}}}{\sqrt{10^{17} \cdot I_{n_{B}}(0.8 \cdot B, 0.5 \cdot B)}} = 0.52 \quad \text{1st pulse worst ISI} \)
\( \frac{v_{max_{B2}} - v_{min1_{B2}}}{\sqrt{10^{17} \cdot I_{n_{B}}(0.8 \cdot B, 0.5 \cdot B)}} = 0.56 \quad \text{2nd pulse} \)
\( \frac{v_{max_{B2}} - v_{min2_{B2}}}{\sqrt{10^{17} \cdot I_{n_{B}}(0.8 \cdot B, 0.5 \cdot B)}} = 0.56 \quad \text{3rd pulse} \)
\[ t_{B3} := 0.6 \times t_s \]
\[ t_{pk3} := \text{root} (xvout (t_{B3}, 0.9 B, 0.5 B), t_{B3}) \]
\[ \frac{t_{pk3}}{t_s} = 0.789 \]
\[ v_{\text{max, } B3} := \text{vout} \left( t_{pk3}, 0.9 B, 0.5 B \right) = 0.906 \]
\[ v_{\text{min, } B3} := \text{vout} \left( t_{pk3} + 1 \times t_s, 0.9 B, 0.5 B \right) = 0.058 \]
\[ v_{\text{min1, } B3} := \text{vout} \left( t_{pk3} + 2 \times t_s, 0.9 B, 0.5 B \right) = -0.003 \]
\[ v_{\text{min2, } B3} := \text{vout} \left( t_{pk3} + 3 \times t_s, 0.9 B, 0.5 B \right) = 5.407 \times 10^{-4} \]
\[ v_{\text{max, } B3} - v_{\text{min, } B3} = 0.848 \quad v_{\text{max, } B3} - v_{\text{min1, } B3} = 0.909 \quad v_{\text{max, } B3} - v_{\text{min2, } B3} = 0.906 \]

\[ \frac{v_{\text{max, } B3} - v_{\text{min, } B3}}{\sqrt{10^{17} \cdot I_{n,B} (0.9 B, 0.5 B)}} = 0.51 \]

1st pulse  worst ISI

\[ \frac{v_{\text{max, } B3} - v_{\text{min1, } B3}}{\sqrt{10^{17} \cdot I_{n,B} (0.9 B, 0.5 B)}} = 0.54 \]

2nd pulse

\[ \frac{v_{\text{max, } B3} - v_{\text{min2, } B3}}{\sqrt{10^{17} \cdot I_{n,B} (0.9 B, 0.5 B)}} = 0.54 \]

3rd pulse

(rt = 0.6)

\[ t_{B4} := 0.6 \times t_s \]
\[ t_{pk4} := \text{root} (xvout (t_{B4}, 0.7 B, 0.6 B), t_{B4}) \]
\[ \frac{t_{pk4}}{t_s} = 0.734 \]
\[ v_{\text{max, } B4} := \text{vout} \left( t_{pk4}, 0.7 B, 0.6 B \right) = 0.927 \]
\[ v_{\text{min, } B4} := \text{vout} \left( t_{pk4} + 1 \times t_s, 0.7 B, 0.6 B \right) = 0.046 \]
\[ v_{\text{min1, } B4} := \text{vout} \left( t_{pk4} + 2 \times t_s, 0.7 B, 0.6 B \right) = 0.002 \]
\[ v_{\text{min2, } B4} := \text{vout} \left( t_{pk4} + 3 \times t_s, 0.7 B, 0.6 B \right) = -1.222 \times 10^{-4} \]
\[ v_{\text{max, } B4} - v_{\text{min, } B4} = 0.88 \quad v_{\text{max, } B4} - v_{\text{min1, } B4} = 0.925 \quad v_{\text{max, } B4} - v_{\text{min2, } B4} = 0.927 \]

\[ \frac{v_{\text{max, } B4} - v_{\text{min, } B4}}{\sqrt{10^{17} \cdot I_{n,B} (0.7 B, 0.6 B)}} = 0.53 \]

1st pulse  worst ISI

\[ \frac{v_{\text{max, } B4} - v_{\text{min1, } B4}}{\sqrt{10^{17} \cdot I_{n,B} (0.7 B, 0.6 B)}} = 0.55 \]

2nd pulse

\[ \frac{v_{\text{max, } B4} - v_{\text{min2, } B4}}{\sqrt{10^{17} \cdot I_{n,B} (0.7 B, 0.6 B)}} = 0.56 \]

3rd pulse
\[ t_{B5} := 0.8 \cdot ts \]

\[ t_{pk5} := \text{root}(xvout(t_{B5}, 0.8 B, 0.6 B), t_{B5}) \]

\[ \frac{t_{pk5}}{ts} = 0.717 \]

\[ v_{\text{max}, B5} := vout\left(t_{pk5}, 0.8 B, 0.6 B\right) = 0.944 \]

\[ v_{\text{min}, B5} := vout\left(t_{pk5} + 1 \cdot ts, 0.8 B, 0.6 B\right) = 0.032 \]

\[ v_{\text{min1}, B5} := vout\left(t_{pk5} + 2 \cdot ts, 0.8 B, 0.6 B\right) = 0.002 \]

\[ v_{\text{min2}, B5} := vout\left(t_{pk5} + 3 \cdot ts, 0.8 B, 0.6 B\right) = -1.561 \cdot 10^{-4} \]

\[ v_{\text{max}, B5} - v_{\text{min}, B5} = 0.912 \quad v_{\text{max}, B5} - v_{\text{min1}, B5} = 0.942 \quad v_{\text{max}, B5} - v_{\text{min2}, B5} = 0.944 \]

\[
\frac{v_{\text{max}, B5} - v_{\text{min}, B5}}{\sqrt{10^{17} \cdot I_{n,B}(0.8 B, 0.6 B)}} = 0.51
\]

\[
\frac{v_{\text{max}, B5} - v_{\text{min1}, B5}}{\sqrt{10^{17} \cdot I_{n,B}(0.8 B, 0.6 B)}} = 0.53
\]

\[
\frac{v_{\text{max}, B5} - v_{\text{min2}, B5}}{\sqrt{10^{17} \cdot I_{n,B}(0.8 B, 0.6 B)}} = 0.53
\]

1st pulse  worst ISI  2nd pulse  3rd pulse

\[ t_{B6} := 0.5 \cdot ts \]

\[ t_{pk6} := \text{root}(xvout(t_{B4}, 0.9 B, 0.6 B), t_{B4}) \]

\[ \frac{t_{pk6}}{ts} = 0.703 \]

\[ v_{\text{max}, B6} := vout\left(t_{pk6}, 0.9 B, 0.6 B\right) = 0.957 \]

\[ v_{\text{min}, B6} := vout\left(t_{pk6} + 1 \cdot ts, 0.9 B, 0.6 B\right) = 0.022 \]

\[ v_{\text{min1}, B6} := vout\left(t_{pk6} + 2 \cdot ts, 0.9 B, 0.6 B\right) = 0.002 \]

\[ v_{\text{min2}, B6} := vout\left(t_{pk6} + 3 \cdot ts, 0.9 B, 0.6 B\right) = -1.808 \cdot 10^{-4} \]

\[ v_{\text{max}, B6} - v_{\text{min}, B6} = 0.935 \quad v_{\text{max}, B6} - v_{\text{min1}, B6} = 0.955 \quad v_{\text{max}, B6} - v_{\text{min2}, B6} = 0.957 \]

\[
\frac{v_{\text{max}, B6} - v_{\text{min}, B6}}{\sqrt{10^{17} \cdot I_{n,B}(0.9 B, 0.6 B)}} = 0.5 \quad \text{1st pulse \ worst ISI} \]

\[
\frac{v_{\text{max}, B6} - v_{\text{min1}, B6}}{\sqrt{10^{17} \cdot I_{n,B}(0.9 B, 0.6 B)}} = 0.51 \quad \text{2nd pulse} \]

\[
\frac{v_{\text{max}, B6} - v_{\text{min2}, B6}}{\sqrt{10^{17} \cdot I_{n,B}(0.9 B, 0.6 B)}} = 0.51 \quad \text{3rd pulse} \]
\( (t_B = 0.7) \)

\[ t_{B7} := 0.5 \cdot t_s \]

\[ t_{pk7} := \text{root} \left( xvout \left( t_{B7}, 0.7 \cdot B, 0.7 \cdot B \right), t_{B7} \right) \]

\[ \frac{t_{pk7}}{t_s} = 0.673 \]

\[ v_{\text{max}_{B7}} := vout \left( t_{pk7}, 0.7 \cdot B, 0.7 \cdot B \right) = 0.953 \]

\[ v_{\text{min}_{B7}} := vout \left( t_{pk7} + 1 \cdot t_s, 0.7 \cdot B, 0.7 \cdot B \right) = 0.032 \]

\[ v_{\text{min}_{1\cdot B7}} := vout \left( t_{pk7} + 2 \cdot t_s, 0.7 \cdot B, 0.7 \cdot B \right) = 0.002 \]

\[ v_{\text{min}_{2\cdot B7}} := vout \left( t_{pk7} + 3 \cdot t_s, 0.7 \cdot B, 0.7 \cdot B \right) = -2.649 \cdot 10^{-5} \]

\[ v_{\text{max}_{B7}} - v_{\text{min}_{B7}} = 0.922 \quad v_{\text{max}_{B7}} - v_{\text{min}_{1\cdot B7}} = 0.951 \quad v_{\text{max}_{B7}} - v_{\text{min}_{2\cdot B7}} = 0.953 \]

\[ \frac{10^{17} \cdot I_n.B(0.7 \cdot B, 0.7 \cdot B)}{v_{\text{max}_{B7}} - v_{\text{min}_{B7}}} = 0.5 \quad \text{1st pulse} \quad \text{worst ISI} \]

\[ \frac{10^{17} \cdot I_n.B(0.7 \cdot B, 0.7 \cdot B)}{v_{\text{max}_{B7}} - v_{\text{min}_{1\cdot B7}}} = 0.52 \quad \text{2nd pulse} \]

\[ \frac{10^{17} \cdot I_n.B(0.7 \cdot B, 0.7 \cdot B)}{v_{\text{max}_{B7}} - v_{\text{min}_{2\cdot B7}}} = 0.52 \quad \text{3rd pulse} \]

\[ t_{B8} := 0.7 \cdot t_s \]

\[ t_{pk8} := \text{root} \left( xvout \left( t_{B8}, 0.8 \cdot B, 0.7 \cdot B \right), t_{B8} \right) \]

\[ \frac{t_{pk8}}{t_s} = 0.657 \]

\[ v_{\text{max}_{B8}} := vout \left( t_{pk8}, 0.8 \cdot B, 0.7 \cdot B \right) = 0.97 \]

\[ v_{\text{min}_{B8}} := vout \left( t_{pk8} + 1 \cdot t_s, 0.8 \cdot B, 0.7 \cdot B \right) = 0.017 \]

\[ v_{\text{min}_{1\cdot B8}} := vout \left( t_{pk8} + 2 \cdot t_s, 0.8 \cdot B, 0.7 \cdot B \right) = 0.002 \]

\[ v_{\text{min}_{2\cdot B8}} := vout \left( t_{pk8} + 3 \cdot t_s, 0.8 \cdot B, 0.7 \cdot B \right) = -3.467 \cdot 10^{-5} \]

\[ v_{\text{max}_{B8}} - v_{\text{min}_{B8}} = 0.953 \quad v_{\text{max}_{B8}} - v_{\text{min}_{1\cdot B8}} = 0.969 \quad v_{\text{max}_{B8}} - v_{\text{min}_{2\cdot B8}} = 0.97 \]

\[ \frac{10^{17} \cdot I_n.B(0.8 \cdot B, 0.7 \cdot B)}{v_{\text{max}_{B8}} - v_{\text{min}_{B8}}} = 0.49 \quad \text{1st pulse} \]

\[ \frac{10^{17} \cdot I_n.B(0.8 \cdot B, 0.7 \cdot B)}{v_{\text{max}_{B8}} - v_{\text{min}_{1\cdot B8}}} = 0.5 \quad \text{2nd pulse} \]

\[ \frac{10^{17} \cdot I_n.B(0.8 \cdot B, 0.7 \cdot B)}{v_{\text{max}_{B8}} - v_{\text{min}_{2\cdot B8}}} = 0.5 \quad \text{3rd pulse} \quad \text{worst ISI} \]
\( t_{B9} := 0.6 \cdot ts \)
\( t_{pk9} := \text{root}(x\text{out}(t_{B9}, 0.5 B, 0.5 B), t_{B9}) \)
\( \frac{t_{pk9}}{ts} = 0.874 \)

\( v_{\text{max,B9}} := v\text{out}(t_{pk9}, 0.5 B, 0.5 B) = 0.825 \)
\( v_{\text{min,B9}} := v\text{out}(t_{pk9} + 1 \cdot ts, 0.5 \cdot B, 0.5 B) = 0.118 \)
\( v_{\text{min1,B9}} := v\text{out}(t_{pk9} + 2 \cdot ts, 0.5 B, 0.5 B) = 3.259 \cdot 10^{-6} \)
\( v_{\text{min2,B9}} := v\text{out}(t_{pk9} + 3 \cdot ts, 0.5 B, 0.5 B) = 6.872 \cdot 10^{-4} \)

\( v_{\text{max,B9}} - v_{\text{min,B9}} = 0.707 \quad v_{\text{max,B9}} - v_{\text{min1,B9}} = 0.825 \quad v_{\text{max,B9}} - v_{\text{min2,B9}} = 0.824 \)

\[
\frac{v_{\text{max,B9}} - v_{\text{min,B9}}}{\sqrt{10^{17} \cdot I_{n,B}(0.5 B, 0.5 B)}} = 0.55 \quad \text{1st pulse}
\]
\[
\frac{v_{\text{max,B9}} - v_{\text{min1,B9}}}{\sqrt{10^{17} \cdot I_{n,B}(0.5 B, 0.5 B)}} = 0.64 \quad \text{2nd pulse}
\]
\[
\frac{v_{\text{max,B9}} - v_{\text{min2,B9}}}{\sqrt{10^{17} \cdot I_{n,B}(0.5 B, 0.5 B)}} = 0.64 \quad \text{3rd pulse} \quad \text{worst ISI}
\]

\( v_{\text{max,B}} := v_{\text{max,B9}} \quad v_{\text{min,B}} := v_{\text{min,B9}} \)

\( \text{opt rd= 0.5, rt = 0.5>0.6} \)

\( x_{111}(t, rd, rt) := v\text{out}(t, rd, rt) + v\text{out}(t - 2 \cdot ts, rd, rt) + v\text{out}(t - ts, rd, rt) \)
\( x_{110}(t, rd, rt) := v\text{out}(t, rd, rt) + v\text{out}(t - ts, rd, rt) \)
\( x_{100}(t, rd, rt) := v\text{out}(t, rd, rt) \)
\( x_{001}(t, rd, rt) := v\text{out}(t - 2 \cdot ts, rd, rt) \)
\( x_{010}(t, rd, rt) := v\text{out}(t - ts, rd, rt) \)
\( x_{011}(t, rd, rt) := v\text{out}(t - ts, rd, rt) + v\text{out}(t - 2 \cdot ts, rd, rt) \)
\( x_{101}(t, rd, rt) := v\text{out}(t, rd, rt) + v\text{out}(t - 2 \cdot ts, rd, rt) \)
Check ratio<1 is ok 100, >1 re-opt

\[
x_{101}(t_{pk1} + ts, 0.5 B, 0.5 B) = 0.18 \\
x_{101}(t_{pk1}, 0.5 B, 0.5 B) = 0.82 \\
x_{101}(t_{pk1} + 2 ts, 0.5 B, 0.5 B) = 0.82 \\
x_{100}(t_{pk1}, 0.5 B, 0.5 B) = 0.82 \\
x_{100}(t_{pk1} + ts, 0.5 B, 0.5 B) = 0.145 \\
x_{100}(t_{pk1} + 2 ts, 0.5 B, 0.5 B) = -5.457 \cdot 10^{-4}
\]

Check ratio<1 is ok 100, >1 re-opt

\[
x_{100}(t_{pk1}, 0.5 B, 0.5 B) - x_{100}(t_{pk1} + 2 ts, 0.5 B, 0.5 B) \over v_{max_B1} - v_{min_B1} = 1.028
\]

Sens PINFET

\[
\lambda := 650 \cdot 10^{-9}
\]

\[
\text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda}
\]

\[
nq := 1.6 \cdot 10^{-19}
\]

\[
Q_N(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{max_B} - v_{min_B}}{2} \right) \over \sqrt{I_{n,B}(0.5 B, 0.5 B)}
\]

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\[ P_e_N(b) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right) \]

\[ P_c_N(b) = \left( \log(P_e_N(b)) + 9 \right) \]

\[ b \equiv 3 \cdot 10^3 \]

\[ b_N := \text{root}(P_c_N(b), b) \]

\[ \text{minimum}_B := \min(b_N) = 4.32 \cdot 10^3 \]

**APD-1FET**

\[ APD_N(b) := \frac{1}{ts} \cdot b \cdot nq \cdot (v_{max,B}) \]

\[ APD_B(b) := \frac{1}{ts} \cdot b \cdot nq \cdot (v_{max,B}) \]

\[ I_{APD,d} := 10 \cdot 10^{-9} \]

\[ M_{APD1} := 100 \]

\[ F_M1 := 7.9 \]

\[ \text{Noise}_{APD1_B}(b, rd, rt) := \left( 2 \cdot q \cdot APD_B(b) \cdot M_{APD1^2} \cdot F_M1 \right) \cdot \text{NEB}_B(rd, rt) \]

\[ + 2 \cdot q \cdot I_{APD,d} \cdot M_{APD1^2} \cdot F_M1 \]

\[ Q_{APD1_B}(b) := \frac{M_{APD1}}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{max,B} - v_{min,B}}{2} \right) \]

\[ \sqrt{I_{n,B}(0.5B, 0.5B) + \text{Noise}_{APD1_B}(b, 0.5B, 0.5B)} \]

\[ P_e_{APD1_B}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD1_B}(b)}{\sqrt{2}} \right) \]

\[ p_{c_{APD1_B}}(b) := \left( \log(P_e_{APD1_B}(b)) + 9 \right) \]

\[ \text{APD1}_B := \text{root}(p_{c_{APD1_B}}(b), b) \]

\[ \text{NEB}_B(0.7B, 0.6B) = 4.845 \cdot 10^7 \]

\[ \text{minimum}_{APD1_B} := \min(\text{APD1}_B) = 1.979 \cdot 10^3 \]

**APD-2FET**

\[ M_{APD2} := 10 \quad F_M2 := 5.5 \]

\[ \text{Noise}_{APD2_B}(b, rd, rt) := \left( 2 \cdot q \cdot APD_B(b) \cdot M_{APD2^2} \cdot F_M2 \right) \cdot \text{NEB}_B(rd, rt) \]

\[ + 2 \cdot q \cdot I_{APD,d} \cdot M_{APD2^2} \cdot F_M2 \]

\[ Q_{APD2_B}(b) := \frac{M_{APD2}}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{max,B} - v_{min,B}}{2} \right) \]

\[ \sqrt{I_{n,B}(0.5B, 0.5B) + \text{Noise}_{APD2_B}(b, 0.5B, 0.5B)} \]
\[ P_{e\_APD2\_B}(b) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\_APD2\_B}(b)}{\sqrt{2}} \right) \]

\[ pc_{\_APD2\_B}(b) := (\log(P_{e\_APD2\_B}(b)) + 9) \]

\[ APD2\_b\_B := \text{root}(pc_{\_APD2\_B}(b), b) \]

\[ \text{minimum\_APD2\_B} := \min(APD2\_b\_B) = 1.588 \cdot 10^3 \]

---

**APD-3FET**

\[ M_{\_APD3} := 10 \quad F_{\_M3} := 9.2 \]

\[ \text{Noise\_APD3\_B}(b, rd, rt) := \left( 2 \cdot q \cdot \text{APD\_B}(b) \cdot M_{\_APD3}^2 \cdot F_{\_M3} \right) \cdot \text{NEB\_B}(rd, rt) \]

\[ + 2 \cdot q \cdot I_{\_APD\_d} \cdot M_{\_APD3}^2 \cdot F_{\_M3} \]

\[ Q_{\_APD3\_B}(b) := \frac{M_{\_APD3}}{ts} \cdot b \cdot nq \cdot \frac{v_{\text{max\_B}} - v_{\text{min\_B}}}{2} \]

\[ \sqrt{I_{\_n\_B}(0.5\_B, 0.5\_B) + \text{Noise\_APD3\_B}(b, 0.5\_B, 0.5\_B)} \]

\[ P_{e\_APD3\_B}(b) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\_APD3\_B}(b)}{\sqrt{2}} \right) \]

\[ pc_{\_APD3\_B}(b) := (\log(P_{e\_APD3\_B}(b)) + 9) \]

\[ APD3\_b\_B := \text{root}(pc_{\_APD3\_B}(b), b) \]

\[ \text{minimum\_APD3\_B} := \min(APD3\_b\_B) = 2.296 \cdot 10^3 \]

---

**Sens\_B\_PIN** := \(10 \cdot \log\left(\frac{\text{minimum\_B} \cdot \text{photon\_energy} \cdot 1}{2 \cdot ts} \cdot 10^{-3}\right) = -41.799 \)

**Sens\_B\_APD\_silicon** := \(10 \cdot \log\left(\frac{\text{minimum\_APD1\_B} \cdot \text{photon\_energy} \cdot 1}{2 \cdot ts} \cdot 10^{-3}\right) = -45.19 \)

**Sens\_B\_APD\_InGaAs** := \(10 \cdot \log\left(\frac{\text{minimum\_APD2\_B} \cdot \text{photon\_energy} \cdot 1}{2 \cdot ts} \cdot 10^{-3}\right) = -46.145 \)

**Sens\_B\_APD\_germanium** := \(10 \cdot \log\left(\frac{\text{minimum\_APD3\_B} \cdot \text{photon\_energy} \cdot 1}{2 \cdot ts} \cdot 10^{-3}\right) = -44.543 \)

#
B.2.6 Tuned B a=0.3

Tuned B optimum Performance (PIN-FET APD-FET 100 Mbit/s) a=0.3

Rx performance opt PIN-FET and APD-FET input configurations, 1st order LPF pre-detection filter. opt as follows

- TIA 3-db bandwidth rt/Filter 3-db bandwidth rd
- range cut-off
- min. noise
- examine ISI
- highest SNR
- BER
- APD

opt rd = 0.5, rd = 0.5

Bit-rate, pulse duration

\[ C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]
\[ B := 100 \cdot 10^6 \quad \text{Bit-rate} \]

pre-dec filter

\[ \omega_B(\text{rd}) := 2 \cdot \pi \cdot \text{rd} \]
\[ H_{\text{b}}(\omega, \text{rd}) := \frac{1}{1 + j \frac{\omega}{2 \cdot \pi \cdot \text{rd}}} \]

TIA

Feedback value for (R) Tuned B

\[ \Delta_L := 2.4 \quad \Delta_R := 2.52 \quad \text{feedback } \Delta_R, \text{ time constant ratio } \Delta_L \]
\[ A_v := 10 \quad \begin{array}{c|c|c}
\alpha & \Delta_L & \Delta_R \\
\hline
0 & 2 & 1.41 \\
0.1 & 1.8 & 1.58 \\
0.2 & 1.8 & 1.87 \\
0.3 & 2.4 & 2.52 \\
0.4 & 1.9 & 2.75 \\
0.5 & 2.5 & 3.17 \\
\end{array} \]

\[ Rf_B(\text{rt}) := \Delta_R \cdot \frac{A_v + 1}{2 \cdot \pi \cdot \text{rt} \cdot C_T} \quad \text{feedback for Tuned B} \]

\[ \alpha := 0.3 \quad \text{splitting ratio} \]

\[ Lc(\text{rt}) := \left( \frac{Rf_B(\text{rt})}{1 + A_v} \right)^2 \cdot C_T \]
\[ C1 := (1 - \alpha) \cdot (C_T) = 1.05 \cdot 10^{-12} \]
\[ C2 := \alpha \cdot (C_T) = 4.5 \cdot 10^{-13} \]
TUNED B (R) Transimpedance

\[ Z_{TIA,B}(\omega, rt) = \frac{1}{\left((1-\omega^2 \cdot Lc(rt) \cdot C1) + \frac{Rf_B(rt)}{(1+Av)} \cdot (\omega \cdot i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc(rt) \cdot C1 \cdot C2)\right)} \]

**Receiver freq-response**

\[ Z_B(\omega, rd, rt) = H_{but}(\omega, rd) \cdot Z_{TIA,B}(\omega, rt) \]

\[ \omega := 1 \cdot 10^6, 10 \cdot 10^6 .. 1 \cdot 10^{11} \]

\[ rd := 0.5 \cdot B, 0.75 \cdot B .. 2 \cdot B \]

\[ rt := 0.5 \cdot B, 0.55 \cdot B .. 1 \cdot B \]

**Noise equivalent bandwidth (NEB and I2)**

\[ NEB_B(rd, rt) := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{11}} \left( \left| Z_B(\omega, rd, rt) \right| \right)^2 \, d\omega \right) \]

\[ I_{2,B}(rd, rt) := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{11}} \left( \left| Z_B(\omega, rd, rt) \right| \right)^2 \, d\omega \right) \]
**Noise equivalent bandwidth (13)**

\[
I_{3,B}(rd, rt) = \frac{1}{2\pi} \cdot \left. \int_{0}^{10^{12}} \left( \left( \frac{1 - \omega^2 \cdot Lc(rt) \cdot C1}{\omega^2 + Rf_B(rt) \cdot \omega} \cdot \left( C1 + C2 - \omega^2 \cdot Lc(rt) \cdot C1 \cdot C2 \right) \cdot 1 \right)^2 \cdot Z_B(\omega, rd, rt) \cdot Rf_B(rt) \right) \, d\omega
\]

**Feedback noise**

\[q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298\]

\[
I_{nRf,B}(rd, rt) = \frac{4 \cdot k \cdot T}{Rf_B(rt)} \cdot I_{2,B}(rd, rt)
\]
\( I_g \) noise (gate current)

\[ I_g := 10 \cdot 10^{-9} \]

\[ I_{nG, B}(rd, rt) := 2 \cdot q \cdot I_g \cdot I_{2, B}(rd, rt) \]

Channel noise (Gate-source)

\[ g_m 1 := 30 \cdot 10^{-3} = 0.03 \quad \text{noise factor} := 1 \]

\[ I_{nD} := 4 \cdot k \cdot T \cdot \frac{1}{g_m 1} \]

\[ I_{nD, B}(rd, rt) := I_{nD} \cdot I_{3, B}(rd, rt) \]

Total noise - PIN-FET

\[ I_{n, B}(rd, rt) := I_{nRf, B}(rd, rt) + I_{nG, B}(rd, rt) + I_{nD, B}(rd, rt) \]
\[ I_{n_B}(rd, rt) \times 10^{-17} \]
\[
I_{n,B}(0.7 \ B, 0.5 \ B) = 1.33 \cdot 10^{-17}
\]
\[
I_{n,B}(0.8 \ B, 0.5 \ B) = 1.47 \cdot 10^{-17}
\]
\[
I_{n,B}(0.9 \ B, 0.5 \ B) = 1.60 \cdot 10^{-17}
\]
\[
I_{n,B}(1 \ B, 0.5 \ B) = 1.71 \cdot 10^{-17}
\]
\[
I_{n,B}(0.7 \ B, 0.55 \ B) = 1.51 \cdot 10^{-17}
\]
\[
I_{n,B}(0.8 \ B, 0.55 \ B) = 1.67 \cdot 10^{-17}
\]
\[
I_{n,B}(0.9 \ B, 0.55 \ B) = 1.82 \cdot 10^{-17}
\]
\[
I_{n,B}(1 \ B, 0.55 \ B) = 1.96 \cdot 10^{-17}
\]
\[
I_{n,B}(0.7 \ B, 0.7 \ B) = 2.07 \cdot 10^{-17}
\]
\[
I_{n,B}(0.8 \ B, 0.7 \ B) = 2.3 \cdot 10^{-17}
\]
\[
I_{n,B}(0.9 \ B, 0.7 \ B) = 2.53 \cdot 10^{-17}
\]
\[
I_{n,B}(1 \ B, 0.7 \ B) = 2.74 \cdot 10^{-17}
\]

0.8/0.7 > rd, range rt 0.55-0.75

**Output pulse shape**

\[
\begin{align*}
\theta_s &:= \frac{1}{B} \quad B = 1 \cdot 10^8 \\
\frac{1}{\theta_s} &\cdot 10^3 \\
\text{vout}(t, rd, rt) &:= \frac{\theta_s}{\pi} \cdot \int_0^{\frac{1}{\theta_s} \cdot 10^4} \frac{\sin \left( \frac{\omega \cdot ts}{2} \right)}{\left( \omega \cdot \frac{ts}{2} \right)} \cdot \text{Re} \left( Z_B(\omega, rd, rt) \cdot \exp \left( \frac{1i \cdot \omega \cdot t \cdot (t)) \right) \right) \, d\omega
\end{align*}
\]

\[
\begin{align*}
\theta_s &:= \frac{ts}{\pi} \cdot \int_0^{\frac{1}{\theta_s} \cdot 10^4} \frac{\sin \left( \frac{\omega \cdot ts}{2} \right)}{\left( \omega \cdot \frac{ts}{2} \right)} \cdot \text{Re} \left( \exp \left( \frac{1i \cdot \omega \cdot t \cdot (t)) \right) \right) \, d\omega
\end{align*}
\]
\[ x_{\text{out}}(t, rd, rt) := ts^2 \cdot \frac{1}{\pi} \int_{0}^{\frac{\omega \cdot ts}{2}} \frac{\sin\left(\frac{\omega \cdot ts}{2}\right)}{\left(\frac{\omega \cdot ts}{2}\right)} d\omega \cdot \Re(1i \cdot \omega \cdot Z_B(\omega, rd, rt) \cdot \exp(1i \cdot \omega \cdot t)) \]

\[ t := -1 ts, -0.99 ts, 0.4 ts \]

\[ \| ISI \|
\]

\[ t := -1 ts, -0.99 ts, 0.6 ts \]

\[ V_1(t, rd, rt) := v_{\text{out}}(t, rd, rt) \]

\[ v_{\text{out}}(t, 0.7 B, 0.6 B) \]
\[ x_{\text{Bt}}(t) \]
\[ x_{\text{out}}(t, 0.7 B, 0.6 B) \]
\[ \frac{1}{2} \]

\[ V_1(t, 0.7 B, 0.5 B) \]
\[ V_1(t, 0.7 B, 0.6 B) \]
\[ V_1(t, 0.7 B, 0.7 B) \]
\[ V_{1}(t, 0.8 B, 0.5 B) \]
\[ V_{1}(t, 0.8 B, 0.6 B) \]
\[ V_{1}(t, 0.8 B, 0.7 B) \]
\[ V_{1}(t, 0.8 B, 0.8 B) \]
\[ V_{1}(t, 0.8 B, 0.9 B) \]
\[ V_{1}(t, 0.8 B, 1 B) \]

\[ V_{1}(t, 0.9 B, 0.5 B) \]
\[ V_{1}(t, 0.9 B, 0.6 B) \]
\[ V_{1}(t, 0.9 B, 0.7 B) \]
\[ V_{1}(t, 0.9 B, 0.8 B) \]
\[ V_{1}(t, 1 B, 0.5 B) \]

\[ V_{101}(t, rd, rt) := vout(t, rd, rt) + vout(t - 2 \ ts, rd, rt) \]

\[ V_{101}(t, 0.7 B, 0.7 B) \]
\[ V_{101}(t, 0.7 B, 0.9 B) \]
\[ V_{101}(t, 0.9 B, 0.6 B) \]
\[ V_{101}(t, 0.9 B, 0.8 B) \]
\[ t_{B1} := 0.6 \cdot ts \]
\[ t_{pk1} := \text{root} \left( xvout \left( t_{B1}, 0.7B, 0.5B \right), t_{B1} \right) \]
\[ \frac{t_{pk1}}{ts} = 0.914 \]
\[ v_{\text{max, } B1} := vout \left( t_{pk1}, 0.7B, 0.5B \right) = 0.814 \]
\[ v_{\text{min, } B1} := vout \left( t_{pk1} + 1 \cdot ts, 0.7B, 0.5B \right) = 0.076 \]
\[ v_{\text{min1, } B1} := vout \left( t_{pk1} + 2 \cdot ts, 0.7B, 0.5B \right) = 0.053 \]
\[ v_{\text{min2, } B1} := vout \left( t_{pk1} + 3 \cdot ts, 0.7B, 0.5B \right) = -0.002 \]
\[ v_{\text{max, } B1} - v_{\text{min, } B1} = 0.738 \quad v_{\text{max, } B1} - v_{\text{min1, } B1} = 0.761 \quad v_{\text{max, } B1} - v_{\text{min2, } B1} = 0.816 \]
\[ \frac{v_{\text{max, } B1} - v_{\text{min, } B1}}{\sqrt{10^{17} \cdot I_{n, B}(0.7B, 0.5B)}} = 0.64 \quad \text{1st pulse} \quad \text{worst ISI} \]
\[ \frac{v_{\text{max, } B1} - v_{\text{min1, } B1}}{\sqrt{10^{17} \cdot I_{n, B}(0.7B, 0.5B)}} = 0.66 \quad \text{2nd pulse} \]
\[ \frac{v_{\text{max, } B1} - v_{\text{min2, } B1}}{\sqrt{10^{17} \cdot I_{n, B}(0.7B, 0.5B)}} = 0.708 \quad \text{3rd pulse} \]

\[ t_{B2} := 1 \cdot ts \]
\[ t_{pk2} := \text{root} \left( xvout \left( t_{B2}, 0.8B, 0.5B \right), t_{B2} \right) \]
\[ \frac{t_{pk2}}{ts} = 0.895 \]
\[ v_{\text{max, } B2} := vout \left( t_{pk2}, 0.8B, 0.5B \right) = 0.829 \]
\[ v_{\text{min, } B2} := vout \left( t_{pk2} + 1 \cdot ts, 0.8B, 0.5B \right) = 0.064 \]
\[ v_{\text{min1, } B2} := vout \left( t_{pk2} + 2 \cdot ts, 0.8B, 0.5B \right) = 0.054 \]
\[ v_{\text{min2, } B2} := vout \left( t_{pk2} + 3 \cdot ts, 0.8B, 0.5B \right) = -0.003 \]
\[ v_{\text{max, } B2} - v_{\text{min, } B2} = 0.765 \quad v_{\text{max, } B2} - v_{\text{min1, } B2} = 0.775 \quad v_{\text{max, } B2} - v_{\text{min2, } B2} = 0.832 \]
\[ \frac{v_{\text{max, } B2} - v_{\text{min, } B2}}{\sqrt{10^{17} \cdot I_{n, B}(0.8B, 0.5B)}} = 0.63 \quad \text{1st pulse} \quad \text{worst ISI} \]
\[ \frac{v_{\text{max, } B2} - v_{\text{min1, } B2}}{\sqrt{10^{17} \cdot I_{n, B}(0.8B, 0.5B)}} = 0.64 \quad \text{2nd pulse} \]
\[ \frac{v_{\text{max, } B2} - v_{\text{min2, } B2}}{\sqrt{10^{17} \cdot I_{n, B}(0.8B, 0.5B)}} = 0.69 \quad \text{3rd pulse} \]
\[ t_{B3} := 0.6 \cdot ts \]
\[ t_{pk3} := \text{root}(xvout(t_{B3}, 0.9 B, 0.5 B), t_{B3}) \]
\[ \frac{t_{pk3}}{ts} = 0.879 \]
\[ v_{\text{max}_{B3}} := vout(t_{pk3}, 0.9 B, 0.5 B) = 0.84 \]
\[ v_{\text{min}_{B3}} := vout(t_{pk3} + 1 \cdot ts, 0.9 B, 0.5 B) = 0.055 \]
\[ v_{\text{min1}_{B3}} := vout(t_{pk3} + 2 \cdot ts, 0.9 B, 0.5 B) = 0.055 \]
\[ v_{\text{min2}_{B3}} := vout(t_{pk3} + 3 \cdot ts, 0.9 B, 0.5 B) = -0.003 \]
\[ v_{\text{max}_{B3}} - v_{\text{min}_{B3}} = 0.785 \quad v_{\text{max}_{B3}} - v_{\text{min1}_{B3}} = 0.785 \quad v_{\text{max}_{B3}} - v_{\text{min2}_{B3}} = 0.843 \]

\[ \frac{v_{\text{max}_{B3}} - v_{\text{min}_{B3}}}{\sqrt{10^{17} \cdot I_{nB}(0.9 B, 0.5 B)}} = 0.62 \quad \text{1st pulse} \quad \text{worst ISI} \]
\[ \frac{v_{\text{max}_{B3}} - v_{\text{min1}_{B3}}}{\sqrt{10^{17} \cdot I_{nB}(0.9 B, 0.5 B)}} = 0.62 \quad \text{2nd pulse} \]
\[ \frac{v_{\text{max}_{B3}} - v_{\text{min2}_{B3}}}{\sqrt{10^{17} \cdot I_{nB}(0.9 B, 0.5 B)}} = 0.67 \quad \text{3rd pulse} \]

\[ \text{(rt } = 0.6) \]

\[ t_{B4} := 0.6 \cdot ts \]
\[ t_{pk4} := \text{root}(xvout(t_{B4}, 0.7 B, 0.6 B), t_{B4}) \]
\[ \frac{t_{pk4}}{ts} = 0.799 \]
\[ v_{\text{max}_{B4}} := vout(t_{pk4}, 0.7 B, 0.6 B) = 0.863 \]
\[ v_{\text{min}_{B4}} := vout(t_{pk4} + 1 \cdot ts, 0.7 B, 0.6 B) = 0.072 \]
\[ v_{\text{min1}_{B4}} := vout(t_{pk4} + 2 \cdot ts, 0.7 B, 0.6 B) = 0.031 \]
\[ v_{\text{min2}_{B4}} := vout(t_{pk4} + 3 \cdot ts, 0.7 B, 0.6 B) = 0.003 \]
\[ v_{\text{max}_{B4}} - v_{\text{min}_{B4}} = 0.79 \quad v_{\text{max}_{B4}} - v_{\text{min1}_{B4}} = 0.831 \quad v_{\text{max}_{B4}} - v_{\text{min2}_{B4}} = 0.859 \]

\[ \frac{v_{\text{max}_{B4}} - v_{\text{min}_{B4}}}{\sqrt{10^{17} \cdot I_{nB}(0.7 B, 0.6 B)}} = 0.61 \quad \text{1st pulse} \quad \text{worst ISI} \]
\[ \frac{v_{\text{max}_{B4}} - v_{\text{min1}_{B4}}}{\sqrt{10^{17} \cdot I_{nB}(0.7 B, 0.6 B)}} = 0.64 \quad \text{2nd pulse} \]
\[ \frac{v_{\text{max}_{B4}} - v_{\text{min2}_{B4}}}{\sqrt{10^{17} \cdot I_{nB}(0.7 B, 0.6 B)}} = 0.66 \quad \text{3rd pulse} \]
\[ t_{B5} := 0.8 \cdot ts \]
\[ t_{pk5} := \text{root}(xvout(t_{B5}, 0.8 \ B, 0.6 \ B), t_{B5}) \]
\[ \frac{t_{pk5}}{ts} = 0.781 \]
\[ v_{\text{max, } B5} := vout(t_{pk5}, 0.8 \ B, 0.6 \ B) = 0.879 \]
\[ v_{\text{min, } B5} := vout(t_{pk5}+1 \cdot ts, 0.8 \ B, 0.6 \ B) = 0.059 \]
\[ v_{\text{min1, } B5} := vout(t_{pk5}+2 \cdot ts, 0.8 \ B, 0.6 \ B) = 0.031 \]
\[ v_{\text{min2, } B5} := vout(t_{pk5}+3 \cdot ts, 0.8 \ B, 0.6 \ B) = 0.003 \]
\[ v_{\text{max, } B5} - v_{\text{min, } B5} = 0.8 \quad v_{\text{max, } B5} - v_{\text{min1, } B5} = 0.848 \quad v_{\text{max, } B5} - v_{\text{min2, } B5} = 0.876 \]

\[
\frac{\sqrt{10^{17} \cdot I_{n,B}(0.8 \ B, 0.6 \ B)}}{v_{\text{max, } B5} - v_{\text{min, } B5}} = 0.6 \quad \text{1st pulse} \quad \text{(worst ISI)}
\]
\[
\frac{\sqrt{10^{17} \cdot I_{n,B}(0.8 \ B, 0.6 \ B)}}{v_{\text{max, } B5} - v_{\text{min1, } B5}} = 0.62 \quad \text{2nd pulse}
\]
\[
\frac{\sqrt{10^{17} \cdot I_{n,B}(0.8 \ B, 0.6 \ B)}}{v_{\text{max, } B5} - v_{\text{min2, } B5}} = 0.64 \quad \text{3rd pulse}
\]

\[ t_{B6} := 0.5 \cdot ts \]
\[ t_{pk6} := \text{root}(xvout(t_{B4}, 0.9 \ B, 0.6 \ B), t_{B4}) \]
\[ \frac{t_{pk6}}{ts} = 0.766 \]
\[ v_{\text{max, } B6} := vout(t_{pk6}, 0.9 \ B, 0.6 \ B) = 0.892 \]
\[ v_{\text{min, } B6} := vout(t_{pk6}+1 \cdot ts, 0.9 \ B, 0.6 \ B) = 0.05 \]
\[ v_{\text{min1, } B6} := vout(t_{pk6}+2 \cdot ts, 0.9 \ B, 0.6 \ B) = 0.031 \]
\[ v_{\text{min2, } B6} := vout(t_{pk6}+3 \cdot ts, 0.9 \ B, 0.6 \ B) = 0.003 \]
\[ v_{\text{max, } B6} - v_{\text{min, } B6} = 0.842 \quad v_{\text{max, } B6} - v_{\text{min1, } B6} = 0.861 \quad v_{\text{max, } B6} - v_{\text{min2, } B6} = 0.888 \]

\[
\frac{\sqrt{10^{17} \cdot I_{n,B}(0.9 \ B, 0.6 \ B)}}{v_{\text{max, } B6} - v_{\text{min, } B6}} = 0.59 \quad \text{1st pulse} \quad \text{(worst ISI)}
\]
\[
\frac{\sqrt{10^{17} \cdot I_{n,B}(0.9 \ B, 0.6 \ B)}}{v_{\text{max, } B6} - v_{\text{min1, } B6}} = 0.6 \quad \text{2nd pulse}
\]
\[
\frac{\sqrt{10^{17} \cdot I_{n,B}(0.9 \ B, 0.6 \ B)}}{v_{\text{max, } B6} - v_{\text{min2, } B6}} = 0.62 \quad \text{3rd pulse}
\]
$t_{B7} := 0.5 \cdot ts$

$t_{pk7} := \text{root}(xvout(t_{B7}, 0.7 B, 0.7 B), t_{B7})$

$\frac{t_{pk7}}{ts} = 0.72$

$v_{max_{B7}} := vout(t_{pk7}, 0.7 B, 0.7 B) = 0.888$

$v_{min_{B7}} := vout(t_{pk7} + 1 \cdot ts, 0.7 B, 0.7 B) = 0.081$

$v_{min1_{B7}} := vout(t_{pk7} + 2 \cdot ts, 0.7 B, 0.7 B) = 0.012$

$v_{min2_{B7}} := vout(t_{pk7} + 3 \cdot ts, 0.7 B, 0.7 B) = 0.003$

$v_{max_{B7}} - v_{min_{B7}} = 0.807 \quad v_{max_{B7}} - v_{min1_{B7}} = 0.876 \quad v_{max_{B7}} - v_{min2_{B7}} = 0.885$

\[
\frac{v_{max_{B7}} - v_{min_{B7}}}{\sqrt{10^{17} \cdot I_n(B)(0.7 B, 0.7 B)}} = 0.56 \\
\frac{v_{max_{B7}} - v_{min1_{B7}}}{\sqrt{10^{17} \cdot I_n(B)(0.7 B, 0.7 B)}} = 0.61 \\
\frac{v_{max_{B7}} - v_{min2_{B7}}}{\sqrt{10^{17} \cdot I_n(B)(0.7 B, 0.7 B)}} = 0.62
\]

1st pulse (worst ISI)

2nd pulse

3rd pulse

$t_{B8} := 0.7 \cdot ts$

$t_{pk8} := \text{root}(xvout(t_{B8}, 0.8 B, 0.7 B), t_{B8})$

$\frac{t_{pk8}}{ts} = 0.702$

$v_{max_{B8}} := vout(t_{pk8}, 0.8 B, 0.7 B) = 0.905$

$v_{min_{B8}} := vout(t_{pk8} + 1 \cdot ts, 0.8 B, 0.7 B) = 0.068$

$v_{min1_{B8}} := vout(t_{pk8} + 2 \cdot ts, 0.8 B, 0.7 B) = 0.011$

$v_{min2_{B8}} := vout(t_{pk8} + 3 \cdot ts, 0.8 B, 0.7 B) = 0.003$

$v_{max_{B8}} - v_{min_{B8}} = 0.836 \quad v_{max_{B8}} - v_{min1_{B8}} = 0.894 \quad v_{max_{B8}} - v_{min2_{B8}} = 0.902$

\[
\frac{v_{max_{B8}} - v_{min_{B8}}}{\sqrt{10^{17} \cdot I_n(B)(0.8 B, 0.7 B)}} = 0.55 \\
\frac{v_{max_{B8}} - v_{min1_{B8}}}{\sqrt{10^{17} \cdot I_n(B)(0.8 B, 0.7 B)}} = 0.59 \\
\frac{v_{max_{B8}} - v_{min2_{B8}}}{\sqrt{10^{17} \cdot I_n(B)(0.8 B, 0.7 B)}} = 0.59
\]

1st pulse

2nd pulse

3rd pulse (worst ISI)
\[
t_{B9} := 0.6 \cdot ts \\
t_{pk9} := \text{root}(x\text{vout}(t_{B9}, 0.5 B, 0.5 B), t_{B9}) \\
\frac{t_{pk9}}{ts} = 0.968 \\
v_{\text{max}_B9} := v\text{out}(t_{pk9}, 0.5 B, 0.5 B) = 0.766 \\
v_{\text{min}_B9} := v\text{out}(t_{pk9} + 1 \cdot ts, 0.5 \cdot B, 0.5 B) = 0.113 \\
v_{\text{min}_1_B9} := v\text{out}(t_{pk9} + 2 \cdot ts, 0.5 B, 0.5 B) = 0.052 \\
v_{\text{min}_2_B9} := v\text{out}(t_{pk9} + 3 \cdot ts, 0.5 B, 0.5 B) = 6.434 \cdot 10^{-4} \\
v_{\text{max}_B9} - v_{\text{min}_B9} = 0.653 \quad v_{\text{max}_B9} - v_{\text{min}_1_B9} = 0.713 \quad v_{\text{max}_B9} - v_{\text{min}_2_B9} = 0.765 \\
\]

\[
\begin{align*}
\frac{v_{\text{max}_B9} - v_{\text{min}_B9}}{\sqrt{10^{17} \cdot I_{n_B}(0.5 B, 0.5 B)}} &= 0.65 \quad \text{1st pulse} \\
\frac{v_{\text{max}_B9} - v_{\text{min}_1_B9}}{\sqrt{10^{17} \cdot I_{n_B}(0.5 B, 0.5 B)}} &= 0.71 \quad \text{2nd pulse} \\
\frac{v_{\text{max}_B9} - v_{\text{min}_2_B9}}{\sqrt{10^{17} \cdot I_{n_B}(0.5 B, 0.5 B)}} &= 0.76 \quad \text{3rd pulse} \quad \text{[worst ISI]} \\
\end{align*}
\]

\[
v_{\text{max}_B} := v_{\text{max}_B9} \quad v_{\text{min}_B} := v_{\text{min}_B9}
\]

**opt rd = 0.5, rt = 0.5 > 0.6**

\[
x_{111}(t, rd, rt) := v\text{out}(t, rd, rt) + v\text{out}(t - 2 \cdot ts, rd, rt) + v\text{out}(t - ts, rd, rt) \\
x_{110}(t, rd, rt) := v\text{out}(t, rd, rt) + v\text{out}(t - ts, rd, rt) \\
x_{100}(t, rd, rt) := v\text{out}(t, rd, rt) \\
x_{001}(t, rd, rt) := v\text{out}(t - 2 \cdot ts, rd, rt) \\
x_{010}(t, rd, rt) := v\text{out}(t - ts, rd, rt) \\
x_{011}(t, rd, rt) := v\text{out}(t - ts, rd, rt) + v\text{out}(t - 2 \cdot ts, rd, rt) \\
x_{101}(t, rd, rt) := v\text{out}(t, rd, rt) + v\text{out}(t - 2 \cdot ts, rd, rt)
\]
\( x_{101}(t_{pk1} + ts, 0.5B, 0.5B) = 0.18 \)
\( x_{101}(t_{pk1}, 0.5B, 0.5B) = 0.761 \)
\( x_{101}(t_{pk1} + 2ts, 0.5B, 0.5B) = 0.815 \)
\( x_{100}(t_{pk1}, 0.5B, 0.5B) = 0.761 \)
\( x_{100}(t_{pk1} + ts, 0.5B, 0.5B) = 0.136 \)
\( x_{100}(t_{pk1} + 2ts, 0.5B, 0.5B) = 0.054 \)

Check ratio < 1 is ok 100, >1 re-opt

\[
\frac{x_{100}(t_{pk1}, 0.5B, 0.5B) - x_{100}(t_{pk1} + 2ts, 0.5B, 0.5B)}{v_{\text{max}_B1} - v_{\text{min}_B1}} = 0.958
\]

Sens PINFET
\( \lambda = 650 \cdot 10^{-9} \)

\[
\text{photon\_energy} = \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda}
\]
\( nq = 1.6 \cdot 10^{-19} \)

\[
Q_N(b) = \frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max}_B} - v_{\text{min}_B}}{2} \right)
\]
\[
\begin{align*}
    P_{e,N}(b) & := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{N}(b)}{\sqrt{2}} \right) \\
    P_{c,N}(b) & := \log \left( P_{e,N}(b) \right) + 9 \\
    b & \equiv 3 \cdot 10^3 \\
    b_N & := \text{root} \left( P_{c,N}(b), b \right) \\
    \text{minimum}_B & := \min \left( b_N \right) = 3.673 \cdot 10^3
\end{align*}
\]

**APD-1FET**

\[
\begin{align*}
    APD_{N}(b) & := \frac{1}{ts} \cdot b \cdot nq \cdot \left( v_{\text{max},B} \right) \\
    APD_{B}(b) & := \frac{1}{ts} \cdot b \cdot nq \cdot \left( v_{\text{max},B} \right) \\
    I_{APD_d} & := 10 \cdot 10^{-9}
\end{align*}
\]

\[
M_{APD1} := 100 \\
F_{M1} := 7.9
\]

\[
\begin{align*}
    \text{Noise}_{APD1_B}(b, rd, rt) & := \left( 2 \cdot q \cdot APD_B(b) \cdot M_{APD1}^2 \cdot F_{M1} \right) \cdot \text{NEB}_B \left( rd, rt \right) \\
    & \quad + 2 \cdot q \cdot I_{APD_d} \cdot M_{APD1}^2 \cdot F_{M1}
\end{align*}
\]

\[
Q_{APD1_B}(b) := \frac{M_{APD1}}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max},B} - v_{\text{min},B}}{2} \right)
\]

\[
\begin{align*}
    P_{e,APD1_B}(b) & := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD1_B}(b)}{\sqrt{2}} \right) \\
    p_{c,APD1_B}(b) & := \left( \log \left( P_{e,APD1_B}(b) \right) + 9 \right) \\
    \text{APD1}_{-b_B} & := \text{root} \left( p_{c,APD1_B}(b), b \right) \\
\text{NEB}_B \left( 0.7 \ B, 0.6 \ B \right) & = 4.185 \cdot 10^7
\end{align*}
\]

\[
\text{minimum}_{APD1_B} := \min \left( \text{APD1}_{-b_B} \right) = 1.92 \cdot 10^3
\]

**APD-2FET**

\[
\begin{align*}
    M_{APD2} & := 10 \quad \quad F_{M2} := 5.5
\end{align*}
\]

\[
\begin{align*}
    \text{Noise}_{APD2_B}(b, rd, rt) & := \left( 2 \cdot q \cdot APD_B(b) \cdot M_{APD2}^2 \cdot F_{M2} \right) \cdot \text{NEB}_B \left( rd, rt \right) \\
    & \quad + 2 \cdot q \cdot I_{APD_d} \cdot M_{APD2}^2 \cdot F_{M2}
\end{align*}
\]

\[
Q_{APD2_B}(b) := \frac{M_{APD2}}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max},B} - v_{\text{min},B}}{2} \right)
\]

\[
\begin{align*}
\text{minimum}_{APD2_B} & := \min \left( \text{APD2}_{-b_B} \right) = 1.92 \cdot 10^3
\end{align*}
\]
\[ P_{c_{-\text{APD2}_{-B}}} (b) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD2}_{-B}} (b)}{\sqrt{2}} \right) \]

\[ p_{c_{\text{APD2}_{-B}}}(b) := (\log (P_{c_{-\text{APD2}_{-B}}} (b)) + 9) \]

\[ \text{APD2}_{-B}_{-B} := \text{root} \left( p_{c_{\text{APD2}_{-B}}}(b), b \right) \]

\[ \text{minimum}_{-\text{APD2}_{-B}} := \min(\text{APD2}_{-B}_{-B}) = 1.527 \cdot 10^3 \]

**APD-3FET**

\[ M_{\text{APD3}} := 10 \quad F_{M3} := 9.2 \]

\[ \text{Noise}_{-\text{APD3}_{-B}}(b, rd, rt) := \left[ 2 \cdot q \cdot \text{APD}_{-B}(b) \cdot M_{\text{APD3}}^2 \cdot F_{M3} \right] \cdot \text{NEB}_{-B}(rd, rt) + 2 \cdot q \cdot I_{\text{APD}_{-d}} \cdot M_{\text{APD3}}^2 \cdot F_{M3} \]

\[ Q_{\text{APD3}_{-B}} (b) := \frac{M_{\text{APD3}}}{ts} \cdot b \cdot nq \left( \frac{v_{\text{max}_{-B}} - v_{\text{min}_{-B}}}{2} \right) \]

\[ I_{n, B}(0.5 B, 0.5 B) + \text{Noise}_{-\text{APD3}_{-B}}(b, 0.5 B, 0.5 B) \]

\[ P_{c_{-\text{APD3}_{-B}}} (b) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD3}_{-B}}(b)}{\sqrt{2}} \right) \]

\[ p_{c_{\text{APD3}_{-B}}}(b) := (\log (P_{c_{-\text{APD3}_{-B}}} (b)) + 9) \]

\[ \text{APD3}_{-B}_{-B} := \text{root} \left( p_{c_{\text{APD3}_{-B}}}(b), b \right) \]

\[ \text{minimum}_{-\text{APD3}_{-B}} := \min(\text{APD3}_{-B}_{-B}) = 2.208 \cdot 10^3 \]

\[ \text{Sens}_{-B}_{-\text{PIN}} := 10 \cdot \log \left( \frac{\text{minimum}_{-B} \cdot \text{photon}_{-energy}}{10^{-3} \cdot \frac{1}{ts}} \right) = -42.503 \]

\[ \text{Sens}_{-B}_{-\text{APD}_{-silicon}} := 10 \cdot \log \left( \frac{\text{minimum}_{-\text{APD1}_{-B}} \cdot \text{photon}_{-energy}}{10^{-3} \cdot \frac{1}{ts}} \right) = -45.321 \]

\[ \text{Sens}_{-B}_{-\text{APD}_{-InGaAs}} := 10 \cdot \log \left( \frac{\text{minimum}_{-\text{APD2}_{-B}} \cdot \text{photon}_{-energy}}{10^{-3} \cdot \frac{1}{ts}} \right) = -46.315 \]

\[ \text{Sens}_{-B}_{-\text{APD}_{-germanium}} := 10 \cdot \log \left( \frac{\text{minimum}_{-\text{APD3}_{-B}}}{10^{-3} \cdot \frac{1}{ts}} \right) = -44.712 \]
B.2.7 Tuned B a=0.4 (1.9/2.5)

Tuned B Optimum Performance (PIN-FET 100 Mbit/s) a=0.4

Rx performance opt PIN-FET and APD-FET input configurations, 1st order LPF pre-detection filter. opt as follows

- TIA 3-db bandwidth rt/Filter 3-dB bandwidth rd
- range cut-off
- minimum noise
- examine ISI
- highest SNR
- BER
- APD

opt rd= 0.5,0.6 rt = 0.5,0.6

Bit-rate, pulse duration

\[ C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]
\[ B := 100 \cdot 10^6 \quad \text{Bit-rate} \]

pre-dec filter

\[ \omega_B(rd) := 2 \cdot \pi \cdot rd \]
\[ H_{b_{in}}(\omega, rd) := \frac{1}{1 + j \cdot \frac{\omega}{2 \cdot \pi \cdot rd}} \]

TIA

Feedback value for (R) Tuned B

\[ \Delta_L := 1.9 \quad \Delta_R := 2.75 \quad \text{feedback } \Delta R, \text{ time constant ratio } \Delta L \]

\[ Rf_B(rt) := \Delta_R \cdot \frac{Av + 1}{2 \cdot \pi \cdot rt \cdot C_T} \quad \text{feedback for Tuned B} \]

\[ \alpha := 0.4 \quad \text{splitting ratio} \]

\[ \frac{\left( Rf_B(rt) \right)^2}{1 + Av} \cdot C_T \]

\[ C1 := (1 - \alpha) \cdot (C_T) = 9 \cdot 10^{-13} \]

\[ C2 := \alpha \cdot (C_T) = 6 \cdot 10^{-13} \]
TUNED B (R) Transimpedance

\[ Z_{TIA,B}(\omega, rt) := \frac{1}{\left( \left( 1 - \omega^2 \cdot Lc(rt) \cdot C1 \right) + \frac{Rf_B(rt)}{(1+Av)} \cdot (\omega \cdot 1i) \cdot \left( C1 + C2 - (\omega)^2 \cdot Lc(rt) \cdot C1 \cdot C2 \right) \right)} \]

**Receiver freq-response**

\[ Z_B(\omega, rd, rt) := H_{but}(\omega, rd) \cdot Z_{TIA,B}(\omega, rt) \]

\[ \omega := 1 \cdot 10^6, 10 \cdot 10^6 \ldots 1 \cdot 10^{11} \]

\[ rd := 0.5 \cdot B, 0.75 \cdot B, 2 \cdot B \]

\[ rt := 0.5 \cdot B, 0.55 \cdot B, 1 \cdot B \]

**Noise equivalent bandwidth (NEB and I2)**

\[
NEB_B(rd, rt) := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{11}} \left( |Z_B(\omega, rd, rt)|^2 \right) d\omega \right)
\]

\[
I_2_B(rd, rt) := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{11}} \left( |Z_B(\omega, rd, rt)|^2 \right) d\omega \right)
\]
Noise equivalent bandwidth (I3)

\[
I_{3\_B}(rd, rt) = \frac{1}{2 \cdot \pi} \cdot \int_{0}^{10^12} \left( \left( \left( 1 - \omega^2 \cdot Lc(rt) \cdot C1 \right) \right) \sqrt{\frac{\left( Rf\_B(rt) \cdot \left( \omega \right) \right)^2 + \left( C1 + C2 - \left( \omega \right)^2 \cdot Lc(rt) \cdot C1 \cdot C2 \right)}{Z\_B(\omega, rd, rt)}} \right)^2 d\omega
\]

Feedback noise

\[
q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298
\]

\[
I_{nRf\_B}(rd, rt) := \frac{4 \cdot k \cdot T}{Rf\_B(rt)} \cdot I_{2\_B}(rd, rt)
\]
**Ig noise (gate current)**

\[ I_g = 10 \cdot 10^{-9} \]

\[ \quad I_{nG,B}(rd, rt) := 2 \cdot q \cdot I_g \cdot I_2_B(rd, rt) \]

**Channel noise (Gate-source)**

\[ g_{m1} = 30 \cdot 10^{-3} = 0.03 \quad \text{noise factor} := 1 \]

\[ I_{nD} := 4 \cdot k \cdot T \cdot \frac{1}{g_{m1}} \]

\[ I_{nD,B}(rd, rt) := I_{nD} \cdot I_{3_B}(rd, rt) \]

**Total noise- PIN-FET**

\[ I_{n_B}(rd, rt) := I_{nRF,B}(rd, rt) + I_{nG,B}(rd, rt) + I_{nD,B}(rd, rt) \]

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\[ \frac{I_{nB}(rd, rt)}{10^{-17}} \]

opt-total noise- PIN-FET
\[ I_{n_B}(0.7 \, B, 0.5 \, B) = 1.09 \cdot 10^{-17} \]
\[ I_{n_B}(0.8 \, B, 0.5 \, B) = 1.18 \cdot 10^{-17} \]
\[ I_{n_B}(0.9 \, B, 0.5 \, B) = 1.26 \cdot 10^{-17} \]
\[ I_{n_B}(1 \, B, 0.5 \, B) = 1.34 \cdot 10^{-17} \]
\[ I_{n_B}(0.7 \, B, 0.55 \, B) = 1.25 \cdot 10^{-17} \]
\[ I_{n_B}(0.8 \, B, 0.55 \, B) = 1.36 \cdot 10^{-17} \]
\[ I_{n_B}(0.9 \, B, 0.55 \, B) = 1.46 \cdot 10^{-17} \]
\[ I_{n_B}(1 \, B, 0.55 \, B) = 1.56 \cdot 10^{-17} \]
\[ \frac{1}{B} = 1 \cdot 10^8 \]

Output pulse shape

\[ v_{out}(t, rd, rt) := \frac{ts}{\pi} \cdot \left[ \frac{1}{ts} \cdot 10^3 \right] \cdot \sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \Re \left( Z_B(\omega, rd, rt) \cdot \exp \left( i \cdot \omega \cdot t \right) \right) \, d\omega \]

\[ x_{Bt}(t) := \frac{ts}{\pi} \cdot \left[ \frac{1}{ts} \cdot 10^4 \right] \cdot \sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \Re \left( \exp \left( i \cdot \omega \cdot t \right) \right) \, d\omega \]
\[ x_{\text{vout}}(t, rd, rt) = ts^2 \cdot \frac{1}{\pi} \int_0^{\frac{1}{ts} \cdot 10^7} \frac{\sin \left( \frac{\omega \cdot ts}{2} \right)}{\left( \frac{\omega \cdot ts}{2} \right)} d\omega \cdot \text{Re} \left( i \cdot \omega \cdot Z_B(\omega, rd, rt) \cdot \exp \left( i \cdot \omega \cdot t \right) \right) \]

\[ t := -1 \, ts, -0.99 \, ts, \ldots, 4 \, ts \]

\[ v_{\text{out}}(t, 0.7 \, B, 0.6 \, B) \]
\[ x_{\text{Bt}}(t) \]
\[ x_{\text{vout}}(t, 0.7 \, B, 0.6 \, B) \]
\[ \frac{1}{2} \]

\[ |S| \]

\[ t := -1 \, ts, -0.99 \, ts, \ldots, 6 \, ts \]

\[ V_1(t, rd, rt) := v_{\text{out}}(t, rd, rt) \]

\[ V_1(t, 0.7 \, B, 0.5 \, B) \]
\[ V_1(t, 0.7 \, B, 0.6 \, B) \]
\[ V_1(t, 0.7 \, B, 0.7 \, B) \]
$V_1(t, 0.8\ B, 0.5\ B)$
$V_1(t, 0.8\ B, 0.6\ B)$
$V_1(t, 0.8\ B, 0.7\ B)$
$V_1(t, 0.8\ B, 0.8\ B)$
$V_1(t, 0.8\ B, 0.9\ B)$
$V_1(t, 0.8\ B, 1\ B)$

$V_1(t, 0.9\ B, 0.5\ B)$
$V_1(t, 0.9\ B, 0.6\ B)$
$V_1(t, 0.9\ B, 0.7\ B)$
$V_1(t, 0.9\ B, 0.8\ B)$
$V_1(t, 1\ B, 0.5\ B)$

$V_{101}(t, rd, rt) := vout(t, rd, rt) + vout(t - 2\ ts, rd, rt)$

$V_{101}(t, 0.7\ B, 0.7\ B)$
$V_{101}(t, 0.7\ B, 0.9\ B)$
$V_{101}(t, 0.9\ B, 0.6\ B)$
$V_{101}(t, 0.9\ B, 0.8\ B)$
\( t_{B1} := 1.2 \cdot t_s \)

\[ t_{pk1} := \text{root}(xvout(t_{B1}, 0.7 B, 0.5 B), t_{B1}) \]

\[ \frac{t_{pk1}}{t_s} = 1.072 \]

\[ v_{max_{B1}} := vout(t_{pk1}, 0.7 B, 0.5 B) = 0.833 \]

\[ v_{min_{B1}} := vout(t_{pk1} + 1 \cdot t_s, 0.7 B, 0.5 B) = 0.014 \]

\[ v_{min_{B1}} := vout(t_{pk1} + 2 \cdot t_s, 0.7 B, 0.5 B) = 0.043 \]

\[ v_{min_{B1}} := vout(t_{pk1} + 3 \cdot t_s, 0.7 B, 0.5 B) = 0.02 \]

\[ v_{max_{B1}} - v_{min_{B1}} = 0.819 \]

\[ v_{max_{B1}} - v_{min_{B1}} = 0.79 \]

\[ v_{max_{B1}} - v_{min_{B1}} = 0.813 \]

\[ \frac{v_{max_{B1}} - v_{min_{B1}}}{\sqrt{10^{17} \cdot I_{n_B}(0.7 B, 0.5 B)}} = 0.79 \quad \text{1st pulse} \]

\[ \frac{v_{max_{B1}} - v_{min_{B1}}}{\sqrt{10^{17} \cdot I_{n_B}(0.7 B, 0.5 B)}} = 0.758 \quad \text{2nd pulse} \]

\[ \text{worst ISI} \]

\[ \frac{v_{max_{B1}} - v_{min_{B1}}}{\sqrt{10^{17} \cdot I_{n_B}(0.7 B, 0.5 B)}} = 0.78 \quad \text{3rd pulse} \]

\[ t_{B2} := 1 \cdot t_s \]

\[ t_{pk2} := \text{root}(xvout(t_{B2}, 0.8 B, 0.5 B), t_{B2}) \]

\[ \frac{t_{pk2}}{t_s} = 1.051 \]

\[ v_{max_{B2}} := vout(t_{pk2}, 0.8 B, 0.5 B) = 0.846 \]

\[ v_{min_{B2}} := vout(t_{pk2} + 1 \cdot t_s, 0.8 B, 0.5 B) = 0.002 \]

\[ v_{min_{B2}} := vout(t_{pk2} + 2 \cdot t_s, 0.8 B, 0.5 B) = 0.045 \]

\[ v_{min_{B2}} := vout(t_{pk2} + 3 \cdot t_s, 0.8 B, 0.5 B) = 0.019 \]

\[ v_{max_{B2}} - v_{min_{B2}} = 0.844 \]

\[ v_{max_{B2}} - v_{min_{B1}} = 0.802 \]

\[ v_{max_{B2}} - v_{min_{B2}} = 0.827 \]

\[ \frac{v_{max_{B2}} - v_{min_{B2}}}{\sqrt{10^{17} \cdot I_{n_B}(0.8 B, 0.5 B)}} = 0.78 \quad \text{1st pulse} \]

\[ \frac{v_{max_{B2}} - v_{min_{B2}}}{\sqrt{10^{17} \cdot I_{n_B}(0.8 B, 0.5 B)}} = 0.74 \quad \text{2nd pulse} \]

\[ \text{worst ISI} \]

\[ \frac{v_{max_{B2}} - v_{min_{B2}}}{\sqrt{10^{17} \cdot I_{n_B}(0.8 B, 0.5 B)}} = 0.76 \quad \text{3rd pulse} \]
\( t_{B3} := 1 \cdot ts \)

\[
\frac{t_{pk3}}{ts} = 1.035
\]

\( v_{max_{B3}} := vout(t_{pk3}, 0.9 B, 0.5 B) = 0.856 \)

\( v_{min_{B3}} := vout(t_{pk3} + 1 \cdot ts, 0.9 B, 0.5 B) = -0.007 \)

\( v_{min1_{B3}} := vout(t_{pk3} + 2 \cdot ts, 0.9 B, 0.5 B) = 0.046 \)

\( v_{min2_{B3}} := vout(t_{pk3} + 3 \cdot ts, 0.9 B, 0.5 B) = 0.018 \)

\[ v_{max_{B3}} - v_{min_{B3}} = 0.863 \quad v_{max_{B3}} - v_{min1_{B3}} = 0.81 \quad v_{max_{B3}} - v_{min2_{B3}} = 0.838 \]

\[
\sqrt{10^{17} \cdot I_{n_B}(0.9 B, 0.5 B)} = 0.77 \quad \text{1st pulse}
\]

\[
\sqrt{10^{17} \cdot I_{n_B}(0.9 B, 0.5 B)} = 0.72 \quad \text{2nd pulse} \quad \text{worst ISI}
\]

\[
\sqrt{10^{17} \cdot I_{n_B}(0.9 B, 0.5 B)} = 0.75 \quad \text{3rd pulse}
\]

\((rt = 0.6)\)

\( t_{B4} := 0.6 \cdot ts \)

\[
\frac{t_{pk4}}{ts} = 0.922
\]

\( v_{max_{B4}} := vout(t_{pk4}, 0.7 B, 0.6 B) = 0.9 \)

\( v_{min_{B4}} := vout(t_{pk4} + 1 \cdot ts, 0.7 B, 0.6 B) = -0.035 \)

\( v_{min1_{B4}} := vout(t_{pk4} + 2 \cdot ts, 0.7 B, 0.6 B) = 0.095 \)

\( v_{min2_{B4}} := vout(t_{pk4} + 3 \cdot ts, 0.7 B, 0.6 B) = -0.03 \)

\[ v_{max_{B4}} - v_{min_{B4}} = 0.935 \quad v_{max_{B4}} - v_{min1_{B4}} = 0.805 \quad v_{max_{B4}} - v_{min2_{B4}} = 0.93 \]

\[
\sqrt{10^{17} \cdot I_{n_B}(0.7 B, 0.6 B)} = 0.79 \quad \text{1st pulse}
\]

\[
\sqrt{10^{17} \cdot I_{n_B}(0.7 B, 0.6 B)} = 0.68 \quad \text{2nd pulse} \quad \text{worst ISI}
\]

\[
\sqrt{10^{17} \cdot I_{n_B}(0.7 B, 0.6 B)} = 0.78 \quad \text{3rd pulse}
\]
\[ t_{B5} := 0.8 \cdot t \]
\[ t_{pk5} := \text{root}(xvout(t_{B5}, 0.8 \ B, 0.6 \ B), t_{B5}) \]
\[ \frac{t_{pk5}}{t} = 0.903 \]
\[ v_{\text{max\_B5}} := vout(t_{pk5}, 0.8 \ B, 0.6 \ B) = 0.917 \]
\[ v_{\text{min\_B5}} := vout(t_{pk5} + 1 \cdot t, 0.8 \ B, 0.6 \ B) = -0.051 \]
\[ v_{\text{min1\_B5}} := vout(t_{pk5} + 2 \cdot t, 0.8 \ B, 0.6 \ B) = 0.099 \]
\[ v_{\text{min2\_B5}} := vout(t_{pk5} + 3 \cdot t, 0.8 \ B, 0.6 \ B) = -0.032 \]
\[ v_{\text{max\_B5}} - v_{\text{min\_B5}} = 0.968 \quad v_{\text{max\_B5}} - v_{\text{min1\_B5}} = 0.818 \quad v_{\text{max\_B5}} - v_{\text{min2\_B5}} = 0.949 \]

\[
\frac{v_{\text{max\_B5}} - v_{\text{min\_B5}}}{\sqrt{10^{17} \cdot I_{n\_B}(0.8 \ B, 0.6 \ B)}} = 0.78 \quad \text{(1st pulse)}
\]
\[
\frac{v_{\text{max\_B5}} - v_{\text{min1\_B5}}}{\sqrt{10^{17} \cdot I_{n\_B}(0.8 \ B, 0.6 \ B)}} = 0.66 \quad \text{(2nd pulse \ worst ISI)}
\]
\[
\frac{v_{\text{max\_B5}} - v_{\text{min2\_B5}}}{\sqrt{10^{17} \cdot I_{n\_B}(0.8 \ B, 0.6 \ B)}} = 0.76 \quad \text{(3rd pulse)}
\]

\[ t_{B6} := 0.9 \cdot t \]
\[ t_{pk6} := \text{root}(xvout(t_{B4}, 0.9 \ B, 0.6 \ B), t_{B4}) \]
\[ \frac{t_{pk6}}{t} = ? \]
\[ v_{\text{max\_B6}} := vout(t_{pk6}, 0.9 \ B, 0.6 \ B) = ? \]
\[ v_{\text{min\_B6}} := vout(t_{pk6} + 1 \cdot t, 0.9 \ B, 0.6 \ B) = ? \]
\[ v_{\text{min1\_B6}} := vout(t_{pk6} + 2 \cdot t, 0.9 \ B, 0.6 \ B) = ? \]
\[ v_{\text{min2\_B6}} := vout(t_{pk6} + 3 \cdot t, 0.9 \ B, 0.6 \ B) = ? \]
\[ v_{\text{max\_B6}} - v_{\text{min\_B6}} = ? \quad v_{\text{max\_B6}} - v_{\text{min1\_B6}} = ? \quad v_{\text{max\_B6}} - v_{\text{min2\_B6}} = ? \]

\[
\frac{v_{\text{max\_B6}} - v_{\text{min\_B6}}}{\sqrt{10^{17} \cdot I_{n\_B}(0.9 \ B, 0.6 \ B)}} = ? \quad \text{(1st pulse \ worst ISI)}
\]
\[
\frac{v_{\text{max\_B6}} - v_{\text{min1\_B6}}}{\sqrt{10^{17} \cdot I_{n\_B}(0.9 \ B, 0.6 \ B)}} = ? \quad \text{(2nd pulse)}
\]
\[
\frac{v_{\text{max\_B6}} - v_{\text{min2\_B6}}}{\sqrt{10^{17} \cdot I_{n\_B}(0.9 \ B, 0.6 \ B)}} = ? \quad \text{(3rd pulse)}
\]
\( t_{B7} := 0.5 \cdot ts \)
\[ t_{pk7} := \text{root} \left( xvout \left( t_{B7}, 0.7 \cdot B, 0.7 \cdot B \right), t_{B7} \right) \]
\[ \frac{t_{pk7}}{ts} = 0.812 \]
\[ v_{\text{max}_{B7}} := vout \left( t_{pk7}, 0.7 \cdot B, 0.7 \cdot B \right) = 0.94 \]
\[ v_{\text{min}_{B7}} := vout \left( t_{pk7} + 1 \cdot ts, 0.7 \cdot B, 0.7 \cdot B \right) = -0.031 \]
\[ v_{\text{min1}_{B7}} := vout \left( t_{pk7} + 2 \cdot ts, 0.7 \cdot B, 0.7 \cdot B \right) = 0.064 \]
\[ v_{\text{min2}_{B7}} := vout \left( t_{pk7} + 3 \cdot ts, 0.7 \cdot B, 0.7 \cdot B \right) = -0.004 \]
\[ v_{\text{max}_{B7}} - v_{\text{min}_{B7}} = 0.972 \quad v_{\text{max}_{B7}} - v_{\text{min1}_{B7}} = 0.877 \quad v_{\text{max}_{B7}} - v_{\text{min2}_{B7}} = 0.944 \]
\[ \frac{v_{\text{max}_{B7}} - v_{\text{min}_{B7}}}{\sqrt{10^{17} \cdot I_{nB}(0.7 \cdot B, 0.7 \cdot B)}} = 0.73 \quad \text{1st pulse} \]
\[ \frac{v_{\text{max}_{B7}} - v_{\text{min1}_{B7}}}{\sqrt{10^{17} \cdot I_{nB}(0.7 \cdot B, 0.7 \cdot B)}} = 0.66 \quad \text{2nd pulse} \quad \text{(worst ISI)} \]
\[ \frac{v_{\text{max}_{B7}} - v_{\text{min2}_{B7}}}{\sqrt{10^{17} \cdot I_{nB}(0.7 \cdot B, 0.7 \cdot B)}} = 0.71 \quad \text{3rd pulse} \]
\( t_{B8} := 0.7 \cdot ts \)
\[ t_{pk8} := \text{root} \left( xvout \left( t_{B8}, 0.8 \cdot B, 0.7 \cdot B \right), t_{B8} \right) \]
\[ \frac{t_{pk8}}{ts} = 0.794 \]
\[ v_{\text{max}_{B8}} := vout \left( t_{pk8}, 0.8 \cdot B, 0.7 \cdot B \right) = 0.959 \]
\[ v_{\text{min}_{B8}} := vout \left( t_{pk8} + 1 \cdot ts, 0.8 \cdot B, 0.7 \cdot B \right) = -0.049 \]
\[ v_{\text{min1}_{B8}} := vout \left( t_{pk8} + 2 \cdot ts, 0.8 \cdot B, 0.7 \cdot B \right) = 0.066 \]
\[ v_{\text{min2}_{B8}} := vout \left( t_{pk8} + 3 \cdot ts, 0.8 \cdot B, 0.7 \cdot B \right) = -0.004 \]
\[ v_{\text{max}_{B8}} - v_{\text{min}_{B8}} = 1.008 \quad v_{\text{max}_{B8}} - v_{\text{min1}_{B8}} = 0.893 \quad v_{\text{max}_{B8}} - v_{\text{min2}_{B8}} = 0.963 \]
\[ \frac{v_{\text{max}_{B8}} - v_{\text{min}_{B8}}}{\sqrt{10^{17} \cdot I_{nB}(0.8 \cdot B, 0.7 \cdot B)}} = 0.73 \quad \text{1st pulse} \]
\[ \frac{v_{\text{max}_{B8}} - v_{\text{min1}_{B8}}}{\sqrt{10^{17} \cdot I_{nB}(0.8 \cdot B, 0.7 \cdot B)}} = 0.64 \quad \text{2nd pulse} \quad \text{(worst ISI)} \]
\[ \frac{v_{\text{max}_{B8}} - v_{\text{min2}_{B8}}}{\sqrt{10^{17} \cdot I_{nB}(0.8 \cdot B, 0.7 \cdot B)}} = 0.69 \quad \text{3rd pulse} \]
\[ t_{B9} = 0.7 \cdot ts \]
\[ t_{pk9} := \text{root}(xout(t_{B9}, 0.5 \cdot B, 0.5 \cdot B), t_{B9}) \]
\[ \frac{t_{pk9}}{ts} = 1.129 \]

\[ v_{\text{max}B9} := \text{vout}(t_{pk9}, 0.5 \cdot B, 0.5 \cdot B) = 0.787 \]
\[ v_{\text{min}B9} := \text{vout}(t_{pk9} + 1 \cdot ts, 0.5 \cdot B, 0.5 \cdot B) = 0.051 \]
\[ v_{\text{min}1B9} := \text{vout}(t_{pk9} + 2 \cdot ts, 0.5 \cdot B, 0.5 \cdot B) = 0.04 \]
\[ v_{\text{min}2B9} := \text{vout}(t_{pk9} + 3 \cdot ts, 0.5 \cdot B, 0.5 \cdot B) = 0.021 \]

\[
\begin{align*}
\frac{v_{\text{max}B9} - v_{\text{min}B9}}{10^{17} \cdot I_{nB}(0.5 \cdot B, 0.5 \cdot B)} & = 0.79 \\
\frac{v_{\text{max}B9} - v_{\text{min}1B9}}{10^{17} \cdot I_{nB}(0.5 \cdot B, 0.5 \cdot B)} & = 0.8 \\
\frac{v_{\text{max}B9} - v_{\text{min}2B9}}{10^{17} \cdot I_{nB}(0.5 \cdot B, 0.5 \cdot B)} & = 0.82
\end{align*}
\]

1st pulse  worst ISI
2nd pulse
3rd pulse

opt rd: 0.5, 0.6, rt = 0.5 > 0.6

\[ x_{111}(t, rd, rt) := vout(t, rd, rt) + vout(t - 2 \cdot ts, rd, rt) + vout(t - ts, rd, rt) \]
\[ x_{110}(t, rd, rt) := vout(t, rd, rt) + vout(t - ts, rd, rt) \]
\[ x_{100}(t, rd, rt) := vout(t, rd, rt) \]
\[ x_{001}(t, rd, rt) := vout(t - 2 \cdot ts, rd, rt) \]
\[ x_{010}(t, rd, rt) := vout(t - ts, rd, rt) \]
\[ x_{011}(t, rd, rt) := vout(t - ts, rd, rt) + vout(t - 2 \cdot ts, rd, rt) \]
\[ x_{101}(t, rd, rt) := vout(t, rd, rt) + vout(t - 2 \cdot ts, rd, rt) \]
Check ratio < 1 is ok 100, > 1 re-opt

\[
x_{101}(t_{pk1} + ts, 0.5 B, 0.5 B) = 0.168 \\
x_{101}(t_{pk1}, 0.5 B, 0.5 B) = 0.783 \\
x_{101}(t_{pk1} + 2 ts, 0.5 B, 0.5 B) = 0.81 \\
x_{100}(t_{pk1}, 0.5 B, 0.5 B) = 0.783 \\
x_{100}(t_{pk1} + ts, 0.5 B, 0.5 B) = 0.09 \\
x_{100}(t_{pk1} + 2 ts, 0.5 B, 0.5 B) = 0.027
\]

\[
\frac{x_{100}(t_{pk1}, 0.5 B, 0.5 B) - x_{100}(t_{pk1} + 2 ts, 0.5 B, 0.5 B)}{v_{max_{B1}} - v_{min_{B1}}} = 0.923
\]

**Sens PINFET**

\[
\lambda := 650 \cdot 10^{-9}
\]

\[
photon\_energy := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda}
\]

\[
nq := 1.6 \cdot 10^{-19}
\]

\[
Q_N(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left(\frac{v_{max_{B}} - v_{min_{B}}}{2}\right)
\]

\[
\sqrt{I_{n,B}(0.5 B, 0.5 B)}
\]

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\[
Pe_N(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right) \\
Pc_N(b) := (\log(Pe_N(b)) + 9) \\
b \equiv 3 \cdot 10^3 \\
b_N := \text{root}(Pc_N(b), b) \\
minimum_B := \text{min}(b_N) = 2.998 \cdot 10^3
\]

**APD-1FET**

\[
APD_N(b) := \frac{1}{ts} \cdot b \cdot nq \cdot (v_{max} - b) \\
APD_B(b) := \frac{1}{ts} \cdot b \cdot nq \cdot (v_{max} - b) \\
I_{APD, d} := 10 \cdot 10^{-9}
\]

\[
M_{APD1} := 100 \\
F_M1 := 7.9
\]

\[
\text{Noise}_{APD1, B}(b, rd, rt) := \left( 2 \cdot q \cdot \text{APD}_B(b) \cdot M_{APD1}^2 \cdot F_{M1} \right) \cdot \text{NEB}_B(rd, rt) + 2 \cdot q \cdot I_{APD, d} \cdot M_{APD1}^2 \cdot F_{M1}
\]

\[
Q_{APD1, B}(b) := \frac{M_{APD1}}{ts} \cdot b \cdot nq \cdot \frac{v_{max} - v_{min, B}}{2} \sqrt{I_{n, B}(0.5 B, 0.5 B) + \text{Noise}_{APD1, B}(b, 0.5 B, 0.5 B)}
\]

\[
P_{e, APD1, B}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD1, B}(b)}{\sqrt{2}} \right) \\
p_{c, APD1, B}(b) := (\log(P_{e, APD1, B}(b)) + 9)
\]

\[
APD1, b_B := \text{root}(p_{c, APD1, B}(b), b) \\
\text{NEB}_B(0.7 B, 0.6 B) = 4.591 \cdot 10^7 \\
minimum_{APD1, B} := \text{min}(APD1, b_B) = 1.742 \cdot 10^3
\]

**APD-2FET**

\[
M_{APD2} := 10 \\
F_M2 := 5.5
\]

\[
\text{Noise}_{APD2, B}(b, rd, rt) := \left( 2 \cdot q \cdot \text{APD}_B(b) \cdot M_{APD2}^2 \cdot F_{M2} \right) \cdot \text{NEB}_B(rd, rt) + 2 \cdot q \cdot I_{APD, d} \cdot M_{APD2}^2 \cdot F_{M2}
\]

\[
Q_{APD2, B}(b) := \frac{M_{APD2}}{ts} \cdot b \cdot nq \cdot \frac{v_{max} - v_{min, B}}{2} \sqrt{I_{n, B}(0.5 B, 0.5 B) + \text{Noise}_{APD2, B}(b, 0.5 B, 0.5 B)}
\]
\[ P_{e\_APD2\_B}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\_APD2\_B}(b)}{\sqrt{2}} \right) \]

\[ p_{c\_APD2\_B}(b) := (\log(P_{e\_APD2\_B}(b)) + 9) \]

\[ \text{APD2\_b\_B} := \text{root}(p_{c\_APD2\_B}(b), b) \]

\[ \text{minimum\_APD2\_B} := \min(\text{APD2\_b\_B}) = 1.378 \times 10^3 \]

**APD-3FET**

\[ M_{\_APD3} := 10 \quad F_{\_M3} := 9.2 \]

\[ \text{Noise\_APD3\_B}(b, rd, rt) := \left( 2 \cdot q \cdot \text{APD\_B}(b) \cdot M_{\_APD3}^2 \cdot F_{\_M3} \right) \cdot \text{NEB\_B}(rd, rt) + 2 \cdot q \cdot I_{\_APD\_d} \cdot M_{\_APD3}^2 \cdot F_{\_M3} \]

\[ Q_{\_APD3\_B}(b) := \frac{M_{\_APD3}}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max\_B}} - v_{\text{min\_B}}}{2} \right) \]

\[ \text{I}_{\text{n,B}}(0.5 B, 0.5 B) + \text{Noise\_APD3\_B}(b, 0.5 B, 0.5 B) \]

\[ P_{e\_APD3\_B}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\_APD3\_B}(b)}{\sqrt{2}} \right) \]

\[ p_{c\_APD3\_B}(b) := (\log(P_{e\_APD3\_B}(b)) + 9) \]

\[ \text{APD3\_b\_B} := \text{root}(p_{c\_APD3\_B}(b), b) \]

\[ \text{minimum\_APD3\_B} := \min(\text{APD3\_b\_B}) = 1.993 \times 10^3 \]

\[ \text{Sens\_B\_PIN} := 10 \cdot \log \left( \frac{\text{minimum\_B} \cdot \text{photon\_energy} \cdot \frac{1}{ts}}{2 \times 10^{-3}} \right) = -43.385 \]

\[ \text{Sens\_B\_APD\_silicon} := 10 \cdot \log \left( \frac{\text{minimum\_APD1\_B} \cdot \text{photon\_energy} \cdot \frac{1}{ts}}{2 \times 10^{-3}} \right) = -45.743 \]

\[ \text{Sens\_B\_APD\_InGaAs} := 10 \cdot \log \left( \frac{\text{minimum\_APD2\_B} \cdot \text{photon\_energy} \cdot \frac{1}{ts}}{2 \times 10^{-3}} \right) = -46.761 \]

\[ \text{Sens\_B\_APD\_germanium} := 10 \cdot \log \left( \frac{\text{minimum\_APD3\_B} \cdot \text{photon\_energy} \cdot \frac{1}{ts}}{2 \times 10^{-3}} \right) = -45.158 \]

#
Rx performance opt PIN-FET and APD-FET input configurations, 1st order LPF pre-detection filter. Opt as follows

- TIA 3-db bandwidth rt/Filter 3-db bandwidth rd
- range cut-off
- minimum noise
- examine ISI
- highest SNR
- BER
- APD

**opt rd = 0.5 rt = 0.5**

### Bit-rate, pulse duration

\[ C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]
\[ B := 100 \cdot 10^6 \quad \text{Bit-rate} \]

**pre-dec filter**

\[ \omega_B(rt) := 2 \cdot \pi \cdot rd \]
\[ H_{\text{bt}}(\omega, rd) := \frac{1}{1 + j \cdot \frac{\omega}{2 \cdot \pi \cdot rd}} \]

### TIA

Feedback value for (R) Tuned B

\[ \Delta_L := 2.5 \quad \Delta_R := 3.17 \quad \text{feedback} \Delta R, \text{time constant ratio} \Delta L \]
\[ Av := 10 \]

\[ Rf_B(rt) := \Delta_R \cdot \frac{Av + 1}{2 \cdot \pi \cdot rt \cdot C_T} \quad \text{feedback for Tuned B} \]

\[ \alpha := 0.4 \quad \text{splitting ratio} \]

\[ Lc(rt) := \left( \frac{Rf_B(rt)}{1 + Av} \right)^2 \cdot C_T \]

\[ C1 := (1 - \alpha) \cdot (C_T) = 9 \cdot 10^{-13} \]

\[ C2 := \alpha \cdot (C_T) = 6 \cdot 10^{-13} \]
TUNED B (R) Transimpedance

\[ Z_{TIA,B}(\omega, rt) = \frac{1}{\left( (1 - \omega^2 \cdot Lc \cdot C1) + \frac{Rf_B(rt)}{1 + Av} \cdot (\omega \cdot i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \right)} \]

**Receiver freq-response**

\[ Z_B(\omega, rd, rt) = H_{but}(\omega, rd) \cdot Z_{TIA,B}(\omega, rt) \]

\[ \omega := 1 \cdot 10^6, 10 \cdot 10^6, 1 \cdot 10^{11} \]

\[ rd := 0.5 \cdot B, 0.75 \cdot B, 2 \cdot B \]

\[ rt := 0.5 \cdot B, 0.55 \cdot B, 1 \cdot B \]

**Noise equivalent bandwidth (NEB and I2)**

\[ NEB_B(rd, rt) = \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{12}} \left( \left| Z_B(\omega, rd, rt) \right| \right)^2 d\omega \right) \]

\[ I_{2,B}(rd, rt) = \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{12}} \left( \left| Z_B(\omega, rd, rt) \right| \right)^2 d\omega \right) \]
Noise equivalent bandwidth \([13]\)

\[
I_{3\ B}(rd, rt) = \frac{1}{2 \cdot \pi} \cdot \left[ \int_{0}^{10^{12}} \left( \frac{\left( (1 - \omega^2 \cdot Lc(rt) \cdot C1) \right) \left( Rf_B(rt) \cdot \left( \frac{\omega}{C1 + C2 - \left( \frac{\omega}{Lc(rt) \cdot C1 \cdot C2} \right)} \right) \right)^{1/2}}{Z_B(\omega, rd, rt)} \right)^2 d\omega 
\]

Feedback noise

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \]

\[
I_{nRf\_B}(rd, rt) := \frac{4 \cdot k \cdot T}{Rf_B(rt)} \cdot I_{2\ B}(rd, rt)
\]
\( I_{\text{RF,B}}(rd, rt) \)

**Ig noise (gate current)**

\( I_{g} := 10 \cdot 10^{-9} \)

\( I_{nG,B}(rd, rt) := 2 \cdot q \cdot I_{g} \cdot I_{2\_B}(rd, rt) \)

**Channel noise (Gate-source)**

\( gm1 := 30 \cdot 10^{-3} = 0.03 \quad \text{noise\_factor} := 1 \)

\( I_{nD} := 4 \cdot k \cdot T \cdot \frac{1}{gm1} \)

\( I_{nD,B}(rd, rt) := I_{nD} \cdot I_{3\_B}(rd, rt) \)

**Total noise- PIN-FET**

\( I_{n}(rd, rt) := I_{nRF,B}(rd, rt) + I_{nG,B}(rd, rt) + I_{nD,B}(rd, rt) \)
opt-total noise- PIN-FET
\[ I_{n,B}(0.7 B, 0.5 B) = 8.58 \times 10^{-18} \]
\[ I_{n,B}(0.8 B, 0.5 B) = 9.29 \times 10^{-18} \]
\[ I_{n,B}(0.9 B, 0.5 B) = 9.93 \times 10^{-18} \]
\[ I_{n,B}(1 B, 0.5 B) = 1.05 \times 10^{-17} \]

\[ I_{n,B}(0.7 B, 0.55 B) = 9.86 \times 10^{-18} \]
\[ I_{n,B}(0.8 B, 0.55 B) = 1.07 \times 10^{-17} \]
\[ I_{n,B}(0.9 B, 0.55 B) = 1.15 \times 10^{-17} \]
\[ I_{n,B}(1 B, 0.55 B) = 1.22 \times 10^{-17} \]

\[ I_{n,B}(0.7 B, 0.7 B) = 1.39 \times 10^{-17} \]
\[ I_{n,B}(0.8 B, 0.7 B) = 1.53 \times 10^{-17} \]
\[ I_{n,B}(0.9 B, 0.7 B) = 1.65 \times 10^{-17} \]
\[ I_{n,B}(1 B, 0.7 B) = 1.77 \times 10^{-17} \]

0.8/0.7 \text{rd}, \text{ range } rt \text{ 0.55-0.75}

**Output pulse shape**

\[ ts := \frac{1}{B} \quad B = 1 \times 10^8 \]

\[ v_{out}(t, rd, rt) := \frac{ts}{\pi} \cdot \frac{1}{ts^{10^3}} \cdot \sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \text{Re} \left( Z_B(\omega, rd, rt) \cdot \exp \left( i \cdot \omega \cdot (t) \right) \right) \text{d}\omega \]

\[ x_{Bt}(t) := \frac{ts}{\pi} \cdot \frac{1}{ts^{10^4}} \cdot \sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \text{Re} \left( \exp \left( i \cdot \omega \cdot (t) \right) \right) \text{d}\omega \]
\[ x_{\text{vout}}(t, rd, rt) := ts^2 \cdot \frac{1}{\pi} \int_0^{\frac{1}{ts}} 10^2 \cdot \frac{\sin \left( \omega \cdot \frac{ts}{2} \right)}{\left( \omega \cdot \frac{ts}{2} \right)^2} \, d\omega \cdot \text{Re} \left( i \cdot \omega \cdot Z_B(\omega, rd, rt) \cdot \exp \left( i \cdot \omega \cdot (t) \right) \right) \]

\[ t := -1 \, ts, -0.99 \, ts, 0.4 \, ts \]

\[ v_{\text{out}}(t, 0.7 \, B, 0.6 \, B) \]
\[ x_B(t) \]
\[ x_{\text{vout}}(t, 0.7 \, B, 0.6 \, B) \]
\[ 2 \]

\[ V_1(t, rd, rt) := v_{\text{out}}(t, rd, rt) \]

\[ V_1(t, 0.7 \, B, 0.5 \, B) \]
\[ V_1(t, 0.7 \, B, 0.6 \, B) \]
\[ V_1(t, 0.7 \, B, 0.7 \, B) \]
\[
\begin{align*}
V_1(t, 0.8, 0.5 B) \\
V_1(t, 0.8, 0.6 B) \\
V_1(t, 0.8, 0.7 B) \\
V_1(t, 0.8, 0.8 B) \\
V_1(t, 0.8, 0.9 B) \\
V_1(t, 0.8, 1 B) \\
V_1(t, 0.9, 0.5 B) \\
V_1(t, 0.9, 0.6 B) \\
V_1(t, 0.9, 0.7 B) \\
V_1(t, 0.9, 0.8 B) \\
V_1(t, 1, 0.5 B) \\
\end{align*}
\]

\[
V_{101}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - 2 \; ts, rd, rt)
\]

\[
\begin{align*}
V_{101}(t, 0.7, 0.7 B) \\
V_{101}(t, 0.7, 0.9 B) \\
V_{101}(t, 0.9, 0.6 B) \\
V_{101}(t, 0.9, 0.8 B) \\
\end{align*}
\]
\[ t_{B1} := 1.2 \cdot ts \]
\[ t_{pk1} := \text{root}(xvout(t_{B1}, 0.7 \ B, 0.5 \ B), t_{B1}) \]
\[ \frac{t_{pk1}}{ts} = 1.083 \]
\[ v_{\text{max}_1} := \text{vout}(t_{pk1}, 0.7 \ B, 0.5 \ B) = 0.776 \]
\[ v_{\text{min},1} := \text{vout}(t_{pk1} + 1 \cdot ts, 0.7 \ B, 0.5 \ B) = 0.033 \]
\[ v_{\text{min},1} := \text{vout}(t_{pk1} + 2 \cdot ts, 0.7 \ B, 0.5 \ B) = 0.083 \]
\[ v_{\text{min},2} := \text{vout}(t_{pk1} + 3 \cdot ts, 0.7 \ B, 0.5 \ B) = 0.01 \]
\[ v_{\text{max},1} - v_{\text{min},1} = 0.743 \quad v_{\text{max},1} - v_{\text{min},1} = 0.693 \quad v_{\text{max},1} - v_{\text{min},2} = 0.766 \]
\[ \frac{v_{\text{max},1} - v_{\text{min},1}}{\sqrt{10^{17} \cdot I_{n,B}(0.7 \ B, 0.5 \ B)}} = 0.8 \quad \text{1st pulse} \]
\[ \frac{v_{\text{max},1} - v_{\text{min},1}}{\sqrt{10^{17} \cdot I_{n,B}(0.7 \ B, 0.5 \ B)}} = 0.748 \quad \text{2nd pulse} \quad \text{(worst ISI)} \]
\[ \frac{v_{\text{max},1} - v_{\text{min},2}}{\sqrt{10^{17} \cdot I_{n,B}(0.7 \ B, 0.5 \ B)}} = 0.827 \quad \text{3rd pulse} \]
\[ t_{B2} := 1.2 \cdot ts \]
\[ t_{pk2} := \text{root}(xvout(t_{B2}, 0.8 \ B, 0.5 \ B), t_{B2}) \]
\[ \frac{t_{pk2}}{ts} = 1.063 \]
\[ v_{\text{max},2} := \text{vout}(t_{pk2}, 0.8 \ B, 0.5 \ B) = 0.789 \]
\[ v_{\text{min},2} := \text{vout}(t_{pk2} + 1 \cdot ts, 0.8 \ B, 0.5 \ B) = 0.022 \]
\[ v_{\text{min},2} := \text{vout}(t_{pk2} + 2 \cdot ts, 0.8 \ B, 0.5 \ B) = 0.086 \]
\[ v_{\text{min},2} := \text{vout}(t_{pk2} + 3 \cdot ts, 0.8 \ B, 0.5 \ B) = 0.009 \]
\[ v_{\text{max},2} - v_{\text{min},2} = 0.766 \quad v_{\text{max},2} - v_{\text{min},1} = 0.703 \quad v_{\text{max},2} - v_{\text{min},2} = 0.78 \]
\[ \frac{v_{\text{max},2} - v_{\text{min},2}}{\sqrt{10^{17} \cdot I_{n,B}(0.8 \ B, 0.5 \ B)}} = 0.79 \quad \text{1st pulse} \]
\[ \frac{v_{\text{max},2} - v_{\text{min},2}}{\sqrt{10^{17} \cdot I_{n,B}(0.8 \ B, 0.5 \ B)}} = 0.73 \quad \text{2nd pulse} \quad \text{(worst ISI)} \]
\[ \frac{v_{\text{max},2} - v_{\text{min},2}}{\sqrt{10^{17} \cdot I_{n,B}(0.8 \ B, 0.5 \ B)}} = 0.81 \quad \text{3rd pulse} \]
\[
t_{B3} := 1 \cdot ts \\
t_{pk3} := \text{root}(xvout(t_{B3}, 0.9 \ B, 0.5 \ B), t_{B3}) \\
\frac{t_{pk3}}{ts} = 1.046 \\
v_{\text{max} \_B3} := vout(t_{pk3}, 0.9 \ B, 0.5 \ B) = 0.798 \\
v_{\text{min} \_B3} := vout(t_{pk3} + 1 \cdot ts, 0.9 \ B, 0.5 \ B) = 0.014 \\
v_{\text{min1} \_B3} := vout(t_{pk3} + 2 \cdot ts, 0.9 \ B, 0.5 \ B) = 0.087 \\
v_{\text{min2} \_B3} := vout(t_{pk3} + 3 \cdot ts, 0.9 \ B, 0.5 \ B) = 0.008 \\
v_{\text{max} \_B3} - v_{\text{min} \_B3} = 0.784 \\
v_{\text{max} \_B3} - v_{\text{min1} \_B3} = 0.711 \\
v_{\text{max} \_B3} - v_{\text{min2} \_B3} = 0.791 \\
\]

\[
\frac{\sqrt{10^{17} \cdot I_{n,B}(0.9 \ B, 0.5 \ B)}}{v_{\text{max} \_B3} - v_{\text{min} \_B3}} = 0.79 \\
\quad \text{(1st pulse)} \\
\frac{\sqrt{10^{17} \cdot I_{n,B}(0.9 \ B, 0.5 \ B)}}{v_{\text{max} \_B3} - v_{\text{min1} \_B3}} = 0.71 \\
\quad \text{(2nd pulse)} \\
\frac{\sqrt{10^{17} \cdot I_{n,B}(0.9 \ B, 0.5 \ B)}}{v_{\text{max} \_B3} - v_{\text{min2} \_B3}} = 0.79 \\
\quad \text{(3rd pulse)} \\
\]

(rt = 0.6)

\[
t_{B4} := 1 \cdot ts \\
t_{pk4} := \text{root}(xvout(t_{B4}, 0.7 \ B, 0.6 \ B), t_{B4}) \\
\frac{t_{pk4}}{ts} = 0.931 \\
v_{\text{max} \_B4} := vout(t_{pk4}, 0.7 \ B, 0.6 \ B) = 0.839 \\
v_{\text{min} \_B4} := vout(t_{pk4} + 1 \cdot ts, 0.7 \ B, 0.6 \ B) = 0.003 \\
v_{\text{min1} \_B4} := vout(t_{pk4} + 2 \cdot ts, 0.7 \ B, 0.6 \ B) = 0.113 \\
v_{\text{min2} \_B4} := vout(t_{pk4} + 3 \cdot ts, 0.7 \ B, 0.6 \ B) = 0.025 \\
v_{\text{max} \_B4} - v_{\text{min} \_B4} = 0.836 \\
v_{\text{max} \_B4} - v_{\text{min1} \_B4} = 0.727 \\
v_{\text{max} \_B4} - v_{\text{min2} \_B4} = 0.865 \\
\]

\[
\frac{\sqrt{10^{17} \cdot I_{n,B}(0.7 \ B, 0.6 \ B)}}{v_{\text{max} \_B4} - v_{\text{min} \_B4}} = 0.79 \\
\quad \text{(1st pulse)} \\
\frac{\sqrt{10^{17} \cdot I_{n,B}(0.7 \ B, 0.6 \ B)}}{v_{\text{max} \_B4} - v_{\text{min1} \_B4}} = 0.69 \\
\quad \text{(2nd pulse)} \\
\frac{\sqrt{10^{17} \cdot I_{n,B}(0.7 \ B, 0.6 \ B)}}{v_{\text{max} \_B4} - v_{\text{min2} \_B4}} = 0.82 \\
\quad \text{(3rd pulse)} \\
\]
\[ t_{B5} := 0.8 \cdot ts \]
\[ t_{pk5} := \text{root}(xvout(t_{B5}, 0.8 B, 0.6 B), t_{B5}) \]
\[ \frac{t_{pk5}}{ts} = 0.913 \]
\[ v_{max,B5} := vout(t_{pk5}, 0.8 B, 0.6 B) = 0.855 \]
\[ v_{min,B5} := vout(t_{pk5} + 1 \cdot ts, 0.8 B, 0.6 B) = -0.011 \]
\[ v_{min1,B5} := vout(t_{pk5} + 2 \cdot ts, 0.8 B, 0.6 B) = 0.116 \]
\[ v_{min2,B5} := vout(t_{pk5} + 3 \cdot ts, 0.8 B, 0.6 B) = -0.027 \]
\[ v_{max,B5} - v_{min,B5} = 0.866 \quad v_{max,B5} - v_{min1,B5} = 0.739 \quad v_{max,B5} - v_{min2,B5} = 0.883 \]

\[ \frac{v_{max,B5} - v_{min,B5}}{\sqrt{10^{17} \cdot I_{n,B}(0.8 B, 0.6 B)}} = 0.78 \quad \text{1st pulse} \]
\[ \frac{v_{max,B5} - v_{min1,B5}}{\sqrt{10^{17} \cdot I_{n,B}(0.8 B, 0.6 B)}} = 0.67 \quad \text{2nd pulse} \quad \text{worst ISI} \]
\[ \frac{v_{max,B5} - v_{min2,B5}}{\sqrt{10^{17} \cdot I_{n,B}(0.8 B, 0.6 B)}} = 0.8 \quad \text{3rd pulse} \]

\[ t_{B6} := 0.9 \cdot ts \]
\[ t_{pk6} := \text{root}(xvout(t_{B4}, 0.9 B, 0.6 B), t_{B4}) \]
\[ \frac{t_{pk6}}{ts} = 0.897 \]
\[ v_{max,B6} := vout(t_{pk6}, 0.9 B, 0.6 B) = 0.867 \]
\[ v_{min,B6} := vout(t_{pk6} + 1 \cdot ts, 0.9 B, 0.6 B) = -0.022 \]
\[ v_{min1,B6} := vout(t_{pk6} + 2 \cdot ts, 0.9 B, 0.6 B) = 0.119 \]
\[ v_{min2,B6} := vout(t_{pk6} + 3 \cdot ts, 0.9 B, 0.6 B) = -0.029 \]
\[ v_{max,B6} - v_{min,B6} = 0.889 \quad v_{max,B6} - v_{min1,B6} = 0.747 \quad v_{max,B6} - v_{min2,B6} = 0.896 \]

\[ \frac{v_{max,B6} - v_{min,B6}}{\sqrt{10^{17} \cdot I_{n,B}(0.9 B, 0.6 B)}} = 0.78 \quad \text{1st pulse} \]
\[ \frac{v_{max,B6} - v_{min1,B6}}{\sqrt{10^{17} \cdot I_{n,B}(0.9 B, 0.6 B)}} = 0.65 \quad \text{2nd pulse} \quad \text{worst ISI} \]
\[ \frac{v_{max,B6} - v_{min2,B6}}{\sqrt{10^{17} \cdot I_{n,B}(0.9 B, 0.6 B)}} = 0.78 \quad \text{3rd pulse} \]
\( t_{B7} = 0.5 \cdot t_s \)

\( t_{pk7} = \sqrt{xvout(t_{B7}, 0.7 B, 0.7 B), t_{B7}} \)

\[
\frac{t_{pk7}}{t_s} = 0.821
\]

\( v_{\text{max}_{B7}} = vout(t_{pk7}, 0.7 B, 0.7 B) = 0.878 \)

\( v_{\text{min}_{B7}} = vout(t_{pk7} + 1 \cdot t_s, 0.7 B, 0.7 B) = 0.021 \)

\( v_{\text{min1}_{B7}} = vout(t_{pk7} + 2 \cdot t_s, 0.7 B, 0.7 B) = 0.065 \)

\( v_{\text{min2}_{B7}} = vout(t_{pk7} + 3 \cdot t_s, 0.7 B, 0.7 B) = 0.009 \)

\[
\begin{align*}
\frac{v_{\text{max}_{B7}} - v_{\text{min}_{B7}}}{\sqrt{10^{17} \cdot I_n B (0.7 B, 0.7 B)}} &= 0.73 \\
\frac{v_{\text{max}_{B7}} - v_{\text{min1}_{B7}}}{\sqrt{10^{17} \cdot I_n B (0.7 B, 0.7 B)}} &= 0.69 & \text{2nd pulse} \\
\frac{v_{\text{max}_{B7}} - v_{\text{min2}_{B7}}}{\sqrt{10^{17} \cdot I_n B (0.7 B, 0.7 B)}} &= 0.74 & \text{3rd pulse}
\end{align*}
\]

\( t_{B8} = 0.7 \cdot t_s \)

\( t_{pk8} = \sqrt{xvout(t_{B8}, 0.8 B, 0.7 B), t_{B8}} \)

\[
\frac{t_{pk8}}{t_s} = 0.803
\]

\( v_{\text{max}_{B8}} = vout(t_{pk8}, 0.8 B, 0.7 B) = 0.895 \)

\( v_{\text{min}_{B8}} = vout(t_{pk8} + 1 \cdot t_s, 0.8 B, 0.7 B) = 0.005 \)

\( v_{\text{min1}_{B8}} = vout(t_{pk8} + 2 \cdot t_s, 0.8 B, 0.7 B) = 0.066 \)

\( v_{\text{min2}_{B8}} = vout(t_{pk8} + 3 \cdot t_s, 0.8 B, 0.7 B) = 0.009 \)

\[
\begin{align*}
\frac{v_{\text{max}_{B8}} - v_{\text{min}_{B8}}}{\sqrt{10^{17} \cdot I_n B (0.8 B, 0.7 B)}} &= 0.829 \\
\frac{v_{\text{max}_{B8}} - v_{\text{min1}_{B8}}}{\sqrt{10^{17} \cdot I_n B (0.8 B, 0.7 B)}} &= 0.72 \\
\frac{v_{\text{max}_{B8}} - v_{\text{min2}_{B8}}}{\sqrt{10^{17} \cdot I_n B (0.8 B, 0.7 B)}} &= 0.67 & \text{(worst ISI)} \\
\frac{v_{\text{max}_{B8}} - v_{\text{min2}_{B8}}}{\sqrt{10^{17} \cdot I_n B (0.8 B, 0.7 B)}} &= 0.72 & \text{3rd pulse}
\end{align*}
\]
$t_{B9} := 0.8 \cdot ts$

$t_{pk9} := \text{root}(xvout(t_{B9}, 0.5 B, 0.5 B), t_{B9})$

$\frac{t_{pk9}}{ts} = 1.141$

$v_{\max_{B9}} := xvout(t_{pk9}, 0.5 B, 0.5 B) = 0.734$

$v_{\min_{B9}} := xvout(t_{pk9} + ts, 0.5 \cdot B, 0.5 B) = 0.068$

$v_{\min_{1:B9}} := xvout(t_{pk9} + 2 \cdot ts, 0.5 B, 0.5 B) = 0.079$

$v_{\min_{2:B9}} := xvout(t_{pk9} + 3 \cdot ts, 0.5 B, 0.5 B) = 0.014$

$v_{\max_{B9}} - v_{\min_{B9}} = 0.666$

$v_{\max_{B9}} - v_{\min_{1:B9}} = 0.655$

$v_{\max_{B9}} - v_{\min_{2:B9}} = 0.72$

\[
\begin{align*}
\frac{v_{\max_{B9}} - v_{\min_{B9}}}{\sqrt{10^{17} \cdot I_n(B)(0.5 B, 0.5 B)}} &= 0.8 \\ 
\frac{v_{\max_{B9}} - v_{\min_{1:B9}}}{\sqrt{10^{17} \cdot I_n(B)(0.5 B, 0.5 B)}} &= 0.79 \\ 
\frac{v_{\max_{B9}} - v_{\min_{2:B9}}}{\sqrt{10^{17} \cdot I_n(B)(0.5 B, 0.5 B)}} &= 0.87
\end{align*}
\]

$v_{\max_{B}} := v_{\max_{B9}}$

$v_{\min_{B}} := v_{\min_{1:B9}}$

opt rd = 0.5>0.6, rt = 0.5>0.6

$x_{111}(t, rd, rt) := xvout(t, rd, rt) + xvout(t - ts, rd, rt) + xvout(t - ts, rd, rt)$

$x_{110}(t, rd, rt) := xvout(t, rd, rt) + xvout(t - ts, rd, rt)$

$x_{100}(t, rd, rt) := xvout(t, rd, rt)$

$x_{001}(t, rd, rt) := xvout(t - ts, rd, rt)$

$x_{010}(t, rd, rt) := xvout(t - ts, rd, rt)$

$x_{011}(t, rd, rt) := xvout(t - ts, rd, rt) + xvout(t - 2ts, rd, rt)$

$x_{101}(t, rd, rt) := xvout(t, rd, rt) + xvout(t - 2ts, rd, rt)$
\[
x_{101}(t_{pk1} + ts, 0.5 B, 0.5 B) = 0.176
\]
\[
x_{101}(t_{pk1}, 0.5 B, 0.5 B) = 0.73
\]
\[
x_{101}(t_{pk1} + 2 ts, 0.5 B, 0.5 B) = 0.798
\]
\[
x_{100}(t_{pk1}, 0.5 B, 0.5 B) = 0.73
\]
\[
x_{100}(t_{pk1} + ts, 0.5 B, 0.5 B) = 0.101
\]
\[
x_{100}(t_{pk1} + 2 ts, 0.5 B, 0.5 B) = 0.069
\]

Check ratio <1 is ok 100, >1 re-opt

\[
\frac{x_{100}(t_{pk1}, 0.5 B, 0.5 B) - x_{100}(t_{pk1} + 2 ts, 0.5 B, 0.5 B)}{v_{max,B1} - v_{min,B1}} = 0.89
\]

**Sens PINFET**

\[
\lambda := 650 \cdot 10^{-9}
\]

\[
photon\_energy := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^{8}}{\lambda}
\]

\[
nq := 1.6 \cdot 10^{-19}
\]

\[
Q_N(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot \left(\frac{v_{max,B} - v_{min,B}}{2}\right) \sqrt{I_{n,B}(0.5 B, 0.5 B)}
\]
\[ P_{e\_N}(b) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\_N}(b)}{\sqrt{2}} \right) \]
\[ P_{c\_N}(b) = (\log(P_{e\_N}(b)) + 9) \]
\[ b \equiv 3 \cdot 10^3 \]
\[ b\_N := \text{root} \left( P_{c\_N}(b), b \right) \]

**minimum\_B := \min(b\_N) = 3.005 \cdot 10^3**

**APD-1FET**

\[ APD\_N(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot v_{\text{max}, B} \]
\[ APD\_B(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot v_{\text{max}, B} \]
\[ I_{\text{APD}\_d} := 10 \cdot 10^{-9} \]

\[ M_{\text{APD}1} := 100 \]
\[ F_{\text{M1}} := 7.9 \]

\[ \text{Noise\_APD1\_B}(b, rd, rt) := \left( 2 \cdot q \cdot APD\_B(b) \cdot M_{\text{APD}1^2} \cdot F_{\text{M1}} \right) \cdot \text{NEB\_B}(rd, rt) \]
\[ + 2 \cdot q \cdot I_{\text{APD}\_d} \cdot M_{\text{APD}1^2} \cdot F_{\text{M1}} \]

\[ Q_{\text{APD1\_B}}(b) := \frac{M_{\text{APD}1}}{t_s} \cdot b \cdot nq \cdot \frac{v_{\text{max}, B} - v_{\text{min}, B}}{2} \]
\[ \sqrt{I_n \cdot B(0.5 \ B, 0.5 \ B) + \text{Noise\_APD1\_B}(b, 0.5 \ B, 0.5 \ B)} \]

\[ P_{e\_\text{APD1\_B}}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD1\_B}}(b)}{\sqrt{2}} \right) \]
\[ p_{\text{APD1\_b}}(b) := (\log(P_{e\_\text{APD1\_B}}(b)) + 9) \]

\[ \text{APD1\_b\_B} := \text{root}(p_{\text{APD1\_B}}(b), b) \]

**NEB\_B(0.7 \ B, 0.6 \ B) = 3.998 \cdot 10^7**

**minimum\_APD1\_B := \min(\text{APD1\_b\_B}) = 1.809 \cdot 10^3**

**APD-2FET**

\[ M_{\text{APD}2} := 10 \quad F_{\text{M2}} := 5.5 \]

\[ \text{Noise\_APD2\_B}(b, rd, rt) := \left( 2 \cdot q \cdot APD\_B(b) \cdot M_{\text{APD}2^2} \cdot F_{\text{M2}} \right) \cdot \text{NEB\_B}(rd, rt) \]
\[ + 2 \cdot q \cdot I_{\text{APD}\_d} \cdot M_{\text{APD}2^2} \cdot F_{\text{M2}} \]

\[ Q_{\text{APD2\_B}}(b) := \frac{M_{\text{APD}2}}{t_s} \cdot b \cdot nq \cdot \frac{v_{\text{max}, B} - v_{\text{min}, B}}{2} \]
\[ \sqrt{I_n \cdot B(0.5 \ B, 0.5 \ B) + \text{Noise\_APD2\_B}(b, 0.5 \ B, 0.5 \ B)} \]
\[ P_{\text{e-APD2_B}}(b) := \frac{1}{2} \cdot \text{erfc}\left( \frac{Q_{\text{APD2_B}}(b)}{\sqrt{2}} \right) \]

\[ p_{\text{APD2_B}}(b) := (\log(P_{\text{e-APD2_B}}(b)) + 9) \]

\[ \text{APD2_B} := \text{root}(p_{\text{APD2_B}}(b), b) \]

\[ \text{minimum}_{\text{APD2_B}} := \min(\text{APD2_B}) = 1.429 \times 10^3 \]

**APD-3FET**

\[ M_{\text{APD3}} = 10 \quad F_{M3} = 9.2 \]

\[ \text{Noise}_{\text{APD3_B}}(b, rd, rt) := \left( 2 \cdot q \cdot \text{APD_B}(b) \cdot M_{\text{APD3}}^2 \cdot F_{M3} \right) \cdot \text{NEB_B}(rd, rt) \]

\[ + 2 \cdot q \cdot I_{\text{APD_B}}(b) \cdot M_{\text{APD3}}^2 \cdot F_{M3} \]

\[ Q_{\text{APD3_B}}(b) := \frac{M_{\text{APD3}}}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max}_B} - v_{\text{min}_B}}{2} \right) \]

\[ \sqrt{I_{n,B}(0.5 B, 0.5 B) + \text{Noise}_{\text{APD3_B}}(b, 0.5 B, 0.5 B)} \]

\[ P_{\text{e-APD3_B}}(b) := \frac{1}{2} \cdot \text{erfc}\left( \frac{Q_{\text{APD3_B}}(b)}{\sqrt{2}} \right) \]

\[ p_{\text{APD3_B}}(b) := (\log(P_{\text{e-APD3_B}}(b)) + 9) \]

\[ \text{APD3_B} := \text{root}(p_{\text{APD3_B}}(b), b) \]

\[ \text{minimum}_{\text{APD3_B}} := \min(\text{APD3_B}) = 2.066 \times 10^3 \]

- \[ \text{Sens}_{B\_PIN} := 10 \cdot \log\left( \frac{\text{minimum}_B \cdot \text{photon\_energy}}{2 \cdot 10^{-3} \cdot \frac{1}{ts}} \right) = -43.374 \]

- \[ \text{Sens}_{B\_APD\_silicon} := 10 \cdot \log\left( \frac{\text{minimum}_{\text{APD1_B}} \cdot \text{photon\_energy}}{2 \cdot 10^{-3} \cdot \frac{1}{ts}} \right) = -45.579 \]

- \[ \text{Sens}_{B\_APD\_InGaAs} := 10 \cdot \log\left( \frac{\text{minimum}_{\text{APD2_B}} \cdot \text{photon\_energy}}{2 \cdot 10^{-3} \cdot \frac{1}{ts}} \right) = -46.602 \]

- \[ \text{Sens}_{B\_APD\_germanium} := 10 \cdot \log\left( \frac{\text{minimum}_{\text{APD3_B}} \cdot \text{photon\_energy}}{2 \cdot 10^{-3} \cdot \frac{1}{ts}} \right) = -45.002 \]
B.2.8 Performance comparison PINFET

Performance comparison (PIN-FET 1st order LPF 100 Mbit/s)

Rx performance with PIN-FET input configuration, 1st order LPF pre-detection filter, calculations are as follow:
- Frequency response (opt rd,rt for each receiver)
- Noise integrals
- Total noise
- Pulse shaping, peak voltage, ISI
- Error bit rate, minimum number of photons, receiver sensitivity

**Bit-rate, pulse duration**

\[ C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]
\[ B := 100 \cdot 10^6 \quad \text{Bit-rate} \]

**pre-dec filter**
\[ rd := 0.7 \cdot B \]
\[ \omega_B := 2 \cdot \pi \cdot rd \]
\[ H_{but}(\omega) := \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_B}} \]

**TIA(Non-tuned)**
\[ f_c := 0.5 \cdot B \]
\[ A_v := 10 \]
\[ \omega_c := 2 \cdot \pi \cdot 0.5 \cdot B \]
\[ Rf_N := \frac{A_v + 1}{2 \cdot \pi \cdot 0.5 \cdot B \cdot C_T} = 2.334 \cdot 10^4 \]
\[ Z_{TIA,N}(\omega) := \frac{1}{1 + 1j \cdot \frac{\omega}{2 \cdot \pi \cdot 0.5 \cdot B}} \]

**TIA(Tuned-A)**
\[ m := 1.8 \quad \text{time constant ratio of L/R and RC} \]
\[ y := \sqrt{\left(\frac{-m^2}{2} + m + 1\right) + \sqrt{\left(\frac{-m^2}{2} + m + 1\right)^2 + m^2}} = 1.825 \]
\[ d := \frac{1}{y} \cdot B \]
\[ fc := 0.5 \cdot d = 2.739 \cdot 10^7 \quad \text{cut-off of half the bit rate, before BWER} \]

\[ RA := \frac{1}{(2 \cdot \pi \cdot C_T \cdot fc)} = 3.874 \cdot 10^3 \]

\[ L := RA^2 \cdot \frac{C_T}{m} = 1.25 \cdot 10^{-5} \]

\[ H_{but,A}(\omega) := \frac{1}{1 + j \cdot \frac{\omega}{2 \cdot \pi \cdot 0.7 \cdot B}} \]

\[ ts := \frac{1}{y \cdot d}, \quad B := \frac{1}{ts} = 1 \cdot 10^8 \]

\[ f_c := 0.5 \cdot B = 5 \cdot 10^7 \]

\[ R_{f,A} := RA \cdot (1 + Av) = 4.261 \cdot 10^4 \]

\[ Lf := L \cdot (1 + Av) = 1.376 \cdot 10^{-4} \]

\[ Z_{TIA,A}(\omega) := \frac{1 + i \cdot \omega \cdot \frac{Lf}{R_{f,A}}}{\left( 1 - \omega^2 \cdot \frac{C_T}{1 + Av} \right) + R_{f,A} \cdot \omega \cdot \frac{C_T}{(1 + Av)} \cdot 1i} \]

**TIA (Tuned B)**

Feedback value for (R) Tuned B

\[ \Delta_L := 1.9 \]

\[ \Delta_R := 2.75 \quad \text{feedback} \Delta R, \ \text{time constant ratio} \ \Delta L \]

\[ R_{f,B} := \frac{Av + 1}{2 \cdot \pi \cdot 0.5 \cdot B \cdot C_T} = 6.419 \cdot 10^4 \quad \text{feedback for Tuned B} \]

\[ \alpha := 0.4 \quad \text{splitting ratio} \]

\[ Lc := \left( \frac{R_{f,B}}{1 + Av} \right)^2 \cdot \frac{C_T}{\Delta_L} = 2.689 \cdot 10^{-5} \]

\[ C1 := (1 - \alpha) \cdot (C_T) = 9 \cdot 10^{-13} \]

\[ C2 := \alpha \cdot (C_T) = 6 \cdot 10^{-13} \]

**TUNED B (R) Transimpedance**

\[ Z_{TIA,B}(\omega) := \frac{1}{\left( (1 - \omega^2 \cdot Lc \cdot C1) + \frac{R_{f,B}}{(1 + Av)} \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \right)} \]

**Receiver freq-response**

\[ Z_{nontuned}(\omega) := H_{but,N}(\omega) \cdot Z_{TIA,N}(\omega) \]

\[ Z_{tuned,A}(\omega) := H_{but,A}(\omega) \cdot Z_{TIA,A}(\omega) \]
\[ Z_{\text{tuned\_B}}(\omega) := H_{\text{bti\_B}}(\omega) \cdot Z_{\text{TIA\_B}}(\omega) \]
\[ \omega := 1 \cdot 10^6, 10 \cdot 10^6 \ldots 1 \cdot 10^{11} \]
Noise equivalent bandwidth

\[ NEB_N := \frac{1}{2 \cdot \pi} \left( \int_0^{10^{12}} \left( |Z_{nontuned}(\omega)|^2 \right) d\omega \right) = 4.581 \cdot 10^7 \]

\[ NEB_A := \frac{1}{2 \cdot \pi} \left( \int_0^{10^{12}} \left( |Z_{tuned_A}(\omega)|^2 \right) d\omega \right) = 4.911 \cdot 10^7 \]

\[ NEB_B := \frac{1}{2 \cdot \pi} \left( \int_0^{10^{12}} \left( |Z_{tuned_B}(\omega)|^2 \right) d\omega \right) = 3.861 \cdot 10^7 \]

Noise equivalent bandwidth - Check

\[ \frac{1}{2 \cdot \pi} \left( \int_0^{10^{12}} \left( |Z_{TIA,N}(\omega)|^2 \right) d\omega \right) = 7.852 \cdot 10^7 \]

\[ B = 1 \cdot 10^8 \]

\[ \frac{\pi}{2} \cdot 0.5 \cdot B = 7.854 \cdot 10^7 \]

Noise integrals

<table>
<thead>
<tr>
<th>Receiver</th>
<th>Rf</th>
<th>lg</th>
<th>channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-tuned</td>
<td>INEB</td>
<td>INEB</td>
<td>I3</td>
</tr>
<tr>
<td>Tuned A</td>
<td>I2</td>
<td>INEB</td>
<td>I3</td>
</tr>
<tr>
<td>tuned B</td>
<td>I2</td>
<td>I2</td>
<td>I3</td>
</tr>
</tbody>
</table>

\[ I_{2,N} = NEB_N = 4.581 \cdot 10^7 \]

\[ I_{2,A} := \frac{1}{2 \cdot \pi} \left( \int_0^{10^{12}} \left( \left( \frac{1}{1 + \omega \cdot \frac{L_f}{R_f A}} \cdot Z_{tuned_A}(\omega) \right)^2 \right) d\omega \right) = 3.704 \cdot 10^7 \]

\[ I_{2,B} := \frac{1}{2 \cdot \pi} \left( \int_0^{10^{12}} \left( \left( 1 - \omega^2 \cdot L_e \cdot C_1 \right) \cdot Z_{tuned_B}(\omega) \right)^2 \right) d\omega \right) = 3.407 \cdot 10^7 \]

\[ I_{3,N} := \frac{1}{2 \pi} \left( \int_0^{10^{12}} \left( \frac{1 + j \cdot (\omega) \cdot R_f N \cdot C_T}{1 \cdot R_f N} \cdot Z_{nontuned}(\omega) \right)^2 d\omega \right) = 14.321 \]
\[
I_{3A} = \frac{1}{2\pi} \int_0^{10^2} \left( \left( \left( \left( 1 - \omega^2 \cdot L_f \cdot C_T \right) + Rf_A \cdot \omega \cdot C_T \cdot 1i \right) \cdot \left( Rf_A + 1i \cdot \omega \cdot L_f \right) \right) \cdot Z_{\text{tuned}_A}(\omega) \right)^2 d\omega = 7.511
\]

\[
I_{3B} = \frac{1}{2\pi} \int_0^{10^2} \left( \left( \left( 1 - \omega^2 \cdot Lc \cdot C1 \right) + Rf_B \cdot (\omega) \cdot (C1+C2-(\omega)^2 \cdot Lc \cdot C1 \cdot C2) \cdot 1i \right) \cdot \left( Rf_B \cdot Z_{\text{tuned}_B}(\omega) \right) \right)^2 d\omega = 1.775
\]

Feedback noise

\[q = 1.6 \cdot 10^{-19}, \quad k = 1.38 \cdot 10^{-23}, \quad T = 298\]

\[I_{nRf,N} = \frac{4 \cdot k \cdot T}{Rf_N} \cdot I_{2N} = 3.229 \cdot 10^{-17}\]

\[I_{nRf,A} = \frac{4 \cdot k \cdot T}{Rf_A} \cdot I_{2A} = 1.43 \cdot 10^{-17}\]

\[I_{nRf,B} = \frac{4 \cdot k \cdot T}{Rf_B} \cdot I_{2B} = 8.731 \cdot 10^{-18}\]

\[10 \log \left( \frac{I_{nRf,B}}{I_{nRf,N}} \right) = -5.679\]

\[10 \log \left( \frac{I_{nRf,A}}{I_{nRf,N}} \right) = -3.538\]

Ig noise (gate current)

\[I_g = 10 \cdot 10^{-9}\]

\[I_{nG,N} = 2 \cdot q \cdot Ig \cdot I_{2N} = 1.466 \cdot 10^{-19}\]

\[I_{nG,A} = 2 \cdot q \cdot Ig \cdot NEB_A = 1.571 \cdot 10^{-19}\]

\[I_{nG,B} = 2 \cdot q \cdot Ig \cdot I_{2B} = 1.09 \cdot 10^{-19}\]

\[10 \log \left( \frac{I_{nG,B}}{I_{nG,N}} \right) = -1.286\]

\[10 \log \left( \frac{I_{nG,A}}{I_{nG,N}} \right) = 0.301\]

Tuned A NEB > non-tuned NEB
**Channel noise (Gate-source)**

\[
\begin{align*}
gm_1 &= 30 \cdot 10^{-3} = 0.03 \quad \textit{noise\_factor} := 1 \\
I_{nD} &= 4 \cdot k \cdot T \cdot \frac{1}{gm_1} \\
I_{nD,N} &= I_{nD} \cdot I_{3,N} = 7.852 \cdot 10^{-18} \\
I_{nD,A} &= I_{nD} \cdot I_{3,A} = 4.118 \cdot 10^{-18} \\
I_{nD,B} &= I_{nD} \cdot I_{3,B} = 9.73 \cdot 10^{-19} \\
10 \log \left( \frac{I_{nD,B}}{I_{nD,N}} \right) &= -9.069 \\
10 \log \left( \frac{I_{nD,A}}{I_{nD,N}} \right) &= -2.803 \\
\end{align*}
\]

**Total noise - PIN-FET**

\[
\begin{align*}
I_{n,N} &= I_{nRf,N} + I_{nG,N} + I_{nD,N} = 4.028 \cdot 10^{-17} \\
I_{n,A} &= I_{nRf,A} + I_{nG,A} + I_{nD,A} = 1.857 \cdot 10^{-17} \\
I_{n,B} &= I_{nRf,B} + I_{nG,B} + I_{nD,B} = 9.813 \cdot 10^{-18} \\
10 \log \left( \frac{I_{n,B}}{I_{n,N}} \right) &= -6.133 \\
10 \log \left( \frac{I_{n,A}}{I_{n,N}} \right) &= -3.363 \\
\end{align*}
\]

**Output pulse shape**

\[
\begin{align*}
ts &= \frac{1}{B} \quad B = 1 \cdot 10^8 \\
t &= -6 \cdot ts, -5.99 \cdot ts .. 6 \cdot ts \\
1 \left( \frac{ts}{10^3} \right) &\left[ \frac{\sin \left( \frac{\omega \cdot ts}{2} \right)}{\left( \frac{\omega \cdot ts}{2} \right)} \right] \cdot \text{Re} \left( Z_{\text{tuned}}(\omega) \cdot \exp \left( \frac{1}{\omega \cdot \left( t - \frac{ts}{2} \right)} \right) \right) \, d\omega
\end{align*}
\]
\[ vout_A(t) = \frac{ts}{\pi} \cdot \frac{1}{ts^{10^3}} \left[ \sin\left(\frac{\omega \cdot ts}{2}\right) \cdot \text{Re}\left(\frac{Z_{tuned\_A}(\omega) \cdot \exp\left(1i \cdot \omega \cdot \left(t - \frac{ts}{2}\right)\right)}{\left(\frac{\omega \cdot ts}{2}\right)}\right) \right] d\omega \]

\[ vout_B(t) = \frac{ts}{\pi} \cdot \frac{1}{ts^{10^3}} \left[ \sin\left(\frac{\omega \cdot ts}{2}\right) \cdot \text{Re}\left(\frac{Z_{tuned\_B}(\omega) \cdot \exp\left(1i \cdot \omega \cdot \left(t - \frac{ts}{2}\right)\right)}{\left(\frac{\omega \cdot ts}{2}\right)}\right) \right] d\omega \]

**peak voltage and time**

\[ xvout_N(t) = \frac{ts^2}{\pi} \cdot \frac{1}{ts^{10^3}} \left[ \sin\left(\frac{\omega \cdot ts}{2}\right) \cdot \text{Re}\left(1i \cdot \omega \cdot Z_{nontuned}(\omega) \cdot \exp\left(1i \cdot \omega \cdot \left(t - \frac{ts}{2}\right)\right)\right) \right] d\omega \]

\[ xvout_A(t) = \frac{ts^2}{\pi} \cdot \frac{1}{ts^{10^3}} \left[ \sin\left(\frac{\omega \cdot ts}{2}\right) \cdot \text{Re}\left(1i \cdot \omega \cdot Z_{tuned\_A}(\omega) \cdot \exp\left(1i \cdot \omega \cdot \left(t - \frac{ts}{2}\right)\right)\right) \right] d\omega \]

\[ xvout_B(t) = \frac{ts^2}{\pi} \cdot \frac{1}{ts^{10^3}} \left[ \sin\left(\frac{\omega \cdot ts}{2}\right) \cdot \text{Re}\left(1i \cdot \omega \cdot Z_{tuned\_B}(\omega) \cdot \exp\left(1i \cdot \omega \cdot \left(t - \frac{ts}{2}\right)\right)\right) \right] d\omega \]
\[ t_N := 0.5 \cdot ts \]
\[ t_{pk1} := \text{root}(xvout_N(t_N), t_N) \]
\[ \frac{t_{pk1}}{ts} = 1.026 \]
\[ v_{\text{max},N} := vout_N(t_{pk1}) = 0.884 \]
\[ v_{\text{min},N} := vout_N(t_{pk1} + 1 \cdot ts) = 0.106 \]
\[ v_{\text{min}2ts,N} := vout_N(t_{pk1} + 2 \cdot ts) = 0.005 \]
\[ v_{\text{min}3ts,N} := vout_N(t_{pk1} + 3 \cdot ts) = 2.451 \cdot 10^{-4} \]
\[ v_{\text{max},N} - v_{\text{min},N} = 0.777 \]
\[ t_A := 0.9 \cdot ts \]
\[ t_{pk1} := \text{root}(xvout_A(t_A), t_A) \]
\[ \frac{t_{pk1}}{ts} = 1.076 \]
\[ v_{\text{max},A} := vout_A(t_{pk1}) = 0.94 \]
\[ v_{\text{min},A} := vout_A(t_{pk1} + 1 \cdot ts) = 0.094 \]
\[ v_{\text{min}2ts,A} := vout_A(t_{pk1} + 2 \cdot ts) = -0.055 \]
\[ v_{\text{min}3ts,A} := vout_A(t_{pk1} + 3 \cdot ts) = -0.001 \]
\[ v_{\text{max},A} - v_{\text{min},A} = 0.846 \]
\[ t_B := 1.2 \cdot ts \]
\[ t_{pk1} := \text{root}(xvout_B(t_B), t_B) \]
\[ \frac{t_{pk1}}{ts} = 1.597 \]
\[ v_{\text{max},B} := vout_B(t_{pk1}) = 0.814 \]
\[ v_{\text{min},B} := vout_B(t_{pk1} + 2 \cdot ts) = 0.041 \]
\[ v_{\text{min}2ts,B} := vout_B(t_{pk1} + 1 \cdot ts) = 0.029 \]
\[ v_{\text{min}3ts,B} := vout_B(t_{pk1} + 3 \cdot ts) = 0.02 \]
\[ v_{\text{max},B} - v_{\text{min},B} = 0.773 \]
\[ v_{\text{max},B} - v_{\text{min}2ts,B} = 0.784 \]
\[ v_{\text{max},B} - v_{\text{min}3ts,B} = 0.794 \]
\[ nq := 1.6 \times 10^{-10} \quad \text{Test}_b_{\text{photons}} := 1100, 1200 \ldots 12000 \]

\[
\text{test}_Q_N (\text{Test}_b_{\text{photons}}) := \frac{1}{ts} \cdot \text{Test}_b_{\text{photons}} \cdot nq \cdot \left( \frac{v_{\text{max}_N} - v_{\text{min}_N}}{2} \right) \sqrt{I_{n_N}}
\]

\[
\text{test}_Q_A (\text{Test}_b_{\text{photons}}) := \frac{1}{ts} \cdot \text{Test}_b_{\text{photons}} \cdot nq \cdot \left( \frac{v_{\text{max}_A} - v_{\text{min}_A}}{2} \right) \sqrt{I_{n_A}}
\]

\[
\text{test}_Q_B (\text{Test}_b_{\text{photons}}) := \frac{1}{ts} \cdot \text{Test}_b_{\text{photons}} \cdot nq \cdot \left( \frac{v_{\text{max}_B} - v_{\text{min}_B}}{2} \right) \sqrt{I_{n_B}}
\]

\[
\text{Test}1 (\text{Test}_b_{\text{photons}}) := \frac{1}{2} \cdot \text{erfc} \left( \frac{\text{test}_Q_N (\text{Test}_b_{\text{photons}})}{\sqrt{2}} \right)
\]

\[
\text{Test}2 (\text{Test}_b_{\text{photons}}) := \frac{1}{2} \cdot \text{erfc} \left( \frac{\text{test}_Q_A (\text{Test}_b_{\text{photons}})}{\sqrt{2}} \right)
\]

\[
\text{Test}3 (\text{Test}_b_{\text{photons}}) := \frac{1}{2} \cdot \text{erfc} \left( \frac{\text{test}_Q_B (\text{Test}_b_{\text{photons}})}{\sqrt{2}} \right)
\]
number of photons per bit for $10^{-9} $

\[
Q_N(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max}_N} - v_{\text{min}_N}}{2} \right) \sqrt{I_{n,N}}
\]

\[
Q_A(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max}_A} - v_{\text{min}_A}}{2} \right) \sqrt{I_{n,A}}
\]

\[
Q_B(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max}_B} - v_{\text{min}_B}}{2} \right) \sqrt{I_{n,B}}
\]

\[
Pe_N(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right)
\]

\[
Pe_A(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_A(b)}{\sqrt{2}} \right)
\]

\[
Pe_B(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_B(b)}{\sqrt{2}} \right)
\]

\[
Pc_N(b) := (\log(Pe_N(b)) + 9)
\]

\[
Pc_A(b) := (\log(Pe_A(b)) + 9)
\]

\[
Pc_B(b) := (\log(Pe_B(b)) + 9)
\]

\[
b \equiv 3 \cdot 10^3
\]

\[
b_N := \text{root} (Pc_N(b), b)
\]

\[
b_A := \text{root} (Pc_A(b), b)
\]

\[
b_B := \text{root} (Pc_B(b), b)
\]

\[
\text{minimum}_N := \min(b_N) = 6.12 \cdot 10^3
\]

\[
\text{minimum}_A := \min(b_A) = 3.82 \cdot 10^3
\]

\[
\text{minimum}_B := \min(b_B) = 3.04 \cdot 10^3
\]

**BER-check for minimum $b$**

| $Q_N(\text{minimum}_N) = 5.998$ | $Pe_N(\text{minimum}_N) = 1 \cdot 10^{-9}$ |
| $Q_A(\text{minimum}_A) = 5.998$ | $Pe_A(\text{minimum}_A) = 1 \cdot 10^{-9}$ |
| $Q_B(\text{minimum}_B) = 5.998$ | $Pe_B(\text{minimum}_B) = 10 \cdot 10^{-10}$ |
Sens PIN-FET

\[ \lambda := 650 \cdot 10^{-9} \]

\[ \text{photon} \_\text{energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ \text{Sens}_N := 10 \cdot \log \left( \frac{\text{minimum}_N}{2} \cdot \frac{\text{photon} \_\text{energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -40.285 \]

\[ \text{Sens}_A := 10 \cdot \log \left( \frac{\text{minimum}_A}{2} \cdot \frac{\text{photon} \_\text{energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -42.332 \]

\[ \text{Sens}_B := 10 \cdot \log \left( \frac{\text{minimum}_B}{2} \cdot \frac{\text{photon} \_\text{energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -43.325 \]

\[ \text{Sens}_B - \text{Sens}_N = -3.039 \]

\[ \text{Sens}_A - \text{Sens}_N = -2.047 \]
B.2.9 Performance comparison APDFET

**Silicon_APD:** Performance comparison (APD-FET 1st order LPF 100 Mbit/s)

Rx performance with APD-FET input configuration, 1st order LPF pre-detection filter. Calculations are as follows:

- APD noise, APD BER, APD receiver sensitivity
- [Silicon Epitaxial: APD M= 10 F(M)=5.5]

### APD noise

\[ M_{APD} = 10 \]
\[ F_M = 5.5 \]

\[ APD_N(b) := \frac{1}{ts} \cdot b \cdot nq \cdot (v_{max,N}) \]

\[ APD_A(b) := \frac{1}{ts} \cdot b \cdot nq \cdot (v_{max,A}) \]

\[ APD_B(b) := \frac{1}{ts} \cdot b \cdot nq \cdot (v_{max,B}) \]

\[ I_{APD,d} := 10 \cdot 10^{-9} \]

\[ Noise_{APD,N}(b) := \left( 2 \cdot q \cdot APD_N(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_N + \left( 2 \cdot q \cdot I_{APD,d} \cdot M_{APD}^2 \cdot F_M \right) \]

\[ Noise_{APD,A}(b) := \left( 2 \cdot q \cdot APD_A(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_A + \left( 2 \cdot q \cdot I_{APD,d} \cdot M_{APD}^2 \cdot F_M \right) \]

\[ Noise_{APD,B}(b) := \left( 2 \cdot q \cdot APD_B(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_B + \left( 2 \cdot q \cdot I_{APD,d} \cdot M_{APD}^2 \cdot F_M \right) \]

### BER+APD

\[ Q_{APD,N}(b) := \frac{M_{APD}}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{max,N} - v_{min,N}}{2} \right) \frac{\sqrt{I_{n,N} + Noise_{APD,N}(b)}}{\sqrt{2}} \]

\[ Q_{APD,A}(b) := \frac{M_{APD}}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{max,A} - v_{min,A}}{2} \right) \frac{\sqrt{I_{n,A} + Noise_{APD,A}(b)}}{\sqrt{2}} \]

\[ Q_{APD,B}(b) := \frac{M_{APD}}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{max,B} - v_{min,B}}{2} \right) \frac{\sqrt{I_{n,B} + Noise_{APD,B}(b)}}{\sqrt{2}} \]

\[ P_{e,APD,N}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD,N}(b)}{\sqrt{2}} \right) \]

\[ P_{e,APD,A}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD,A}(b)}{\sqrt{2}} \right) \]
\[
P_{e,APD_B}(b) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD_B}(b)}{\sqrt{2}} \right)
\]

\[
PC_{APD_N}(b) := (\log(P_{e,APD_N}(b)) + 9)
\]

\[
PC_{APD_A}(b) := (\log(P_{e,APD_A}(b)) + 9)
\]

\[
PC_{APD_B}(b) := (\log(P_{e,APD_B}(b)) + 9)
\]

\[
APD_b_N := \text{root}(PC_{APD_N}(b), b)
\]

\[
APD_b_A := \text{root}(PC_{APD_A}(b), b)
\]

\[
APD_b_B := \text{root}(PC_{APD_B}(b), b)
\]

\[
\text{minimum}_{APD_N} := \min(APD_b_N) = 1.716 \cdot 10^3
\]

\[
\text{minimum}_{APD_A} := \min(APD_b_A) = 1.553 \cdot 10^3
\]

\[
\text{minimum}_{APD_B} := \min(APD_b_B) = 1.368 \cdot 10^3
\]

**Check BER, Q, and b APD**

\[
Q_{APD_N}(\min(APD_b_N)) = 5.998
\]

\[
Q_{APD_A}(\min(APD_b_A)) = 5.998
\]

\[
Q_{APD_B}(\min(APD_b_B)) = 5.998
\]

**Sens**

\[
APD_N := 10 \cdot \log \left( \frac{\text{minimum}_{APD_N} \cdot \text{photon\_energy}}{2 \cdot 10^{-3} \cdot \frac{1}{ts}} \right) = -45.809
\]

\[
APD_A := 10 \cdot \log \left( \frac{\text{minimum}_{APD_A} \cdot \text{photon\_energy}}{2 \cdot 10^{-3} \cdot \frac{1}{ts}} \right) = -46.242
\]

\[
APD_B := 10 \cdot \log \left( \frac{\text{minimum}_{APD_B} \cdot \text{photon\_energy}}{2 \cdot 10^{-3} \cdot \frac{1}{ts}} \right) = -46.792
\]

\[
\text{Sens}_N - APD_N = 5.524 \quad \text{Noise}_{APD_N}(\text{minimum}_{APD_N}) = 2.762 \cdot 10^{-16}
\]

\[
\text{Sens}_A - APD_A = 3.91 \quad \text{Noise}_{APD_A}(\text{minimum}_{APD_A}) = 2.882 \cdot 10^{-16}
\]

\[
\text{Sens}_B - APD_B = 3.467 \quad \text{Noise}_{APD_B}(\text{minimum}_{APD_B}) = 1.89 \cdot 10^{-16}
\]

\[
I_n_N = 4.028 \cdot 10^{-17} \quad I_n_A = 1.857 \cdot 10^{-17} \quad I_n_B = 9.813 \cdot 10^{-18}
\]

\[
x_{\text{Noise}_{APD_N}(bx)} = \left( 2 \cdot q \cdot APD_N(bx) \cdot M_{APD^2} \cdot F_M \right) \cdot \text{NEB}_N
\]

\[
x_{\text{Noise}_{APD_A}(bx)} = \left( 2 \cdot q \cdot APD_A(bx) \cdot M_{APD^2} \cdot F_M \right) \cdot \text{NEB}_A
\]

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Check BER performance of APD receiver

\[ xQ_{APD_N}(bx) = \frac{M_{APD}}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{max_N} - v_{min_N}}{2} \right) \sqrt{I_{n,N} + Noise_{APD_N}(bx)} \]

\[ xQ_{APD_A}(bx) = \frac{M_{APD}}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{max_A} - v_{min_A}}{2} \right) \sqrt{I_{n,A} + Noise_{APD_A}(bx)} \]

\[ xQ_{APD_B}(bx) = \frac{M_{APD}}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{max_B} - v_{min_B}}{2} \right) \sqrt{I_{n,B} + Noise_{APD_B}(bx)} \]

\[ xP_{e_{APD_N}}(bx) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD_N}(bx)}{\sqrt{2}} \right) \]

\[ xP_{e_{APD_A}}(bx) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD_A}(bx)}{\sqrt{2}} \right) \]

\[ xP_{e_{APD_B}}(bx) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD_B}(bx)}{\sqrt{2}} \right) \]

\[ bx := 100, 200, ..., 8000 \]
InGaAs APD: Performance comparison (APD-FET 1st order LPF 100 Mbit/s)

Rx performance with APD-FET input configuration, 1st order LPF pre-detection filter. calculations are as follow

- APD noise, APD BER, APD receiver sensitivity
- \[ \text{InGaAs: } M = 100 \text{ F(M)=7.9} \]

APD noise

\[ M_{\text{APD}} := 100 \]
\[ F_{\text{M}} := 7.9 \]
\[ APD_N(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( v_{\text{max},N} \right) \]
\[ APD_A(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( v_{\text{max},A} \right) \]
\[ APD_B(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( v_{\text{max},B} \right) \]

\[ I_{\text{APD}_d} := 10 \cdot 10^{-9} \]

\[ \text{Noise}_{\text{APD}_N}(b) := \left( 2 \cdot q \cdot APD_N(b) \cdot M_{\text{APD}}^2 \cdot F_{\text{M}} \right) \cdot NEB_N \]
\[ + 2 \cdot q \cdot I_{\text{APD}_d} \cdot M_{\text{APD}}^2 \cdot F_{\text{M}} \]

\[ \text{Noise}_{\text{APD}_A}(b) := \left( 2 \cdot q \cdot APD_A(b) \cdot M_{\text{APD}}^2 \cdot F_{\text{M}} \right) \cdot NEB_A \]
\[ + 2 \cdot q \cdot I_{\text{APD}_d} \cdot M_{\text{APD}}^2 \cdot F_{\text{M}} \]

\[ \text{Noise}_{\text{APD}_B}(b) := \left( 2 \cdot q \cdot APD_B(b) \cdot M_{\text{APD}}^2 \cdot F_{\text{M}} \right) \cdot NEB_B \]
\[ + 2 \cdot q \cdot I_{\text{APD}_d} \cdot M_{\text{APD}}^2 \cdot F_{\text{M}} \]

BER+APD

\[ Q_{\text{APD}_N}(b) := \frac{M_{\text{APD}} \cdot b \cdot nq \cdot \left( v_{\text{max},N} - v_{\text{min},N} \right)}{\sqrt{I_{n,N} + \text{Noise}_{\text{APD}_N}(b)}} \]

\[ Q_{\text{APD}_A}(b) := \frac{M_{\text{APD}} \cdot b \cdot nq \cdot \left( v_{\text{max},A} - v_{\text{min},A} \right)}{\sqrt{I_{n,A} + \text{Noise}_{\text{APD}_A}(b)}} \]

\[ Q_{\text{APD}_B}(b) := \frac{M_{\text{APD}} \cdot b \cdot nq \cdot \left( v_{\text{max},B} - v_{\text{min},B} \right)}{\sqrt{I_{n,B} + \text{Noise}_{\text{APD}_B}(b)}} \]

\[ P_{e,\text{APD}_N}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD}_N}(b)}{\sqrt{2}} \right) \]

\[ P_{e,\text{APD}_A}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD}_A}(b)}{\sqrt{2}} \right) \]
\[ P_{e-APD_B}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{-APD_B}(b)}{\sqrt{2}} \right) \]

\[ p_{APD_N}(b) := (\log(P_{e-APD_N}(b)) + 9) \]

\[ p_{APD_A}(b) := (\log(P_{e-APD_A}(b)) + 9) \]

\[ p_{APD_B}(b) := (\log(P_{e-APD_B}(b)) + 9) \]

\[ APD_b_N := \text{root}(p_{APD_N}(b), b) \]

\[ APD_b_A := \text{root}(p_{APD_A}(b), b) \]

\[ APD_b_B := \text{root}(p_{APD_B}(b), b) \]

\[ \text{minimum}_{APD_N} := \min(\text{APD}_b_N) = 2.05 \cdot 10^3 \]

\[ \text{minimum}_{APD_A} := \min(\text{APD}_b_A) = 1.964 \cdot 10^3 \]

\[ \text{minimum}_{APD_B} := \min(\text{APD}_b_B) = 1.729 \cdot 10^3 \]

Check BER, Q, and b APD

\[ Q_{APD_N}(\min(\text{APD}_b_N)) = 5.998 \]

\[ Q_{APD_A}(\min(\text{APD}_b_A)) = 5.998 \]

\[ Q_{APD_B}(\min(\text{APD}_b_B)) = 5.998 \]

\[ \text{Sens} \]

\[ APD_N := 10 \cdot \log \left( \frac{\text{minimum}_{APD_N}}{2} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -45.036 \]

\[ APD_A := 10 \cdot \log \left( \frac{\text{minimum}_{APD_A}}{2} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -45.221 \]

\[ APD_B := 10 \cdot \log \left( \frac{\text{minimum}_{APD_B}}{2} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -45.776 \]

\[ \text{Sens}_N - \text{APD}_N = 4.75 \quad \text{Noise}_{APD_N}(\text{minimum}_{APD_N}) = 4.515 \cdot 10^{-14} \]

\[ \text{Sens}_A - \text{APD}_A = 2.889 \quad \text{Noise}_{APD_A}(\text{minimum}_{APD_A}) = 4.909 \cdot 10^{-14} \]

\[ \text{Sens}_B - \text{APD}_B = 2.451 \quad \text{Noise}_{APD_B}(\text{minimum}_{APD_B}) = 3.173 \cdot 10^{-14} \]

\[ I_{n,N} = 4.028 \cdot 10^{-17} \quad I_{n,A} = 1.857 \cdot 10^{-17} \quad I_{n,B} = 9.813 \cdot 10^{-18} \]

\[ x_{\text{Noise}_{APD_N}(bx)} = \left( 2 \cdot q \cdot \text{APD}_N(bx) \cdot M_{APD}^2 \cdot F_M \right) \cdot \text{NEB}_N \]

\[ = \left( 2 \cdot q \cdot I_{APD_d} \cdot M_{APD}^2 \cdot F_M \right) \cdot \text{NEB}_N \]

\[ x_{\text{Noise}_{APD_A}(bx)} = \left( 2 \cdot q \cdot \text{APD}_A(bx) \cdot M_{APD}^2 \cdot F_M \right) \cdot \text{NEB}_A \]

\[ = \left( 2 \cdot q \cdot I_{APD_d} \cdot M_{APD}^2 \cdot F_M \right) \cdot \text{NEB}_A \]

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\[ \text{Noise}_{\text{APD}_B}(bx) := \left( 2 \cdot q \cdot \text{APD}_B(bx) \cdot M_{\text{APD}^2} \cdot F_M \right) \cdot \text{NEB}_B + 2 \cdot q \cdot I_{\text{APD}_d} \cdot M_{\text{APD}^2} \cdot F_M \]

**Check BER performance of APD receiver**

\[ xQ_{\text{APD}_N}(bx) := \frac{M_{\text{APD}} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max}_N} - v_{\text{min}_N}}{2} \right)}{\sqrt{I_{n_N} + \text{Noise}_{\text{APD}_N}(bx)}} \]

\[ xQ_{\text{APD}_A}(bx) := \frac{M_{\text{APD}} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max}_A} - v_{\text{min}_A}}{2} \right)}{\sqrt{I_{n_A} + \text{Noise}_{\text{APD}_A}(bx)}} \]

\[ xQ_{\text{APD}_B}(bx) := \frac{M_{\text{APD}} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max}_B} - v_{\text{min}_B}}{2} \right)}{\sqrt{I_{n_B} + \text{Noise}_{\text{APD}_B}(bx)}} \]

\[ xP_{e_{\text{APD}_N}}(bx) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD}_N}(bx)}{\sqrt{2}} \right) \]

\[ xP_{e_{\text{APD}_A}}(bx) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD}_A}(bx)}{\sqrt{2}} \right) \]

\[ xP_{e_{\text{APD}_B}}(bx) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD}_B}(bx)}{\sqrt{2}} \right) \]

\[ bx := 100, 200 \ldots 8000 \]
Germanium APD: Performance comparison (APD-FET 1st order LPF 100 Mbit/s)

Rx performance with APD-FET input configuration, 1st order LPF pre-detection filter. Calculations are as follow:

- APD noise, APD BER, APD receiver sensitivity
- [Germanium: M=10 F(M)=9.2]

**APD noise**

\[ M_{APD} := 10 \]
\[ F_M := 9.2 \]

\[ APD_N(b) := \frac{1}{t_s} b \cdot nq \cdot (v_{max,N}) \]

\[ APD_A(b) := \frac{1}{t_s} b \cdot nq \cdot (v_{max,A}) \]

\[ APD_B(b) := \frac{1}{t_s} b \cdot nq \cdot (v_{max,B}) \]

\[ I_{APD,d} := 10 \cdot 10^{-9} \]

\[ Noise_{APD,N}(b) := \left( 2 \cdot q \cdot APD_N(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_N \]
\[ Noise_{APD,A}(b) := \left( 2 \cdot q \cdot APD_A(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_A \]
\[ Noise_{APD,B}(b) := \left( 2 \cdot q \cdot APD_B(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_B \]

**BER+APD**

\[ Q_{APD,N}(b) := \frac{M_{APD}}{t_s} \cdot b \cdot nq \cdot \left( \frac{v_{max,N} - v_{min,N}}{2} \right) \sqrt{I_n,N + Noise_{APD,N}(b)} \]

\[ Q_{APD,A}(b) := \frac{M_{APD}}{t_s} \cdot b \cdot nq \cdot \left( \frac{v_{max,A} - v_{min,A}}{2} \right) \sqrt{I_n,A + Noise_{APD,A}(b)} \]

\[ Q_{APD,B}(b) := \frac{M_{APD}}{t_s} \cdot b \cdot nq \cdot \left( \frac{v_{max,B} - v_{min,B}}{2} \right) \sqrt{I_n,B + Noise_{APD,B}(b)} \]

\[ P_{e,APD,N}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD,N}(b)}{\sqrt{2}} \right) \]

\[ P_{e,APD,A}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD,A}(b)}{\sqrt{2}} \right) \]
\[ P_{e\text{-APD}\_B}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{-APD}\_B}(b)}{\sqrt{2}} \right) \]

\[ pc_{\text{-APD}\_N}(b) := (\log(P_{e\text{-APD}\_N}(b)) + 9) \]

\[ pc_{\text{-APD}\_A}(b) := (\log(P_{e\text{-APD}\_A}(b)) + 9) \]

\[ pc_{\text{-APD}\_B}(b) := (\log(P_{e\text{-APD}\_B}(b)) + 9) \]

\[ \text{APD}\_b\_N := \text{root}(pc_{\text{-APD}\_N}(b), b) \]

\[ \text{APD}\_b\_A := \text{root}(pc_{\text{-APD}\_A}(b), b) \]

\[ \text{APD}\_b\_B := \text{root}(pc_{\text{-APD}\_B}(b), b) \]

\[ \text{minimum}\_\text{APD}\_N := \min(\text{APD}\_b\_N) = 2.441 \cdot 10^3 \]

\[ \text{minimum}\_\text{APD}\_A := \min(\text{APD}\_b\_A) = 2.272 \cdot 10^3 \]

\[ \text{minimum}\_\text{APD}\_B := \min(\text{APD}\_b\_B) = 1.98 \cdot 10^3 \]

**Check BER, Q, and b APD**

\[ Q_{\text{-APD}\_N}(\min(\text{APD}\_b\_N)) = 5.998 \]

\[ Q_{\text{-APD}\_A}(\min(\text{APD}\_b\_A)) = 5.998 \]

\[ Q_{\text{-APD}\_B}(\min(\text{APD}\_b\_B)) = 5.998 \]

**Sens**

\[ \text{APD}\_N := 10 \cdot \log \left( \frac{\text{minimum}\_\text{APD}\_N}{2} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -44.278 \]

\[ \text{APD}\_A := 10 \cdot \log \left( \frac{\text{minimum}\_\text{APD}\_A}{2} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -44.588 \]

\[ \text{APD}\_B := 10 \cdot \log \left( \frac{\text{minimum}\_\text{APD}\_B}{2} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -45.185 \]

\[ \text{Sens}\_N – \text{APD}\_N = 3.993 \quad \text{Noise}\_\text{APD}\_N(\text{minimum}\_\text{APD}\_N) = 6.003 \cdot 10^{-16} \]

\[ \text{Sens}\_A – \text{APD}\_A = 2.256 \quad \text{Noise}\_\text{APD}\_A(\text{minimum}\_\text{APD}\_A) = 6.386 \cdot 10^{-16} \]

\[ \text{Sens}\_B – \text{APD}\_B = 1.861 \quad \text{Noise}\_\text{APD}\_B(\text{minimum}\_\text{APD}\_B) = 4.067 \cdot 10^{-16} \]

\[ I_{n,N} = 4.028 \cdot 10^{-17} \quad I_{n,A} = 1.857 \cdot 10^{-17} \quad I_{n,B} = 9.813 \cdot 10^{-18} \]

\[ x_{\text{Noise}\_\text{APD}\_N(bx)} := \left( 2 \cdot q \cdot \text{APD}\_N(bx) \cdot M_{\text{APD}}^2 \cdot F_{\text{M}} \right) \cdot \text{NEB}\_N \]

\[ + 2 \cdot q \cdot I_{\text{APD}\_d} \cdot M_{\text{APD}}^2 \cdot F_{\text{M}} \]

\[ x_{\text{Noise}\_\text{APD}\_A(bx)} := \left( 2 \cdot q \cdot \text{APD}\_A(bx) \cdot M_{\text{APD}}^2 \cdot F_{\text{M}} \right) \cdot \text{NEB}\_A \]

\[ + 2 \cdot q \cdot I_{\text{APD}\_d} \cdot M_{\text{APD}}^2 \cdot F_{\text{M}} \]
Check BER performance of APD receiver

\[
\begin{align*}
\text{Noise}_{\text{APD}}_B (bx) &:= \left( 2 \cdot q \cdot \text{APD}_B (bx) \cdot M_{\text{APD}}^2 \cdot F_M \right) \cdot \text{NEB}_B \\
&\quad + 2 \cdot \frac{q \cdot I_{\text{APD}_d} \cdot M_{\text{APD}}^2 \cdot F_M}{ts} \\
\end{align*}
\]

\[
\begin{align*}
xQ_{\text{APD}}_N (bx) &:= \frac{M_{\text{APD}} \cdot b \cdot nq \cdot \left( v_{\text{max}_N} - v_{\text{min}_N} \right)}{\sqrt{I_{n,N} + \text{Noise}_{\text{APD}}_N (bx)}} \\
xQ_{\text{APD}}_A (bx) &:= \frac{M_{\text{APD}} \cdot b \cdot nq \cdot \left( v_{\text{max}_A} - v_{\text{min}_A} \right)}{\sqrt{I_{n,A} + \text{Noise}_{\text{APD}}_A (bx)}} \\
xQ_{\text{APD}}_B (bx) &:= \frac{M_{\text{APD}} \cdot b \cdot nq \cdot \left( v_{\text{max}_B} - v_{\text{min}_B} \right)}{\sqrt{I_{n,B} + \text{Noise}_{\text{APD}}_B (bx)}} \\

xP_{e,\text{APD}}_N (bx) &:= \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD}}_N (bx)}{\sqrt{2}} \right) \\
xP_{e,\text{APD}}_A (bx) &:= \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD}}_A (bx)}{\sqrt{2}} \right) \\
xP_{e,\text{APD}}_B (bx) &:= \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD}}_B (bx)}{\sqrt{2}} \right) \\
\end{align*}
\]

\[bx := 100, 200 .. 8000\]
B.3 Receiver optimisation (BJT/ 1st order LPF pre-detection filter)

B.3.1 Non-tuned receiver

(Non-tuned optimum Performance (PIN-BJT APD-BJT 100 Mbit/s)

Rx performance opt PIN-BJT input configurations, 1st order LPF pre-detection filter, opt as follows

- TIA 3-db bandwidth $r_t$/Filter 3-dB bandwidth $r_d$
- range cut-off
- minimum noise
- examine ISI
- highest SNR
- BER

\[ \text{opt } r_d = 0.7, \text{ opt } r_t = 0.5 \]

Bit-rate, pulse duration

\[ C_T := 1.5 \cdot 10^{-12} \text{ total C} \]
\[ B := 100 \cdot 10^6 \text{ Bit-rate} \]

pre-dec filter

\[ \omega_B (r_d) := 2 \cdot \pi \cdot r_d \]
\[ H_{but}(\omega, r_d) := \frac{1}{1 + 1j \cdot \frac{\omega}{2 \cdot \pi \cdot r_d}} \]

TIA(Non-tuned)

\[ \omega_c (r_t) := 2 \cdot \pi \cdot r_t \]
\[ A_v := 10 \]
\[ Rf_N(r_t) := \frac{A_v + 1}{2 \cdot \pi \cdot r_t \cdot C_T} \]
\[ Z_{TIA,N}(\omega, r_t) := \frac{1}{1 + 1j \cdot \frac{\omega}{2 \cdot \pi \cdot r_t}} \]
\[ Rf_N(0.5 \cdot B) = 2.334 \cdot 10^4 \]

Receiver freq-response

\[ Z_N(\omega, r_d, r_t) := H_{but}(\omega, r_d) \cdot Z_{TIA,N}(\omega, r_t) \]
\[ \omega := 1 \cdot 10^6, 10 \cdot 10^6 .. 1 \cdot 10^{11} \]
\[ r_d := 0.7 \cdot B, 0.75 \cdot B .. 2 \cdot B \]
\[ r_t := 0.5 \cdot B, 0.55 \cdot B .. 1 \cdot B \]

Noise equivalent bandwidth (NEB and I2)

\[ \text{NEB}_N(r_d, r_t) := \frac{1}{2 \cdot \pi} \left( \int_0^{10^{12}} \left( |Z_N(\omega, r_d, r_t)|^2 \right) d\omega \right) \]
Noise equivalent bandwidth \cite{13}

\[
I_{3,N}(rd,rt) := \frac{1}{2 \cdot \pi} \int_0^{10^{12}} \left( 1 + j \cdot \omega \cdot Rf_N(rt) \cdot CT \cdot Z_N(\omega,rd,rt) \right)^2 d\omega
\]

Total noise- PIN-BJT

\[q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100\]

\[I_{copt,N}(rd,rt) := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3,N}(rd,rt)}{NEB_N(rd,rt)}}\]

\[I_{n,B}(rd,rt) := \left( \frac{4 \cdot k \cdot T}{Rf_N(rt)} \cdot NEB_N(rd,rt) \right) \cdot \left( \frac{2 \cdot q \cdot I_{copt,N}(rd,rt)}{hfe} \cdot NEB_N(rd,rt) \right)^{\frac{1}{2}} \cdot \left( \frac{2 \cdot q}{I_{copt,N}(rd,rt)} \cdot I_{3,N}(rd,rt) \right)^{\frac{1}{2}} \cdot \left( \frac{I_{copt,N}(rd,rt)}{25 \cdot 10^{-3}} \right)\]

\[
I_{n,B}(0.7 B, 0.5 B) = 7.327 \cdot 10^{-17} \quad #
\]

Output pulse shape

\[ts := \frac{1}{B} \quad B = 1 \cdot 10^8\]
\[ v_{out}(t, rd, rt) = ts \cdot \frac{1}{ts} \int_{0}^{\frac{t}{ts} \cdot 10^3} \sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \text{Re} \left( Z_N(\omega, rd, rt) \cdot \exp \left( 1i \cdot \omega \cdot (t) \right) \right) d\omega \]
\[ x\text{v}_{out}(t, rd, rt) = ts^2 \cdot \frac{1}{ts} \int_{0}^{\frac{t}{ts} \cdot 10^3} \sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \text{Re} \left( 1i \cdot \omega \cdot Z_N(\omega, rd, rt) \cdot \exp \left( 1i \cdot \omega \cdot (t) \right) \right) d\omega \]

\[ t := -1 \cdot ts, -0.99 \cdot ts, 4 \cdot ts \]

\[ |SI| \]

\[ t_{B1} := 0.7 \cdot ts \]
\[ t_{pk1} := \text{root} \left( xv_{out}(t_{B1}, 0.7 \cdot B, 0.5 \cdot B), t_{B1} \right) \]
\[ \frac{t_{pk1}}{ts} = 0.526 \]
\[ v_{\text{max}, B1} := v_{out}(t_{pk1}, 0.7 \cdot B, 0.5 \cdot B) = 0.884 \]
\[ v_{\text{min}, B1} := v_{out}(t_{pk1} + 1 \cdot ts, 0.7 \cdot B, 0.5 \cdot B) = 0.106 \]
\[ v_{\text{min1}, B1} := v_{out}(t_{pk1} + 2 \cdot ts, 0.7 \cdot B, 0.5 \cdot B) = 0.005 \]
\[ v_{\text{min2}, B1} := v_{out}(t_{pk1} + 3 \cdot ts, 0.7 \cdot B, 0.5 \cdot B) = 2.451 \cdot 10^{-4} \]
\[ v_{\text{max}, B1} - v_{\text{min}, B1} = 0.777 \quad v_{\text{max}, B1} - v_{\text{min1}, B1} = 0.878 \quad v_{\text{max}, B1} - v_{\text{min2}, B1} = 0.883 \]

\[
\frac{\sqrt{10^{16} \cdot I_{n,B}(0.7 \cdot B, 0.5 \cdot B)}}{v_{\text{max}, B1} - v_{\text{min}, B1}} = 0.91 \quad \text{1st pulse} \quad \text{[worst ISI]} \]
\[
\frac{\sqrt{10^{16} \cdot I_{n,B}(0.7 \cdot B, 0.5 \cdot B)}}{v_{\text{max}, B1} - v_{\text{min1}, B1}} = 1.026 \quad \text{2nd pulse} \]
\[
\frac{\sqrt{10^{16} \cdot I_{n,B}(0.7 \cdot B, 0.5 \cdot B)}}{v_{\text{max}, B1} - v_{\text{min2}, B1}} = 1.032 \quad \text{3rd pulse} \]

Highest SNR @ rd=0.7 rt=0.5
\[ v_{\text{max}, B} := v_{\text{max}, B1} \quad v_{\text{min}, B} := v_{\text{min}, B1} \]

Eye-diagram check
\[ x_{111}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - 2 \cdot ts, rd, rt) + v_{out}(t - ts, rd, rt) \]
\[ x_{110}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - ts, rd, rt) \]
\[ x_{00}(t) := v_{out}(t-2ts, rd, rt) \]
\[ x_{01}(t) := v_{out}(t-ts, rd, rt) + v_{out}(t-2ts, rd, rt) \]
\[ x_{10}(t) := v_{out}(t, rd, rt) + v_{out}(t-2ts, rd, rt) \]

\[ x_{11}(t) := v_{out}(t, 0.7B, 0.5B) \]
\[ x_{100}(t) := v_{out}(t, 0.7B, 0.5B) \]
\[ x_{110}(t) := v_{out}(t, 0.7B, 0.5B) \]
\[ x_{001}(t) := v_{out}(t, 0.7B, 0.5B) \]
\[ x_{011}(t) := v_{out}(t, 0.7B, 0.5B) \]
\[ x_{101}(t) := v_{out}(t, 0.7B, 0.5B) \]

**Sens PINFET**

\[ \lambda := 650 \times 10^{-9} \]
\[ photon\_energy := \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda} \]
\[ nq := 1.6 \times 10^{-19} \]
\[ Q_N(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \frac{v_{max,B} - v_{min,B}}{2} \]
\[ I_{n,B}(0.7B, 0.5B) \]
\[ Pe_N(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right) \]
\[ Pc_N(b) := (\log(Pe_N(b)) + 9) \]

\[ b \equiv 3 \times 10^3 \]
\[ b_N := \text{root}(Pc_N(b), b) \]
\[ \text{minimum}_B := \text{min}(b_N) = 8.254 \times 10^3 \]

\[ \text{Sens}_N\_PIN := 10 \cdot \log \left( \frac{\text{minimum}_B \cdot photon\_energy \cdot 1}{ts} \right) = -38.986 \]
B.3.2 Tuned A receiver

Tuned A Optimum Performance (PIN-BJT 100 Mbit/s)

Rx performance opt PIN-BJT input configurations, 1st order LPF pre-detection filter. opt as follows

- TIA 3-db bandwidth rt/Filter 3-db bandwidth rd
- range cut-off
- minimum noise
- examine ISI
- highest SNR
- BER

\[
\text{opt rd} = 0.7, \text{opt rt} = 0.6
\]

Bit-rate, pulse duration

\[
C_T := 1.5 \cdot 10^{-12} \quad \text{total C}
\]

\[
B := 100 \cdot 10^6 \quad \text{Bit-rate}
\]

pre-dec filter

\[
\omega_B(\text{rd}) := 2 \cdot \pi \cdot \text{rd}
\]

\[
H_{\text{but}}(\omega, \text{rd}) := \frac{1}{1 + \frac{1}{2} \cdot \pi \cdot \text{rd}}
\]

TIA(tuned A)

\[
Av := 10
\]

\[
m := 1.8 \quad \text{time constant ratio of L/R and RC}
\]

\[
y := \sqrt{\left(\frac{-m^2}{2} + m + 1\right) + \left(\frac{-m^2}{2} + m + 1\right)^2 + m^2} = 1.825
\]

\[
Rf_A(\text{rt}) := y \cdot \frac{Av + 1}{2 \cdot \pi \cdot \text{rt} \cdot C_T}
\]

\[
Rf_A(0.6 \ B) = 3.551 \cdot 10^4
\]

\[
L(\text{rt}) := \frac{Rf_A(\text{rt})^2 \cdot C_T}{m}
\]

\[
L(0.6 \cdot B) = 0.001
\]

\[
L(f(\text{rt}) := \left(\frac{Rf_A(\text{rt})}{Av + 1}\right) \cdot \frac{(Av + 1) \cdot C_T}{m}
\]

\[
L(f(0.6 \cdot B) = 9.552 \cdot 10^{-5}
\]

\[
Z_{\text{TIA-A}}(\omega, \text{rt}) := \frac{1 + 1i \cdot \omega \cdot \frac{Lf(\text{rt})}{Rf_A(\text{rt})}}{\left(1 - \omega^2 \cdot Lf(\text{rt}) \cdot \frac{C_T}{1 + Av}\right) + Rf_A(\text{rt}) \cdot \omega \cdot \frac{C_T}{(1 + Av)} \cdot 1i}
\]

Receiver freq-response

\[
Z_A(\omega, \text{rd}, \text{rt}) := H_{\text{but}}(\omega, \text{rd}) \cdot Z_{\text{TIA-A}}(\omega, \text{rt})
\]

\[
\omega := 1 \cdot 10^6, 10 \cdot 10^6, \ldots 1 \cdot 10^{11}
\]

\[
\text{rd} := 0.7 \cdot R, 0.75 \cdot R, \ldots 2 \cdot R
\]
\( rt := 0.5 \cdot B, 0.55 \cdot B, 1 B \)

**Noise equivalent bandwidth (NEB and I2)**

\[
NEB_A(rd, rt) := \frac{1}{2 \pi} \cdot \int_0^{10^{-12}} \left( (\left| Z_A(\omega, rd, rt) \right|)^2 \right) d\omega
\]

\( NEB_A(0.7 B, 0.5 B) = 4.911 \cdot 10^7 \)

**Noise equivalent bandwidth (I2)**

\[
I_2_A(rd, rt) := \frac{1}{2 \pi} \cdot \int_0^{10^{-12}} \left( \left( \frac{1}{1 + i \omega \cdot \frac{L_f(rt)}{R_f(A(rt)}} \right)^2 \cdot Z_A(\omega, rd, rt) \right) d\omega
\]

**Noise equivalent bandwidth (I3)**

\[
I_3_A(rd, rt) := \frac{1}{2 \pi} \cdot \int_0^{10^{-12}} \left( \left( \left( 1 - \frac{\omega^2 \cdot L_f(rt) \cdot C_T}{R_f(A(rt))} \right) + \frac{R_f(A(rt)) \cdot \omega \cdot C_T \cdot i \cdot l_f}{R_f(A(rt)) + i \cdot \omega \cdot L_f(rt)} \right) \cdot Z_A(\omega, rd, rt) \right)^2 d\omega
\]

**Total noise - PIN-BJT**

\( q = 1.6 \cdot 10^{-19} \quad k = 1.38 \cdot 10^{-23} \quad T = 298 \quad hfe := 100 \)

\[
I_{\text{opt}}(rd, rt) := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_3_A(rd, rt)}{NEB_A(rd, rt)}}
\]

\[
I_{n_B}(rd, rt) := \left( \frac{4 \cdot k \cdot T}{R_f(A(rt))} \cdot I_2_A(rd, rt) \right) \cdot \left( 2 \cdot q \cdot \frac{I_{\text{opt}}(rd, rt)}{hfe} \right) \cdot NEB_A(rd, rt) + 2 \cdot q \cdot \left( \frac{I_{\text{opt}}(rd, rt)}{25 \cdot 10^{-3}} \right)^2 \cdot \frac{I_{\text{opt}}(rd, rt)}{I_3_A(rd, rt)}
\]

**Graph**

\( \frac{I_{n_B}(rd, rt)}{10^{-16}} \)
Output pulse shape

\[ I_{n_B}(0.7 \, B, 0.5 \, B) = 4.503 \cdot 10^{-17} \]

\[ ts := \frac{1}{B} \]

\[ v_{\text{out}}(t, rd, rt) := \frac{ts}{\pi} \cdot \left[ \sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \Re \{ Z_A(\omega, rd, rt) \cdot \exp \{ i \cdot \omega \cdot (t) \} \} \right] d\omega \]

\[ x_{\text{out}}(t, rd, rt) := \frac{ts^2}{\pi} \cdot \left[ \sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \Re \{ i \cdot \omega \cdot Z_A(\omega, rd, rt) \cdot \exp \{ i \cdot \omega \cdot (t) \} \} \right] d\omega \]

\[ t := -1 \, ts, -0.99 \, ts, -0.4 \, ts \]

\[ |S| \]

\[ t_{B1} := 0.7 \cdot ts \]

\[ t_{pk1} := \text{root} \left( x_{\text{out}}(t_{B1}, 0.7 \, B, 0.6 \, B), t_{B1} \right) \]

\[ \frac{t_{pk1}}{ts} = 0.547 \]

\[ v_{\text{max}, B1} := v_{\text{out}}(t_{pk1}, 0.7 \, B, 0.6 \, B) = 0.992 \]

\[ v_{\text{min}, B1} := v_{\text{out}}(t_{pk1} + 1 \cdot ts, 0.7 \, B, 0.6 \, B) = 0.025 \]

\[ v_{\text{min}, B1} := v_{\text{out}}(t_{pk1} + 2 \cdot ts, 0.7 \, B, 0.6 \, B) = -0.031 \]

\[ v_{\text{max}, B1} := v_{\text{out}}(t_{pk1} + 3 \cdot ts, 0.7 \, B, 0.6 \, B) = 0.004 \]

\[ v_{\text{max}, B1} - v_{\text{min}, B1} = 0.967 \quad v_{\text{max}, B1} - v_{\text{min}, B1} = 1.023 \quad v_{\text{max}, B1} - v_{\text{min}, B1} = 0.988 \]

\[ \frac{v_{\text{max}, B1} - v_{\text{min}, B1}}{\sqrt{10} \cdot I_{n_B}(0.7 \, B, 0.6 \, B)} = 1.27 \quad \text{1st pulse} \quad |S| \]

\[ \frac{v_{\text{max}, B1} - v_{\text{min}, B1}}{\sqrt{10} \cdot I_{n_B}(0.7 \, B, 0.6 \, B)} = 1.34 \quad \text{2nd pulse} \]

\[ \frac{v_{\text{max}, B1} - v_{\text{min}, B1}}{\sqrt{10} \cdot I_{n_B}(0.7 \, B, 0.6 \, B)} = 1.295 \quad \text{3rd pulse} \]

Highest SNR @ rd=0.7 rt=0.6

\[ v_{\text{max}, B} := v_{\text{max}, B1} \quad v_{\text{min}, B} := v_{\text{min}, B1} \]

Eye-diagram check

\[ x_{111}(t, rd, rt) := v_{\text{out}}(t, rd, rt) + v_{\text{out}}(t - 2 \cdot ts, rd, rt) + v_{\text{out}}(t - ts, rd, rt) \]

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\[ x_{110}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - ts, rd, rt) \]
\[ x_{100}(t, rd, rt) := v_{out}(t, rd, rt) \]
\[ x_{001}(t, rd, rt) := v_{out}(t - 2 ts, rd, rt) \]
\[ x_{010}(t, rd, rt) := v_{out}(t - ts, rd, rt) \]
\[ x_{011}(t, rd, rt) := v_{out}(t - ts, rd, rt) + v_{out}(t - 2 ts, rd, rt) \]
\[ x_{101}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - 2 ts, rd, rt) \]

\[ x_{111}(t, 0.7 B, 0.6 B) \]
\[ x_{100}(t, 0.7 B, 0.6 B) \]
\[ x_{110}(t, 0.7 B, 0.6 B) \]
\[ x_{001}(t, 0.7 B, 0.6 B) \]
\[ x_{010}(t, 0.7 B, 0.6 B) \]
\[ x_{011}(t, 0.7 B, 0.6 B) \]
\[ x_{101}(t, 0.7 B, 0.6 B) \]

Sens PINFET

\[ \lambda := 650 \cdot 10^{-9} \]
\[ \text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]
\[ nq := 1.6 \cdot 10^{-19} \]
\[ Q_N(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{max,B} - v_{min,B}}{2} \right) \]
\[ \sqrt{I_{n,B}(0.7 B, 0.6 B)} \]
\[ Pe_N(b) := \frac{1}{2} \cdot \text{erfc}\left( \frac{Q_N(b)}{\sqrt{2}} \right) \]
\[ Pc_N(b) := (\log(Pe_N(b)) + 9) \]
\[ b \equiv 3 \cdot 10^3 \]
\[ b_N := \text{root}(Pc_N(b), b) \]
\[ \text{minimum}_B := \text{min}(b_N, b) = 5.919 \cdot 10^3 \]

\[ \text{Sens}_N\_PIN := 10 \cdot \log\left( \frac{\text{minimum}_B \cdot \text{photon\_energy} \cdot \frac{1}{ts}}{2} \right) = -40.43 \]
B.3.3 Tuned B receiver $a=0.2$

Tuned B Optimum Performance (PIN-BJT 100 Mbit/s)

Rx performance opt PIN-BJT input configurations, 1st order LPF pre-detection filter, opt as follows

- TIA 3-db bandwidth $rt$/Filter 3-dB bandwidth $rd$
- range cut-off
- minimum noise
- examine ISI
- highest SNR
- BER

**Opt $rd = 0.7$, opt $rt = 0.5**

**Bit-rate, pulse duration**

$C_T := 1.5 \times 10^{-12}$ total C

$B := 100 \times 10^6$ Bit-rate

**pre-dec filter**

$\omega_B (rd) := 2 \cdot \pi \cdot rd$

$H_{but}(\omega , rd) := \frac{1}{1 + 1j \cdot \frac{\omega}{2 \cdot \pi \cdot rd}}$

**TIA(tuned B)**

Feedback value for (R) Tuned B

$\Delta_L := 1.8$  
$\Delta_R := 1.87$

$Rf_B (rt) := \Delta_R \cdot \frac{Av + 1}{2 \cdot \pi \cdot rt \cdot C_T}$

$\alpha := 0.2$

$C1 := (1 - \alpha) \cdot (C_T) = 1.2 \times 10^{-12}$

$C2 := \alpha \cdot (C_T) = 3 \times 10^{-13}$

$Z_{TIA,B}(\omega , rt) := \frac{1}{\left[ (1 - \omega^2 \cdot Lc (rt) \cdot C1) + \frac{Rf_B (rt)}{(1 + Av)} \cdot (\omega \cdot 1i) \right]^\frac{1}{2} \cdot (C1 + C2 - (\omega)^2 \cdot Lc (rt) \cdot C1 \cdot C2)}$

**Receiver freq-response**

$Z_B (\omega , rd , rt) := H_{but}(\omega , rd) \cdot Z_{TIA,B}(\omega , rt)$

$\omega := 1 \times 10^6, 10 \times 10^6, 1 \times 10^{11}$

$rd := 0.7 \cdot B, 0.75 \cdot B, 0.2 \cdot B$

$rt := 0.5 \cdot B, 0.55 \cdot B, 1 \cdot B$
Noise equivalent bandwidth (NEB and I2)

\[
NEB_B(rd, rt) := \frac{1}{2 \pi} \left( \int_0^{10^{12}} \left( \left[ \frac{(1 - \omega^2 \cdot Lc(rt) \cdot C1)}{Z_B(\omega, rd, rt)} \right] \right)^2 d\omega \right)
\]

\[
NEB_B(0.7 B, 0.5 B) = 4.282 \cdot 10^7
\]

Noise equivalent bandwidth (I2)

\[
I_{2B}(rd, rt) := \frac{1}{2 \pi} \left( \int_0^{10^{12}} \left( \left[ \frac{(1 - \omega^2 \cdot Lc(rt) \cdot C1)}{Z_B(\omega, rd, rt)} \right] \right)^2 d\omega \right)
\]

Noise equivalent bandwidth (I3)

\[
I_{3B}(rd, rt) := \frac{1}{2 \pi} \left( \int_0^{10^{12}} \left( \frac{\left( \left(1 - \omega^2 \cdot Lc(rt) \cdot C1\right) \div \left( \frac{Rf_B(rt) \cdot (\omega) \div \left( (C1+C2-(\omega)^2 \cdot Lc(rt) \cdot C1 \cdot C2) \div 1i \right) \right) \right)^2}{Z_B(\omega, rd, rt)} \right) d\omega \right)
\]

Total noise - PIN-BJT

\[ q = 1.6 \cdot 10^{-19} \quad k = 1.38 \cdot 10^{-23} \quad T = 298 \quad hfe = 100 \]

\[
I_{opt}(rd, rt) := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \frac{I_{3B}(rd, rt)}{NEB_B(rd, rt)}
\]

\[
I_{nB}(rd, rt) := \left( \frac{4 \cdot k \cdot T}{Rf_B(rt)} \cdot I_{2B}(rd, rt) \right) + \left( 2 \cdot q \cdot \frac{I_{opt}(rd, rt)}{hfe} \right) \cdot I_{2B}(rd, rt)
\]

\[
+ 2 \cdot q \cdot \frac{I_{opt}(rd, rt)}{ \left( \frac{I_{opt}(rd, rt)}{25 \cdot 10^{-3}} \right)^2} \cdot I_{3B}(rd, rt)
\]
\[ I_{n,B}(0.7\,B, 0.5\,B) = 4.269 \cdot 10^{-17} \]

**Output pulse shape**

\[ t_s = \frac{1}{B} \]

\[
vout(t, rd, rt) := \frac{t_s}{\pi} \cdot \left[ \sin\left(\frac{\omega \cdot t_s}{2}\right) \cdot \text{Re}\left(Z_B(\omega, rd, rt) \cdot \exp(1i \cdot \omega \cdot t)\right) \right] d\omega
\]

\[
xvout(t, rd, rt) := \frac{t_s^2}{\pi} \cdot \left[ \sin\left(\frac{\omega \cdot t_s}{2}\right) \cdot \text{Re}\left(Z_B(\omega, rd, rt) \cdot \exp(1i \cdot \omega \cdot t)\right) \right] d\omega
\]

\[ t := -1\,t_s, -0.99\,t_s..4\,t_s \]

**ISI**

\[ t_{B1} := 0.7 \cdot t_s \]

\[ t_{pk1} := \text{root}(xvout(t_{B1}, 0.5\,B, 0.5\,B), t_{B1}) \]

\[ \frac{t_{pk1}}{t_s} = 0.874 \]

\[ v_{max_{B1}} := vout(t_{pk1}, 0.5\,B, 0.5\,B) = 0.825 \]

\[ v_{min_{B1}} := vout(t_{pk1} + 1\,t_s, 0.5\,B, 0.5\,B) = 0.118 \]

\[ v_{min_{1,B1}} := vout(t_{pk1} + 2\,t_s, 0.5\,B, 0.5\,B) = 3.259 \cdot 10^{-6} \]

\[ v_{min_{2,B1}} := vout(t_{pk1} + 3\,t_s, 0.5\,B, 0.5\,B) = 6.872 \cdot 10^{-4} \]

\[ v_{max_{B1}} - v_{min_{B1}} = 0.707 \quad v_{max_{B1}} - v_{min_{1,B1}} = 0.825 \quad v_{max_{B1}} - v_{min_{2,B1}} = 0.824 \]

\[
\begin{align*}
\sqrt{10^{16} \cdot I_{n,B}(0.5\,B, 0.5\,B)} &= 1.27 & \text{1st pulse} & \text{worst ISI} \\
\sqrt{10^{16} \cdot I_{n,B}(0.5\,B, 0.5\,B)} &= 1.481 & \text{2nd pulse} \\
\sqrt{10^{16} \cdot I_{n,B}(0.5\,B, 0.5\,B)} &= 1.48 & \text{3rd pulse} \\
\end{align*}
\]

**Highest SNR @ rd=0.5 rt=0.5**

\[ v_{max_{B}} := v_{max_{B1}} \quad v_{min_{B}} := v_{min_{B1}} \]

**Eye-diagram check**

\[ x_{111}(t, rd, rt) := vout(t, rd, rt) + vout(t-2\,t_s, rd, rt) + vout(t-t_s, rd, rt) \]
\[ x_{110}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - ts, rd, rt) \]
\[ x_{00}(t, rd, rt) := v_{out}(t, rd, rt) \]
\[ x_{001}(t, rd, rt) := v_{out}(t - 2ts, rd, rt) \]
\[ x_{010}(t, rd, rt) := v_{out}(t - ts, rd, rt) \]
\[ x_{011}(t, rd, rt) := v_{out}(t - ts, rd, rt) + v_{out}(t - 2ts, rd, rt) \]
\[ x_{101}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - 2ts, rd, rt) \]

\[ x_{111}(t, 0.5B, 0.5B) \]
\[ x_{100}(t, 0.5B, 0.5B) \]
\[ x_{110}(t, 0.5B, 0.5B) \]
\[ x_{001}(t, 0.5B, 0.5B) \]
\[ x_{010}(t, 0.5B, 0.5B) \]
\[ x_{011}(t, 0.5B, 0.5B) \]
\[ x_{101}(t, 0.5B, 0.5B) \]

**Sens PINFET**

\[ \lambda := 650 \cdot 10^{-9} \]
\[ photon\_energy := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]
\[ nq := 1.6 \cdot 10^{-19} \]
\[ Q_N(b) := \frac{1}{ts} \cdot nq \cdot \frac{v_{max,B} - v_{min,B}}{2} \]
\[ I_{n,B}(0.5B, 0.5B) \]
\[ Pe_N(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right) \]
\[ Pe_N(b) := (\log(Pe_N(b)) + 9) \]
\[ b \equiv 3 \cdot 10^3 \]
\[ b_N \equiv \text{root}(Pe_N(b), b) \]
\[ \text{minimum}_B := \text{min}(b_N, b) = 5.907 \cdot 10^3 \]
\[ Sens\_N\_PIN := 10 \cdot \log \left( \frac{\text{minimum}_B \cdot photon\_energy \cdot \frac{1}{ts}}{2} \right) = -40.44 \]

#
B.3.4 Tuned B receiver $a=0.3$

**Tuned B Optimum Performance (PIN-BJT 100 Mbit/s)**

Rx performance opt PIN-BJT input configurations, 1st order LPF pre-detection filter, opt as follows:

- TIA 3-db bandwidth $rt$/Filter 3-dB bandwidth $rd$
- range cut-off
- minimum noise
- examine ISI
- highest SNR
- BER

opt $rd = 0.5$, opt $rt = 0.5$

**Bit-rate, pulse duration**

$C_T := 1.5 \cdot 10^{-12}$ total C

$B := 100 \cdot 10^6$ Bit-rate

**pre-dec filter**

$\omega_B (rd) := 2 \cdot \pi \cdot rd$

$H_{but} (\omega, rd) := \frac{1}{1 + 1j \cdot \frac{\omega}{2 \cdot \pi \cdot rd}}$

**TIA(tuned B)**

Feedback value for (R) Tuned B

$\Delta_L := 2.4$  $\Delta_R := 2.52$

$Av := 10$

$Rf_B (rt) := \Delta_R \cdot \frac{Av + 1}{2 \cdot \pi \cdot rt \cdot C_T}$

$\alpha := 0.3$

$C1 := (1 - \alpha) \cdot (C_T) = 1.05 \cdot 10^{-12}$

$C2 := \alpha \cdot (C_T) = 4.5 \cdot 10^{-13}$

$Z_{TIA,B} (\omega, rt) := \frac{1}{((1 - \omega^2 \cdot Lc (rt) \cdot C1) + \frac{Rf_B (rt)}{1 + Av} \cdot (\omega + 1i)^{-1}) \cdot (C1 + C2 - (\omega)^2 \cdot Lc (rt) \cdot C1 \cdot C2)}$

**Receiver freq-response**

$Z_B (\omega, rd, rt) := H_{but} (\omega, rd) \cdot Z_{TIA,B} (\omega, rt)$

$\omega := 1 \cdot 10^6, 10 \cdot 10^6 .. 1 \cdot 10^{11}$

$rd := 0.7 \cdot B, 0.75 \cdot B .. 2 \cdot B$

$rt := 0.5 \cdot B, 0.55 \cdot B .. 1 \cdot B$
Noise equivalent bandwidth (NEB and I2)

$$NEB_B(r, r') := \frac{1}{2 \pi} \left( \int_0^{10^{12}} \left( \frac{\left| Z_B(\omega, r, r') \right|^2}{\int_0^{10^{12}} \left( \left| \frac{1-\omega^2 \cdot Lc(r) \cdot C1}{Z_B(\omega, r, r')} \right|^2 d\omega \right)} d\omega \right)^2$$

$$NEB_B(0.7 B, 0.5 B) = 3.689 \cdot 10^7 \#$$

Noise equivalent bandwidth (I2)

$$I_{2B}(r, r') := \frac{1}{2 \pi} \left( \int_0^{10^{12}} \left( \frac{\left| \frac{1-\omega^2 \cdot Lc(r) \cdot C1}{Z_B(\omega, r, r')} \right|^2 d\omega \right)^2 \right)$$

Noise equivalent bandwidth (I3)

$$I_{3B}(r, r') := \frac{1}{2 \pi} \left( \int_0^{10^{12}} \left( \frac{\left| \frac{(1-\omega^2 \cdot Lc(r) \cdot C1)}{Rf_B(r)} + \frac{\omega}{Rf_B(r)} \cdot \frac{1}{C1+C2-(\omega)^2 Lc(r) \cdot C1 \cdot C2} \right|^2 d\omega \right)^2 \right)$$

Total noise- PIN-BJT

$$q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100$$

$$I_{optB}(r, r') := 25 \cdot 10^{-3} \cdot hfe \cdot \sqrt{I_{B}(r, r') \cdot NEB_B(r, r')}$$

$$I_{nB}(r, r') := \left\{ \begin{array}{l} \frac{4 \cdot k \cdot T}{Rf_B(r')} \cdot I_{2B}(r, r') + \left( 2 \cdot q \cdot \frac{I_{optB}(r, r')}{hfe} \right) \cdot I_{2B}(r, r') \\ + 2 \cdot q \cdot \frac{I_{optB}(r, r')}{hfe} \cdot I_{3B}(r, r') \end{array} \right\} \left( \begin{array}{l} \frac{I_{optB}(r, r')}{25 \cdot 10^{-3}} \end{array} \right)^2$$
\[ I_{n, B}(0.7 \, B, 0.5 \, B) = 2.796 \cdot 10^{-17} \]

Output pulse shape

\[ t_s = \frac{1}{B} \]

\[ v_{out}(t, r_d, r_t) := \frac{t_s}{\pi} \cdot \left[ \frac{\sin \left( \frac{\omega \cdot t_s}{2} \right)}{\left( \frac{\omega \cdot t_s}{2} \right)^2} \cdot \text{Re} \left[ Z_B(\omega, r_d, r_t) \cdot \exp \left( i \cdot \omega \cdot t \right) \right] \right] d\omega \]

\[ x_{out}(t, r_d, r_t) := \frac{t_s^2}{\pi} \cdot \left[ \frac{\sin \left( \frac{\omega \cdot t_s}{2} \right)}{\left( \frac{\omega \cdot t_s}{2} \right)^2} \right] d\omega \]

\[ t := -1 \cdot t_s, -0.99 \cdot t_s, \ldots, 4 \cdot t_s \]

ISI

\[ t_{B1} := 0.9 \cdot t_s \]

\[ t_{pk1} := \text{root} \left[ x_{out} \left( t_{B1}, 0.5 \, B, 0.5 \, B \right), t_{B1} \right] \]

\[ t_{pk1} = 0.968 \]

\[ v_{max_{B1}} := v_{out} \left( t_{pk1}, 0.5 \, B, 0.5 \, B \right) = 0.766 \]

\[ v_{min_{B1}} := v_{out} \left( t_{pk1} + 1 \cdot t_s, 0.5 \, B, 0.5 \, B \right) = 0.113 \]

\[ v_{min_{B1}} := v_{out} \left( t_{pk1} + 2 \cdot t_s, 0.5 \, B, 0.5 \, B \right) = 0.052 \]

\[ v_{min_{B1}} := v_{out} \left( t_{pk1} + 3 \cdot t_s, 0.5 \, B, 0.5 \, B \right) = 6.434 \cdot 10^{-4} \]

\[ v_{max_{B1}} - v_{min_{B1}} = 0.653 \quad v_{max_{B1}} - v_{min_{B1}} = 0.713 \quad v_{max_{B1}} - v_{min_{B1}} = 0.765 \]

\[ \sqrt{10^{16} \cdot I_{n, B}(0.5 \, B, 0.5 \, B)} = 1.43 \quad \text{1st pulse \ worst ISI} \]

\[ \sqrt{10^{16} \cdot I_{n, B}(0.5 \, B, 0.5 \, B)} = 1.561 \quad \text{2nd pulse} \]

\[ \sqrt{10^{16} \cdot I_{n, B}(0.5 \, B, 0.5 \, B)} = 1.674 \quad \text{3rd pulse} \]

\[ v_{max_{B1}} - 0.183 = 0.583 \]

\[ v_{max_{B1}} := v_{max_{B1}} \quad v_{min_{B1}} := v_{min_{B1}} \]

Eye-diagram check

\[ x_{111}(t, r_d, r_t) := v_{out}(t, r_d, r_t) + v_{out}(t - 2 \cdot t_s, r_d, r_t) + v_{out}(t - t_s, r_d, r_t) \]

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\[ x_{110}(t, rd, rt) := vout(t, rd, rt) + vout(t - ts, rd, rt) \]
\[ x_{100}(t, rd, rt) := vout(t, rd, rt) \]
\[ x_{001}(t, rd, rt) := vout(t - 2 \cdot ts, rd, rt) \]
\[ x_{010}(t, rd, rt) := vout(t - ts, rd, rt) \]
\[ x_{011}(t, rd, rt) := vout(t - ts, rd, rt) + vout(t - 2 \cdot ts, rd, rt) \]
\[ x_{101}(t, rd, rt) := vout(t, rd, rt) + vout(t - 2 \cdot ts, rd, rt) \]

**Sens PINFET**

\[ \lambda := 650 \cdot 10^{-9} \]

\[ photon\_energy := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ nq := 1.6 \cdot 10^{-10} \]

\[ Q_N(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{max,B} - v_{min,B}}{2} \right) \sqrt{I_{n,B}(0.5B, 0.5B)} \]

\[ Pe_N(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right) \]

\[ Pc_N(b) := (\log(Pe_N(b)) + 9) \]

\[ b \equiv 3 \cdot 10^3 \]

\[ b_N := \text{root}(Pc_N(b), b) \]

\[ minimum_B := \min(b_N) = 5.247 \cdot 10^3 \]

\[ Sens\_N\_PIN := 10 \cdot \log \left( \frac{minimum_B \cdot photon\_energy \cdot 1}{ts} \right) = -40.954 \]

#
B.3.5 Tuned B receiver a=0.4

Tuned B Optimum Performance (PIN-BJT 100 Mbit/s)

Rx performance opt PIN-BJT input configurations, 1st order LPF pre-detection filter, opt as follows

- TIA 3-db bandwidth rt/Filter 3-dB bandwidth rd
- range cut-off
- minimum noise
- examine ISI
- highest SNR
- BER

opt rd = 0.5, opt rt = 0.5

Bit-rate, pulse duration
\[ C_T := 1.5 \times 10^{-12} \quad \text{total C} \]
\[ B := 100 \times 10^6 \quad \text{Bit-rate} \]

pre-dec filter
\[ \omega_B (rd) := 2 \cdot \pi \cdot rd \]
\[ H_{\text{but}} (\omega, rd) := \frac{1}{1 + 1j \cdot \frac{\omega}{2 \cdot \pi \cdot rd}} \]

TIA(tuned B)
Feedback value for (R) Tuned B
\[ \Delta_L := 1.9 \quad \Delta_R := 2.75 \]

\[ Av := 10 \]

\[ Rf_B (rt) := \Delta_R \cdot \frac{Av + 1}{2 \cdot \pi \cdot rt \cdot C_T} \]
\[ \alpha := 0.4 \]

\[ Lc(rt) := \left( \frac{(Rf_B(rt))^2}{1 + Av} \right) \cdot C_T \]

\[ C1 := (1 - \alpha) \cdot (C_T) = 9 \cdot 10^{-13} \]
\[ C2 := \alpha \cdot (C_T) = 6 \cdot 10^{-13} \]

\[ Z_{\text{TIA,B}}(\omega, rt) := \frac{1}{\left( \left( (1 - \omega^2 \cdot Lc(rt) \cdot C1) + \frac{Rf_B(rt)}{1 + Av} \cdot (\omega \cdot 1i)^{\phi} \right) \cdot (C1 + C2 - (\omega)^2 \cdot Lc(rt) \cdot C1 \cdot C2) \right)} \]

Receiver freq-response
\[ Z_B(\omega, rd, rt) := H_{\text{but}} (\omega, rd) \cdot Z_{\text{TIA,B}}(\omega, rt) \]
\[ \omega := 1 \cdot 10^6, 10 \cdot 10^6 .. 1 \cdot 10^{11} \]
\[ rd := 0.7 \cdot B, 0.75 \cdot B .. 2 \cdot B \]
\[ rt := 0.5 \cdot B, 0.55 \cdot B .. 1 \cdot B \]
Noise equivalent bandwidth (NEB and I2)

\[
NEB_B(rd, rt) := \frac{1}{2\pi} \cdot \left( \int_0^{10^3} \left( |Z_B(\omega, rd, rt)|^2 \right) d\omega \right)
\]

\[NEB_B(0.7 B, 0.5 B) = 4.038 \cdot 10^7 \quad \#\]

Noise equivalent bandwidth (I2)

\[
I_{2,B}(rd, rt) := \frac{1}{2\pi} \cdot \left( \int_0^{10^3} \left( \left| \left( 1 - \omega^2 \cdot L_c(rt) \cdot C1 \right) \cdot Z_B(\omega, rd, rt) \right|^2 \right) d\omega \right)
\]

Noise equivalent bandwidth (I3)

\[
I_{3,B}(rd, rt) := \frac{1}{2\pi} \cdot \left( \int_0^{10^3} \left( \left| \left( 1 - \omega^2 \cdot L_c(rt) \cdot C1 \right) \cdot Z_B(\omega, rd, rt) \right|^2 \right) d\omega \right)
\]

Total noise- PIN-BJT

\[q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100\]

\[
I_{\text{opt},B}(rd, rt) := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3,B}(rd, rt)}{NEB_B(rd, rt)}}
\]

\[
I_{n,B}(rd, rt) := \begin{cases} 
4 \cdot k \cdot T \cdot I_{2,B}(rd, rt) + 2 \cdot q \cdot \frac{I_{\text{opt},B}(rd, rt)}{hfe} \cdot I_{2,B}(rd, rt) \\
+ 2 \cdot q \cdot \frac{I_{\text{opt},B}(rd, rt)}{25 \cdot 10^{-3}} \cdot I_{3,B}(rd, rt)
\end{cases}
\]

\[
\frac{I_{n,B}(rd, rt)}{10^{-16}}
\]

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\[ I_{n,B}(0.7 B, 0.5 B) = 2.388 \cdot 10^{-17} \]  

**Output pulse shape**

\[
t_s := \frac{1}{B}
\]

\[
v_{out}(t, rd, rt) := \frac{t_s}{\pi} \cdot \frac{1}{t_s} \cdot 10^5 \left[ \frac{\sin \left( \omega \cdot \frac{t_s}{2} \right)}{\omega \cdot \frac{t_s}{2}} \cdot \text{Re} \left( Z_B(\omega, rd, rt) \cdot \exp \left( i \cdot \omega \cdot t \right) \right) \right] d\omega
\]

\[
x_{vout}(t, rd, rt) := \frac{t_s^2}{\pi} \cdot \frac{1}{t_s} \cdot 10^5 \left[ \frac{\sin \left( \omega \cdot \frac{t_s}{2} \right)}{\omega \cdot \frac{t_s}{2}} \cdot \text{Re} \left( i \cdot \omega \cdot Z_B(\omega, rd, rt) \cdot \exp \left( i \cdot \omega \cdot t \right) \right) \right] d\omega
\]

\[
t := -1 \cdot t_s, -0.99 \cdot t_s, 0.4 \cdot t_s
\]

**ISI**

\[ t_{B1} := 1 \cdot t_s \]

\[
t_{pk1} := \text{root} \left( x_{vout}(t_{B1}, 0.5 B, 0.5 B), t_{B1} \right)
\]

\[
t_{pk1} = 1.129
\]

\[
v_{max_{B1}} := v_{out}(t_{pk1}, 0.5 B, 0.5 B) = 0.787
\]

\[
v_{min_{B1}} := v_{out}(t_{pk1} + 1 \cdot t_s, 0.5 B, 0.5 B) = 0.051
\]

\[
v_{min_{1,B1}} := v_{out}(t_{pk1} + 2 \cdot t_s, 0.5 B, 0.5 B) = 0.04
\]

\[
v_{min_{2,B1}} := v_{out}(t_{pk1} + 3 \cdot t_s, 0.5 B, 0.5 B) = 0.021
\]

\[
v_{max_{B1}} - v_{min_{B1}} = 0.736 \quad v_{max_{B1}} - v_{min_{1,B1}} = 0.747 \quad v_{max_{B1}} - v_{min_{2,B1}} = 0.766
\]

\[
\frac{v_{max_{B1}} - v_{min_{B1}}}{\sqrt{10^{16} \cdot I_{n,B}(0.5 B, 0.5 B)}} = 1.72 \quad \text{(1st pulse, worst ISI)}
\]

\[
\frac{v_{max_{B1}} - v_{min_{1,B1}}}{\sqrt{10^{16} \cdot I_{n,B}(0.5 B, 0.5 B)}} = 1.744 \quad \text{(2nd pulse)}
\]

\[
\frac{v_{max_{B1}} - v_{min_{2,B1}}}{\sqrt{10^{16} \cdot I_{n,B}(0.5 B, 0.5 B)}} = 1.79 \quad \text{(3rd pulse)}
\]

**Highest SNR @ rd=0.7 rt=0.6**

\[
v_{max_B} := v_{max_{B1}} \quad v_{min_B} := v_{min_{B1}}
\]

**Eye-diagram check**

\[ x_{111}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - 2 \cdot t_s, rd, rt) + v_{out}(t - t_s, rd, rt) \]

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\[ x_{110}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - ts, rd, rt) \]
\[ x_{100}(t, rd, rt) := v_{out}(t, rd, rt) \]
\[ x_{001}(t, rd, rt) := v_{out}(t - 2 ts, rd, rt) \]
\[ x_{010}(t, rd, rt) := v_{out}(t - ts, rd, rt) \]
\[ x_{011}(t, rd, rt) := v_{out}(t - ts, rd, rt) + v_{out}(t - 2 ts, rd, rt) \]
\[ x_{101}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - 2 ts, rd, rt) \]

\[ x_{111}(t, 0.5 B, 0.5 B) \]
\[ x_{100}(t, 0.5 B, 0.5 B) \]
\[ x_{110}(t, 0.5 B, 0.5 B) \]
\[ x_{001}(t, 0.5 B, 0.5 B) \]
\[ x_{010}(t, 0.5 B, 0.5 B) \]
\[ x_{011}(t, 0.5 B, 0.5 B) \]
\[ x_{101}(t, 0.5 B, 0.5 B) \]

**Sens PINFET**

\[ \lambda := 650 \times 10^{-9} \]

\[ photon\_energy := \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda} \]

\[ nq := 1.6 \times 10^{-19} \]

\[ Q_N(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{max,B} - v_{min,B}}{2} \right) \]

\[ Q_N(b) := \frac{1}{ts} \cdot nq \cdot \left( \frac{v_{max,B} - v_{min,B}}{2} \right) \]

\[ Pe_N(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right) \]

\[ Pe_N(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right) \]

\[ b \equiv 3 \cdot 10^3 \]

\[ b_N := \text{root} (Pe_N(b), b) \]

\[ \text{minimum}_B := \text{min} (b_N) = 4.298 \times 10^3 \]

\[ \text{Sens}_N\_\text{PIN} := 10 \cdot \log \left( \frac{\text{maximum}_B \cdot photon\_energy \cdot 1}{ts} \right) = -41.82 \]
B.3.6 Performance comparison PINBJT

**Performance comparison (PIN-BJT 1st order LPF 100 Mbit/s)**

Rx performance with PIN-BJT input configuration, 1st order LPF pre-detection filter. Calculations are as follow:

- Frequency response (opt rd, rt for each receiver)
- Noise integrals
- Total noise
- Pulse shaping, peak voltage, ISI
- Error bit rate, minimum number of photons, receiver sensitivity

**Bit-rate, pulse duration**

\[ C_T := 1.5 \times 10^{-12} \quad \text{total C} \]
\[ B := 100 \times 10^6 \quad \text{Bit-rate} \]

**pre-dec filter**

\[ rd := 0.7 \cdot B \]
\[ \omega_B := 2 \cdot \pi \cdot rd \]
\[ H_{but, N, A}(\omega) := \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_B}} \quad H_{but, B}(\omega) := \frac{1}{1 + 1j \cdot \frac{\omega}{2 \cdot \pi \cdot 0.5 \cdot B}} \]

**TIA (Non-tuned)**

\[ f_c := 0.5 \cdot B \]
\[ Av := 10 \]
\[ \omega_c := 2 \cdot \pi \cdot f_c \]
\[ Rf_{N} := \frac{Av + 1}{2 \cdot \pi \cdot f_c \cdot C_T} = 2.334 \times 10^4 \]
\[ Z_{TIA, N}(\omega) := \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_c}} \]

**TIA (Tuned-A)**

\[ m := 1.8 \quad \text{time constant ratio of L/R and RC} \]
\[ y := \sqrt{\left(-\frac{m^2}{2} + m + 1\right) + \left(-\frac{m^2}{2} + m + 1\right)^2 + m^2} = 1.825 \]
\[ d := \frac{1}{y} \cdot B \]
\[ f_c := 0.6 \cdot d = 3.287 \times 10^7 \]
\[ RA := \frac{1}{(2 \cdot \pi \cdot C_T \cdot f_c)} = 3.228 \times 10^3 \]
\[ L := RA^2 \cdot \frac{C_T}{m} = 8.684 \times 10^{-6} \]
\[ ts := \frac{1}{v \cdot d} \quad B := \frac{1}{ts} = 1 \cdot 10^8 \]

\[ f_c := 0.6 \cdot B = 6 \cdot 10^7 \]

\[ Rf_A := RA \cdot (1 + Av) = 3.551 \cdot 10^4 \]

\[ Lf := L \cdot (1 + Av) = 9.552 \cdot 10^{-5} \]

\[ Z_{TIA, A}(\omega) := \frac{1 + 1i \cdot \omega \cdot \frac{Lf}{Rf_A}}{\left( \left( 1 - \omega^2 \cdot Lf \cdot \frac{C_T}{1 + Av} \right) + Rf_A \cdot \omega \cdot \frac{C_T}{(1 + Av) \cdot 1i} \right)} \]

**TIA(Tuned-B)**

Feedback value for (R) Tuned B

\[ \Delta_L := 1.9 \quad \Delta_R := 2.75 \]

\[ Rf_B := \Delta_R \cdot Rf_N = 6.419 \cdot 10^4 \]

\[ \alpha := 0.4 \quad \text{splitting ratio} \]

\[ Lc := \frac{\left( \frac{Rf_B}{1 + Av} \right)^2 \cdot C_T}{\Delta_L} = 2.689 \cdot 10^{-5} \]

\[ C1 := (1 - \alpha) \cdot (C_T) = 9 \cdot 10^{-13} \]

\[ C2 := \alpha \cdot (C_T) = 6 \cdot 10^{-13} \]

\[ Z_{TIA, B}(\omega) := \frac{1}{\left( \left( 1 - \omega^2 \cdot Lc \cdot C1 \right) + \frac{Rf_B}{(1 + Av)} \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \right)} \]

**Receiver freq-response**

\[ Z_{\text{nontuned}}(\omega) := H_{\text{bun, N, A}}(\omega) \cdot Z_{TIA, N}(\omega) \]

\[ Z_{\text{tuned, A}}(\omega) := H_{\text{bun, N, A}}(\omega) \cdot Z_{TIA, A}(\omega) \]

\[ Z_{\text{tuned, B}}(\omega) := H_{\text{bun, B}}(\omega) \cdot Z_{TIA, B}(\omega) \]

**Noise equivalent bandwidth**

\[ NEB_N := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{13}} \left( \left| Z_{\text{nontuned}}(\omega) \right| \right)^2 \, d\omega \right) = 4.581 \cdot 10^7 \]

\[ NEB_A := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{13}} \left( \left| Z_{\text{tuned, A}}(\omega) \right| \right)^2 \, d\omega \right) = 5.528 \cdot 10^7 \]

\[ NEB_B := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{13}} \left( \left| Z_{\text{tuned, B}}(\omega) \right| \right)^2 \, d\omega \right) = 3.62 \cdot 10^7 \]
Noise integrals

\( I_{2,N} = NEB_N = 4.581 \cdot 10^{7} \)

\[
I_{2,A} := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{12}} \left( \left( \frac{1}{(1 + 1i \cdot \omega \cdot \frac{L_f}{Rf_A}) \cdot Z_{\text{tuned}_A}(\omega)} \right)^2 \right) d\omega \right) = 4.249 \cdot 10^{7}
\]

\[
I_{2,B} := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{12}} \left( \left( \frac{(1 - \omega^2 \cdot Lc \cdot C1)}{Z_{\text{tuned}_B}(\omega)} \right)^2 \right) d\omega \right) = 3.036 \cdot 10^{7}
\]

\[
I_{3,N} := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{12}} \left( \frac{1 + 1j \cdot (\omega) \cdot Rf_N \cdot C_T}{Rf_N \cdot Z_{\text{nontuned}}(\omega)} \right)^2 d\omega \right) = 14.321
\]

\[
I_{3,A} := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{12}} \left( \left( \frac{(1 - \omega^2 \cdot Lf \cdot C_T)}{(Rf_A + 1i \cdot \omega \cdot Lf)} \cdot Z_{\text{tuned}_A}(\omega) \right)^2 \right) d\omega \right) = 10.523
\]

\[
I_{3,B} := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{12}} \left( \left( \frac{(1 - \omega^2 \cdot Lc \cdot C1)}{Rf_B} \right)^2 \right) d\omega \right) = 1.422
\]

Feedback noise

\( q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \)

\( I_{nRf,N} := \frac{4 \cdot k \cdot T}{Rf_N} \cdot I_{2,N} = 3.229 \cdot 10^{-17} \)

\( I_{nRf,A} := \frac{4 \cdot k \cdot T}{Rf_A} \cdot I_{2,A} = 1.968 \cdot 10^{-17} \)

\( I_{nRf,B} := \frac{4 \cdot k \cdot T}{Rf_B} \cdot I_{2,B} = 7.78 \cdot 10^{-18} \)

\[
10 \log \left( \frac{I_{nRf,B}}{I_{nRf,N}} \right) = -6.18 \quad 10 \log \left( \frac{I_{nRf,A}}{I_{nRf,N}} \right) = -2.149
\]
BJT input stage

\[ h_{fe} = 100 \]

noise-opt BJT (Ic re-calculations)

\[
x_N(x) := \left( \frac{4 \cdot k \cdot T}{Rf_N} \cdot I_{2,N} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{2,N} + 2 \cdot q \cdot \frac{x}{\left( 25 \cdot 10^{-3} \right)^2} \cdot I_{3,N} \right)
\]

\[
x_A(x) := \left( \frac{4 \cdot k \cdot T}{Rf_A} \cdot I_{2,A} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot NEB_A + 2 \cdot q \cdot \frac{x}{\left( 25 \cdot 10^{-3} \right)^2} \cdot I_{3,A} \right)
\]

\[
x_B(x) := \left( \frac{4 \cdot k \cdot T}{Rf_B} \cdot I_{2,B} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{2,B} + 2 \cdot q \cdot \frac{x}{\left( 25 \cdot 10^{-3} \right)^2} \cdot I_{3,B} \right)
\]

\[ x := 0.00001, 0.00002, 0.01 \]

noise-opt BJT (ex)

\[ I_{opt,N} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3,N}}{I_{2,N}}} = 1.398 \cdot 10^{-4} \]

\[ I_{opt,A} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3,A}}{NEB_A}} = 1.091 \cdot 10^{-4} \]

\[ I_{opt,B} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3,B}}{I_{2,B}}} = 5.41 \cdot 10^{-5} \]

noise-opt BJT

\[ I_{n,N} := x_N(I_{opt,N}) = 7.327 \cdot 10^{-17} \]

\[ I_{n,A} := x_A(I_{opt,A}) = 5.827 \cdot 10^{-17} \]

\[ I_{n,B} := x_B(I_{opt,B}) = 1.829 \cdot 10^{-17} \]

B-220
Output pulse shape

\[ t_s := \frac{1}{B} \]

\[ vout_N(t) := \frac{t_s}{\pi} \int_0^{\frac{1}{t_s} \cdot 10^3} \frac{\sin\left(\frac{\omega \cdot t_s}{2}\right)}{\left(\frac{\omega \cdot t_s}{2}\right)} \cdot \text{Re}\left(Z_{\text{nontuned}}(\omega) \cdot \exp\left(1i \cdot \omega \cdot \left(\frac{t - t_s}{2}\right)\right)\right) d\omega \]

\[ vout_A(t) := \frac{t_s}{\pi} \int_0^{\frac{1}{t_s} \cdot 10^3} \frac{\sin\left(\frac{\omega \cdot t_s}{2}\right)}{\left(\frac{\omega \cdot t_s}{2}\right)} \cdot \text{Re}\left(Z_{\text{tuned}}(\omega) \cdot \exp\left(1i \cdot \omega \cdot \left(\frac{t - t_s}{2}\right)\right)\right) d\omega \]

\[ vout_B(t) := \frac{t_s}{\pi} \int_0^{\frac{1}{t_s} \cdot 10^3} \frac{\sin\left(\frac{\omega \cdot t_s}{2}\right)}{\left(\frac{\omega \cdot t_s}{2}\right)} \cdot \text{Re}\left(Z_{\text{tuned}}(\omega) \cdot \exp\left(1i \cdot \omega \cdot \left(\frac{t - t_s}{2}\right)\right)\right) d\omega \]

peak voltage and time

\[ xvout_N(t) := \frac{t_s^2}{\pi} \int_0^{\frac{1}{t_s} \cdot 10^3} \frac{\sin\left(\frac{\omega \cdot t_s}{2}\right)}{\left(\frac{\omega \cdot t_s}{2}\right)} \cdot \text{Re}\left(1i \cdot \omega \cdot Z_{\text{nontuned}}(\omega) \cdot \exp\left(1i \cdot \omega \cdot \left(\frac{t - t_s}{2}\right)\right)\right) d\omega \]

\[ xvout_A(t) := \frac{t_s^2}{\pi} \int_0^{\frac{1}{t_s} \cdot 10^3} \frac{\sin\left(\frac{\omega \cdot t_s}{2}\right)}{\left(\frac{\omega \cdot t_s}{2}\right)} \cdot \text{Re}\left(1i \cdot \omega \cdot Z_{\text{tuned}}(\omega) \cdot \exp\left(1i \cdot \omega \cdot \left(\frac{t - t_s}{2}\right)\right)\right) d\omega \]

\[ xvout_B(t) := \frac{t_s^2}{\pi} \int_0^{\frac{1}{t_s} \cdot 10^3} \frac{\sin\left(\frac{\omega \cdot t_s}{2}\right)}{\left(\frac{\omega \cdot t_s}{2}\right)} \cdot \text{Re}\left(1i \cdot \omega \cdot Z_{\text{tuned}}(\omega) \cdot \exp\left(1i \cdot \omega \cdot \left(\frac{t - t_s}{2}\right)\right)\right) d\omega \]
\[ t_N := 0.7 \cdot ts \]

\[ t_{pk1} := \text{root}(xvout_N(t_N), t_N) \]

\[ \frac{t_{pk1}}{ts} = 1.026 \]

\[ v_{\text{max}_N} := \text{vout}_N(t_{pk1}) = 0.884 \]

\[ v_{\text{min}_N} := \text{vout}_N(t_{pk1} + 1 \cdot ts) = 0.106 \]

\[ v_{\text{min2ts}_N} := \text{vout}_N(t_{pk1} + 2 \cdot ts) = 0.005 \]

\[ v_{\text{min3ts}_N} := \text{vout}_N(t_{pk1} + 3 \cdot ts) = 2.451 \cdot 10^{-4} \]

\[ t_A := 1 \cdot ts \]

\[ t_{pk1} := \text{root}(xvout_A(t_A), t_A) \]

\[ \frac{t_{pk1}}{ts} = 1.047 \]

\[ v_{\text{max}_A} := \text{vout}_A(t_{pk1}) = 0.992 \]

\[ v_{\text{min}_A} := \text{vout}_A(t_{pk1} + 1 \cdot ts) = 0.025 \]

\[ v_{\text{min2ts}_A} := \text{vout}_A(t_{pk1} + 2 \cdot ts) = -0.031 \]

\[ v_{\text{min3ts}_A} := \text{vout}_A(t_{pk1} + 3 \cdot ts) = 0.004 \]

\[ t_B := 1.2 \cdot ts \]

\[ t_{pk1} := \text{root}(xvout_B(t_B), t_B) \]

\[ \frac{t_{pk1}}{ts} = 1.629 \]

\[ v_{\text{max}_B} := \text{vout}_B(t_{pk1}) = 0.787 \]

\[ v_{\text{min}_B} := \text{vout}_B(t_{pk1} + 1 \cdot ts) = 0.051 \]

\[ v_{\text{min2ts}_B} := \text{vout}_B(t_{pk1} + 2 \cdot ts) = 0.04 \]

\[ v_{\text{min3ts}_B} := \text{vout}_B(t_{pk1} + 3 \cdot ts) = 0.021 \]

**BER**

\[ nq := 1.6 \cdot 10^{-19} \quad \text{Test}_{b\text{-photons}} := 1100, 1200, \ldots, 15000 \]

\[
\text{test}_{Q_N}(\text{Test}_{b\text{-photons}}) = \frac{1}{ts} \cdot \text{Test}_{b\text{-photons}} \cdot nq \cdot \left( \frac{v_{\text{max}_N} - v_{\text{min}_N}}{2} \right) \sqrt{I_{n_N}}
\]

\[
\text{test}_{Q_A}(\text{Test}_{b\text{-photons}}) = \frac{1}{ts} \cdot \text{Test}_{b\text{-photons}} \cdot nq \cdot \left( \frac{v_{\text{max}_A} - v_{\text{min}_A}}{2} \right) \sqrt{I_{n_A}}
\]
\[
\text{test}_Q (\text{Test}_b) := \frac{1}{ts} \cdot \text{Test}_b \cdot nq \cdot \left( \frac{v_{\text{max}_B} - v_{\text{min}_B}}{2} \right) = \sqrt{I_{n_B}}
\]
\[
\text{Test1 (Test}_b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{\text{test}_Q (\text{Test}_b)}{\sqrt{2}} \right)
\]
\[
\text{Test2 (Test}_b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{\text{test}_Q (\text{Test}_b)}{\sqrt{2}} \right)
\]
\[
\text{Test3 (Test}_b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{\text{test}_Q (\text{Test}_b)}{\sqrt{2}} \right)
\]

\[
\text{number of photons per bit for } 10^9
\]
\[
Q_N(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max}_N} - v_{\text{min}_N}}{2} \right) = \sqrt{I_{n_N}}
\]
\[
Q_A(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max}_A} - v_{\text{min}_A}}{2} \right) = \sqrt{I_{n_A}}
\]
\[
Q_B(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max}_B} - v_{\text{min}_B}}{2} \right) = \sqrt{I_{n_B}}
\]
\[
\text{Pe}_N(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right)
\]
\[
\text{Pe}_A(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_A(b)}{\sqrt{2}} \right)
\]
\[
\text{Pe}_B(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_B(b)}{\sqrt{2}} \right)
\]
\[ P_{e,N}(b) = (\log(P_{e,N}(b)) + 9) \]
\[ P_{e,A}(b) = (\log(P_{e,A}(b)) + 9) \]
\[ P_{e,B}(b) = (\log(P_{e,B}(b)) + 9) \]
\[ b \equiv 3 \cdot 10^3 \]
\[ b_N := \text{root}(P_{e,N}(b), b) \]
\[ b_A := \text{root}(P_{e,A}(b), b) \]
\[ b_B := \text{root}(P_{e,B}(b), b) \]

minimum\_N := \min(b_N) = 8.254 \cdot 10^3
minimum\_A := \min(b_A) = 5.919 \cdot 10^3
minimum\_B := \min(b_B) = 4.357 \cdot 10^3

**BER-check for minimum b**

<table>
<thead>
<tr>
<th>Q_N(minimum_N) = 5.998</th>
<th>Pe_N(minimum_N) = 1 \cdot 10^{-9}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q_A(minimum_A) = 5.998</td>
<td>Pe_A(minimum_A) = 1 \cdot 10^{-9}</td>
</tr>
<tr>
<td>Q_B(minimum_B) = 5.998</td>
<td>Pe_B(minimum_B) = 1 \cdot 10^{-9}</td>
</tr>
</tbody>
</table>

**Sens PIN-BJT**

\[ \lambda := 650 \cdot 10^{-9} \]
\[ \text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]
\[ \text{Sens\_N} := 10 \cdot \log \left( \frac{\text{minimum\_N}}{2} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -38.986 \]
\[ \text{Sens\_A} := 10 \cdot \log \left( \frac{\text{minimum\_A}}{2} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -40.43 \]
\[ \text{Sens\_B} := 10 \cdot \log \left( \frac{\text{minimum\_B}}{2} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -41.761 \]

Sens\_B – Sens\_N = -2.775

Sens\_A – Sens\_N = -1.444

#
B.3.7 Performance comparison APDBJT

Silicon_APD: Performance comparison (APD-BJT 1st order LPF 100 Mbit/s)

Rx performance with APD-BJT input configuration, 1st order LPF, pre-detection filter. Calculations are as follows:

- APD noise, APD BER, APD receiver sensitivity
- [Silicon Epitaxial: APD M= 10 F(M)=5.5]

**APD noise**

\[ M_{APD} := 10 \]

\[ F_M := 5.5 \]

\[ APD_N(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot \left( v_{\text{max}_N} - v_{\text{min}_N} \right) \]

\[ APD_A(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot \left( v_{\text{max}_A} - v_{\text{min}_A} \right) \]

\[ APD_B(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot \left( v_{\text{max}_B} - v_{\text{min}_B} \right) \]

\[ I_{APD,d} := 10 \cdot 10^{-9} \]

\[ \text{Noise}_{APD,N}(b) := \left( 2 \cdot q \cdot APD_N(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_N \]

\[ \text{Noise}_{APD,A}(b) := \left( 2 \cdot q \cdot APD_A(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_A \]

\[ \text{Noise}_{APD,B}(b) := \left( 2 \cdot q \cdot APD_B(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_B \]

**BER + APD**

\[ Q_{APD,N}(b) := \frac{M_{APD} \cdot b \cdot nq \cdot \left( v_{\text{max}_N} - v_{\text{min}_N} \right)}{2 \cdot \sqrt{I_{n,N} + \text{Noise}_{APD,N}(b)}} \]

\[ Q_{APD,A}(b) := \frac{M_{APD} \cdot b \cdot nq \cdot \left( v_{\text{max}_A} - v_{\text{min}_A} \right)}{2 \cdot \sqrt{I_{n,A} + \text{Noise}_{APD,A}(b)}} \]

\[ Q_{APD,B}(b) := \frac{M_{APD} \cdot b \cdot nq \cdot \left( v_{\text{max}_B} - v_{\text{min}_B} \right)}{2 \cdot \sqrt{I_{n,B} + \text{Noise}_{APD,B}(b)}} \]

\[ P_{e,APD,N}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD,N}(b)}{\sqrt{2}} \right) \]

\[ P_{e,APD,A}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD,A}(b)}{\sqrt{2}} \right) \]

\[ P_{e,APD,B}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD,B}(b)}{\sqrt{2}} \right) \]
\[ p_{APD,N}(b) := \log(P_{c\_APD\_N}(b)) + 9 \]
\[ p_{APD,A}(b) := \log(P_{c\_APD\_A}(b)) + 9 \]
\[ p_{APD,B}(b) := \log(P_{c\_APD\_B}(b)) + 9 \]
\[ APD\_b\_N := \text{root}(p_{APD,N}(b), b) \]
\[ APD\_b\_A := \text{root}(p_{APD,A}(b), b) \]
\[ APD\_b\_B := \text{root}(p_{APD,B}(b), b) \]
\[ minimum\_APD\_N := \min(APD\_b\_N) = 1.839 \cdot 10^3 \]
\[ minimum\_APD\_A := \min(APD\_b\_A) = 1.537 \cdot 10^3 \]
\[ minimum\_APD\_B := \min(APD\_b\_B) = 1.429 \cdot 10^3 \]

Check BER, Q, and b APD

\[ Q\_APD\_N(\min(APD\_b\_N)) = 5.998 \]
\[ Q\_APD\_A(\min(APD\_b\_A)) = 5.998 \]
\[ Q\_APD\_B(\min(APD\_b\_B)) = 5.998 \]

**Sens**

\[ APD\_N := 10 \cdot \log \left( \frac{minimum\_APD\_N \cdot photon\_energy \cdot 1}{2 \cdot ts \cdot 10^{-3}} \right) = -45.508 \]
\[ APD\_A := 10 \cdot \log \left( \frac{minimum\_APD\_A \cdot photon\_energy \cdot 1}{2 \cdot ts \cdot 10^{-3}} \right) = -46.286 \]
\[ APD\_B := 10 \cdot \log \left( \frac{minimum\_APD\_B \cdot photon\_energy \cdot 1}{2 \cdot ts \cdot 10^{-3}} \right) = -46.604 \]

Sens\_N − APD\_N = 6.522
Noise\_APD\_N(\text{minimum\_APD\_N}) = 2.902 \cdot 10^{-16}

Sens\_A − APD\_A = 5.856
Noise\_APD\_A(\text{minimum\_APD\_A}) = 3.347 \cdot 10^{-16}

Sens\_B − APD\_B = 4.843
Noise\_APD\_B(\text{minimum\_APD\_B}) = 1.783 \cdot 10^{-16}

\[ I_{n,N} = 7.327 \cdot 10^{-17} \quad I_{n,A} = 5.827 \cdot 10^{-17} \quad I_{n,B} = 1.829 \cdot 10^{-17} \]
**InGaAs APD:** Performance comparison (APD-BJT 1st order LPF 100 Mbit/s)

Rx performance with APD-BJT input configuration, 1st order LPF pre-detection filter. calculations are as follow

- APD noise, APD BER, APD receiver sensitivity
- \( \text{InGaAs: } M = 100 \quad F(M) = 7.9 \)

**APD noise**

\[
M_{\text{APD}} := 100 \\
F_M := 7.9
\]

\[
APD_N(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot (v_{\text{max},N})
\]

\[
APD_A(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot (v_{\text{max},A})
\]

\[
APD_B(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot (v_{\text{max},B})
\]

\[
I_{\text{APD},d} := 10 \cdot 10^{-9}
\]

\[
\text{Noise}_{\text{APD},N}(b) := \left( 2 \cdot q \cdot APD_N(b) \cdot M_{\text{APD}}^2 \cdot F_M \right) \cdot \text{NEB}_{N} \]

\[
\text{Noise}_{\text{APD},A}(b) := \left( 2 \cdot q \cdot APD_A(b) \cdot M_{\text{APD}}^2 \cdot F_M \right) \cdot \text{NEB}_{A} \]

\[
\text{Noise}_{\text{APD},B}(b) := \left( 2 \cdot q \cdot APD_B(b) \cdot M_{\text{APD}}^2 \cdot F_M \right) \cdot \text{NEB}_{B}
\]

\[
Q_{\text{APD},N}(b) := \frac{M_{\text{APD}} \cdot b \cdot nq \cdot (v_{\text{max},N} - v_{\text{min},N})}{\sqrt{I_{n,N} + \text{Noise}_{\text{APD},N}(b)}}
\]

\[
Q_{\text{APD},A}(b) := \frac{M_{\text{APD}} \cdot b \cdot nq \cdot (v_{\text{max},A} - v_{\text{min},A})}{\sqrt{I_{n,A} + \text{Noise}_{\text{APD},A}(b)}}
\]

\[
Q_{\text{APD},B}(b) := \frac{M_{\text{APD}} \cdot b \cdot nq \cdot (v_{\text{max},B} - v_{\text{min},B})}{\sqrt{I_{n,B} + \text{Noise}_{\text{APD},B}(b)}}
\]

\[
P_{e,\text{APD},N}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD},N}(b)}{\sqrt{2}} \right)
\]

\[
P_{e,\text{APD},A}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD},A}(b)}{\sqrt{2}} \right)
\]
\[ P_{e,APD_B}(b) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD_B}(b)}{\sqrt{2}} \right) \]

\[ p_{APD_N}(b) = (\log(P_{e,APD_N}(b)) + 9) \]

\[ p_{APD_A}(b) = (\log(P_{e,APD_A}(b)) + 9) \]

\[ p_{APD_B}(b) = (\log(P_{e,APD_B}(b)) + 9) \]

\[ APD_b_N := \text{root} \left( p_{APD_N}(b), b \right) \]

\[ APD_b_A := \text{root} \left( p_{APD_A}(b), b \right) \]

\[ APD_b_B := \text{root} \left( p_{APD_B}(b), b \right) \]

\[ \text{minimum}_{APD_N} := \min(\text{APD}_b_N) = 2.051 \cdot 10^3 \]

\[ \text{minimum}_{APD_A} := \min(\text{APD}_b_A) = 1.802 \cdot 10^3 \]

\[ \text{minimum}_{APD_B} := \min(\text{APD}_b_B) = 1.742 \cdot 10^3 \]

Check BER, Q, and b APD

\[ Q_{APD_N}(\min(\text{APD}_b_N)) = 5.998 \]

\[ Q_{APD_A}(\min(\text{APD}_b_A)) = 5.998 \]

\[ Q_{APD_B}(\min(\text{APD}_b_B)) = 5.998 \]

Sens

\[ \text{APD}_N := 10 \cdot \log \left( \frac{\text{minimum}_{APD_N}}{2} \cdot \frac{\text{photon}_\text{energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -45.033 \]

\[ \text{APD}_A := 10 \cdot \log \left( \frac{\text{minimum}_{APD_A}}{2} \cdot \frac{\text{photon}_\text{energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -45.595 \]

\[ \text{APD}_B := 10 \cdot \log \left( \frac{\text{minimum}_{APD_B}}{2} \cdot \frac{\text{photon}_\text{energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -45.742 \]

\[ \text{Sens}_N - \text{APD}_N = 6.047 \]

\[ \text{Noise}_{APD_N}(\text{minimum}_{APD_N}) = 4.517 \cdot 10^{-14} \]

\[ \text{Sens}_A - \text{APD}_A = 5.165 \]

\[ \text{Noise}_{APD_A}(\text{minimum}_{APD_A}) = 5.395 \cdot 10^{-14} \]

\[ \text{Sens}_B - \text{APD}_B = 3.981 \]

\[ I_{n,N} = 7.327 \cdot 10^{-17} \quad I_{n,A} = 5.827 \cdot 10^{-17} \quad I_{n,B} = 1.829 \cdot 10^{-17} \]
Germanium_APD: Performance comparison (APD-FET 1st order LPF 100 Mbit/s)

Rx performance with APD-FET input configuration, 1st order LPF pre-detection filter. Calculations are as follows:

- APD noise, APD BER, APD receiver sensitivity
- [Germanium: M=10 F(M)=9.2]

### APD noise

\[ M_{APD} := 10 \]
\[ F_M := 9.2 \]

\[ APD_N(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot (v_{\text{max},N}) \]

\[ APD_A(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot (v_{\text{max},A}) \]

\[ APD_B(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot (v_{\text{max},B}) \]

\[ I_{APD_d} := 10 \cdot 10^{-9} \]

\[ Noise_{APD_N}(b) := \left( 2 \cdot q \cdot APD_N(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_N + 2 \cdot q \cdot I_{APD_d} \cdot M_{APD}^2 \cdot F_M \]

\[ Noise_{APD_A}(b) := \left( 2 \cdot q \cdot APD_A(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_A + 2 \cdot q \cdot I_{APD_d} \cdot M_{APD}^2 \cdot F_M \]

\[ Noise_{APD_B}(b) := \left( 2 \cdot q \cdot APD_B(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_B + 2 \cdot q \cdot I_{APD_d} \cdot M_{APD}^2 \cdot F_M \]

### BER + APD

\[ Q_{APD_N}(b) := \frac{M_{APD}}{t_s} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max},N} - v_{\text{min},N}}{2} \right) \]

\[ Q_{APD_A}(b) := \frac{M_{APD}}{t_s} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max},A} - v_{\text{min},A}}{2} \right) \]

\[ Q_{APD_B}(b) := \frac{M_{APD}}{t_s} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max},B} - v_{\text{min},B}}{2} \right) \]

\[ P_{APD_N}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD_N}(b)}{\sqrt{2}} \right) \]
\[ P_{e,\text{APD}_A}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD}_A}(b)}{\sqrt{2}} \right) \]

\[ P_{e,\text{APD}_B}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD}_B}(b)}{\sqrt{2}} \right) \]

\[ p_{\text{APD}_N}(b) := (\log (P_{e,\text{APD}_N}(b))) + 9 \]

\[ p_{\text{APD}_A}(b) := (\log (P_{e,\text{APD}_A}(b))) + 9 \]

\[ p_{\text{APD}_B}(b) := (\log (P_{e,\text{APD}_B}(b))) + 9 \]

\[ \text{APD}_b_N := \text{root}(p_{\text{APD}_N}(b), b) \]

\[ \text{APD}_b_A := \text{root}(p_{\text{APD}_A}(b), b) \]

\[ \text{APD}_b_B := \text{root}(p_{\text{APD}_B}(b), b) \]

\[ \text{minimum}_{\text{APD}_N} := \min(\text{APD}_b_N) = 2.536 \cdot 10^3 \]

\[ \text{minimum}_{\text{APD}_A} := \min(\text{APD}_b_A) = 2.167 \cdot 10^3 \]

\[ \text{minimum}_{\text{APD}_B} := \min(\text{APD}_b_B) = 2.031 \cdot 10^3 \]

Check BER, Q, and b APD

\[ Q_{\text{APD}_N}(\min(\text{APD}_b_N)) = 5.998 \]

\[ Q_{\text{APD}_A}(\min(\text{APD}_b_A)) = 5.998 \]

\[ Q_{\text{APD}_B}(\min(\text{APD}_b_B)) = 5.998 \]

\textbf{Sens}

\[ \text{APD}_N := 10 \cdot \log \left( \frac{\text{minimum}_{\text{APD}_N}}{2}, \frac{\text{photon\_energy}}{10^{-3}}, \frac{1}{ts} \right) = -44.111 \]

\[ \text{APD}_A := 10 \cdot \log \left( \frac{\text{minimum}_{\text{APD}_A}}{2}, \frac{\text{photon\_energy}}{10^{-3}}, \frac{1}{ts} \right) = -44.795 \]

\[ \text{APD}_B := 10 \cdot \log \left( \frac{\text{minimum}_{\text{APD}_B}}{2}, \frac{\text{photon\_energy}}{10^{-3}}, \frac{1}{ts} \right) = -45.076 \]

\[ \text{Sens}_N - \text{APD}_N = 5.124 \quad \text{Noise}_{\text{APD}_N}(\text{minimum}_{\text{APD}_N}) = 6.186 \cdot 10^{-16} \]

\[ \text{Sens}_A - \text{APD}_A = 4.365 \quad \text{Noise}_{\text{APD}_A}(\text{minimum}_{\text{APD}_A}) = 7.225 \cdot 10^{-16} \]

\[ \text{Sens}_B - \text{APD}_B = 3.315 \quad \text{Noise}_{\text{APD}_B}(\text{minimum}_{\text{APD}_B}) = 3.791 \cdot 10^{-16} \]

\[ I_n_N = 7.327 \cdot 10^{-17} \quad I_n_A = 5.827 \cdot 10^{-17} \quad I_n_B = 1.829 \cdot 10^{-17} \]
B.4 Receiver optimisation (FET/ 3rd order Butterworth pre-detection filter)

B.4.1 Non-tuned receiver

(Non-tuned optimum Performance (PIN-FET APD-FET 100 Mbit/s))

Rx performance opt PIN-FET and APD-FET input configurations, 3rd order Butterworth pre-detection filter. opt as follows

- TIA 3-dB bandwidth rt/Filter 3-dB bandwidth rd
- range cut-off
- minimum noise
- highest SNR
- BER
- Rx sens

**Bit-rate, pulse duration**

\[ C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]

\[ B := 100 \cdot 10^6 \quad \text{Bit-rate} \]

**pre-dec filter**

\[ \omega_B(rd) := 2 \cdot \pi \cdot rd \]

\[ H_{but}(\omega, rd) := \frac{\omega_B(rd)^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B(rd) + 2 \cdot (1j \cdot \omega) \cdot \omega_B(rd)^2 + \omega_B(rd)^3} \]

**TIA(Non-tuned)**

\[ \omega_c(rt) := 2 \cdot \pi \cdot rt \]

\[ Av := 10 \]

\[ Rf_N(rt) := \frac{Av + 1}{2 \cdot \pi \cdot rt \cdot C_T} \]

\[ Z_{TIA,N}(\omega, rt) := \frac{1}{1 + 1j \cdot \frac{\omega}{2 \cdot \pi \cdot rt}} \]

**Receiver freq-response**

\[ Z_N(\omega, rd, rt) := H_{but}(\omega, rd) \cdot Z_{TIA,N}(\omega, rt) \]

\[ \omega := 1 \cdot 10^6, 10 \cdot 10^6 \ldots 1 \cdot 10^{11} \]

\[ rd := 0.7 \cdot B, 0.75 \cdot B \ldots 2 \cdot B \]

\[ rt := 0.5 \cdot B, 0.55 \cdot B \ldots 1 \cdot B \]

**Noise equivalent bandwidth (NEB and I2)**

\[ NEB_N(rd, rt) := \frac{1}{2 \cdot \pi} \int_0^{10^{12}} \left( |Z_N(\omega, rd, rt)|^2 \right) d\omega \]

\[ NEB_N(0.7 \cdot B, 0.5 \cdot B) = 4.722 \cdot 10^7 \quad NEB_B(rd, rt) := NEB_N(rd, rt) \]
Noise equivalent bandwidth (13)

\[ I_{3\,N}(rd, rt) = \frac{1}{2 \cdot \pi} \int_0^{10^2} \left( \frac{1 + 1 \cdot j \cdot (\omega) \cdot Rf_N(rt) \cdot C_T \cdot Z_N(\omega, rd, rt)}{1 \cdot Rf_N(rt)} \right)^2 d\omega \]

Feedback noise

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \]

\[ I_{n_{RF,N}}(rd, rt) = \frac{4 \cdot k \cdot T}{Rf_N(rt)} \cdot NEB_N(rd, rt) \]

\[ I_{n_{RF,N}}(0.7 \, B, 0.5 \, B) = 3.327 \cdot 10^{-17} \]

Ig noise (gate current)

\[ Ig := 10 \cdot 10^{-9} \]

\[ I_{n_{G,N}}(rd, rt) = 2 \cdot q \cdot Ig \cdot NEB_N(rd, rt) \]
\[ I_{nG,N}(0.7\, B, 0.5\, B) = 1.511 \cdot 10^{-19} \]

**Channel noise (Gate-source)**

\[ g_{m1} := 30 \cdot 10^{-3} = 0.03 \quad \text{noise\_factor} := 1 \]

\[ I_{nD} := 4 \cdot k \cdot T \cdot \frac{1}{g_{m1}} \]

\[ I_{nD,N}(rd, rt) := I_{nD} \cdot I_{3,N}(rd, rt) \]

\[ I_{nD,N}(0.7\, B, 0.5\, B) = 3.224 \cdot 10^{-18} \]

**Total noise - PIN-FET**

\[ I_{n_B}(rd, rt) := I_{nRf,N}(rd, rt) + I_{nG,N}(rd, rt) + I_{nD,N}(rd, rt) \]

\[ I_{n_B}(0.7\, B, 0.5\, B) = 3.665 \cdot 10^{-17} \]

**Output pulse shape**

\[ ts := \frac{1}{B} \quad B = 1 \cdot 10^8 \]

\[ \frac{1}{t_s} = 10^3 \]

\[ vout(t, rd, rt) := ts \cdot \frac{\sin \left( \frac{\omega \cdot ts}{2} \right)}{\pi} \cdot \frac{\exp \left( 1i \cdot \omega \cdot t \right) \cdot \Re \left( Z_N(\omega, rd, rt) \right)}{\left( \frac{\omega \cdot ts}{2} \right)} \]

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\[ x_{Bt}(t) := \frac{ts}{\pi} \cdot \frac{1}{ts} \cdot \int_0^{1/10^6} \sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \frac{1}{\omega} \cdot \Re \left( \exp \left( i \cdot \omega \cdot (t) \right) \right) \, d\omega \]

\[ x_{vout}(t, rd, rt) := \frac{ts^2}{\pi} \cdot \int_0^{1/10^6} \sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \frac{1}{\omega} \cdot \Re \left( i \cdot \omega \cdot Z_N(\omega, rd, rt) \cdot \exp \left( i \cdot \omega \cdot (t) \right) \right) \, d\omega \]

\[ t := -1 \cdot ts, -0.99 \cdot ts, 4 \cdot ts \]

<table>
<thead>
<tr>
<th>ISI</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_{B1} := 0.5 \cdot ts )</td>
</tr>
<tr>
<td>( t_{pk1} := \text{root} (x_{vout}(t_{B1}, 0.7 B, 0.5 B), t_{B1}) )</td>
</tr>
<tr>
<td>( \frac{t_{pk1}}{ts} = 0.77 )</td>
</tr>
<tr>
<td>( v_{max,B1} := v_{out}(t_{pk1}, 0.7 B, 0.5 B) = 0.918 )</td>
</tr>
<tr>
<td>( v_{min,B1} := v_{out}(t_{pk1} + 1 \cdot ts, 0.7 B, 0.5 B) = 0.043 )</td>
</tr>
<tr>
<td>( v_{min1,B1} := v_{out}(t_{pk1} + 2 \cdot ts, 0.7 B, 0.5 B) = 0.009 )</td>
</tr>
<tr>
<td>( v_{min2,B1} := v_{out}(t_{pk1} + 3 \cdot ts, 0.7, 0.5 B) = -2.32 \cdot 10^{-21} )</td>
</tr>
<tr>
<td>( v_{max,B1} - v_{min,B1} = 0.875 )</td>
</tr>
<tr>
<td>( v_{max,B1} - v_{min1,B1} = 0.909 )</td>
</tr>
<tr>
<td>( v_{max,B1} - v_{min2,B1} = 0.918 )</td>
</tr>
</tbody>
</table>

\[ \sqrt{10^{16} \cdot I_{n,B}(0.7 B, 0.5 B)} \]

\[ \frac{v_{max,B1} - v_{min,B1}}{\sqrt{10^{16} \cdot I_{n,B}(0.7 B, 0.5 B)}} = 1.45 \quad \text{1st pulse} \]

\[ \frac{v_{max,B1} - v_{min1,B1}}{\sqrt{10^{16} \cdot I_{n,B}(0.7 B, 0.5 B)}} = 1.501 \quad \text{2nd pulse} \]

\[ \frac{v_{max,B1} - v_{min2,B1}}{\sqrt{10^{16} \cdot I_{n,B}(0.7 B, 0.5 B)}} = 1.516 \quad \text{3rd pulse} \]

**Highest SNR @ rd=0.7 rt=0.5**

\[ v_{max,B} := v_{max,B1} \]

\[ v_{min,B} := v_{min,B1} \]

**Eye-diagram check**

\[ x_{111}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - 2 \cdot ts, rd, rt) + v_{out}(t - ts, rd, rt) \]

\[ x_{110}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - ts, rd, rt) \]

\[ x_{000}(t, rd, rt) := v_{out}(t, rd, rt) \]

\[ x_{001}(t, rd, rt) := v_{out}(t - 2 \cdot ts, rd, rt) \]

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\[ x_{00}(t, rd, rt) := v_{out}(t - ts, rd, rt) \]
\[ x_{01}(t, rd, rt) := v_{out}(t - ts, rd, rt) + v_{out}(t - 2ts, rd, rt) \]
\[ x_{01}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - 2ts, rd, rt) \]

**Sens PINFET**

\[ \lambda := 650 \cdot 10^{-9} \]

\[ \text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ nq := 1.6 \cdot 10^{-10} \]

\[ Q_N(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{max\_B} - v_{min\_B}}{2} \right) \]

\[ I_n(B) \]

\[ Q_N(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right) \]

\[ P_N(b) := \log(P_N(b)) + 9 \]

\[ b \equiv 3 \cdot 10^3 \]

\[ b_N := \text{root}(P_N(b), b) \]

**minimum\_B := min(b, N) = 5.187 \cdot 10^3**

\[ \text{Sens\_N\_PIN} := 10 \cdot \log \left( \frac{\text{minimum\_B} \cdot \text{photon\_energy} \cdot 1}{2 \cdot ts} \right) = -41.004 \]

#
B.4.2 Tuned A receiver

Tuned A Optimum Performance (PIN-FET APD-FET 100 Mbit/s)

Rx performance opt PIN-FET and APD-FET input configurations, 3rd order Butterworth pre-detection filter. opt as follows

- TIA 3-db bandwidth rt/Filter 3-D bandwidth rd
- range cut-off
- minimum noise
- examine ISI
- highest SNR
- BER
- Rx sens

Bit-rate, pulse duration

\[ C_T := 1.5 \cdot 10^{-12} \]  \quad total C

\[ B := 100 \cdot 10^6 \]  \quad Bit-rate

pre-dec filter

\[ \omega_B (rd) := 2 \cdot \pi \cdot rd \]

\[ H_{but}(\omega, rd) := \frac{\omega_B (rd)^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B (rd) + 2 \cdot (1j \cdot \omega) \cdot \omega_B (rd)^2 + \omega_B (rd)^3} \]

TIA (Non-tuned)

\[ A_v := 10 \]
\[ m := 1.8 \]  \quad time constant ratio of L/R and RC

\[ y := \sqrt{\frac{-m^2}{2} + m + 1} + \sqrt{\left(\frac{-m^2}{2} + m + 1\right)^2 + m^2} = 1.825 \]

\[ Rf_N(rt) := y \cdot \frac{A_v + 1}{2 \cdot \pi \cdot rt \cdot C_T} \]

\[ L(rt) := Rf_N(rt)^2 \cdot \frac{C_T}{m} \]

\[ Lf(rt) := \left(\frac{Rf_N(rt)}{A_v + 1}\right)^2 \cdot \frac{(A_v + 1) \cdot C_T}{m} \]

\[ Z_{TIA,N}(\omega, rt) := \frac{1 + 1i \cdot \omega \cdot \frac{Lf(rt)}{Rf_N(rt)}}{\left(\left(1 - \omega^2 \cdot Lf(rt) \cdot \frac{C_T}{1 + A_v}\right) + Rf_N(rt) \cdot \omega \cdot \frac{C_T}{1 + A_v} \cdot 1i\right)} \]

Receiver freq-response

\[ Z_N(\omega, rd, rt) := H_{but}(\omega, rd) \cdot Z_{TIA,N}(\omega, rt) \]

\[ \omega := 1 \cdot 10^6, 10 \cdot 10^6 \ldots 1 \cdot 10^{11} \]

\[ rd := 0.7 \cdot B, 0.75 \cdot B \ldots 2 \cdot B \quad rt := 0.5 \cdot B, 0.55 \cdot B \ldots 1 \cdot B \]
Noise equivalent bandwidth (NEB and I2)

\[ NEB_N(rd, rt) := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{12}} \left( \left| Z_N(\omega, rd, rt) \right|^2 \right) d\omega \right) \]

\[ I2_A(rd, rt) := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{12}} \left( \left( \frac{1}{1 + i \cdot \omega \cdot Lf(rt) Rf_N(rt)} \right) \cdot Z_N(\omega, rd, rt) \right)^2 d\omega \right) \]

Noise equivalent bandwidth (I3)

\[ I_3N(rd, rt) := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{12}} \left( \left( \frac{\left( 1 - \omega^2 \cdot Lf(rt) \cdot C_T \right) + Rf_N(rt) \cdot \omega \cdot C_T \cdot i}{Rf_N(rt) + i \cdot \omega \cdot Lf(rt)} \right) \cdot Z_N(\omega, rd, rt) \right)^2 d\omega \right) \]
Feedback noise

\[ q := 1.6 \cdot 10^{-10} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \]

\[ I_{nRF,N}(rd, rt) := \frac{4 \cdot k \cdot T}{Rf_N(rt)} \cdot I_{2 \_A}(rd, rt) \]

 Ig noise  

\[ I_G := 10 \cdot 10^{-9} \]

\[ I_{nG,N}(rd, rt) := 2 \cdot q \cdot I_G \cdot NEB_N(rd, rt) \]

Channel noise  

\[ gm1 := 30 \cdot 10^{-3} = 0.03 \quad noise\_factor := 1 \]

\[ I_{nD} := 4 \cdot k \cdot T \cdot \frac{1}{gm1} \]

\[ I_{nD,N}(rd, rt) := I_{nD} \cdot I_{3 \_N}(rd, rt) \]

\[ I_{nD,N}(0.7 B, 0.5 B) = 2.744 \cdot 10^{-18} \]

Total noise - PIN-FET

\[ I_{n_B}(rd, rt) := I_{nRF,N}(rd, rt) + I_{nG,N}(rd, rt) + I_{nD,N}(rd, rt) \]
Output pulse shape

\[ t_s := \frac{1}{B} \]

\[ v_{out}(t, r_d, r_t) := t_s \cdot \frac{\sin \left( \frac{\omega \cdot t_s}{2} \right)}{\pi} \cdot \Re \left\{ Z_N(\omega, r_d, r_t) \cdot \exp(1i \cdot \omega \cdot (t)) \right\} d\omega \]

\[ x_{out}(t, r_d, r_t) := t_s^2 \cdot \frac{\sin \left( \frac{\omega \cdot t_s}{2} \right)}{\pi} \cdot \Re \left\{ Z_N(\omega, r_d, r_t) \cdot \exp(1i \cdot \omega \cdot (t)) \right\} d\omega \]

\[ t := -1 \cdot ts, -0.99 \cdot ts, 4 \cdot ts \]

(rt = 0.5)

\[ t_{B1} := 0.8 \cdot ts \]

\[ t_{pk1} := \text{root} \left( x_{out} \left( t_{B1}, 0.7 \cdot B, 0.5 \cdot B \right), t_{B1} \right) \]

\[ \frac{t_{pk1}}{ts} = 0.87 \]

\[ v_{max_{B1}} := v_{out} \left( t_{pk1}, 0.7 \cdot B, 0.5 \cdot B \right) = 0.978 \]

\[ v_{min_{B1}} := v_{out} \left( t_{pk1} + 1 \cdot ts, 0.7 \cdot B, 0.5 \cdot B \right) = 0.015 \]

\[ v_{min_{1,B1}} := v_{out} \left( t_{pk1} + 2 \cdot ts, 0.7 \cdot B, 0.5 \cdot B \right) = -0.054 \]

\[ v_{min_{2,B1}} := v_{out} \left( t_{pk1} + 3 \cdot ts, 0.7 \cdot B, 0.5 \cdot B \right) = 0.002 \]

\[ v_{max_{B1}} - v_{min_{B1}} = 0.962 \quad v_{max_{B1}} - v_{min_{1,B1}} = 1.032 \quad v_{max_{B1}} - v_{min_{2,B1}} = 0.975 \]

\[ \frac{v_{max_{B1}} - v_{min_{B1}}}{\sqrt{10^{17} \cdot I_{n,B}(0.7 \cdot B, 0.5 \cdot B)}} = 0.71 \quad \text{1st pulse \ worst ISI} \]

\[ \frac{v_{max_{B1}} - v_{min_{1,B1}}}{\sqrt{10^{17} \cdot I_{n,B}(0.7 \cdot B, 0.5 \cdot B)}} = 0.757 \quad \text{2nd pulse} \]

\[ \frac{v_{max_{B1}} - v_{min_{2,B1}}}{\sqrt{10^{17} \cdot I_{n,B}(0.7 \cdot B, 0.5 \cdot B)}} = 0.716 \quad \text{3rd pulse} \]

\[ v_{max_{B}} := v_{max_{B1}} \quad v_{min_{B}} := v_{min_{B1}} \]

Eye-diagram check

\[ x_{111}(t, r_d, r_t) := v_{out}(t, r_d, r_t) + v_{out}(t - 2 \cdot ts, r_d, r_t) + v_{out}(t - ts, r_d, r_t) \]

\[ x_{110}(t, r_d, r_t) := v_{out}(t, r_d, r_t) + v_{out}(t - ts, r_d, r_t) \]
\[ x_{100}(t, rd, rt) := v_{out}(t, rd, rt) \]
\[ x_{001}(t, rd, rt) := v_{out}(t - 2 \cdot ts, rd, rt) \]
\[ x_{010}(t, rd, rt) := v_{out}(t - ts, rd, rt) \]
\[ x_{011}(t, rd, rt) := v_{out}(t - ts, rd, rt) + v_{out}(t - 2 \cdot ts, rd, rt) \]
\[ x_{101}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - 2 \cdot ts, rd, rt) \]

\[ \lambda := 650 \cdot 10^{-9} \]
\[ \text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]
\[ nq := 1.6 \cdot 10^{-19} \]
\[ Q_N(b) := \frac{\frac{1}{ts} \cdot b \cdot nq \cdot \left( v_{max,B} - v_{min,B} \right)}{2} \]
\[ \frac{1}{\sqrt{I_{n,B}(0.7 B, 0.5 B)}} \]
\[ P_{e_N}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right) \]
\[ P_{c_N}(b) := (\log(P_{e_N}(b)) + 9) \]
\[ b \equiv 3 \cdot 10^3 \]
\[ b_N := \text{root}(P_{c_N}(b), b) \]
\[ \text{minimum}_B := \min(b_N) = 3.359 \cdot 10^3 \]
\[ \text{Sens}_N_{\text{PIN}} := 10 \cdot \log \left( \frac{\text{minimum}_B \cdot \text{photon\_energy} \cdot 1}{2 \cdot ts} \right) = -42.892 \]
B.4.3 Tuned B receiver $a=0.2$

**Tuned B Optimum Performance (PIN-FET 100 Mbit/s) $a=0.2$**

Rx performance opt PIN-FET input configurations, 3rd order Butterworth pre-detection filter. opt as follows:

- TIA 3-db bandwidth $rt$/Filter 3-dB bandwidth $rd$
- range cut-off
- minimum noise
- examine ISI
- highest SNR
- BER
- Rx sens

**opt $rd=0.6$ $rt=0.5$**

**Bit-rate, pulse duration**

$$C_T := 1.5 \cdot 10^{-12} \quad \text{total } C$$

$$B := 100 \cdot 10^6 \quad \text{Bit-rate}$$

**pre-dec filter**

$$\omega_B(rd) := 2 \cdot \pi \cdot rd$$

$$H_{bunt}(\omega, rd) := \frac{\omega_B(rd)^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B(rd) + 2 \cdot (1j \cdot \omega) \cdot \omega_B(rd)^2 + \omega_B(rd)^3}$$

**TIA**

Feedback value for (R) Tuned B

$$\Delta_L := 1.8 \quad \Delta_R := 1.87 \quad \alpha := 0.2 \quad Av := 10$$

$$Rf_B(rt) := \Delta_R \cdot \frac{Av + 1}{2 \cdot \pi \cdot rt \cdot C_T}$$

$$Lc(rt) := \frac{\left(\frac{Rf_B(rt)}{1 + Av}\right)^2 \cdot C_T}{\Delta_L}$$

$$C1 := (1 - \alpha) \cdot (C_T) = 1.2 \cdot 10^{-12}$$

$$C2 := \alpha \cdot (C_T) = 3 \cdot 10^{-13}$$

$$Z_{TIA,B}(\omega, rt) := \frac{1}{\left(\frac{\left(1 - \omega^2 \cdot Lc(rt) \cdot C1\right)^{\downarrow}}{Rf_B(rt) / (1 + Av)} \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc(rt) \cdot C1 \cdot C2)\right)}$$

**Receiver freq-response**

$$Z_B(\omega, rd, rt) := H_{bunt}(\omega, rd) \cdot Z_{TIA,B}(\omega, rt)$$

$$\omega := 1 \cdot 10^6, 10 \cdot 10^6, .. 1 \cdot 10^{11}$$

$$rd := 0.5 \cdot B, 0.75 \cdot B, .. 2 \cdot B$$

$$rt := 0.5 \cdot B, 0.55 \cdot B, .. 1 \cdot B$$

**Noise equivalent bandwidth (NEB and I2)**

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Noise equivalent bandwidth \((13)\)

\[
NEB_B(rd, rt) = \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{12}} \left( \frac{\left| Z_B(\omega, rd, rt) \right|^2}{\left| (1 - \omega^2 \cdot Lc(rt) \cdot C1) \right|^2} \right) d\omega \right)
\]

\[
I_2_B(rd, rt) = \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{12}} \left( \frac{\left| \frac{(1 - \omega^2 \cdot Lc(rt) \cdot C1)}{\left| \frac{Z_B(\omega, rd, rt) + Rf_B(rt) \cdot \omega}{Rf_B(rt)} \right|^2 \cdot \left| C1 + C2 - \left( \omega \right)^2 \cdot Lc(rt) \cdot C1 \cdot C2 \right|^2 \cdot \left( \frac{1}{\left( \omega \right)^2} \right) \right|^2}{\left| Z_B(\omega, rd, rt) \right|^2} \right) d\omega \right)
\]

\[
I_3_B(rd, rt) = \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{12}} \left( \frac{\left| \frac{(1 - \omega^2 \cdot Lc(rt) \cdot C1)}{\left| \frac{Z_B(\omega, rd, rt) + Rf_B(rt) \cdot \omega}{Rf_B(rt)} \right|^2 \cdot \left| C1 + C2 - \left( \omega \right)^2 \cdot Lc(rt) \cdot C1 \cdot C2 \right|^2 \cdot \left( \frac{1}{\left( \omega \right)^2} \right) \right|^2}{\left| Z_B(\omega, rd, rt) \right|^2} \right) d\omega \right)
\]
**Feedback noise**

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \]

\[ I_{n_{RF,B}}(rd, rt) := \frac{4 \cdot k \cdot T}{R_{f_B}(rt)} \cdot I_{2_B}(rd, rt) \]

\[ \frac{I_{n_{RF,B}}(rd, rt)}{10^{-17}} \]

**Ig noise (gate current)**

\[ Ig := 10 \cdot 10^{-9} \]

\[ I_{n_{G,B}}(rd, rt) := 2 \cdot q \cdot Ig \cdot I_{2_B}(rd, rt) \]

**Channel noise (Gate-source)**

\[ g_{m1} := 30 \cdot 10^{-3} \approx 0.03 \quad noise\_factor := 1 \]

\[ I_{n_{D}} = 4 \cdot k \cdot T \cdot \frac{1}{g_{m1}} \]

\[ I_{n_{D,B}}(rd, rt) := I_{n_{D}} \cdot I_{3_B}(rd, rt) \]

\[ \frac{I_{n_{D,B}}(rd, rt)}{10^{-18}} \]

**Total noise - PIN-FET**

\[ I_{n_{B}}(rd, rt) = I_{n_{RF,B}}(rd, rt) + I_{n_{G,B}}(rd, rt) + I_{n_{D,B}}(rd, rt) \]

\[ \frac{I_{n_{B}}(rd, rt)}{10^{-17}} \]
Output pulse shape

\[ t_s := \frac{1}{B} \]

\[ vout(t, rd, rt) := \frac{ts}{\pi} \int_0^{1/10^3} \sin\left(\frac{\omega \cdot ts}{2}\right) \cdot \text{Re}\{Z_B(\omega, rd, rt) \cdot \exp(1i \cdot \omega \cdot t)\} \, d\omega \]

\[ xvout(t, rd, rt) := \frac{ts^2}{\pi} \int_0^{1/10^2} \sin\left(\frac{\omega \cdot ts}{2}\right) \cdot \text{Re}\{1i \cdot \omega \cdot Z_B(\omega, rd, rt) \cdot \exp(1i \cdot \omega \cdot t)\} \, d\omega \]

\[ t := -1 \cdot ts, -0.99 \cdot ts, 4 \cdot ts \]

\[ |ISI| \]

\[ t_{pk_1} = 1.2 \cdot ts \]

\[ t_{pk_1} = \text{root}(xvout(t_{B1}, 0.6 B, 0.5 B), t_{B1}) \]

\[ t_{pk_1} = 1.21 \]

\[ v_{max_{B1}} := vout(t_{pk_1}, 0.6 B, 0.5 B) = 0.896 \]

\[ v_{min_{B1}} := vout(t_{pk_1} + 1 \cdot ts, 0.6 B, 0.5 B) = 0.019 \]

\[ v_{min1_{B1}} := vout(t_{pk_1} + 2 \cdot ts, 0.6 B, 0.5 B) = -0.004 \]

\[ v_{min2_{B1}} := vout(t_{pk_1} + 3 \cdot ts, 0.6 B, 0.5 B) = 9.097 \cdot 10^{-4} \]

\[ v_{max_{B1}} - v_{min_{B1}} = 0.877 \quad v_{max_{B1}} - v_{min1_{B1}} = 0.899 \quad v_{max_{B1}} - v_{min2_{B1}} = 0.895 \]

\[ \frac{v_{max_{B1}} - v_{min_{B1}}}{\sqrt{10^{17} \cdot I_n B(0.6 B, 0.5 B)}} = 0.79 \quad \text{1st pulse worst ISI} \]

\[ \frac{v_{max_{B1}} - v_{min1_{B1}}}{\sqrt{10^{17} \cdot I_n B(0.6 B, 0.5 B)}} = 0.81 \quad \text{2nd pulse} \]

\[ \frac{v_{max_{B1}} - v_{min2_{B1}}}{\sqrt{10^{17} \cdot I_n B(0.6 B, 0.5 B)}} = 0.8 \quad \text{3rd pulse} \]

\[ v_{max_{B1}} = v_{max_{B1}} \quad v_{min_{B1}} = v_{min_{B1}} \]

\[ x_{111}(t, rd, rt) := vout(t, rd, rt) + vout(t - 2 \cdot ts, rd, rt) + vout(t - ts, rd, rt) \]

\[ x_{110}(t, rd, rt) := vout(t, rd, rt) + vout(t - ts, rd, rt) \]

\[ x_{100}(t, rd, rt) := vout(t, rd, rt) \]

\[ x_{001}(t, rd, rt) := vout(t - 2 \cdot ts, rd, rt) \]
\[ x_{010}(t, rd, rt) := v_{out}(t - ts, rd, rt) \]
\[ x_{011}(t, rd, rt) := v_{out}(t - ts, rd, rt) + v_{out}(t - 2\ ts, rd, rt) \]
\[ x_{101}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - 2\ ts, rd, rt) \]

**Sens PINFET**

\[ \lambda := 650 \cdot 10^{-9} \]

\[ \text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ nq := 1.6 \cdot 10^{-19} \]

\[ Q_N(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \frac{v_{max,B} - v_{min,B}}{2} \]

\[ Q_N(b) := \frac{1}{\sqrt{I_{n,B}(0.6\ B, 0.5\ B)}} \]

\[ Pe_N(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right) \]

\[ Pc_N(b) := (\log(Pe_N(b)) + 9) \]

\[ b := 3 \cdot 10^3 \]

\[ b_N := \text{root}(Pc_N(b), b) \]

\[ \text{minimum}_B := \text{min}(b_N) = 3.017 \cdot 10^3 \]

\[ \text{Sens}_B\_PIN := 10 \cdot \log \left( \frac{\text{minimum}_B}{2} \cdot \text{photon\_energy} \cdot \frac{1}{ts} \right) = -43.357 \]

#
B.4.4 Tuned B receiver a=0.3

**Tuned B Optimum Performance (PIN-FET 100 Mbit/s) a=0.3**

Rx performance opt PIN-FET input configurations, 3rd order Butterworth pre-detection filter. opt as follows:

- TIA 3-dB bandwidth rt/Filter 3-dB bandwidth rd
- minimum noise
- examine ISI
- highest SNR
- BER
- Rx sens

opt rd = 0.5, 0.6 rt = 0.5

**Bit-rate, pulse duration**

\[ C_T := 1.5 \cdot 10^{-12} \] \hspace{1cm} \text{total C}
\[ B := 100 \cdot 10^6 \] \hspace{1cm} \text{Bit-rate}

\[ \omega_B(rd) := 2 \cdot \pi \cdot rd \]

\[ H_{\text{but}}(\omega, rd) := \frac{\omega_B(rd)^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B(rd) + 2 \cdot (1j \cdot \omega) \cdot \omega_B(rd)^2 + \omega_B(rd)^3} \]

**TIA**

Feedback value for (R) Tuned B

\[ \Delta_L := 2.4 \quad \Delta_R := 2.52 \quad \alpha := 0.3 \]

\[ Av := 10 \]

\[ \begin{bmatrix} \alpha & \Delta_L & \Delta_R \\ 0 & 2 & 1.41 \\ 0.1 & 1.8 & 1.58 \\ 0.2 & 1.8 & 1.87 \\ 0.3 & 2.4 & 2.52 \\ 0.4 & 1.9 & 2.75 \\ 0.4 & 2.5 & 3.17 \\ 0.5 & 1.5 & 2.65 \end{bmatrix} \]

\[ C1 := (1-\alpha) \cdot (C_T) = 1.05 \cdot 10^{-12} \]
\[ C2 := \alpha \cdot (C_T) = 4.5 \cdot 10^{-13} \]

\[ Z_{\text{TIA,B}}(\omega, rt) := \frac{1}{((1-\omega^2 \cdot Lc(rt) \cdot C1)^{\frac{1}{2}} + \frac{Rf_B(rt)}{1+Av} \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc(rt) \cdot C1 \cdot C2)^{\frac{1}{2}})} \]

**Receiver freq-response**

\[ Z_B(\omega, rd, rt) := H_{\text{but}}(\omega, rd) \cdot Z_{\text{TIA,B}}(\omega, rt) \]
\[ \omega := 1 \cdot 10^6, 10 \cdot 10^6 \ldots 1 \cdot 10^{11} \]
\[ rd := 0.5 \cdot B, 0.75 \cdot B \ldots 2 \cdot B \]
\[ rt := 0.5 \cdot B, 0.55 \cdot B \ldots 1 \cdot B \]

**Noise equivalent bandwidth (NEB and I2)**

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Noise equivalent bandwidth (L3)

\[ NEB_B(rd, rt) = \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{12}} \left( \left| \frac{1 - \omega^2 \cdot Lc(rt) \cdot C1}{\cdot Z_B(\omega, rd, rt)} \right| \right)^2 d\omega \right) \]

\[ I_2_B(rd, rt) = \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{12}} \left( \left( \frac{1 - \omega^2 \cdot Lc(rt) \cdot C1}{\cdot Z_B(\omega, rd, rt)} \right)^2 + \frac{Rf_B(rt) \cdot \omega \cdot (C1 + C2 - \frac{C1 \cdot C2}{\omega^2}) \cdot Lc(rt) \cdot C1 \cdot C2 \cdot 1i}{\cdot Z_B(\omega, rd, rt)} \right)^2 d\omega \right) \]

\[ I_3_B(rd, rt) = \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{12}} \left( \frac{1 - \omega^2 \cdot Lc(rt) \cdot C1}{\cdot Z_B(\omega, rd, rt)} \right)^2 + \frac{Rf_B(rt) \cdot \omega \cdot (C1 + C2 - \frac{C1 \cdot C2}{\omega^2}) \cdot Lc(rt) \cdot C1 \cdot C2 \cdot 1i}{\cdot Z_B(\omega, rd, rt)} \right)^2 d\omega \]
Feedback noise
\[ q := 1.6 \times 10^{-19} \quad k := 1.38 \times 10^{-23} \quad T := 298 \]
\[ I_{n_{RF,B}}(rd, rt) := \frac{4 \cdot k \cdot T}{\frac{R_{f,B}(rt)}{\cdot I_{2, B}(rd, rt)}} \]

Ig noise (gate current)
\[ Ig := 10 \cdot 10^{-9} \]
\[ I_{n_{G, B}}(rd, rt) := 2 \cdot q \cdot Ig \cdot I_{2, B}(rd, rt) \]

Channel noise (Gate-source)
\[ g_{m1} := 30 \cdot 10^{-9} = 0.03 \quad \text{noise_factor} := 1 \]
\[ I_{n_{D}} := 4 \cdot k \cdot T \cdot \frac{1}{g_{m1}} \]
\[ I_{n_{D, B}}(rd, rt) := I_{n_{D}} \cdot I_{3, B}(rd, rt) \]

Total noise - PIN-FET
\[ I_{n_{B}}(rd, rt) := I_{n_{RF, B}}(rd, rt) + I_{n_{G, B}}(rd, rt) + I_{n_{D, B}}(rd, rt) \]
Output pulse shape

\[ ts = \frac{1}{B} \]

\[
v_{out}(t, rd, rt) = ts \cdot \frac{1}{ts} \cdot \frac{\sin \left( \omega \cdot \frac{ts}{2} \right)}{\frac{\omega}{2}} \cdot \text{Re} \left( Z_B(\omega, rd, rt) \cdot \exp(1i \cdot \omega \cdot (t)) \right) d\omega
\]

\[
v_{xout}(t, rd, rt) = ts^2 \cdot \frac{1}{ts} \cdot \frac{\sin \left( \omega \cdot \frac{ts}{2} \right)}{\frac{\omega}{2}} \cdot \text{Re} \left( 1i \cdot \omega \cdot Z_B(\omega, rd, rt) \cdot \exp(1i \cdot \omega \cdot (t)) \right) d\omega
\]

\[ t := -1 \cdot ts, -0.99 \cdot ts, 4 \cdot ts \]

\[ |ISI| \]

\[ t_{pk1} := 1.1 \cdot ts \]

\[ \frac{t_{pk1}}{ts} = 1.424 \]

\[ v_{max_{B1}} := v_{out}(t_{pk1}, 0.5 B, 0.5 B) = 0.789 \]

\[ v_{min_{B1}} := v_{out}(t_{pk1} + 1 \cdot ts, 0.5 B, 0.5 B) = 0.064 \]

\[ v_{min1_{B1}} := v_{out}(t_{pk1} + 2 \cdot ts, 0.5 B, 0.5 B) = 0.028 \]

\[ v_{min2_{B1}} := v_{out}(t_{pk1} + 3 \cdot ts, 0.5 B, 0.5 B) = 0.003 \]

\[ v_{max_{B1}} - v_{min_{B1}} = 0.725 \]

\[ v_{max_{B1}} - v_{min1_{B1}} = 0.761 \]

\[ v_{max_{B1}} - v_{min2_{B1}} = 0.786 \]

\[ \sqrt{10^{17} \cdot I_{n_B}(0.5 B, 0.5 B)} - v_{max_{B1}} - v_{min_{B1}} = 0.86 \]

\[ \sqrt{10^{17} \cdot I_{n_B}(0.5 B, 0.5 B)} - v_{max_{B1}} - v_{min1_{B1}} = 0.9 \]

\[ \sqrt{10^{17} \cdot I_{n_B}(0.5 B, 0.5 B)} - v_{max_{B1}} - v_{min2_{B1}} = 0.93 \]

\[ x_{111}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - 2 \cdot ts, rd, rt) + v_{out}(t - ts, rd, rt) \]

\[ x_{110}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - ts, rd, rt) \]

\[ x_{100}(t, rd, rt) := v_{out}(t, rd, rt) \]

\[ x_{001}(t, rd, rt) := v_{out}(t - 2 \cdot ts, rd, rt) \]
\[ x_{010}(t, rd, rt) = vout(t - ts, rd, rt) \]
\[ x_{011}(t, rd, rt) = vout(t - ts, rd, rt) + vout(t - 2\, ts, rd, rt) \]
\[ x_{101}(t, rd, rt) = vout(t, rd, rt) + vout(t - 2\, ts, rd, rt) \]

**Sens PINFET**

\[ \lambda = 650 \cdot 10^{-9} \]

\[ \text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^{8}}{\lambda} \]

\[ nq := 1.6 \cdot 10^{-19} \]

\[ Q_N(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max}_B} - v_{\text{min}_B}}{2} \right) \]

\[ P_{e\_N}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right) \]

\[ P_{c\_N}(b) := (\log(P_{e\_N}(b)) + 9) \]

\[ b \equiv 3 \cdot 10^3 \]

\[ b_{\_N} := \sqrt[3]{P_{c\_N}(b)} \]

\[ \text{minimum}_B := \min(b_{\_N}) = 2.768 \cdot 10^3 \]

\[ \text{Sens\_B\_PIN} := 10 \cdot \log \left( \frac{\text{minimum}_B \cdot \text{photon\_energy}}{2} \cdot \frac{1}{ts} \right) = -43.732 \]
B.4.5 Tuned B receiver $a=0.4$

Tuned B Optimum Performance (PIN-FET 100 Mbit/s) $a=0.4$

Rx performance opt PIN-FET input configurations, 3rd order Butterworth pre-detection filter. opt as follows:
- TIA 3-dB bandwidth $rt/\text{Filter 3-dB bandwidth } rd$
- range cut-off
- minimum noise
- examine ISI
- highest SNR
- BER
- Rx sens

Bit-rate, pulse duration

$C_T := 1.5 \cdot 10^{-12}$  total C

$B := 100 \cdot 10^6$  Bit-rate

pre-dec filter

$\omega_B(rd) := 2 \pi \cdot rd$

$H_{but}(\omega, rd) := \frac{\omega_B(rd)^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B(rd) + 2 \cdot (1j \cdot \omega) \cdot \omega_B(rd)^2 + \omega_B(rd)^3}$

TIA

Feedback value for (R) Tuned B

$\Delta_L := 1.9$  $\Delta_R := 2.75$  $\alpha := 0.4$

$A_v := 10$

$\begin{array}{c|ccc}
\alpha & \Delta_L & \Delta_R \\
0 & 2 & 1.41 \\
0.1 & 1.8 & 1.58 \\
0.2 & 1.8 & 1.87 \\
0.3 & 2.4 & 2.52 \\
0.4 & 1.9 & 2.75 \\
0.5 & 2.5 & 3.17 \\
\end{array}$

$Rf_B(rt) := \Delta_R \cdot \frac{A_v + 1}{2 \pi \cdot rt \cdot C_T}$

$Lc(rt) := \Delta_L \cdot \frac{(Rf_B(rt))^2 \cdot C_T}{1 + A_v}$

$C1 := (1 - \alpha) \cdot (C_T) = 9 \cdot 10^{-13}$

$C2 := \alpha \cdot (C_T) = 6 \cdot 10^{-13}$

$Z_{TIA,B}(\omega, rt) := \frac{1}{((1 - \omega^2 \cdot Lc(rt) \cdot C1) + \frac{Rf_B(rt)}{1 + A_v}) \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc(rt) \cdot C1 \cdot C2)}$

Receiver freq-response

$Z_B(\omega, rd, rt) := H_{but}(\omega, rd) \cdot Z_{TIA,B}(\omega, rt)$

$\omega := 1 \cdot 10^6, 10 \cdot 10^6 \ldots 1 \cdot 10^{11}$

$rd := 0.5 \cdot B, 0.75 \cdot B \ldots 2 \cdot B$

$rt := 0.5 \cdot B, 0.55 \cdot B \ldots 1 \cdot B$

Noise equivalent bandwidth (NEB and I2)
\[ \text{NEB}_B(rd, rt) = \frac{1}{2 \cdot \pi} \int_0^{10^{12}} \left( \frac{1}{\left| Z_B(\omega, rd, rt) \right|^2} \right) d\omega \]

\[ I_{2,B}(rd, rt) = \frac{1}{2 \cdot \pi} \int_0^{10^{12}} \left( \frac{\left( (1 - \omega^2 \cdot Lc(rt) \cdot C1) \right)}{\left| Z_B(\omega, rd, rt) \right|^2} \right) d\omega \]

**Noise equivalent bandwidth (I3)**

\[ I_{3,B}(rd, rt) = \frac{1}{2 \cdot \pi} \int_0^{10^{12}} \left( \frac{\left( \left( (1 - \omega^2 \cdot Lc(rt) \cdot C1) \right) + RF_B(rt) \cdot (\omega) \cdot \left( C1 + C2 - (\omega)^2 \cdot Lc(rt) \cdot C1 \cdot C2 \cdot 1\right) \right)}{RF_B(rt) \cdot Z_B(\omega, rd, rt)} \right) d\omega \]

**Feedback noise**

\[ q = 1.6 \cdot 10^{-19} \quad k = 1.38 \cdot 10^{-23} \quad T = 298 \]

\[ I_{nRF,B}(rd, rt) = \frac{4 \cdot k \cdot T}{RF_B(rt)} \cdot I_{2,B}(rd, rt) \]

**Ig noise (gate current)**

\[ Ig = 10 \cdot 10^{-9} \]

\[ I_{nG,B}(rd, rt) = 2 \cdot q \cdot Ig \cdot I_{2,B}(rd, rt) \]

**Channel noise (Gate-source)**

\[ gm1 = 30 \cdot 10^{-3} = 0.03 \quad \text{noise_factor} = 1 \]

\[ I_{nD} = 4 \cdot k \cdot T \cdot \frac{1}{gm1} \]

\[ I_{nD,B}(rd, rt) = I_{nD} \cdot I_{3,B}(rd, rt) \]

**Total noise- PIN-FET**

\[ I_{n,B}(rd, rt) = I_{nRF,B}(rd, rt) + I_{nG,B}(rd, rt) + I_{nD,B}(rd, rt) \]
Output pulse shape

\[ t_s := \frac{1}{B} \]

\[
vout(t, rd, rt) := t_s \cdot \frac{1}{t_s \cdot 10^3} \int_{-1}^{1} \sin \left( \frac{\omega \cdot t_s}{2} \right) \cdot \text{Re} \left( Z_B(\omega, rd, rt) \cdot \exp \left( 1i \cdot \omega \cdot (t) \right) \right) d\omega
\]

\[
xvout(t, rd, rt) := t_s^2 \cdot \frac{1}{\pi} \int_{0}^{\frac{1}{t_s \cdot 10^3}} \sin \left( \frac{\omega \cdot t_s}{2} \right) \cdot \text{Re} \left( 1i \cdot \omega \cdot Z_B(\omega, rd, rt) \cdot \exp \left( 1i \cdot \omega \cdot (t) \right) \right) d\omega
\]

\[ t := -1 \cdot t_s, -0.99 \cdot t_s, \ldots, 8 \cdot t_s \]

\[ t_{pk1} := \text{root} \left( xvout(t_{B1}, 0.5 B, 0.5 B), t_{B1} \right) \]

\[ t_{pk1} = 1.577 \cdot t_s \]

\[ v_{max_{B1}} := vout(t_{pk1}, 0.5 B, 0.5 B) = 0.816 \]

\[ v_{min_{B1}} := vout(t_{pk1} + 1 \cdot t_s, 0.5 B, 0.5 B) = 0.001 \]

\[ v_{min_{1_{B1}}} := vout(t_{pk1} + 2 \cdot t_s, 0.5 B, 0.5 B) = 0.008 \]

\[ v_{min_{2_{B1}}} := vout(t_{pk1} + 3 \cdot t_s, 0.5 B, 0.5 B) = 0.039 \]

\[ v_{max_{B1}} - v_{min_{B1}} = 0.815 \quad v_{max_{B1}} - v_{min_{1_{B1}}} = 0.808 \quad v_{max_{B1}} - v_{min_{2_{B1}}} = 0.777 \]

\[ \frac{v_{max_{B1}} - v_{min_{B1}}}{\sqrt{10^{17} \cdot I_{n_B}(0.5 B, 0.5 B)}} = 0.96 \quad \text{1st pulse} \]

\[ \frac{v_{max_{B1}} - v_{min_{B1}}}{\sqrt{10^{17} \cdot I_{n_B}(0.5 B, 0.5 B)}} = 0.96 \quad \text{2nd pulse} \]

\[ \frac{v_{max_{B1}} - v_{min_{2_{B1}}}}{\sqrt{10^{17} \cdot I_{n_B}(0.5 B, 0.5 B)}} = 0.92 \quad \text{3rd pulse \ worst ISI} \]

\[ v_{max_B} := v_{max_{B1}} \quad v_{min_B} := v_{min_{2_{B1}}} \]

\[ x_{111}(t, rd, rt) := vout(t, rd, rt) + vout(t - 2 \cdot ts, rd, rt) + vout(t - ts, rd, rt) \]

\[ x_{110}(t, rd, rt) := vout(t, rd, rt) + vout(t - ts, rd, rt) \]

\[ x_{100}(t, rd, rt) := vout(t, rd, rt) \]

\[ x_{001}(t, rd, rt) := vout(t - 2 \cdot ts, rd, rt) \]

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\[ x_{010}(t, \text{rd}, \text{rt}) = v_{out}(t - t_s, \text{rd}, \text{rt}) \]
\[ x_{011}(t, \text{rd}, \text{rt}) = v_{out}(t - t_s, \text{rd}, \text{rt}) + v_{out}(t - 2 \times t_s, \text{rd}, \text{rt}) \]
\[ x_{101}(t, \text{rd}, \text{rt}) = v_{out}(t, \text{rd}, \text{rt}) + v_{out}(t - 2 \times t_s, \text{rd}, \text{rt}) \]

\[ \text{Sens PINFET} \]
\[ \lambda := 650 \times 10^{-9} \]
\[ \text{photon}_\text{energy} := \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda} \]
\[ n_q := 1.6 \times 10^{-19} \]
\[ Q_N(b) := \frac{1}{t_s} \cdot b \cdot n_q \cdot \frac{(v_{\text{max}, B} - v_{\text{min}, B})}{\sqrt{I_{n,B}(0.5 \text{ B}, 0.5 \text{ B})}} \]
\[ P_e_N(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right) \]
\[ P_c_N(b) := (\log(P_e_N(b)) + 9) \]
\[ b \equiv 3 \times 10^3 \]
\[ b_N := \text{root}(P_c_N(b), b) \]
\[ \text{minimum}_B := \min(b_N) = 2.577 \times 10^3 \]
\[ \text{Sens}_B\text{ PIN} := 10 \cdot \log \left( \frac{\text{minimum}_B}{2}, \frac{\text{photon}_\text{energy}}{10^{-3}}, \frac{1}{t_s} \right) = -44.042 \]
B.4.6 Performance comparison PINFET

Performance comparison (PIN-FET 3rd order Butterworth 100 Mbit/s)

Rx performance with PIN-FET input configuration, 3rd order Butterworth pre-detection filter. Calculations are as follows:

- Frequency response (opt rd, rt for each receiver)
- Noise integrals
- Total noise
- Pulse shaping, peak voltage, ISI
- Error bit rate, minimum number of photons, receiver sensitivity

**Bit-rate, pulse duration**

\[ C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]

\[ B := 100 \cdot 10^6 \quad \text{Bit-rate} \]

**TIA(Non-tuned)**

\[ f_c := 0.5 \cdot B \]

\[ Av := 10 \]

\[ \omega_c := 2 \cdot \pi \cdot 0.5 \cdot B \]

\[ Rf_N := \frac{Av + 1}{2 \cdot \pi \cdot 0.5 \cdot B \cdot C_T} = 2.334 \cdot 10^4 \]

\[ Z_{TIA,N}(\omega) := \frac{1}{1 + \frac{\omega}{2 \cdot \pi \cdot 0.5 \cdot B}} \]

**TIA(Tuned-A)**

\[ m := 1.8 \]

\[ y := \sqrt{\left(\frac{-m^2}{2} + m + 1\right) + \sqrt{\left(\frac{-m^2}{2} + m + 1\right)^2 + m^2}} = 1.825 \]

\[ d := \frac{1}{y} \cdot B \]

\[ f_c := 0.5 \cdot d = 2.739 \cdot 10^7 \]

\[ RA := \frac{1}{(2 \cdot \pi \cdot C_T \cdot f_c)} = 3.874 \cdot 10^3 \]

\[ L := RA^2 \cdot \frac{C_T}{m} = 1.25 \cdot 10^{-5} \]

\[ ts := \frac{1}{y \cdot d} \quad B := \frac{1}{ts} = 1 \cdot 10^8 \]

\[ f_c := 0.5 \cdot B = 5 \cdot 10^7 \]

\[ Rf_A := RA \cdot (1 + Av) = 4.261 \cdot 10^4 \]

\[ Lf := L \cdot (1 + Av) = 1.376 \cdot 10^{-4} \]
\[ Z_{TIA,A}(\omega) := \frac{1 + i\omega \cdot \frac{L_f}{Rf_A}}{\left(1 - \omega^2 \cdot \frac{C_T}{1 + Av}\right) + Rf_A \omega \cdot \frac{C_T}{(1 + Av)} \cdot i} \]

**TIA**

Feedback value for (R) Tuned B

\[ \Delta_L := 1.9 \quad \Delta_R := 2.75 \]

feedback \( \Delta R \), time constant ratio \( \Delta L \)

\[ Rf_B := \Delta_R \cdot \frac{Av + 1}{2 \cdot \pi \cdot 0.5 \cdot B \cdot C_T} = 6.419 \cdot 10^4 \]

feedback for Tuned B

\( \alpha := 0.4 \)

splitting ratio

\[ Lc := \frac{(Rf_B)^2}{1 + Av} \cdot C_T \]

\[ \Delta_L = 2.689 \cdot 10^{-5} \]

\[ C1 := (1 - \alpha) \cdot (C_T) = 9 \cdot 10^{-13} \]

\[ C2 := \alpha \cdot (C_T) = 6 \cdot 10^{-13} \]

**TUNED B (R) Transimpedance**

\[ Z_{TIA,B}(\omega) := \frac{1}{\left(1 - \omega^2 \cdot Lc \cdot C1\right) + \frac{Rf_B}{1 + Av} \cdot (\omega \cdot i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2)} \]

**Receiver freq-response**

\[ \omega_{B,1} := 2 \cdot \pi \cdot 0.7 \cdot B \]

\[ \omega_{B,2} := 2 \cdot \pi \cdot 0.5 \cdot B \]

\[ H_{bnt,N,A}(\omega) := \frac{\omega_{B,1}^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_{B,1} + 2 \cdot (1j \cdot \omega) \cdot \omega_{B,1}^2 + \omega_{B,1}^3} \]

\[ H_{bnt,B}(\omega) := \frac{\omega_{B,2}^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_{B,2} + 2 \cdot (1j \cdot \omega) \cdot \omega_{B,2}^2 + \omega_{B,2}^3} \]

\[ Z_{nontuned}(\omega) := H_{bnt,N,A}(\omega) \cdot Z_{TIA,N}(\omega) \]

\[ Z_{tuned,A}(\omega) := H_{bnt,N,A}(\omega) \cdot Z_{TIA,A}(\omega) \]

\[ Z_{tuned,B}(\omega) := H_{bnt,B}(\omega) \cdot Z_{TIA,B}(\omega) \]

\[ \omega := 1 \cdot 10^6, 10 \cdot 10^6, \ldots, 1 \cdot 10^{11} \]

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Noise equivalent bandwidth

\[
NEB_N := \frac{1}{2 \cdot \pi} \int_0^{10^{12}} (|Z_{\text{nontuned}}(\omega)|^2) \, d\omega = 4.722 \times 10^7
\]

\[
NEB_A := \frac{1}{2 \cdot \pi} \int_0^{10^{12}} (|Z_{\text{tuned A}}(\omega)|^2) \, d\omega = 5.341 \times 10^7
\]

\[
NEB_B := \frac{1}{2 \cdot \pi} \int_0^{10^{12}} (|Z_{\text{tuned B}}(\omega)|^2) \, d\omega = 4.09 \times 10^7
\]
Noise integrals

\[ I_{2,N} = NEB_N = 4.722 \cdot 10^7 \]

\[ I_{2,A} := \frac{1}{2 \cdot \pi} \int_{0}^{10^{12}} \left( \left( \frac{1}{1 + 1i \cdot \omega} \cdot \frac{L_f}{Rf_A} \right) \cdot Z_{tuned_A}(\omega) \right)^2 d\omega = 4.057 \cdot 10^7 \]

\[ I_{2,B} := \frac{1}{2 \cdot \pi} \int_{0}^{10^{12}} \left( \left( 1 - \omega^2 \cdot Lc \cdot C1 \right) \cdot Z_{tuned_B}(\omega) \right)^2 d\omega = 2.587 \cdot 10^7 \]

\[ I_{3,N} := \frac{1}{2 \cdot \pi} \int_{0}^{10^{12}} \left( \left( 1 + 1j \cdot (\omega \cdot Rf_N \cdot C_T) \cdot Z_{nontuned}(\omega) \right) \right)^2 d\omega = 5.88 \]

\[ I_{3,A} := \frac{1}{2 \cdot \pi} \int_{0}^{10^{12}} \left( \left( \left( 1 - \omega^2 \cdot Lf \cdot C_T \right) + Rf_A \cdot \omega \cdot C_T \cdot 1i \right) \cdot Z_{tuned_A}(\omega) \right)^2 d\omega = 5.004 \]

\[ I_{3,B} := \frac{1}{2 \cdot \pi} \int_{0}^{10^{12}} \left( \left( \left( 1 - \omega^2 \cdot Lc \cdot C1 \right) \right) \cdot \left( \frac{1}{Rf_B} \cdot \left( C1 + C2 - (\omega^2 \cdot Lc \cdot C1 \cdot C2) \cdot 1i \right) \right) \right)^2 d\omega = 0.784 \]

Feedback noise

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \]

\[ I_{nRf,N} = \frac{4 \cdot k \cdot T}{Rf_N} \cdot I_{2,N} = 3.327 \cdot 10^{-17} \]

\[ I_{nRf,A} = \frac{4 \cdot k \cdot T}{Rf_A} \cdot I_{2,A} = 1.566 \cdot 10^{-17} \]

\[ I_{nRf,B} = \frac{4 \cdot k \cdot T}{Rf_B} \cdot I_{2,B} = 6.629 \cdot 10^{-18} \]

\[ 10 \log \left( \frac{I_{nRf,B}}{I_{nRf,N}} \right) = -7.007 \]

\[ 10 \log \left( \frac{I_{nRf,A}}{I_{nRf,N}} \right) = -3.273 \]

Ig noise (gate current)

\[ Ig := 10 \cdot 10^{-9} \]
\[ I_{nG,N} := 2 \cdot q \cdot I_g \cdot I_{2,N} = 1.511 \cdot 10^{-19} \]
\[ I_{nG,A} := 2 \cdot q \cdot I_g \cdot NEB_A = 1.709 \cdot 10^{-19} \]
\[ I_{nG,B} := 2 \cdot q \cdot I_g \cdot I_{2,B} = 8.278 \cdot 10^{-20} \]

\[ 10 \log \left( \frac{I_{nG,B}}{I_{nG,N}} \right) = -2.613 \]
\[ 10 \log \left( \frac{I_{nG,A}}{I_{nG,N}} \right) = 0.535 \]

\text{Tuned A NEB > non-tuned NEB}

\begin{align*}
\text{Channel noise (Gate-source)} \\
g_{m1} &= 30 \cdot 10^{-3} = 0.03 \quad \text{noise\_factor} := 1 \\
I_{nD} &= 4 \cdot k \cdot T \cdot \frac{1}{g_{m1}} \\
I_{nD,N} &= I_{nD} \cdot I_{3,N} = 3.224 \cdot 10^{-18} \\
I_{nD,A} &= I_{nD} \cdot I_{3,A} = 2.744 \cdot 10^{-18} \\
I_{nD,B} &= I_{nD} \cdot I_{3,B} = 4.3 \cdot 10^{-19} \\
10 \log \left( \frac{I_{nD,B}}{I_{nD,N}} \right) &= -8.749 \\
10 \log \left( \frac{I_{nD,A}}{I_{nD,N}} \right) &= -0.7
\end{align*}

\text{Total noise- PIN-FET}

\begin{align*}
I_{n,N} &= I_{nRF,N} + I_{nG,N} + I_{nD,N} = 3.665 \cdot 10^{-17} \\
I_{n,A} &= I_{nRF,A} + I_{nG,A} + I_{nD,A} = 1.858 \cdot 10^{-17} \\
I_{n,B} &= I_{nRF,B} + I_{nG,B} + I_{nD,B} = 7.141 \cdot 10^{-18} \\
10 \log \left( \frac{I_{n,B}}{I_{n,N}} \right) &= -7.103 \\
10 \log \left( \frac{I_{n,A}}{I_{n,N}} \right) &= -2.951
\end{align*}

\text{Output pulse shape}
\[ t_s := \frac{1}{B} \]
\[ t := -6 \cdot t_s, -5.99 \cdot t_s, \ldots, 6 \cdot t_s \]
\[ \frac{1}{t_s} \cdot 10^3 \]
\[ v_{out,N}(t) := \frac{t_s}{\pi} \cdot \sin \left( \frac{\omega \cdot t_s}{2} \right) \cdot \text{Re} \left[ Z_{nontuned}(\omega) \cdot \exp \left( 1i \cdot \omega \cdot \left( t - \frac{t_s}{2} \right) \right) \right] \text{d}\omega \]

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\[
v_{out,A}(t) := \frac{ts}{\pi} \cdot \frac{1}{ts \cdot 10^3} \int_0^{\frac{1}{ts} \cdot 10^3} \left( \frac{\sin \left( \frac{\omega \cdot ts}{2} \right)}{\left( \frac{\omega \cdot ts}{2} \right)} \cdot \text{Re}\left( Z_{tuned,A}(\omega) \cdot \exp \left( \frac{1i \cdot \omega \left( t - \frac{ts}{2} \right)}{2} \right) \right) \right) d\omega
\]

\[
v_{out,B}(t) := \frac{ts}{\pi} \cdot \frac{1}{ts \cdot 10^3} \int_0^{\frac{1}{ts} \cdot 10^3} \left( \frac{\sin \left( \frac{\omega \cdot ts}{2} \right)}{\left( \frac{\omega \cdot ts}{2} \right)} \cdot \text{Re}\left( Z_{tuned,B}(\omega) \cdot \exp \left( \frac{1i \cdot \omega \left( t - \frac{ts}{2} \right)}{2} \right) \right) \right) d\omega
\]

**peak voltage and time**

\[
x_{out,N}(t) := \frac{ts^2}{\pi} \cdot \frac{1}{ts \cdot 10^3} \int_0^{\frac{1}{ts} \cdot 10^3} \left( \frac{\sin \left( \frac{\omega \cdot ts}{2} \right)}{\left( \frac{\omega \cdot ts}{2} \right)} \cdot \text{Re}\left( 1i \cdot \omega \cdot Z_{nontuned}(\omega) \cdot \exp \left( \frac{1i \cdot \omega \left( t - \frac{ts}{2} \right)}{2} \right) \right) \right) d\omega
\]

\[
x_{out,A}(t) := \frac{ts^2}{\pi} \cdot \frac{1}{ts \cdot 10^3} \int_0^{\frac{1}{ts} \cdot 10^3} \left( \frac{\sin \left( \frac{\omega \cdot ts}{2} \right)}{\left( \frac{\omega \cdot ts}{2} \right)} \cdot \text{Re}\left( 1i \cdot \omega \cdot Z_{tuned,A}(\omega) \cdot \exp \left( \frac{1i \cdot \omega \left( t - \frac{ts}{2} \right)}{2} \right) \right) \right) d\omega
\]

\[
x_{out,B}(t) := \frac{ts^2}{\pi} \cdot \frac{1}{ts \cdot 10^3} \int_0^{\frac{1}{ts} \cdot 10^3} \left( \frac{\sin \left( \frac{\omega \cdot ts}{2} \right)}{\left( \frac{\omega \cdot ts}{2} \right)} \cdot \text{Re}\left( 1i \cdot \omega \cdot Z_{tuned,B}(\omega) \cdot \exp \left( \frac{1i \cdot \omega \left( t - \frac{ts}{2} \right)}{2} \right) \right) \right) d\omega
\]

\[t_N := t_s\]

\[t_{pk1} := \text{root}(x_{out,N}(t_N), t_N)\]

\[\frac{t_{pk1}}{ts} = 1.27\]

\[v_{max,N} := x_{out,N}(t_{pk1}) = 0.918\]
\[ v_{\text{min}N} := v_{\text{out}}(t_{pk1} + 1 \cdot ts) = 0.043 \]
\[ v_{\text{min2ts}N} := v_{\text{out}}(t_{pk1} + 2 \cdot ts) = 0.009 \]
\[ v_{\text{min3ts}N} := v_{\text{out}}(t_{pk1} + 3 \cdot ts) = -6.008 \cdot 10^{-4} \]
\[ t_A := 1 \cdot ts \]
\[ t_{pk1} := \text{root}(x_{\text{vout}}(t_A), t_A) \]
\[ \frac{t_{pk1}}{ts} = 1.37 \]
\[ v_{\text{max}A} := v_{\text{out}}(t_{pk1}) = 0.978 \]
\[ v_{\text{min}A} := v_{\text{out}}(t_{pk1} + 1 \cdot ts) = 0.015 \]
\[ v_{\text{min2ts}A} := v_{\text{out}}(t_{pk1} + 2 \cdot ts) = -0.054 \]
\[ v_{\text{min3ts}A} := v_{\text{out}}(t_{pk1} + 3 \cdot ts) = 0.002 \]

\[ t_B := 2 \cdot ts \]
\[ t_{pk1} := \text{root}(x_{\text{vout}}(t_B), t_B) \]
\[ \frac{t_{pk1}}{ts} = 2.077 \]
\[ v_{\text{max}B} := v_{\text{out}}(t_{pk1}) = 0.816 \]
\[ v_{\text{min1ts}B} := v_{\text{out}}(t_{pk1} + 1 \cdot ts) = 0.001 \]
\[ v_{\text{min2ts}B} := v_{\text{out}}(t_{pk1} + 2 \cdot ts) = 0.008 \]
\[ v_{\text{min3ts}B} := v_{\text{out}}(t_{pk1} + 3 \cdot ts) = 0.039 \]
\[ v_{\text{min}B} := v_{\text{min3ts}B} \]

**BER**

\[ nq := 1.6 \cdot 10^{-19} \quad \text{Test}_{\text{b photons}} := 1100, 1200 \ldots 12000 \]

\[ \text{test}_{Q N}(\text{Test}_{\text{b photons}}) := \frac{1}{ts} \cdot \text{Test}_{\text{b photons}} \cdot nq \cdot \left( \frac{v_{\text{max}N} - v_{\text{min}N}}{2} \right) \sqrt{I_{nN}} \]
\[ \text{test}_{Q A}(\text{Test}_{\text{b photons}}) := \frac{1}{ts} \cdot \text{Test}_{\text{b photons}} \cdot nq \cdot \left( \frac{v_{\text{max}A} - v_{\text{min}A}}{2} \right) \sqrt{I_{nA}} \]

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\[ test_{Q,B}(Test_{b\text{photons}}) := \frac{1}{ts} \cdot Test_{b\text{photons}} \cdot nq \cdot \left( \frac{v_{\text{max}_B} - v_{\text{min}_B}}{2} \right) \sqrt{I_{n_B}} \]

\[ Test1(Test_{b\text{photons}}) := \frac{1}{2} \cdot \text{erfc} \left( \frac{test_{Q,N}(Test_{b\text{photons}})}{\sqrt{2}} \right) \]

\[ Test2(Test_{b\text{photons}}) := \frac{1}{2} \cdot \text{erfc} \left( \frac{test_{Q,A}(Test_{b\text{photons}})}{\sqrt{2}} \right) \]

\[ Test3(Test_{b\text{photons}}) := \frac{1}{2} \cdot \text{erfc} \left( \frac{test_{Q,B}(Test_{b\text{photons}})}{\sqrt{2}} \right) \]

\[ Q_N(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max}_N} - v_{\text{min}_N}}{2} \right) \sqrt{I_{n_N}} \]

\[ Q_A(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max}_A} - v_{\text{min}_A}}{2} \right) \sqrt{I_{n_A}} \]

\[ Q_B(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max}_B} - v_{\text{min}_B}}{2} \right) \sqrt{I_{n_B}} \]
\[ P_{e,N}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right) \]
\[ P_{e,A}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_A(b)}{\sqrt{2}} \right) \]
\[ P_{e,B}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_B(b)}{\sqrt{2}} \right) \]
\[ P_{c,N}(b) := (\log(P_{e,N}(b))) + 9 \]
\[ P_{c,A}(b) := (\log(P_{e,A}(b))) + 9 \]
\[ P_{c,B}(b) := (\log(P_{e,B}(b))) + 9 \]
\[ b \equiv 3 \cdot 10^3 \]
\[ b_N := \text{root} (P_{c,N}(b), b) \]
\[ b_A := \text{root} (P_{c,A}(b), b) \]
\[ b_B := \text{root} (P_{c,B}(b), b) \]
\[ \text{minimum}_N := \min (b_N) = 5.187 \cdot 10^3 \]
\[ \text{minimum}_A := \min (b_A) = 3.359 \cdot 10^3 \]
\[ \text{minimum}_B := \min (b_B) = 2.577 \cdot 10^3 \]

**BER-check for minimum \( b \)**

| \( Q_N(\text{minimum}_N) = 5.998 \) | \( P_{e,N}(\text{minimum}_N) = 10 \cdot 10^{-10} \) |
| \( Q_A(\text{minimum}_A) = 5.998 \) | \( P_{e,A}(\text{minimum}_A) = 1 \cdot 10^{-9} \) |
| \( Q_B(\text{minimum}_B) = 5.998 \) | \( P_{e,B}(\text{minimum}_B) = 10 \cdot 10^{-10} \) |

**Sens PIN-FET**

\[ \lambda := 650 \cdot 10^{-9} \]

\[ \text{photon} \_\text{energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ \text{Sens}_N := 10 \cdot \log \left( \frac{\text{minimum}_N}{2} \cdot \frac{\text{photon} \_\text{energy}}{10^{-3}} \cdot \frac{1}{\text{ts}} \right) = -41.004 \]

\[ \text{Sens}_A := 10 \cdot \log \left( \frac{\text{minimum}_A}{2} \cdot \frac{\text{photon} \_\text{energy}}{10^{-3}} \cdot \frac{1}{\text{ts}} \right) = -42.892 \]

\[ \text{Sens}_B := 10 \cdot \log \left( \frac{\text{minimum}_B}{2} \cdot \frac{\text{photon} \_\text{energy}}{10^{-3}} \cdot \frac{1}{\text{ts}} \right) = -44.042 \]

\[ \text{Sens}_B - \text{Sens}_N = -3.038 \]

\[ \text{Sens}_A - \text{Sens}_N = -1.887 \]

#
B.4.7 Performance comparison APDFET

**Silicon APD:** (Performance comparison (APD-FET 3rd order Butterworth 100 Mbit/s))

Rx performance with APD-FET input configuration, 3rd order Butterworth pre-detection filter, calculations are as follow

- APD noise, APD BER, APD receiver sensitivity
- [Silicon Epitaxial: APD M= 10 F(M)=5.5]

**APD noise**

\[ M_{APD} = 10 \]
\[ F_M = 5.5 \]
\[ APD_N(b) = \frac{1}{t_s} \cdot b \cdot nq \cdot \left( v_{max,N} \right) \]
\[ APD_A(b) = \frac{1}{t_s} \cdot b \cdot nq \cdot \left( v_{max,A} \right) \]
\[ APD_B(b) = \frac{1}{t_s} \cdot b \cdot nq \cdot \left( v_{max,B} \right) \]
\[ I_{APD,d} = 10 \cdot 10^{-9} \]
\[ Noise_{APD_N}(b) = \left( 2 \cdot q \cdot APD_N(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_N \]
\[ + 2 \cdot q \cdot I_{APD,d} \cdot M_{APD}^2 \cdot F_M \]
\[ Noise_{APD_A}(b) = \left( 2 \cdot q \cdot APD_A(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_A \]
\[ + 2 \cdot q \cdot I_{APD,d} \cdot M_{APD}^2 \cdot F_M \]
\[ Noise_{APD_B}(b) = \left( 2 \cdot q \cdot APD_B(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_B \]
\[ + 2 \cdot q \cdot I_{APD,d} \cdot M_{APD}^2 \cdot F_M \]

**BER+APD**

\[ Q_{APD_N}(b) = \frac{M_{APD} \cdot b \cdot nq \cdot \left( v_{max,N} - v_{min,N} \right)}{\sqrt{I_{n,N} + Noise_{APD_N}(b)}} \]
\[ Q_{APD_A}(b) = \frac{M_{APD} \cdot b \cdot nq \cdot \left( v_{max,A} - v_{min,A} \right)}{\sqrt{I_{n,A} + Noise_{APD_A}(b)}} \]
\[ Q_{APD_B}(b) = \frac{M_{APD} \cdot b \cdot nq \cdot \left( v_{max,B} - v_{min,B} \right)}{\sqrt{I_{n,B} + Noise_{APD_B}(b)}} \]

\[ P_{e,APD_N}(b) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD_N}(b)}{\sqrt{2}} \right) \]
\[ P_{e,APD_A}(b) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD_A}(b)}{\sqrt{2}} \right) \]
\[ P_{e,APD_B}(b) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD_B}(b)}{\sqrt{2}} \right) \]
\[ \text{pc}_{APD_N}(b) := \log(P_e \cdot APD_N(b)) + 9 \]
\[ \text{pc}_{APD_A}(b) := \log(P_e \cdot APD_A(b)) + 9 \]
\[ \text{pc}_{APD_B}(b) := \log(P_e \cdot APD_B(b)) + 9 \]
\[ APD_b_N := \text{root}(\text{pc}_{APD_N}(b), b) \]
\[ APD_b_A := \text{root}(\text{pc}_{APD_A}(b), b) \]
\[ APD_b_B := \text{root}(\text{pc}_{APD_B}(b), b) \]
\[ \text{minimum}_{APD_N} := \min(\text{APD}_b_N) = 1.487 \cdot 10^3 \]
\[ \text{minimum}_{APD_A} := \min(\text{APD}_b_A) = 1.386 \cdot 10^3 \]
\[ \text{minimum}_{APD_B} := \min(\text{APD}_b_B) = 1.4 \cdot 10^3 \]

**Check BER, Q, and b APD**

\[ Q_{APD_N}(\min(\text{APD}_b_N)) = 5.998 \]
\[ Q_{APD_A}(\min(\text{APD}_b_A)) = 5.998 \]
\[ Q_{APD_B}(\min(\text{APD}_b_B)) = 5.998 \]

**Sens**

\[ APD_N := 10 \cdot \log \left( \frac{\text{minimum}_{APD_N} \cdot \text{photon\_energy} \cdot \frac{1}{ts}}{10^{-3}} \right) = -46.43 \]
\[ APD_A := 10 \cdot \log \left( \frac{\text{minimum}_{APD_A} \cdot \text{photon\_energy} \cdot \frac{1}{ts}}{10^{-3}} \right) = -46.735 \]
\[ APD_B := 10 \cdot \log \left( \frac{\text{minimum}_{APD_B} \cdot \text{photon\_energy} \cdot \frac{1}{ts}}{10^{-3}} \right) = -46.692 \]
\[ \text{Sens}_N - APD_N = 5.426 \quad \text{Noise}_{APD_N}(\text{minimum}_{APD_N}) = 2.646 \cdot 10^{-16} \]
\[ \text{Sens}_A - APD_A = 3.844 \quad \text{Noise}_{APD_A}(\text{minimum}_{APD_A}) = 2.978 \cdot 10^{-16} \]
\[ \text{Sens}_B - APD_B = 2.65 \quad \text{Noise}_{APD_B}(\text{minimum}_{APD_B}) = 2.036 \cdot 10^{-16} \]
\[ I_{n,N} = 3.665 \cdot 10^{-17} \quad I_{n,A} = 1.858 \cdot 10^{-17} \quad I_{n,B} = 7.141 \cdot 10^{-18} \]
**InGaAs_APD**: Performance comparison (APD-FET 3rd order Butterworth 100 Mbit/s)

Rx performance with APD-FET input configuration, 3rd order Butterworth pre-detection filter, calculations are as follow

- APD noise, APD BER, APD receiver sensitivity
- \([\text{InGaAs; } M= 100 \text{ F(M)=7.9}]\)

**APD noise**

\[ M_{\text{APD}} := 100 \]
\[ F_M := 7.9 \]
\[ APD_N(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot v_{\text{max},N} \]
\[ APD_A(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot v_{\text{max},A} \]
\[ APD_B(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot v_{\text{max},B} \]
\[ I_{\text{APD,d}} := 10 \cdot 10^{-9} \]

\[ \begin{align*}
\text{Noise}_{APD,N}(b) &= \left( 2 \cdot q \cdot APD_N(b) \cdot M_{\text{APD}}^2 \cdot F_M \right) \cdot \text{NEB}_N \\
&\quad + 2 \cdot q \cdot I_{\text{APD,d}} \cdot M_{\text{APD}}^2 \cdot F_M \\
\text{Noise}_{APD,A}(b) &= \left( 2 \cdot q \cdot APD_A(b) \cdot M_{\text{APD}}^2 \cdot F_M \right) \cdot \text{NEB}_A \\
&\quad + 2 \cdot q \cdot I_{\text{APD,d}} \cdot M_{\text{APD}}^2 \cdot F_M \\
\text{Noise}_{APD,B}(b) &= \left( 2 \cdot q \cdot APD_B(b) \cdot M_{\text{APD}}^2 \cdot F_M \right) \cdot \text{NEB}_B \\
&\quad + 2 \cdot q \cdot I_{\text{APD,d}} \cdot M_{\text{APD}}^2 \cdot F_M 
\end{align*} \]

**BER+APD**

\[ Q_{APD,N}(b) := \frac{M_{\text{APD}} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max},N} - v_{\text{min},N}}{2} \right)}{\sqrt{I_{n,N} + \text{Noise}_{APD,N}(b)}} \]
\[ Q_{APD,A}(b) := \frac{M_{\text{APD}} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max},A} - v_{\text{min},A}}{2} \right)}{\sqrt{I_{n,A} + \text{Noise}_{APD,A}(b)}} \]
\[ Q_{APD,B}(b) := \frac{M_{\text{APD}} \cdot b \cdot nq \cdot \left( \frac{v_{\text{max},B} - v_{\text{min},B}}{2} \right)}{\sqrt{I_{n,B} + \text{Noise}_{APD,B}(b)}} \]

\[ P_{e,APD,N}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD,N}(b)}{\sqrt{2}} \right) \]
\[ P_{e,APD,A}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD,A}(b)}{\sqrt{2}} \right) \]
\[ P_{e, APD_B}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD_B}(b)}{\sqrt{2}} \right) \]

\[ pc_{APD_N}(b) := (\log(P_{e, APD_N}(b)) + 9) \]

\[ pc_{APD_A}(b) := (\log(P_{e, APD_A}(b)) + 9) \]

\[ pc_{APD_B}(b) := (\log(P_{e, APD_B}(b)) + 9) \]

\[ APD_b_N := \text{root}(pc_{APD_N}(b), b) \]

\[ APD_b_A := \text{root}(pc_{APD_A}(b), b) \]

\[ APD_b_B := \text{root}(pc_{APD_B}(b), b) \]

\[ \text{minimum}_{APD_N} := \min(APD_b_N) = 1.781 \cdot 10^3 \]

\[ \text{minimum}_{APD_A} := \min(APD_b_A) = 1.751 \cdot 10^3 \]

\[ \text{minimum}_{APD_B} := \min(APD_b_B) = 1.793 \cdot 10^3 \]

Check BER, Q, and b APD

\[ Q_{APD_N}(\min(APD_b_N)) = 5.998 \]

\[ Q_{APD_A}(\min(APD_b_A)) = 5.998 \]

\[ Q_{APD_B}(\min(APD_b_B)) = 5.998 \]

**Sens**

\[ APD_N := 10 \cdot \log \left( \frac{\text{minimum}_{APD_N}}{2} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -45.647 \]

\[ APD_A := 10 \cdot \log \left( \frac{\text{minimum}_{APD_A}}{2} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -45.72 \]

\[ APD_B := 10 \cdot \log \left( \frac{\text{minimum}_{APD_B}}{2} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -45.618 \]

\[ Sens_N - APD_N = 4.643 \quad Noise_{APD_N}(\text{minimum}_{APD_N}) = 4.315 \cdot 10^{-14} \]

\[ Sens_A - APD_A = 2.828 \quad Noise_{APD_A}(\text{minimum}_{APD_A}) = 5.049 \cdot 10^{-14} \]

\[ Sens_B - APD_B = 1.576 \quad Noise_{APD_B}(\text{minimum}_{APD_B}) = 3.456 \cdot 10^{-14} \]

\[ I_{n,N} = 3.665 \cdot 10^{-17} \quad I_{n,A} = 1.858 \cdot 10^{-17} \quad I_{n,B} = 7.141 \cdot 10^{-18} \]
**Germanium_APD:** Performance comparison APD-FET 3rd order Butterworth 100 Mbit/s

Rx performance with APD-FET input configuration, 3rd order Butterworth pre-detection filter, calculations are as follow

- APD noise, APD BER, APD receiver sensitivity
- [Germanium: M=10 F(M)=9.2]

**APD noise**

\[
M_{APD} := 10 \\
F_M := 9.2
\]

\[
APD_N(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot (v_{max,N})
\]

\[
APD_A(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot (v_{max,A})
\]

\[
APD_B(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot (v_{max,B})
\]

\[
I_{APD_d} := 10 \cdot 10^{-9}
\]

\[
Noise_{APD,N}(b) := \left(2 \cdot q \cdot APD_N(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_N + 2 \cdot q \cdot I_{APD_d} \cdot M_{APD}^2 \cdot F_M
\]

\[
Noise_{APD,A}(b) := \left(2 \cdot q \cdot APD_A(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_A + 2 \cdot q \cdot I_{APD_d} \cdot M_{APD}^2 \cdot F_M
\]

\[
Noise_{APD,B}(b) := \left(2 \cdot q \cdot APD_B(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_B + 2 \cdot q \cdot I_{APD_d} \cdot M_{APD}^2 \cdot F_M
\]

**BER+APD**

\[
Q_{APD,N}(b) := \frac{M_{APD}}{t_s} \cdot b \cdot nq \cdot \left(\frac{v_{max,N} - v_{min,N}}{2}\right) \sqrt{I_{n,N} + Noise_{APD,N}(b)}
\]

\[
Q_{APD,A}(b) := \frac{M_{APD}}{t_s} \cdot b \cdot nq \cdot \left(\frac{v_{max,A} - v_{min,A}}{2}\right) \sqrt{I_{n,A} + Noise_{APD,A}(b)}
\]

\[
Q_{APD,B}(b) := \frac{M_{APD}}{t_s} \cdot b \cdot nq \cdot \left(\frac{v_{max,B} - v_{min,B}}{2}\right) \sqrt{I_{n,B} + Noise_{APD,B}(b)}
\]

\[
P_{c-APD,N}(b) := \frac{1}{2} \cdot \text{erfc} \left(\frac{Q_{APD,N}(b)}{\sqrt{2}}\right)
\]

\[
P_{c-APD,A}(b) := \frac{1}{2} \cdot \text{erfc} \left(\frac{Q_{APD,A}(b)}{\sqrt{2}}\right)
\]
\[ P_{e,APD\_B}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD\_B}(b)}{\sqrt{2}} \right) \]

\[ p_{APD\_N}(b) := (\log(P_{e,APD\_N}(b)) + 9) \]

\[ p_{APD\_A}(b) := (\log(P_{e,APD\_A}(b)) + 9) \]

\[ p_{APD\_B}(b) := (\log(P_{e,APD\_B}(b)) + 9) \]

\[ APD\_b\_N := \text{root}(p_{APD\_N}(b), b) \]

\[ APD\_b\_A := \text{root}(p_{APD\_A}(b), b) \]

\[ APD\_b\_B := \text{root}(p_{APD\_B}(b), b) \]

\[ \text{minimum}_{\_APD\_N} := \min(APD\_b\_N) = 2.11 \cdot 10^3 \]

\[ \text{minimum}_{\_APD\_A} := \min(APD\_b\_A) = 2.022 \cdot 10^3 \]

\[ \text{minimum}_{\_APD\_B} := \min(APD\_b\_B) = 2.043 \cdot 10^3 \]

Check BER, Q, and b APD

\[ Q_{APD\_N}(\min(APD\_b\_N)) = 5.998 \]

\[ Q_{APD\_A}(\min(APD\_b\_A)) = 5.998 \]

\[ Q_{APD\_B}(\min(APD\_b\_B)) = 5.998 \]

\[ \text{Sens} \]

\[ APD\_N := 10 \cdot \log \left( \frac{\text{minimum}_{\_APD\_N} \cdot \text{photon\_energy}}{2 \cdot 10^{-3} \cdot ts} \right) = -44.91 \]

\[ APD\_A := 10 \cdot \log \left( \frac{\text{minimum}_{\_APD\_A} \cdot \text{photon\_energy}}{2 \cdot 10^{-3} \cdot ts} \right) = -45.096 \]

\[ APD\_B := 10 \cdot \log \left( \frac{\text{minimum}_{\_APD\_B} \cdot \text{photon\_energy}}{2 \cdot 10^{-3} \cdot ts} \right) = -45.05 \]

\[ \text{Sens\_N – APD\_N} = 3.906 \quad \text{Noise\_APD\_N}(\text{minimum\_APD\_N}) = 5.698 \cdot 10^{-16} \]

\[ \text{Sens\_A – APD\_A} = 2.205 \quad \text{Noise\_APD\_A}(\text{minimum\_APD\_A}) = 6.544 \cdot 10^{-16} \]

\[ \text{Sens\_B – APD\_B} = 1.008 \quad \text{Noise\_APD\_B}(\text{minimum\_APD\_B}) = 4.418 \cdot 10^{-16} \]

\[ I_{n,N} = 3.665 \cdot 10^{-17} \quad I_{n,A} = 1.858 \cdot 10^{-17} \quad I_{n,B} = 7.141 \cdot 10^{-18} \]
B.5 Receiver optimisation (BJT/3\textsuperscript{rd} order Butterworth pre-detection filter)

B.5.1 Non-tuned receiver

Non-tuned Optimum Performance (PIN-BJT 100 Mbit/s)

Rx performance opt PIN-BJT input configurations, 3rd order Butterworth pre-detection filter, opt as follows

- TIA 3-db bandwidth rt/Filter 3-dB bandwidth rd
- range cut-off
- minimum noise
- examine ISI
- highest SNR
- BER

\[ \text{opt rd} = 0.6, \text{opt rt} = 0.5 \]

Bit-rate, pulse duration

\[ C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]
\[ B := 100 \cdot 10^6 \quad \text{Bit-rate} \]

\[ \omega_B (rd) := 2 \cdot \pi \cdot rd \]
\[ H_{\text{bun}} (\omega, rd) := \dfrac{\omega_B (rd)^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B (rd) + 2 \cdot (1j \cdot \omega) \cdot \omega_B (rd)^2 + \omega_B (rd)^3} \]

TIA (Non-tuned)

\[ \omega_c (rt) := 2 \cdot \pi \cdot rt \]
\[ Av := 10 \]
\[ Rf_N (rt) := \dfrac{Av + 1}{2 \pi \cdot rt \cdot C_T} \]
\[ Z_{\text{TIA}} (\omega, rt) := \dfrac{1}{1 + 1j \cdot \omega \cdot \frac{2 \cdot \pi \cdot rt}{2 \cdot \pi \cdot rt}} \]
\[ Rf_N (0.5 \cdot B) = 2.334 \cdot 10^4 \]

Receiver freq-response

\[ Z_N (\omega, rd, rt) := H_{\text{bun}} (\omega, rd) \cdot Z_{\text{TIA}} (\omega, rt) \]
\[ \omega := 1 \cdot 10^6, 10 \cdot 10^6 \ldots 1 \cdot 10^{11} \]
\[ rd := 0.7 \cdot B, 0.75 \cdot B \ldots 2 \cdot B \]
\[ rt := 0.5 \cdot B, 0.55 \cdot B \ldots 1 \cdot B \]

Noise equivalent bandwidth (NEB and I2)

\[ \text{NEB}_N (rd, rt) := \dfrac{1}{2 \pi} \int_0^{10^{12}} \left( |Z_N (\omega, rd, rt)|^2 \right) d\omega \]
\[ \text{Noise equivalent bandwidth (13)} \]

\[
I_{\text{3}_N}(rd, rt) := \frac{1}{2 \pi} \int_0^{10^{12}} \left| \left( \frac{1 + \frac{j \cdot r(\omega)}{R_f N(rt) \cdot C_T \cdot Z_N(\omega, rd, rt)}}{1 \cdot R_f N(rt)} \right)^2 \right| d\omega
\]

**Total noise- PIN-BJT**

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100 \]

\[ I_{\text{c,opt}}(rd, rt) := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{\text{3}_N}(rd, rt)}{\text{NEB}_N(rd, rt)}} \]

\[
I_{n_B}(rd, rt) := \left( \frac{4 \cdot k \cdot T}{R_f N(rt)} \cdot \text{NEB}_N(rd, rt) \right) + \left( \frac{2 \cdot q \cdot I_{\text{c,opt}}(rd, rt)}{hfe} \right) \cdot \text{NEB}_N(rd, rt) + \left( \frac{2 \cdot q \cdot I_{\text{c,opt}}(rd, rt)}{\left( I_{\text{c,opt}}(rd, rt) \right)^2} \right) \cdot I_{\text{3}_N}(rd, rt)
\]

\[ I_{n_B}(0.7 \ B, 0.5 \ B) = 5.993 \cdot 10^{-17} \]

**Output pulse shape**

\[ ts := \frac{1}{B} \quad B = 1 \cdot 10^8 \]
\[
vout(t, rd, rt) := \frac{ts}{\pi} \cdot \frac{1}{ts} \cdot 10^3 \cdot \frac{\sin \left( \frac{\omega \cdot ts}{2} \right)}{\left( \frac{\omega \cdot ts}{2} \right)} \cdot \text{Re} \{ Z_N(\omega, rd, rt) \cdot \exp (i \cdot \omega \cdot (t)) \} \text{d}\omega
\]

\[
xvout(t, rd, rt) := \frac{ts^2}{\pi} \cdot \frac{1}{ts} \cdot 10^3 \cdot \frac{\sin \left( \frac{\omega \cdot ts}{2} \right)}{\left( \frac{\omega \cdot ts}{2} \right)} \cdot \text{Re} \{ i \cdot \omega \cdot Z_N(\omega, rd, rt) \cdot \exp (i \cdot \omega \cdot (t)) \} \text{d}\omega
\]

\[t := -1 \cdot ts, -0.99 \cdot ts, 4 \cdot ts\]

\[|SI|

\[t_{B1} := 0.6 \cdot ts\]

\[t_{pk1} := \text{root}(xvout(t_{B1}, 0.6 B, 0.5 B), t_{B1})\]

\[\frac{t_{pk1}}{ts} = 0.853\]

\[v_{max_{B1}} := vout(t_{pk1}, 0.6 B, 0.5 B) = 0.887\]

\[v_{min_{B1}} := vout(t_{pk1} + 1 \cdot ts, 0.6 B, 0.5 B) = 0.057\]

\[v_{min1_{B1}} := vout(t_{pk1} + 2 \cdot ts, 0.6 B, 0.5 B) = 0.006\]

\[v_{min2_{B1}} := vout(t_{pk1} + 3 \cdot ts, 0.6 B, 0.5 B) = -5.432 \cdot 10^{-4}\]

\[v_{max_{B1}} - v_{min_{B1}} = 0.83\]

\[v_{max_{B1}} - v_{min1_{B1}} = 0.881\]

\[v_{max_{B1}} - v_{min2_{B1}} = 0.888\]

\[
\begin{align*}
\frac{v_{max_{B1}} - v_{min_{B1}}}{\sqrt{10^{16} \cdot I_{n.B}(0.6 B, 0.5 B)}} &= 1.14 & \text{1st pulse} \\
\frac{v_{max_{B1}} - v_{min1_{B1}}}{\sqrt{10^{16} \cdot I_{n.B}(0.6 B, 0.5 B)}} &= 1.212 & \text{2nd pulse} \\
\frac{v_{max_{B1}} - v_{min2_{B1}}}{\sqrt{10^{16} \cdot I_{n.B}(0.6 B, 0.5 B)}} &= 1.222 & \text{3rd pulse}
\end{align*}
\]

\[\text{Highest SNR @ rd=0.7 rt=0.5}\]

\[v_{max_{B}} := v_{max_{B1}} \quad v_{min_{B}} := v_{min_{B1}}\]

\[\text{Eye-diagram check}\]

\[x_{111}(t, rd, rt) := vout(t, rd, rt) + vout(t - 2 \cdot ts, rd, rt) + vout(t - ts, rd, rt)\]

\[x_{110}(t, rd, rt) := vout(t, rd, rt) + vout(t - ts, rd, rt)\]

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\[ x_{100}(t, rd, rt) := v_{out}(t, rd, rt) \]
\[ x_{001}(t, rd, rt) := v_{out}(t - 2\, ts, rd, rt) \]
\[ x_{010}(t, rd, rt) := v_{out}(t - ts, rd, rt) \]
\[ x_{011}(t, rd, rt) := v_{out}(t - ts, rd, rt) + v_{out}(t - 2\, ts, rd, rt) \]
\[ x_{101}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - 2\, ts, rd, rt) \]

\[ \begin{align*}
  &x_{111}(t, 0.6\, B, 0.5\, B) \\
  &x_{100}(t, 0.6\, B, 0.5\, B) \\
  &x_{110}(t, 0.6\, B, 0.5\, B) \\
  &x_{001}(t, 0.6\, B, 0.5\, B) \\
  &x_{010}(t, 0.6\, B, 0.5\, B) \\
  &x_{011}(t, 0.6\, B, 0.5\, B) \\
  &x_{101}(t, 0.6\, B, 0.5\, B)
\end{align*} \]

**Sens PINFET**

\[ \lambda := 650 \cdot 10^{-9} \]
\[ \text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]
\[ nq := 1.6 \cdot 10^{-19} \]
\[ Q_N(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot \left( \frac{v_{max\_B} - v_{min\_B}}{2} \right) \]
\[ \sqrt{I_{n,B}(0.6\, B, 0.5\, B)} \]
\[ Pe_N(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right) \]
\[ Pe_C(b) := (\log(Pe_N(b)) + 9) \]
\[ b \equiv 3 \cdot 10^3 \]
\[ b \_N := \sqrt[n]{Pe\_N(b) \cdot b} \]
\[ \text{minimum}_B := \min(b \_N) = 6.562 \cdot 10^3 \]

\[ Sens\_N\_PIN := 10 \cdot \log \left( \frac{\text{minimum}_B \cdot \text{photon\_energy}}{2 \cdot 10^{-3} \cdot t_s} \right) = -39.983 \]
B.5.2 Tuned A receiver

Tuned A Optimum Performance (PIN-BJT 100 Mbit/s)

Rx performance opt PIN-BJT input configurations, 3rd order Butterworth pre-detection filter, opt as follows:
- TIA 3-dB bandwidth rt/Filter 3-dB bandwidth rd
- range cut-off
- minimum noise
- examine ISI
- highest SNR
- BER

**opt rd = 0.6, opt rt = 0.5**

**Bit-rate, pulse duration**

\[ C_T = 1.5 \cdot 10^{-12} \quad \text{total C} \]

\[ B = 100 \cdot 10^6 \quad \text{Bit-rate} \]

**pre-dec filter**

\[ \omega_B(rd) = 2 \cdot \pi \cdot rd \]

\[ H_{\text{but}}(\omega, rd) = \frac{\omega_B(rd)^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B(rd) + 2 \cdot (1j \cdot \omega) \cdot \omega_B(rd)^2 + \omega_B(rd)^3} \]

**TIA(tuned A)**

\[ A_v = 10 \]

\[ m = 1.8 \quad \text{time constant ratio of L/R and RC} \]

\[ y = \sqrt{\left( \frac{-m^2}{2} + m + 1 \right) + \left( \frac{-m^2}{2} + m + 1 \right)^2 + m^2} = 1.825 \]

\[ Rf_A(rt) = y \cdot \frac{A_v + 1}{2 \cdot \pi \cdot rt \cdot C_T} \]

\[ L(rt) = Rf_A(rt) \cdot \frac{C_T}{m} \]

\[ Lf(rt) = \left( \frac{Rf_A(rt)}{A_v + 1} \right) \cdot \left( \frac{A_v + 1}{C_T} \right) \]

\[ Z_{\text{TIA}_A}(\omega, rt) = \frac{1 + 1j \cdot \omega \cdot \frac{Lf(rt)}{Rf_A(rt)}}{\left( 1 - \omega^2 \cdot Lf(rt) \cdot \frac{C_T}{1 + A_v} \right) + Rf_A(rt) \cdot \omega \cdot \frac{C_T}{(1 + A_v) \cdot 1j} } \]

**Receiver freq-response**

\[ Z_A(\omega, rd, rt) := H_{\text{but}}(\omega, rd) \cdot Z_{\text{TIA}_A}(\omega, rt) \]

\[ \omega = 1 \cdot 10^6, 10 \cdot 10^6 \ldots 1 \cdot 10^{11} \]

\[ rd = 0.7 \cdot B, 0.75 \cdot B, 0.2 \cdot B \]
\[ rt := 0.5 \cdot B, 0.55 \cdot B, 1 B \]

**Noise equivalent bandwidth (NEB and I2)**

\[
NEB_A(\omega, rd, rt) := \frac{1}{2 \pi} \left[ \frac{1}{\left(1 + 1i \cdot \omega \cdot \frac{L(f(rt))}{Rf_A(rt)} \right)^2} \cdot Z_A(\omega, rd, rt) \right] d\omega
\]

\[
NEB_A(0.7 B, 0.5 B) = 5.341 \cdot 10^7
\]

**Noise equivalent bandwidth (I3)**

\[
I_3_A(\omega, rd, rt) := \frac{1}{2 \pi} \left[ \frac{1}{\left(\frac{(1 - \omega^2 \cdot L(f(rt)) \cdot C_T) \cdot Rf_A(rt) \cdot \omega \cdot C_T \cdot 1i}{(Rf_A(rt) + 1i \cdot \omega \cdot L(f(rt))) \cdot Z_A(\omega, rd, rt)}\right)^2} \right] d\omega
\]

**Total noise - PIN-BJT**

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100 \]

\[ I_{opt}(\omega, rd, rt) = 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_3_A(\omega, rd, rt)}{NEB_A(\omega, rd, rt)}} \]

\[ I_{n_B}(\omega, rd, rt) = \frac{4 \cdot k \cdot T}{Rf_A(rt)} \cdot I_2_A(\omega, rd, rt) \]

\[ + \frac{2 \cdot q \cdot I_{opt} A(\omega, rd, rt)}{hfe} \cdot NEB_A(\omega, rd, rt) \]

\[ + 2 \cdot q \cdot I_{opt} A(\omega, rd, rt) \cdot \frac{I_3_A(\omega, rd, rt)}{(25 \cdot 10^{-3})^2} \]

\[ \frac{I_{n_B}(\omega, rd, rt)}{10^{-16}} \]
Output pulse shape

\[ t_s := \frac{1}{B} \]

\[ v_{out}(t, rd, rt) := \frac{t_s}{\pi} \cdot \sin \left( \frac{\omega \cdot t_s}{2} \right) \cdot \frac{\Re(Z_A(\omega, rd, rt) \cdot \exp(1i \cdot \omega \cdot (t)))}{\left( \frac{\omega \cdot t_s}{2} \right)} \mathrm{d}\omega \]

\[ x_{vout}(t, rd, rt) := \frac{t_s^2}{\pi} \cdot \sin \left( \frac{\omega \cdot t_s}{2} \right) \cdot \frac{1}{\left( \frac{\omega \cdot t_s}{2} \right)} \cdot \Re(1i \cdot \omega \cdot Z_A(\omega, rd, rt) \cdot \exp(1i \cdot \omega \cdot (t))) \mathrm{d}\omega \]

\[ t := -1 \cdot t_s, -0.99 \cdot t_s, -0.4 \cdot t_s \]

\[ |S_t| \]

\[ t_{B1} := 0.8 \cdot t_s \]

\[ t_{pk1} := \sqrt{x_{vout}(t_{B1}, 0.6 \cdot B, 0.5 \cdot B)} \]

\[ \frac{t_{pk1}}{t_s} = 0.958 \]

\[ v_{max_{B1}} := v_{out}(t_{pk1}, 0.6 \cdot B, 0.5 \cdot B) = 0.947 \]

\[ v_{min_{B1}} := v_{out}(t_{pk1} + 1 \cdot t_s, 0.6 \cdot B, 0.5 \cdot B) = 0.028 \]

\[ v_{min_{1B1}} := v_{out}(t_{pk1} + 2 \cdot t_s, 0.6 \cdot B, 0.5 \cdot B) = -0.06 \]

\[ v_{min_{2B1}} := v_{out}(t_{pk1} + 3 \cdot t_s, 0.6 \cdot B, 0.5 \cdot B) = 0.004 \]

\[ v_{max_{B1}} - v_{min_{B1}} = 0.919 \quad v_{max_{B1}} - v_{min_{1B1}} = 1.007 \quad v_{max_{B1}} - v_{min_{2B1}} = 0.943 \]

\[ \frac{v_{max_{B1}} - v_{min_{B1}}}{\sqrt{10^{16} \cdot I_{n_B}(0.6 \cdot B, 0.5 \cdot B)}} = 1.49 \] 1st pulse worst ISI

\[ \frac{v_{max_{B1}} - v_{min_{1B1}}}{\sqrt{10^{16} \cdot I_{n_B}(0.6 \cdot B, 0.5 \cdot B)}} = 1.633 \] 2nd pulse

\[ \frac{v_{max_{B1}} - v_{min_{2B1}}}{\sqrt{10^{16} \cdot I_{n_B}(0.6 \cdot B, 0.5 \cdot B)}} = 1.529 \] 3rd pulse

Highest SNR @ rd=0.7 rt=0.6

\[ v_{max_{B}} := v_{max_{B1}} \quad v_{min_{B}} := v_{min_{B1}} \]

Eye-diagram check

\[ x_{111}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - 2 \cdot t_s, rd, rt) + v_{out}(t - t_s, rd, rt) \]
\[ x_{110}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - ts, rd, rt) \]
\[ x_{100}(t, rd, rt) := v_{out}(t, rd, rt) \]
\[ x_{001}(t, rd, rt) := v_{out}(t - 2ts, rd, rt) \]
\[ x_{010}(t, rd, rt) := v_{out}(t - ts, rd, rt) \]
\[ x_{00}(t, rd, rt) := v_{out}(t - ts, rd, rt) + v_{out}(t - 2ts, rd, rt) \]
\[ x_{10}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - 2ts, rd, rt) \]

**Sens PINFET**
\[ \lambda := 650 \cdot 10^{-9} \]
\[ photon\_energy := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]
\[ nq := 1.6 \cdot 10^{-19} \]
\[ Q_N(b) := \frac{1}{ts} \cdot nq \cdot \left( v_{max\_B} - v_{min\_B} \right) \frac{2}{I_{n,B}(0.6\ B, 0.5\ B)} \]
\[ Pe_N(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right) \]
\[ Pc_N(b) := (\log(Pe_N(b)) + 9) \]
\[ b \equiv 3 \cdot 10^3 \]
\[ b\ N := \text{root}(Pc\ N(b), b) \]
\[ \text{minimum}\_B := \min(b\ N) = 5.035 \cdot 10^3 \]
\[ Sens\_N\_PIN := 10 \cdot \log \left( \frac{\text{minimum}\_B \cdot \text{photon}\_energy \cdot \frac{1}{ts}}{2} \right) = -41.133 \]
**B.5.3 Tuned B receiver a=0.2**

**Tuned B Optimum Performance (PIN-BJT 100 Mbit/s)**

Rx performance opt PIN-BJT input configurations, 3rd order Butterworth pre-detection filter. opt as follows

- TIA 3-dB bandwidth rt/Filter 3-dB bandwidth rd
- range cut-off
- minimum noise
- examine ISI
- highest SNR
- BER

\[ \text{opt rd = 0.6, opt rt = 0.5} \]

**Bit-rate, pulse duration**

\[ C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]

\[ B := 100 \cdot 10^6 \quad \text{Bit-rate} \]

**pre-dec filter**

\[ \omega_B (rd) := 2 \cdot \pi \cdot rd \]

\[ H_{but}(\omega, rd) := \frac{1}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B(rd) + 2 \cdot (1j \cdot \omega) \cdot \omega_B(rd)^2 + \omega_B(rd)^3} \]

**TIA(tuned B)**

Feedback value for (R) Tuned B

\[ \Delta_L := 1.8 \quad \Delta_R := 1.87 \quad \text{Av := 10} \]

\[ Rf_B(rt) := \Delta_R \cdot \frac{Av + 1}{2 \cdot \pi \cdot rt \cdot C_T} \]

\[ \alpha := 0.2 \]

\[ Lc(rt) := \left( \frac{Rf_B(rt)}{1 + Av} \right)^2 \cdot C_T \]

\[ C1 := (1 - \alpha) \cdot (C_T) = 1.2 \cdot 10^{-12} \]

\[ C2 := \alpha \cdot (C_T) = 3 \cdot 10^{-13} \]

\[ Z_{TIA,B}(\omega, rt) := \frac{1}{(1 - \omega^2 \cdot Lc(rt) \cdot C1) + \frac{Rf_B(rt)}{1 + Av} \cdot (\omega \cdot 1i)^2 \cdot (C1 + C2 - (\omega)^3 \cdot Lc(rt) \cdot C1 \cdot C2)} \]

**Receiver freq-response**

\[ Z_B(\omega, rd, rt) := H_{but}(\omega, rd) \cdot Z_{TIA,B}(\omega, rt) \]

\[ \omega := 1 \cdot 10^6, 10 \cdot 10^6 \ldots 1 \cdot 10^{11} \]

\[ rd := 0.7 \cdot B, 0.75 \cdot B \ldots 2 \cdot B \]

\[ rt := 0.5 \cdot B, 0.55 \cdot B \ldots 1 \cdot B \]

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Noise equivalent bandwidth (NEB and \(I_2\))

\[
NEB_B(rd, rt) := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{14}} \left( \frac{(1 - \omega^2 \cdot Lc(rt) \cdot C1) \cdot d\omega}{\cdot Z_B(\omega, rd, rt)} \right)^2 \right) d\omega
\]

\[
NEB_B(0.7 B, 0.5 B) = 4.757 \cdot 10^7
\]

Noise equivalent bandwidth (\(I_3\))

\[
I_{_2B}(rd, rt) := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{14}} \left( \frac{(1 - \omega^2 \cdot Lc(rt) \cdot C1) \cdot d\omega}{\cdot Z_B(\omega, rd, rt)} \right)^2 \right) d\omega
\]

Noise equivalent bandwidth (\(I_3\))

\[
I_{_3B}(rd, rt) := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{14}} \left( \frac{(1 - \omega^2 \cdot Lc(rt) \cdot C1)}{\cdot Z_B(\omega, rd, rt)} \right)^2 \right) d\omega
\]

Total noise- PIN-BJT

\[
q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100
\]

\[
I_{icomp}(rd, rt) := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \left( \frac{I_{_3B}(rd, rt)}{NEB_B(rd, rt)} \right)
\]

\[
I_{nB}(rd, rt) := \left( \frac{4 \cdot k \cdot T}{Rf_B(rt)} \cdot I_{_2B}(rd, rt) + \left( \frac{2 \cdot q \cdot I_{icomp}(rd, rt)}{hfe} \right) \cdot I_{_2B}(rd, rt) \right)^{\frac{1}{2}} + \left( \frac{2 \cdot q \cdot I_{icomp}(rd, rt)}{hfe} \right) \cdot I_{_3B}(rd, rt)
\]

\[
\frac{I_{nB}(rd, rt)}{10^{-16}}
\]
\[ I_{n,B}(0.7 \ B, 0.5 \ B) = 2.878 \cdot 10^{-17} \]

**Output pulse shape**

\[
t_s := \frac{1}{B} \]

\[
vout(t, rd, rt) := \frac{ts}{\pi} \cdot \frac{1}{ts \cdot 10^5} \cdot \sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \text{Re} \left( Z_B(\omega, rd, rt) \cdot \exp(1i \cdot \omega \cdot t) \right) d\omega
\]

\[
xvout(t, rd, rt) := \frac{ts^2}{\pi} \cdot \frac{1}{ts \cdot 10^5} \cdot \sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \left( \frac{\omega \cdot ts}{2} \right)^{-1} \cdot \text{Re} \left( 1i \cdot \omega \cdot Z_B(\omega, rd, rt) \cdot \exp(1i \cdot \omega \cdot t) \right) d\omega
\]

\[ t := -1 \ ts, -0.99 \ ts .. 4 \ ts \]

**ISI**

\[ t_B1 = 1.2 \cdot ts \]

\[ t_{pk1} := \text{root}(xvout(t_B1, 0.6 \ B, 0.5 \ B), t_B1) \]

\[ \frac{t_{pk1}}{ts} = 1.21 \]

\[ v_{max,B1} := vout(t_{pk1}, 0.6 \ B, 0.5 \ B) = 0.896 \]

\[ v_{min,B1} := vout(t_{pk1} + 1 \cdot ts, 0.6 \ B, 0.5 \ B) = 0.019 \]

\[ v_{min_{B1}} := vout(t_{pk1} + 2 \cdot ts, 0.6 \ B, 0.5 \ B) = -0.004 \]

\[ v_{min_{B2}} := vout(t_{pk1} + 3 \cdot ts, 0.6 \ B, 0.5 \ B) = 9.097 \cdot 10^{-4} \]

\[ v_{max,B1} - v_{min,B1} = 0.877 \quad v_{max,B1} - v_{min_{B1}} = 0.899 \quad v_{max,B1} - v_{min_{B2}} = 0.895 \]

\[
\frac{v_{max,B1} - v_{min,B1}}{\sqrt{10^{16} \cdot I_{n,B}(0.6 \ B, 0.5 \ B)}} = 1.78 \quad \text{1st pulse \ worst ISI} \\
\frac{v_{max,B1} - v_{min_{B1}}}{\sqrt{10^{16} \cdot I_{n,B}(0.6 \ B, 0.5 \ B)}} = 1.83 \quad \text{2nd pulse} \\
\frac{v_{max,B1} - v_{min_{B2}}}{\sqrt{10^{16} \cdot I_{n,B}(0.6 \ B, 0.5 \ B)}} = 1.82 \quad \text{3rd pulse} \\
\]

\[ v_{max,B1} - 0.183 = 0.713 \]

**Highest SNR @ rd=0.7 rt=0.6**

\[ v_{max,B} := v_{max,B1} \quad v_{min,B} := v_{min,B1} \]

**Eye-diagram check**

\[ x_{111}(t, rd, rt) := vout(t, rd, rt) + vout(t - 2 \ ts, rd, rt) + vout(t - ts, rd, rt) \]

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\[ x_{110}(t, rd, rt) := vout(t, rd, rt) + vout(t - ts, rd, rt) \]
\[ x_{100}(t, rd, rt) := vout(t, rd, rt) \]
\[ x_{001}(t, rd, rt) := vout(t - 2 ts, rd, rt) \]
\[ x_{010}(t, rd, rt) := vout(t - ts, rd, rt) \]
\[ x_{011}(t, rd, rt) := vout(t - ts, rd, rt) + vout(t - 2 ts, rd, rt) \]
\[ x_{101}(t, rd, rt) := vout(t, rd, rt) + vout(t - 2 ts, rd, rt) \]

\[ \lambda := 650 \cdot 10^{-9} \]

\[ photon\_energy := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^{8}}{\lambda} \]

\[ nq := 1.6 \cdot 10^{-19} \]

\[ Q_N(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \frac{\left( v_{\text{max}_B} - v_{\text{min}_B} \right)}{2} \]

\[ Q_N(b) := \frac{1}{\sqrt{I_{n,B}(0.6 B, 0.5 B)}} \]

\[ Pe_N(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right) \]

\[ Pc_N(b) := (\log(Pe_N(b)) + 9) \]

\[ b := 3 \cdot 10^3 \]

\[ b_N := \text{root}(Pc_N(b), b) \]

\[ \text{minimum}_B := \min(b_N, b) = 4.203 \cdot 10^3 \]

\[ \text{Sens}_N\_PIN := 10 \cdot \log \left( \frac{\text{minimum}_B \cdot \text{photon\_energy} \cdot \frac{1}{ts}}{2} \right) = -41.917 \]
B.5.4 Tuned B receiver $a=0.3$

**Tuned B Optimum Performance (PIN-BJT 100 Mbit/s)**

Rx performance opt PIN-BJT input configurations, 3rd order Butterworth pre-detection filter, opt as follows

- TIA 3-dB bandwidth $r_t$ / Filter 3-dB bandwidth $r_d$
- range cut-off
- minimum noise
- examine ISI
- highest SNR
- BER

**opt $r_d = 0.5$, opt $r_t = 0.5$**

**Bit-rate, pulse duration**

$C_T := 1.5 \cdot 10^{-12}$ total C

$B := 100 \cdot 10^6$ Bit-rate

**pre-dec filter**

$\omega_B (rd) = 2 \cdot \pi \cdot rd$

$H_{but}(\omega, rd) := \frac{\omega_B (rd)^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B (rd) + 2 \cdot (1j \cdot \omega) \cdot \omega_B (rd)^2 + \omega_B (rd)^3}$

**TIA(tuned B)**

Feedback value for (R) Tuned B

$\Delta_L := 2.4 \quad \Delta_R := 2.52$

$A_v := 10$

$\alpha := 0.3$

$Rf_B (rt) := \Delta_R \cdot \frac{A_v + 1}{2 \cdot \pi \cdot r_t \cdot C_T}$

$C_1 := (1 - \alpha) \cdot (C_T) = 1.05 \cdot 10^{-12}$

$C_2 := \alpha \cdot (C_T) = 4.5 \cdot 10^{-13}$

$Z_{TIA,B}(\omega, rt) := \frac{1}{\left( (1 - \omega^2 \cdot Lc (rt) \cdot C_1) + \frac{Rf_B (rt)}{(1 + A_v)} \cdot (\omega \cdot 1i) \right) \left( C_1 + C_2 - (\omega)^2 \cdot Lc (rt) \cdot C_1 \cdot C_2 \right)}$

**Receiver freq-response**

$Z_B (\omega, rd, rt) := H_{but} (\omega, rd) \cdot Z_{TIA,B}(\omega, rt)$

$\omega := 1 \cdot 10^6 , 10 \cdot 10^6 .. 1 \cdot 10^{11}$

$rd := 0.7 \cdot B , 0.75 \cdot B .. 2 \cdot B$

$rt := 0.5 \cdot B , 0.55 \cdot B .. 1 \cdot B$
Noise equivalent bandwidth (NEB and I2)

\[ NEB_B(rd, rt) := \frac{1}{2 \pi} \left( \int_{0}^{10^{13}} \left( \frac{1}{Z_B(\omega, rd, rt)} \right)^2 d\omega \right) \]

\[ NEB_B(0.7 B, 0.5 B) = 4.118 \cdot 10^7 \quad \# \]

Noise equivalent bandwidth (I2)

\[ I_{2_B}(rd, rt) := \frac{1}{2 \pi} \left( \int_{0}^{10^{13}} \left( \frac{1}{Z_B(\omega, rd, rt)} \right)^2 d\omega \right) \left( 1 - \omega^2 \cdot Lc(rt) \cdot C1 \right) \]

Noise equivalent bandwidth (I3)

\[ I_{3_B}(rd, rt) := \frac{1}{2 \pi} \left( \int_{0}^{10^{12}} \left( \frac{1}{Z_B(\omega, rd, rt)} \right)^2 d\omega \right) \left( \frac{\left(\frac{1}{Rf_B(rt)}\right)^2}{1 + C1 + C2 - (\omega^2 \cdot Lc(rt) \cdot C1 \cdot C2)} \right) \]

Total noise - PIN-BJT

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100 \]

\[ I_{opt_B}(rd, rt) := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3_B}(rd, rt)}{NEB_B(rd, rt)}} \]

\[ I_{n_B}(rd, rt) := \left\{ \begin{array}{l}
4 \cdot k \cdot T \cdot I_{2_B}(rd, rt) + 2 \cdot q \cdot \frac{I_{opt_B}(rd, rt)}{hfe} \cdot I_{2_B}(rd, rt) \\
+ 2 \cdot q \cdot \frac{I_{opt_B}(rd, rt)}{hfe} \cdot I_{3_B}(rd, rt) \\
\left( \frac{I_{opt_B}(rd, rt)}{25 \cdot 10^{-3}} \right) \left( \frac{I_{opt_B}(rd, rt)}{25 \cdot 10^{-3}} \right)
\end{array} \right. \]
\[ I_{n,B}(0.7 B, 0.5 B) = 2.13 \cdot 10^{-17} \]  #

**Output pulse shape**

\[ ts := \frac{1}{B} \]

\[ vout(t, rd, rt) := \frac{ts}{\pi} \int_0^{1/ts} \frac{\sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \text{Re} \left( Z_B(\omega, rd, rt) \cdot \exp \left( i \cdot \omega \cdot t \right) \right)}{\left( \frac{\omega \cdot ts}{2} \right)} \, d\omega \]

\[ xvout(t, rd, rt) := \frac{ts^2}{\pi} \int_0^{1/ts} \frac{\sin \left( \frac{\omega \cdot ts}{2} \right) \cdot \text{Re} \left( 1 \cdot i \cdot \omega \cdot Z_B(\omega, rd, rt) \cdot \exp \left( i \cdot \omega \cdot t \right) \right)}{\left( \frac{\omega \cdot ts}{2} \right)} \, d\omega \]

\[ t := -1 \, ts, -0.99 \, ts, 0.4 \, ts \]

**ISI**

\[ t_{B1} = 1.1 \cdot ts \]

\[ t_{pk1} := \text{root} \left( xvout \left( t_{B1}, 0.5 B, 0.5 B \right), t_{B1} \right) \]

\[ \frac{t_{pk1}}{ts} = 1.424 \]

\[ v_{\text{max}, B1} := vout \left( t_{pk1}, 0.5 B, 0.5 B \right) = 0.789 \]

\[ v_{\text{min}, B1} := vout \left( t_{pk1} + 1 \cdot ts, 0.5 B, 0.5 B \right) = 0.064 \]

\[ v_{\text{min}, B1} := vout \left( t_{pk1} + 2 \cdot ts, 0.5 B, 0.5 B \right) = 0.028 \]

\[ v_{\text{min}, B1} := vout \left( t_{pk1} + 3 \cdot ts, 0.5 B, 0.5 B \right) = 0.003 \]

\[ v_{\text{max}, B1} - v_{\text{min}, B1} = 0.725 \quad v_{\text{max}, B1} - v_{\text{min}, B1} = 0.761 \quad v_{\text{max}, B1} - v_{\text{min}, B1} = 0.786 \]

\[ \frac{v_{\text{max}, B1} - v_{\text{min}, B1}}{\sqrt{10^{16} \cdot I_{n,B}(0.5 B, 0.5 B)}} = 1.9 \quad \text{1st pulse, worst ISI} \]

\[ \frac{v_{\text{max}, B1} - v_{\text{min}, B1}}{\sqrt{10^{16} \cdot I_{n,B}(0.5 B, 0.5 B)}} = 1.998 \quad \text{2nd pulse} \]

\[ \frac{v_{\text{max}, B1} - v_{\text{min}, B1}}{\sqrt{10^{16} \cdot I_{n,B}(0.5 B, 0.5 B)}} = 2.063 \quad \text{3rd pulse} \]

**Highest SNR @ rd=0.7 rt=0.6**

\[ v_{\text{max}} := v_{\text{max}, B1} \quad v_{\text{min}} := v_{\text{min}, B1} \]

**Eye-diagram check**

\[ x_{111}(t, rd, rt) := vout(t, rd, rt) + vout(t - 2 \cdot ts, rd, rt) + vout(t - ts, rd, rt) \]
\[ x_{110}(t, rd, rt) = vout(t, rd, rt) + vout(t - ts, rd, rt) \]
\[ x_{100}(t, rd, rt) = vout(t, rd, rt) \]
\[ x_{001}(t, rd, rt) = vout(t - 2 ts, rd, rt) \]
\[ x_{010}(t, rd, rt) = vout(t - ts, rd, rt) \]
\[ x_{011}(t, rd, rt) = vout(t - ts, rd, rt) + vout(t - 2 ts, rd, rt) \]
\[ x_{101}(t, rd, rt) = vout(t, rd, rt) + vout(t - 2 ts, rd, rt) \]

\[ x_{111}(t, 0.5 B, 0.5 B) \]
\[ x_{100}(t, 0.5 B, 0.5 B) \]
\[ x_{110}(t, 0.5 B, 0.5 B) \]
\[ x_{001}(t, 0.5 B, 0.5 B) \]
\[ x_{010}(t, 0.5 B, 0.5 B) \]
\[ x_{011}(t, 0.5 B, 0.5 B) \]
\[ x_{101}(t, 0.5 B, 0.5 B) \]

**Sens PINFET**

\[ \lambda := 650 \cdot 10^{-9} \]

\[ photon\_energy := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^{8}}{\lambda} \]

\[ nq := 1.6 \cdot 10^{-19} \]

\[ Q_N(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{max\_B} - v_{min\_B}}{2} \right) \]

\[ I_n.B(0.5 B, 0.5 B) \]

\[ Pe_N(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right) \]

\[ Pc_N(b) := (\log(Pe_N(b)) + 9) \]

\[ b \equiv 3 \cdot 10^3 \]

\[ b_N := \text{root}(Pc_N(b), b) \]

**minimum\_B := \text{min}(b_N) = 3.941 \cdot 10^3**

\[ Sens\_N\_PIN := 10 \cdot \log \left( \frac{\text{minimum\_B}, photon\_energy}{2 \cdot 10^{-3}, \frac{1}{ts}} \right) = -42.197 \]
B.5.5 Tuned B receiver a=0.4

Tuned B Optimum Performance (PIN-BJT 100 Mbit/s)

Rx performance opt PIN-BJT input configurations, 3rd order Butterworth pre-detection filter. opt as follows

- TIA 3-db bandwidth rt/Filter 3-dB bandwidth rd
- range cut-off
- minimum noise
- examine ISI
- highest SNR
- BER

opt rd = 0.5, opt rt = 0.5

Bit-rate, pulse duration

\[ C_T := 1.5 \times 10^{-12} \quad \text{total C} \]

\[ B := 100 \times 10^6 \quad \text{Bit-rate} \]

pre-dec filter

\[ \omega_B (rd) := 2 \pi \cdot rd \]

\[ H_{but}(\omega, rd) := \frac{\omega_B (rd)^3}{(1j \cdot \omega)^3 + 2(1j \cdot \omega)^2 \cdot \omega_B (rd) + 2(1j \cdot \omega) \cdot \omega_B (rd)^2 + \omega_B (rd)^3} \]

TIA(tuned B)

Feedback value for (R) Tuned B

\[ \Delta_L := 1.9 \quad \Delta_R := 2.75 \]

\[ Av := 10 \]

\[ Rf_B(rt) := \Delta_R \cdot \frac{Av + 1}{2 \pi \cdot rt \cdot C_T} \]

\[ \alpha := 0.4 \]

\[ Lc(rt) := \left( \frac{Rf_B(rt)}{1 + Av} \right)^2 \cdot C_T \]

\[ C1 := (1 - \alpha) \cdot (C_T) = 9 \times 10^{-13} \]

\[ C2 := \alpha \cdot (C_T) = 6 \times 10^{-13} \]

\[ Z_{TIA,B} (\omega, rt) := \frac{1}{((1 - \omega^2 \cdot Lc(rt) \cdot C1)) + \frac{Rf_B(rt)}{(1 + Av)} \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc(rt) \cdot C1 \cdot C2)} \]

Receiver freq-response

\[ Z_B (\omega, rd, rt) := H_{but}(\omega, rd) \cdot Z_{TIA,B}(\omega, rt) \]

\[ \omega := 1 \times 10^6, 10 \times 10^6 \ldots 1 \times 10^{11} \]

\[ rd := 0.7 \cdot B, 0.75 \cdot B \ldots 2 \cdot B \]

\[ rt := 0.5 \cdot B, 0.55 \cdot B \ldots 1 \cdot B \]
Noise equivalent bandwidth (NEB and I2)

\[ NEB_B(\omega, rd, rt) = \frac{1}{2 \pi} \left( \int_0^{10^{12}} \left( \frac{1}{Z_B(\omega, rd, rt)} \right)^2 d\omega \right) \]

\[ NEB_B(0.5 B, 0.5 B) = 4.09 \cdot 10^7 \]

Noise equivalent bandwidth (I2)

\[ I_{2-B}(\omega, rd, rt) = \frac{1}{2 \pi} \left( \int_0^{10^{12}} \left( \frac{\left( \left( 1 - \omega^2 \cdot Lc(rt) \cdot C1 \right) \left| Z_B(\omega, rd, rt) \right| \right)^2}{R_f_B(rt)} d\omega \right) \]

Noise equivalent bandwidth (I3)

\[ I_{3-B}(\omega, rd, rt) = \frac{1}{2 \pi} \left( \int_0^{10^{12}} \left( \frac{\left( \left( 1 - \omega^2 \cdot Lc(rt) \cdot C1 \right) + Rf_B(rt) \cdot (\omega) \left| Z_B(\omega, rd, rt) \right| \right) \left( C1 + C2 - (\omega)^2 \cdot Lc(rt) \cdot C1 \cdot C2 \right) \cdot 1i}{R_f_B(rt)} d\omega \right)^2 \]

Total noise- PIN-BJT

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100 \]

\[ I_{opt-B}(\omega, rd, rt) = 25 \cdot 10^{-6} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3-B}(\omega, rd, rt)}{I_{2-B}(\omega, rd, rt)}} \]

\[ I_{n-B}(\omega, rd, rt) = \frac{4 \cdot k \cdot T}{R_f_B(rt)} \cdot I_{2-B}(\omega, rd, rt) \left( 1 + 2 \cdot q \cdot \frac{I_{opt-B}(rd, rt)}{hfe} \right) \cdot I_{2-B}(\omega, rd, rt) \]

\[ + 2 \cdot q \cdot \left( \frac{I_{opt-B}(rd, rt)}{25 \cdot 10^{-6}} \right)^2 \cdot I_{3-B}(rd, rt) \]

\[ I_{n-B}(rd, rt) = \frac{4 \cdot k \cdot T}{R_f_B(rt)} \cdot I_{2-B}(\omega, rd, rt) \left( 1 + 2 \cdot q \cdot \frac{I_{opt-B}(rd, rt)}{hfe} \right) \cdot I_{2-B}(\omega, rd, rt) \]

\[ + 2 \cdot q \cdot \left( \frac{I_{opt-B}(rd, rt)}{25 \cdot 10^{-6}} \right)^2 \cdot I_{3-B}(rd, rt) \]
Output pulse shape

\[ I_{n,B}(0.5 B, 0.5 B) = 1.383 \cdot 10^{-17} \]  

\[ t_s := \frac{1}{B} \]

\[ v_{out}(t, rd, rt) := \frac{t_s}{\pi} \int_{0}^{\frac{1}{t_s} \cdot 10^5} \sin \left( \frac{\omega \cdot t_s}{2} \right) \cdot \text{Re} \left( Z_B(\omega, rd, rt) \cdot \exp(1i \cdot \omega \cdot t) \right) d\omega \]

\[ x_{vout}(t, rd, rt) := \frac{t_s^2}{\pi} \int_{0}^{\frac{1}{t_s} \cdot 10^5} \sin \left( \frac{\omega \cdot t_s}{2} \right) \cdot \frac{1}{\omega \cdot t_s} \cdot \text{Re} \left( 1i \cdot \omega \cdot Z_B(\omega, rd, rt) \cdot \exp(1i \cdot \omega \cdot t) \right) d\omega \]

\[ t := -1 ts, -0.99 ts, 4 ts \]

\[ t_{B1} := 1.2 \cdot ts \]

\[ t_{pk1} := \text{root}(x_{vout}(t_{B1}, 0.5 B, 0.5 B), t_{B1}) \]

\[ \frac{t_{pk1}}{ts} = 1.577 \]

\[ v_{max_{B1}} := v_{out}(t_{pk1}, 0.5 B, 0.5 B) = 0.816 \]

\[ v_{min_{B1}} := v_{out}(t_{pk1} + 1 \cdot ts, 0.5 B, 0.5 B) = 0.001 \]

\[ v_{min_{1,B1}} := v_{out}(t_{pk1} + 2 \cdot ts, 0.5 B, 0.5 B) = 0.008 \]

\[ v_{min_{2,B1}} := v_{out}(t_{pk1} + 3 \cdot ts, 0.5 B, 0.5 B) = 0.039 \]

\[ v_{max_{B1}} - v_{min_{B1}} = 0.815 \quad v_{max_{B1}} - v_{min_{1,B1}} = 0.808 \quad v_{max_{B1}} - v_{min_{2,B1}} = 0.777 \]

\[ \frac{v_{max_{B1}} - v_{min_{B1}}}{\sqrt{10^{16} \cdot I_{n,B}(0.5 B, 0.5 B)}} = 2.19 \quad \text{1st pulse} \]

\[ \frac{v_{max_{B1}} - v_{min_{1,B1}}}{\sqrt{10^{16} \cdot I_{n,B}(0.5 B, 0.5 B)}} = 2.172 \quad \text{2nd pulse} \]

\[ \frac{v_{max_{B1}} - v_{min_{2,B1}}}{\sqrt{10^{16} \cdot I_{n,B}(0.5 B, 0.5 B)}} = 2.09 \quad \text{3rd pulse, worst ISI} \]

Highest SNR @ rd=0.5 rt=0.5

\[ v_{max_{B}} := v_{max_{B1}} \quad v_{min_{B}} := v_{min_{2,B1}} \]

Eye-diagram check

\[ x_{111}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - 2 ts, rd, rt) + v_{out}(t - ts, rd, rt) \]
\[ x_{110}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - ts, rd, rt) \]
\[ x_{100}(t, rd, rt) := v_{out}(t, rd, rt) \]
\[ x_{001}(t, rd, rt) := v_{out}(t - 2 ts, rd, rt) \]
\[ x_{010}(t, rd, rt) := v_{out}(t - ts, rd, rt) \]
\[ x_{011}(t, rd, rt) := v_{out}(t - ts, rd, rt) + v_{out}(t - 2 ts, rd, rt) \]
\[ x_{101}(t, rd, rt) := v_{out}(t, rd, rt) + v_{out}(t - 2 ts, rd, rt) \]

**Sens PINFET**

\[ \lambda := 650 \cdot 10^{-9} \]
\[ \text{photon} \text{ energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]
\[ nq := 1.6 \cdot 10^{-10} \]

\[ Q_N(b) := \frac{1}{ts} \cdot b \cdot nq \cdot \left( \frac{v_{max,B} - v_{min,B}}{2} \right) \]
\[ I_{n,B}(0.5 B, 0.5 B) = 1.383 \cdot 10^{-17} \]

\[ P_e_N(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_N(b)}{\sqrt{2}} \right) \]
\[ P_{c,N}(b) := (\log(P_e_N(b)) + 9) \]
\[ b \equiv 3 \cdot 10^3 \]

\[ b_N := \text{root}(P_{c,N}(b) \cdot b) \]
\[ \text{minimum}_B := \min(b_N) = 3.587 \cdot 10^3 \]

\[ \text{Sens}_N \text{PIN} := 10 \cdot \log \left( \frac{\text{minimum}_B \cdot \text{photon} \text{ energy} \cdot 1}{ts} \right) = -42.606 \]
B.5.6 Performance comparison PINBJT

Performance comparison (PIN-BJT 100 Mbit/s)

Rx performance with PIN-BJT input configuration, 3rd order Butterworth pre-detection filter. Calculations are as follow:

- Frequency response (opt rd, rt for each receiver)
- Noise integrals
- Total noise
- Pulse shaping, peak voltage, ISI
- Error bit rate, minimum number of photons, receiver sensitivity

**Bit-rate, pulse duration**

\[ C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]
\[ B := 100 \cdot 10^6 \quad \text{Bit-rate} \]

**TIA(Non-tuned)**

\( f_c := 0.5 \cdot B \)
\( Av := 10 \)
\( \omega_c := 2 \cdot \pi \cdot f_c \)
\( R_{f-N} := \frac{Av + 1}{2 \cdot \pi \cdot f_c \cdot C_T} = 2.334 \cdot 10^4 \)
\( Z_{TIA-N}(\omega) := \frac{1}{1 + j \cdot \omega / \omega_c} \)

**TIA(Tuned-A)**

\( m := 1.8 \quad \text{time constant ratio of } L/R \text{ and } RC \)
\[ y := \sqrt{\frac{-m^2}{2} + m + 1} + \sqrt{\left(\frac{-m^2}{2} + m + 1\right)^2 + m^2} = 1.825 \]
\( d := \frac{1}{y} \cdot B \)
\( f_c := 0.5 \cdot d = 2.739 \cdot 10^7 \)
\( R_A := \frac{1}{(2 \cdot \pi \cdot C_T \cdot f_c)} = 3.874 \cdot 10^3 \)
\( L := R_A^2 \cdot \frac{C_T}{m} = 1.25 \cdot 10^{-5} \)
\( ts := \frac{1}{y \cdot d} \quad B := \frac{1}{ts} = 1 \cdot 10^8 \)
\( f_c := 0.5 \cdot B = 5 \cdot 10^7 \)
\( R_{f-A} := R_A \cdot (1 + Av) = 4.261 \cdot 10^4 \)
\( L_f := L \cdot (1 + Av) = 1.376 \cdot 10^{-4} \)
\[ Z_{TIA,A}(\omega) := \frac{1 + 1i \cdot \omega \cdot \frac{L_f}{Rf_A}}{\left( \left( 1 - \omega^2 \cdot L_f \cdot \frac{C_T}{1 + Av} \right) + Rf_A \cdot \omega \cdot \frac{C_T}{1 + Av} \cdot 1i \right)} \]

**TIA (Tuned-B)**

\[ \Delta_L := 1.9 \quad \Delta_R := 2.75 \quad \alpha := 0.4 \]

\[ Rf_B := \Delta_R \cdot Rf_N = 6.419 \cdot 10^4 \]

\[ Lc := \frac{(Rf_B)^2}{1 + Av} \cdot \frac{C_T}{\Delta_L} = 2.689 \cdot 10^{-5} \]

\[ C1 := (1 - \alpha) \cdot C_T = 9 \cdot 10^{-13} \]

\[ C2 := \alpha \cdot C_T = 6 \cdot 10^{-13} \]

\[ Z_{TIA,B}(\omega) := \frac{1}{\left( \left( 1 - \omega^2 \cdot Lc \cdot C1 \right) + \frac{Rf_B}{1 + Av} \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \right)} \]

**Receiver freq-response**

\[ \omega_{B,1} := 2 \cdot \pi \cdot 0.6 \, B \]

\[ \omega_{B,2} := 2 \cdot \pi \cdot 0.5 \, B \]

\[ H_{btl,N_A}(\omega) := \frac{\omega_{B,1}^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_{B,1} + 2 \cdot (1j \cdot \omega) \cdot \omega_{B,1}^2 + \omega_{B,1}^3} \]

\[ H_{btl,B}(\omega) := \frac{\omega_{B,2}^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_{B,2} + 2 \cdot (1j \cdot \omega) \cdot \omega_{B,2}^2 + \omega_{B,2}^3} \]

\[ Z_{nontuned}(\omega) := H_{btl,N_A}(\omega) \cdot Z_{TIA,A}(\omega) \]

\[ Z_{tuned.A}(\omega) := H_{btl,N_A}(\omega) \cdot Z_{TIA,B}(\omega) \]

\[ Z_{tuned.B}(\omega) := H_{btl,B}(\omega) \cdot Z_{TIA,B}(\omega) \]

**Noise equivalent bandwidth**

\[ NEB_N := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{13}} \left( \left| Z_{nontuned}(\omega) \right| \right)^2 \, d\omega \right) = 4.362 \cdot 10^7 \]

\[ NEB_A := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{13}} \left( \left| Z_{tuned,A}(\omega) \right| \right)^2 \, d\omega \right) = 5.059 \cdot 10^7 \]

\[ NEB_B := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{13}} \left( \left| Z_{tuned.B}(\omega) \right| \right)^2 \, d\omega \right) = 4.09 \cdot 10^7 \]

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Noise integrals

\[ I_{2\_N} = NEB\_N = 4.362 \cdot 10^7 \]

\[ I_{2\_A} = \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{12}} \left( \left( \frac{1}{\left( 1 + 1i \cdot \omega \cdot \frac{L_f}{R_{f\_A}} \right)} \cdot Z_{\_tuned\_A}(\omega) \right)^2 \right) d\omega \right) = 3.938 \cdot 10^7 \]

\[ I_{2\_B} = \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{12}} \left( \left( \left( 1 - \omega^2 \cdot L_c \cdot C_1 \right) \cdot Z_{\_tuned\_B}(\omega) \right)^2 \right) d\omega \right) = 2.587 \cdot 10^7 \]

\[ I_{3\_N} = \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{12}} \left( \left( \frac{1 \cdot \omega \cdot R_{f\_N} \cdot C_T}{1 \cdot R_{f\_N} \cdot Z_{\_nontuned}(\omega)} \right) \right)^2 d\omega \right) = 4.345 \]

\[ I_{3\_A} = \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{12}} \left( \left( \frac{\left( \left( 1 - \omega^2 \cdot L_c \cdot C_1 \right) + R_{f\_A} \cdot \omega \cdot C_T \cdot 1i \right)}{R_{f\_A} + 1i \cdot \omega \cdot L_f} \right) \cdot Z_{\_tuned\_A}(\omega) \right)^2 d\omega \right) = 4.029 \]

\[ I_{3\_B} = \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{12}} \left( \left( \frac{\left( \left( 1 - \omega^2 \cdot L_c \cdot C_1 \right) + R_{f\_B} \cdot (\omega) \cdot 1i \right)}{R_{f\_B}} \cdot \frac{\left( C_1 + C_2 - (\omega)^2 \cdot L_c \cdot C_1 \cdot C_2 \right)}{1i} \right) \right)^2 d\omega \right) = 0.784 \]

Feedback noise

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \]

\[ I_{nRf\_N} := \frac{4 \cdot k \cdot T}{R_{f\_N}} \cdot I_{2\_N} = 3.074 \cdot 10^{-17} \]

\[ I_{nRf\_A} := \frac{4 \cdot k \cdot T}{R_{f\_A}} \cdot I_{2\_A} = 1.52 \cdot 10^{-17} \]

\[ I_{nRf\_B} := \frac{4 \cdot k \cdot T}{R_{f\_B}} \cdot I_{2\_B} = 6.629 \cdot 10^{-18} \]

\[ 10 \log \left( \frac{I_{nRf\_B}}{I_{nRf\_N}} \right) = -6.663 \quad 10 \log \left( \frac{I_{nRf\_A}}{I_{nRf\_N}} \right) = -3.058 \]
BJT input stage

\[ h_{fe} = 100 \]

noise-opt BJT (Ic re-calculation)

\[
x_N(x) := \frac{4 \cdot k \cdot T}{Rf_N} \cdot I_{2\ N} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{2\ N} + 2 \cdot q \cdot \frac{x}{(25 \cdot 10^{-3})^2} \cdot I_{3\ N}
\]

\[
x_A(x) := \frac{4 \cdot k \cdot T}{Rf_A} \cdot I_{2\ A} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot NEB_A + 2 \cdot q \cdot \frac{x}{(25 \cdot 10^{-3})^2} \cdot I_{3\ A}
\]

\[
x_B(x) := \frac{4 \cdot k \cdot T}{Rf_B} \cdot I_{2\ B} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{2\ B} + 2 \cdot q \cdot \frac{x}{(25 \cdot 10^{-3})^2} \cdot I_{3\ B}
\]

\[ x := 0.00001, 0.00002, 0.01 \]

noise-opt BJT (ex)

\[ I_{opt\ N} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3\ N}}{I_{2\ N}}} = 7.89 \cdot 10^{-5} \]

\[ I_{opt\ A} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3\ A}}{NEB_A}} = 7.056 \cdot 10^{-5} \]

\[ I_{opt\ B} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3\ B}}{I_{2\ B}}} = 4.353 \cdot 10^{-5} \]

noise-opt BJT

\[ I_{n\ N} := x_N(I_{opt\ N}) = 5.277 \cdot 10^{-17} \]

\[ I_{n\ A} := x_A(I_{opt\ A}) = 3.805 \cdot 10^{-17} \]

\[ I_{n\ B} := x_B(I_{opt\ B}) = 1.383 \cdot 10^{-17} \]
Output pulse shape

\[ \tau_s := \frac{1}{B} \]

\[ \nu_{out\ N}(t) := \tau_s \cdot \sqrt{\frac{1}{\tau_s}} \cdot \sin\left(\frac{\omega \cdot \tau_s}{2}\right) \cdot \Re\left(\frac{Z_{\text{nontuned}}(\omega) \cdot \exp\left(\frac{\tau_s}{2} \cdot t - \frac{1}{2}\right)}{\omega \cdot \tau_s} \right) \, d\omega \]

\[ \nu_{out\ A}(t) := \tau_s \cdot \sqrt{\frac{1}{\tau_s}} \cdot \sin\left(\frac{\omega \cdot \tau_s}{2}\right) \cdot \Re\left(\frac{Z_{\text{tuned\ A}}(\omega) \cdot \exp\left(\frac{\tau_s}{2} \cdot t - \frac{1}{2}\right)}{\omega \cdot \tau_s} \right) \, d\omega \]

\[ \nu_{out\ B}(t) := \tau_s \cdot \sqrt{\frac{1}{\tau_s}} \cdot \sin\left(\frac{\omega \cdot \tau_s}{2}\right) \cdot \Re\left(\frac{Z_{\text{tuned\ B}}(\omega) \cdot \exp\left(\frac{\tau_s}{2} \cdot t - \frac{1}{2}\right)}{\omega \cdot \tau_s} \right) \, d\omega \]

peak voltage and time

\[ \xi_{out\ N}(t) := \tau_s^2 \cdot \sqrt{\frac{1}{\tau_s}} \cdot \sin\left(\frac{\omega \cdot \tau_s}{2}\right) \cdot \Re\left(\frac{\tau_s \cdot Z_{\text{nontuned}}(\omega) \cdot \exp\left(\frac{\tau_s}{2} \cdot t - \frac{1}{2}\right)}{\omega \cdot \tau_s} \right) \, d\omega \]

\[ \xi_{out\ A}(t) := \tau_s^2 \cdot \sqrt{\frac{1}{\tau_s}} \cdot \sin\left(\frac{\omega \cdot \tau_s}{2}\right) \cdot \Re\left(\frac{\tau_s \cdot Z_{\text{tuned\ A}}(\omega) \cdot \exp\left(\frac{\tau_s}{2} \cdot t - \frac{1}{2}\right)}{\omega \cdot \tau_s} \right) \, d\omega \]

\[ \xi_{out\ B}(t) := \tau_s^2 \cdot \sqrt{\frac{1}{\tau_s}} \cdot \sin\left(\frac{\omega \cdot \tau_s}{2}\right) \cdot \Re\left(\frac{\tau_s \cdot Z_{\text{tuned\ B}}(\omega) \cdot \exp\left(\frac{\tau_s}{2} \cdot t - \frac{1}{2}\right)}{\omega \cdot \tau_s} \right) \, d\omega \]
\[ t_N := 1 \cdot ts \]

\[ t_{pk1} := \text{root}(xvout_N(t_N), t_N) \]

\[
\frac{t_{pk1}}{ts} = 1.353
\]

\[ v_{\text{max}_N} := \text{vout}_N(t_{pk1}) = 0.887 \]

\[ v_{\text{min}_N} := \text{vout}_N(t_{pk1} + 1 \cdot ts) = 0.057 \]

\[ v_{\text{min2ts}_N} := \text{vout}_N(t_{pk1} + 2 \cdot ts) = 0.006 \]

\[ v_{\text{min3ts}_N} := \text{vout}_N(t_{pk1} + 3 \cdot ts) = -5.432 \cdot 10^{-4} \]

\[ t_A := 1.5 \cdot ts \]

\[ t_{pk1} := \text{root}(xvout_A(t_A), t_A) \]

\[
\frac{t_{pk1}}{ts} = 1.458
\]

\[ v_{\text{max}_A} := \text{vout}_A(t_{pk1}) = 0.947 \]

\[ v_{\text{min}_A} := \text{vout}_A(t_{pk1} + 1 \cdot ts) = 0.028 \]

\[ v_{\text{min2ts}_A} := \text{vout}_A(t_{pk1} + 2 \cdot ts) = -0.06 \]

\[ v_{\text{min3ts}_A} := \text{vout}_A(t_{pk1} + 3 \cdot ts) = 0.004 \]

\[ t_B := 2 \cdot ts \]

\[ t_{pk1} := \text{root}(xvout_B(t_B), t_B) \]

\[
\frac{t_{pk1}}{ts} = 2.077
\]

\[ v_{\text{max}_B} := \text{vout}_B(t_{pk1}) = 0.816 \]

\[ v_{\text{min1ls}_B} := \text{vout}_B(t_{pk1} + 1 \cdot ts) = 0.001 \]

\[ v_{\text{min2ts}_B} := \text{vout}_B(t_{pk1} + 2 \cdot ts) = 0.008 \]

\[ v_{\text{min3ts}_B} := \text{vout}_B(t_{pk1} + 3 \cdot ts) = 0.039 \quad v_{\text{min}_B} := v_{\text{min3ts}_B} \]

\[ nq := 1.6 \cdot 10^{-19} \quad \text{Test}_\text{b}_\text{photons} := 1100, 1200 \ldots 15000 \]

\[
\text{test}_Q_N(\text{Test}_b\text{photons}) := \frac{1}{ts} \cdot \text{Test}_b\text{photons} \cdot nq \cdot \frac{v_{\text{max}_N} - v_{\text{min}_N}}{2} \sqrt{I_{n_N}}
\]

\[
\text{test}_Q_A(\text{Test}_b\text{photons}) := \frac{1}{ts} \cdot \text{Test}_b\text{photons} \cdot nq \cdot \frac{v_{\text{max}_A} - v_{\text{min}_A}}{2} \sqrt{I_{n_A}}
\]
number of photons per bit for $10^9$

\[
Q_N(b) = \frac{1}{t_s} \cdot b \cdot nq \cdot \left(\frac{v_{\text{max},N} - v_{\text{min},N}}{2}\right) \sqrt{I_{n,N}}
\]

\[
Q_A(b) = \frac{1}{t_s} \cdot b \cdot nq \cdot \left(\frac{v_{\text{max},A} - v_{\text{min},A}}{2}\right) \sqrt{I_{n,A}}
\]

\[
Q_B(b) = \frac{1}{t_s} \cdot b \cdot nq \cdot \left(\frac{v_{\text{max},B} - v_{\text{min},B}}{2}\right) \sqrt{I_{n,B}}
\]

\[
Pe_N(b) = \frac{1}{2} \cdot \text{erfc} \left(\frac{Q_N(b)}{\sqrt{2}} \right)
\]

\[
Pe_A(b) = \frac{1}{2} \cdot \text{erfc} \left(\frac{Q_A(b)}{\sqrt{2}} \right)
\]

\[
Pe_B(b) = \frac{1}{2} \cdot \text{erfc} \left(\frac{Q_B(b)}{\sqrt{2}} \right)
\]
\[ P_{c_N}(b) = (\log(P_{c_N}(b)) + 9) \]
\[ P_{c_A}(b) = (\log(P_{c_A}(b)) + 9) \]
\[ P_{c_B}(b) = (\log(P_{c_B}(b)) + 9) \]
\[ b = 3 \cdot 10^3 \]
\[ b_N = \text{root}(P_{c_N}(b), b) \]
\[ b_A = \text{root}(P_{c_A}(b), b) \]
\[ b_B = \text{root}(P_{c_B}(b), b) \]
\[ \text{minimum}_N = \text{min}(b_N) = 6.562 \cdot 10^3 \]
\[ \text{minimum}_A = \text{min}(b_A) = 5.035 \cdot 10^3 \]
\[ \text{minimum}_B = \text{min}(b_B) = 3.587 \cdot 10^3 \]

**BER-check for minimum b**

\[
\begin{align*}
Q_N(\text{minimum}_N) &= 5.998 & P_{e_N}(\text{minimum}_N) &= 10 \cdot 10^{-10} \\
Q_A(\text{minimum}_A) &= 5.998 & P_{e_A}(\text{minimum}_A) &= 1 \cdot 10^{-9} \\
Q_B(\text{minimum}_B) &= 5.998 & P_{e_B}(\text{minimum}_B) &= 1 \cdot 10^{-9}
\end{align*}
\]

**Sens PIN-BJT**

\[ \lambda = 650 \cdot 10^{-9} \]
\[ \text{photon} \_\text{energy} = \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]
\[ \text{Sens}_N = 10 \cdot \log \left( \frac{\text{minimum}_N}{2} \cdot \frac{\text{photon} \_\text{energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -39.983 \]
\[ \text{Sens}_A = 10 \cdot \log \left( \frac{\text{minimum}_A}{2} \cdot \frac{\text{photon} \_\text{energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -41.133 \]
\[ \text{Sens}_B = 10 \cdot \log \left( \frac{\text{minimum}_B}{2} \cdot \frac{\text{photon} \_\text{energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -42.606 \]

\[ \text{Sens}_B - \text{Sens}_N = -2.623 \]
\[ \text{Sens}_A - \text{Sens}_N = -1.151 \]
B.5.7 Performance comparison APDBJT

**Silicon APD: Performance comparison (APD-BJT 1st order LPF 100 Mbit/s)**

Rx performance with APD-BJT input configuration, 1st order LPF pre-detection filter. Calculations are as follows:

- APD noise, APD BER, APD receiver sensitivity
- [Silicon Epitaxial: APD M= 10 F(M)=5.5]

### APD noise

\[ M_{APD} := 10 \]

\[ F_M := 5.5 \]

\[ APD_N(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot (v_{max,N}) \]

\[ APD_A(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot (v_{max,A}) \]

\[ APD_B(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot (v_{max,B}) \]

\[ I_{APD,d} := 10 \cdot 10^{-9} \]

\[ Noise_{APD,N}(b) := \left( 2 \cdot q \cdot APD_N(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_N \]

\[ + \left( 2 \cdot q \cdot I_{APD,d} \cdot M_{APD}^2 \cdot F_M \right) \]

\[ Noise_{APD,A}(b) := \left( 2 \cdot q \cdot APD_A(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_A \]

\[ + \left( 2 \cdot q \cdot I_{APD,d} \cdot M_{APD}^2 \cdot F_M \right) \]

\[ Noise_{APD,B}(b) := \left( 2 \cdot q \cdot APD_B(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_B \]

\[ + \left( 2 \cdot q \cdot I_{APD,d} \cdot M_{APD}^2 \cdot F_M \right) \]

### BER+APD

\[ Q_{APD,N}(b) := \frac{2 \cdot q \cdot APD_N(b) \cdot M_{APD}^2 \cdot F_M \cdot I_{n,N} + Noise_{APD,N}(b)}{2} \]

\[ Q_{APD,A}(b) := \frac{2 \cdot q \cdot APD_A(b) \cdot M_{APD}^2 \cdot F_M \cdot I_{n,A} + Noise_{APD,A}(b)}{2} \]

\[ Q_{APD,B}(b) := \frac{2 \cdot q \cdot APD_B(b) \cdot M_{APD}^2 \cdot F_M \cdot I_{n,B} + Noise_{APD,B}(b)}{2} \]

\[ P_e_{APD,N}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD,N}(b)}{\sqrt{2}} \right) \]

\[ P_e_{APD,A}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD,A}(b)}{\sqrt{2}} \right) \]

\[ P_e_{APD,B}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD,B}(b)}{\sqrt{2}} \right) \]
\[ p_{\text{APD}_N}(b) := \log (P_{\text{e-APD}_N}(b)) + 9 \]
\[ p_{\text{APD}_A}(b) := \log (P_{\text{e-APD}_A}(b)) + 9 \]
\[ p_{\text{APD}_B}(b) := \log (P_{\text{e-APD}_B}(b)) + 9 \]
\[ \text{APD}_b_N := \text{root}(p_{\text{APD}_N}(b), b) \]
\[ \text{APD}_b_A := \text{root}(p_{\text{APD}_A}(b), b) \]
\[ \text{APD}_b_B := \text{root}(p_{\text{APD}_B}(b), b) \]

minimum\_APD\_N := \min(\text{APD}_b_N) = 1.565 \cdot 10^3
minimum\_APD\_A := \min(\text{APD}_b_A) = 1.473 \cdot 10^3
minimum\_APD\_B := \min(\text{APD}_b_B) = 1.432 \cdot 10^3

Check BER, Q, and b APD

\[ Q_{\text{APD}_N}(\min(\text{APD}_b_N)) = 5.998 \]
\[ Q_{\text{APD}_A}(\min(\text{APD}_b_A)) = 5.998 \]
\[ Q_{\text{APD}_B}(\min(\text{APD}_b_B)) = 5.998 \]

Sens

\[ \text{APD}_N := 10 \cdot \log \left( \frac{\text{minimum\_APD\_N}}{2} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -46.209 \]
\[ \text{APD}_A := 10 \cdot \log \left( \frac{\text{minimum\_APD\_A}}{2} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -46.47 \]
\[ \text{APD}_B := 10 \cdot \log \left( \frac{\text{minimum\_APD\_B}}{2} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \frac{1}{ts} \right) = -46.594 \]

\[ \text{Sens}_N - \text{APD}_N = 6.226 \quad \text{Noise}_{\text{APD}_N}(\text{minimum\_APD\_N}) = 2.473 \cdot 10^{-16} \]
\[ \text{Sens}_A - \text{APD}_A = 5.336 \quad \text{Noise}_{\text{APD}_A}(\text{minimum\_APD\_A}) = 2.878 \cdot 10^{-16} \]
\[ \text{Sens}_B - \text{APD}_B = 3.988 \quad \text{Noise}_{\text{APD}_B}(\text{minimum\_APD\_B}) = 2.066 \cdot 10^{-16} \]

\[ I_{n_N} = 5.277 \cdot 10^{-17} \quad I_{n_A} = 3.805 \cdot 10^{-17} \quad I_{n_B} = 1.383 \cdot 10^{-17} \]

#
InGaAs_APD: Performance comparison (APD-FET 3rd order Butterworth 100 Mbit/s)

Rx performance with APD-BJT input configuration. 3rd order Butterworth pre-detection filter. calculations are as follow

- APD noise, APD BER, APD receiver sensitivity
- [InGaAs: M= 100 F(M)=7.9]

**APD noise**

\[ M_{APD} := 100 \]
\[ F_M := 7.9 \]

\[ APD_N(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot (v_{max_N}) \]

\[ APD_A(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot (v_{max_A}) \]

\[ APD_B(b) := \frac{1}{t_s} \cdot b \cdot nq \cdot (v_{max_B}) \]

\[ I_{APD_d} := 10 \cdot 10^{-6} \]

\[ Noise_{APD_N}(b) := \left( 2 \cdot q \cdot APD_N(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_N \]

\[ Noise_{APD_A}(b) := \left( 2 \cdot q \cdot APD_A(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_A \]

\[ Noise_{APD_B}(b) := \left( 2 \cdot q \cdot APD_B(b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_B \]

**BER+APD**

\[ Q_{APD_N}(b) := \frac{M_{APD} \cdot b \cdot nq \cdot \left( v_{max_N} - v_{min_N} \right)}{\sqrt{I_n + Noise_{APD_N}(b)}} \]

\[ Q_{APD_A}(b) := \frac{M_{APD} \cdot b \cdot nq \cdot \left( v_{max_A} - v_{min_A} \right)}{\sqrt{I_n + Noise_{APD_A}(b)}} \]

\[ Q_{APD_B}(b) := \frac{M_{APD} \cdot b \cdot nq \cdot \left( v_{max_B} - v_{min_B} \right)}{\sqrt{I_n + Noise_{APD_B}(b)}} \]

\[ P_{APD_N}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD_N}(b)}{\sqrt{2}} \right) \]

\[ P_{APD_A}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD_A}(b)}{\sqrt{2}} \right) \]
\[ P_{e_{\text{APD-B}}}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD-B}}(b)}{\sqrt{2}} \right) \]

\[ p_{c_{\text{APD-N}}}(b) := (\log (P_{e_{\text{APD-N}}}(b)) + 9) \]

\[ p_{c_{\text{APD-A}}}(b) := (\log (P_{e_{\text{APD-A}}}(b)) + 9) \]

\[ p_{c_{\text{APD-B}}}(b) := (\log (P_{e_{\text{APD-B}}}(b)) + 9) \]

\[ \text{APD}_b_N := \text{root} (p_{c_{\text{APD-N}}}(b), b) \]

\[ \text{APD}_b_A := \text{root} (p_{c_{\text{APD-A}}}(b), b) \]

\[ \text{APD}_b_B := \text{root} (p_{c_{\text{APD-B}}}(b), b) \]

\[ \text{minimum}_{\text{APD-N}} := \min (\text{APD}_b_N) = 1.784 \cdot 10^3 \]

\[ \text{minimum}_{\text{APD-A}} := \min (\text{APD}_b_A) = 1.773 \cdot 10^3 \]

\[ \text{minimum}_{\text{APD-B}} := \min (\text{APD}_b_B) = 1.793 \cdot 10^3 \]

**Check BER, Q, and b APD**

\[ Q_{\text{APD-N}}(\min (\text{APD}_b_N)) = 5.998 \]

\[ Q_{\text{APD-A}}(\min (\text{APD}_b_A)) = 5.998 \]

\[ Q_{\text{APD-B}}(\min (\text{APD}_b_B)) = 5.998 \]

**Sens**

\[ \text{APD}_N := 10 \cdot \log \left( \frac{\text{minimum}_{\text{APD-N}} \cdot \text{photon\_energy} \cdot \frac{1}{ts}}{2 \cdot 10^{-3}} \right) = -45.639 \]

\[ \text{APD}_A := 10 \cdot \log \left( \frac{\text{minimum}_{\text{APD-A}} \cdot \text{photon\_energy} \cdot \frac{1}{ts}}{2 \cdot 10^{-3}} \right) = -45.666 \]

\[ \text{APD}_B := 10 \cdot \log \left( \frac{\text{minimum}_{\text{APD-B}} \cdot \text{photon\_energy} \cdot \frac{1}{ts}}{2 \cdot 10^{-3}} \right) = -45.617 \]

\[ \text{Sens}_N - \text{APD}_N = 5.656 \quad \text{Noise}_{\text{APD-N}}(\text{minimum}_{\text{APD-N}}) = 3.896 \cdot 10^{-14} \]

\[ \text{Sens}_A - \text{APD}_A = 4.533 \quad \text{Noise}_{\text{APD-A}}(\text{minimum}_{\text{APD-A}}) = 4.714 \cdot 10^{-14} \]

\[ \text{Sens}_B - \text{APD}_B = 3.011 \quad \text{Noise}_{\text{APD-B}}(\text{minimum}_{\text{APD-B}}) = 3.456 \cdot 10^{-14} \]

\[ I_{n,N} = 5.277 \cdot 10^{-17} \quad I_{n,A} = 3.805 \cdot 10^{-17} \quad I_{n,B} = 1.383 \cdot 10^{-17} \]
**Germanium APD: Performance comparison APD-FET 3rd order Butterworth 100 Mbit/s**

Rx performance with APD-FET input configuration, 3rd order Butterworth pre-detection filter. Calculations are as follows:

- APD noise, APD BER, APD receiver sensitivity
- [Germanium: M=10 F(M)=9.2]

**APD noise**

\[ M_{APD} := 10 \]
\[ F_M := 9.2 \]

\[ APD_N (b) := \frac{1}{ts} \cdot b \cdot n_q \cdot (v_{max,N}) \]

\[ APD_A (b) := \frac{1}{ts} \cdot b \cdot n_q \cdot (v_{max,A}) \]

\[ APD_B (b) := \frac{1}{ts} \cdot b \cdot n_q \cdot (v_{max,B}) \]

\[ I_{APD_d} := 10 \cdot 10^{-9} \]

\[ Noise_{APD_N} (b) := \left( 2 \cdot q \cdot APD_N (b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_N + 2 \cdot q \cdot I_{APD_d} \cdot M_{APD}^2 \cdot F_M \]

\[ Noise_{APD_A} (b) := \left( 2 \cdot q \cdot APD_A (b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_A + 2 \cdot q \cdot I_{APD_d} \cdot M_{APD}^2 \cdot F_M \]

\[ Noise_{APD_B} (b) := \left( 2 \cdot q \cdot APD_B (b) \cdot M_{APD}^2 \cdot F_M \right) \cdot NEB_B + 2 \cdot q \cdot I_{APD_d} \cdot M_{APD}^2 \cdot F_M \]

**BER+APD**

\[ Q_{APD_N} (b) := \frac{M_{APD}}{ts} \cdot b \cdot n_q \cdot \left( \frac{v_{max,N} - v_{min,N}}{2} \right) \]

\[ Q_{APD_A} (b) := \frac{M_{APD}}{ts} \cdot b \cdot n_q \cdot \left( \frac{v_{max,A} - v_{min,A}}{2} \right) \]

\[ Q_{APD_B} (b) := \frac{M_{APD}}{ts} \cdot b \cdot n_q \cdot \left( \frac{v_{max,B} - v_{min,B}}{2} \right) \]

\[ P_{e_{APD_N}} (b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD_N} (b)}{\sqrt{2}} \right) \]

\[ P_{e_{APD_A}} (b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{APD_A} (b)}{\sqrt{2}} \right) \]
\[ P_{c\text{-}APD\_B}(b) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{APD}\_B}(b)}{\sqrt{2}} \right) \]

\[ p_{c\text{-}APD\_N}(b) := \log(P_{c\text{-}APD\_N}(b)) + 9 \]

\[ p_{c\text{-}APD\_A}(b) := \log(P_{c\text{-}APD\_A}(b)) + 9 \]

\[ p_{c\text{-}APD\_B}(b) := \log(P_{c\text{-}APD\_B}(b)) + 9 \]

\[ \text{APD\_b\_N} := \text{root}(p_{c\text{-}APD\_N}(b), b) \]

\[ \text{APD\_b\_A} := \text{root}(p_{c\text{-}APD\_A}(b), b) \]

\[ \text{APD\_b\_B} := \text{root}(p_{c\text{-}APD\_B}(b), b) \]

\[ \text{minimum\_APD\_N} := \min(\text{APD\_b\_N}) = 2.169 \cdot 10^3 \]

\[ \text{minimum\_APD\_A} := \min(\text{APD\_b\_A}) = 2.097 \cdot 10^3 \]

\[ \text{minimum\_APD\_B} := \min(\text{APD\_b\_B}) = 2.067 \cdot 10^3 \]

**Check BER, Q, and b APD**

\[ Q_{\text{APD}\_N}(\min(\text{APD\_b\_N})) = 5.998 \]

\[ Q_{\text{APD}\_A}(\min(\text{APD\_b\_A})) = 5.998 \]

\[ Q_{\text{APD}\_B}(\min(\text{APD\_b\_B})) = 5.998 \]

**Sens**

\[ \text{APD\_N} := 10 \cdot \log \left( \frac{\text{minimum\_APD\_N} \cdot \text{photon\_energy} \cdot \frac{1}{ts}}{2 \cdot 10^{-3}} \right) = -44.79 \]

\[ \text{APD\_A} := 10 \cdot \log \left( \frac{\text{minimum\_APD\_A} \cdot \text{photon\_energy} \cdot \frac{1}{ts}}{2 \cdot 10^{-3}} \right) = -44.936 \]

\[ \text{APD\_B} := 10 \cdot \log \left( \frac{\text{minimum\_APD\_B} \cdot \text{photon\_energy} \cdot \frac{1}{ts}}{2 \cdot 10^{-3}} \right) = -45 \]

\[ \text{Sens\_N} - \text{APD\_N} = 4.808 \quad \text{Noise\_APD\_N}(\text{minimum\_APD\_N}) = 5.238 \cdot 10^{-16} \]

\[ \text{Sens\_A} - \text{APD\_A} = 3.803 \quad \text{Noise\_APD\_A}(\text{minimum\_APD\_A}) = 6.222 \cdot 10^{-16} \]

\[ \text{Sens\_B} - \text{APD\_B} = 2.394 \quad \text{Noise\_APD\_B}(\text{minimum\_APD\_B}) = 4.455 \cdot 10^{-16} \]

\[ I_{n\_N} = 5.277 \cdot 10^{-17} \quad I_{n\_A} = 3.805 \cdot 10^{-17} \quad I_{n\_B} = 1.383 \cdot 10^{-17} \]
B.6 Summary of APD results

Table A-2 APD-based receiver comparison (100 Mbits/s). the terms in brackets are the avalanche gain and noise factor of APD.

<table>
<thead>
<tr>
<th>FET/1\textsuperscript{st} order LPF</th>
<th>Non-tuned</th>
<th>Tuned A</th>
<th>Tuned B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon (10/5.5)</td>
<td>-45.8</td>
<td>-46.24</td>
<td>-46.79</td>
</tr>
<tr>
<td>InGaAs (100/7.9)</td>
<td>-45.03</td>
<td>-45.22</td>
<td>-45.77</td>
</tr>
<tr>
<td>Germanium (10/9.2)</td>
<td>-44.27</td>
<td>-44.58</td>
<td>-45.18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FET/3\textsuperscript{rd} order Butterworth</th>
<th>Non-tuned</th>
<th>Tuned A</th>
<th>Tuned B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon (10/5.5)</td>
<td>-46.43</td>
<td>-44.74</td>
<td>-46.69</td>
</tr>
<tr>
<td>InGaAs (100/7.9)</td>
<td>-45.65</td>
<td>-45.72</td>
<td>-45.61</td>
</tr>
<tr>
<td>Germanium (10/9.2)</td>
<td>-44.91</td>
<td>-45.09</td>
<td>-45.05</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BJT/1\textsuperscript{st} order LPF</th>
<th>Non-tuned</th>
<th>Tuned A</th>
<th>Tuned B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon (10/5.5)</td>
<td>-45.5</td>
<td>-46.28</td>
<td>-46.6</td>
</tr>
<tr>
<td>InGaAs (100/7.9)</td>
<td>-45.03</td>
<td>-45.56</td>
<td>-45.74</td>
</tr>
<tr>
<td>Germanium (10/9.2)</td>
<td>-44.11</td>
<td>-44.79</td>
<td>-45.07</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BJT/3\textsuperscript{rd} order Butterworth</th>
<th>Non-tuned</th>
<th>Tuned A</th>
<th>Tuned B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon (10/5.5)</td>
<td>-46.2</td>
<td>-46.47</td>
<td>-46.59</td>
</tr>
<tr>
<td>InGaAs (100/7.9)</td>
<td>-45.63</td>
<td>-45.66</td>
<td>-45.61</td>
</tr>
<tr>
<td>Germanium (10/9.2)</td>
<td>-44.79</td>
<td>-44.93</td>
<td>-45.8</td>
</tr>
</tbody>
</table>
Appendix C: Results II (Tuned PPM receiver)

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C.1 Pulse shape (Gaussian)

Gaussian input pulse shape and matched filter output pulse shape

\[ PCM_B := 1 \cdot 10^9 \]
\[ M := 3 \]

\[ T_n := \frac{M}{PCM_B} \]
Frame time - DPPM

\[ m \equiv 1 \]
Modulation depth - DPPM

\[ n := 2^M \]
Number of DPPM active slots

\[ T_s := \frac{m \cdot T_n}{n} = 3.75 \cdot 10^{-10} \]
Slot width

\[ T_b := \frac{1}{PCM_B} \]
This is the PCM bit time

\[ f_{n0.5} := 0.5 \]
\[ \alpha_{0.5} := \frac{0.1874 \cdot T_b}{f_{n0.5}} = 3.748 \cdot 10^{-10} \]

\[ f_{n1} := 0.7 \]
\[ \alpha_1 := \frac{0.1874 \cdot T_b}{f_{n1}} = 2.677 \cdot 10^{-10} \]

\[ f_{n2} := 1 \]
\[ \alpha_2 := \frac{0.1874 \cdot T_b}{f_{n2}} = 1.874 \cdot 10^{-10} \]

\[ f_{n4} := 2 \]
\[ \alpha_5 := \frac{0.1874 \cdot T_b}{f_{n4}} = 9.37 \cdot 10^{-11} \]

\[ I_{0.5}(t) := \frac{1}{\pi} \int_0^{\frac{1}{T_s \cdot 10^5}} \exp \left( -\alpha_{0.5}^2 \cdot \omega^2 \right) \cdot \text{Re} \left( \exp \left( 1i \cdot \omega \cdot (t) \right) \right) \, d\omega \]

\[ I_1(t) := \frac{1}{\pi} \int_0^{\frac{1}{T_s \cdot 10^5}} \exp \left( -\alpha_1^2 \cdot \omega^2 \right) \cdot \text{Re} \left( \exp \left( 1i \cdot \omega \cdot (t) \right) \right) \, d\omega \]

\[ I_2(t) := \frac{1}{\pi} \int_0^{\frac{1}{T_s \cdot 10^5}} \exp \left( -\alpha_2^2 \cdot \omega^2 \right) \cdot \text{Re} \left( \exp \left( 1i \cdot \omega \cdot (t) \right) \right) \, d\omega \]
\[
I_5(t) := \frac{1}{\pi} \cdot \int_0^{1 \cdot 10^7} \exp\left(\frac{-\alpha_5^2 \cdot \omega^2}{2}\right) \cdot \text{Re}\left(\exp\left(\frac{i \cdot \omega \cdot (t)}{2}\right)\right) d\omega
\]

\[
t := -5 \cdot T_s, -4.99 \cdot T_s, 5 \cdot T_s
\]

\[
oI_{0.5}(t) := \frac{1}{\pi} \cdot \int_0^{1 \cdot 10^7} \exp\left(\frac{-\alpha^{0.5}_2 \cdot \omega^2}{2}\right) \cdot \exp\left(\frac{-\alpha^{0.5}_5 \cdot \omega^2}{2}\right) d\omega
\]

\[
oI_1(t) := \frac{1}{\pi} \cdot \int_0^{1 \cdot 10^7} \exp\left(\frac{-\alpha^1_2 \cdot \omega^2}{2}\right) \cdot \exp\left(\frac{-\alpha^1_5 \cdot \omega^2}{2}\right) d\omega
\]

\[
oI_2(t) := \frac{1}{\pi} \cdot \int_0^{1 \cdot 10^7} \exp\left(\frac{-\alpha^2_2 \cdot \omega^2}{2}\right) \cdot \exp\left(\frac{-\alpha^2_5 \cdot \omega^2}{2}\right) d\omega
\]

\[
oI_5(t) := \frac{1}{\pi} \cdot \int_0^{1 \cdot 10^7} \exp\left(\frac{-\alpha^5_2 \cdot \omega^2}{2}\right) \cdot \exp\left(\frac{-\alpha^5_5 \cdot \omega^2}{2}\right) d\omega
\]
C.2 PPM receiver with Matched filter (Optical fibre/without equalizer)

C.2.1 non-tuned PPM receiver with matched filter (PIN-FET)

PPM (PIN-FET Matched filter 1 Gbit/s)

PPM Rx performance with PIN-FET input configuration, Matched filter pre-detection filter. calculations are as follow

- PPM terms
- Rx terms (noise + TF)
- Pulse shaping, voltages
- Error bit rate
- Optimum threshold/minimum number of photons
- receiver sensitivity

Bit-rate, pulse duration, and input pulse

\[ PCM_B := 1 \cdot 10^9 \]
\[ M := 3 \]
\[ T_n := \frac{M}{PCM_B} \quad \text{Frame time - DPPM} \]
\[ m \equiv 1 \quad \text{Modulation depth - DPPM} \]
\[ n := 2^M \quad \text{Number of DPPM active slots} \]
\[ T_s := \frac{m \cdot T_n}{n} = 3.75 \cdot 10^{-10} \quad \text{Slot width} \]
\[ \eta q := 1.6 \cdot 10^{-10} \quad \text{Quantum energy} \]
\[ \lambda := 1.55 \cdot 10^{-6} \quad \text{wavelength of operation} \]

\[ \text{photon} \_\text{energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]
\[ T_b := \frac{1}{PCM_B} \quad \text{This is the PCM bit time} \]
\[ f_n := 5 \]
\[ \alpha := \frac{0.1874 \cdot T_b}{f_n} = 3.748 \cdot 10^{-11} \]
\[ \tau_R := \alpha \]

Preamplifier terms

\[ \omega_c := 2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} = 3.351 \cdot 10^{10} \quad Av := 10 \]
\[ C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]
\[ Z_{TIA,N}(\omega) := \frac{1}{1 + j \cdot \omega \cdot \frac{2}{\omega_c}} \]
\[ \text{filter}(\omega) := \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \]
\[ \frac{1}{T_s} = 2.667 \cdot 10^9 \]
\[ R_{f\_N} := \frac{A_v + 1}{2 \cdot \pi \cdot 2 \cdot \left( \frac{1}{T_s} \cdot C_T \right)} = 218.838 \]

\[ Z_{\text{nontuned}}(\omega) := \text{filter}(\omega) \cdot Z_{\text{TIA\_N}}(\omega) \quad \mu R := \alpha \]

**Receiver noise**

\[ \text{NEB\_N} := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^3} \left( \left| Z_{\text{nontuned}}(\omega) \right| \right)^2 d\omega \right) = 3.071 \cdot 10^9 \]

\[ I_{2\_N} := \text{NEB\_N} = 3.071 \cdot 10^9 \]

\[ I_{3\_N} := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^3} \left( \left| \frac{1 + 1j \cdot (\omega) \cdot R_{f\_N} \cdot C_T \cdot Z_{\text{nontuned}}(\omega)}{1 \cdot R_{f\_N}} \right| \right)^2 d\omega \right) = 1.813 \cdot 10^6 \]

**Feedback noise**

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \]

\[ I_{\text{nRF\_N}} := \frac{4 \cdot k \cdot T}{R_{f\_N}} \cdot I_{2\_N} = 2.308 \cdot 10^{-13} \]

**Ig noise** (gate current)

\[ I_{\text{g\_N}} := 10 \cdot 10^{-9} \]

\[ I_{\text{g\_N}} := 2 \cdot q \cdot I_g \cdot I_{2\_N} = 9.827 \cdot 10^{-18} \]

**Channel noise (Gate-source)**

\[ g_{m1} := 30 \cdot 10^{-3} = 0.03 \quad \text{noise\_factor} := 1 \]

\[ I_{\text{nD\_N}} := 4 \cdot k \cdot T \cdot \frac{1}{g_{m1}} \]

\[ I_{\text{nD\_N}} := I_{\text{nD\_N}} = 9.941 \cdot 10^{-13} \]

**Total noise- PIN-FET**

\[ I_{\text{n\_N}} := I_{\text{nRF\_N}} + I_{\text{nG\_N}} + I_{\text{nD\_N}} = 1.225 \cdot 10^{-12} \]

**noise** := \[ I_{\text{n\_N}} = 1.225 \cdot 10^{-12} \]

**Pulse shape**

\[ I_1(t) := \frac{T_s}{\pi} \cdot \frac{1}{10^5} \cdot \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right)^j \]

\[ \cdot \text{Re} \left( \frac{1}{10} \cdot \omega \cdot \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \cdot \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_c}} \cdot \exp (1i \cdot \omega \cdot (t)) \right) \]

C-7
\[ I_0(t) := \frac{1}{\pi} \cdot \frac{1}{T_s} \cdot \left[ \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \cdot \text{Re} \left( \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \cdot \frac{1}{1 + j \cdot \omega / \omega_c} \right) \right] \text{d}\omega \]

\[ t := 0.1 \cdot T_s \]

\[ t_{pk} := \text{root} \left( T_s \cdot I_1(t), t \right) \]

\[ \frac{t_{pk}}{T_s} = 0.066 \]

\[ t := -5 \cdot T_s, -4.90 \cdot T_s, \ldots, 6 \cdot T_s \]

**Voltages**

\[ v_o(t) := I_0(t) \]

\[ v_o(t_{pk}) = 6.742 \times 10^9 \]

---

**Graphs:**

- **\( I_0(t) \)**
- **\( I_0(t) + I_0(t - T_s) \)**
- **\( I_0(t) + I_0(t - 2T_s) \)**
- **\( I_0(t) + I_0(t - 3T_s) \)**
- **\( I_0(t) + I_0(t - 4T_s) \)**

---

**Graphs:**

- **\( I_1(t) \)**
\[ v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \]
\[ \text{temp}(v, t) := \frac{I_0(t)}{v \cdot I_0(t_{pk})} - 1 \]
\[ t := -0.1 \cdot T_s \]
\[ t_d(v) := \text{root}(\text{temp}(v, t), t) \]
\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \]
\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \]
\[ \text{slope}(b, v) := b \cdot \eta q \cdot \frac{I_1(t_d(v))}{T_s} \]
\[ v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \]

**Q values**
\[ Q_r(b, v) := \frac{v_{pk}(b) - v_d(b, v)}{\sqrt{\text{noise}}} \]
\[ Q_s(b, v) := \left( \frac{m \cdot T_n}{2 \cdot n} \right) \cdot \frac{\text{slope}(b, v)}{\sqrt{\text{noise}}} \]
\[ Q_t(b, v) := \frac{v_d(b, v)}{\sqrt{\text{noise}}} \]

**Error probabilities**
\[ P_r(b, v) := 0.5 \cdot \text{erfc} \left( \frac{Q_r(b, v)}{\sqrt{2}} \right) \]
\[ P_s(b, v) := \text{erfc} \left( \frac{Q_s(b, v)}{\sqrt{2}} \right) \]
\[ P_f(b, v) := \frac{m \cdot T_n}{n \cdot \alpha} \cdot 0.5 \cdot \text{erfc} \left( \frac{Q_t(b, v)}{\sqrt{2}} \right) \]
\[ P_{es}(b, v) := P_r(b, v) + P_s(b, v) + \frac{n-1}{2} \cdot P_f(b, v) \]
\[ P_{eb}(b, v) := \frac{n}{2 \cdot (n-1)} \cdot P_{es}(b, v) \]
\[ b := 0.25 \cdot 10^4 \]
\[ pc(b, v) := \log(P_{eb}(b, v)) + 9 \]
\[ a(v) := \text{root}(pc(b, v), b) \]

**Set for 1 in 10^9 errors**

Find the root to give 1 in 10^9
\( v := 0.2, 0.21 \ldots 0.9 \)

\( b_{(v-0.2) \cdot 100} := a(v) \)

minimum := \( \min(b) \)

minimum = \( 1.292 \cdot 10^4 \)

result := \( \text{minimum} \cdot \text{photon\_energy} \cdot \frac{\text{PCM\_B}}{M} \)

\( \omega := 1 \cdot 10^7, 10 \cdot 10^7 \ldots 5 \cdot 10^{11} \)

\( \frac{1.292 \cdot 10^4}{M} = 4.307 \cdot 10^3 \)

\( dBM := 10 \cdot \log \left( \frac{\text{result}}{10^{-3}} \right) \)

\( dBM = -32.574 \)

\( |Z_{TIA,N}(\omega)| \)

\( |filter(\omega)| \)

\( |Z_{TIA,N}(\omega)| \cdot |\text{filter}(\omega)| \)
\[ v = 0.49, 0.5 \ldots 0.55 \quad b = 1.2 \cdot 10^4, 1.21 \cdot 10^4 \ldots 1.4 \cdot 10^4 \]

\[ P_{ex}(b, v) \]
C.2.2 Tuned-B PPM receiver with matched filter (PIN-FET)

PPM (PIN-FET Matched filter 1 Gbit/s)

PPM Rx performance with PIN-FET input configuration. Matched filter pre-detection filter. Calculations are as follow

- PPM terms
- Rx terms (noise + TF)
- Pulse shaping, voltages
- Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

**Bit-rate, pulse duration, and input pulse**

\[ PCM_B := 1 \cdot 10^9 \]

\[ M := 3 \]

\[ T_n := \frac{M}{PCM_B} \]  
Frame time - DPPM

\[ m \equiv 1 \]  
Modulation depth - DPPM

\[ n := 2^M \]  
Number of DPPM active slots

\[ T_s := \frac{m \cdot T_n}{n} = 3.75 \cdot 10^{-10} \]  
Slot width

\[ \eta q := 1.6 \cdot 10^{-10} \]  
Quantum energy

\[ \lambda := 1.55 \cdot 10^{-6} \]  
Wavelength of operation

\[ photon\_energy := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ T_b := \frac{1}{PCM_B} \]  
This is the PCM bit time

\[ f_n := 5 \]

\[ \alpha := \frac{0.1874 \cdot T_b}{f_n} = 3.748 \cdot 10^{-11} \]

**Preamplifier terms**

\[ A v := 10 \quad C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]

Feedback value for (R) Tuned B

\[ \Delta_L := 1.9 \quad \Delta_R := 2.75 \]  
Feedback \( \Delta R \), time constant ratio \( \Delta L \)

\[ Rf_B := \Delta_R \cdot \frac{A v + 1}{2 \cdot \pi \cdot \frac{1}{T_s} \cdot C_T} \]

\[ \alpha_B := 0.4 \]  
Splitting ratio
C1 := \( (1 - \alpha_b) \cdot (C_T) = 9 \cdot 10^{-13} \)

C2 := \( \alpha_b \cdot (C_T) = 6 \cdot 10^{-13} \)

\[
Z_{TIA,B}(\omega) := \frac{1}{((1 - \omega^2 \cdot Lc \cdot C1) + \frac{Rf_B}{1+Av} \cdot (\omega \cdot 1i) \cdot (C1+C2-(\omega)^2 \cdot Lc \cdot C1 \cdot C2))}
\]

\[
filter(\omega) := \exp\left(\frac{-\alpha^2 \cdot \omega^2}{2}\right)
\]

**Receiver noise**

\[
Z_{tuned,B}(\omega) := filter(\omega) \cdot Z_{TIA,B}(\omega)
\]

NEB_B := \( \frac{1}{2 \cdot \pi} \cdot \int_0^{10^{10}} \left( \left| Z_{tuned,B}(\omega) \right|^2 \right) d\omega \) = 3.131 \cdot 10^9

I_{2,B} := \( \frac{1}{2 \cdot \pi} \cdot \int_0^{10^{10}} \left( \left( \frac{(1 - \omega^2 \cdot Lc \cdot C1)}{Z_{tuned,B}(\omega)} \right)^2 \right) d\omega \) = 1.995 \cdot 10^9

I_{3,B} := \( \frac{1}{2 \cdot \pi} \cdot \int_0^{10^{10}} \left( \left( \frac{(1 - \omega^2 \cdot Lc \cdot C1)}{Z_{tuned,B}(\omega)} \right)^2 \right) d\omega \) = 5.964 \cdot 10^5

**Feedback noise**

q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298

I_{nRf,B} := \frac{4 \cdot k \cdot T}{Rf_B} \cdot I_{2,B} = 5.452 \cdot 10^{-14}

**Ig noise** \( (\text{gate current}) \)

I_g := 10 \cdot 10^{-9}

I_{nG,B} := 2 \cdot q \cdot 1g \cdot I_{2,B} = 6.383 \cdot 10^{-18}

**Channel noise (Gate-source)**

\( gml := 30 \cdot 10^{-3} = 0.03 \quad noise\_factor := 1 \)

I_{nD} := 4 \cdot k \cdot T \cdot \frac{1}{gml}
\[ I_{nD,B} := I_{nD} \cdot I_{3,B} = 3.27 \cdot 10^{-13} \]

**Total noise - PIN-FET**

\[ I_{nB} := I_{nRf,B} + I_{nG,B} + I_{nD,B} = 3.815 \cdot 10^{-13} \]

\[ noise := I_{nB} \]

\[
Pulse(t) := \frac{1}{\pi} \cdot \frac{1}{T_s} \cdot 10^5 \cdot \int_{0}^{\frac{T_s}{2}} \frac{\sin(\omega \cdot \frac{T_s}{2})}{\left(\frac{T_s}{2}\right)} \cdot \text{Re}\left(1 \cdot \exp\left(1i \cdot \omega \cdot (t)\right)\right) \, d\omega
\]

**Pulse shape**

\[
I_1(t) := \frac{T_s}{\pi} \cdot \exp\left(\frac{-\alpha^2 \cdot \omega^2}{2}\right) \cdot \frac{1}{\left(\frac{Rf_B}{1 + Av}\right) \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2)}
\]

\[
I_0(t) := \frac{1}{\pi} \cdot \exp\left(\frac{-\alpha^2 \cdot \omega^2}{2}\right) \cdot \frac{1}{\left(\frac{Rf_B}{1 + Av}\right) \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2)}
\]

\[ t := 0.1 \cdot T_s \]

\[ t_{pk} := \text{root}\left(T_s \cdot I_1(t), t\right) \]

\[ \frac{t_{pk}}{T_s} = 0.212 \]
Voltages

\[ v_o(t) := I_0(t) \]

\[ v_o(t_{pk}) = 6.714 \cdot 10^3 \]

\[ t := -5 \cdot T_s, -4.99 \cdot T_s, 5 \cdot T_s \]

\[ \frac{I_0(t)}{T_s} \]

\[ \frac{I_0(t) + I_0(t - T_s)}{T_0} \]

\[ \frac{I_0(t) + I_0(t - 2 T_s)}{T_0} \]

\[ \frac{I_0(t) + I_0(t - 3 T_s)}{T_0} \]

\[ \frac{I_0(t) + I_0(t - 4 T_s)}{T_0} \]

\[ \frac{I_1(t)}{T_s} \]

\[ v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \]

\[ \text{temp}(v, t) := \frac{I_0(t)}{v \cdot I_0(t_{pk})} - 1 \]

\[ t := 0.08 \cdot T_s \]

\[ \tau_d(v) := \text{root}(\text{temp}(v, t), t) \]

\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \]
\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \]
\[ \text{slope}(b, v) := b \cdot \eta q \cdot \frac{I_1(t_d(v))}{T_s} \]
\[ v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \]

**Q values**
\[ Q_r(b, v) := \frac{v_{pk}(b) - v_d(b, v)}{\sqrt{\text{noise}}} \]
\[ Q_s(b, v) := \left( \frac{m \cdot T_n}{2 \cdot n} \right) \cdot \frac{\text{slope}(b, v)}{\sqrt{\text{noise}}} \]
\[ Q_t(b, v) := \frac{v_d(b, v)}{\sqrt{\text{noise}}} \]

**Error probabilities**
\[ P_r(b, v) := 0.5 \cdot \text{erfc} \left( \frac{Q_r(b, v)}{\sqrt{2}} \right) \]
\[ P_s(b, v) := \text{erfc} \left( \frac{Q_s(b, v)}{\sqrt{2}} \right) \]
\[ P_f(b, v) := \frac{m \cdot T_n}{n \cdot \alpha} \cdot 0.5 \cdot \text{erfc} \left( \frac{Q_t(b, v)}{\sqrt{2}} \right) \]
\[ P_{es}(b, v) := P_r(b, v) + P_s(b, v) + \frac{n - 1}{2} \cdot P_f(b, v) \]
\[ P_{eb}(b, v) := \frac{n}{2 \cdot (n - 1)} \cdot P_{es}(b, v) \]

\[ b := 0.25 \cdot 10^4 \quad v := 0.5 \quad M = 3 \]
\[ Q_r(b, v) = 2.174 \quad P_r(b, v) = 0.015 \]
\[ Q_s(b, v) = 7.855 \quad P_s(b, v) = 3.991 \cdot 10^{-15} \]
\[ Q_t(b, v) = 2.174 \quad P_f(b, v) = 0.149 \]
\[ P_{eb}(b, v) = 0.306 \]
\[ p_c(b, v) := \log(P_{eb}(b, v)) + 9 \]

Set for 1 in $10^9$ errors
\[ a(v) := \text{root}(pc(b, v), b) \]

Find the root to give 1 in \(10^9\)

\[ v = 0.2, 0.21 \ldots 0.7 \]

Scan for Optimum threshold

\[ b_{(v - 0.2) \cdot 100} := a(v) \]

Photons per bit

\[ \text{minimum} := \min(b) \]

Search for minimum and store it

\[ \text{minimum} = 7.243 \cdot 10^3 \]

\[ \frac{\text{minimum}}{M} = 2.414 \cdot 10^3 \]

\[ a(v) \]

\[ 0.2 \quad 0.25 \quad 0.3 \quad 0.35 \quad 0.4 \quad 0.45 \quad 0.5 \quad 0.55 \quad 0.6 \quad 0.65 \quad 0.7 \]

\[ \text{result} := \text{minimum} \cdot \text{photon}\_\text{energy} \cdot \frac{\text{PCM}\_\text{B}}{M} \]

\[ dBm := 10 \cdot \log \left( \frac{\text{result}}{10^{-3}} \right) \]

\[ dBm = -35.089 \]
\[ \omega = 1 \cdot 10^7, 10 \cdot 10^7 \ldots 5 \cdot 10^{11} \]

\[
\begin{align*}
|Z_{TIA,B}(\omega)| \quad |\text{filter}(\omega)| \\
|Z_{TIA,B}(\omega)| \cdot |\text{filter}(\omega)| \\
\end{align*}
\]

\[ v = 0.47, 0.48 \ldots 0.55 \quad b = 7.1 \cdot 10^3, 7.2 \cdot 10^3 \ldots 7.5 \cdot 10^3 \]

\[ P_{\phi}(b, v) \]
C.2.3 Non-tuned PPM receiver with matched filter (PIN-BJT)

PPM (PIN-BJT Matched filter 1 Gbit/s)

PPM Rx performance with PIN-BJT input configuration, Matched filter pre-detection filter. calculations are as follow

- PPM terms
- Rx terms (noise + TF)
- Pulse shaping, voltages
- Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

Bit-rate, pulse duration, and input pulse

\[ PCM_B := 1 \cdot 10^9 \]
\[ M := 3 \]
\[ T_n := \frac{M}{PCM_B} \] Frame time - DPPM
\[ m \equiv 1 \] Modulation depth - DPPM
\[ n := 2^M \] Number of DPPM active slots
\[ T_s := \frac{m \cdot T_n}{n} = 3.75 \cdot 10^{-10} \] Slot width
\[ \eta q := 1.6 \cdot 10^{-10} \] Quantum energy
\[ \lambda := 1.55 \cdot 10^{-6} \] wavelength of operation

\[ photon\_energy := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]
\[ T_b := \frac{1}{PCM_B} \] This is the PCM bit time
\[ f_n := 5 \]
\[ \alpha := \frac{0.1874 \cdot T_b}{f_n} = 3.748 \cdot 10^{-11} \]
\[ \tau_R := \alpha \]

Preamplifier terms

\[ \omega_c := 2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} = 3.351 \cdot 10^{10} \]
\[ A v := 10 \]
\[ C_T := 1.5 \cdot 10^{-12} \]
\[ \frac{1}{T_s} = 2.667 \cdot 10^9 \]

\[ Z_{TIA,N}(\omega) := \frac{1}{1 + j \frac{\omega}{\omega_c}} \]
\[ \text{filter}(\omega) := \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \]
\[
Rf_N = \frac{Av + 1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} = 218.838
\]

\[
Z_{\text{nontuned}}(\omega) = \frac{\text{filter}(\omega) \cdot Z_{\text{TIA}(\omega)}}{\omega_R = \alpha}
\]

\[
NEB_N = \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^3} \left( |Z_{\text{nontuned}}(\omega)| \right)^2 d\omega \right) = 3.071 \cdot 10^9
\]

\[
I_{\text{2\_N}} = NEB_N = 3.071 \cdot 10^9
\]

\[
I_{\text{3\_N}} = \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^3} \left( \frac{1 + j \cdot (\omega) \cdot Rf_N \cdot C_T}{1 \cdot Rf_N \cdot Z_{\text{nontuned}}(\omega)} \right)^2 d\omega \right) = 1.813 \cdot 10^6
\]

- noise-opt BJT
- \( q = 1.6 \cdot 10^{-19} \)
- \( k = 1.38 \cdot 10^{-23} \)
- \( T = 298 \)
- \( hfe = 100 \)

\[
x_N(x) = \left( \frac{4 \cdot k \cdot T}{Rf_N \cdot I_{\text{2\_N}}} \cdot I_{\text{2\_N}} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{\text{2\_N}} + 2 \cdot q \cdot \frac{x}{\left( 25 \cdot 10^{-3} \right)^2} \cdot I_{\text{3\_N}} \right)
\]

\[
I_{\text{opt\_N}} = 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{\text{3\_N}}}{I_{\text{2\_N}}}} = 0.006
\]

\[
I_{\text{n\_N}} = x_N(I_{\text{opt\_N}}) = 3.502 \cdot 10^{-13}
\]

\[
x = 0.00001, 0.00002, 0.1
\]

\[
x_N(x)
\]
noise := \( I_{n,N} = 3.502 \cdot 10^{-13} \)

**Pulse shape**

\[
I_1(t) := \frac{T_s}{\pi} \cdot \frac{1}{T_s \cdot 10^9} \int_{0}^{\frac{1}{T_s} \cdot 10^9} \exp\left(\frac{-\alpha^2 \cdot \omega^2}{2}\right) \cdot \operatorname{Re}\left(\frac{1}{1 + \frac{\omega}{\omega_c}} \cdot \exp\left(\frac{1i \cdot \omega \cdot (t)}{2}\right)\right) \cdot \frac{1}{1 + \frac{\omega}{\omega_c}} \cdot \exp\left(\frac{1i \cdot \omega \cdot (t)}{2}\right) \, d\omega
\]

\[
I_0(t) := \frac{1}{\pi} \cdot \frac{1}{T_s \cdot 10^9} \int_{0}^{\frac{1}{T_s} \cdot 10^9} \exp\left(\frac{-\alpha^2 \cdot \omega^2}{2}\right) \cdot \operatorname{Re}\left(\frac{1}{1 + \frac{\omega}{\omega_c}} \cdot \exp\left(\frac{1i \cdot \omega \cdot (t)}{2}\right)\right) \cdot \frac{1}{1 + \frac{\omega}{\omega_c}} \cdot \exp\left(\frac{1i \cdot \omega \cdot (t)}{2}\right) \, d\omega
\]

\[t := 0.1 \cdot T_s\]

\[t_{pk} := \text{root}(T_s \cdot I_1(t), t)\]

\[t_{pk} = 0.066\]

\[t := -5 \cdot T_s, -4.99 \cdot T_s, -6 \cdot T_s\]

**Voltages**

\[v_o(t) := I_0(t)\]

\[v_o(t_{pk}) = 6.742 \cdot 10^9\]
\( v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \)
\( \text{temp}(v, t) := \frac{I_0(t)}{v \cdot I_0(t_{pk})} - 1 \)
\( t := -0.1 \cdot T_s \)
\( t_d(v) := \text{root}(\text{temp}(v, t), t) \)
\( v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \)
\( v_d(b, v) := b \cdot \eta q \cdot I_0(t_{pk}) \)
\( \text{slope}(b, v) := b \cdot \eta q \cdot \frac{I_1(t_d(v))}{T_s} \)
\( v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \)

**Q values**
\( Q_r(b, v) := \frac{v_{pk}(b) - v_d(b, v)}{\sqrt{\text{noise}}} \)
\( Q_s(b, v) := \left( \frac{m \cdot T_n}{2 \cdot n} \right) \cdot \frac{\text{slope}(b, v)}{\sqrt{\text{noise}}} \)
\( Q_t(b, v) := \frac{v_d(b, v)}{\sqrt{\text{noise}}} \)

**Error probabilities**
\( P_r(b, v) := 0.5 \cdot \text{erfc}\left( \frac{Q_r(b, v)}{\sqrt{2}} \right) \)
\[
P_s(b, v) = \text{erfc} \left( \frac{Q_s(b, v)}{\sqrt{2}} \right)
\]

\[
P_f(b, v) = \frac{m \cdot T_n}{n \cdot \alpha} \cdot 0.5 \cdot \text{erfc} \left( \frac{Q_i(b, v)}{\sqrt{2}} \right)
\]

\[
P_{es}(b, v) = P_r(b, v) + P_s(b, v) + \frac{n-1}{2} \cdot P_f(b, v)
\]

\[
P_{eb}(b, v) = \frac{n}{2 \cdot (n-1)} \cdot P_{es}(b, v)
\]

\[b := 2.5 \cdot 10^4\]

\[p_c(b, v) := \log \left( P_{eb}(b, v) \right) + 9\]

\[a(v) := \text{root} \left( p_c(b, v), b \right)\]

\[v := 0.2, 0.21, 0.3, 0.21, 0.9\]

\[b \left( v - 0.2 \right) \cdot 100 := a(v)\]

Set for 1 in 10^9 errors

Find the root to give 1 in 10^9

Scan for Optimum threshold

Photons per bit

Search for minimum and store it

\[
\text{minimum} := \text{min} \left( b \right)
\]

\[
\text{minimum} = 6.91 \cdot 10^3
\]

\[
\text{result} := \text{minimum} \cdot \text{photon\_energy} \cdot \frac{PCM\_B}{M}
\]

\[
dBm := 10 \cdot \log \left( \frac{\text{result}}{10^{-3}} \right)
\]

\[
dBm = -35.293
\]

\[
\frac{PCM\_B}{M} = 3.333 \cdot 10^8
\]

\[
\frac{1.292 \cdot 10^4}{M} = 4.307 \cdot 10^3
\]

\[
#\]
C.2.4 Tuned-B PPM receiver with matched filter (PIN-BJT)

- PPM (PIN-BJT Matched filter 1 Gbit/s)

PPM Rx performance with PIN-BJT input configuration, Matched filter pre-detection filter. calculations are as follow

- PPM terms
- Rx terms (noise + TF)
- Pulse shaping, voltages
- Error bit rate
- Optimum threshold/minimum number of photons
- receiver sensitivity

**Bit-rate, pulse duration, and input pulse**

*PCM_B := 1 \cdot 10^9

\[ M := 3 \]

\[ T_n := \frac{M}{PCM_B} \]  
Frame time - DPPM

\[ m := 1 \]  
Modulation depth - DPPM

\[ n := 2^M \]  
Number of DPPM active slots

\[ T_s := \frac{m \cdot T_n}{n} = 3.75 \cdot 10^{-10} \]  
Slot width

\[ \eta q := 1.6 \cdot 10^{-10} \]  
Quantum energy

\[ \lambda := 1.55 \cdot 10^{-6} \]  
wavelength of operation

\[ photon\_energy := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ T_b := \frac{1}{PCM_B} \]  
This is the PCM bit time

\[ f_n := 5 \]

\[ \alpha := \frac{0.1874 \cdot T_b}{f_n} = 3.748 \cdot 10^{-11} \]

**Preamplifier terms**

\[ Av := 10 \quad C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]

Feedback value for (R) Tuned B

\[ \Delta_L := 1.9 \quad \Delta_R := 2.75 \quad \text{feedback } \Delta R \text{, time constant ratio } \Delta L \]

\[ Rf_B := \Delta_R \cdot \frac{Av + 1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} = 601.8046 \]  
feedback for Tuned B

\[ \alpha_b := 0.4 \]  
splitting ratio
\[ L_c := \left( \frac{R_f \cdot B}{1 + A_v} \right)^2 \cdot C_T \cdot \Delta_L = 2.363 \cdot 10^{-9} \]

\[ C_1 := (1 - \alpha_b) \cdot (C_T) = 9 \cdot 10^{-13} \]

\[ C_2 := \alpha_b \cdot (C_T) = 6 \cdot 10^{-13} \]

\[ Z_{TIA,B}(\omega) := \frac{1}{\left( (1 - \omega^2 \cdot L_c \cdot C_1) + \frac{R_f \cdot B}{1 + A_v} \cdot (\omega \cdot i) \cdot (C_1 + C_2 - (\omega)^2 \cdot L_c \cdot C_1 \cdot C_2) \right)} \]

\[ \text{filter}(\omega) := \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \]

\[ \text{Receiver noise} \]

\[ Z_{tuned,B}(\omega) := \text{filter}(\omega) \cdot Z_{TIA,B}(\omega) \]

\[ NEB_B := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{10}} \left( \frac{\left( \left| Z_{tuned,B}(\omega) \right| \right)^2}{\left| \left( 1 - \omega^2 \cdot L_c \cdot C_1 \right) \right|} \right) \, d\omega \right) = 3.131 \cdot 10^9 \]

\[ I_{2,B} := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{10}} \left( \frac{\left( \left| Z_{tuned,B}(\omega) \right| \right)^2}{\left( \left( 1 - \omega^2 \cdot L_c \cdot C_1 \right) \right) + \frac{R_f \cdot B}{1 + A_v} \cdot (C_1 + C_2 - (\omega)^2 \cdot L_c \cdot C_1 \cdot C_2) \cdot (\omega \cdot i)} \right) \, d\omega \right) = 1.995 \cdot 10^9 \]

\[ I_{3,B} := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{10}} \left( \frac{\left( \left| Z_{tuned,B}(\omega) \right| \right)^2}{\left( \left( 1 - \omega^2 \cdot L_c \cdot C_1 \right) \right) + \frac{R_f \cdot B}{1 + A_v} \cdot (C_1 + C_2 - (\omega)^2 \cdot L_c \cdot C_1 \cdot C_2) \cdot (\omega \cdot i)} \right) \, d\omega \right)^2 = 5.964 \cdot 10^5 \]

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad h_{fe} := 100 \]

\[ \text{noise-opt BJT} \]

\[ x_B(x) := \frac{4 \cdot k \cdot T}{R_f \cdot B} \cdot I_{2,B} + \left( 2 \cdot q \cdot \frac{x}{h_{fe}} \right) \cdot I_{2,B} + 2 \cdot q \cdot \frac{x}{\left( 25 \cdot 10^{-3} \right)} \cdot I_{3,B} \]

\[ I_{copt,B} := 25 \cdot 10^{-3} \cdot \sqrt{h_{fe}} \cdot \sqrt{\frac{I_{3,B}}{I_{2,B}}} = 0.004 \]

\[ \text{noise-opt BJT} \]

\[ I_{n,B} := x_B(I_{copt,B}) = 1.097 \cdot 10^{-13} \]

\[ \frac{0.004}{10^{-3}} = 4 \]

noise := I_{n,B}
\[ x := 0.00001, 0.00002 \ldots 0.1 \]

**Pulse shape**

\[ I_1(t) := \frac{T_s}{\pi} \int_0^{\frac{1}{T_s} \cdot 10^5} \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \mathrm{d}\omega \quad \cdot \text{Re} \left( 1 \cdot \omega \cdot \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \right) \]

\[ \cdot \frac{1}{\left( \left( 1 - \omega^2 \cdot Lc \cdot C1 \right) \right)^{\frac{1}{2}}} \]

\[ \cdot \frac{1}{\left( \frac{Rf \cdot B}{(1 + Av)} \right) \cdot (\omega \cdot i) \cdot \left( C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2 \right) \}

\[ \cdot \exp \left( 1i \cdot \omega \cdot (t) \right) \]

\[ I_0(t) := \frac{1}{\pi} \int_0^{\frac{1}{T_s} \cdot 10^5} \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \mathrm{d}\omega \quad \cdot \text{Re} \left( \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \right) \]

\[ \cdot \frac{1}{\left( \left( 1 - \omega^2 \cdot Lc \cdot C1 \right) \right)^{\frac{1}{2}}} \]

\[ \cdot \frac{1}{\left( \frac{Rf \cdot B}{(1 + Av)} \right) \cdot (\omega \cdot i) \cdot \left( C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2 \right) \}

\[ \cdot \exp \left( 1i \cdot \omega \cdot (t) \right) \]

\[ t := 0.1 \cdot T_s \]

\[ t_{pk} := \text{root} \left( T_s \cdot I_1(t), t \right) \]

\[ \frac{t_{pk}}{T_s} = 0.212 \]
Voltages

\[ v_o(t) := I_0(t) \]

\[ v_o(t_{pk}) = 6.714 \cdot 10^3 \]

\[ t := -5 \cdot T_s, -4.99 \cdot T_s, \ldots, 5 \cdot T_s \]

\[ I_0(t) \]
\[ I_0(t) + I_0(t - T_s) \]
\[ I_0(t) + I_0(t - 2T_s) \]
\[ I_0(t) + I_0(t - 3T_s) \]
\[ I_0(t) + I_0(t - 4T_s) \]

\[ v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \]

\[ temp(v, t) := \frac{I_0(t)}{v \cdot I_0(t_{pk})} - 1 \]

\[ t := 0.08 \cdot T_s \]

\[ t_d(v) := \text{root}(temp(v, t), t) \]

\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \]
\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \]
\[ \text{slope}(b, v) := b \cdot \eta q \cdot \frac{I_1(t_d(v))}{T_s} \]
\[ v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \]

**Q values**

\[ Q_r(b, v) := \frac{v_{pk}(b) - v_d(b, v)}{\sqrt{\text{noise}}} \]
\[ Q_s(b, v) := \left( \frac{m \cdot T_n}{2 \cdot n} \right) \cdot \frac{\text{slope}(b, v)}{\sqrt{\text{noise}}} \]
\[ Q_t(b, v) := \frac{v_d(b, v)}{\sqrt{\text{noise}}} \]

**Error probabilities**

\[ P_r(b, v) := 0.5 \cdot \text{erfc} \left( \frac{Q_r(b, v)}{\sqrt{2}} \right) \]
\[ P_s(b, v) := \text{erfc} \left( \frac{Q_s(b, v)}{\sqrt{2}} \right) \]
\[ P_f(b, v) := \frac{m \cdot T_n}{n \cdot \alpha} \cdot 0.5 \cdot \text{erfc} \left( \frac{Q_t(b, v)}{\sqrt{2}} \right) \]
\[ P_{es}(b, v) := P_r(b, v) + P_s(b, v) + \frac{n-1}{2} \cdot P_f(b, v) \]
\[ P_{eb}(b, v) := \frac{n}{2 \cdot (n-1)} \cdot P_{es}(b, v) \]

\[ b := 0.25 \cdot 10^4 \quad v := 0.5 \quad M = 3 \]

\[ Q_r(b, v) = 4.054 \quad P_r(b, v) = 2.517 \cdot 10^{-5} \]
\[ Q_s(b, v) = 14.649 \quad P_s(b, v) = 1.36 \cdot 10^{-48} \]
\[ Q_t(b, v) = 4.054 \quad P_f(b, v) = 2.518 \cdot 10^{-4} \]
\[ P_{eb}(b, v) = 5.18 \cdot 10^{-4} \]

\[ pc(b, v) := \log \left( \frac{P_{eb}(b, v)}{5.18 \cdot 10^{-4}} \right) + 9 \]
\[ a(v) := \text{root}(pc(b, v), b) \]

Set for 1 in $10^9$ errors

Find the root to give 1 in $10^9$
\[ v := 0.2, 0.21 \ldots 0.7 \]  

Scan for Optimum threshold

\[ b_{(v-0.2) \cdot 100} := a(v) \]  

Photons per bit

\[ \text{minimum} := \min(b) \]  

Search for minimum and store it

\[ \text{minimum} = 3.884 \cdot 10^3 \]  

\[ \frac{\text{minimum}}{M} = 1.295 \cdot 10^3 \]

\[ \text{result} := \text{minimum} \cdot \text{photon\_energy} \cdot \frac{\text{PCM\_B}}{M} \]

\[ dBm := 10 \cdot \log \left( \frac{\text{result}}{10^{-3}} \right) \]

\[ dBm = -37.796 \]
C.3 PPM receiver with Matched filter (Optical fibre/with equalizer)

C.3.1 Non-tuned PPM receiver with matched filter (PIN-FET/f_s = 2)

PPM (PIN-FET Matched filter 1 Gbit/s)

PPM Rx performance with PIN-FET input configuration, Matched filter pre-detection filter. calculations are as follow

- PPM terms
- Rx terms (noise+TF)
- Pulse shaping, voltages
- Error bit rate
- Optimum threshold/minimum number of photons

receiver sensitivity

Bit-rate, pulse duration, and input pulse

\[ PCM_B := 1 \cdot 10^9 \]

\[ M := 3 \]

\[ T_n := \frac{M}{PCM_B} \]

Frame time - DPPM

\[ m \equiv 1 \]

Modulation depth - DPPM

\[ n := 2^M \]

Number of DPPM active slots

\[ T_s := \frac{m \cdot T_n}{n} = 3.75 \cdot 10^{-10} \]

Slot width

\[ \eta q := 1.6 \cdot 10^{-10} \]

Quantum energy

\[ \lambda := 1.55 \cdot 10^{-6} \]

wavelength of operation

\[ photon\_energy := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ T_b := \frac{1}{PCM_B} \]

This is the PCM bit time

\[ f_n := 2 \]

\[ \alpha := \frac{0.1874 \cdot T_b}{f_n} = 9.37 \cdot 10^{-11} \]

\[ \tau_R := \alpha \]

Preamplifier terms

\[ \omega_c := 2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} = 3.351 \cdot 10^{10} \]

\[ Av := 10 \]

\[ C_T := 1.5 \cdot 10^{-12} \]

\[ \frac{1}{T_s} = 2.667 \cdot 10^9 \]

\[ Z_{TIA,N}(\omega) := \frac{1}{1 + j \frac{\omega}{\omega_c}} \]

\[ filter(\omega) := \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \]
\[ Rf_N := \frac{Av + 1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} = 218.838 \quad \omega := 1 \cdot 10^6, 10 \cdot 10^6 \ldots 1 \cdot 10^{11} \]

**Receiver noise**

\[ NEB_N := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^3} \left( |Z_{\text{nontuned}}(\omega)|^2 \right) d\omega \right) = 1.505 \cdot 10^9 \]

\[ I_{2,N} := NEB_N = 1.505 \cdot 10^9 \]

\[ I_{3,N} := \frac{1}{2 \cdot \pi} \int_0^{10^9} \left( \frac{1 + 1j \cdot (\omega) \cdot Rf_N \cdot C_T}{1 \cdot Rf_N} \cdot Z_{\text{nontuned}}(\omega) \right)^2 d\omega = 2.243 \cdot 10^5 \]

**Feedback noise**

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \]

\[ I_{nRF,N} := \frac{4 \cdot k \cdot T}{Rf_N} \cdot I_{2,N} = 1.132 \cdot 10^{-13} \]

**Ig noise (gate current)**

\[ Ig := 10 \cdot 10^{-9} \]

\[ I_{nG,N} := 2 \cdot q \cdot Ig \cdot I_{2,N} = 4.817 \cdot 10^{-18} \]

**Channel noise (Gate-source)**

\[ gm1 := 30 \cdot 10^{-3} = 0.03 \quad \text{noise factor} := 1 \]

\[ I_{nD,N} := 4 \cdot k \cdot T \cdot \frac{1}{gm1} \]

\[ I_{nD,N} := I_{nD,N} \cdot I_{3,N} = 1.23 \cdot 10^{-13} \]

**Total noise- PIN-FET**

\[ I_{n,N} := I_{nRF,N} + I_{nG,N} + I_{nD,N} = 2.362 \cdot 10^{-13} \]

**noise** := \[ I_{n,N} = 2.362 \cdot 10^{-13} \]

**Pulse shape**

\[ I_1(t) := \frac{T_s}{\pi} \cdot \frac{1}{T_s} \cdot 10^7 \left[ \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \right] \int_0^{10^3} \left[ \text{Re} \left( 1i \cdot \omega \cdot \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \cdot \exp \left( 1i \cdot \omega \cdot (t) \right) \right) \right] d\omega \]
\[ I_0(t) := \frac{1}{\pi} \cdot \frac{1}{T_s} \cdot 10^{17} \cdot \exp\left(\frac{-\alpha^2 \cdot \omega^2}{2}\right) \cdot \operatorname{Re}\left(\exp\left(\frac{-\alpha^2 \cdot \omega^2}{2}\right) \cdot \exp(1i \cdot \omega \cdot t)\right) \, d\omega \]

\[ t := 0.1 \cdot T_s \]

\[ t_{pk} := \text{root}(T_s \cdot I_1(t), t) \]

\[ \frac{t_{pk}}{T_s} = 3.994 \cdot 10^{-14} \]

\[ t := -5 \cdot T_s, -4.99 \cdot T_s, 6 \cdot T_s \]

**Voltages**

\[ v_o(t) := I_0(t) \]

\[ v_o(t_{pk}) = 3.011 \cdot 10^9 \]
\[ v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \]
\[ \text{temp}(v, t) := \frac{I_0(t)}{v \cdot I_0(t_{pk})} - 1 \]
\[ t := -0.3 \cdot T_s \]
\[ t_d(v) := \text{root}(\text{temp}(v, t), t) \]
\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \]
\[ \text{slope}(b, v) := b \cdot \eta q \cdot \frac{I_1(t_d(v))}{T_s} \]
\[ v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \]

**Q values**
\[ Q_r(b, v) := \frac{v_{pk}(b) - v_d(b, v)}{\sqrt{\text{noise}}} \]
\[ Q_s(b, v) := \left( \frac{m \cdot T_n}{2 \cdot n} \right) \cdot \frac{\text{slope}(b, v)}{\sqrt{\text{noise}}} \]
\[ Q_t(b, v) := \frac{v_d(b, v)}{\sqrt{\text{noise}}} \]

**Error probabilities**
\[ P_r(b, v) := 0.5 \cdot \text{erfc} \left( \frac{Q_r(b, v)}{\sqrt{2}} \right) \]
\[ P_s(b, v) := \text{erfc} \left( \frac{Q_s(b, v)}{\sqrt{2}} \right) \]
\[ P_f(b, v) := \frac{m \cdot T_n}{n \cdot \alpha} \cdot 0.5 \cdot \text{erfc} \left( \frac{Q_t(b, v)}{\sqrt{2}} \right) \]
\[ P_{es}(b, v) := P_r(b, v) + P_s(b, v) + \frac{n - 1}{2} \cdot P_f(b, v) \]
\[ P_{eb}(b, v) := \frac{n}{2 \cdot (n - 1)} \cdot P_{es}(b, v) \]

\[ b := 0.25 \cdot 10^4 \]
\[ pc(b, v) := \log(P_{eb}(b, v)) + 9 \]
\[ a(v) := \text{root}(pc(b, v), b) \]

Set for 1 in $10^9$ errors

Find the root to give 1 in $10^9$
\[ v := 0.2, 0.21 \ldots 0.9 \]

\[ b_{(v \cdot 0.2) \cdot 100} := a(v) \]

\[ \text{minimum} := \min(b) \]

\[ \text{minimum} = 1.257 \cdot 10^4 \]

Scan for Optimum threshold

Photons per bit

Search for minimum and store it

\[ \text{result} := \text{minimum} \cdot \text{photon\_energy} \cdot \frac{\text{PCM\_B}}{M} \]

\[ dBm := 10 \cdot \log \left( \frac{\text{result}}{10^{-3}} \right) \]

\[ \omega := 1 \cdot 10^7, 10 \cdot 10^7 \ldots 5 \cdot 10^{11} \]

\[ \frac{\text{PCM}_B}{M} = 4.307 \cdot 10^3 \]

\[ \frac{\text{PCM}_B}{M} = 3.333 \cdot 10^8 \]

\[ dBm = -32.694 \]

\[ |Z_{\text{TIA\_N}}(\omega)| \]

\[ |\text{filter}(\omega)| \]

\[ |Z_{\text{TIA\_N}}(\omega) \cdot |\text{filter}(\omega)| |
$v := 0.45, 0.46 \ldots 0.55 \quad b := 1.1 \cdot 10^4, 1.11 \cdot 10^4 \ldots 1.3 \cdot 10^4$

$P_{ex}(b, v)$

$P_{es}(b, v)$
C.3.2 Non-tuned PPM receiver with matched filter (PIN-FET/$f_n = 5$)

PPM (PIN-FET Matched filter 1 Gbit/s)

PPM Rx performance with PIN-FET input configuration, Matched filter pre-detection filter. calculations are as follow

- PPM terms
- Rx terms (noise+TF)
- Pulse shaping, voltages
- Error bit rate
- Optimum threshold/minimum number of photons

receiver sensitivity

Bit-rate, pulse duration, and input pulse

$PCM_B := 1 \cdot 10^9$

$M := 3$

$T_n := \frac{M}{PCM_B}$ \hspace{1cm} Frame time - DPPM

$m \equiv 1$ \hspace{1cm} Modulation depth - DPPM

$n := 2^M$ \hspace{1cm} Number of DPPM active slots

$T_s := \frac{m \cdot T_n}{n} = 3.75 \cdot 10^{-10}$ \hspace{1cm} Slot width

$\eta q := 1.6 \cdot 10^{-10}$ \hspace{1cm} Quantum energy

$\lambda := 1.55 \cdot 10^{-6}$ \hspace{1cm} wavelength of operation

$\text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda}$

$T_b := \frac{1}{PCM_B}$ \hspace{1cm} This is the PCM bit time

$f_n := 5$

$\alpha := \frac{0.1874 \cdot T_b}{f_n} = 3.748 \cdot 10^{-11}$

$\tau_R := \alpha$

Preamplifier terms

$\omega_c := 2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} = 3.351 \cdot 10^{10}$ \hspace{1cm} $Av := 10$

$C_T := 1.5 \cdot 10^{-12}$ \hspace{0.5cm} total C

$Z_{TIA,N}(\omega) := \frac{1}{1 + j \cdot \frac{\omega}{\omega_c}}$

$\frac{1}{T_s} = 2.667 \cdot 10^9$

$filter(\omega) := \exp\left(\frac{-\alpha^2 \cdot \omega^2}{2}\right)$
\[ R_{f\_N} := \frac{A_v + 1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} = 218.838 \quad \omega := 1 \cdot 10^6, 10 \cdot 10^6 \ldots 1 \cdot 10^{11} \]

\[ Z_{\text{nontuned}}(\omega) := \text{filter}(\omega) \quad \nu_R := \alpha \]

**Receiver noise**

\[ \text{NEB}_N := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^9} \left( |Z_{\text{nontuned}}(\omega)|^2 \right) d\omega \right) = 3.763 \cdot 10^9 \]

\[ I_{2\_N} := \text{NEB}_N = 3.763 \cdot 10^9 \]

\[ I_{3\_N} := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^9} \left( \frac{1 + 1j \cdot (\omega) \cdot Rf\_N \cdot C_T \cdot Z_{\text{nontuned}}(\omega)}{1 \cdot Rf\_N} \right)^2 d\omega \right) = 3.092 \cdot 10^6 \]

**Feedback noise**

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \]

\[ I_{nRf\_N} := \frac{4 \cdot k \cdot T}{Rf\_N} \cdot I_{2\_N} = 2.829 \cdot 10^{-13} \]

**Ig noise (gate current)**

\[ I_{nG\_N} := 2 \cdot q \cdot Ig \cdot I_{2\_N} = 1.204 \cdot 10^{-17} \]

**Channel noise (Gate-source)**

\[ gm1 := 30 \cdot 10^{-3} = 0.03 \quad \text{noise\_factor} := 1 \]

\[ I_{nD\_N} := 4 \cdot k \cdot T \cdot \frac{1}{gm1} \]

\[ I_{nD\_N} := I_{nD\_N} \cdot I_{3\_N} = 1.696 \cdot 10^{-12} \]

**Total noise- PIN-FET**

\[ I_{n\_N} := I_{nRf\_N} + I_{nG\_N} + I_{nD\_N} = 1.979 \cdot 10^{-12} \]

\[ \text{noise} := I_{n\_N} = 1.979 \cdot 10^{-12} \]

**Pulse shape**

\[ I_1(t) := \frac{T_s}{\pi} \cdot \left( \int_0^{10^5} \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \cdot \text{Re} \left( 11 \cdot \omega \cdot \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \cdot \exp \left( 11 \cdot \omega \cdot (t) \right) \right) d\omega \right) \]
\[ I_0(t) := \frac{1}{\pi} \cdot \frac{1}{T_s} \cdot 10^7 \left( \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \right) \cdot \text{Re} \left( \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \cdot \exp \left( 1i \cdot \omega^1 \cdot t(t) \right) \right) \, d\omega \]

\[ t := 0.1 \cdot T_s \]

\[ t_{pk} := \text{root} \left( T_s \cdot I_1(t) , t \right) \]

\[ \frac{t_{pk}}{T_s} = 2.318 \cdot 10^{-19} \]

\[ t := -5 \cdot T_s , -4.99 \cdot T_s , -6 \cdot T_s \]

**Voltages**

\[ v_o(t) := I_0(t) \]

\[ v_o \left( t_{pk} \right) = 7.527 \cdot 10^9 \]
\[ v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \]
\[ \text{temp}(v, t) := \frac{I_0(t)}{v \cdot I_0(t_{pk})} - 1 \]
\[ t := -0.1 \cdot T_s \]
\[ t_d(v) := \text{root}(\text{temp}(v, t), t) \]
\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \]
\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \]
\[ \text{slope}(b, v) := b \cdot \eta q \cdot \frac{I_1(t_d(v))}{T_s} \]
\[ v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \]

**Q values**

\[ Q_r(b, v) := \frac{v_{pk}(b) - v_d(b, v)}{\sqrt{\text{noise}}} \]
\[ Q_s(b, v) := \left( \frac{m \cdot T_n}{2 \cdot n} \right) \cdot \frac{\text{slope}(b, v)}{\sqrt{\text{noise}}} \]
\[ Q_t(b, v) := \frac{v_d(b, v)}{\sqrt{\text{noise}}} \]

**Error probabilities**

\[ P_r(b, v) := 0.5 \cdot \text{erfc} \left( \frac{Q_r(b, v)}{\sqrt{2}} \right) \]
\[ P_s(b, v) := \text{erfc} \left( \frac{Q_s(b, v)}{\sqrt{2}} \right) \]
\[ P_f(b, v) := \frac{m \cdot T_n}{n \cdot \alpha} \cdot 0.5 \cdot \text{erfc} \left( \frac{Q_t(b, v)}{\sqrt{2}} \right) \]
\[ P_{es}(b, v) := P_r(b, v) + P_s(b, v) + \frac{n-1}{2} \cdot P_f(b, v) \]
\[ P_{eb}(b, v) := \frac{n}{2 \cdot (n-1)} \cdot P_{es}(b, v) \]

\[ b := 0.25 \cdot 10^4 \]
\[ pc(b, v) := \log(P_{eb}(b, v)) + 9 \]

\[ a(v) := \text{root}(pc(b, v), b) \]

Set for 1 in \(10^9\) errors

Find the root to give 1 in \(10^9\)
\[ v := 0.2, 0.21, 0.22, 0.23, 0.24, 0.25, 0.26, 0.27, 0.28, 0.29, 0.3, 0.31, 0.32, 0.33, 0.34, 0.35, 0.36, 0.37, 0.38, 0.39, 0.4, 0.41, 0.42, 0.43, 0.44, 0.45, 0.46, 0.47, 0.48, 0.49, 0.5, 0.51, 0.52, 0.53, 0.54, 0.55, 0.56, 0.57, 0.58, 0.59, 0.6, 0.61, 0.62, 0.63, 0.64, 0.65, 0.66, 0.67, 0.68, 0.69, 0.7, 0.71, 0.72, 0.73, 0.74, 0.75, 0.76, 0.77, 0.78, 0.79, 0.8, 0.81, 0.82, 0.83, 0.84, 0.85, 0.86, 0.87, 0.88, 0.89, 0.9, 0.91, 0.92, 0.93, 0.94, 0.95, 0.96, 0.97, 0.98, 0.99, 1.0 \]

Scan for Optimum threshold

\[ b_{(v - 0.2) \cdot 100} := a(v) \]

Photons per bit

\[ \text{minimum} := \min(b) \]

Search for minimum and store it

\[ \text{minimum} = 1.471 \cdot 10^4 \]

\[ \text{result} := \text{minimum} \cdot \text{photon}\_\text{energy} \cdot \frac{\text{PCM}\_B}{M} \]

\[ dBm = 10 \cdot \log \left( \frac{\text{result}}{10^{-3}} \right) \]

\[ \frac{1.292 \cdot 10^4}{M} = 4.307 \cdot 10^3 \]

\[ \omega := 1 \cdot 10^7, 10 \cdot 10^7, 5 \cdot 10^{11} \]

\[ \frac{\text{PCM}\_B}{M} = 3.333 \cdot 10^8 \]

\[ dBm = -32.011 \]

\[ \left| Z_{TIA\_N}(\omega) \right| \]

\[ \left| \text{filter}(\omega) \right| \]

\[ \left| Z_{TIA\_N}(\omega) \right| \cdot \left| \text{filter}(\omega) \right| \]

\[ \omega \]
\[ v = 0.49, 0.5 \ldots 0.55 \quad \quad \quad b = 1.2 \times 10^4, 1.21 \times 10^4 \ldots 1.4 \times 10^4 \]

\[ P_{ex}(b, v) \]

\[ P_{ex}(b, v) \]
C.3.3 Non-tuned PPM receiver with matched filter (PIN-BJT/\(f_n = 2\))

**PPM (PIN-BJT Matched filter 1 Gbit/s)**

PPM Rx performance with PIN-BJT input configuration. Matched filter pre-detection filter. calculations are as follow

- PPM terms
- Rx terms (noise+TF)
- Pulse shaping, voltages
- Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

**Bit-rate, pulse duration, and input pulse**

**PCM\(_B\) := 1 \cdot 10^9**

\(\bar{M} := 3\)

\[ T_n := \frac{M}{PCM_B} \quad \text{Frame time - DPPM} \]

\[ m \equiv 1 \quad \text{Modulation depth - DPPM} \]

\[ n := 2^\bar{M} \quad \text{Number of DPPM active slots} \]

\[ T_s := \frac{m \cdot T_n}{n} = 3.75 \cdot 10^{-10} \quad \text{Slot width} \]

\[ \eta q := 1.6 \cdot 10^{-10} \quad \text{Quantum energy} \]

\[ \lambda := 1.55 \cdot 10^{-6} \quad \text{wavelength of operation} \]

\[ photon\_energy := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ T_b := \frac{1}{PCM_B} \quad \text{This is the PCM bit time} \]

\[ f_n := 2 \]

\[ \alpha := \frac{0.1874 \cdot T_b}{f_n} = 9.37 \cdot 10^{-11} \]

\[ \tau_R := \alpha \quad \text{Preamplifier terms} \]

\[ \omega_c := 2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} = 3.351 \cdot 10^{10} \quad Av := 10 \]

\[ C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]

\[ Z_{TIA,N}(\omega) := \frac{1}{1 + j \cdot \frac{\omega}{\omega_c}} \]

\[ filter(\omega) := \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \]

\[ \frac{1}{T_s} = 2.667 \cdot 10^9 \]
\[
Rf_N := \frac{Av + 1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot Ct} = 218.838
\]

\[
\omega := 1 \cdot 10^6, 10 \cdot 10^6 \ldots 1 \cdot 10^{11}
\]

\[
Z_{\text{nontuned}}(\omega) := \text{filter}(\omega)
\]

\[
\nu_R := \alpha
\]

\[
NEB_N := \frac{1}{2 \cdot \pi} \left( \int_0^{10^9} \left( |Z_{\text{nontuned}}(\omega)| \right)^2 d\omega \right) = 1.505 \cdot 10^9
\]

\[
I_{2\_N} := NEB_N = 1.505 \cdot 10^9
\]

\[
I_{3\_N} := \frac{1}{2 \cdot \pi} \left( \int_0^{10^9} \left( \left| \frac{1 + j \cdot \omega \cdot Rf_N \cdot Ct \cdot Z_{\text{nontuned}}(\omega)}{1 \cdot Rf_N} \right|^2 \right) d\omega \right) = 2.243 \cdot 10^5
\]

\[\text{noise-opt BJT}\]

\[
q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100
\]

\[
x_N(x) := \left( \frac{4 \cdot k \cdot T}{Rf_N} \cdot I_{2\_N} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{2\_N} + 2 \cdot q \cdot \frac{x}{\left( 25 \cdot 10^{-3} \right)^3} \cdot I_{3\_N} \right)
\]

\[
I_{\text{opt-N}} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3\_N}}{I_{2\_N}}} = 0.003
\]

\[
\frac{0.006}{10^{-3}} = 6
\]

\[
I_{n\_N} := x_N(I_{\text{opt-N}}) = 1.426 \cdot 10^{-13}
\]

\[
x := 0.00001, 0.00002, \ldots, 0.1
\]

\[x_N(x)\]

\[x\]

\[\text{noise} := I_{n\_N} = 1.426 \cdot 10^{-13}\]

\[\text{noise} := I_{n\_N} = 1.426 \cdot 10^{-13}\]
Pulse shape

\[ I_1(t) := \frac{T_s}{\pi} \cdot \frac{1}{T_s} \cdot 10^5 \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \cdot \text{Re} \left( \frac{1}{10} \cdot \omega \cdot \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \cdot \exp (1i \cdot \omega \cdot t) \right) \]

\[ I_0(t) := \frac{1}{\pi} \cdot \frac{1}{T_s} \cdot 10^5 \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \cdot \text{Re} \left( \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \cdot \exp (1i \cdot \omega \cdot t) \right) \]

t := 0.1 \cdot T_s

t_{pk} := \text{root} \left( T_s \cdot I_1(t), t \right)

\frac{t_{pk}}{T_s} = 3.994 \cdot 10^{-14}

t := -5 \cdot T_s, -4.99 \cdot T_s, 6 \cdot T_s

Voltagess
\[ v_o(t) := I_0(t) \]
\[ v_o(t_{pk}) = 3.011 \cdot 10^9 \]
\[ v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \]
\[ \text{temp}(v, t) := \frac{I_0(t)}{v \cdot I_0(t_{pk})} - 1 \]
\[ t := -0.3 \cdot T_s \]
\[ t_d(v) := \text{root}(\text{temp}(v, t), t) \]
\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \]
\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \]
\[ \text{slope}(b, v) := b \cdot \eta q \cdot \frac{I_1(t_d(v))}{T_s} \]
\[ v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \]

**Q values**
\[ Q_r(b, v) := \frac{v_{pk}(b) - v_d(b, v)}{\sqrt{\text{noise}}} \]
\[ Q_s(b, v) := \left( \frac{m \cdot T_n}{2 \cdot n} \right) \cdot \frac{\text{slope}(b, v)}{\sqrt{\text{noise}}} \]
\[ Q_t(b, v) := \frac{v_d(b, v)}{\sqrt{\text{noise}}} \]

**Error probabilities**
\[ P_r(b, v) := 0.5 \cdot \text{erfc} \left( \frac{Q_r(b, v)}{\sqrt{2}} \right) \]
\[ P_s(b, v) = \text{erfc} \left( \frac{Q_s(b, v)}{\sqrt{2}} \right) \]
\[ P_f(b, v) = \frac{m \cdot T_n}{n \cdot \alpha} \cdot 0.5 \cdot \text{erfc} \left( \frac{Q_t(b, v)}{\sqrt{2}} \right) \]
\[ P_{es}(b, v) := P_r(b, v) + P_s(b, v) + \frac{n-1}{2} \cdot P_f(b, v) \]
\[ P_{eb}(b, v) := \frac{n}{2(n-1)} \cdot P_{es}(b, v) \]
\[ b := 0.25 \cdot 10^4 \]
\[ pc(b, v) := \log \left( P_{eb}(b, v) \right) + 9 \]
\[ a(v) := \text{root} \left( pc(b, v), b \right) \]

Set for 1 in $10^9$ errors  
Find the root to give 1 in $10^9$  

Scan for Optimum threshold  
Photons per bit  
Search for minimum and store it

\[ v := 0.2, 0.21, ..., 0.9 \]
\[ b_{(v-0.2) \cdot 100} := a(v) \]
\[ \text{minimum} := \text{min} \left( b \right) \]

\[ \text{minimum} = 9.767 \cdot 10^3 \]

\[ \text{result} := \text{minimum} \cdot \text{photon\_energy} \cdot \frac{\text{PCM}_B}{M} \]
\[ dBm := 10 \cdot \log \left( \frac{\text{result}}{10^{-3}} \right) \]
\[ \omega := 1 \cdot 10^7, 10 \cdot 10^7, ..., 5 \cdot 10^{11} \]
\[ \frac{\text{PCM}_B}{M} = 3.33 \cdot 10^8 \]
\[ v = 0.45, 0.46 \ldots 0.55 \quad b = 9.5 \times 10^3, 9.55 \times 10^3 \ldots 10 \times 10^3 \]

\[ P_{es}(b, v) \]
C.3.4 Non-tuned PPM receiver with matched filter (PIN-BJT/f_n = 5)

PPM (PIN-BJT Matched filter 1 Gbit/s)

PPM Rx performance with PIN-BJT input configuration, Matched filter pre-detection filter. calculations are as follow

- PPM terms
- Rx terms (noise+TF)
- Pulse shaping, voltages
- Error bit rate
- Optimum threshold/minimum number of photons

receiver sensitivity

Bit-rate, pulse duration, and input pulse

\[ PCM_B := 1 \cdot 10^9 \]

\[ M := 3 \]

\[ T_n := \frac{M}{PCM_B} \quad \text{Frame time - DPPM} \]

\[ m \equiv 1 \quad \text{Modulation depth - DPPM} \]

\[ n := 2^M \quad \text{Number of DPPM active slots} \]

\[ T_s := \frac{m \cdot T_n}{n} = 3.75 \cdot 10^{-10} \quad \text{Slot width} \]

\[ \eta q := 1.6 \cdot 10^{-10} \quad \text{Quantum energy} \]

\[ \lambda := 1.55 \cdot 10^{-6} \quad \text{wavelength of operation} \]

\[ \text{photon} \_\text{energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ T_b := \frac{1}{PCM_B} \quad \text{This is the PCM bit time} \]

\[ f_n := 5 \]

\[ \alpha := \frac{0.1874 \cdot T_b}{f_n} = 3.748 \cdot 10^{-11} \]

\[ \tau_R := \alpha \]

Preamplifier terms

\[ \omega_c := 2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} = 3.351 \cdot 10^{10} \]

\[ Av := 10 \]

\[ C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]

\[ Z_{TIA,N}(\omega) := \frac{1}{1 + j \frac{\omega}{\omega_c}} \]

\[ \text{filter}(\omega) := \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \]

\[ \frac{1}{T_s} = 2.667 \cdot 10^9 \]
\[ Rf_N := \frac{Av + 1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} = 218.838 \quad \omega := 1 \cdot 10^6, 10 \cdot 10^6 \ldots 1 \cdot 10^{11} \]

\[ Z_{\text{nontuned}}(\omega) := \text{filter}(\omega) \quad u_R := \alpha \]

\[ NEB_N := \frac{1}{2 \cdot \pi} \int_0^{10^9} \left(\frac{1}{(\mid Z_{\text{nontuned}}(\omega) \mid)^2}\right) d\omega = 3.763 \cdot 10^9 \]

\[ I_{2\,N} := NEB_N = 3.763 \cdot 10^9 \]

\[ I_{3\,N} := \frac{1}{2 \cdot \pi} \int_0^{10^9} \left(\frac{1 + 1j \cdot (\omega) \cdot Rf_N \cdot C_T}{1 \cdot Rf_N} \cdot Z_{\text{nontuned}}(\omega)\right)^2 d\omega = 3.092 \cdot 10^6 \]

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100 \]

\[ x_N(x) := \left(\frac{4 \cdot k \cdot T}{Rf_N} \cdot I_{2\,N} + \left(2 \cdot q \cdot \frac{x}{hfe}\right) \cdot I_{2\,N} + 2 \cdot q \cdot \frac{x}{(25 \cdot 10^{-3})^3} \cdot I_{3\,N}\right) \]

\[ I_{\text{opt,}N} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3\,N}}{I_{2\,N}}} = 0.007 \quad 0.006 \quad \frac{10^{-3}}{6} = 6 \]

\[ I_{n\,N} := x_N(I_{\text{opt,}N}) = 4.555 \cdot 10^{-13} \]

\[ x := 0.00001, 0.00002, \ldots , 0.1 \]

\[ \text{noise} := I_{n\,N} = 4.555 \cdot 10^{-13} \]

\[ \text{noise} := I_{n\,N} = 4.555 \cdot 10^{-13} \]
Pulse shape

\[ I_1(t) := \frac{T_s}{\pi} \cdot \int_0^{\frac{1}{T_s} \cdot 10^5} \exp\left(\frac{-\alpha^2 \cdot \omega^2}{2}\right) \cdot \Re\left(1i \cdot \omega \cdot \exp\left(\frac{-\alpha^2 \cdot \omega^2}{2}\right) \cdot \exp(1i \cdot \omega \cdot (t))\right) d\omega \]

\[ I_0(t) := \frac{1}{\pi} \cdot \int_0^{\frac{1}{T_s} \cdot 10^5} \exp\left(\frac{-\alpha^2 \cdot \omega^2}{2}\right) \cdot \Re\left(\exp\left(\frac{-\alpha^2 \cdot \omega^2}{2}\right) \cdot \exp(1i \cdot \omega \cdot (t))\right) d\omega \]

t := 0.1 \cdot T_s

\[ t_{pk} := \text{root}(T_s \cdot I_1(t), t) \]

\[ \frac{t_{pk}}{T_s} = 2.318 \cdot 10^{-19} \]

\[ t := -5 \cdot T_s, -4.99 \cdot T_s, 6 \cdot T_s \]

**Voltages**

\[ v_o(t) := I_0(t) \quad \quad \quad v_o(t_{pk}) = 7.527 \cdot 10^9 \]
\[ v_{pk}(b) := b \cdot \eta \cdot I_0(t_{pk}) \]
\[ temp(v, t) := \frac{I_0(t)}{v \cdot I_0(t_{pk})} - 1 \]
\[ t := -0.1 \cdot T_s \]
\[ t_d(v) := \text{root}(temp(v, t), t) \]
\[ v_d(b, v) := b \cdot \eta \cdot I_0(t_d(v)) \]
\[ v_d(b, v) := b \cdot \eta \cdot I_0(t_d(v)) \]
\[ \text{slope}(b, v) := b \cdot \eta \cdot \frac{I_1(t_d(v))}{T_s} \]
\[ v_{pk}(b) := b \cdot \eta \cdot I_0(t_{pk}) \]

**Q values**

\[ Q_r(b, v) := \frac{v_{pk}(b) - v_d(b, v)}{\sqrt{\text{noise}}} \]
\[ Q_s(b, v) := \left( \frac{m \cdot T_n}{2 \cdot n} \right) \cdot \frac{\text{slope}(b, v)}{\sqrt{\text{noise}}} \]
\[ Q_e(b, v) := \frac{v_d(b, v)}{\sqrt{\text{noise}}} \]

**Error probabilities**

\[ P_r(b, v) := 0.5 \cdot \text{erfc} \left( \frac{Q_r(b, v)}{\sqrt{2}} \right) \]
\[ P_s(b, v) := \text{erfc} \left( \frac{Q_s(b, v)}{\sqrt{2}} \right) \]
\[ P_f(b, v) := \frac{m \cdot T_n}{n \cdot \alpha} \cdot 0.5 \cdot \text{erfc} \left( \frac{Q_t(b, v)}{\sqrt{2}} \right) \]
\[ P_{es}(b, v) := P_r(b, v) + P_s(b, v) + \frac{n-1}{2} \cdot P_f(b, v) \]
\[ P_{eb}(b, v) := \frac{n}{2 \cdot (n-1)} \cdot P_{es}(b, v) \]
\[ b := 0.25 \cdot 10^4 \]
\[ pc(b, v) := \log \left( P_{eb}(b, v) \right) + 9 \]
\[ a(v) := \text{root} \left( pc(b, v), b \right) \]
\[ v := 0.2, 0.21, \ldots, 0.9 \]
\[ b_{(v-0.2) \cdot 100} := a(v) \]
\[ \text{minimum} := \min(b) \]
\[ \text{minimum} = 7.059 \cdot 10^3 \]

Set for 1 in \(10^9\) errors
Find the root to give 1 in \(10^9\)
Scan for Optimum threshold
Photons per bit
Search for minimum and store it

\[ \text{result} := \text{minimum} \cdot \text{photon\_energy} \cdot \frac{\text{PCM\_B}}{M} \]
\[ dBm := 10 \cdot \log \left( \frac{\text{result}}{10^{-3}} \right) \]
\[ dBm = -35.201 \]
\[ \omega := 1 \cdot 10^7, 10 \cdot 10^7 \ldots 5 \cdot 10^{11} \]
\[ \frac{\text{PCM\_B}}{M} = 3.333 \cdot 10^8 \]
\[ v := 0.45, 0.46, \ldots, 0.55 \quad b := 9.5 \times 10^3, 9.55 \times 10^3, \ldots, 10 \times 10^3 \]

\[ P_{es}(b, v) \]
C.3.5 Tuned B PPM receiver with matched filter (PIN-FET/\(f_n = 2\))

PPM (PIN-FET Tuned/Matched 1 Gbit/s)

PPM Rx performance with PIN-FET input configuration, Matched filter. Calculations are as follows:

- PPM terms
- Rx terms (noise+TF)
- Pulse shaping, voltages
- Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

**Bit-rate, pulse duration, and input pulse**

\(\text{PCM}_B := 1 \cdot 10^9\)

\(M := 3\)

\(T_n := \frac{M}{\text{PCM}_B}\)  Frame time - DPPM

\(m \equiv 1\)  Modulation depth - DPPM

\(n := 2^M\)  Number of DPPM active slots

\(T_s := \frac{m \cdot T_n}{n} = 3.75 \cdot 10^{-10}\)  Slot width

\(\eta_q := 1.6 \cdot 10^{-10}\)  Quantum energy

\(\lambda := 1.55 \cdot 10^{-6}\)  Wavelength of operation

\(\text{photon}_\text{energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda}\)

\(T_b := \frac{1}{\text{PCM}_B}\)  This is the PCM bit time

\(f_n := 2\)

\(\alpha := \frac{0.1874 \cdot T_b}{f_n} = 9.37 \cdot 10^{-11}\)

**Preamplifier terms**

\(A_v := 10\)  \(C_T := 1.5 \cdot 10^{-12}\)  Total C

Feedback value for (R) Tuned B

\(\Delta_L := 2.5\)  \(\Delta_R := 3.17\)  Feedback \(\Delta R\), time constant ratio \(\Delta L\)

\(Rf_B := \Delta_R \cdot \frac{A_v + 1}{2 \pi \cdot \frac{1}{T_s} \cdot C_T} = 693.7166\)  Feedback for Tuned B

\(\alpha_b := 0.4\)  Splitting ratio
\[ L_c := \frac{\left( \frac{R_f \cdot B}{1 + Av} \right)^2 \cdot C_T}{\Delta L} = 2.386 \cdot 10^{-9} \]

\[ C_1 := (1 - \alpha_b) \cdot (C_T) = 9 \cdot 10^{-13} \]

\[ C_2 := \alpha_b \cdot (C_T) = 6 \cdot 10^{-13} \]

\[ Z_{TLA,B}(\omega) := \frac{1}{\left( (1 - \omega^2 \cdot L_c \cdot C_1) + \frac{R_f \cdot B}{(1 + Av)} \cdot (\omega + 1i) \cdot (C_1 + C_2 - (\omega)^2 \cdot L_c \cdot C_1 \cdot C_2) \right)} \]

\[ filter(\omega) := \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \]

Receiver noise

\[ Z_{tuned,B}(\omega) := filter(\omega) \quad \omega := 1 \cdot 10^6, 10 \cdot 10^6 \ldots 1 \cdot 10^{11} \]

\[ N_{EB,B} := \frac{1}{2 \cdot \pi} \int_{0}^{10^9} \left( |Z_{tuned,B}(\omega)|^2 \right) d\omega = 1.505 \cdot 10^9 \]

\[ I_{2,B} := \frac{1}{2 \cdot \pi} \int_{0}^{10^9} \left( \left( 1 - \omega^2 \cdot L_c \cdot C_1 \right) \left( |Z_{tuned,B}(\omega)|^2 \right) \right) d\omega = 1.205 \cdot 10^9 \]

\[ I_{3,B} := \frac{1}{2 \cdot \pi} \int_{0}^{10^9} \left( \left( 1 - \omega^2 \cdot L_c \cdot C_1 \right) \left( \frac{\sqrt{\left( (1 - \omega^2 \cdot C_1) \right) + \frac{R_f \cdot B}{(1 + Av)} \cdot (C_1 + C_2 - (\omega)^2 \cdot L_c \cdot C_1 \cdot C_2) \cdot 1i}{R_f \cdot B} \right) \cdot |Z_{tuned,B}(\omega)|^2 \right) d\omega = 1.457 \cdot 10^5 \]
Feedback noise
\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \]

\[ I_{n_{RF,B}} := \frac{4 \cdot k \cdot T}{R_{f,B}} \cdot I_{2,B} = 2.856 \cdot 10^{-14} \]

**Ig noise (gate current)**
\[ Ig := 10 \cdot 10^{-9} \]

\[ I_{n_{G,B}} := 2 \cdot q \cdot Ig \cdot I_{2,B} = 3.855 \cdot 10^{-18} \]

**Channel noise (Gate-source)**
\[ gm1 := 30 \cdot 10^{-3} = 0.03 \quad noise\_factor := 1 \]

\[ I_{n_{D}} := 4 \cdot k \cdot T \cdot \frac{1}{gm1} \]

\[ I_{n_{D,B}} := I_{n_{D}} \cdot I_{3,B} = 7.989 \cdot 10^{-14} \]

**Total noise- PIN-FET**

\[ I_{n_{B}} := I_{n_{RF,B}} + I_{n_{G,B}} + I_{n_{D,B}} = 1.085 \cdot 10^{-13} \]

\[ noise := I_{n_{B}} \]

**Pulse shape**
\[ \frac{1}{T_s} \cdot 10^5 \]

\[ I_1(t) := \frac{T_s}{\pi} \cdot \left[ \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \right]^d \cdot \exp \left( i \cdot \omega \cdot \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \right) \cdot \exp \left( i \cdot \omega \cdot (t) \right) \]

\[ I_0(t) := \frac{1}{T_s} \cdot 10^5 \]

\[ \left[ \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \right]^d \cdot \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \cdot \exp \left( i \cdot \omega \cdot (t) \right) \]
\[ t := 0.002 \cdot T_s \]
\[ t_{pk} := \text{root}(T_s \cdot I_1(t), t) \]
\[ \frac{t_{pk}}{T_s} = 1.028 \cdot 10^{-12} \]

**Voltages**
\[ v_o(t) := I_0(t) \]
\[ v_o(t_{pk}) = 3.011 \cdot 10^9 \]

\[ t := -5 \cdot T_s, -4.99 \cdot T_s, \ldots, 5 \cdot T_s \]
\[ t_d(v) := \text{root}(\text{temp}(v, t), t) \]
\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \]
\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \]
\[ \text{slope}(b, v) := b \cdot \eta q \cdot \frac{I_1(t_d(v))}{T_s} \]
\[ v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \]

**Q values**
\[ Q_r(b, v) := \frac{v_{pk}(b) - v_d(b, v)}{\sqrt{\text{noise}}} \]
\[ Q_s(b, v) := \left( \frac{m \cdot T_n}{2 \cdot n} \right) \cdot \frac{\text{slope}(b, v)}{\sqrt{\text{noise}}} \]
\[ Q_t(b, v) := \frac{v_d(b, v)}{\sqrt{\text{noise}}} \]

**Error probabilities**
\[ P_r(b, v) := 0.5 \cdot \text{erfc} \left( \frac{Q_r(b, v)}{\sqrt{2}} \right) \]
\[ P_s(b, v) := \text{erfc} \left( \frac{Q_s(b, v)}{\sqrt{2}} \right) \]
\[ P_f(b, v) := \frac{m \cdot T_n}{n \cdot \alpha} \cdot 0.5 \cdot \text{erfc} \left( \frac{Q_t(b, v)}{\sqrt{2}} \right) \]
\[ P_{es}(b, v) := P_r(b, v) + P_s(b, v) + \frac{n-1}{2} \cdot P_f(b, v) \]
\[ P_{eb}(b, v) := \frac{n}{2 \cdot (n-1)} \cdot P_{es}(b, v) \]

\[ b := 0.25 \cdot 10^4 \quad v := 0.5 \quad M = 3 \]
\[ Q_r(b, v) = 1.828 \quad P_r(b, v) = 0.034 \]
\[ Q_s(b, v) = 3.046 \quad P_s(b, v) = 0.002 \]
\[ Q_t(b, v) = 1.828 \quad P_f(b, v) = 0.135 \]
\[ P_{eb}(b, v) = 0.291 \]

\[ pc(b, v) := \log \left( P_{eb}(b, v) \right) + 9 \]

\[ a(v) := \text{root}(pc(b, v), b) \]

\[ v = 0.2, 0.21, 0.7 \]

Set for 1 in \(10^{9}\) errors

Find the root to give 1 in \(10^{9}\)

Scan for Optimum threshold

Photons per bit

Search for minimum and store it

Minimum \(= 8.519 \cdot 10^3\)

\[ \frac{\text{minimum}}{M} = 2.84 \cdot 10^3 \]

Number of photons per bit verses threshold level (BER = \(10^{-9}\)).

Rx Sens

\[ \text{result} := \text{minimum} \cdot \text{photon\_energy} \cdot \frac{\text{PCM\_B}}{M} \]

\[ dBm := 10 \cdot \log \left( \frac{\text{result}}{10^{-3}} \right) \]

\[ dBm = -34.384 \]

Sens compared to non-tuned for all “a” value

Photons per pulse nontuned vs tuned

\[ 1.257 \cdot 10^4 - 8.519 \cdot 10^3 = 4.051 \cdot 10^3 \]

32.69 – 33.39 = –0.7
32.69 – 33.64 = –0.95
32.69 – 34 = –1.31
32.69 – 34.38 = –1.69
32.69 – 34.28 = –1.59
Error check

\[ \omega := 1 \cdot 10^7, 10 \cdot 10^7, \ldots, 5 \cdot 10^{11} \]

\[ |Z_{TIA,B}(\omega)| \]
\[ |\text{filter}(\omega)| \]
\[ |Z_{TIA,B}(\omega)| \cdot |\text{filter}(\omega)| \]

\[ v := 0.47, 0.48, \ldots, 0.55 \]
\[ b := 8.3 \cdot 10^3, 8.4 \cdot 10^3, \ldots, 8.8 \cdot 10^3 \]

\[ P_{es}(b, v) \]
Noise check for all $\alpha_b$

$\Delta_L := 1.8 \quad \Delta_R := 1.58 \quad \text{feedback } \Delta R \cdot \text{time constant ratio } \Delta L$

$Rf_B := \Delta_R \cdot \frac{Av + 1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} = 345.7641$

$\alpha_b := 0.1 \quad \text{splitting ratio}$

$Lc := \frac{\left( \frac{Rf_B}{1 + Av} \right)^2 \cdot C_T}{\Delta_L} = 8.234 \cdot 10^{-10}$

$C1 := (1 - \alpha_b) \cdot (C_T) = 1.35 \cdot 10^{-12}$

$C2 := \alpha_b \cdot (C_T) = 1.5 \cdot 10^{-13}$

$Lc1 := \frac{\left( \frac{1.58}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right)^2 \cdot C_T}{1.8} = 8.234 \cdot 10^{-10} \quad C11 := (1 - 0.1) \cdot (C_T) = 1.35 \cdot 10^{-12}$

$Lc2 := \frac{\left( \frac{1.87}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right)^2 \cdot C_T}{1.8} = 1.153 \cdot 10^{-9} \quad C12 := (1 - 0.2) \cdot (C_T) = 1.2 \cdot 10^{-12}$

$Lc3 := \frac{\left( \frac{2.52}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right)^2 \cdot C_T}{2.4} = 1.571 \cdot 10^{-9} \quad C13 := (1 - 0.3) \cdot (C_T) = 1.05 \cdot 10^{-12}$

$Lc4 := \frac{\left( \frac{2.75}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right)^2 \cdot C_T}{1.9} = 2.363 \cdot 10^{-9} \quad C14 := (1 - 0.4) \cdot (C_T) = 9 \cdot 10^{-13}$

$Lc5 := \frac{\left( \frac{2.65}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right)^2 \cdot C_T}{1.5} = 2.779 \cdot 10^{-9} \quad C15 := (1 - 0.5) \cdot (C_T) = 7.5 \cdot 10^{-13}$
Noise check- IG/RF TF

\[ Z_{\text{In},D,0.1}(\omega) := 4 \cdot k \cdot T \cdot \left( \frac{\left( (1 - \omega^2 \cdot Lc1 \cdot C11) \right)}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \left( 1 + \frac{1 + Av}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right) \cdot \left( C_T - (\omega)^2 \cdot Lc1 \cdot C11 \cdot (C_T - C11) \right) \right)^2 \cdot \frac{1}{gm1} \cdot \frac{1}{Z_{\text{tuned}} B(\omega)} \]

Noise check- ID noise + TF

\[ Z_{\text{In},D,0.2}(\omega) := 4 \cdot k \cdot T \cdot \left( \frac{\left( (1 - \omega^2 \cdot Lc2 \cdot C12) \right)}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \left( 1 + \frac{1 + Av}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right) \cdot \left( C_T - (\omega)^2 \cdot Lc2 \cdot C12 \cdot (C_T - C12) \right) \right)^2 \cdot \frac{1}{gm1} \cdot \frac{1}{Z_{\text{tuned}} B(\omega)} \]
\[ Z_{\text{In.D.0.3}}(\omega) := 4 \cdot k \cdot T \cdot \frac{2.52 \cdot \frac{1}{1+Av}}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s \cdot C_T}} \cdot \left( \frac{1}{1+Av} \cdot \frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s \cdot C_T}} \cdot Z_{\text{tuned}}(\omega) \right)^2 \]

\[ Z_{\text{In.D.0.4}}(\omega) := 4 \cdot k \cdot T \cdot \frac{2.75 \cdot \frac{1}{1+Av}}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s \cdot C_T}} \cdot \left( \frac{1}{1+Av} \cdot \frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s \cdot C_T}} \cdot Z_{\text{tuned}}(\omega) \right)^2 \]

\[ Z_{\text{In.D.0.5}}(\omega) := 4 \cdot k \cdot T \cdot \frac{2.65 \cdot \frac{1}{1+Av}}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s \cdot C_T}} \cdot \left( \frac{1}{1+Av} \cdot \frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s \cdot C_T}} \cdot Z_{\text{tuned}}(\omega) \right)^2 \]
Noise check - Total O/N over RT

\[
V_1(\omega) := \left( \frac{4 \cdot k \cdot T}{1 + A v} + 2 \cdot q \cdot I g \right) \left( 1.58 \cdot \frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right) \cdot \left| (1 - \omega^2 \cdot Lc1 \cdot C11) \cdot Z_{\text{tuned}_B}(\omega) \right| + 4 \cdot k \cdot T \]

\[
\cdot \left( \left( \frac{(1 - \omega^2 \cdot Lc1 \cdot C11)}{1 + A v} \right) + 1.58 \cdot \frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \cdot (\omega) \right) \left| \left( \frac{(c_T - (\omega)^2 \cdot Lc1 \cdot C11 \cdot (c_T - C11)) \cdot 1}{T_s} \right) \right| \cdot \left( \frac{1.58 \cdot (1 + A v)}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \cdot (1) \right) \cdot \left| Z_{\text{tuned}_B}(\omega) \right| \]

\[
V_2(\omega) := \left( \frac{4 \cdot k \cdot T}{1 + A v} + 2 \cdot q \cdot I g \right) \left( 1.87 \cdot \frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right) \cdot \left| (1 - \omega^2 \cdot Lc2 \cdot C12) \cdot Z_{\text{tuned}_B}(\omega) \right|^2 + 4 \cdot k \cdot T \]

\[
\cdot \left( \left( \frac{(1 - \omega^2 \cdot Lc2 \cdot C12)}{1 + A v} \right) + 1.87 \cdot \frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \cdot (\omega) \right) \left| (\omega) \cdot (c_T - (\omega)^2 \cdot Lc2 \cdot C12 \cdot (c_T - C12)) \cdot 1 \right| \cdot \left( \frac{1.87 \cdot (1 + A v)}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \cdot (1) \right) \cdot \left| Z_{\text{tuned}_B}(\omega) \right| \]

C-65
\[ V_5(\omega) := \left( \frac{4 \cdot k \cdot T}{2.65 \cdot \frac{1 + Av}{1 + Av} + 2 \cdot q \cdot I_g} \right)^\frac{1}{d} \]
\[ \cdot \left[ \left( 1 - \omega^2 \cdot Lc5 \cdot C15 \right) \cdot Z_{tuned\_B}(\omega) \right]^2 + 4 \cdot k \cdot T \]
\[ \cdot \left( \frac{\left( \left( 1 - \omega^2 \cdot Lc5 \cdot C15 \right) \right)^\frac{1}{d} + 2.65 \cdot \frac{1 + Av}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}}{\left( \frac{1 + Av}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right)^\frac{1}{d}} \right)^2 \]
\[ \cdot \frac{1}{gm1} \cdot \left( \frac{(\omega) \cdot (C_T - (\omega)^2 \cdot Lc5 \cdot C15 \cdot (C_T - C15)) \cdot 1i}{2.65 \cdot \frac{1 + Av}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}} \right)^\frac{1}{d} \]
\[ \cdot Z_{tuned\_B}(\omega) \]
C.3.6 Tuned B PPM receiver with matched filter (PIN-FET/f_n = 5)

PPM (PIN-FET Tuned/Matched 1 Gbit/s)

PPM Rx performance with PIN-FET input configuration, Matched filter. Calculations are as follows:
- PPM terms
- Rx terms (noise+TF)
- Pulse shaping, voltages
- Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

**Bit-rate, pulse duration, and input pulse**

\[ PCM_B := 1 \cdot 10^9 \]

\[ M := 3 \]

\[ T_n := \frac{M}{PCM_B} \quad \text{Frame time - DPPM} \]

\[ m := 1 \quad \text{Modulation depth - DPPM} \]

\[ n := 2^M \quad \text{Number of DPPM active slots} \]

\[ T_s := \frac{m \cdot T_n}{n} = 3.75 \cdot 10^{-10} \quad \text{Slot width} \]

\[ \eta q := 1.6 \cdot 10^{-10} \quad \text{Quantum energy} \]

\[ \lambda := 1.55 \cdot 10^{-6} \quad \text{Wavelength of operation} \]

\[ \text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ T_b := \frac{1}{PCM_B} \quad \text{This is the PCM bit time} \]

\[ f_n := 5 \]

\[ \alpha := \frac{0.1874 \cdot T_b}{f_n} = 3.748 \cdot 10^{-11} \]

**Preamplifier terms**

\[ A_v := 10 \quad C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]

Feedback value for (R) Tuned B

\[ \Delta_L := 2.4 \quad \Delta_R := 2.52 \quad \text{feedback} \Delta R, \text{time constant ratio} \Delta L \]

\[ Rf_B := \Delta R \cdot \frac{A_v + 1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} = 551.4719 \quad \text{feedback for Tuned B} \]

\[ \alpha_b := 0.3 \quad \text{splitting ratio} \]
\[ \begin{align*}
Lc &:= \frac{\left( \frac{Rf_B}{1 + Av} \right)^2 \cdot C_T}{\Delta_L} = 1.571 \cdot 10^{-9} \\
C1 &:= (1 - \alpha_b) \cdot (C_T) = 1.05 \cdot 10^{-12} \\
C2 &:= \alpha_b \cdot (C_T) = 4.5 \cdot 10^{-13} \\
Z_{TLA,B}(\omega) &:= \frac{1}{\left( \frac{1 - \omega^2 \cdot Lc \cdot C1}{(1 + Av) \cdot (\omega + 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) + \frac{Rf_B}{1 + Av}} \right)} \\
filter(\omega) &:= \exp\left( -\alpha^2 \cdot \omega^2 \right)
\end{align*} \]

Receiver noise

\[ Z_{tuned,B}(\omega) := filter(\omega) \quad \omega := 1 \cdot 10^6, 10 \cdot 10^6 \ldots 1 \cdot 10^{11} \]

\[ NEB_B := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{11}} \left( |Z_{tuned,B}(\omega)| \right)^2 \, d\omega \right) = 3.763 \cdot 10^9 \]

\[ I_{2,B} := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{11}} \left( \frac{\left| (1 - \omega^2 \cdot Lc \cdot C1) \right|}{\left| Z_{tuned,B}(\omega) \right|} \right)^2 \, d\omega \right) = 3.236 \cdot 10^9 \]

\[ I_{3,B} := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{11}} \left( \frac{\left| (1 - \omega^2 \cdot Lc \cdot C1) \right|}{\left( \frac{Rf_B}{1 + Av} \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) + 1i \right)^{\frac{1}{2}}} \right)^2 \, d\omega \right) = 1.242 \cdot 10^6 \]

\[ |(1 - \omega^2 \cdot Lc \cdot C1) \cdot Z_{tuned,B}(\omega)| \]

\[ \omega \]

C-69
Feedback noise
\[ q := 1.6 \cdot 10^{-10} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \]

\[ I_{n_{Rf.B}} := \frac{4 \cdot k \cdot T}{R_{f.B}} \cdot I_{2.B} = 9.652 \cdot 10^{-14} \]

Ig noise (gate current)
\[ I_{g} := 10 \cdot 10^{-9} \]

\[ I_{n_{G.B}} := 2 \cdot q \cdot I_{g} \cdot I_{2.B} = 1.035 \cdot 10^{-17} \]

Channel noise (Gate-source)
\[ g_{m1} := 30 \cdot 10^{-3} = 0.03 \quad noise \_factor := 1 \]

\[ I_{n_{D}} := 4 \cdot k \cdot T \cdot \frac{1}{g_{m1}} \]

\[ I_{n_{D,B}} := I_{n_{D}} \cdot I_{3.B} = 6.81 \cdot 10^{-13} \]

Total noise - PIN-FET
\[ I_{n,B} := I_{n_{Rf.B}} + I_{n_{G.B}} + I_{n_{D,B}} = 7.775 \cdot 10^{-13} \]

\[ noise := I_{n,B} \]

Pulse shape
\[ I_{1}(t) := \frac{1}{\pi} \cdot \frac{T_{s}}{10^{3}} \cdot \frac{1}{T_{s}} \cdot 10^{3} \]

\[ \int_{0}^{1} \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) d\omega \cdot \text{Re} \left[ 1 \cdot i \cdot \omega \cdot \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \cdot \exp \left( 1 \cdot i \cdot \omega \cdot (t) \right) \right] \]

\[ I_{0}(t) := \frac{1}{\pi} \cdot \frac{T_{s}}{10^{3}} \cdot \frac{1}{T_{s}} \cdot 10^{3} \]

\[ \int_{0}^{1} \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) d\omega \cdot \text{Re} \left[ \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \cdot \exp \left( 1 \cdot i \cdot \omega \cdot (t) \right) \right] \]
\[ t := 0.002 \cdot T_s \]
\[ t_{pk} := \text{root} \left( T_s \cdot I_1(t), t \right) \]
\[ \frac{t_{pk}}{T_s} = -1.616 \cdot 10^{-22} \]

**Voltages**

\[ v_o(t) := I_0(t) \]
\[ v_o(t_{pk}) = 7.527 \cdot 10^9 \]

\[ t := -5 \cdot T_s, -4.99 \cdot T_s, \ldots, 5 \cdot T_s \]

**Diagram 1:**

- Graph showing the function \( I_1(t) \) with peaks at various intervals.

**Diagram 2:**

- Graph showing the function \( v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \) with a peak at \( -0.13 \).

\[ \text{temp}(v, t) := \frac{I_0(t)}{v \cdot I_0(t_{pk})} - 1 \]

\[ t := -0.1 \cdot T_s \]
\[ t_d(v) := \sqrt{\text{root}(\text{temp}(v, t), t)} \]
\[ v_d(b, v) := b \cdot \eta_q \cdot I_0(t_d(v)) \]
\[ v_d(b, v) := b \cdot \eta_q \cdot I_0(t_d(v)) \]
\[ \text{slope}(b, v) := b \cdot \eta_q \cdot \frac{I_1(t_d(v))}{T_s} \]
\[ v_{pk}(b) := b \cdot \eta_q \cdot I_0(t_{pk}) \]

**Q values**
\[ Q_r(b, v) := \frac{v_{pk}(b) - v_d(b, v)}{\sqrt{\text{noise}}} \]
\[ Q_s(b, v) := \left( \frac{m \cdot T_n}{2 \cdot n} \right) \cdot \frac{\text{slope}(b, v)}{\sqrt{\text{noise}}} \]
\[ Q_{l}(b, v) := \frac{v_d(b, v)}{\sqrt{\text{noise}}} \]

**Error probabilities**
\[ P_r(b, v) := 0.5 \cdot \text{erfc} \left( \frac{Q_r(b, v)}{\sqrt{2}} \right) \]
\[ P_s(b, v) := \text{erfc} \left( \frac{Q_s(b, v)}{\sqrt{2}} \right) \]
\[ P_{f}(b, v) := \frac{m \cdot T_n}{n \cdot \alpha} \cdot 0.5 \cdot \text{erfc} \left( \frac{Q_{l}(b, v)}{\sqrt{2}} \right) \]
\[ P_{es}(b, v) := P_r(b, v) + P_s(b, v) + \frac{n-1}{2} \cdot P_{f}(b, v) \]
\[ P_{eb}(b, v) := \frac{n}{2 \cdot (n-1)} \cdot P_{es}(b, v) \]

\[
\begin{align*}
  b &:= 0.25 \cdot 10^4 \\
  v &:= 0.5 \\
  M &:= 3
\end{align*}
\]
\[ Q_r(b, v) = 1.707 \quad P_r(b, v) = 0.044 \]
\[ Q_s(b, v) = 7.11 \quad P_s(b, v) = 1.158 \cdot 10^{-12} \]
\[ Q_{l}(b, v) = 1.707 \quad P_{f}(b, v) = 0.439 \]
\[ P_{eb}(b, v) = 0.903 \]
\[ pc(b, v) := \log(P_{eb}(b, v)) + 9 \quad \text{Set for } 1 \text{ in } 10^9 \text{ errors} \]
\[ a(v) := \text{root}(pc(b, v), b) \quad \text{Find the root to give } 1 \text{ in } 10^9 \]
\[ v := 0.2, 0.21 \ldots 0.7 \quad \text{Scan for Optimum threshold} \]
\[ b_{(v-0.2) \cdot 100} := a(v) \quad \text{Photons per bit} \]
\[ \text{minimum} := \min(b) \quad \text{Search for minimum and store it} \]

\[
\frac{\text{minimum}}{M} = 3.074 \cdot 10^3
\]

Number of photons per bit verses threshold level (BER = 10\(^{-9}\)).

Rx Sens

\[ \text{result} := \text{minimum} \cdot \text{photon_energy} \cdot \frac{PCM_B}{M} \]
\[ dBm := 10 \cdot \log\left(\frac{\text{result}}{10^{-3}}\right) \]
\[ dBm = -34.039 \]
Error check

\[ \omega := 1 \cdot 10^7, 10 \cdot 10^7 \ldots 5 \cdot 10^{11} \]

\[
\begin{align*}
|Z_{TIA,B}(\omega)| \\
|\text{filter}(\omega)| \\
|Z_{TIA,B}(\omega)| \cdot |\text{filter}(\omega)|
\end{align*}
\]

\[ v := 0.47, 0.48 \ldots 0.55 \quad b := 9 \cdot 10^3, 9.1 \cdot 10^3 \ldots 9.3 \cdot 10^3 \]

\[ P_{ex}(b, v) \]
Noise check - for all $\alpha_b$

$\Delta_L := 1.8$ \hspace{1em} $\Delta_R := 1.58$ \hspace{1em} feedback $\Delta R$, time constant ratio $\Delta L$

$$R_{f,B} := \Delta_R \cdot \frac{Av + 1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} = 345.7641$$

$\alpha_b := 0.1$ \hspace{1em} splitting ratio

$$Lc := \frac{\left( \frac{R_{f,B}}{1 + Av} \right)^2 \cdot C_T}{\Delta_L} = 8.234 \cdot 10^{-10}$$

$C1 := (1 - \alpha_b) \cdot (C_T) = 1.35 \cdot 10^{-12}$

$C2 := \alpha_b \cdot (C_T) = 1.5 \cdot 10^{-13}$

$$Lc1 := \left( \frac{1.58 \cdot \frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}}{1.8} \right)^2 \cdot C_T = 8.234 \cdot 10^{-10} \hspace{1em} C11 := (1 - 0.1) \cdot (C_T) = 1.35 \cdot 10^{-12}$$

$$Lc2 := \left( \frac{1.87 \cdot \frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}}{1.8} \right)^2 \cdot C_T = 1.153 \cdot 10^{-9} \hspace{1em} C12 := (1 - 0.2) \cdot (C_T) = 1.2 \cdot 10^{-12}$$

$$Lc3 := \left( \frac{2.52 \cdot \frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}}{2.4} \right)^2 \cdot C_T = 1.571 \cdot 10^{-9} \hspace{1em} C13 := (1 - 0.3) \cdot (C_T) = 1.05 \cdot 10^{-12}$$

$$Lc4 := \left( \frac{2.75 \cdot \frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}}{1.9} \right)^2 \cdot C_T = 2.363 \cdot 10^{-9} \hspace{1em} C14 := (1 - 0.4) \cdot (C_T) = 9 \cdot 10^{-13}$$

$$Lc5 := \left( \frac{2.65 \cdot \frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}}{1.5} \right)^2 \cdot C_T = 2.779 \cdot 10^{-9} \hspace{1em} C15 := (1 - 0.5) \cdot (C_T) = 7.5 \cdot 10^{-13}$$
Noise check - IG/RF TF

\[ Z_{\text{In},D,0.1}(\omega) := 4 \cdot k \cdot T \cdot \left( \frac{\left(1 - \omega^2 \cdot Lc1 \cdot C11\right)}{\left(1 + A_{\text{V}}\right)} \cdot \left(\frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}\right) \right)^2 \]

\[ \frac{\left((1 - \omega^2 \cdot Lc1 \cdot C11) \cdot Z_{\text{tuned}_B}(\omega)\right)}{\left(1 + A_{\text{V}}\right)} \cdot \left(\frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}\right) \]

\[ \cdot \frac{1}{g_{m1}} \cdot \frac{\left((1 - \omega^2 \cdot Lc1 \cdot C11) \cdot Z_{\text{tuned}_B}(\omega)\right)}{\left(1 + A_{\text{V}}\right)} \cdot \left(\frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}\right) \]

Noise check - ID noise + TF

\[ Z_{\text{In},D,0.2}(\omega) := 4 \cdot k \cdot T \cdot \left( \frac{\left(1 - \omega^2 \cdot Lc1 \cdot C11\right)}{\left(1 + A_{\text{V}}\right)} \cdot \left(\frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}\right) \right)^2 \]

\[ \frac{\left(1 - \omega^2 \cdot Lc2 \cdot C12\right)}{\left(1 + A_{\text{V}}\right)} \cdot \left(\frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}\right) \]

\[ \cdot \frac{1}{g_{m1}} \cdot \frac{\left(1 - \omega^2 \cdot Lc2 \cdot C12\right)}{\left(1 + A_{\text{V}}\right)} \cdot \left(\frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}\right) \]
\[ Z_{in,D.0.3}(\omega) := 4 \cdot k \cdot T_d \]
\[
= \frac{1}{gm1} \left( \left( \frac{1}{1 + Av} \right) \left( \frac{1}{2 \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right) \right) \]
\[
\left( \left( 1 - \omega^2 \cdot Lc3 \cdot C13 \right) \right) \left( \left( 1 + Av \right) \right) \left( \frac{1}{2 \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right) \]
\[
\cdot \left( \frac{1}{1 + Av} \right) \left( \frac{1}{2 \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right) \]
\[
\cdot Z_{tuned_B}(\omega) \]

\[ Z_{in,D.0.4}(\omega) := 4 \cdot k \cdot T_d \]
\[
= \frac{1}{gm1} \left( \left( \frac{1}{1 + Av} \right) \left( \frac{1}{2 \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right) \right) \]
\[
\left( \left( 1 - \omega^2 \cdot Lc4 \cdot C14 \right) \right) \left( \left( 1 + Av \right) \right) \left( \frac{1}{2 \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right) \]
\[
\cdot \left( \frac{1}{1 + Av} \right) \left( \frac{1}{2 \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right) \]
\[
\cdot Z_{tuned_B}(\omega) \]

\[ Z_{in,D.0.5}(\omega) := 4 \cdot k \cdot T_d \]
\[
= \frac{1}{gm1} \left( \left( \frac{1}{1 + Av} \right) \left( \frac{1}{2 \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right) \right) \]
\[
\left( \left( 1 - \omega^2 \cdot Lc5 \cdot C15 \right) \right) \left( \left( 1 + Av \right) \right) \left( \frac{1}{2 \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right) \]
\[
\cdot \left( \frac{1}{1 + Av} \right) \left( \frac{1}{2 \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right) \]
\[
\cdot Z_{tuned_B}(\omega) \]
Noise check - Total O/N over RT

\[
V_1(\omega) := \left( \frac{4 \cdot k \cdot T}{1 + Av} + 2 \cdot q \cdot Ig \right) \cdot \frac{1.58}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \cdot \left| (1 - \omega^2 \cdot Lc1 \cdot C11) \cdot Z_{tuned\_B}(\omega) \right| + 4 \cdot k \cdot T
\]

\[
\left( \frac{(1 - \omega^2 \cdot Lc1 \cdot C11)}{1 + Av} + 1.58 \cdot \frac{1 + Av}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \cdot \omega \right) \cdot \left( \frac{(C_T - (\omega)^2 \cdot Lc1 \cdot C11 \cdot (C_T - C11)) \cdot 1i}{1.58 \cdot \frac{1 + Av}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \cdot Z_{tuned\_B}(\omega)} \right)^2
\]

\[
V_2(\omega) := \left( \frac{4 \cdot k \cdot T}{1 + Av} + 2 \cdot q \cdot Ig \right) \cdot \frac{1.87}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \cdot \left| (1 - \omega^2 \cdot Lc2 \cdot C12) \cdot Z_{tuned\_B}(\omega) \right|^2 + 4 \cdot k \cdot T
\]

\[
\left( \frac{(1 - \omega^2 \cdot Lc2 \cdot C12)}{1 + Av} + 1.87 \cdot \frac{1 + Av}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \cdot \omega \right) \cdot \left( \frac{(\omega) \cdot (C_T - (\omega)^2 \cdot Lc2 \cdot C12 \cdot (C_T - C12)) \cdot 1i}{1.87 \cdot \frac{1 + Av}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \cdot Z_{tuned\_B}(\omega)} \right)^2
\]
\[ V3(\omega) := \left( \frac{4 \cdot k \cdot T}{2.52 \cdot \frac{1 + Av}{1 + Av} + 2 \cdot q \cdot Ig} \right)^\dagger \]
\[
\cdot \left( 1 - \omega^2 \cdot Lc3 \cdot C13 \right) \cdot Z_{\text{tuned}_B}(\omega) \right)^2 + 4 \cdot k \cdot T \cdot \dagger
\]
\[
\cdot \frac{1}{gm1} \cdot \left( \frac{\left\{ \left( 1 - \omega^2 \cdot Lc3 \cdot C13 \right) \right\}^\dagger}{\frac{1 + Av}{1 + Av} + 2 \cdot \pi \cdot \frac{1}{T_s} \cdot C_T} \right)^\dagger
\]
\[
\cdot \left( (\omega) \cdot (C_T - (\omega)^2 \cdot Lc3 \cdot C13 \cdot (C_T - C13)) \cdot 1i \right)^\dagger
\]
\[
\cdot Z_{\text{tuned}_B}(\omega)
\]

\[ V4(\omega) := \left( \frac{4 \cdot k \cdot T}{2.75 \cdot \frac{1 + Av}{1 + Av} + 2 \cdot q \cdot Ig} \right)^\dagger \]
\[
\cdot \left( 1 - \omega^2 \cdot Lc4 \cdot C14 \right) \cdot Z_{\text{tuned}_B}(\omega) \right)^2 + 4 \cdot k \cdot T \cdot \dagger
\]
\[
\cdot \frac{1}{gm1} \cdot \left( \frac{\left\{ \left( 1 - \omega^2 \cdot Lc4 \cdot C14 \right) \right\}^\dagger}{\frac{1 + Av}{1 + Av} + 2 \cdot \pi \cdot \frac{1}{T_s} \cdot C_T} \right)^\dagger
\]
\[
\cdot \left( (\omega) \cdot (C_T - (\omega)^2 \cdot Lc4 \cdot C14 \cdot (C_T - C14)) \cdot 1i \right)^\dagger
\]
\[
\cdot Z_{\text{tuned}_B}(\omega)
\]
\[V_5(\omega) := \left( \frac{4 \cdot k \cdot T}{2.65 \cdot \frac{1}{1+Av} + 2 \cdot q \cdot I_g} \right)^d \\
\cdot \left| (1 - \omega^2 \cdot Lc_5 \cdot C_{15}) \cdot Z_{\text{tuned B}}(\omega) \right|^2 + 4 \cdot k \cdot T \right) \cdot Z_{\text{tuned B}}(\omega) \]
C.3.7 Tuned B PPM receiver with matched filter (PIN-BJT/f_n = 2)

PPM (PIN-BJT Tuned/Matched 1 Gbit/s)

PPM Rx performance with PIN-BJT input configuration. Matched filter. calculations are as follow:

- PPM terms
- Rx terms (noise+TF)
- Pulse shaping, voltages
- Error bit rate
- Optimum threshold/minimum number of photons
- receiver sensitivity

**Bit-rate, pulse duration, and input pulse**

\[ PCM_B := 1 \cdot 10^9 \]

\[ M := 3 \]

\[ T_n := \frac{M}{PCM_B} \]

Frame time - DPPM

\[ m \equiv 1 \]

Modulation depth - DPPM

\[ n := 2^M \]

Number of DPPM active slots

\[ T_s := \frac{m \cdot T_n}{n} = 3.75 \cdot 10^{-10} \]

Slot width

\[ \eta q := 1.6 \cdot 10^{-10} \]

Quantum energy

\[ \lambda := 1.55 \cdot 10^{-6} \]

wavelength of operation

\[ photon\_energy := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

This is the PCM bit time

\[ T_b := \frac{1}{PCM_B} \]

\[ f_n := 2 \]

\[ \alpha := \frac{0.1874 \cdot T_b}{f_n} = 9.37 \cdot 10^{-11} \]

**Preamplifier terms**

\[ A v := 10 \quad C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]

Feedback value for (R) Tuned B

\[ \Delta L := 2.5 \quad \Delta R := 3.17 \quad \text{feedback } \Delta R, \text{ time constant ratio } \Delta L \]

\[ R f_B := \Delta R \cdot \frac{A v + 1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} = 693.7166 \]

feedback for Tuned B

\[ \alpha_b := 0.4 \]

splitting ratio
\[ L_c := \frac{\left( \frac{R_f \cdot B}{1 + A v} \right)^2 \cdot C_T}{\Delta_L} = 2.386 \cdot 10^{-9} \]

\[ C_1 := (1 - \alpha_b) \cdot (C_T) = 9 \cdot 10^{-13} \]

\[ C_2 := \alpha_b \cdot (C_T) = 6 \cdot 10^{-13} \]

\[ Z_{TIA,B}(\omega) := \frac{1}{\left( (1 - \omega^2 \cdot L_c \cdot C_1) + \frac{R_f \cdot B}{(1 + A v)} \cdot (\omega \cdot 1i) \cdot (C_1 + C_2 - (\omega)^2 \cdot L_c \cdot C_1 \cdot C_2) \right)} \]

\[ \text{filter}(\omega) := \exp\left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \]

**Receiver noise**

\[ Z_{tuned,B}(\omega) := \text{filter}(\omega) \quad \omega := 1 \cdot 10^6, 10 \cdot 10^6 \ldots 1 \cdot 10^{11} \]

\[ NEB_B := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^9} \left( \left| Z_{tuned,B}(\omega) \right| \right)^2 d\omega \right) = 1.505 \cdot 10^9 \]

\[ I_{2,B} := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^9} \left( \left| \frac{(1 - \omega^2 \cdot L_c \cdot C_1) \cdot \text{filter}(\omega)}{Z_{tuned,B}(\omega)} \right| \right)^2 d\omega \right) = 1.205 \cdot 10^9 \]

\[ I_{3,B} := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^9} \left( \left( \frac{(1 - \omega^2 \cdot L_c \cdot C_1) \cdot \text{filter}(\omega) \cdot (C_1 + C_2 - (\omega)^2 \cdot L_c \cdot C_1 \cdot C_2) \cdot 1i}{R_f \cdot B} \right) \cdot \text{filter}(\omega) \cdot Z_{tuned,B}(\omega) \right)^2 \right) d\omega = 1.457 \cdot 10^5 \]

\[ [1 - \omega^2 \cdot L_c \cdot C_1] \cdot Z_{tuned,B}(\omega) \]

\[ \omega \]
noise-opt BJT

\( q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100 \)

\[
x_B(x) := \left(4kT \cdot I_{2\cdot B} + 2q \cdot \frac{x}{hfe} \cdot I_{2\cdot B} + 2q \cdot \frac{x}{(25 \cdot 10^{-3})^2} \cdot I_{3\cdot B}\right)
\]

\( I_{\text{opt, } B} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3\cdot B}}{I_{2\cdot B}}} = 0.003 \)

\[\frac{0.006}{10^{-3}} = 6\]

\( I_{n\cdot B} := x_B(I_{\text{opt, } B}) = 4.976 \cdot 10^{-14} \)

\( x := 0.00001, 0.00002, 0.1 \)

\[x_B(x)\]

noise := \( I_{n\cdot B} \)

Pulse shape

\[
I_1(t) := \frac{T_s}{\pi} \cdot \frac{1}{T_s} \cdot 10^3 \cdot \exp\left(-\frac{\alpha^2 \cdot \omega^2}{2}\right) \cdot \exp\left(\frac{-\alpha^2 \cdot \omega^2}{2}\cdot \exp\left(1i \cdot \omega \cdot (t)\right)\right) \cdot \exp\left(1i \cdot \omega \cdot (t)\right) \cdot \exp\left(1i \cdot \omega \cdot (t)\right)
\]

\[
I_0(t) := \frac{1}{\pi} \cdot \int_0^{1/T_s} \exp\left(-\frac{\alpha^2 \cdot \omega^2}{2}\right) \cdot \exp\left(\frac{-\alpha^2 \cdot \omega^2}{2}\cdot \exp\left(1i \cdot \omega \cdot (t)\right)\right) \cdot \exp\left(1i \cdot \omega \cdot (t)\right) \cdot \exp\left(1i \cdot \omega \cdot (t)\right) \cdot \exp\left(1i \cdot \omega \cdot (t)\right)
\]

C-84
\[ t := 0.002 \cdot T_s \]
\[ t_{pk} := \text{root} \left( T_s \cdot I_1(t), t \right) \]
\[ \frac{t_{pk}}{T_s} = 1.028 \cdot 10^{-12} \]

**Voltages**

\[ v_o(t) := I_0(t) \]
\[ v_o(t_{pk}) = 3.011 \cdot 10^9 \]

\[ t := -5 \cdot T_s, -4.99 \cdot T_s, \ldots 5 \cdot T_s \]

\[ v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \]
\[ \text{temp}(v, t) := \frac{I_0(t)}{v \cdot I_0(t_{pk})} - 1 \]
\( t := -0.1 \cdot T_s \)
\( t_d(v) := \text{root}(\text{temp}(v, t), t) \)
\( v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \)
\( v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \)
\( \text{slope}(b, v) := b \cdot \eta q \cdot \frac{I_1(t_d(v))}{T_s} \)
\( v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \)

**Q values**

\( Q_r(b, v) := \frac{v_{pk}(b) - v_d(b, v)}{\sqrt{\text{noise}}} \)

\( Q_s(b, v) := \left( \frac{m \cdot T_n}{2 \cdot n} \right) \cdot \frac{\text{slope}(b, v)}{\sqrt{\text{noise}}} \)

\( Q_t(b, v) := \frac{v_d(b, v)}{\sqrt{\text{noise}}} \)

**Error probabilities**

\( P_r(b, v) := 0.5 \cdot \text{erfc} \left( \frac{Q_r(b, v)}{\sqrt{2}} \right) \)

\( P_s(b, v) := \text{erfc} \left( \frac{Q_s(b, v)}{\sqrt{2}} \right) \)

\( P_f(b, v) := \frac{m \cdot T_n}{n \cdot \alpha} \cdot 0.5 \cdot \text{erfc} \left( \frac{Q_t(b, v)}{\sqrt{2}} \right) \)

\( P_{cs}(b, v) := P_r(b, v) + P_s(b, v) + \frac{n-1}{2} \cdot P_f(b, v) \)

\( P_{ch}(b, v) := \frac{n}{2 \cdot (n-1)} \cdot P_{cs}(b, v) \)

\( b := 0.25 \cdot 10^4 \quad v := 0.5 \quad M = 3 \)

\( Q_r(b, v) = 2.699 \quad P_r(b, v) = 0.003 \)

\( Q_s(b, v) = 4.497 \quad P_s(b, v) = 6.895 \cdot 10^{-6} \)
Set for 1 in $10^9$ errors

Find the root to give 1 in $10^9$

Scan for Optimum threshold

Photons per bit

Search for minimum and store it

Number of photons per bit verses threshold level (BER = $10^{-9}$).

Rx Sens

```
result := minimum \cdot photon\_energy \cdot \frac{PCM\_B}{M}
```

```
dBm := 10 \cdot \log \left( \frac{result}{10^{-3}} \right)
```

32.694

```
dBm = -36.076
```

Sens compared to non-tuned for all “a” value

33.79 - 34.8 = -1.01
33.79 - 35.13 = -1.34
33.79 - 35.63 = -1.84
33.79 - 36.07 = -2.28
33.79 - 35.85 = -2.06

$9.767 \cdot 10^3 - 5.771 \cdot 10^3 = 3.996 \cdot 10^3$
Error check

$\omega := 1 \cdot 10^7, 10 \cdot 10^7 \ldots 5 \cdot 10^{11}$

$|Z_{TIA,B}(\omega)|$

$|\text{filter}(\omega)|$

$|Z_{TIA,B}(\omega)| \cdot |\text{filter}(\omega)|$

$v := 0.47, 0.48 \ldots 0.55$

$b := 5.6 \cdot 10^3, 5.7 \cdot 10^3 \ldots 6 \cdot 10^3$

$P_{es}(b, v)$
Noise check for all $\alpha_b$

$\Delta_L := 1.8 \quad \Delta_R := 1.58$ feedback $\Delta R$, time constant ratio $\Delta L$

$Rf_B := \Delta_R \cdot \frac{Av + 1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} = 345.7641$

$\alpha_b := 0.1$ splitting ratio

$Lc := \frac{\left(\frac{Rf_B}{1 + Av}\right)^2 \cdot C_T}{\Delta_L} = 8.234 \cdot 10^{-10}$

$C1 := (1 - \alpha_b) \cdot (C_T) = 1.35 \cdot 10^{-12}$

$C2 := \alpha_b \cdot (C_T) = 1.5 \cdot 10^{-13}$

$Lc_1 := \left(\frac{1.58 \cdot \frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}}{1.8}\right)^2 \cdot C_T = 8.234 \cdot 10^{-10}$

$Lc_2 := \left(\frac{1.87 \cdot \frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}}{1.8}\right)^2 \cdot C_T = 1.153 \cdot 10^{-9}$

$Lc_3 := \left(\frac{2.52 \cdot \frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}}{2.4}\right)^2 \cdot C_T = 1.571 \cdot 10^{-9}$

$Lc_4 := \left(\frac{2.75 \cdot \frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}}{1.9}\right)^2 \cdot C_T = 2.363 \cdot 10^{-9}$

$Lc_5 := \left(\frac{2.65 \cdot \frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}}{1.5}\right)^2 \cdot C_T = 2.779 \cdot 10^{-9}$

$C11 := (1 - 0.1) \cdot (C_T) = 1.35 \cdot 10^{-12}$

$C12 := (1 - 0.2) \cdot (C_T) = 1.2 \cdot 10^{-12}$

$C13 := (1 - 0.3) \cdot (C_T) = 1.05 \cdot 10^{-12}$

$C14 := (1 - 0.4) \cdot (C_T) = 9 \cdot 10^{-13}$

$C15 := (1 - 0.5) \cdot (C_T) = 7.5 \cdot 10^{-13}$
Noise check - IG/RF TF

\[ Z_{In.D.0.1}(\omega) := 2q \cdot \frac{I_{opt_B}}{\left( \frac{I_{opt_B}}{25 \cdot 10^{-3}} \right)^2} \cdot \left( \frac{(1 - \omega^2 \cdot Lc1 \cdot C11) \cdot Z_{tuned_B}(\omega)}{(1 + Av) \cdot \left( \frac{1}{2 \pi \cdot 2 \cdot \frac{1}{T_s}} \right) \cdot (C_T - (\omega)^2 \cdot Lc1 \cdot C11 \cdot (C_T - C11))} \right)^2 + 1.58 \cdot \frac{1 + Av}{2 \pi \cdot 2 \cdot \frac{1}{T_s}} \cdot \left( C_T - (\omega)^2 \cdot Lc1 \cdot C11 \cdot (C_T - C11) \right) \cdot 11 \right) \]

\[ Z_{In.D.0.2}(\omega) := 2q \cdot \frac{I_{opt_B}}{\left( \frac{I_{opt_B}}{25 \cdot 10^{-3}} \right)^2} \cdot \left( \frac{(1 - \omega^2 \cdot Lc2 \cdot C12) \cdot Z_{tuned_B}(\omega)}{(1 + Av) \cdot \left( \frac{1}{2 \pi \cdot 2 \cdot \frac{1}{T_s}} \right) \cdot (C_T - (\omega)^2 \cdot Lc2 \cdot C12 \cdot (C_T - C12))} \right)^2 + 1.87 \cdot \frac{1 + Av}{2 \pi \cdot 2 \cdot \frac{1}{T_s}} \cdot \left( C_T - (\omega)^2 \cdot Lc2 \cdot C12 \cdot (C_T - C12) \right) \cdot 11 \right) \]
\[ Z_{In,D,0.3}(\omega) := 2 \cdot q \cdot \frac{I_{opt_B}}{\left( \frac{I_{opt_B}}{25 \cdot 10^{-3}} \right)^2} \cdot \frac{(1 - \omega^2 \cdot \text{Le}3 \cdot C_{13})^j}{1 + Av} \cdot \frac{2.52 \cdot 1 + Av}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \cdot 2.52 \cdot \frac{1 + Av}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \cdot Z_{tuned_B}(\omega) \]

\[ Z_{In,D,0.4}(\omega) := 2 \cdot q \cdot \frac{I_{opt_B}}{\left( \frac{I_{opt_B}}{25 \cdot 10^{-3}} \right)^2} \cdot \frac{(1 - \omega^2 \cdot \text{Le}4 \cdot C_{14})^j}{1 + Av} \cdot \frac{2.75 \cdot 1 + Av}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \cdot 2.75 \cdot \frac{1 + Av}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \cdot Z_{tuned_B}(\omega) \]

\[ Z_{In,D,0.5}(\omega) := 2 \cdot q \cdot \frac{I_{opt_B}}{\left( \frac{I_{opt_B}}{25 \cdot 10^{-3}} \right)^2} \cdot \frac{(1 - \omega^2 \cdot \text{Le}5 \cdot C_{15})^j}{1 + Av} \cdot \frac{2.65 \cdot 1 + Av}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \cdot 2.65 \cdot \frac{1 + Av}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \cdot Z_{tuned_B}(\omega) \]
Noise check - Total O/N over RT

\[ V_1(\omega) := \left( \frac{4 \cdot k \cdot T}{1 + Av} + 2 \cdot q \cdot \frac{I_{opt \cdot B}}{hfe} \right)^2 \]
\[ \cdot \left| \left( \left( 1 - \omega^2 \cdot Lc1 \cdot C11 \right) \cdot Z_{tuned \cdot B}(\omega) \right) + 1 \right|^2 \]
\[ \cdot 2 \cdot q \cdot \frac{I_{opt \cdot B}}{\left( \frac{I_{opt \cdot B}}{25 \cdot 10^{-3}} \right)^2} \cdot \left( \frac{C_T - (\omega)^2 \cdot Lc1 \cdot C11 \cdot (C_T - C11)}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right) \cdot Z_{tuned \cdot B}(\omega) \]

\[ V_2(\omega) := \left( \frac{4 \cdot k \cdot T}{1 + Av} + 2 \cdot q \cdot \frac{I_{opt \cdot B}}{hfe} \right)^2 \]
\[ \cdot \left| \left( \left( 1 - \omega^2 \cdot Lc2 \cdot C12 \right) \cdot Z_{tuned \cdot B}(\omega) \right) + 1 \right|^2 \]
\[ \cdot 2 \cdot q \cdot \frac{I_{opt \cdot B}}{\left( \frac{I_{opt \cdot B}}{25 \cdot 10^{-3}} \right)^2} \cdot \left( \frac{C_T - (\omega)^2 \cdot Lc2 \cdot C12 \cdot (C_T - C12)}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right) \cdot Z_{tuned \cdot B}(\omega) \]
\[ V3(\omega) := \left( \frac{4 \cdot k \cdot T}{2.52 \cdot \frac{1 + A_v}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}} + 2 \cdot q \cdot \frac{I_{opt, B}}{hfe} \right)^{\frac{1}{d}} \]

\[ \cdot \left| (1 - \omega^2 \cdot Lc3 \cdot C13) \cdot Z_{tuned, B}(\omega) \right|^2 + 1 \]

\[ \cdot 2 \cdot q \cdot \frac{I_{opt, B}}{\left( \frac{I_{opt, B}}{(25 \cdot 10^{-3})} \right)^2} \cdot \left| \left( \frac{\left( (1 - \omega^2 \cdot Lc3 \cdot C13) \right)_{\frac{d}{d}}}{\left( 2.52 \cdot \frac{1 + A_v}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right)^{\frac{d}{d}}} + \left( \frac{\left( (1 - \omega^2 \cdot Lc3 \cdot C13) \right)_{\frac{d}{d}}}{\left( 2.52 \cdot \frac{1 + A_v}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right)^{\frac{d}{d}}} \right)^{\frac{1}{d}} \cdot \left( \frac{(\omega)(C_T - (\omega)^2 \cdot Lc3 \cdot C13 \cdot (C_T - C13)) \cdot 1}{2.52 \cdot \frac{1 + A_v}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}} \right)^{\frac{1}{d}} \cdot Z_{tuned, B}(\omega) \right|^{\frac{1}{d}} \]

\[ V4(\omega) := \left( \frac{4 \cdot k \cdot T}{2.75 \cdot \frac{1 + A_v}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}} + 2 \cdot q \cdot \frac{I_{opt, B}}{hfe} \right)^{\frac{1}{d}} \]

\[ \cdot \left| (1 - \omega^2 \cdot Lc4 \cdot C14) \cdot Z_{tuned, B}(\omega) \right|^2 + 1 \]

\[ \cdot 2 \cdot q \cdot \frac{I_{opt, B}}{\left( \frac{I_{opt, B}}{(25 \cdot 10^{-3})} \right)^2} \cdot \left| \left( \frac{\left( (1 - \omega^2 \cdot Lc4 \cdot C14) \right)_{\frac{d}{d}}}{\left( 2.75 \cdot \frac{1 + A_v}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right)^{\frac{d}{d}}} + \left( \frac{\left( (1 - \omega^2 \cdot Lc4 \cdot C14) \right)_{\frac{d}{d}}}{\left( 2.75 \cdot \frac{1 + A_v}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right)^{\frac{d}{d}}} \right)^{\frac{1}{d}} \cdot \left( \frac{(\omega)(C_T - (\omega)^2 \cdot Lc4 \cdot C14 \cdot (C_T - C14)) \cdot 1}{2.75 \cdot \frac{1 + A_v}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}} \right)^{\frac{1}{d}} \cdot Z_{tuned, B}(\omega) \right|^{\frac{1}{d}} \]
\[ V_5(\omega) := \left( \frac{4 \cdot k \cdot T}{2.65 \cdot \frac{1 + Av}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}} + 2 \cdot q \cdot \frac{I_{opt_B}}{hfe} \right)^d \]

\[ \cdot \left| \left( 1 - \omega^2 \cdot Lc5 \cdot C_{15} \right) \cdot Z_{tuned_B}(\omega) \right|^2 + 1^d \]

\[ \cdot 2 \cdot q \cdot \frac{I_{opt_B}}{\left( \frac{I_{opt_B}}{(25 \cdot 10^{-3})} \right)^2} \]

\[ \cdot Z_{tuned_B}(\omega) \]
C.3.8 Tuned B PPM receiver with matched filter (PIN-BJT/\(f_n = 5\))

PPM (PIN-BJT Tuned/Matched 1 Gbit/s)

PPM Rx performance with PIN-BJT input configuration. Matched filter, calculations are as follow

- PPM terms
- Rx terms (noise+TF)
- Pulse shaping, voltages
- Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

Bit-rate, pulse duration, and input pulse

**PCM\(_B\) \equiv 1 \cdot 10^9**

\(M := 3\)

\(T_n := \frac{M}{PCM\_B}\) Frame time - DPPM

\(m \equiv 1\) Modulation depth - DPPM

\(n := 2^M\) Number of DPPM active slots

\(T_s := \frac{m \cdot T_n}{n} = 3.75 \cdot 10^{-10}\) Slot width

\(\eta q := 1.6 \cdot 10^{-10}\) Quantum energy

\(\lambda := 1.55 \cdot 10^{-6}\) Wavelength of operation

\(\text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda}\)

\(T_b := \frac{1}{PCM\_B}\) This is the PCM bit time

\(f_n := 5\)

\(\alpha := \frac{0.1874 \cdot T_b}{f_n} = 3.748 \cdot 10^{-11}\)

**Preamplifier terms**

\(Av := 10\) \([C_T := 1.5 \cdot 10^{-12}\) total C

Feedback value for (R) Tuned B

\(\Delta L := 2.4\) \(\Delta R := 2.52\) Feedback \(\Delta R\), time constant ratio \(\Delta L\)

\(Rf\_B := \Delta R \cdot \frac{Av + 1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} = 551.4719\) Feedback for Tuned B

\(\alpha_b := 0.3\) Splitting ratio
\[
Lc := \frac{\left( \frac{Rf_B}{1 + Av} \right)^2 \cdot C_T}{\Delta_L} = 1.571 \cdot 10^{-9}
\]

\[
C1 := (1 - \alpha_b) \cdot (C_T) = 1.05 \cdot 10^{-12}
\]

\[
C2 := \alpha_b \cdot (C_T) = 4.5 \cdot 10^{-13}
\]

\[
Z_{TLA,B}(\omega) := \frac{1}{\left( (1 - \omega^2 \cdot Lc \cdot C1) + \frac{Rf_B}{(1 + Av)} \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \right)}
\]

\[
filter(\omega) := \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right)
\]

**Receiver noise**

\[
Z_{tuned,B}(\omega) := filter(\omega) \quad \omega := 1 \cdot 10^6, 10 \cdot 10^6 \ldots 1 \cdot 10^{11}
\]

\[
NEB_B := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{10}} \left\{ \left( |Z_{tuned,B}(\omega)| \right)^2 \right\} d\omega \right) = 3.763 \cdot 10^9
\]

\[
I_{2,B} := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{10}} \left\{ \left( \left| (1 - \omega^2 \cdot Lc \cdot C1) \right| + \frac{Rf_B}{Lc \cdot C1 \cdot C2} \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \right) \right\} d\omega \right) = 3.236 \cdot 10^9
\]

\[
I_{3,B} := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^{10}} \left\{ \left( \frac{\left( (1 - \omega^2 \cdot Lc \cdot C1) \right) + \frac{Rf_B}{Lc \cdot C1 \cdot C2} \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \cdot (1i)}{Rf_B} \right) \right\}^2 d\omega \right) = 1.242 \cdot 10^6
\]
noise-opt BJT

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100 \]

\[ x_B(x) := \frac{4 \cdot k \cdot T \cdot I_2 \cdot B}{Rf \cdot B} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_2 \cdot B + 2 \cdot q \cdot \frac{x}{(25 \cdot 10^{-3})^2} \cdot I_3 \cdot B \]

\[ I_{opt \cdot B} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_3 \cdot B}{I_2 \cdot B}} = 0.005 \]

\[ \frac{0.005}{10^{-3}} = 6 \]

\[ I_{n \cdot B} := x_B(I_{opt \cdot B}) = 1.979 \cdot 10^{-13} \]

\[ x := 0.00001, 0.00002, 0.1 \]

\[
\begin{align*}
    I_1(t) & := \frac{T_s}{T} \cdot \int_{0}^{1/T \cdot 10^3} \exp\left(\frac{-\alpha^2 \cdot \omega^2}{2}\right) \cdot \text{Re}\left\{ \frac{1}{\text{exp}\left(\frac{-\alpha^2 \cdot \omega^2}{2}\right)} \cdot \text{exp}\left(1i \cdot \omega \cdot t\right) \right\} \, d\omega \\
    I_0(t) & := \frac{T_s}{T} \cdot \int_{0}^{1/T \cdot 10^3} \exp\left(\frac{-\alpha^2 \cdot \omega^2}{2}\right) \cdot \text{Re}\left\{ \frac{1}{\text{exp}\left(\frac{-\alpha^2 \cdot \omega^2}{2}\right)} \cdot \text{exp}\left(1i \cdot \omega \cdot t\right) \right\} \, d\omega
\end{align*}
\]
\[ t := 0.002 \cdot T_s \]
\[ t_{pk} := \text{root} \left( T_s \cdot I_1(t), t \right) \]
\[ \frac{t_{pk}}{T_s} = -1.616 \cdot 10^{-22} \]

**Voltages**

\[ v_o(t) := I_0(t) \]
\[ v_o(t_{pk}) = 7.527 \cdot 10^9 \]

\[ t := -5 \cdot T_s, -4.99 \cdot T_s, \ldots, 5 \cdot T_s \]
\[ t := -0.1 \cdot T_s \]

\[ t_d(v) := \text{root}(\text{temp}(v, t), t) \]

\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \]

\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \]

\[ \text{slope}(b, v) := b \cdot \eta q \cdot \frac{I_1(t_d(v))}{T_s} \]

\[ v_p(b) := b \cdot \eta q \cdot I_0(t_p(k)) \]

\[ \text{Q values} \]

\[ Q_r(b, v) := \frac{v_p(b) - v_d(b, v)}{\sqrt{\text{noise}}} \]

\[ Q_s(b, v) := \left( \frac{m \cdot T_n}{2 \cdot n} \right) \cdot \frac{\text{slope}(b, v)}{\sqrt{\text{noise}}} \]

\[ Q_t(b, v) := \frac{v_d(b, v)}{\sqrt{\text{noise}}} \]

\[ \text{Error probabilities} \]

\[ P_r(b, v) := 0.5 \cdot \text{erfc} \left( \frac{Q_r(b, v)}{\sqrt{2}} \right) \]

\[ P_s(b, v) := \text{erfc} \left( \frac{Q_s(b, v)}{\sqrt{2}} \right) \]

\[ P_f(b, v) := \frac{m \cdot T_n}{n \cdot \alpha} \cdot 0.5 \cdot \text{erfc} \left( \frac{Q_t(b, v)}{\sqrt{2}} \right) \]

\[ P_{cs}(b, v) := P_r(b, v) + P_s(b, v) + \frac{n - 1}{2} \cdot P_f(b, v) \]

\[ P_{cb}(b, v) := \frac{n}{2 \cdot (n - 1)} \cdot P_{cs}(b, v) \]

\[ b := 0.25 \cdot 10^4 \quad v := 0.5 \quad M = 3 \]

\[ Q_r(b, v) = 3.383 \quad P_r(b, v) = 3.58 \cdot 10^{-4} \]

\[ Q_s(b, v) = 14.092 \quad P_s(b, v) = 4.27 \cdot 10^{-45} \]
\[ Q_t(b, v) = 3.383 \quad P_f(b, v) = 0.004 \]

\[ P_{eb}(b, v) = 0.007 \]

\[ pc(b, v) := \log(P_{eb}(b, v)) + 9 \]

Set for 1 in \(10^9\) errors

Find the root to give \(10^9\)

\[ a(v) := \text{root}(pc(b, v), b) \]

Scan for Optimum threshold

\[ v := 0.2, 0.21, 0.22, \ldots, 0.7 \]

Photons per bit

\[ b_{(v - 0.2) \cdot 100} := a(v) \]

Search for minimum and store it

\[ \text{minimum} := \text{min}(b) \]

\[ \text{minimum} = 4.654 \cdot 10^3 \quad \frac{\text{minimum}}{M} = 1.551 \cdot 10^3 \]

Number of photons per bit verses threshold level (BER = \(10^{-9}\)).

\[
\begin{align*}
Q_t(b, v) &= 3.383 \\
P_f(b, v) &= 0.004 \\
P_{eb}(b, v) &= 0.007 \\
p_c(b, v) &= \log(P_{eb}(b, v)) + 9 \\
a(v) &= \text{root}(pc(b, v), b) \\
v &= 0.2, 0.21, 0.22, \ldots, 0.7 \\
b_{(v - 0.2) \cdot 100} &= a(v) \\
\text{minimum} &= \text{min}(b) \\
\text{minimum} &= 4.654 \cdot 10^3 \\
\frac{\text{minimum}}{M} &= 1.551 \cdot 10^3 \\
\end{align*}
\]

**Rx Sens**

\[ \text{result} := \text{minimum} \cdot \text{photon\_energy} \cdot \frac{\text{PCM\_B}}{M} \]

\[ dBm := 10 \cdot \log \left( \frac{\text{result}}{10^{-3}} \right) \]

\[ dBm = -37.01 \]

Sens compared to non-tuned for all “a” value

35.20 – 36.53 = -1.33

35.20 – 36.76 = -1.56

35.20 – 37.01 = -1.81

35.20 – 36.38 = -1.18

35.20 – 36.15 = -0.95

**Photons per pulse nontuned vs tuned**

\[ 7.059 \cdot 10^3 - 4.654 \cdot 10^3 = 2.405 \cdot 10^3 \]
Error check

\[ \omega := 1 \cdot 10^7, 10 \cdot 10^7 \ldots 5 \cdot 10^{11} \]

\[ |Z_{TIA, B}(\omega)| \]
\[ |\text{filter}(\omega)| \]
\[ |Z_{TIA, B}(\omega)| \cdot |\text{filter}(\omega)| \]

\[ v := 0.47, 0.48 \ldots 0.55 \]
\[ b := 4.5 \cdot 10^3, 4.6 \cdot 10^3 \ldots 5 \cdot 10^3 \]

\[ P_{es}(b, v) \]
Noise check - for all $\alpha_b$

$\Delta_L := 1.8 \quad \Delta_R := 1.58$ feedback $\Delta R$, time constant ratio $\Delta L$

$$R_{f_B} := \Delta_R \cdot \frac{Av + 1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} = 345.7641$$

$\alpha_b := 0.1$ splitting ratio

$$L_c := \frac{(R_{f_B} \cdot B)^2 \cdot C_T}{\Delta_L} = 8.234 \cdot 10^{-10}$$

$C1 := (1 - \alpha_b) \cdot (C_T) = 1.35 \cdot 10^{-12}$

$C2 := \alpha_b \cdot (C_T) = 1.5 \cdot 10^{-13}$

$$Lc1 := \frac{\left(\frac{1.58}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}\right)^2 \cdot C_T}{1.8} = 8.234 \cdot 10^{-10}$$

$C11 := (1 - 0.1) \cdot (C_T) = 1.35 \cdot 10^{-12}$

$$Lc2 := \frac{\left(\frac{1.87}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}\right)^2 \cdot C_T}{1.8} = 1.153 \cdot 10^{-9}$$

$C12 := (1 - 0.2) \cdot (C_T) = 1.2 \cdot 10^{-12}$

$$Lc3 := \frac{\left(\frac{2.52}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}\right)^2 \cdot C_T}{2.4} = 1.571 \cdot 10^{-9}$$

$C13 := (1 - 0.3) \cdot (C_T) = 1.05 \cdot 10^{-12}$

$$Lc4 := \frac{\left(\frac{2.75}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}\right)^2 \cdot C_T}{1.9} = 2.363 \cdot 10^{-9}$$

$C14 := (1 - 0.4) \cdot (C_T) = 9 \cdot 10^{-13}$

$$Lc5 := \frac{\left(\frac{2.65}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}\right)^2 \cdot C_T}{1.5} = 2.779 \cdot 10^{-9}$$

$C15 := (1 - 0.5) \cdot (C_T) = 7.5 \cdot 10^{-13}$
Noise check - IG/RF TF

\[ Z_{\text{In.D.0.1}}(\omega) = 2 \cdot q \cdot \frac{I_{\text{opt.B}}}{(25 \cdot 10^{-3})^2} \left\{ (1 - \omega^2 \cdot Lc1 \cdot C11) \cdot \frac{1 + Av}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \cdot (\omega) \right\} \]

\[ \cdot \left( C_T - (\omega)^2 \cdot Lc1 \cdot C11 \cdot (C_T - C11) \right) \cdot \frac{1.58 \cdot \frac{1 + Av}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}}{1 + Av} \cdot Z_{\text{tuned.B}}(\omega) \]

\[ Z_{\text{In.D.0.2}}(\omega) = 2 \cdot q \cdot \frac{I_{\text{opt.B}}}{(25 \cdot 10^{-3})^2} \left\{ (1 - \omega^2 \cdot Lc2 \cdot C12) \cdot \frac{1 + Av}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \cdot (\omega) \right\} \]

\[ \cdot \left( C_T - (\omega)^2 \cdot Lc2 \cdot C12 \cdot (C_T - C12) \right) \cdot \frac{1.87 \cdot \frac{1 + Av}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}}{1 + Av} \cdot Z_{\text{tuned.B}}(\omega) \]
\[ Z_{\text{In}_{\text{D},0.3}}(\omega) = 2 \cdot q \cdot \frac{I_{\text{opt_B}}}{(I_{\text{opt_B}})^2} \left[ \left(1 - \omega^2 \cdot L_{C3} \cdot C_{13}\right) + 2.52 \cdot \frac{1 + Av}{1 + Av} \cdot \frac{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right]^2 \cdot Z_{\text{tuned_B}}(\omega) \]

\[ Z_{\text{In}_{\text{D},0.4}}(\omega) = 2 \cdot q \cdot \frac{I_{\text{opt_B}}}{(I_{\text{opt_B}})^2} \left[ \left(1 - \omega^2 \cdot L_{C4} \cdot C_{14}\right) + 2.75 \cdot \frac{1 + Av}{1 + Av} \cdot \frac{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right]^2 \cdot Z_{\text{tuned_B}}(\omega) \]

\[ Z_{\text{In}_{\text{D},0.5}}(\omega) = 2 \cdot q \cdot \frac{I_{\text{opt_B}}}{(I_{\text{opt_B}})^2} \left[ \left(1 - \omega^2 \cdot L_{C5} \cdot C_{15}\right) + 2.65 \cdot \frac{1 + Av}{1 + Av} \cdot \frac{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right]^2 \cdot Z_{\text{tuned_B}}(\omega) \]
Noise check - Total O/N over RT

\[ V_1(\omega) := \left( \frac{4kT}{1+Av} \frac{1 + Av}{1 + Av} + 2q \frac{I_{opt,B}}{hfe} \right)^2 \cdot \left( \frac{1}{1 - \frac{1}{T_s}} \cdot C_T \right) \cdot \left( 1 - \omega^2 \right) \cdot C_{tuned,B}(\omega) \]

\[ V_2(\omega) := \left( \frac{4kT}{1+Av} \frac{1 + Av}{1 + Av} + 2q \frac{I_{opt,B}}{hfe} \right)^2 \cdot \left( \frac{1}{1 - \frac{1}{T_s}} \cdot C_T \right) \cdot \left( 1 - \omega^2 \right) \cdot C_{tuned,B}(\omega) \]
\[V_3(\omega) := \left(\frac{4 \cdot k \cdot T}{2.52 \cdot \frac{1 + A_v}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}} + 2 \cdot q \cdot \frac{I_{opt_B}}{hfe}\right)^d \cdot \left(\frac{\left(1 - \omega^2 \cdot Lc3 \cdot C13\right) \cdot Z_{\text{tuned_B}}(\omega)}{2.52 \cdot \frac{1 + A_v}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \cdot \left(\frac{I_{opt_B}}{(25 \cdot 10^{-3})}\right)^2} + 1\right)^d \]

\[V_4(\omega) := \left(\frac{4 \cdot k \cdot T}{2.75 \cdot \frac{1 + A_v}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}} + 2 \cdot q \cdot \frac{I_{opt_B}}{hfe}\right)^d \cdot \left(\frac{\left(1 - \omega^2 \cdot Lc4 \cdot C14\right) \cdot Z_{\text{tuned_B}}(\omega)}{2.75 \cdot \frac{1 + A_v}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \cdot \left(\frac{I_{opt_B}}{(25 \cdot 10^{-3})}\right)^2} + 1\right)^d \]
\[ V_5(\omega) := \left( \frac{4 \cdot k \cdot T}{2.65 \cdot \frac{1 + Av}{1 + Av} \cdot \frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T}} + 2 \cdot q \cdot \frac{I_{opt_B}}{hfe} \right)_d \]

\[ \cdot \left| \left( 1 - \omega^2 \cdot Lc5 \cdot C_{15} \right) \cdot Z_{tuned_B}(\omega) \right|^2 + 1 \]

\[ \cdot 2 \cdot q \cdot \frac{I_{opt_B}}{\left( \frac{I_{opt_B}}{(25 \cdot 10^{-3})} \right)^2} \cdot \left| \left( \frac{(1 - \omega^2 \cdot Lc5 \cdot C_{15})}{d} + 2.65 \cdot \frac{1 + Av}{1 + Av} \cdot \frac{1}{2 \cdot \pi \cdot 2 \cdot \frac{1}{T_s} \cdot C_T} \right) \cdot \left( (C_T - (\omega^2 \cdot Lc5 \cdot C_{15} \cdot (C_T - C_{15})) \cdot 1 \right) \right|_d \]

\[ \cdot Z_{tuned_B}(\omega) \]
C.4 PPM receiver with Butterworth filter (Optical fibre)

C.4.1 Non-tuned PPM receiver with Butterworth filter (PIN-BJT, $f_n = 5$)

PPM (PIN-BJT Butterworth filter 1 Gbit/s)

PPM Rx performance with PIN-BJT input configuration, Butterworth pre-detection filter. Calculations are as follows:

- PPM terms
- Rx terms (noise+TF)
- Pulse shaping, voltages
- Error bit rate
- Optimum threshold/minimum number of photons

**Bit-rate, pulse duration, and input pulse**

\[ PCM_B := 1 \cdot 10^9 \]

\[ M := 3 \]

\[ T_n := \frac{M}{PCM_B} \]

Frame time - DPPM

\[ m \equiv 0.8 \]

Modulation depth - DPPM

\[ n := 2^M \]

Number of DPPM active slots

\[ T_s := \frac{m \cdot T_n}{n} = 3 \cdot 10^{-10} \]

Slot width

\[ \eta q := 1.6 \cdot 10^{-10} \]

Quantum energy

\[ \lambda := 1.55 \cdot 10^{-6} \]

Wavelength of operation

\[ photon\_energy := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ T_b := \frac{1}{PCM_B} \]

This is the PCM bit time

\[ f_n := 5 \]

\[ \alpha := \frac{0.1874 \cdot T_b}{f_n} = 3.748 \cdot 10^{-11} \]

**Preamplifier terms**

\[ \omega_c := 2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} = 1.047 \cdot 10^{10} \]

\[ A v := 10 \]

\[ C_T := 1.5 \cdot 10^{-12} \]

\[ Z_{TIA,N}(\omega) := \frac{1}{1 + j \frac{\omega}{\omega_c}} \]

\[ \omega_B := 2 \cdot \pi \cdot 0.7 \cdot \frac{1}{T_s} \]

\[ filter(\omega) := \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \]
\[ Rf_N := \frac{Av + 1}{2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \cdot C_T} = 700.282 \]

\[ \omega := 1 \cdot 10^6, 10 \cdot 10^6 .. 1 \cdot 10^{11} \]

\[ Z_{\text{nontuned}}(\omega) = Z_{\text{TIA,N}}(\omega) \cdot \text{filter}(\omega) \]

\[ t_R := \frac{8}{\omega_B} \quad \frac{t_R}{T_s} = 1.819 \]

\[ NEB_N := \frac{1}{2 \cdot \pi} \int_0^{10^3} \left( \left| Z_{\text{nontuned}}(\omega) \right| \right)^2 d\omega = 1.574 \cdot 10^9 \]

\[ I_{2,N} := NEB_N = 1.574 \cdot 10^9 \]

\[ I_{3,N} := \frac{1}{2 \cdot \pi} \int_0^{10^3} \left( \left| 1 + j \cdot (\omega) \cdot \frac{Rf_N \cdot C_T}{1 \cdot Rf_N} \cdot Z_{\text{nontuned}}(\omega) \right| \right)^2 d\omega = 2.178 \cdot 10^5 \]

**noise-opt BJT**

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100 \]

\[ x_N(x) := \left( \frac{4 \cdot k \cdot T}{Rf_N} \cdot I_{2,N} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{2,N} + 2 \cdot q \cdot \frac{x}{(25 \cdot 10^{-3})^2} \cdot I_{3,N} \right) \]

\[ I_{\text{opt,N}} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3,N}}{I_{2,N}}} = 0.003 \]

\[ \frac{0.006}{10^{-3}} = 6 \]

\[ I_{n,N} := x_N(I_{\text{opt,N}}) = 6.659 \cdot 10^{-14} \]

\[ x := 0.00001, 0.00002 .. 0.1 \]
Pulse shape

\[ I_1(t) := \frac{T_s}{\pi} \cdot \frac{1}{T_0 \cdot 10^2} \left[ \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \right] \left[ \text{Re} \left( \frac{1}{\frac{1+j \cdot \omega}{\omega_c}} \right) \right] \left( \frac{\omega_B^3}{(1+j \cdot \omega)^3 + 2 \cdot (1+j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1+j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \right) \exp (1i \cdot \omega \cdot (t)) \right) \] \]

\[ I_0(t) := \frac{1}{\pi} \cdot \frac{1}{T_0 \cdot 10^2} \left[ \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \right] \left[ \text{Re} \left( \frac{1}{\frac{1+j \cdot \omega}{\omega_c}} \right) \right] \left( \frac{\omega_B^3}{(1+j \cdot \omega)^3 + 2 \cdot (1+j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1+j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \right) \exp (1i \cdot \omega \cdot (t)) \right) \]

t := 0.5 \cdot T_s
\[ t_{pk} := \text{root} \left( T_s \cdot I_1(t), t \right) \]
\[ \frac{t_{pk}}{T_s} = 0.715 \]
\[ t := -5 \cdot T_s, -4.99 \cdot T_s, 6 \cdot T_s \]
\[ v_0(t) := I_0(t) \]
\[ v_0(t_{pk}) = 3.988 \cdot 10^9 \]

Voltages

\[ \frac{I_0(t)}{I_0(t) + I_0(t - (1-m) \cdot T_s)} \]

\[ \frac{t}{T_s} \]
\[ I_1(t) \]

\[ v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \]

\[ \text{temp}(v, t) := \frac{I_0(t)}{v \cdot I_0(t_{pk})} - 1 \]

\[ t := 0.37 \cdot T_s \]

\[ t_d(v) := \text{root}(\text{temp}(v, t), t) \]

\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_{d(v)}) \]

\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_{d(v)}) \]

\[ \text{slope}(b, v) := b \cdot \eta q \cdot \frac{I_1(t_{d(v)})}{T_s} \]

\[ v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \]

**Q values**

\[ Q_r(b, v) := \frac{v_{pk}(b) - v_d(b, v)}{\sqrt{\text{noise}}} \]

\[ Q_s(b, v) := \left( \frac{m \cdot T_n}{2 \cdot n} \right) \cdot \frac{slope(b, v)}{\sqrt{\text{noise}}} \]

\[ Q_t(b, v) := \frac{v_d(b, v)}{\sqrt{\text{noise}}} \]

**Error probabilities**

\[ P_r(b, v) := 0.5 \cdot \text{erfc} \left( \frac{Q_r(b, v)}{\sqrt{2}} \right) \]

\[ P_s(b, v) := \text{erfc} \left( \frac{Q_s(b, v)}{\sqrt{2}} \right) \]
\[ P_f(b, v) := \frac{m \cdot T_n}{n \cdot t_R} \cdot 0.5 \cdot \text{erfc} \left( \frac{Q_l(b, v)}{\sqrt{2}} \right) \]

\[ P_{es}(b, v) := P_r(b, v) + P_s(b, v) + \frac{n-1}{2} \cdot P_f(b, v) \]

\[ P_{eb}(b, v) := \frac{n}{2 \cdot (n-1)} \cdot P_{es}(b, v) \]

\[ b := 0.25 \cdot 10^4 \]

\[ pc(b, v) := \log(P_{eb}(b, v)) + 9 \]

\[ a(v) := \text{root}(pc(b, v), b) \]

Set for 1 in \(10^9\) errors

Find the root to give 1 in \(10^9\)

\[ v := 0.2, 0.21, \ldots, 0.9 \]

Scan for Optimum threshold

\[ b_{(v \cdot 0.2) \cdot 100} := a(v) \]

Photons per bit

Set for minimum and store it

\[ \text{minimum} := \min(b) \]

Search for minimum and store it

\[ \text{minimum} = 4.918 \cdot 10^3 \]

\[ \text{result} := \text{minimum} \cdot \text{photon\_energy} \cdot \frac{PCM_B}{M} \]

\[ dBm := 10 \cdot \log \left( \frac{\text{result}}{10^{-3}} \right) \]

\[ dBm = -36.77 \]

\[ \frac{PCM_B}{M} = 3.333 \cdot 10^8 \]

\[ \omega := 1 \cdot 10^7, 10 \cdot 10^7, \ldots, 5 \cdot 10^{11} \]
\[ |Z_{TIA,N}(\omega)| \]
\[ |filter(\omega)| \]
\[ |Z_{TIA,N}(\omega)| \cdot |filter(\omega)| \]

\[ \omega \]

\[ v := 0.45, 0.46 \ldots 0.55 \quad b := 4.87 \cdot 10^3, 4.88 \cdot 10^3 \ldots 4.93 \cdot 10^3 \]
C.4.2 Tuned B PPM receiver with Butterworth filter (PIN-BJT/f_n = 5)

PPM (PIN-BJT Tuned/Butterworth 1 Gbit/s)

PPM Rx performance with PIN-BJT input configuration, Butterworth. calculations are as follow:
- PPM terms
- Rx terms (noise+TF)
- Pulse shaping, voltages
- Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

**Bit-rate, pulse duration, and input pulse**

\[ PCM_B := 1 \cdot 10^9 \]

\[ M := 3 \]

\[ T_n := \frac{M}{PCM_B} \quad \text{Frame time - DPPM} \]

\[ m := 0.8 \quad \text{Modulation depth - DPPM} \]

\[ n := 2^M \quad \text{Number of DPPM active slots} \]

\[ T_s := \frac{m \cdot T_n}{n} = 3 \cdot 10^{-10} \quad \text{Slot width} \]

\[ \eta q := 1.6 \cdot 10^{-10} \quad \text{Quantum energy} \]

\[ \lambda := 1.55 \cdot 10^{-6} \quad \text{wavelength of operation} \]

\[ \text{photon}_\text{energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ T_b := \frac{1}{PCM_B} \quad \text{This is the PCM bit time} \]

\[ f_n := 5 \]

\[ \alpha := \frac{0.1874 \cdot T_b}{f_n} = 3.748 \cdot 10^{-11} \]

**Preamplifier terms**

\[ A v := 10 \quad \text{total C} \]

Feedback value for (R) Tuned B

\[ \Delta L := 2.4 \quad \Delta R := 2.52 \quad \text{feedback } \Delta R \text{, time constant ratio } \Delta L \]

\[ R f_B := \Delta R \cdot \frac{A v + 1}{2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \cdot C_T} = 1.7647 \cdot 10^3 \quad \text{feedback for Tuned B} \]

\[ \alpha_b := 0.3 \quad \text{splitting ratio} \]
\[ L_c := \frac{\left( \frac{R_f \cdot B}{1+Av} \right)^2 \cdot C_T}{\Delta_L} = 1.609 \cdot 10^{-8} \]

\[ C_1 := (1-\alpha_b) \cdot (C_T) = 1.05 \cdot 10^{-12} \]

\[ C_2 := \alpha_b \cdot (C_T) = 4.5 \cdot 10^{-13} \]

\[ Z_{TIA,B}(\omega) := \frac{1}{\left( (1-\omega^2 \cdot L_c \cdot C_1) + \frac{R_f \cdot B}{(1+Av)} \cdot (\omega \cdot 1i) \cdot (C_1+C_2-(\omega)^2 \cdot L_c \cdot C_1 \cdot C_2) \right)} \]

\[ \omega_B := 2 \cdot \pi \cdot 0.7 \cdot \frac{1}{T_s} \]

\[ \text{filter}(\omega) := \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \]

**Receiver noise**

\[ Z_{\text{tuned} \cdot B}(\omega) := Z_{TIA,B}(\omega) \cdot \text{filter}(\omega) \]

\[ NEB_B := \frac{1}{2 \cdot \pi} \int_{0}^{10^{12}} \left( \left| Z_{\text{tuned} \cdot B}(\omega) \right|^2 \right) d\omega = 1.373 \cdot 10^9 \]

\[ I_{2,B} := \frac{1}{2 \cdot \pi} \int_{0}^{10^{12}} \left( \left| \left( 1 - \omega^2 \cdot L_c \cdot C_1 \right) + \frac{R_f \cdot B}{(1+Av)} \cdot (C_1+C_2-(\omega)^2 \cdot L_c \cdot C_1 \cdot C_2) \cdot 1i \right| \right) d\omega = 1.225 \cdot 10^9 \]

\[ I_{3,B} := \frac{1}{2 \cdot \pi} \int_{0}^{10^{12}} \left( \left| \frac{\left( (1-\omega^2 \cdot L_c \cdot C_1) + \frac{R_f \cdot B}{(1+Av)} \cdot (C_1+C_2-(\omega)^2 \cdot L_c \cdot C_1 \cdot C_2) \cdot 1i \right) \cdot Z_{\text{tuned} \cdot B}(\omega)}{R_f \cdot B} \right|^2 d\omega = 4.775 \cdot 10^4 \]

**noise-opt BJT**

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100 \]

\[ x_B(x) := \left( \frac{4 \cdot k \cdot T}{R_f \cdot B} \cdot I_{2,B} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{2,B} + 2 \cdot q \cdot \left( \frac{x}{(25 \cdot 10^{-3})^2} \right) \cdot I_{3,B} \right) \]

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\[ I_{\text{opt,B}} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3,B}}{I_{2,B}}} = 1.5612 \cdot 10^{-3} \]

\[ I_{n,B} := x_B I_{\text{opt,B}} = 2.365 \cdot 10^{-14} \]

\[ \text{noise} := I_{n,B} = 2.365 \cdot 10^{-14} \]

\[ 10 \log \left( \frac{I_{n,B}}{6.659 \cdot 10^{-14}} \right) = -4.496 \]

**Pulse shape**

\[ I_1(t) := \frac{T_s}{\pi} \cdot \left[ \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \right]_0^{1/T_s} \cdot \Re \left( \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2(1j \cdot \omega)^2 \cdot \omega_B + 2(1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \right)^{1/10} \cdot \left( \left( 1 - \omega^2 \cdot Lc \cdot C1 \right) \right)_{\omega_B^3}^{1/10} \cdot \exp \left( (1 - \omega^2 \cdot Lc \cdot C1) \cdot \left( 1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2 \right) \right) \cdot \exp \left( 1i \cdot \omega \cdot (t) \right) \]

\[ I_0(t) := \frac{1}{\pi} \cdot \left[ \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \right]_0^{1/T_s} \cdot \Re \left( \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2(1j \cdot \omega)^2 \cdot \omega_B + 2(1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \right)^{1/10} \cdot \left( \left( 1 - \omega^2 \cdot Lc \cdot C1 \right) \right)_{\omega_B^3}^{1/10} \cdot \exp \left( (1 - \omega^2 \cdot Lc \cdot C1) \cdot \left( 1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2 \right) \right) \cdot \exp \left( 1i \cdot \omega \cdot (t) \right) \]

\[ t := 1.1 \cdot T_s \]

\[ t_{pk} := \text{root} \left( T_s \cdot I_1(t), t \right) \]

\[ \frac{t_{pk}}{T_s} = 1.214 \]

**Voltages**

\[ v_o(t) := I_0(t) \quad v_o(t_{pk}) = 3.483 \cdot 10^9 \]

\[ t := -5 \cdot T_s, -4.99 \cdot T_s, 5 \cdot T_s \]
\[ v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \]
\[ \text{temp}(v, t) := \frac{I_0(t)}{v \cdot I_0(t_{pk})} - 1 \]
\[ t = 0.7 \cdot T_s \]
\[ t_d(v) := \text{root}(\text{temp}(v, t), t) \]
\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \]
\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \]
\[ \text{slope}(b, v) := b \cdot \eta q \cdot \frac{I_1(t_d(v))}{T_s} \]
\[ v_{pk}(b) = b \cdot \eta q \cdot I_0(t_{pk}) \]

**Q values**

\[ Q_r(b, v) = \frac{v_{pk}(b) - v_d(b, v)}{\sqrt{\text{noise}}} \]

\[ Q_s(b, v) = \left( \frac{m \cdot T_n}{2 \cdot n} \right) \cdot \frac{\text{slope}(b, v)}{\sqrt{\text{noise}}} \]

\[ Q_t(b, v) = \frac{v_d(b, v)}{\sqrt{\text{noise}}} \]

**Error probabilities**

\[ P_r(b, v) = 0.5 \cdot \text{erfc} \left( \frac{Q_r(b, v)}{\sqrt{2}} \right) \]

\[ P_s(b, v) = \text{erfc} \left( \frac{Q_s(b, v)}{\sqrt{2}} \right) \]

\[ P_f(b, v) = \frac{m \cdot T_n}{8} \cdot 0.5 \cdot \text{erfc} \left( \frac{Q_t(b, v)}{\sqrt{2}} \right) \]

\[ P_{es}(b, v) = P_r(b, v) + P_s(b, v) + \frac{n-1}{2} \cdot P_f(b, v) \]

\[ P_{eb}(b, v) = \frac{n}{2 \cdot (n-1)} \cdot P_{es}(b, v) \]

\[ v = 0.5 \quad M = 3 \]

\[ b = 0.25 \cdot 10^4 \]

\[ P_r(b, v) = 2.947 \cdot 10^{-6} \]

\[ P_s(b, v) = 5.302 \cdot 10^{-13} \]

\[ P_f(b, v) = 1.62 \cdot 10^{-6} \]

\[ P_{eb}(b, v) = 4.925 \cdot 10^{-6} \]

\[ pc(b, v) = \log (P_{eb}(b, v)) + 9 \]

\[ a(v) = \text{root}(pc(b, v), b) \]

\[ v = 0.2, 0.21, 0.22, 0.23, 0.24, 0.25, 0.26, 0.27, 0.28, 0.29, 0.30, 0.31, 0.32, \ldots, 0.7 \]

\[ a(v) = b \quad (v - 0.2) \cdot 100 \]

Set for 1 in 10^9 errors

Find the root to give 1 in 10^9

Scan for Optimum threshold

Photons per bit
minimum := \min(b)

minimum = 3.356 \cdot 10^3

\frac{minimum}{M} = 1.119 \cdot 10^3

Number of photons per bit verses threshold level (BER = 10^{-3}).

Rx Sens

result := minimum \cdot photon\_energy \cdot \frac{PCM\_B}{M}

dBm := 10 \cdot \log\left(\frac{\text{result}}{10^{-3}}\right)

dBm = -38.43

Error check

\omega := 1 \cdot 10^7, 10 \cdot 10^7 .. 5 \cdot 10^{11}
\( v := 0.47, 0.48 \ldots 0.55 \quad b := 3.2 \cdot 10^3, 3.25 \cdot 10^3 \ldots 3.4 \cdot 10^3 \)

Sens compared to non-tuned for all “a” values

\[
\begin{align*}
36.77 - 37.99 &= -1.22 \\
36.77 - 38.16 &= -1.39 \\
36.77 - 38.43 &= -1.66 \\
36.77 - 38.52 &= -1.75 \\
36.77 - 38.73 &= -1.96 \\
\end{align*}
\]

Photons per pulse non-tuned vs tuned (a = 0.5)

\[
4.918 \cdot 10^3 - 3.285 \cdot 10^3 = 1.633 \cdot 10^3
\]

#
C.4.3 Non-tuned PPM receiver with Butterworth filter (PIN-BJT/f_n = 5, filter 3-dB = 0.5 PPM line rate)

PPM Rx performance with PIN-BJT input configuration, Butterworth pre-detection filter. Calculations are as follow:

- PPM terms
- Rx terms (noise+TF)
- Pulse shaping, voltages
- Error bit rate
- Optimum threshold/minimum number of photons

**Bit-rate, pulse duration, and input pulse**

\[ PCM_B := 1 \times 10^9 \]
\[ M := 3 \]

\[ T_n := \frac{M}{PCM_B} \quad \text{Frame time - DPPM} \]
\[ m \equiv 0.8 \quad \text{Modulation depth - DPPM} \]
\[ n := 2^M \quad \text{Number of DPPM active slots} \]
\[ T_s := \frac{m \cdot T_n}{n} = 3 \times 10^{-10} \quad \text{Slot width} \]
\[ \eta q := 1.6 \times 10^{-10} \quad \text{Quantum energy} \]
\[ \lambda := 1.55 \times 10^{-6} \quad \text{Wavelength of operation} \]

\[ \text{photon\_energy} := \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda} \]

\[ T_b := \frac{1}{PCM_B} \quad \text{This is the PCM bit time} \]
\[ f_n := 5 \]
\[ \alpha := \frac{0.1874 \cdot T_b}{f_n} = 3.748 \times 10^{-11} \]

**Preamplifier terms**

\[ \omega_c := 2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} = 1.047 \times 10^{10} \quad A v := 10 \quad C_T := 1.5 \times 10^{-12} \quad \text{total C} \]

\[ Z_{TIA,N}(\omega) := \frac{1}{1 + 1j \frac{\omega}{\omega_c}} \]

\[ \omega_B := 2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \]

\[ \text{filter}(\omega) := \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \]
\[ R_{f,N} = \frac{A v + 1}{2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \cdot C_T} = 700.282 \quad \omega = 1 \cdot 10^6, 10 \cdot 10^6 \ldots 1 \cdot 10^{11} \]

\[ \tau_R = \frac{8}{\omega_B} \quad \frac{\tau_R}{T_s} = 2.546 \]

Receiver noise

\[ Z_{\text{nontuned}}(\omega) = Z_{\text{TIA, N}}(\omega) \cdot \text{filter}(\omega) \]

\[ \text{NEB}_N = \frac{1}{2 \cdot \pi} \int_0^{10^{12}} \left( \left| Z_{\text{nontuned}}(\omega) \right| \right)^2 \ d\omega = 1.309 \cdot 10^9 \]

\[ I_{2,N} = \text{NEB}_N = 1.309 \cdot 10^9 \]

\[ I_{3,N} = \frac{1}{2 \cdot \pi} \int_0^{10^{12}} \left( \left| \frac{1 + j \cdot \omega \cdot R_{f,N} \cdot C_T}{1 \cdot R_{f,N}} \cdot Z_{\text{nontuned}}(\omega) \right| \right)^2 \ d\omega = 1.103 \cdot 10^5 \]

noise-opt BJT

\[ q = 1.6 \cdot 10^{-19} \quad k = 1.38 \cdot 10^{-23} \quad T = 298 \quad hfe = 100 \]

\[ x_N(x) = \left( \frac{4 \cdot k \cdot T}{R_{f,N}} \cdot I_{2,N} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{2,N} + 2 \cdot q \cdot \frac{x}{\left( \frac{25 \cdot 10^{-3}}{25 \cdot 10^{-3}} \right)^2} \cdot I_{3,N} \right) \]

\[ I_{\text{opt},N} = 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3,N}}{I_{2,N}}} = 0.002 \quad \frac{0.006}{10^{-3}} = 6 \]

\[ I_{n,N} := x_N(I_{\text{opt},N}) = 4.998 \cdot 10^{-14} \]

\[ x := 0.00001, 0.00002 \ldots 0.1 \]

\[ \text{noise} := I_{n,N} = 4.998 \cdot 10^{-14} \]
Pulse shape

\[ I_1(t) := \frac{T_s}{\pi} \cdot \frac{1}{T_s} \cdot 10^5 \left[ \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \right] \left[ \text{Re} \left( 1i \cdot \omega \cdot \frac{1}{1 + i \frac{\omega}{\omega_c}} \right) \right] \left[ \exp \left( (1i \cdot \omega(t)) \right) \right] \left( (1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3 \right) \right. \]

\[ I_0(t) := \frac{1}{\pi} \cdot \frac{1}{T_s} \cdot 10^5 \left[ \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \right] \left[ \text{Re} \left( \frac{1}{1 + i \frac{\omega}{\omega_c}} \right) \right] \left[ \exp \left( (1i \cdot \omega(t)) \right) \right] \left( (1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3 \right) \right. \]

\[ t := 1 \cdot T_s \]
\[ t_{pk} := \text{root} (T_s \cdot I_1(t), t) \]
\[ \frac{t_{pk}}{T_s} = 0.932 \]
\[ t := -5 \cdot T_s - 4.99 \cdot T_s \cdot 6 \cdot T_s \]
\[ v_o(t) := I_0(t) \]
\[ v_o(t_{pk}) = 3.269 \cdot 10^9 \]

Voltages

\[ I_0(t) \]
\[ I_0(t) + I_0(t - (1 - m) \cdot T_s) \]
\[ v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \]
\[ temp(v, t) := \frac{I_0(t)}{v \cdot I_0(t_{pk})} - 1 \]
\[ t := 0.4 \cdot T_s \]
\[ t_d(v) := \text{root}(temp(v, t), t) \]
\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \]
\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \]
\[ \text{slope}(b, v) := b \cdot \eta q \cdot \frac{I_1(t_d(v))}{T_s} \]
\[ v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \]

**Q values**

\[ Q_r(b, v) := \frac{v_{pk}(b) - v_d(b, v)}{\sqrt{\text{noise}}} \]
\[ Q_s(b, v) := \left( \frac{m \cdot T_n}{2 \cdot n} \right) \cdot \frac{\text{slope}(b, v)}{\sqrt{\text{noise}}} \]
\[ Q_t(b, v) := \frac{v_d(b, v)}{\sqrt{\text{noise}}} \]

**Error probabilities**

\[ P_r(b, v) := 0.5 \cdot \text{erfc}\left(\frac{Q_r(b, v)}{\sqrt{2}}\right) \]
\[ P_s(b, v) := \text{erfc}\left(\frac{Q_s(b, v)}{\sqrt{2}}\right) \]
\[ P_f(b, v) := \frac{m \cdot T_n \cdot 0.5 \cdot \text{erfc} \left( \frac{Q_i(b, v)}{\sqrt{2}} \right)}{n \cdot tR} \]

\[ P_{es}(b, v) := P_r(b, v) + P_s(b, v) + \frac{n-1}{2} \cdot P_f(b, v) \]

\[ P_{eb}(b, v) := \frac{n}{2 \cdot (n-1)} \cdot P_{es}(b, v) \]

\[ b := 0.25 \cdot 10^4 \]

\[ pc(b, v) := \log \left( P_{eb}(b, v) \right) + 9 \]

\[ a(v) := \text{root}(pc(b, v), b) \]

Set for 1 in \(10^9\) errors

Find the root to give 1 in \(10^9\)

\[ v := 0.2, 0.21, \ldots, 0.9 \]

Scan for Optimum threshold

\[ b_{(v-0.2) \cdot 100} := a(v) \]

Photons per bit

\[ \text{minimum} := \text{min}(b) \]

Search for minimum and store it

\[ \text{minimum} = 5.17 \cdot 10^3 \]

\[ \text{result} := \text{minimum} \cdot \text{photon}\_\text{energy} \cdot \frac{PCM_B}{M} \]

\[ dBm := 10 \cdot \log \left( \frac{\text{result}}{10^{-3}} \right) \]

\[ dBm = -36.553 \]

\[ \frac{PCM_B}{M} = 3.333 \cdot 10^8 \]

\[ \omega := 1 \cdot 10^7, 10 \cdot 10^7 \ldots 5 \cdot 10^{11} \]
\[ |Z_{TIA_N}(\omega)| \]
\[ |filter(\omega)| \]
\[ |Z_{TIA_N}(\omega)| \cdot |filter(\omega)| \]

\[ \omega \]

\[ v := 0.45, 0.46 \ldots 0.55 \quad b := 5.14 \cdot 10^3, 5.15 \cdot 10^3 \ldots 5.2 \cdot 10^3 \]

#
C.4.4 Tuned B PPM receiver with Butterworth filter (PIN-BJT/f_n = 5, filter 3-dB = 0.5
PPM line rate)

PPM Rx performance with PIN-BJT input configuration, Butterworth filter. calculations are as follow
- PPM terms
- Rx terms (noise+TF)
- Pulse shaping, voltages
- Error bit rate
- Optimum threshold/minimum number of photons
- receiver sensitivity

Bit-rate, pulse duration, and input pulse

\[ PCM_B := 1 \cdot 10^9 \]

\[ M := 3 \]

\[ T_n := \frac{M}{PCM_B} \]

Frame time - DPPM

\[ m := 0.8 \]

Modulation depth - DPPM

\[ n := 2^M \]

Number of DPPM active slots

\[ T_s := \frac{m \cdot T_n}{n} = 3 \cdot 10^{-10} \]

Slot width

\[ \eta q := 1.6 \cdot 10^{-10} \]

Quantum energy

\[ \lambda := 1.55 \cdot 10^{-6} \]

Wavelength of operation

\[ photon\_energy := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ T_b := \frac{1}{PCM_B} \]

This is the PCM bit time

\[ f_n := 5 \]

\[ \alpha := \frac{0.1874 \cdot T_b}{f_n} = 3.748 \cdot 10^{-11} \]

Preamplifier terms

\[ Av := 10 \quad C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]

Feedback value for (R) Tuned B

\[ \Delta L := 2.4 \quad \Delta R := 2.52 \quad \text{feedback} \Delta R, \text{ time constant ratio} \ \Delta L \]

\[ Rf_B := \Delta R \cdot \frac{Av + 1}{2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \cdot C_T} = 1.7647 \cdot 10^3 \quad \text{feedback for Tuned B} \]

\[ \alpha_b := 0.3 \quad \text{splitting ratio} \]

C-129
\[
L_c := \frac{\left(\frac{R_f \cdot B}{1 + A v}\right)^2 \cdot C_T}{\Delta_L} = 1.609 \cdot 10^{-8}
\]

\[C_1 := (1 - \alpha_b) \cdot (C_T) = 1.05 \cdot 10^{-12}\]

\[C_2 := \alpha_b \cdot (C_T) = 4.5 \cdot 10^{-13}\]

\[
Z_{TIA, B}(\omega) := \frac{1}{\left(\left(1 - \omega^2 \cdot L_c \cdot C_1\right) + \frac{R_f \cdot B}{1 + A v} \cdot (\omega \cdot 1i) \cdot \left(C_1 + C_2 - (\omega)^2 \cdot L_c \cdot C_1 \cdot C_2\right)\right)}
\]

\[
\omega_B := 2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s}
\]

\[
\text{filter}(\omega) := \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3}
\]

**Receiver noise**

\[
Z_{tuned, B}(\omega) := Z_{TIA, B}(\omega) \cdot \text{filter}(\omega)
\]

\[
NEB_B := \frac{1}{2 \cdot \pi} \cdot \left(\int_0^{10^{12}} \left(\left|Z_{tuned, B}(\omega)\right|^2\right) d\omega\right) = 1.191 \cdot 10^9
\]

\[
I_{2, B} := \frac{1}{2 \cdot \pi} \cdot \left(\int_0^{10^{12}} \left(\left|\left((1 - \omega^2 \cdot L_c \cdot C_1) \cdot \frac{1}{Z_{tuned, B}(\omega)}\right)^2\right| d\omega\right) = 7.775 \cdot 10^8
\]

\[
I_{3, B} := \frac{1}{2 \cdot \pi} \cdot \left(\int_0^{10^{12}} \left(\left|\frac{\left(1 - \omega^2 \cdot L_c \cdot C_1\right) \cdot \left(\frac{R_f \cdot B}{C_1 + C_2 - (\omega)^2 \cdot L_c \cdot C_1 \cdot C_2} \cdot 1i\right)}{\left(Z_{tuned, B}(\omega)\right)}\right)^2 d\omega\right) = 3.758 \cdot 10^4
\]

**noise-opt BJT**

\[
q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100
\]

\[
x_B(x) := \left(\frac{4 \cdot k \cdot T}{R_f \cdot B} \cdot I_{2, B} + \left(2 \cdot q \cdot \frac{x}{hfe}\right) \cdot I_{2, B} + 2 \cdot q \cdot \frac{x}{\left(25 \cdot 10^{-3}\right)^2} \cdot I_{3, B}\right)
\]

C-130
\[ I_{\text{opt}_B} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3\_B}}{I_{2\_B}}} = 1.7444 \cdot 10^{-3} \]

\[ I_{n\_B} := x \cdot B (I_{\text{opt}_B}) = 1.593 \cdot 10^{-14} \]

\[ \text{noise} := I_{n\_B} = 1.593 \cdot 10^{-14} \quad 10 \log \left( \frac{I_{n\_B}}{6.659 \cdot 10^{-14}} \right) = -6.213 \]

**Pulse shape**

\[
I_1(t) := \frac{T_s}{\pi} \left[ \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \right. \\
\cdot \Re \left[ \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \\
\cdot \left( (1 - \omega^2 \cdot Lc \cdot C1) + \frac{Rf_B}{(1 + Av)} \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \right} \\
\left. \cdot \exp(1j \cdot \omega \cdot (t)) \right] \right]_{0}^{T_s \cdot 10^5}
\]

\[
I_0(t) := \frac{1}{\pi} \left[ \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \right. \\
\cdot \Re \left[ \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \\
\cdot \left( (1 - \omega^2 \cdot Lc \cdot C1) + \frac{Rf_B}{(1 + Av)} \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \right} \\
\left. \cdot \exp(1j \cdot \omega \cdot (t)) \right] \right]_{0}^{T_s \cdot 10^5}
\]

\[ t := 1.3 \cdot T_s \]

\[ t_{pk} := \text{root}(T_s \cdot I_1(t), t) \]

\[ \frac{t_{pk}}{T_s} = 1.421 \]

**Voltages**

\[ v_o(t) := I_0(t) \]

\[ v_o(t_{pk}) = 3.052 \cdot 10^9 \]

\[ t := -5 \cdot T_s, -4.99 \cdot T_s, \ldots, 5 \cdot T_s \]

C-131
\[ v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \]

\[ \text{temp}(v, t) := \frac{I_0(t)}{v \cdot I_0(t_{pk})} - 1 \]

\[ t := 0.7 \cdot T_s \]

\[ t_d(v) := \text{root}(\text{temp}(v, t), t) \]

\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \]

\[ v_d(b, v) := b \cdot \eta q \cdot I_0(t_d(v)) \]

\[ \text{slope}(b, v) := b \cdot \eta q \cdot \frac{I_1(t_d(v))}{T_s} \]
\( v_{pk}(b) := b \cdot \eta q \cdot I_0(t_{pk}) \)

**Q values**

\( Q_r(b, v) := \frac{v_{pk}(b) - v_d(b, v)}{\sqrt{\text{noise}}} \)

\( Q_s(b, v) := \left( \frac{m \cdot T_n}{2 \cdot n} \right) \cdot \frac{slope(b, v)}{\sqrt{\text{noise}}} \)

\( Q_t(b, v) := \frac{v_d(b, v)}{\sqrt{\text{noise}}} \)

**Error probabilities**

\( P_r(b, v) := 0.5 \cdot \text{erfc}\left( \frac{Q_r(b, v)}{\sqrt{2}} \right) \)

\( P_s(b, v) := \text{erfc}\left( \frac{Q_s(b, v)}{\sqrt{2}} \right) \)

\( P_f(b, v) := \frac{m \cdot T_n}{8} \cdot 0.5 \cdot \text{erfc}\left( \frac{Q_t(b, v)}{\sqrt{2}} \right) \)

\( P_{es}(b, v) := P_r(b, v) + P_s(b, v) + \frac{n-1}{2} \cdot P_f(b, v) \)

\( P_{eb}(b, v) := \frac{n}{2 \cdot (n-1)} \cdot P_{es}(b, v) \)

\[ b := 0.25 \cdot 10^4 \quad v := 0.5 \quad M = 3 \]

\( Q_r(b, v) = 4.837 \quad P_r(b, v) = 6.593 \cdot 10^{-7} \)

\( Q_s(b, v) = 6.71 \quad P_s(b, v) = 1.945 \cdot 10^{-11} \)

\( Q_t(b, v) = 4.837 \quad P_f(b, v) = 2.589 \cdot 10^{-7} \)

\( P_{eb}(b, v) = 8.946 \cdot 10^{-7} \)

\( pc(b, v) := \log\left( P_{eb}(b, v) \right) + 9 \) \quad Set for 1 in \( 10^9 \) errors

\( a(v) := \text{root}(pc(b, v), b) \) \quad Find the root to give 1 in \( 10^9 \)

\( v := 0.2, 0.21, 0.7 \) \quad Scan for Optimum threshold

\( b \quad (v - 0.2) \cdot 100 := a(v) \) \quad Photons per bit
minimum := min(b)  
Search for minimum and store it

\[
\text{minimum} = 3.126 \cdot 10^3
\]
\[
\frac{\text{minimum}}{M} = 1.042 \cdot 10^3
\]

Number of photons per bit verses threshold level (BER = 10^{-9}).

\[a(v)\]

Rx Sens

\[\text{result} := \text{minimum} \cdot \text{photon\_energy} \cdot \frac{\text{PCM\_B}}{M}\]

\[dBm := 10 \cdot \log \left( \frac{\text{result}}{10^{-3}} \right)\]

\[dBm = -38.74\]

Error check

\[\omega := 1 \cdot 10^7, 10 \cdot 10^7 .. 5 \cdot 10^{11}\]
Sens compared to non-tuned for all “a” value
36.55 – 38.02 = -1.47
36.55 – 38.35 = -1.8
36.55 – 38.74 = -2.19
36.55 – 38.97 = -2.42
36.55 – 39.05 = -2.5
C.5 Di-code PPM receiver with Butterworth filter (Optical fibre)

C.5.1 Non-tuned PPM receiver with Butterworth filter (PIN-BJT/\(f_n = 5\))

Di-code PPM (PIN-BJT Butterworth 1 Gbit/s)

PPM Rx performance with PIN-BJT input configuration, Butterworth pre-detection filter. Calculations are as follows:
- Di-code PPM terms
- Rx terms (noise + TF)
- Noise optimisation
- Pulse shaping, voltages
- ISI/Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

\[ i := 0, 1 \ldots 100 \quad n := 10 \quad j := 0 \ldots 10 \quad \text{Set up the scan limits} \]
\[ v_i = v_{off} + \frac{i}{1000} \]
\[ x := 0 \ldots n \quad \text{This gives the row of the matrix} \quad y := 0 \ldots n \quad \text{This gives the column of the matrix} \]

\[ B := 1 \cdot 10^9 \quad \text{Bit rate} \]
\[ T_b := \frac{1}{B} = 1 \cdot 10^{-9} \quad \text{PCM bit time} \]
\[ T_s := \frac{T_b}{2 + gu} \quad \text{Slot time} \]
\[ n := 10 \quad \text{Number of like symbols in PCM} \]
\[ \eta q := 1.6 \cdot 10^{-10} \quad \text{Quantum energy} \]
\[ \lambda := 1.55 \cdot 10^{-6} \quad \text{This is the wavelength of operation} \]
\[ \text{photon energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]
\[ R_o := \frac{\eta q}{\text{photon energy}} \]
\[ PPM_B := \frac{1}{T_s} = 4 \cdot 10^9 \quad \text{this PPM bit rate, used for filter bandwidth} \]
\[ ts := \frac{1}{PPM_B} = 2.5 \cdot 10^{-10} \]
**Preamplifier terms**

\[ Av := 10 \quad C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]

\[ \omega_c := 2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} = 1.257 \cdot 10^{10} \]

\[ Z_{TIA,N}(\omega) := \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_c}} \]

\[ Rf_N := \frac{Av + 1}{2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \cdot C_T} = 583.568 \]

\[ \omega_B := 2 \cdot \pi \cdot 0.7 \cdot \frac{1}{T_s} \]

\[ \text{filter}(\omega) := \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \]

\[ Z_{\text{nontuned}}(\omega) := Z_{TIA,N}(\omega) \cdot \text{filter}(\omega) \]

**Receiver noise**

\[ NEB_N := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^3} \left[ \left( |Z_{\text{nontuned}}(\omega)| \right)^2 \right] \ d\omega \right) = 1.889 \cdot 10^9 \]

\[ I_{2,N} := NEB_N = 1.889 \cdot 10^9 \]

\[ I_{3,N} := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^3} \left( \left| \frac{1 + 1j \cdot (\omega) \cdot Rf_N \cdot C_T}{1 \cdot Rf_N} \cdot Z_{\text{nontuned}}(\omega) \right| \right)^2 \ d\omega \right) = 3.763 \cdot 10^5 \]

**noise-opt BJT**

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100 \]

\[ x_N(x) := \left( \frac{4 \cdot k \cdot T}{Rf_N} \cdot I_{2,N} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{2,N} + 2 \cdot q \cdot \frac{x}{\left( \frac{x}{25 \cdot 10^{-3}} \right)^2} \cdot I_{3,N} \right) \]

\[ I_{\text{opt,N}} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3,N}}{I_{2,N}}} = 0.004 \]

\[ I_{n,N} := x_N(I_{\text{opt,N}}) = 9.589 \cdot 10^{-14} \]

**noise**

\[ I_{n,N} = 9.589 \cdot 10^{-14} \]
Pulse shape terms

\[ f_n := 5 \]

\[ \alpha := \frac{0.1784 \cdot T_b}{f_n} = 3.568 \cdot 10^{-11} \]

\[ H_p(\omega) := \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \]

\[ \frac{1}{T_s} \cdot 10^3 \]

\[ I_0(t) := \frac{1}{\pi} \cdot \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \cdot \text{Re} \left( \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_c}} \right) \cdot \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \cdot \exp(1i \cdot \omega(t)) \]

\[ \frac{1}{T_s} \cdot 10^3 \]

\[ I_1(t) := \frac{T_s}{\pi} \cdot \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \cdot \text{Re} \left( \frac{1i \cdot \omega \cdot 1j \cdot \frac{\omega}{\omega_c}}{1 + 1j \cdot \frac{\omega}{\omega_c}} \right) \cdot \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \cdot \exp(1i \cdot \omega(t)) \]

\[ t := -1 \cdot T_s, -0.9 \cdot T_s, \ldots, 3 \cdot T_s \]
Peak voltage

\[ t_i := 0.72 \cdot T_s \]

\[ t_{pk} := \text{root} \left( T_s \cdot I_1(t), t \right) \]

\[ t_{pk} = 1.796 \cdot 10^{-10} \]

\[ v_{pk} := I_0 \left( t_{pk} \right) = 4.697 \cdot 10^9 \]

Erasure of pulse

\[ Q_{c_i} := \eta q \cdot \frac{v_{pk} - v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

\[ P_r(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{c_i} \cdot b}{\sqrt{2}} \right) \]

\[ P_{er}(b, i) := 2 \cdot \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_r(b, i) \cdot (x+1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_r(b, i) \cdot (n+1) \]

False alarm

False alarm when pulse appears in slot R can spread into S-slot of following symbol \( P_{fISI1} \) or into previous S-slot of same symbol

\[ v_{oISI1 Ts_i} := v_i \cdot I_0 \left( t_{pk} - T_s \right) \]

\[ v_{oISI2 Ts_i} := v_i \cdot I_0 \left( t_{pk} + T_s \right) \]
\[ Q_{e\text{ISI}_i} := \eta q \cdot \frac{v_i \cdot v_{pk} - v_{o\text{ISI}1 \cdot T_{s_i}}}{\sqrt{\text{noise}}} \]

\[ Q_{e\text{ISI}_2_i} := \eta q \cdot \frac{v_i \cdot v_{pk} - v_{o\text{ISI}2 \cdot T_{s_i}}}{\sqrt{\text{noise}}} \]

\[ P_{e\text{ISI}_1}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{e\text{ISI}_1} \cdot b}{\sqrt{2}} \right) \]

\[ P_{e\text{ISI}_2}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{e\text{ISI}_2} \cdot b}{\sqrt{2}} \right) \]

\[ P_{e\text{R}}(b, i) := \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{e\text{ISI}_1}(b, i) \cdot (x) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{e\text{ISI}_1}(b, i) \cdot (n) \]

\[ + \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{e\text{ISI}_2}(b, i) \cdot (x+1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{e\text{ISI}_2}(b, i) \cdot (n+1) \]

False alarm no ISI occurs between S and R and the error appears within the run of N-symbols where k is the symbol position.

False alarm between R and S pulses - N to SET

\[ Q_{NS}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

\[ P_{NS}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NS}(b, i)}{\sqrt{2}} \right) \]

\[ P_{eNS}(b, i) := \sum_{y=3}^{n-1} \sum_{k=2}^{y-1} \left( \frac{1}{2} \right)^{y+3} \cdot P_{NS}(b, i) \cdot (y+1-k) \]

\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NS}(b, i) \cdot (n+1-k) \]

False alarm between S and R pulses - N to R

\[ Q_{NR}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]
\[ P_{NR}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NR}(b, i)}{\sqrt{2}} \right) \]
\[ P_{eNR}(b, i) := \sum_{x=3}^{n-1} \sum_{k=2}^{x-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{NR}(b, i) \cdot (x+1-k) \]
\[ + \sum_{k=2}^{n+2} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NR}(b, i) \cdot (n+1-k) \]
\[ P_{F^N}(b, i) := P_{eNS}(b, i) + P_{eNR}(b, i) \]
\[ P_{eF^N}(b, i) := \sum_{x=1}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=1}^{x} \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=1}^{n} (P_{F^N}(b, i) \cdot (n+1-k)) \]
\[ + \sum_{x=2}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=2}^{x} (P_{F^N}(b, i) \cdot (x+1-k)) \]
\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=2}^{n} (P_{F^N}(b, i) \cdot (n+1-k)) \]

**Total false alarm**
\[ P_{ef}(b, i) := P_{eF^N}(b, i) + P_{eR}(b, i) \]
\[ b := 5 \cdot 10^3 \]

**Total Error**
\[ P_{eb}(b, i) := P_{er}(b, i) + P_{eR}(b, i) + P_{eF^N}(b, i) \]
\[ p_c(b, i) := (\log(P_{eb}(b, i)) + 9) \]
\[ a := \text{root}(p_c(b, i), b) \]
\[ v_{off} \equiv 0.45 \]

**minimum := min(a)**
\[ \text{minimum} = 5.041 \cdot 10^3 \]
\[ gu \equiv 2 \]
\[ dBm := 10 \cdot \log \left( \frac{\text{minimum} \cdot \text{photon\_energy}}{10^{-3}} \cdot \left( \frac{n+1}{2 \cdot (gu+2) \cdot n} \right) \cdot B \right) = -40.51 \]
\[ i := 20, 21 \ldots 40 \]

\[ b := 4.6 \times 10^3, 4.61 \times 10^3 \ldots 4.8 \times 10^3 \]
C.5.2 Non-tuned PPM receiver with Butterworth filter (PIN-BJT/$f_n=1$)

Di-code PPM (PIN-BJT Butterworth 1 Gbit/s)

PPM Rx performance with PIN-BJT input configuration, Butterworth pre-detection filter. Calculations are as follows:

- Di-code PPM terms
- Rx terms (noise+TF)
- Noise optimisation
- Pulse shaping, voltages
- ISI/Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

**Di-code PPM terms**

$$i := 0, 1 \ldots 100 \quad n := 10 \quad j := 0 \ldots 10$$

Set up the scan limits

$$v_i := v_{\text{off}} + \frac{i}{1000}$$

$$x := 0 \ldots n \quad \text{This gives the row of the matrix} \quad y := 0 \ldots n \quad \text{This gives the column of the matrix}$$

$$B := 1 \cdot 10^9$$

Bit rate

$$T_b := \frac{1}{B} = 1 \cdot 10^{-9}$$

PCM bit time

$$T_s := \frac{T_b}{2 + gu}$$

Slot time

$$n := 10$$

Number of like symbols in PCM

$$\eta q := 1.6 \cdot 10^{-10}$$

Quantum energy

$$\lambda := 1.55 \cdot 10^{-6}$$

This is the wavelength of operation

$$\text{photon\textunderscore energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda}$$

$$R_o := \frac{\eta q}{\text{photon\textunderscore energy}}$$

$$\text{PPM}_B := \frac{1}{T_s} = 4 \cdot 10^9$$

This PPM bit rate, used for filter bandwidth

$$ts := \frac{1}{\text{PPM}_B} = 2.5 \cdot 10^{-10}$$
**Preamplifier terms**

\[ Av := 10 \quad C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]

\[ \omega_c := 2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} = 1.257 \cdot 10^{10} \]

\[ Z_{TIA,N}(\omega) := \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_c}} \]

\[ Rf_N := \frac{Av + 1}{2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \cdot C_T} = 583.568 \]

\[ \omega_B := 2 \cdot \pi \cdot 0.7 \cdot \frac{1}{T_s} \]

\[ \text{filter}(\omega) := \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \]

\[ Z_{\text{nontuned}}(\omega) := Z_{TIA,N}(\omega) \cdot \text{filter}(\omega) \]

**Receiver noise**

\[ NEB_N := \frac{1}{2 \cdot \pi} \int_0^{10^3} \left( \left| Z_{\text{nontuned}}(\omega) \right|^2 \right) d\omega = 1.889 \cdot 10^9 \]

\[ I_{\text{2.N}} := NEB_N = 1.889 \cdot 10^9 \]

\[ I_{\text{3.N}} := \frac{1}{2 \cdot \pi} \int_0^{10^3} \left( \left| \frac{1 + 1j \cdot (\omega) \cdot Rf_N \cdot C_T}{1 \cdot Rf_N} \cdot Z_{\text{nontuned}}(\omega) \right|^2 \right) d\omega = 3.763 \cdot 10^5 \]

**noise-opt BJT**

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100 \]

\[ x_N(x) := \left( \frac{4 \cdot k \cdot T}{Rf_N} \cdot I_{\text{2.N}} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{\text{2.N}} + 2 \cdot q \cdot \frac{x}{\left( \frac{25 \cdot 10^{-3}}{(25 \cdot 10^{-3})} \right)} \cdot I_{\text{3.N}} \right) \]

\[ I_{\text{copt}} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{\text{3.N}}}{I_{\text{2.N}}}} = 0.004 \]

\[ I_{n,N} := x_N(I_{\text{copt}}) = 9.589 \cdot 10^{-14} \]

\[ noise := I_{n,N} = 9.589 \cdot 10^{-14} \]
Pulse shape terms

\[ f_n := 1 \]
\[ \alpha := \frac{0.1784 \cdot T_b}{f_n} = 1.784 \cdot 10^{-10} \]
\[ H_p(\omega) := \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \]

\[ I_0(t) := \frac{1}{\pi} \cdot \frac{1}{T_s \cdot 10^2} \cdot \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \cdot \frac{1}{1 + j \cdot \frac{\omega}{\omega_c}} \cdot \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \cdot \exp(1i \cdot \omega \cdot (t)) \]

\[ I_1(t) := \frac{T_s}{\pi} \cdot \frac{1}{T_s \cdot 10^2} \cdot \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \cdot \frac{1}{1 + j \cdot \frac{\omega}{\omega_c}} \cdot \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \cdot \exp(1i \cdot \omega \cdot (t)) \]

\[ t := -1 \cdot T_s, -0.9 \cdot T_s \ldots 3 \cdot T_s \]
Peak voltage

\[ t := 0.72 \cdot T_s \]
\[ t_{pk} := \text{root}(T_s \cdot I_1(t), t) \]
\[ t_{pk} = 1.898 \cdot 10^{-10} \]

\[ v_{pk} := I_0(t_{pk}) = 2.063 \cdot 10^9 \]

Erasure of pulse

\[ Q_{e_i} := \eta q \cdot \frac{v_{pk} - v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

\[ P_r(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{e_i} \cdot b}{\sqrt{2}} \right) \]

\[ P_{er}(b, i) := 2 \cdot \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_r(b, i) \cdot (x+1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_r(b, i) \cdot (n+1) \]

False alarm

False alarm when pulse appears in slot R can spread into S-slot of following symbol \( P_{\text{SISI}} \) or into previous S-slot of same symbol

\[ v_{\text{SISI}_1 \cdot T_s} := v_i \cdot I_0(t_{pk} - T_s) \]

\[ v_{\text{SISI}_2 \cdot T_s} := v_i \cdot I_0(t_{pk} + T_s) \]
\[ Q_{e\text{ISI}_1} := \eta q \cdot \frac{v_i \cdot v_{pk} - v_{\text{ISI1}_{T_1}}}{\sqrt{\text{noise}}} \]

\[ Q_{e\text{ISI}_2} := \eta q \cdot \frac{v_i \cdot v_{pk} - v_{\text{ISI2}_{T_2}}}{\sqrt{\text{noise}}} \]

\[ P_{e\text{ISI}_1}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{e\text{ISI}_1} \cdot b}{\sqrt{2}} \right) \]

\[ P_{e\text{ISI}_2}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{e\text{ISI}_2} \cdot b}{\sqrt{2}} \right) \]

\[ P_{e\text{R}}(b, i) := \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{e\text{ISI}_1}(b, i) \cdot (x) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{e\text{ISI}_1}(b, i) \cdot (n) \]

\[ + \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{e\text{ISI}_2}(b, i) \cdot (x+1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{e\text{ISI}_2}(b, i) \cdot (n+1) \]

False alarm no ISI occurs between S and R and the error appears within the run of N symbols where k is the symbol position.

False alarm between R and S pulses - N to SET

\[ Q_{NS}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

\[ P_{NS}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NS}(b, i)}{\sqrt{2}} \right) \]

\[ P_{eNS}(b, i) := \sum_{y=3}^{n-1} \sum_{k=2}^{y-1} \left( \frac{1}{2} \right)^{y+3} \cdot P_{NS}(b, i) \cdot (y+1-k) \]

\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NS}(b, i) \cdot (n+1-k) \]

False alarm between S and R pulses - N to R

\[ Q_{NR}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]
\[ P_{NR}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NR}(b, i)}{\sqrt{2}} \right) \]

\[ P_{eNR}(b, i) := \sum_{x=3}^{n-1} \sum_{k=2}^{x-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{NR}(b, i) \cdot (x+1-k) \]
\[ + \sum_{k=2}^{n+2} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NR}(b, i) \cdot (n+1-k) \]

\[ P_{FN}(b, i) := P_{eNS}(b, i) + P_{eNR}(b, i) \]

\[ P_{efN}(b, i) := \sum_{x=1}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=1}^{x} (P_{FN}(b, i) \cdot (x+1-k)) \]
\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=1}^{n} (P_{FN}(b, i) \cdot (n+1-k)) \]
\[ + \sum_{x=2}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=2}^{x} (P_{FN}(b, i) \cdot (x+1-k)) \]
\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=2}^{n} (P_{FN}(b, i) \cdot (n+1-k)) \]

**Total false alarm**

\[ P_{ef}(b, i) := P_{efN}(b, i) + P_{efR}(b, i) \]

\[ b := 5 \cdot 10^3 \]

**Total Error**

\[ P_{eb}(b, i) := P_{er}(b, i) + P_{efR}(b, i) + P_{efN}(b, i) \]

\[ pc(b, i) := (\log(P_{eb}(b, i)) + 9) \]

\[ a_i := \text{root}(pc(b, i), b) \]

\[ v_{off} \equiv 0.6 \]

**minimum := min(a)**

\[ \text{minimum} = 1.575 \cdot 10^4 \]

\[ gu \equiv 2 \]

\[ dBm := 10 \cdot \log \left( \frac{\text{minimum} \cdot \text{photon\_energy}}{10^{-3}} \cdot \left( \frac{n+1}{2 \cdot (gu + 2) \cdot n} \right) \cdot B \right) = -35.56 \]
\[ i := 20, 25 \ldots 40 \quad \text{and} \quad b := 1.5 \cdot 10^4, 1.51 \cdot 10^4 \ldots 1.6 \cdot 10^4 \]
C.5.3 Non-tuned PPM receiver with Butterworth filter (PIN-BJT/\(f_n=0.7\))

Di-code PPM (PIN-BJT Butterworth 1 Gbit/s)

PPM Rx performance with PIN-BJT input configuration, Butterworth pre-detection filter, calculations are as follow:
- Di-code PPM terms
- Rx terms (noise+TF)
- Noise optimisation
- Pulse shaping, voltages
- ISI/Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

**Di-code PPM terms**

\[ i := 0, 1 \ldots 100 \quad n := 10 \quad j := 0 \ldots 10 \quad \text{Set up the scan limits} \]

\[ v_i := v_{\text{off}} + \frac{i}{1000} \]

\[ x := 0 \ldots n \quad \text{This gives the row of the matrix} \quad y := 0 \ldots n \quad \text{This gives the column of the matrix} \]

\[ B := 1 \cdot 10^9 \quad \text{Bit rate} \]

\[ T_b := \frac{1}{B} = 1 \cdot 10^{-9} \quad \text{PCM bit time} \]

\[ T_s := \frac{T_b}{2 + gu} \quad \text{Slot time} \]

\[ n := 10 \quad \text{Number of like symbols in PCM} \]

\[ \eta q := 1.6 \cdot 10^{-10} \quad \text{Quantum energy} \]

\[ \lambda := 1.55 \cdot 10^{-6} \quad \text{This is the wavelength of operation} \]

\[ \text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ R_o := \frac{\eta q}{\text{photon\_energy}} \]

\[ \text{PPM}_B := \frac{1}{T_s} = 4 \cdot 10^9 \quad \text{this PPM bit rate, used for filter bandwidth} \]

\[ t_s := \frac{1}{\text{PPM}_B} = 2.5 \cdot 10^{-10} \]
Preamplifier terms

\[ Av := 10 \quad C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]

\[ \omega_c := 2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} = 1.257 \cdot 10^{10} \]

\[ Z_{TIA,N}(\omega) := \frac{1}{1 + j \cdot \frac{\omega}{\omega_c}} \quad Rf_N := \frac{Av + 1}{2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \cdot C_T} = 583.568 \]

\[ \omega_B := 2 \cdot \pi \cdot 0.7 \cdot \frac{1}{T_s} \]

\[ \text{filter}(\omega) := \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2(1j \cdot \omega)^2 \cdot \omega_B + 2(1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \]

\[ Z_{\text{nontuned}}(\omega) := Z_{TIA,N}(\omega) \cdot \text{filter}(\omega) \]

Receiver noise

\[ NEB_N := \frac{1}{2 \cdot \pi} \int_0^{10^{10}} \left( \left| Z_{\text{nontuned}}(\omega) \right|^2 \right) d\omega = 1.889 \cdot 10^9 \]

\[ I_{2,N} := NEB_N = 1.889 \cdot 10^9 \]

\[ I_{3,N} := \frac{1}{2 \cdot \pi} \int_0^{10^{10}} \left( \left| \frac{1 + j \cdot (\omega) \cdot Rf_N \cdot C_T}{1 \cdot Rf_N} \cdot Z_{\text{nontuned}}(\omega) \right|^2 \right) d\omega = 3.763 \cdot 10^5 \]

noise-opt BJT

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100 \]

\[ x_N(x) := \left( \frac{4 \cdot k \cdot T}{Rf_N} \cdot I_{2,N} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{2,N} + 2 \cdot q \cdot \frac{x}{(25 \cdot 10^{-3})^2} \cdot I_{3,N} \right) \]

\[ I_{\text{copt}} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3,N}}{I_{2,N}}} = 0.004 \]

\[ I_{n,N} := x_N(I_{\text{copt}}) = 9.589 \cdot 10^{-14} \]

\[ \text{noise} := I_{n,N} = 9.589 \cdot 10^{-14} \]
Pulse shape terms

\[ f_n := 0.7 \]
\[ \alpha := \frac{0.1784 \cdot T_b}{f_n} = 2.549 \cdot 10^{-10} \]

\[ H_p(\omega) := \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \]

\[ \frac{1}{T_s \cdot 10^5} \]
\[ I_0(t) := \frac{1}{\pi} \cdot \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \cdot \left( 1 + 1j \cdot \frac{\omega}{\omega_c} \right) \cdot \omega_B^3 \cdot \left( (1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3 \right) \cdot \exp(1i \cdot \omega \cdot (t)) \]

\[ \frac{1}{T_s \cdot 10^5} \]
\[ I_1(t) := \frac{T_s}{\pi} \cdot \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \cdot \left( 1i \cdot \omega \cdot \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_c}} \right) \cdot \omega_B^3 \cdot \left( (1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3 \right) \cdot \exp(1i \cdot \omega \cdot (t)) \]

\[ t := -1 \cdot T_s, -0.9 \cdot T_s, .., 3 \cdot T_s \]
Peak voltage

t := 0.72 \cdot T_s

t_{pk} := \text{root}(T_s \cdot I_1(t), t)

\begin{align*}
t_{pk} &= 1.906 \cdot 10^{-10} \\
v_{pk} &:= I_0(t_{pk}) = 1.5 \cdot 10^9
\end{align*}

Erasure of pulse

\begin{align*}
Q_{e_i} &:= \eta q \cdot \frac{v_{pk} - v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \\
P_r(b, i) &:= \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{e_i} \cdot b}{\sqrt{2}} \right) \\
P_{er}(b, i) &:= 2 \cdot \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_r(b, i) \cdot (x+1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_r(b, i) \cdot (n+1)
\end{align*}

False alarm

False alarm when pulse appears in slot R can spread into S-slot of following symbol \( P_{sflst} \) or into previous S-slot of same symbol

\begin{align*}
v_{sflst1} &:= v_i \cdot I_0(t_{pk} - T_s) \\
v_{sflst2} &:= v_i \cdot I_0(t_{pk} + T_s)
\end{align*}
\[ Q_{e\text{ISI}_1} := \eta q \cdot \frac{v_i \cdot v_{pk} - v_{\text{ISI}_1 \cdot T_{S,i}}}{\sqrt{\text{noise}}} \]

\[ Q_{e\text{ISI}_2} := \eta q \cdot \frac{v_i \cdot v_{pk} - v_{\text{ISI}_2 \cdot T_{S,i}}}{\sqrt{\text{noise}}} \]

\[ P_{e\text{ISI}_1}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{e\text{ISI}_1} \cdot b}{\sqrt{2}} \right) \]

\[ P_{e\text{ISI}_2}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{e\text{ISI}_2} \cdot b}{\sqrt{2}} \right) \]

\[ P_{eR}(b, i) := \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{e\text{ISI}_1}(b, i) \cdot (x) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{e\text{ISI}_1}(b, i) \cdot (n) \]

\[ + \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{e\text{ISI}_2}(b, i) \cdot (x+1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{e\text{ISI}_2}(b, i) \cdot (n+1) \]

False alarm no ISI occurs between S and R and the error appears within the run of N-symbols where k is the symbol position

False alarm between R and S pulses - N to SET

\[ Q_{NS}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

\[ P_{NS}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NS}(b, i)}{\sqrt{2}} \right) \]

\[ P_{eNS}(b, i) := \sum_{y=3}^{n-1} \sum_{k=2}^{y-1} \left( \frac{1}{2} \right)^{y+3} \cdot P_{NS}(b, i) \cdot (y+1-k) \]

\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NS}(b, i) \cdot (n+1-k) \]

False alarm between S and R pulses - N to R

\[ Q_{NR}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]
\[ P_{NR}(b,i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NR}(b,i)}{\sqrt{2}} \right) \]

\[ P_{eNR}(b,i) := \sum_{x=3}^{n-1} \sum_{k=2}^{x-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{NR}(b,i) \cdot (x+1-k) \]
\[ + \sum_{k=2}^{n} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NR}(b,i) \cdot (n+1-k) \]

\[ P_{FN}(b,i) := P_{eNS}(b,i) + P_{eNR}(b,i) \]

\[ P_{eFN}(b,i) := \sum_{x=1}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=1}^{x} \left( P_{FN}(b,i) \cdot (x+1-k) \right) \]
\[ + \sum_{k=1}^{n} \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{x=1}^{n-1} \left( P_{FN}(b,i) \cdot (n+1-k) \right) \]
\[ + \sum_{x=2}^{n} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=2}^{x} \left( P_{FN}(b,i) \cdot (x+1-k) \right) \]
\[ + \sum_{k=2}^{n} \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=2}^{n} \left( P_{FN}(b,i) \cdot (n+1-k) \right) \]

**Total false alarm**

\[ P_{ef}(b,i) := P_{eFN}(b,i) + P_{eFR}(b,i) \]

\[ b := 5 \cdot 10^3 \]

**Total Error**

\[ P_{eb}(b,i) := P_{er}(b,i) + P_{eFR}(b,i) + P_{eFN}(b,i) \]

\[ pc(b,i) := \log (P_{eb}(b,i)) + 9 \]

\[ a_i := \text{root}(pc(b,i), b) \]

\[ v_{off} \equiv 0.7 \]

minimum := min(a)

minimum = 2.968 \cdot 10^4

\[ gu \equiv 2 \]

\[ dBm := 10 \cdot \log \left( \frac{\text{minimum} \cdot \text{photon energy} \cdot (n+1)}{10^{-3} \cdot 2 \cdot (gu+2) \cdot n \cdot B} \right) = -32.81 \]

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\[ i := 20, 25 \ldots 40 \quad b := 2.91 \times 10^4, 2.92 \times 10^4 \ldots 2.99 \times 10^4 \]
C.5.4 Tuned B di-code PPM receiver with Butterworth filter (PIN-BJT/f_n=0.7)

Di-code PPM (PIN-BJT Butterworth 1 Gbit/s)

PPM Rx performance with PIN-BJT input configuration, Butterworth pre-detection filter. Calculations are as follows:

- Di-code PPM terms
- Rx terms (noise+TF)
- Noise optimisation
- Pulse shaping, voltages
- ISI/Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

**Di-code PPM terms**

\[ i := 0, 1 \ldots 100 \quad n := 10 \quad j := 0 \ldots 10 \quad \text{Set up the scan limits} \]

\[ v_i := v_{\text{off}} + \frac{i}{1000} \]

\[ x := 0 \ldots n \quad \text{This gives the row of the matrix} \quad y := 0 \ldots n \quad \text{This gives the column of the matrix} \]

**Set up the scan limits**

\[ B := 1 \cdot 10^9 \quad \text{Bit rate} \]

\[ T_b := \frac{1}{B} = 1 \cdot 10^{-9} \quad \text{PCM bit time} \]

\[ T_s := \frac{T_b}{2 + gu} \quad \text{Slot time} \]

\[ n := 10 \quad \text{Number of like symbols in PCM} \]

\[ \eta q := 1.6 \cdot 10^{-19} \quad \text{Quantum energy} \]

\[ \lambda := 1.55 \cdot 10^{-6} \quad \text{This is the wavelength of operation} \]

\[ \text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ R_o := \frac{\eta q}{\text{photon\_energy}} \]

\[ \text{PPM\_B} := \frac{1}{T_s} = 4 \cdot 10^9 \quad \text{This PPM bit rate, used for filter bandwidth} \]
$$t_s := \frac{1}{PPM_B} = 2.5 \cdot 10^{-10}$$

**Preamplifier terms**

$$Av := 10 \quad C_T := 1.5 \cdot 10^{-12} \quad \text{total C}$$

Feedback value for (R) Tuned B

$$\Delta_L := 1.5 \quad \Delta_R := 2.65 \quad \text{feedback } \Delta R \text{, time constant ratio } \Delta L$$

$$Rf_B := \Delta_r \cdot \frac{Av + 1}{2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \cdot C_T} = 1.5465 \cdot 10^3 \quad \text{feedback for Tuned B}$$

$$\alpha_b := 0.5 \quad \text{splitting ratio}$$

$$L_c := \frac{(\frac{Rf_B}{1 + Av})^2 \cdot C_T}{\Delta_L} = 1.976 \cdot 10^{-8}$$

$$C1 := (1 - \alpha_b) \cdot (C_T) = 7.5 \cdot 10^{-13}$$

$$C2 := \alpha_b \cdot (C_T) = 7.5 \cdot 10^{-13}$$

$$Z_{TIA,B}(\omega) := \frac{1}{\left( (1 - \omega^2 \cdot L_c \cdot C1) + \frac{Rf_B}{(1 + Av)}(\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot L_c \cdot C1 \cdot C2) \right)}$$

$$\omega_B := 2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s}$$

$$\text{filter}(\omega) := \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3}$$

**Receiver noise**

$$Z_{\text{tuned},B}(\omega) := Z_{TIA,B}(\omega) \cdot \text{filter}(\omega)$$

$$NEB_B := \frac{1}{2 \cdot \pi} \left( \int_0^{10^6} \left( \left| Z_{\text{tuned},B}(\omega) \right|^2 \right) d\omega \right) = 2.048 \cdot 10^9$$

$$I_{2,B} := \frac{1}{2 \cdot \pi} \left( \int_0^{10^6} \left( \left| (1 - \omega^2 \cdot L_c \cdot C1) \cdot Z_{\text{tuned},B}(\omega) \right|^2 \right) d\omega \right) = 1.068 \cdot 10^9$$
\[ I_{3B} = \frac{1}{2\pi} \int_{0}^{10^9} \left( \frac{\left( \frac{(1 - \omega^2 \cdot Lc \cdot C1)}{Rf_B \cdot (\omega + (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \cdot \omega)} \right)}{Z_{tuned \_ B}(\omega)} \right)^2 d\omega = 5.239 \cdot 10^4 \]

**noise-opt BJT**

\( q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100 \)

\[ x_B(x) := 4 \cdot k \cdot T \cdot I_{2B} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{2B} + 2 \cdot q \cdot \left( \frac{x}{x} \right)^2 \cdot I_{3B} \]

\[ I_{opt \_ B} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3B}}{I_{2B}}} = 1.7513 \cdot 10^{-3} \]

\[ I_{nB} := x_B(I_{opt \_ B}) = 2.332 \cdot 10^{-14} \]

**noise** := \( I_{nB} = 2.332 \cdot 10^{-14} \)

**Pulse shape terms**

\[ f_n := 0.7 \]

\[ \alpha := \frac{0.1784 \cdot T_b}{f_n} = 2.549 \cdot 10^{-10} \]

\[ H_p(\omega) := \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \]

\[ I_0(t) := \frac{1}{\pi} \cdot \int_{0}^{10^3} \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) d\omega \]

\[ \cdot \text{Re}\left( \frac{1}{\omega^3 \cdot \left( (1 - \omega^2 \cdot Lc \cdot C1) \right) + \frac{Rf_B}{(1 + A_v)} \cdot \omega \cdot (\omega \cdot \omega)} \cdot \exp(1 \cdot \omega \cdot t) + 2 \cdot (1 \cdot \omega^2) \cdot \omega_B + 2 \cdot (1 \cdot \omega) \cdot \omega_B^2 + \omega_B^3 \right) \]
\[ I_1(t) := \frac{T_s}{\pi} \cdot \frac{1}{10^7} \cdot \frac{1}{\omega} \cdot \exp\left(\frac{-\alpha^2 \cdot \omega^2}{2}\right) \int_0^\omega \cdot \Re \left[ \frac{\left(1 - \omega^2 \cdot Lc \cdot C1\right)}{(1 + Av)} \cdot \left(\omega \cdot 1i \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2)\right)\right] \cdot \omega_B^3 \cdot \omega_B^3 \cdot \exp(1i \cdot \omega \cdot t) \, d\omega \]

\[ t := -1 \cdot T_s, -0.9 \cdot T_s, 0.6 \cdot T_s \]

---

**Peak voltage**

\[ t := 1.4 \cdot T_s \]

\[ t_{pk} := \text{root}(T_s \cdot I_1(t), t) \]

\[ t_{pk} = 3.655 \cdot 10^{-10} \]

\[ v_{pk} := I_0(t_{pk}) = 1.522 \cdot 10^9 \]

**Erasure of pulse**

\[ Q_{e_i} := \eta q \cdot \frac{v_{pk} - v \cdot v_{pk}}{\sqrt{\text{noise}}} \]
\[ P_r(b, i) := \frac{1}{2} \cdot \text{erfc}\left(\frac{Q_{ei} \cdot b}{\sqrt{2}}\right) \]

\[ P_{er}(b, i) := 2 \cdot \sum_{x=0}^{n-1} \left(\frac{1}{2}\right)^{x+3} \cdot P_r(b, i) \cdot (x+1) + \left(\frac{1}{2}\right)^{n+2} \cdot P_r(b, i) \cdot (n+1) \]

**False alarm**

False alarm when pulse appears in slot R can spread into S-slot of following symbol or into previous S-slot of same symbol

\[ v_{oISI1-Ts_i} := v_i \cdot I_0 \left( t_{pk} - T_s \right) \]

\[ v_{oISI2-Ts_i} := v_i \cdot I_0 \left( t_{pk} + T_s \right) \]

\[ Q_{elSI1_i} := \frac{v_i \cdot v_{pk} - v_{oISI1-Ts_i}}{\sqrt{\text{noise}}} \]

\[ Q_{elSI2_i} := \frac{v_i \cdot v_{pk} - v_{oISI2-Ts_i}}{\sqrt{\text{noise}}} \]

\[ P_{efSI1}(b, i) := \frac{1}{2} \cdot \text{erfc}\left(\frac{Q_{elSI1_i} \cdot b}{\sqrt{2}}\right) \]

\[ P_{efSI2}(b, i) := \frac{1}{2} \cdot \text{erfc}\left(\frac{Q_{elSI2_i} \cdot b}{\sqrt{2}}\right) \]

\[ P_{efR}(b, i) := \sum_{x=0}^{n-1} \left(\frac{1}{2}\right)^{x+3} \cdot P_{efSI1}(b, i) \cdot (x) + \left(\frac{1}{2}\right)^{n+2} \cdot P_{efSI1}(b, i) \cdot (n+1) \]

False alarm no ISI occurs between S and R and the error appears within the run of N-symbols where k is the symbol position

False alarm between R and S pulses - N to SET

\[ Q_{NS}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

C-161
\[ P_{NS}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NS}(b, i)}{\sqrt{2}} \right) \]

\[ P_{eNS}(b, i) := \sum_{y=2}^{n-1} \sum_{k=2}^{y-1} \left( \frac{1}{2} \right)^{y+k} \cdot P_{NS}(b, i) \cdot (y+1-k) \]

\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+k} \cdot P_{NS}(b, i) \cdot (n+1-k) \]

False alarm between S and R pulses - N to R

\[ Q_{NR}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

\[ P_{NR}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NR}(b, i)}{\sqrt{2}} \right) \]

\[ P_{eNR}(b, i) := \sum_{x=3}^{n-1} \sum_{k=2}^{x-1} \left( \frac{1}{2} \right)^{x+k} \cdot P_{NR}(b, i) \cdot (x+1-k) \]

\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+k} \cdot P_{NR}(b, i) \cdot (n+1-k) \]

\[ P_{FN}(b, i) := P_{eNS}(b, i) + P_{eNR}(b, i) \]

\[ P_{efN}(b, i) := \sum_{x=1}^{n-1} \left( \frac{1}{2} \right)^{x+1} \cdot \sum_{k=1}^{x} \left( P_{FN}(b, i) \cdot (x+1-k) \right) \]

\[ + \sum_{x=2}^{n} \left( \frac{1}{2} \right)^{n+k} \cdot \sum_{k=1}^{n} \left( P_{FN}(b, i) \cdot (n+1-k) \right) \]

\[ + \sum_{x=1}^{n-1} \left( \frac{1}{2} \right)^{n+k} \cdot \sum_{k=2}^{n} \left( P_{FN}(b, i) \cdot (n+1-k) \right) \]

Total false alarm

\[ P_{ef}(b, i) := P_{efN}(b, i) + P_{eFR}(b, i) \]

\[ b := 5 \cdot 10^3 \]

Total Error

\[ P_{eb}(b, i) := P_{er}(b, i) + P_{eFR}(b, i) + P_{efN}(b, i) \]

\[ pc(b, i) := \log \left( P_{eb}(b, i) \right) + 9 \]

\[ a_i := \text{root} \left( pc(b, i), b \right) \]
\( v_{off} \equiv 0.7 \)

\[ \text{minimum} := \min(a) \quad \text{minimum} = 1.392 \cdot 10^4 \]

\( gu \equiv 2 \)

\[ \text{noise} = 2.332 \cdot 10^{-14} \]

\[ dBm := 10 \cdot \log \left( \frac{\text{minimum} \cdot \text{photon\_energy}}{10^{-3}} \cdot \left( \frac{n + 1}{2 \cdot (gu + 2) \cdot n} \right) \cdot B \right) = -36.098 \]
C.5.5 Tuned B di-code PPM receiver with Butterworth filter (PIN-BJT/\(f_n = 1\))

Di-code PPM (PIN-BJT Butterworth 1 Gbit/s)

PPM Rx performance with PIN-BJT input configuration, Butterworth pre-detection filter. Calculations are as follow:
- Di-code PPM terms
- Rx terms (noise+TF)
- Noise optimisation
- Pulse shaping, voltages
- ISI/Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

**Di-code PPM terms**

\[
i := 0..100 \quad n := 10 \quad j := 0..10 \quad \text{Set up the scan limits}
\]

\[
v_i := v_{off} + \frac{i}{1000}
\]

\[
x := 0..n \quad \text{This gives the row of the matrix}
\]

\[
y := 0..n \quad \text{This gives the column of the matrix}
\]

\[
B := 1 \cdot 10^9 \quad \text{Bit rate}
\]

\[
T_b := \frac{1}{B} = 1 \cdot 10^{-9} \quad \text{PCM bit time}
\]

\[
T_s := \frac{T_b}{2 + g u} \quad \text{Slot time}
\]

\[
n := 10 \quad \text{Number of like symbols in PCM}
\]

\[
\eta q := 1.6 \cdot 10^{-10} \quad \text{Quantum energy}
\]

\[
\lambda := 1.55 \cdot 10^{-6} \quad \text{This is the wavelength of operation}
\]

\[
\text{photon}_\text{energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda}
\]

\[
R_o := \frac{\eta q}{\text{photon}_\text{energy}}
\]

\[
PPM_B := \frac{1}{T_s} = 4 \cdot 10^9 \quad \text{this PPM bit rate, used for filter bandwidth}
\]
\[ ts := \frac{1}{PPM_B} = 2.5 \cdot 10^{-10} \]

**Preamplifier terms**

\[ Av := 10 \quad C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]

Feedback value for (R) Tuned B

\[ \Delta_L := 1.5 \quad \Delta_R := 2.65 \quad \text{feedback \( \Delta R \), time constant ratio \( \Delta L \)} \]

\[ Rf_B := \Delta_R \cdot \frac{Av + 1}{2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \cdot C_T} = 1.5465 \cdot 10^3 \quad \text{feedback for Tuned B} \]

\[ \alpha_b := 0.5 \quad \text{splitting ratio} \]

\[ Lc := \frac{\left( \frac{Rf_B}{1 + Av} \right)^2 \cdot C_T}{\Delta_L} = 1.976 \cdot 10^{-8} \]

\[ C1 := (1 - \alpha_b) \cdot (C_T) = 7.5 \cdot 10^{-13} \]

\[ C2 := \alpha_b \cdot (C_T) = 7.5 \cdot 10^{-13} \]

\[ Z_{TIA,B}(\omega) := \frac{1}{\left( (1 - \omega^2 \cdot Lc \cdot C1) + \frac{Rf_B}{(1 + Av)} \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \right)} \]

\[ \omega_B := 2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \]

\[ filter(\omega) := \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \]

**Receiver noise**

\[ Z_{tuned,B}(\omega) := Z_{TIA,B}(\omega) \cdot filter(\omega) \]

\[ NEB_B := \frac{1}{2 \cdot \pi} \left( \int_0^{10^\pi} \left( \left| Z_{tuned,B}(\omega) \right|^2 \right)^2 d\omega \right) = 2.048 \cdot 10^9 \]

\[ I_{2,B} := \frac{1}{2 \cdot \pi} \left( \int_0^{10^\pi} \left( \left| (1 - \omega^2 \cdot Lc \cdot C1) \cdot \left| Z_{tuned,B}(\omega) \right| \right|^2 \right)^2 d\omega \right) = 1.068 \cdot 10^9 \]
\[ I_{3B} := \frac{1}{2\pi} \int_{0}^{10^{12}} \left[ \left( \frac{\left( \left( \left( 1 - \omega^2 \cdot Lc \cdot C1 \right) \right)^{\frac{1}{2}} + Rf_B \cdot \omega \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \cdot i}{Rf_B \cdot Z_{tuned_B}(\omega)} \right)^{\frac{1}{2}} \right)^2 \right] d\omega = 5.239 \cdot 10^4 \]

**noise-opt BJT**

\( q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100 \)

\[ x_B(x) := \left( \frac{4 \cdot k \cdot T}{Rf_B} \cdot I_{2B} + \left( \frac{2 \cdot q \cdot x}{hfe} \right) \cdot I_{2B} + 2 \cdot q \cdot \left( \frac{x}{25 \cdot 10^{-3}} \right)^2 \cdot I_{3B} \right) \]

\[ I_{optB} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3B}}{I_{2B}}} = 1.7513 \cdot 10^{-3} \]

\[ I_{nB} := x_B(I_{optB}) = 2.332 \cdot 10^{-14} \]

**noise** := \( I_{nB} = 2.332 \cdot 10^{-14} \)

**Pulse shape terms**

\[ f_n := 1 \]

\[ \alpha := \frac{0.1784 \cdot T_b}{f_n} = 1.784 \cdot 10^{-10} \]

\[ H_p(\omega) := \exp\left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \]

\[ \int_{0}^{10^{3}} \frac{1}{T} \cdot 10^{3} \]

\[ I_0(t) := \frac{1}{\pi} \cdot \int_{0}^{10^{3}} \exp\left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \cdot d\omega \]

\[ \left( \frac{\left( (1 - \omega^2 \cdot Lc \cdot C1) \right)^{\frac{1}{2}} + Rf_B \cdot \left( (\omega \cdot i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \right) \cdot \omega_B^3}{(1 + Av)^{\frac{1}{2}}} \right) \]

\[ \left( (1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3 \right) \cdot \exp(1i \cdot \omega \cdot (t)) \]
\[ I_1(t) := \frac{T_s}{\pi} \cdot \frac{1}{10^5} \cdot \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \cdot \text{Re} \left\{ \frac{1}{1 + \omega^2 \cdot Lc \cdot C1} \cdot \frac{Rf \cdot B}{(1 + Av)} \cdot \left( \omega \cdot 1i \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \right) \cdot \frac{\omega^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \cdot \exp(1i \cdot \omega(t)) \right\} \]  

\[ t := -1 \cdot T_s, -0.9 \cdot T_s, 6 \cdot T_s \]  

\[ I_0(t) \]  

Peak voltage  
\[ t := 1.4 \cdot T_s \]  
\[ t_{pk} := \text{root}(T_s \cdot I_1(t), t) \]  
\[ t_{pk} = 3.825 \cdot 10^{-10} \]  
\[ v_{pk} := I_0(t_{pk}) = 2.126 \cdot 10^9 \]  

Erasure of pulse  
\[ Q_{ei} := \eta q \cdot \frac{v_{pk} - v \cdot v_{pk}}{\sqrt{\text{noise}}} \]
\[ P_r(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{e_i} \cdot b}{\sqrt{2}} \right) \]

\[ P_{er}(b, i) := 2 \cdot \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_r(b, i) \cdot (x+1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_r(b, i) \cdot (n+1) \]

**False alarm**

False alarm when pulse appears in slot R can spread into S-slot of following symbol or into previous S-slot of same symbol

\[ v_{o\text{ISI}1-Ts_i} := v_i \cdot I_0(t_{pk} - T_s) \]

\[ v_{o\text{ISI}2-Ts_i} := v_i \cdot I_0(t_{pk} + T_s) \]

\[ Q_{eIsI_1} := \eta q \cdot \frac{v_i \cdot v_{pk} - v_{o\text{ISI}1-Ts_i}}{\sqrt{\text{noise}}} \]

\[ Q_{eIsI_2} := \eta q \cdot \frac{v_i \cdot v_{pk} - v_{o\text{ISI}2-Ts_i}}{\sqrt{\text{noise}}} \]

\[ P_{ef\text{ISI}1}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{eIsI_1} \cdot b}{\sqrt{2}} \right) \]

\[ P_{ef\text{ISI}2}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{eIsI_2} \cdot b}{\sqrt{2}} \right) \]

\[ P_{efR}(b, i) := \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{ef\text{ISI}1}(b, i) \cdot (x) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{ef\text{ISI}1}(b, i) \cdot (n) \]

**False alarm no ISI accurs between S and R and the error apperas within the run of N-symbols where k is the symbol position**

**False alarm between R and S pulses - N to SET**

\[ Q_{NS}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

C-168
\[ P_{NS}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NS}(b, i)}{\sqrt{2}} \right) \]

\[ P_{eNS}(b, i) := \sum_{y=2}^{n-1} \sum_{k=2}^{y-1} \left( \frac{1}{2} \right)^{y+3} \cdot P_{NS}(b, i) \cdot (y+1-k) \]

\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NS}(b, i) \cdot (n+1-k) \]

False alarm between S and R pulses - N to R

\[ Q_{NR}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

\[ P_{NR}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NR}(b, i)}{\sqrt{2}} \right) \]

\[ P_{eNR}(b, i) := \sum_{x=3}^{n-1} \sum_{k=2}^{x-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{NR}(b, i) \cdot (x+1-k) \]

\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NR}(b, i) \cdot (n+1-k) \]

\[ P_{FN}(b, i) := P_{eNS}(b, i) + P_{eNR}(b, i) \]

\[ P_{efN}(b, i) := \sum_{x=1}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=1}^{x} (P_{FN}(b, i) \cdot (x+1-k)) \]

\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=1}^{n} (P_{FN}(b, i) \cdot (n+1-k)) \]

\[ + \sum_{x=2}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=2}^{x} (P_{FN}(b, i) \cdot (x+1-k)) \]

\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=2}^{n} (P_{FN}(b, i) \cdot (n+1-k)) \]

Total false alarm

\[ P_{ef}(b, i) := P_{efN}(b, i) + P_{efR}(b, i) \]

\[ b := 5 \cdot 10^3 \]

Total Error

\[ P_{eb}(b, i) := P_{er}(b, i) + P_{efR}(b, i) + P_{efN}(b, i) \]

\[ pc(b, i) := \langle \log(P_{eb}(b, i)) \rangle + 9 \]

\[ a_i := \text{root}(pc(b, i), b) \]
\[ v_{off} \equiv 0.55 \]

\[ \min := \min(a) \]

\[ \text{minimum} = 7.626 \times 10^3 \]

\[ \text{noise} = 2.332 \times 10^{-14} \]

\[ gu \equiv 2 \]

\[ dBm := 10 \cdot \log\left( \text{minimum} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \left( \frac{n + 1}{2 \cdot (gu + 2) \cdot n} \right) \cdot B \right) = -38.711 \]
C.5.6 Tuned B di-code PPM receiver with Butterworth filter (PIN-BJT/$f_n = 5$)

Di-code PPM (PIN-BJT Butterworth 1 Gbit/s)

PPM Rx performance with PIN-BJT input configuration, Butterworth pre-detection filter, calculations are as follow

- Di-code PPM terms
- Rx terms (noise+TF)
- Noise optimisation
- Pulse shaping, volatages
- ISI/Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

**Di-code PPM terms**

\[ i := 0,1 \ldots 100 \quad n := 10 \quad j := 0,1 \ldots 10 \quad \text{Set up the scan limits} \]

\[ v_i := v_{\text{off}} + \frac{i}{1000} \]

\[ x := 0 \ldots n \quad \text{This gives the row of the matrix} \quad y := 0 \ldots n \quad \text{This gives the column of the matrix} \]

\[ B := 1 \cdot 10^9 \quad \text{Bit rate} \]

\[ T_b := \frac{1}{B} = 1 \cdot 10^{-9} \quad \text{PCM bit time} \]

\[ T_s := \frac{T_b}{2 + gn} \quad \text{Slot time} \]

\[ n := 10 \quad \text{Number of like symbols in PCM} \]

\[ \eta q := 1.6 \cdot 10^{-10} \quad \text{Quantum energy} \]

\[ \lambda := 1.55 \cdot 10^{-6} \quad \text{This is the wavelength of operation} \]

\[ \text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ R_o := \frac{\eta q}{\text{photon\_energy}} \]

\[ PPM_B := \frac{1}{T_s} = 4 \cdot 10^9 \quad \text{this PPM bit rate, used for filter bandwidth} \]
Preamplifier terms

\[ t_s := \frac{1}{PPM_B} = 2.5 \cdot 10^{-10} \]

Feedback value for (R) Tuned B

\[ \Delta_L := 1.5 \quad \Delta_R := 2.65 \quad \text{feedback } \Delta_R, \text{ time constant ratio } \Delta_L \]

\[ R_{f,B} := \Delta_R \cdot \frac{A_v + 1}{2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \cdot C_T} = 1.5465 \cdot 10^3 \quad \text{feedback for Tuned B} \]

\[ \alpha_b := 0.5 \quad \text{splitting ratio} \]

\[ L_c := \left( \frac{R_{f,B}}{1 + A_v} \right)^2 \cdot C_T \cdot \Delta_L = 1.976 \cdot 10^{-8} \]

\[ C1 := (1 - \alpha_b) \cdot (C_T) = 7.5 \cdot 10^{-13} \]

\[ C2 := \alpha_b \cdot (C_T) = 7.5 \cdot 10^{-13} \]

\[ Z_{TIA,B}(\omega) := \frac{1}{\left( (1 - \omega^2 \cdot L_c \cdot C1) + \frac{R_{f,B}}{(1 + A_v)} \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot L_c \cdot C1 \cdot C2) \right)} \]

\[ \omega_B := 2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \]

\[ \text{filter}(\omega) := \frac{\omega_B^3}{(1i \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \]

Receiver noise

\[ Z_{tuned,B}(\omega) := Z_{TIA,B}(\omega) \cdot \text{filter}(\omega) \]

\[ NEB_B := \frac{1}{2 \cdot \pi} \cdot \int_0^{10^9} \left( \left( \left| Z_{tuned,B}(\omega) \right| \right)^2 \right) d\omega \approx 2.048 \cdot 10^9 \]

\[ I_{2,B} := \frac{1}{2 \cdot \pi} \cdot \int_0^{10^9} \left( \left| \left( \left( \left| Z_{tuned,B}(\omega) \right| \right) \right) \right|^2 \right) d\omega \approx 1.068 \cdot 10^9 \]
$$I_{3B} := \frac{1}{2\pi} \int_{0}^{10^3} \left( \left( \frac{\left( \frac{1}{1 - \omega^2 \cdot Lc \cdot C1} \right)}{Rf_B} \cdot (\frac{C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2)}{1i} \right) \cdot Z_{tuned_B}(\omega) \right)^2 d\omega = 5.239 \cdot 10^4$$

noise-opt BJT

$$q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100$$

$$x_B(x) := \left( 4 \cdot k \cdot T \cdot I_{2B} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{2B} + 2 \cdot q \cdot \frac{x}{25 \cdot 10^{-3}} \right) \cdot I_{3B}$$

$$I_{optB} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3B}}{I_{2B}}} = 1.7513 \cdot 10^{-3}$$

$$I_{nB} := x_B(I_{optB}) = 2.332 \cdot 10^{-14}$$

noise := $$I_{nB} = 2.332 \cdot 10^{-14}$$

Pulse shape terms

$$f_n := 1$$

$$\alpha := \frac{0.1784 \cdot T_b}{f_n} = 1.784 \cdot 10^{-10}$$

$$H_p(\omega) := \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right)$$

$$I_0(t) := \frac{1}{\pi} \cdot \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \cdot \text{Re} \left( \frac{1}{\left( (1 - \omega^2 \cdot Lc \cdot C1) \right)} \cdot \left( \frac{Rf_B}{(1 + A)^v} \cdot (\omega \cdot i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \right) \cdot \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \cdot \exp(1i \cdot \omega \cdot t) \right)$$
\[ I_1(t) := \frac{T_s}{\pi} \cdot \frac{1}{10^3} \cdot \exp \left( \frac{-\alpha^2 \cdot \omega^2}{2} \right) \mathrm{d}\omega \]

\[ \cdot \mathrm{Re} \left( \frac{1}{\left( \frac{Rf \cdot B}{1 + Av} \right) \cdot \left( \omega \cdot 1i \right) \cdot \left( C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2 \right) + \omega_B^3} \cdot \frac{1}{\left( 1j \cdot \omega \right)^3 + 2 \cdot \left( 1j \cdot \omega \right)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \cdot \exp(1i \cdot \omega(t)) \right) \]

\[ t := -1 \cdot T_s, -0.9 \cdot T_s, 6 \cdot T_s \]

**Peak voltage**

\[ t := 1.4 \cdot T_s \]

\[ t_{pk} := \text{root}(T_s \cdot I_1(t), t) \]

\[ t_{pk} = 3.825 \cdot 10^{-10} \]

\[ v_{pk} := I_0(t_{pk}) = 2.126 \cdot 10^9 \]

**Erasure of pulse**

\[ Q_{e_i} := \eta q \cdot \frac{v_{pk} - v \cdot v_{pk}}{\sqrt{\text{noise}}} \]
\[ P_r(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{ei} \cdot b}{\sqrt{2}} \right) \]

\[ P_{er}(b, i) := 2 \cdot \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_r(b, i) \cdot (x+1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_r(b, i) \cdot (n+1) \]

**False alarm**

False alarm when pulse appears in slot R can spread into S-slot of following symbol or into previous S-slot of same symbol.

\[ v_{oISI1-Ts_i} := v_i \cdot I_0(t_{pk-Ts}) \]

\[ v_{oISI2-Ts_i} := v_i \cdot I_0(t_{pk+Ts}) \]

\[ Q_{eISI1_i} := \eta q \cdot \frac{v_i \cdot v_{pk} - v_{oISI1-Ts_i}}{\sqrt{\text{noise}}} \]

\[ Q_{eISI2_i} := \eta q \cdot \frac{v_i \cdot v_{pk} - v_{oISI2-Ts_i}}{\sqrt{\text{noise}}} \]

\[ P_{efSI1}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{eISI1} \cdot b}{\sqrt{2}} \right) \]

\[ P_{efSI2}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{eISI2} \cdot b}{\sqrt{2}} \right) \]

\[ P_{efR}(b, i) := \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{efSI1}(b, i) \cdot (x) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{efSI1}(b, i) \cdot (n) + \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{efSI2}(b, i) \cdot (x+1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{efSI2}(b, i) \cdot (n+1) \]

**False alarm no ISI occurs between S and R and the error appears within the run of N-symbols where k is the symbol position**

**False alarm between R and S pulses - N to SET**

\[ Q_{NS}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]
\[ P_{\text{NS}}(b,i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{NS}}(b,i)}{\sqrt{2}} \right) \]

\[ P_{\text{eNS}}(b,i) := \sum_{y=2}^{n-1} \sum_{k=2}^{y-1} \left( \frac{1}{2} \right)^{y+3} \cdot P_{\text{NS}}(b,i) \cdot (y+1-k) \cdot \text{erfc} \left( \frac{Q_{\text{NS}}(b,i)}{\sqrt{2}} \right) \]

\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{\text{NS}}(b,i) \cdot (n+1-k) \cdot \text{erfc} \left( \frac{Q_{\text{NS}}(b,i)}{\sqrt{2}} \right) \]

**False alarm between S and R pulses - N to R**

\[ Q_{\text{NR}}(b,i) := b \cdot \eta_q \cdot \frac{v_i \cdot v_{pk}}{\text{noise}} \]

\[ P_{\text{NR}}(b,i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{NR}}(b,i)}{\sqrt{2}} \right) \]

\[ P_{\text{eNR}}(b,i) := \sum_{x=3}^{n-1} \sum_{k=2}^{x-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{\text{NR}}(b,i) \cdot (x+1-k) \cdot \text{erfc} \left( \frac{Q_{\text{NR}}(b,i)}{\sqrt{2}} \right) \]

\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{\text{NR}}(b,i) \cdot (n+1-k) \cdot \text{erfc} \left( \frac{Q_{\text{NR}}(b,i)}{\sqrt{2}} \right) \]

\[ P_{\text{FN}}(b,i) := P_{\text{eNS}}(b,i) + P_{\text{eNR}}(b,i) \]

\[ P_{\text{efN}}(b,i) := \sum_{x=1}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=1}^{x} P_{\text{FN}}(b,i) \cdot (x+1-k) \cdot \text{erfc} \left( \frac{Q_{\text{FN}}(b,i)}{\sqrt{2}} \right) \]

\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=1}^{n} P_{\text{FN}}(b,i) \cdot (n+1-k) \cdot \text{erfc} \left( \frac{Q_{\text{FN}}(b,i)}{\sqrt{2}} \right) \]

\[ + \sum_{x=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=2}^{x} P_{\text{FN}}(b,i) \cdot (x+1-k) \cdot \text{erfc} \left( \frac{Q_{\text{FN}}(b,i)}{\sqrt{2}} \right) \]

\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=2}^{n} P_{\text{FN}}(b,i) \cdot (n+1-k) \cdot \text{erfc} \left( \frac{Q_{\text{FN}}(b,i)}{\sqrt{2}} \right) \]

**Total false alarm**

\[ P_{ef}(b,i) := P_{\text{efN}}(b,i) + P_{\text{efR}}(b,i) \]

\[ b := 5 \cdot 10^3 \]

**Total Error**

\[ P_{eb}(b,i) := P_{er}(b,i) + P_{efR}(b,i) + P_{efN}(b,i) \]

\[ pc(b,i) := \log \left( P_{eb}(b,i) \right) + 9 \]

\[ a_i := \text{root} \left( pc(b,i), b \right) \]
\( v_{off} \equiv 0.55 \)

\[
\text{minimum} := \min(a)
\]

\( \text{minimum} = 7.626 \cdot 10^3 \)

\( \text{noise} = 2.332 \cdot 10^{-14} \)

\( gu \equiv 2 \)

\[
\text{dBm} := 10 \cdot \log \left( \frac{\text{minimum} \cdot \text{photon\_energy}}{10^{-3}} \cdot \left( \frac{n + 1}{2 \cdot (gu + 2) \cdot n} \right) \cdot B \right) = -38.711
\]
C.6 Di-code PPM receiver with 1st order filter (Optical fibre)

C.6.1 Non-tuned di-code PPM receiver with 1st order filter (PIN-BJT/f_n = 5)

Di-code PPM (PIN-BJT 1st order 1 Gbit/s)

PPM Rx performance with PIN-BJT input configuration, 1st order pre-detection filter, calculations are as follow

- Di-code PPM terms
- Rx terms (noise+TF)
- Noise optimisation
- Pulse shaping, voltages
- ISI/Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

**Di-code PPM terms**

\[ i := 0, 1 \ldots 100 \quad n := 10 \quad j := 0 \ldots 10 \quad \text{Set up the scan limits} \]

\[ v_i := v_{\text{off}} + \frac{i}{1000} \]

\[ x := 0 \ldots n \quad \text{This gives the row of the matrix} \quad y := 0 \ldots n \quad \text{This gives the column of the matrix} \]

\[ B := 1 \cdot 10^9 \quad \text{Bit rate} \]

\[ T_b := \frac{1}{B} = 1 \cdot 10^{-9} \quad \text{PCM bit time} \]

\[ T_s := \frac{T_b}{2 + g} \quad \text{Slot time} \]

\[ n := 10 \quad \text{Number of like symbols in PCM} \]

\[ \eta q := 1.6 \cdot 10^{-10} \quad \text{Quantum energy} \]

\[ \lambda := 1.55 \cdot 10^{-6} \quad \text{This is the wavelength of operation} \]

\[ \text{photon}\_\text{energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ R_o := \frac{\eta q}{\text{photon}\_\text{energy}} \]

\[ \text{PPM}\_B := \frac{1}{T_s} = 4 \cdot 10^9 \quad \text{this PPM bit rate, used for filter bandwidth} \]

\[ t_s := \frac{1}{\text{PPM}\_B} = 2.5 \cdot 10^{-10} \]
Preamplifier terms

\[ A_v := 10 \quad C_T := 1.5 \cdot 10^{-12} \quad \text{total} \ C \]

\[ \omega_c := 2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} = 1.257 \cdot 10^{10} \]

\[ Z_{TIA,N}(\omega) := \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_c}} \]

\[ \omega_B := 2 \cdot \pi \cdot 0.7 \cdot \frac{1}{T_s} \]

\[ Rf_N := \frac{Av + 1}{2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \cdot C_T} = 583.568 \]

\[ \text{filter}(\omega) := \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_B}} \]

\[ Z_{nontuned}(\omega) := Z_{TIA,N}(\omega) \cdot \text{filter}(\omega) \]

Receiver noise

\[ NEB_N := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{11}} \left( |Z_{nontuned}(\omega)|^2 \right) \text{d} \omega \right) = 1.833 \cdot 10^9 \]

\[ I_{2,N} := NEB_N = 1.833 \cdot 10^9 \]

\[ I_{3,N} := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{11}} \left( |1 + 1j \cdot (\omega) \cdot \frac{Rf_N \cdot C_T}{1 \cdot Rf_N} \cdot Z_{nontuned}(\omega)|^2 \right) \text{d} \omega \right) = 8.995 \cdot 10^5 \]

noise-opt BJT

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100 \]

\[ x_N(x) := \left( \frac{4 \cdot k \cdot T}{Rf_N} \cdot I_{2,N} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{2,N} + 2 \cdot q \cdot \frac{x}{\left(25 \cdot 10^{-3}\right)^2} \cdot I_{3,N} \right) \]

\[ I_{\text{copt},N} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3,N}}{I_{2,N}}} = 0.006 \]

\[ I_{n,N} := x_N(I_{\text{copt},N}) = 1.166 \cdot 10^{-13} \]

\[ \text{noise} := I_{n,N} = 1.166 \cdot 10^{-13} \]
**Pulse shape terms**

\[ f_n := 5 \]

\[ \alpha := \frac{0.1784 \cdot T_b}{f_n} = 3.568 \cdot 10^{-11} \]

\[ H_p(\omega) := \exp\left(-\frac{\alpha^2 \cdot \omega^2}{2}\right) \]

\[
I_0(t) := \frac{1}{\pi} \cdot \frac{1}{T_s \cdot 10^3} \left[ \exp\left(-\frac{\alpha^2 \cdot \omega^2}{2}\right) \right]_0^d d\omega
\cdot \text{Re}\left( \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_c}} \right)^d
\cdot \text{Re}\left( \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_B}} \right)^d
\cdot \exp(1j \cdot \omega \cdot (t))
\]

\[
I_1(t) := \frac{T_s}{\pi} \cdot \frac{1}{T_s \cdot 10^3} \left[ \exp\left(-\frac{\alpha^2 \cdot \omega^2}{2}\right) \right]_0^d d\omega
\cdot \text{Re}\left( 1j \cdot \omega \cdot \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_c}} \right)^d
\cdot \text{Re}\left( \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_B}} \right)^d
\cdot \exp(1j \cdot \omega \cdot (t))
\]

\[ t := -1 \cdot T_s, -0.9 \cdot T_s .. 6 \cdot T_s \]
Peak voltage

\[ t_i := 0.3 \cdot T_s \]

\[ t_{pk} := \text{root}(T_s \cdot I_1(t), t) \]

\[ t_{pk} = 8.316 \cdot 10^{-11} \]

\[ v_{pk} := I_0(t_{pk}) = 4.74 \cdot 10^0 \]

Erasure of pulse

\[ Q_{e_i} := \eta q \cdot \frac{v_{pk} - v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

\[ P_r(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{e_i} \cdot b}{\sqrt{2}} \right) \]

\[ P_{er}(b, i) := 2 \sum_{x=0}^{n-1} \left( \left( \frac{1}{2} \right)^{x+3} \cdot P_r(b, i) \cdot (x+1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_r(b, i) \cdot (n+1) \right) \]

False alarm

False alarm when pulse appears in slot R can spread into S-slot of following symbol \( P_{efIS1} \) or into previous S-slot of same symbol

\[ v_{oIS1.T_s} := v \cdot I_0(t_{pk} - T_s) \]

\[ v_{oIS2.T_s} := v \cdot I_0(t_{pk} + T_s) \]
\[ Q_{\text{elISI}_i} = \eta q \cdot \frac{v_i \cdot v_{pk} - v_{\text{elISI}_i}}{\sqrt{\text{noise}}} \]

\[ Q_{\text{elISI}_2} = \eta q \cdot \frac{v_i \cdot v_{pk} - v_{\text{elISI}_2}}{\sqrt{\text{noise}}} \]

\[ P_{\text{elISI}_1}(b, i) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{elISI}_1} \cdot b}{\sqrt{2}} \right) \]

\[ P_{\text{elISI}_2}(b, i) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{elISI}_2} \cdot b}{\sqrt{2}} \right) \]

\[ P_{\text{elR}}(b, i) = \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{\text{elISI}_1}(b, i) \cdot (x) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{\text{elISI}_1}(b, i) \cdot (n) \]
\[ + \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{\text{elISI}_2}(b, i) \cdot (x+1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{\text{elISI}_2}(b, i) \cdot (n+1) \]

False alarm no ISI occurs between S and R and the error appears within the run of N-symbols where k is the symbol position

False alarm between R and S pulses - N to SET

\[ Q_{\text{NS}}(b, i) = b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

\[ P_{\text{NS}}(b, i) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{NS}}(b, i)}{\sqrt{2}} \right) \]

\[ P_{\text{elNS}}(b, i) = \sum_{y=3}^{n-1} \sum_{k=2}^{y-1} \left( \frac{1}{2} \right)^{y+3} \cdot P_{\text{NS}}(b, i) \cdot (y+1-k) \]
\[ + \sum_{k=2}^{y-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{\text{NS}}(b, i) \cdot (n+1-k) \]

False alarm between S and R pulses - N to R

\[ Q_{\text{NR}}(b, i) = b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]
\[ P_{\text{NR}}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{\text{NR}}(b, i)}{\sqrt{2}} \right) \]

\[ P_{\text{eNR}}(b, i) := \sum_{x=3}^{n-1} \sum_{k=2}^{x-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{\text{NR}}(b, i) \cdot (x+1-k) \cdot \left( \frac{1}{2} \right)^{n+2} \cdot P_{\text{NR}}(b, i) \cdot (n+1-k) \]

\[ P_{\text{FN}}(b, i) := P_{\text{eNS}}(b, i) + P_{\text{eNR}}(b, i) \]

\[ P_{\text{efN}}(b, i) := \sum_{x=1}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=1}^{x} \left( P_{\text{FN}}(b, i) \cdot (x+1-k) \right) \cdot \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=1}^{n} \left( P_{\text{FN}}(b, i) \cdot (n+1-k) \right) \]

\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=1}^{n} \left( P_{\text{FN}}(b, i) \cdot (n+1-k) \right) \]

\[ P_{\text{ef}}(b, i) := P_{\text{efN}}(b, i) + P_{\text{efR}}(b, i) \]

\[ b := 5 \cdot 10^3 \]

Total false alarm

\[ P_{\text{ef}}(b, i) := P_{\text{efN}}(b, i) + P_{\text{efR}}(b, i) \]

Total Error

\[ P_{\text{eb}}(b, i) := P_{\text{er}}(b, i) + P_{\text{efR}}(b, i) + P_{\text{efN}}(b, i) \]

\[ p_c(b, i) := (\log (P_{\text{eb}}(b, i)) + 9) \]

\[ a := \text{root}(p_c(b, i), b) \]

\[ v_{\text{off}} \equiv 0.5 \]

minimum := min(a)

minimum = 5.84 \cdot 10^3

\[ gu \equiv 2 \]

\[ dBm := 10 \cdot \log \left( \frac{\text{minimum} \cdot \text{photon\_energy}}{10^{-3}} \cdot \left( \frac{n+1}{2 \cdot (gu+2) \cdot n} \right) \cdot B \right) = -39.87 \]
C.6.2 Non-tuned di-code PPM receiver with 1st order filter (PIN-BJT/f_n = 0.7)

Di-code PPM (PIN-BJT 1st order 1 Gbit/s)

PPM Rx performance with PIN-BJT input configuration, 1st order pre-detection filter. Calculations are as follow:

- Di-code PPM terms
- Rx terms (noise+TF)
- Noise optimisation
- Pulse shaping, voltages
- ISI/Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

**Di-code PPM terms**

\[
i := 0, 1..100 \quad n := 10 \quad j := 0..10 \quad \text{Set up the scan limits}
\]

\[
v_i := v_{\text{off}} + \frac{i}{1000}
\]

\[
x := 0..n \quad \text{This gives the row of the matrix} \quad y := 0..n \quad \text{This gives the column of the matrix}
\]

\[
B := 1 \cdot 10^9 \quad \text{Bit rate}
\]

\[
T_b := \frac{1}{B} = 1 \cdot 10^{-9} \quad \text{PCM bit time}
\]

\[
T_s := \frac{T_b}{2 + gn} \quad \text{Slot time}
\]

\[
n := 10 \quad \text{Number of like symbols in PCM}
\]

\[
\eta q := 1.6 \cdot 10^{-10} \quad \text{Quantum energy}
\]

\[
\lambda := 1.55 \cdot 10^{-6} \quad \text{This is the wavelength of operation}
\]

\[
\text{photon_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda}
\]

\[
R_o := \frac{\eta q}{\text{photon_energy}}
\]

\[
PPM_B := \frac{1}{T_s} = 4 \cdot 10^9 \quad \text{this PPM bit rate, used for filter bandwidth}
\]

\[
t_s := \frac{1}{PPM_B} = 2.5 \cdot 10^{-10}
\]
**Preamplifier terms**

\[ Av := 10 \quad C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]

\[ \omega_c := 2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} = 1.257 \cdot 10^{10} \]

\[ Z_{TIA,N}(\omega) := \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_c}} \]

\[ \omega_B := 2 \cdot \pi \cdot 0.7 \cdot \frac{1}{T_s} \]

\[ Rf_N := \frac{Av + 1}{2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \cdot C_T} = 583.568 \]

\[ \text{filter}(\omega) := \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_B}} \]

\[ Z_{nontuned}(\omega) := Z_{TIA,N}(\omega) \cdot \text{filter}(\omega) \]

**Receiver noise**

\[ \text{NEB}_N := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^9} \left( \frac{1}{|Z_{nontuned}(\omega)|} \right)^2 \, d\omega \right) = 1.833 \cdot 10^9 \]

\[ I_{2,N} := \text{NEB}_N = 1.833 \cdot 10^9 \]

\[ I_{3,N} := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^9} \left( \frac{1}{1 + 1j \cdot (\omega) \cdot Rf_N \cdot C_T} \cdot \frac{Z_{nontuned}(\omega)}{1 \cdot Rf_N} \right)^2 \, d\omega \right) = 8.995 \cdot 10^5 \]

**noise-opt BJT**

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100 \]

\[ x_N(x) := \left( \frac{4 \cdot k \cdot T}{Rf_N} \cdot I_{2,N} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{2,N} + 2 \cdot q \cdot \frac{x}{x} \left( \frac{25 \cdot 10^{-3}}{(25 \cdot 10^{-3})} \right)^2 \cdot I_{3,N} \right) \]

\[ I_{c,opt,N} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3,N}}{I_{2,N}}} = 0.006 \]

\[ I_{n,N} := x_N(I_{c,opt,N}) = 1.166 \cdot 10^{-13} \]

**noise**

\[ := I_{n,N} = 1.166 \cdot 10^{-13} \]
Pulse shape terms

\[ f_n := 0.7 \]

\[ \alpha := \frac{0.1784 \cdot T_b}{f_n} = 2.549 \cdot 10^{-10} \]

\[ H_p(\omega) := \exp\left(\frac{-\alpha^2 \cdot \omega^2}{2}\right) \]

\[ \frac{1}{T_s} \cdot 10^3 \]

\[ I_0(t) := \frac{1}{\pi} \cdot \left[ \begin{array}{c} \exp\left(\frac{-\alpha^2 \cdot \omega^2}{2}\right) \cdot \text{Re}\left(\frac{1}{1 + 1j \cdot \frac{\omega}{\omega_c}}\right) \cdot \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_B}} \cdot \exp(1i \cdot \omega \cdot t) \end{array} \right] \]

\[ \frac{1}{T_s} \cdot 10^3 \]

\[ I_1(t) := \frac{T_s}{\pi} \cdot \left[ \begin{array}{c} \exp\left(\frac{-\alpha^2 \cdot \omega^2}{2}\right) \cdot \text{Re}\left(1i \cdot \omega \cdot \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_c}}\right) \cdot \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_B}} \cdot \exp(1i \cdot \omega \cdot t) \end{array} \right] \]

\[ t := -1 \cdot T_s, -0.9 \cdot T_s, 6 \cdot T_s \]
Peak voltage

\[ t := 0.3 \cdot T_s \]
\[ t_{pk} := \text{root}(T_s \cdot I_1(t), t) \]
\[ t_{pk} = 1.285 \cdot 10^{-10} \]
\[ v_{pk} := I_0(t_{pk}) = 1.469 \cdot 10^0 \]

Erasure of pulse

\[ Q_{e_i} := \eta q \cdot \frac{v_{pk} - v \cdot v_{pk}}{\sqrt{\text{noise}}} \]
\[ P_r(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{e_i} \cdot b}{\sqrt{2}} \right) \]
\[ P_{er}(b, i) := 2 \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_r(b, i) \cdot (x+1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_r(b, i) \cdot (n+1) \]

False alarm

False alarm when pulse appears in slot R can spread into S-slot of following symbol \( P_{\text{efIISI}} \) or into previous S-slot of same symbol

\[ v_{dSI_{1,Ts}} := v \cdot I_0(t_{pk} - T_s) \]
\[ v_{dSI_{2,Ts}} := v \cdot I_0(t_{pk} + T_s) \]
\[ Q_{eISI_{i}} := \eta q \cdot \frac{v_{i} \cdot v_{pk} - v_{oISI_{1,TS}i}}{\sqrt{\text{noise}}} \]

\[ Q_{eISI_{2,i}} := \eta q \cdot \frac{v_{i} \cdot v_{pk} - v_{oISI_{2,TS}i}}{\sqrt{\text{noise}}} \]

\[ P_{efISI_{i}}(b,i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{eISI_{i}} \cdot b}{\sqrt{2}} \right) \]

\[ P_{efISI_{2,i}}(b,i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{eISI_{2,i}} \cdot b}{\sqrt{2}} \right) \]

\[ P_{efR}(b,i) := \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{efISI_{1}}(b,i) \cdot (x) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{efISI_{1}}(b,i) \cdot (n) \]

\[ + \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{efISI_{2}}(b,i) \cdot (x+1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{efISI_{2}}(b,i) \cdot (n+1) \]

False alarm no ISI occurs between S and R and the error appears within the run of N-symbols where k is the symbol position.

False alarm between R and S pulses - N to SET

\[ Q_{NS}(b,i) := b \cdot \eta q \cdot \frac{v_{i} \cdot v_{pk}}{\sqrt{\text{noise}}} \]

\[ P_{NS}(b,i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NS}(b,i)}{\sqrt{2}} \right) \]

\[ P_{eNS}(b,i) := \sum_{y=3}^{n-1} \sum_{k=0}^{y-1} \left( \frac{1}{2} \right)^{y+3} \cdot P_{NS}(b,i) \cdot (y+1-k) \]

\[ + \sum_{k=0}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NS}(b,i) \cdot (n+1-k) \]

False alarm between S and R pulses - N to R

\[ Q_{NR}(b,i) := b \cdot \eta q \cdot \frac{v_{i} \cdot v_{pk}}{\sqrt{\text{noise}}} \]
\[ P_{NR}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NR}(b, i)}{\sqrt{2}} \right) \]

\[ P_{eNR}(b, i) := \sum_{x=3}^{n-1} \sum_{k=2}^{x-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{NR}(b, i) \cdot (x+1-k) \]

\[ + \sum_{k=2}^{n+2} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NR}(b, i) \cdot (n+1-k) \]

\[ P_{fN}(b, i) := P_{eNS}(b, i) + P_{eNR}(b, i) \]

\[ P_{efN}(b, i) := \sum_{x=1}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=1}^{x} \left( P_{fN}(b, i) \cdot (x+1-k) \right) \]

\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=1}^{n} \left( P_{fN}(b, i) \cdot (n+1-k) \right) \]

\[ + \sum_{x=2}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=2}^{x} \left( P_{fN}(b, i) \cdot (x+1-k) \right) \]

\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=2}^{n} \left( P_{fN}(b, i) \cdot (n+1-k) \right) \]

**Total false alarm**

\[ P_{ef}(b, i) := P_{efN}(b, i) + P_{efR}(b, i) \]

\[ b := 5 \cdot 10^3 \]

**Total Error**

\[ P_{eb}(b, i) := P_{er}(b, i) + P_{efR}(b, i) + P_{efN}(b, i) \]

\[ p_c(b, i) := \log(P_{eb}(b, i)) + 9 \]

\[ a_i := \text{root}(p_c(b, i), b) \]

\[ v_{off} \equiv 0.7 \]

minimum := \text{min}(a)

minimum = 3.432 \cdot 10^4

\[ gu \equiv 2 \]

\[ dBm := 10 \cdot \log \left( \text{minimum} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \left( \frac{n+1}{2 \cdot (gu+2) \cdot n} \right) \cdot B \right) \]

= -32.18
C.6.3 Tuned B di-code PPM receiver with 1st order filter (PIN-BJT/$f_n=5$)

Di-code PPM (PIN-BJT 1st order 1 Gbit/s)

PPM Rx performance with PIN-BJT input configuration, 1st order pre-detection filter, calculations are as follow
- Di-code PPM terms
- Rx terms (noise+TF)
- Noise optimisation
- Pulse shaping, voltages
- ISI/Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

**Di-code PPM terms**

$i := 0, \ldots, 100 \quad n := 10 \quad j := 0, \ldots, 10 \quad$ Set up the scan limits

$v_i := v_{off} + \frac{i}{1000}$

$x := 0, \ldots, n \quad$ This gives the row of the matrix

$y := 0, \ldots, n \quad$ This gives the column of the matrix

$B := 1 \cdot 10^9 \quad$ Bit rate

$T_b := \frac{1}{B} = 1 \cdot 10^{-9} \quad$ PCM bit time

$T_s := \frac{T_b}{2 + gn} \quad$ Slot time

$n := 10 \quad$ Number of like symbols in PCM

$\eta q := 1.6 \cdot 10^{-19} \quad$ Quantum energy

$\lambda := 1.55 \cdot 10^{-6} \quad$ This is the wavelength of operation

$\text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda}$

$R_o := \frac{\eta q}{\text{photon\_energy}}$

$PPM_B := \frac{1}{T_s} = 4 \cdot 10^9 \quad$ This PPM bit rate, used for filter bandwidth
\[ ts := \frac{1}{PPM_B} = 2.5 \cdot 10^{-10} \]

**Preamplifier terms**

\[ Av := 10 \quad C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]

Feedback value for (R) Tuned B

\[ \Delta_L := 1.5 \quad \Delta_R := 2.65 \quad \text{feedback } \Delta R, \text{ time constant ratio } \Delta L \]

\[ Rf_B := \Delta_R \cdot \frac{Av + 1}{2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \cdot C_T} = 1.5465 \cdot 10^3 \quad \text{feedback for Tuned B} \]

\[ \alpha_b := 0.5 \quad \text{splitting ratio} \]

\[ L_c := \frac{Rf_B^2}{1 + Av} \cdot C_T \left( \frac{1}{\Delta_L} \right) = 1.976 \cdot 10^{-8} \]

\[ C1 := (1 - \alpha_b) \cdot (C_T) = 7.5 \cdot 10^{-13} \]

\[ C2 := \alpha_b \cdot (C_T) = 7.5 \cdot 10^{-13} \]

\[ Z_{TIA_B}(\omega) := \frac{1}{\left( (1 - \omega^2 \cdot L_c \cdot C1) + \frac{Rf_B}{(1 + Av)} \cdot (\omega \cdot i) \cdot (C1 + C2 - (\omega^2 \cdot L_c \cdot C1 \cdot C2) \right)} \]

\[ \omega_B := 2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \]

\[ \text{filter } (\omega) := \frac{1}{1 + 1i \cdot \frac{\omega}{\omega_B}} \]

**Receiver noise**

\[ Z_{tuned_B}(\omega) := Z_{TIA_B}(\omega) \cdot \text{filter } (\omega) \]

\[ NEB_B := \frac{1}{2 \cdot \pi} \cdot \left\{ 10^{10} \int_{0}^{10^{10}} \left( |Z_{tuned_B}(\omega)|^2 \right) d\omega \right\} = 1.765 \cdot 10^9 \]

\[ I_{2_B} := \frac{1}{2 \cdot \pi} \cdot \left\{ 10^{10} \int_{0}^{10^{10}} \left( |(1 - \omega^2 \cdot L_c \cdot C1) \cdot Z_{tuned_B}(\omega)|^2 \right) d\omega \right\} = 1.154 \cdot 10^9 \]

C-193
\[ I_{3\,B} := \frac{1}{2\pi} \int_{0}^{10^3} \left( \frac{\left((1 - \omega^2 \cdot Lc \cdot C1)\right) - Rf_B \cdot (\omega) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \cdot 1i}{Rf_B} \cdot Z_{\text{tuned}\,B}(\omega) \right)^2 d\omega = 9.976 \cdot 10^4 \]

\textit{noise-opt BJT}

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100 \]

\[ x_B(x) := \left( \frac{4 \cdot k \cdot T}{Rf_B} \cdot I_{2\,B} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{2\,B} + 2 \cdot q \cdot \left( \frac{x}{25 \cdot 10^{-3}} \right)^2 \cdot I_{3\,B} \right) \]

\[ I_{\text{opt}\,B} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3\,B}}{I_{2\,B}}} = 2.3241 \cdot 10^{-3} \]

\[ I_{n\,B} := x_B(I_{\text{opt}\,B}) = 2.945 \cdot 10^{-14} \]

\textit{noise} := \[ I_{n\,B} = 2.945 \cdot 10^{-14} \]

\textbf{Pulse shape terms}

\[ f_n := 5 \]

\[ \alpha := \frac{0.1784 \cdot T_b}{f_n} = 3.568 \cdot 10^{-11} \]

\[ H_p(\omega) := \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \]

\[ I_0(t) := \frac{1}{\pi} \int_{-10^3}^{10^3} \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \cdot \text{Re} \left( \frac{1}{\left((1 - \omega^2 \cdot Lc \cdot C1)\right) - Rf_B \cdot (\omega) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \cdot 1i} \right) \left( \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_B}} \cdot \exp(1i \cdot \omega \cdot (t)) \right) d\omega \]
\[ I_1(t) := \frac{T_s}{\pi} \int_0^{1/T_s} \exp\left(-\frac{\alpha^2 \cdot \omega^2}{2}\right) \, d\omega \]

\[ \cdot \text{Re} \left( \frac{1}{\left(1 - \omega^2 \cdot Lc \cdot C1\right)} \cdot \frac{1}{1 + \omega \cdot 1i \cdot \left(1 + C2 - (\omega^2 \cdot Lc \cdot C1 \cdot C2)\right) + \frac{Rf_B}{\sqrt{1 + Av}}} \cdot \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_B}} \cdot \exp(1i \cdot \omega \cdot (t)) \right) \]

\[ t := -1 \cdot T_s, -0.9 \cdot T_s, 0.6 \cdot T_s \]

**Peak voltage**

\[ t := 1.2 \cdot T_s \]

\[ t_{pk} := \text{root} \left( T_s \cdot I_1(t), t \right) \]

\[ t_{pk} = 2.953 \cdot 10^{-10} \]

\[ v_{pk} := I_0 \left( t_{pk} \right) = 3.813 \cdot 10^9 \]

**Erasure of pulse**

\[ Q_{e_i} := \eta q \cdot \frac{v_{pk} - v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]
\[ P_r(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{ei} \cdot b}{\sqrt{2}} \right) \]

\[ P_{er}(b, i) := 2 \cdot \sum_{x=0}^{n-1} \frac{1}{2} \cdot P_r(b, i) \cdot (x + 1) + \frac{1}{2} \cdot P_r(b, i) \cdot (n + 1) \]

**False alarm**

False alarm when pulse appears in slot R can spread into S-slot of following symbol or into previous S-slot of same symbol

\[ v_{oISI1_{Ts_i}} := v_i \cdot I_0 (t_{pk} - T_s) \]

\[ v_{oISI2_{Ts_i}} := v_i \cdot I_0 (t_{pk} + T_s) \]

\[ Q_{eISI1_i} := \eta q \cdot \frac{v_i \cdot v_{pk} - v_{oISI1_{Ts_i}}}{\sqrt{\text{noise}}} \]

\[ Q_{eISI2_i} := \eta q \cdot \frac{v_i \cdot v_{pk} - v_{oISI2_{Ts_i}}}{\sqrt{\text{noise}}} \]

\[ P_{efISI1}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{eISI1_i} \cdot b}{\sqrt{2}} \right) \]

\[ P_{efISI2}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{eISI2_i} \cdot b}{\sqrt{2}} \right) \]

\[ P_{efR}(b, i) := \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{efISI1}(b, i) \cdot (x) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{efISI1}(b, i) \cdot (n) \]

\[ + \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{efISI2}(b, i) \cdot (x + 1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{efISI2}(b, i) \cdot (n + 1) \]

False alarm no ISI occurs between S and R and the error appears within the run of N-symbols where \( k \) is the symbol position

False alarm between R and S pulses - N to SET

\[ Q_{NS}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

C-196
\[ P_{NS}(b, i) = \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NS}(b, i)}{\sqrt{2}} \right) \]

\[ P_{eNS}(b, i) := \sum_{y=2}^{n-1} \sum_{k=2}^{y-1} \left( \frac{1}{2} \right)^{y+3} \cdot P_{NS}(b, i) \cdot (y+1-k) \]
\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NS}(b, i) \cdot (n+1-k) \]

False alarm between S and R pulses - N to R

\[ Q_{NR}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

\[ P_{NR}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NR}(b, i)}{\sqrt{2}} \right) \]

\[ P_{eNR}(b, i) := \sum_{x=3}^{n-1} \sum_{k=2}^{x-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{NR}(b, i) \cdot (x+1-k) \]
\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NR}(b, i) \cdot (n+1-k) \]

\[ P_{FN}(b, i) := P_{eNS}(b, i) + P_{eNR}(b, i) \]

\[ P_{efN}(b, i) := \sum_{x=1}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=1}^{x} \left( P_{FN}(b, i) \cdot (x+1-k) \right) \]
\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=1}^{n} \left( P_{FN}(b, i) \cdot (n+1-k) \right) \]
\[ + \sum_{x=2}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=2}^{x} \left( P_{FN}(b, i) \cdot (x+1-k) \right) \]
\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=2}^{n} \left( P_{FN}(b, i) \cdot (n+1-k) \right) \]

Total false alarm

\[ P_{ef}(b, i) := P_{efN}(b, i) + P_{eFR}(b, i) \]

\[ b := 5 \cdot 10^3 \]

Total Error

\[ P_{eb}(b, i) := P_{er}(b, i) + P_{eFR}(b, i) + P_{efN}(b, i) \]

\[ pc(b, i) := \log (P_{eb}(b, i)) + 9 \]

\[ a_i := \text{root}(pc(b, i), b) \]
\[ v_{off} \equiv 0.45 \]

\[ \text{minimum} = \min(a) \]

\[ \text{minimum} = 3.482 \cdot 10^3 \]

\[ gu \equiv 2 \]

\[ \text{noise} = 2.945 \cdot 10^{-14} \]

\[ dBm := 10 \cdot \log \left( \frac{\text{minimum} \cdot \text{photon} \_ \text{energy}}{10^{-3}} \cdot \frac{n + 1}{2 \cdot (gu + 2) \cdot n} \cdot B \right) = -42.116 \]
C.6.4 Tuned B di-code PPM receiver with 1st order filter (PIN-BJT/\(f_n=0.7\))

**Di-code PPM (PIN-BJT 1st order 1 Gbit/s)**

PPM Rx performance with PIN-BJT input configuration, 1st order pre-detection filter. calculations are as follow

- Di-code PPM terms
- Rx terms (noise+TF)
- Noise optimisation
- Pulse shaping, voltages
- ISI/Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

**Di-code PPM terms**

\[ i := 0, 1 \ldots 100 \quad n := 10 \quad j := 0 \ldots 10 \quad \text{Set up the scan limits} \]

\[ v_i := v_{\text{off}} + \frac{i}{1000} \]

\[ x := 0 \ldots n \quad \text{This gives the row of the matrix} \quad y := 0 \ldots n \quad \text{This gives the column of the matrix} \]

\[ B := 1 \cdot 10^9 \quad \text{Bit rate} \]

\[ T_b := \frac{1}{B} = 1 \cdot 10^{-9} \quad \text{PCM bit time} \]

\[ T_s := \frac{T_b}{2 + g\nu} \quad \text{Slot time} \]

\[ n := 10 \quad \text{Number of like symbols in PCM} \]

\[ \eta q := 1.6 \cdot 10^{-19} \quad \text{Quantum energy} \]

\[ \lambda := 1.55 \cdot 10^{-6} \quad \text{This is the wavelength of operation} \]

\[ \text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ R_o := \frac{\eta q}{\text{photon\_energy}} \]

\[ PPM_B := \frac{1}{T_s} = 4 \cdot 10^9 \quad \text{this PPM bit rate, used for filter bandwidth} \]

C-199
\[
C = 2.5 \times 10^{-10}
\]

**Preamplifier terms**

\[Av := 10 \quad C_T := 1.5 \times 10^{-12} \quad \text{total C}\]

Feedback value for (R) Tuned B

\[
\Delta_L := 1.5 \quad \Delta_R := 2.65 \quad \text{feedback } \Delta R \quad \text{time constant ratio } \Delta L
\]

\[Rf_B := \frac{\Delta_R \cdot Av + 1}{2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \cdot C_T} = 1.5465 \times 10^{3} \quad \text{feedback for Tuned B}\]

\[\alpha_b := 0.5 \quad \text{splitting ratio}\]

\[L_c := \frac{\left(\frac{Rf_B}{1 + Av}\right)^2 \cdot C_T}{\Delta_L} = 1.976 \times 10^{-8}\]

\[C1 := (1 - \alpha_b) \cdot (C_T) = 7.5 \times 10^{-13}\]

\[C2 := \alpha_b \cdot (C_T) = 7.5 \times 10^{-13}\]

\[Z_{TIA_B}(\omega) := \frac{1}{\left(\left(1 - \omega^2 \cdot L_c \cdot C1\right) + \frac{Rf_B}{\left(1 + Av\right)} \cdot (\omega \cdot i) \cdot (C1 + C2 - (\omega)^2 \cdot L_c \cdot C1 \cdot C2)\right)}\]

\[\omega_B := 2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s}\]

\[\text{filter}(\omega) := \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_B}}\]

**Receiver noise**

\[Z_{\text{tuned}_B}(\omega) := Z_{TIA_B}(\omega) \cdot \text{filter}(\omega)\]

\[\text{NEB}_B := \frac{1}{2 \cdot \pi} \cdot \left(\int_{0}^{10^{11}} \left(\left|Z_{\text{tuned}_B}(\omega)\right|\right)^2 \, d\omega\right) = 1.765 \times 10^{9}\]

\[I_{\cdot 2\cdot B} := \frac{1}{2 \cdot \pi} \cdot \left(\int_{0}^{10^{11}} \left(\left|\left(1 - \omega^2 \cdot L_c \cdot C1\right) \cdot Z_{\text{tuned}_B}(\omega)\right|^2 \right) \, d\omega\right) = 1.154 \times 10^{9}\]
\[
I_{3\cdot B} = \frac{1}{2\pi} \int_{0}^{10^6} \left( \left( \frac{\omega^2}{Rf_B} \cdot (1 + \frac{C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2}{Rf_B}) \cdot Z_{tuned\cdot B}(\omega) \right)^\frac{1}{4} \right)^2 d\omega = 9.976 \cdot 10^4
\]

\text{noise-opt BJT}
\[
q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100
\]

\[
x_B(x) := 4 \cdot k \cdot T \cdot I_{2\cdot B} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right)^2 \cdot I_{2\cdot B} + 2 \cdot q \cdot \left( \frac{x}{(25 \cdot 10^{-3})} \right)^2 \cdot I_{3\cdot B}
\]

\[
I_{\text{opt} \cdot B} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3\cdot B}}{I_{2\cdot B}}} = 2.3241 \cdot 10^{-3}
\]

\[
I_{n\cdot B} := x_B(I_{\text{opt} \cdot B}) = 2.945 \cdot 10^{-14}
\]

\text{noise} := I_{n\cdot B} = 2.945 \cdot 10^{-14}

\text{Pulse shape terms}
\]

\[
f_n := 0.7
\]

\[
\alpha := \frac{0.1784 \cdot T_b}{f_n} = 2.549 \cdot 10^{-10}
\]

\[
H_p(\omega) := \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right)
\]

\[
I_0(t) := \frac{1}{\pi} \int_{0}^{T_b \cdot 10^3} \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \cdot \text{Re} \left( \frac{1}{(1 - \omega^2 \cdot Lc \cdot C1) \cdot \left( 1 + \frac{C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2}{Rf_B} \right)} \cdot \left( \frac{1}{1 + \frac{\omega}{\omega_B}} \cdot \exp \left( 1i \cdot \omega \cdot t \right) \right) \right) d\omega
\]
$$I_1(t) := \frac{T_s}{\pi} \left[ \frac{1}{T_s} \cdot 10^5 \right] \exp \left( -\frac{\alpha^2 \cdot \omega^2}{2} \right) \cdot \Re \left( \frac{1}{\left( \frac{1 - \omega^2 \cdot Lc \cdot C1}{(1 + Av)} \right) \cdot \left( \omega \cdot 1i \cdot (C1 + C2 - (\omega^2 \cdot Lc \cdot C1 \cdot C2) \right) \cdot \exp \left( 1i \cdot \omega \cdot (t) \right) }{1 + 1j \cdot \frac{\omega}{\omega B}} \right) \frac{1}{\omega} \right) d\omega$$

$$t := -1 \cdot T_s, -0.9 \cdot T_s, 0.6 \cdot T_s$$

Peak voltage

$$t := 1.2 \cdot T_s$$

$$t_{pk} := \text{root} (T_s \cdot I_1(t), t)$$

$$t_{pk} = 2.708 \cdot 10^{-10}$$

$$v_{pk} := I_0(t_{pk}) = 1.47 \cdot 10^9$$

Erasure of pulse

$$Q_{e_i} = \eta q \cdot \frac{v_{pk} - v \cdot v_{pk}}{\sqrt{\text{noise}}}$$
\[ P_r(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{e1} \cdot b}{\sqrt{2}} \right) \]

\[ P_{er}(b, i) := 2 \cdot \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_r(b, i) \cdot (x + 1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_r(b, i) \cdot (n + 1) \]

**False alarm**

False alarm when pulse appears in slot R can spread into S-slot of following symbol or into previous S-slot of same symbol

\[ v_{oIS1 \_Ts_i} := v_i \cdot I_0 (t_{pk} - T_s) \]

\[ v_{oIS2 \_Ts_i} := v_i \cdot I_0 (t_{pk} + T_s) \]

\[ Q_{eIS1_i} := \eta_q \cdot \frac{v_i \cdot v_{pk} - v_{oIS1 \_Ts_i}}{\sqrt{\text{noise}}} \]

\[ Q_{eIS2_i} := \eta_q \cdot \frac{v_i \cdot v_{pk} - v_{oIS2 \_Ts_i}}{\sqrt{\text{noise}}} \]

\[ P_{efIS1}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{eIS1_i} \cdot b}{\sqrt{2}} \right) \]

\[ P_{efIS2}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{eIS2_i} \cdot b}{\sqrt{2}} \right) \]

\[ P_{efR}(b, i) := \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{efIS1}(b, i) \cdot (x + 1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{efIS1}(b, i) \cdot (n + 1) \]

False alarm no ISI occurs between S and R and the error appears within the run of N-symbols where k is the symbol position

False alarm between R and S pulses - N to SET

\[ Q_{NS}(b, i) := b \cdot \eta_q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]
\[ P_{NS}(b,i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NS}(b,i)}{\sqrt{2}} \right) \]

\[ P_{eNS}(b,i) := \sum_{y=2}^{n-1} \sum_{k=2}^{y-1} \left( \frac{1}{2} \right)^{y+3} \cdot P_{NS}(b,i) \cdot (y+1-k) \]
\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NS}(b,i) \cdot (n+1-k) \]

\text{False alarm between S and R pulses - N to R}

\[ Q_{NR}(b,i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

\[ P_{NR}(b,i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NR}(b,i)}{\sqrt{2}} \right) \]

\[ P_{eNR}(b,i) := \sum_{x=3}^{n-1} \sum_{k=2}^{x-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{NR}(b,i) \cdot (x+1-k) \]
\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NR}(b,i) \cdot (n+1-k) \]

\[ P_{JN}(b,i) := P_{eNS}(b,i) + P_{eNR}(b,i) \]

\[ P_{eJN}(b,i) := \sum_{x=1}^{n-1} \left( \frac{1}{2} \right)^{x+3} \sum_{k=1}^{x} \left( P_{JN}(b,i) \cdot (x+1-k) \right) \]
\[ + \left( \frac{1}{2} \right)^{n+2} \sum_{k=1}^{n} \left( P_{JN}(b,i) \cdot (n+1-k) \right) \]
\[ + \sum_{x=2}^{n-1} \left( \frac{1}{2} \right)^{x+3} \sum_{k=2}^{x} \left( P_{JN}(b,i) \cdot (x+1-k) \right) \]
\[ + \left( \frac{1}{2} \right)^{n+2} \sum_{k=2}^{n} \left( P_{JN}(b,i) \cdot (n+1-k) \right) \]

\text{Total false alarm}

\[ P_{ef}(b,i) := P_{eJN}(b,i) + P_{eNR}(b,i) \]
\[ b := 5 \cdot 10^3 \]

\text{Total Error}

\[ P_{eb}(b,i) := P_{er}(b,i) + P_{eJN}(b,i) + P_{eNR}(b,i) \]
\[ pc(b,i) := \log \left( P_{eb}(b,i) \right) + 9 \]
\[ a_i := \text{root} \left( pc(b,i), b \right) \]
$v_{off} = 0.7$

$minimum := \min(a)$

$gu = 2$

$minimum = 1.648 \cdot 10^4$

$noise = 2.945 \cdot 10^{-14}$

$dBm := 10 \cdot \log\left(\frac{minimum \cdot photon\_energy}{10^{-3}} \cdot \left(\frac{n+1}{2 \cdot (gu + 2) \cdot n}\right) \cdot B\right) = -35.363$

Tuned B vs non-tuned

$32.18 - 35.363 = -3.183$
C.7 Di-code PPM receiver (Optical wireless)

C.7.1 Non-tuned di-code PPM receiver (ideal LOS)

Di-code PPM (PIN-BJT Butterworth 1 Gbit/s)

PPM Rx performance with PIN-BJT input configuration, Butterworth pre-detection filter. Calculations are as follows:

- Di-code PPM terms
- Rx terms (noise+TF)
- Noise optimisation
- Pulse shaping, voltages
- ISI/Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

**Di-code PPM terms**

\[ i := 0, 1 \ldots 100 \quad n := 10 \quad j := 0 \ldots 10 \quad \text{Set up the scan limits} \]

\[ v_i := v_{\text{off}} + \frac{i}{1000} \]

\[ x := 0 \ldots n \quad \text{This gives the row of the matrix} \quad y := 0 \ldots n \quad \text{This gives the column of the matrix} \]

\[ B := 1 \cdot 10^9 \quad \text{Bit rate} \]

\[ T_b := \frac{1}{B} = 1 \cdot 10^{-9} \quad \text{PCM bit time} \]

\[ T_s := \frac{T_b}{2 + gu} \quad \text{Slot time} \]

\[ n := 10 \quad \text{Number of like symbols in PCM} \]

\[ \eta q := 1.6 \cdot 10^{-10} \quad \text{Quantum energy} \]

\[ \lambda := 1.55 \cdot 10^{-6} \quad \text{This is the wavelength of operation} \]

\[ \text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ R_o := \frac{\eta q}{\text{photon\_energy}} \]

\[ PPM_B := \frac{1}{T_s} = 4 \cdot 10^9 \quad \text{this PPM bit rate, used for filter bandwidth} \]

\[ ts := \frac{1}{PPM_B} = 2.5 \cdot 10^{-10} \]
Preamplifier terms

\[ Av := 10 \quad C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]

\[ \omega_c := 2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} = 1.257 \cdot 10^{10} \]

\[ Z_{TIA_N} (\omega) := \frac{1}{1 + j \frac{\omega}{\omega_c}} \]

\[ R_{f_{N}} := \frac{Av + 1}{2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \cdot C_T} = 583.568 \]

\[ \omega_B := 2 \cdot \pi \cdot 0.7 \cdot \frac{1}{T_s} \]

\[ \text{filter} (\omega) := \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \]

\[ Z_{\text{nontuned}} (\omega) := Z_{TIA_N} (\omega) \cdot \text{filter} (\omega) \]

Receiver noise

\[ N_E_{B_N} := \frac{1}{2 \cdot \pi} \cdot \left( \int_{10^{13}}^{10^{14}} (|Z_{\text{nontuned}} (\omega)|)^2 \, d\omega \right) = 1.889 \cdot 10^9 \]

\[ I_{2_{N}} := N_{E_B_N} = 1.889 \cdot 10^9 \]

\[ I_{3_{N}} := \frac{1}{2 \cdot \pi} \cdot \left( \int_{0}^{10^{13}} \left( \frac{1 + j \cdot (\omega) \cdot R_{f_N} \cdot C_T}{1 \cdot R_{f_N} \cdot Z_{\text{nontuned}} (\omega)} \right)^2 \, d\omega \right) = 3.763 \cdot 10^5 \]

noise-opt BJT

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad h_{fe} := 100 \]

\[ x_\text{N} (x) := \left( \frac{4 \cdot k \cdot T}{R_{f_N} \cdot I_{2_N}} \right) + \left( 2 \cdot q \cdot \frac{x}{h_{fe}} \cdot I_{2_N} + 2 \cdot q \cdot \frac{x}{(25 \cdot 10^{-3})} \cdot I_{3_N} \right) \]

\[ I_{\text{copt}_N} := 25 \cdot 10^{-3} \cdot \sqrt{h_{fe}} \cdot \sqrt{\frac{I_{3_N}}{I_{2_N}}} = 0.004 \]

\[ I_{n_N} := x_\text{N} (I_{\text{copt}_N}) = 9.589 \cdot 10^{-14} \]

\[ \text{noise} := I_{n_N} = 9.589 \cdot 10^{-14} \]

C-207
Pulse shape terms

\[ I_0(t) := \frac{1}{\pi} \int_0^{\frac{1}{T_s} \cdot 10^9} \left[ \sin \left( \frac{\omega \cdot T_s}{2} \right) \cdot \text{Re} \left( \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_c}} \cdot \frac{\omega^3_B}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \right) \right] d\omega \]

\[ I_1(t) := \frac{T_s}{\pi} \int_0^{\frac{1}{T_s} \cdot 10^9} \left[ \sin \left( \frac{\omega \cdot T_s}{2} \right) \cdot \text{Re} \left( \frac{1i \cdot \omega}{1 + 1j \cdot \frac{\omega}{\omega_c}} \cdot \frac{\omega^3_B}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \right) \right] d\omega \]

\[ t := -1 \cdot T_s, -0.9 \cdot T_s, \ldots, 6 \cdot T_s \]

\[ I_0(t) \]
\[ I_0(t - 3T_s) \]
Peak voltage

\[ t := 0.77 \cdot T_s \]
\[ t_{pk} := \sqrt[3]{T_s \cdot I_1(t), t} \]
\[ t_{pk} = 1.924 \cdot 10^{-10} \]
\[ v_{pk} := I_0(t_{pk}) = 3.672 \cdot 10^9 \]

Erasure of pulse

\[ Q_{e_i} := \eta q \cdot \frac{v_{pk} - v_{i} \cdot v_{pk}}{\sqrt{\text{noise}}} \]
\[ P_r(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{e_i} \cdot b}{\sqrt{2}} \right) \]
\[ P_{er}(b, i) := 2 \cdot \sum_{x=0}^{n-1} \left( \left( \frac{1}{2} \right)^{x+3} \cdot P_r(b, i)^{x+1} + \left( \frac{1}{2} \right)^{n+2} \cdot P_r(b, i) \cdot (n+1) \right) \]

False alarm

False alarm when pulse appears in slot R can spread into S-slot of following symbol \( P_{effIS1} \) or into previous S-slot of same symbol

\[ v_{oIS1 Ts_i} := v_{i} \cdot I_0(t_{pk} - T_s) \]
\[ v_{oIS2 Ts_i} := v_{i} \cdot I_0(t_{pk} + T_s) \]

\[ Q_{eIS1i} := \eta q \cdot \frac{v_{i} \cdot v_{pk} - v_{oIS1 Ts_i}}{\sqrt{\text{noise}}} \]

\[ Q_{eIS2i} := \eta q \cdot \frac{v_{i} \cdot v_{pk} - v_{oIS2 Ts_i}}{\sqrt{\text{noise}}} \]

\[ P_{effIS1}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{eIS1i} \cdot b}{\sqrt{2}} \right) \]
\[ P_{effIS2}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{eIS2i} \cdot b}{\sqrt{2}} \right) \]
\[ P_{efR}(b, i) := \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{efISI_1}(b, i) \cdot (x) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{efISI_1}(b, i) \cdot (n) \]
\[ + \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{efISI_2}(b, i) \cdot (x+1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{efISI_2}(b, i) \cdot (n+1) \]

False alarm no ISI occurs between S and R and the error appears within the run of N-symbols where k is the symbol position

False alarm between R and S pulses - N to SET

\[ Q_{NS}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]
\[ P_{NS}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NS}(b, i)}{\sqrt{2}} \right) \]
\[ P_{eNS}(b, i) := \sum_{y=3}^{n-1} \sum_{k=2}^{y-1} \left( \frac{1}{2} \right)^{y+3} \cdot P_{NS}(b, i) \cdot (y+1-k) \]
\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NS}(b, i) \cdot (n+1-k) \]

False alarm between S and R pulses - N to R

\[ Q_{NR}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]
\[ P_{NR}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NR}(b, i)}{\sqrt{2}} \right) \]
\[ P_{eNR}(b, i) := \sum_{x=3}^{n-1} \sum_{k=2}^{x-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{NR}(b, i) \cdot (x+1-k) \]
\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NR}(b, i) \cdot (n+1-k) \]
\[ P_{fN}(b, i) := P_{eNS}(b, i) + P_{eNR}(b, i) \]
\[ P_{eFN}(b, i) := \sum_{x=1}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=1}^{x} (P_{fN}(b, i) \cdot (x+1-k)) \]
\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=1}^{n} (P_{fN}(b, i) \cdot (n+1-k)) \]
\[ + \sum_{x=2}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=2}^{x} (P_{fN}(b, i) \cdot (x+1-k)) \]
\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=2}^{n} (P_{fN}(b, i) \cdot (n+1-k)) \]

C-210
Total false alarm

\[ P_{ef}(b, i) := P_{efN}(b, i) + P_{efR}(b, i) \]

\[ b := 5 \cdot 10^3 \]

Total Error

\[ P_{eb}(b, i) := P_{er}(b, i) + P_{efR}(b, i) + P_{efN}(b, i) \]

\[ pc(b, i) := (\log P_{eb}(b, i)) + 9 \]

\[ a_i := \text{root}(pc(b, i), b) \]

\[ v_{off} \equiv 0.45 \]

\[ \text{minimum} := \min(a) \]

\[ \text{minimum} = 6.573 \cdot 10^3 \]

\[ gu \equiv 2 \]

\[ dBm := 10 \cdot \log \left( \frac{\text{minimum} \cdot \text{photon\_energy}}{10^{-3}} \cdot \left( \frac{n + 1}{2 \cdot \left( gu + 2 \right) \cdot n} \right) \cdot B \right) = -39.36 \]
C.7.2 Tuned B (0.5) di-code PPM receiver (ideal LOS)

Di-code PPM (PIN-BJT Butterworth 1 Gbit/s)

PPM Rx performance with PIN-BJT input configuration, Butterworth pre-detection filter. Calculations are as follows:
- Di-code PPM terms
- Rx terms (noise+TF)
- Noise optimisation
- Pulse shaping, voltages
- ISI/Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

**Di-code PPM terms**

\[ i := 0, 1 \ldots 100 \quad n := 10 \quad j := 0 \ldots 10 \quad \text{Set up the scan limits} \]

\[ v_i := v_{\text{off}} + \frac{i}{1000} \]

\[ x := 0 \ldots n \quad \text{This gives the row of the matrix} \quad y := 0 \ldots n \quad \text{This gives the column of the matrix} \]

\[ B := 1 \cdot 10^9 \quad \text{Bit rate} \]

\[ T_b := \frac{1}{B} = 1 \cdot 10^{-9} \quad \text{PCM bit time} \]

\[ T_s := \frac{T_b}{2 + \gamma u} \quad \text{Slot time} \]

\[ n := 10 \quad \text{Number of like symbols in PCM} \]

\[ \eta q := 1.6 \cdot 10^{-10} \quad \text{Quantum energy} \]

\[ \lambda := 1.55 \cdot 10^{-6} \quad \text{This is the wavelength of operation} \]

\[ \text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ R_o := \frac{\eta q}{\text{photon\_energy}} \]

\[ \text{PPM\_B} := \frac{1}{T_s} = 4 \cdot 10^9 \quad \text{this PPM bit rate, used for filter bandwidth} \]
$$\text{Preamplifier terms}$$

$$A_v := 10 \quad C_T := 1.5 \cdot 10^{-12} \quad \text{total C}$$

Feedback value for (R) Tuned B

$$\Delta_L := 1.5 \quad \Delta_R := 2.65$$ feedback $$\Delta R$$, time constant ratio $$\Delta L$$

$$Rf_B := \Delta_R \frac{A_v + 1}{2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \cdot C_T} = 1.5465 \cdot 10^3$$ feedback for Tuned B

$$\alpha_b := 0.5$$ splitting ratio

$$L_c := \frac{\left( \frac{Rf_B}{1 + A_v} \right)^2 \cdot C_T}{\Delta_L} = 1.976 \cdot 10^{-8}$$

$$C1 := (1 - \alpha_b) \cdot (C_T) = 7.5 \cdot 10^{-13}$$

$$C2 := \alpha_b \cdot (C_T) = 7.5 \cdot 10^{-13}$$

$$Z_{TIA,B}(\omega) := \frac{1}{\left( \left( 1 - \omega^2 \cdot L_c \cdot C1 \right) + \frac{Rf_B}{(1 + A_v)} \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot L_c \cdot C1 \cdot C2) \right)}$$

$$\omega_B := 2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s}$$

$$\text{filter}(\omega) := \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3}$$

$$Z_{tuned,B}(\omega) := Z_{TIA,B}(\omega) \cdot \text{filter}(\omega)$$

$$NEB_B := \frac{1}{2 \cdot \pi} \left( \int_{0}^{T_s} \left( \left| Z_{tuned,B}(\omega) \right|^2 \right) d\omega \right) = 2.048 \cdot 10^9$$

$$I_{2,B} := \frac{1}{2 \cdot \pi} \left( \int_{0}^{T_s} \left( \left| \left( 1 - \omega^2 \cdot L_c \cdot C1 \right) \cdot Z_{tuned,B}(\omega) \right|^2 \right) d\omega \right) = 1.068 \cdot 10^9$$
\( I_{3B} := \frac{1}{2\pi} \int_{0}^{10^3} \left( \frac{\left((1 - \omega^2 \cdot Lc \cdot C1) \right)_+}{\text{If}_{-}B \cdot (\omega \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \cdot i)} \cdot Z_{tuned_B}(\omega) \right) \, d\omega = 5.239 \cdot 10^4 \)

noise-opt BJT

\( q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100 \)

\( I_{B} := \frac{4 \cdot k \cdot T}{\text{If}_{-}B} \cdot I_{2B} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{2B} + 2 \cdot q \cdot \left( \frac{x}{x} \right) \cdot \sqrt{25 \cdot 10^{-3}} \cdot I_{3B} \)

\( I_{opt_B} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3B}}{I_{2B}}} = 1.7513 \cdot 10^{-3} \)

\( I_n := I_{B}(I_{opt_B}) = 2.332 \cdot 10^{-14} \)

\( \text{noise} := I_n = 2.332 \cdot 10^{-14} \)

Pulse shape terms

\( \frac{1}{T_s} \cdot 10^2 \)

\( I_0(t) := \frac{1}{\pi} \int_{0}^{10^2} \sin \left( \frac{\omega \cdot T_s}{2} \right) \cdot \left( \frac{\omega \cdot T_s}{2} \right) \cdot \text{Re} \left[ \frac{1}{\left((1 - \omega^2 \cdot Lc \cdot C1) \right)_+} \cdot \frac{\text{If}_{-}B \cdot (\omega \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2)}{(1 + Av)} \cdot \left( \frac{1}{\omega_B^3} \right) \cdot (1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3 \cdot \exp(11i \cdot \omega \cdot (t)) \right] \, d\omega \)
\[ I_1(t) = \frac{T_s}{\pi} \cdot \frac{1}{2^{10^7}} \cdot \sin \left( \frac{\omega \cdot T_s}{2} \right) \left( \frac{\omega \cdot T_s}{2} \right)^d \cdot \text{Re} \left( \frac{1}{\left( 1 - \omega^2 \cdot Lc \cdot C1 \right)^d} + \frac{Rf \cdot B}{(1 + Av)} \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \right) \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \cdot \exp \left( 1i \cdot \omega \cdot (t) \right) \right) \]

\[ t = -1 \cdot T_s \cdot -0.9 \cdot T_s \cdot 6 \cdot T_s \]

**Peak voltage**

\[ t := 1.2 \cdot T_s \]

\[ t_{pk} := \text{root} \left( T_s \cdot I_1(t), t \right) \]

\[ t_{pk} = 4.062 \cdot 10^{-10} \]

\[ v_{pk} := I_0 \left( t_{pk} \right) = 3.483 \cdot 10^9 \]

**Erasure of pulse**

\[ Q_e := \eta q \cdot \frac{v_{pk} - v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]
\[ P_r(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{c_i} \cdot b}{\sqrt{2}} \right) \]

\[ P_{er}(b, i) := 2 \cdot \sum_{x=0}^{n-1} \left( \left( \frac{1}{2} \right)^{x+3} \cdot P_r(b, i) \cdot (x+1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_r(b, i) \cdot (n+1) \right) \]

**False alarm**

False alarm when pulse appears in slot R can spread into S-slot of following symbol or into previous S-slot of same symbol

\[ v_{ISI1_{T_s}} := v_i \cdot I_0(t_{pk} - T_s) \]

\[ v_{ISI2_{T_s}} := v_i \cdot I_0(t_{pk} + T_s) \]

\[ Q_{eISI1_i} = \eta q \cdot \frac{v_i \cdot v_{pk} - v_{ISI1_{T_s}}}{\text{noise}} \]

\[ Q_{eISI2_i} = \eta q \cdot \frac{v_i \cdot v_{pk} - v_{ISI2_{T_s}}}{\text{noise}} \]

\[ P_{eISI1}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{eISI1_i} \cdot b}{\sqrt{2}} \right) \]

\[ P_{eISI2}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{eISI2_i} \cdot b}{\sqrt{2}} \right) \]

\[ P_{eR}(b, i) := \sum_{x=0}^{n-1} \left( \left( \frac{1}{2} \right)^{x+3} \cdot P_{eISI1}(b, i) \cdot (x) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{eISI1}(b, i) \cdot (n) \right) + \sum_{x=0}^{n-1} \left( \left( \frac{1}{2} \right)^{x+3} \cdot P_{eISI2}(b, i) \cdot (x+1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{eISI2}(b, i) \cdot (n+1) \right) \]

**False alarm no ISI occurs between S and R and the error appears within the run of N-symbols where k is the symbol position**

**False alarm between R and S pulses - N to SET**

\[ Q_{NS}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\text{noise}} \]

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\[ P_{NS}(b,i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NS}(b,i)}{\sqrt{2}} \right) \]

\[ P_{eNS}(b,i) := \sum_{y=3}^{n-1} \sum_{k=2}^{y-1} \left( \frac{1}{2} \right)^{y+3-1} \cdot P_{NS}(b,i) \cdot (y+1-k) \]
\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2-1} \cdot P_{NS}(b,i) \cdot (n+1-k) \]

False alarm between S and R pulses - N to R

\[ Q_{NR}(b,i) := b \cdot \eta \cdot \frac{\nu_i \cdot \nu_{pk}}{\text{noise}} \]

\[ P_{NR}(b,i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NR}(b,i)}{\sqrt{2}} \right) \]

\[ P_{eNR}(b,i) := \sum_{x=1}^{n-1} \sum_{k=2}^{x-1} \left( \frac{1}{2} \right)^{x+3-1} \cdot P_{NR}(b,i) \cdot (x+1-k) \]
\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2-1} \cdot P_{NR}(b,i) \cdot (n+1-k) \]

\[ P_{FN}(b,i) := P_{eNS}(b,i) + P_{eNR}(b,i) \]

\[ P_{efN}(b,i) := \sum_{x=1}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=1}^{x} \left( P_{FN}(b,i) \cdot (x+1-k) \right) \]
\[ + \frac{1}{2} \cdot \sum_{k=1}^{n} \left( P_{FN}(b,i) \cdot (n+1-k) \right) \]
\[ + \sum_{x=2}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=2}^{x} \left( P_{FN}(b,i) \cdot (x+1-k) \right) \]
\[ + \frac{1}{2} \cdot \sum_{k=2}^{n} \left( P_{FN}(b,i) \cdot (n+1-k) \right) \]

Total false alarm

\[ P_{ef}(b,i) := P_{efN}(b,i) + P_{eFR}(b,i) \]

\[ b := 5 \cdot 10^3 \]

Total Error

\[ P_{eb}(b,i) := P_{er}(b,i) + P_{efR}(b,i) + P_{efN}(b,i) \]

\[ pc(b,i) := (\log(P_{eb}(b,i)) + 9) \]

\[ a_i := \text{root}(pc(b,i), b) \]
\[ v_{off} \equiv 0.5 \]

\[ \text{minimum} := \min(a) \quad \text{minimum} = 3.673 \cdot 10^3 \]

\[ gu \equiv 2 \]

\[ dBm := 10 \cdot \log \left( \frac{\text{minimum} \cdot photon\_energy}{10^{-3}} \cdot \left( \frac{n+1}{2 \cdot (gu+2) \cdot n} \right) \cdot B \right) = -41.884 \]
C.7.3 Non-tuned di-code PPM receiver (with wireless channel effect)

**Di-code PPM (PIN-BJT Butterworth 1 Gbit/s)**

PPM Rx performance with PIN-BJT input configuration, Butterworth pre-detection filter. calculations are as follow

- Di-code PPM terms
- Rx terms (noise+TF)
- Noise optimisation
- Channel effect, Convolution, pulse shaping, voltages
- ISI/Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

**Di-code PPM terms**

\[ i := 0, 1 \ldots 100 \quad n := 10 \quad j := 0 \ldots 10 \quad \text{Set up the scan limits} \]

\[ v_i := v_{\text{off}} + \frac{i}{1000} \]

\[ x := 0 \ldots n \quad \text{This gives the row of the matrix} \quad y := 0 \ldots n \quad \text{This gives the column of the matrix} \]

\[ B := 1 \cdot 10^9 \quad \text{Bit rate} \]

\[ T_b := \frac{1}{B} = 1 \cdot 10^{-9} \quad \text{PCM bit time} \]

\[ T_s := \frac{T_b}{2 + gu} \quad \text{Slot time} \]

\[ n := 10 \quad \text{Number of like symbols in PCM} \]

\[ \eta q := 1.6 \cdot 10^{-10} \quad \text{Quantum energy} \]

\[ \lambda := 1.55 \cdot 10^{-6} \quad \text{This is the wavelength of operation} \]

\[ \text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ R_o := \frac{\eta q}{\text{photon\_energy}} \]

\[ PPM_B := \frac{1}{T_s} = 4 \cdot 10^9 \quad \text{this PPM bit rate, used for filter bandwidth} \]

\[ ts := \frac{1}{PPM_B} = 2.5 \cdot 10^{-10} \]
### Preamplifier terms

\[ Av := 10 \quad C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]

\[ \omega_c := 2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} = 1.257 \cdot 10^{10} \]

\[ Z_{TIA,N}(\omega) := \frac{1}{1 + j \cdot \frac{\omega}{\omega_c}} \]

\[ R_{f,N} := \frac{Av + 1}{2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \cdot C_T} = 583.568 \]

\[ \omega_B := 2 \cdot \pi \cdot 0.7 \cdot \frac{1}{T_s} \]

\[ \text{filter}(\omega) := \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \]

\[ Z_{\text{nontuned}}(\omega) := Z_{TIA,N}(\omega) \cdot \text{filter}(\omega) \]

### Receiver noise

\[ NEB_N := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^3} \left( |Z_{\text{nontuned}}(\omega)| \right)^2 d\omega \right) = 1.889 \cdot 10^9 \]

\[ I_{2,N} := NEB_N = 1.889 \cdot 10^9 \]

\[ I_{3,N} := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^3} \left( 1 + j \cdot (\omega \cdot R_{f,N} \cdot C_T \cdot Z_{\text{nontuned}}(\omega)) \right)^2 d\omega \right) = 3.763 \cdot 10^5 \]

#### noise-opt BJT

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100 \]

\[ x_N(x) := \frac{4 \cdot k \cdot T}{Rf_N} \cdot I_{2,N} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{2,N} + 2 \cdot q \cdot \frac{x}{\left( \frac{25 \cdot 10^{-3}}{(25 \cdot 10^{-3})} \right)} \cdot I_{3,N} \]

\[ I_{\text{copt}} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{I_{2,N} I_{3,N}} = 0.004 \]

\[ I_{n,N} := x_N(I_{\text{copt}}) = 9.589 \cdot 10^{-14} \]

\[ \text{noise} := I_{n,N} = 9.589 \cdot 10^{-14} \]

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C-220
Pulse shape terms

\[ \text{Pulse} (t) : = \frac{1}{\pi} \cdot \left( \frac{\sin \left( \frac{\omega \cdot T_s}{2} \right)}{\omega \cdot T_s} \right) \cdot \text{Re} \left( \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_c}} \cdot \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \cdot \exp \left( 1i \cdot \omega \cdot (t) \right) \right) \] 

\[ \text{I}_1 (t) : = \frac{T_s}{\pi} \cdot \left( \frac{\sin \left( \frac{\omega \cdot T_s}{2} \right)}{\omega \cdot T_s} \right) \cdot \text{Re} \left( \frac{1i \cdot \omega \cdot \frac{1}{1 + 1j \cdot \frac{\omega}{\omega_c}} \cdot \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \cdot \exp \left( 1i \cdot \omega \cdot (t) \right) \right) \] 

\[ t : = -1 \cdot T_s, -0.9 \cdot T_s, 6 \cdot T_s \]

Convolution (non-directed)

\[ u (t) : = \text{if } (t < 0, 0, 1) \quad \text{Drms} : = 10 \cdot 10^{-9} \quad \text{ts} : = 2.5 \cdot 10^{-10} \quad \frac{1.1}{12} \cdot \sqrt{\frac{13}{11}} = 0.1 \]

\[ D_{nor} : = \text{Drms} \]

Channel impulse response

\[ h (t) : = \left( \frac{6 \cdot (0.1 \cdot D_{nor})^6}{(t + 0.1 \cdot D_{nor})^7} \right) \cdot u (t) \]

\[ I_0(t) : = \int_0^t \text{Pulse} (\tau) \cdot h (t - \tau) \, d\tau \]

C-221
pulse shape with Channel effect (OWC)

Peak voltage

\[ t := 1.19 \cdot T_s \]

\[ t_{pk} := \text{root} \left( T_s^2 \cdot I_1(t), t \right) \]

\[ t_{pk} = 2.975 \cdot 10^{-10} \]

\[ v_{pk} := I_0(t_{pk}) = 2.342 \cdot 10^9 \]

Erasure of pulse

\[ Q_{c_i} := \eta q \cdot \frac{v_{pk} - v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

\[ P_r(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{c_i} \cdot b}{\sqrt{2}} \right) \]

\[ P_{er}(b, i) := 2 \cdot \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_r(b, i) \cdot (x+1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_r(b, i) \cdot (n+1) \]

False alarm

False alarm when pulse appears in slot R can spread into S-slot of following symbol \( P_{oIS1} \) or into previous S-slot of same symbol

\[ v_{oIS1-T_s} := v_i \cdot I_0(t_{pk} - T_s) \]

\[ v_{oIS2-T_s} := v_i \cdot I_0(t_{pk} + T_s) \]
\[ Q_{eISI_1_i} := \eta q \cdot \frac{v_i \cdot v_{pk} - v_{oISI1} \cdot T_{s_i}}{\sqrt{\text{noise}}} \]

\[ Q_{eISI_2_i} := \eta q \cdot \frac{v_i \cdot v_{pk} - v_{oISI2} \cdot T_{s_i}}{\sqrt{\text{noise}}} \]

\[ P_{eISI_1}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{eISI_1} \cdot b}{\sqrt{2}} \right) \]

\[ P_{eISI_2}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{eISI_2} \cdot b}{\sqrt{2}} \right) \]

\[ P_{eIR}(b, i) := \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{eISI_1}(b, i) \cdot (x) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{eISI_1}(b, i) \cdot (n) + \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{eISI_2}(b, i) \cdot (x+1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{eISI_2}(b, i) \cdot (n+1) \]

False alarm no ISI occurs between S and R and the error appears within the run of N-symbols where k is the symbol position.

False alarm between R and S pulses - N to SET

\[ Q_{NS}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

\[ P_{NS}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NS}(b, i)}{\sqrt{2}} \right) \]

\[ P_{eNS}(b, i) := \sum_{y=3}^{n-1} \sum_{k=2}^{y-1} \left( \frac{1}{2} \right)^{y+3} \cdot P_{NS}(b, i) \cdot (y + 1 - k) + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NS}(b, i) \cdot (n + 1 - k) \]

False alarm between S and R pulses - N to R

\[ Q_{NR}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]
\[ P_{NR}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NR}(b, i)}{\sqrt{2}} \right) \]

\[ P_{eNR}(b, i) := \sum_{x=3}^{n-1} \sum_{k=2}^{x-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{NR}(b, i) \cdot (x+1-k) \]

\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NR}(b, i) \cdot (n+1-k) \]

\[ P_{fN}(b, i) := P_{eNS}(b, i) + P_{eNR}(b, i) \]

\[ P_{efN}(b, i) := \sum_{x=1}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=1}^{x} \left( P_{fN}(b, i) \cdot (x+1-k) \right) \]

\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=1}^{n} \left( P_{fN}(b, i) \cdot (n+1-k) \right) \]

\[ + \sum_{x=2}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=2}^{x} \left( P_{fN}(b, i) \cdot (x+1-k) \right) \]

\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=2}^{n} \left( P_{fN}(b, i) \cdot (n+1-k) \right) \]

**Total false alarm**

\[ P_{ef}(b, i) := P_{efN}(b, i) + P_{efR}(b, i) \]

\[ b := 5 \cdot 10^3 \]

**Total Error**

\[ P_{eb}(b, i) := P_{er}(b, i) + P_{efR}(b, i) + P_{efN}(b, i) \]

\[ pc(b, i) := \log \left( P_{eb}(b, i) \right) + 9 \]

\[ a := \text{root} \left( pc(b, i), b \right) \]

\[ v_{off} := 0.55 \]

**minimum := min(a)**

\[ \text{minimum} = 1.268 \cdot 10^4 \]

\[ gu := 2 \]

\[ dBm := 10 \cdot \log \left( \text{minimum} \cdot \frac{\text{photon\_energy}}{10^{-3}} \cdot \left( \frac{n+1}{2 \cdot (gu+2) \cdot n} \right) \cdot B \right) = -36.5 \]
C.7.4 Tuned B (0.5) di-code PPM receiver (with wireless channel effect)

Di-code PPM (PIN-BJT Butterworth 1 Gbit/s)
PPM Rx performance with PIN-BJT input configuration, Butterworth pre-detection filter. Calculations are as follow
- Di-code PPM terms
- Rx terms (noise+TF)
- Noise optimisation
- Channel effect, Convolution, pulse shaping, voltages
- ISI/Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

**Di-code PPM terms**

\[ i := 0, 1 \ldots 100 \quad n := 10 \quad j := 0 \ldots 10 \quad \text{Set up the scan limits} \]

\[ v_i := v_{\text{off}} + \frac{i}{1000} \]

\[ x := 0 \ldots n \quad \text{This gives the row of the matrix} \quad y := 0 \ldots n \quad \text{This gives the column of the matrix} \]

\[ B := 1 \cdot 10^9 \quad \text{Bit rate} \]

\[ T_b := \frac{1}{B} = 1 \cdot 10^{-9} \quad \text{PCM bit time} \]

\[ T_s := \frac{T_b}{2 + g\mu} \quad \text{Slot time} \]

\[ n := 10 \quad \text{Number of like symbols in PCM} \]

\[ \eta q := 1.6 \cdot 10^{-19} \quad \text{Quantum energy} \]

\[ \lambda := 1.55 \cdot 10^{-6} \quad \text{This is the wavelength of operation} \]

\[ \text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ R_o := \frac{\eta q}{\text{photon\_energy}} \]

\[ PPM\_B := \frac{1}{T_s} = 4 \cdot 10^9 \quad \text{this PPM bit rate, used for filter bandwidth} \]
Preamplifier terms

\[ Av := 10 \quad C_T := 1.5 \cdot 10^{-12} \quad \text{total C} \]

Feedback value for (R) Tuned B

\[ \Delta_L := 2.4 \quad \Delta_R := 2.52 \quad \text{feedback} \Delta_R, \text{time constant ratio} \Delta_L \]

\[ Rf_B := \Delta_R \cdot \frac{Av + 1}{2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \cdot C_T} = 1.4706 \cdot 10^3 \quad \text{feedback for Tuned B} \]

\[ \alpha_b := 0.3 \quad \text{splitting ratio} \]

\[ Lc := \left( \frac{Rf_B}{1 + Av} \right)^2 \cdot C_T \]

\[ \frac{\Delta_L}{\Delta_L} = 1.117 \cdot 10^{-8} \]

\[ C1 := (1 - \alpha_b) \cdot (C_T) = 1.05 \cdot 10^{-12} \]

\[ C2 := \alpha_b \cdot (C_T) = 4.5 \cdot 10^{-13} \]

\[ Z_{TIA,B}(\omega) := \frac{1}{\left( (1 - \omega^2 \cdot Lc \cdot C1) + \frac{Rf_B}{(1 + Av)} \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \right)} \]

\[ \omega_B := 2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \]

\[ \text{filter} (\omega) := \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \]

Receiver noise

\[ Z_{tuned,B}(\omega) := Z_{TIA,B}(\omega) \cdot \text{filter}(\omega) \]

\[ NEB_B := \frac{1}{2 \cdot \pi} \left( \int_0^{10^{13}} \left( \left| Z_{tuned,B}(\omega) \right|^2 \right) d\omega \right) = 1.429 \cdot 10^9 \]

\[ I_{2,B} := \frac{1}{2 \cdot \pi} \left( \int_0^{10^{13}} \left( \left| \left( 1 - \omega^2 \cdot Lc \cdot C1 \right) \cdot Z_{tuned,B}(\omega) \right|^2 \right) d\omega \right) = 9.33 \cdot 10^8 \]
\[ I_{3B} = \frac{1}{2\pi} \cdot \int_{0}^{10^2} \left( \frac{I_{1-2B} \cdot (1 - \omega^2 \cdot Lc \cdot C1)}{+Rf_B \cdot (\omega) \cdot (C1+C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \cdot i} \right)^2 \cdot Z_{tuned_B}(\omega) \, d\omega = 6.541 \cdot 10^4 \]

**noise-opt BJT**

\[ q = 1.6 \cdot 10^{-19} \quad k = 1.38 \cdot 10^{-23} \quad T = 298 \quad hfe = 100 \]

\[ x_B(x) = \left( \frac{4 \cdot k \cdot T}{Rf_B} \cdot I_{2B} + 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{2B} + 2 \cdot q \cdot \frac{x}{x} \cdot \left( \frac{x}{(25 \cdot 10^{-3})} \right)^2 \cdot I_{3B} \]

\[ I_{opt_B} = 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3B}}{I_{2B}}} = 2.0933 \cdot 10^{-3} \]

\[ I_{nB} = x_B(I_{opt_B}) = 2.294 \cdot 10^{-14} \]

**noise** \( = I_{nB} = 2.294 \cdot 10^{-14} \)

**Pulse shape terms**

\[ Pulse(t) = \frac{1}{\pi} \cdot \int_{0}^{\frac{1}{T_s} \cdot 10^2} \frac{\sin \left( \frac{\omega \cdot T_s}{2} \right)}{\left( \frac{\omega \cdot T_s}{2} \right)} \, d\omega \]

\[ \cdot \operatorname{Re} \left( \frac{1}{\left( (1 - \omega^2 \cdot Lc \cdot C1) + \frac{Rf_B}{(1 + Av)} \cdot (\omega \cdot 1i) \cdot (C1+C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \right)^2 \cdot \omega_B^3 \cdot (1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B^2 + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \cdot \exp \left( 1i \cdot \omega \cdot (t) \right) \right) \]
\[ I_1(t) := \frac{T_s}{\pi} \int_0^{\frac{T_s \cdot 10^{-7}}{T_s}} \sin \left( \frac{\omega \cdot T_s}{2} \right) d\omega \]

\[ \cdot \text{Re} \left[ \frac{1}{i \cdot \omega \cdot \left( (1 - \omega^2 \cdot Lc \cdot C1) \right) \cdot \left( (1 + \frac{Rf \cdot B}{1 + Av}) \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \right)}{\omega B^3 \cdot \exp(1i \cdot \omega \cdot (t))} \right] \]

\[ t := -1 \cdot T_s, -0.9 \cdot T_s \ldots 6 \cdot T_s \]

Convolution (non-directed)

\[ u(t) := \begin{cases} 1, & t < 0, 0, 1 \\ 0, & \text{otherwise} \end{cases} \quad Drms := 10 \cdot 10^{-9} \quad ts = 2.5 \cdot 10^{-10} \]

\[ Dnor := Drms \]

Channel impulse response

\[ h(t) := \frac{6 \cdot (0.1 \cdot Dnor)^6}{(t + 0.1 \cdot Dnor)^7} \cdot u(t) \]

\[ I_0(t) := \int_0^t \text{Pulse}(\tau) \cdot h(t - \tau) d\tau \]

pulse shape with Channel effect (OWC)
Peak voltage

t := 1.88 \cdot T_s

\( t_{pk} := \text{root}(I_s^2 \cdot I_1(t), t) \)

\( t_{pk} = 4.7 \cdot 10^{-10} \)

\( v_{pk} := I_0(t_{pk}) = 2.165 \cdot 10^9 \)

Erasure of pulse

\[ Q_{ei} := \eta q \cdot \frac{v_{pk} - v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

\[ P_r(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{ei} \cdot b}{\sqrt{2}} \right) \]

\[ P_{er}(b, i) := 2 \cdot \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+1} \cdot P_r(b, i) \cdot (x+1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_r(b, i) \cdot (n+1) \]

False alarm

False alarm when pulse appears in slot R can spread into S-slot of following symbol or into previous S-slot of same symbol

\( v_{olSI1_Ts} := v_i \cdot I_0(t_{pk} - T_s) \)

\( v_{olSI2_Ts} := v_i \cdot I_0(t_{pk} + T_s) \)

\[ Q_{eSI1_i} := \eta q \cdot \frac{v_i \cdot v_{pk} - v_{olSI1_Ts}}{\sqrt{\text{noise}}} \]

\[ Q_{eSI2_i} := \eta q \cdot \frac{v_i \cdot v_{pk} - v_{olSI2_Ts}}{\sqrt{\text{noise}}} \]

\[ P_{efSI1}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{eSI1_i} \cdot b}{\sqrt{2}} \right) \]

\[ P_{efSI2}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{eSI2_i} \cdot b}{\sqrt{2}} \right) \]
False alarm no ISI occurs between S and R and the error appears within the run of N-symbols where k is the symbol position

False alarm between R and S pulses - N to SET

\[ Q_{NS}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

\[ P_{NS}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NS}(b, i)}{\sqrt{2}} \right) \]

\[ P_{eNS}(b, i) := \sum_{y=3}^{n-1} \sum_{k=2}^{y-1} \left( \frac{1}{2} \right)^{y+3} \cdot P_{NS}(b, i) \cdot (y+1-k) \]

\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NS}(b, i) \cdot (n+1-k) \]

False alarm between S and R pulses - N to R

\[ Q_{NR}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

\[ P_{NR}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NR}(b, i)}{\sqrt{2}} \right) \]

\[ P_{eNR}(b, i) := \sum_{x=3}^{n-1} \sum_{k=2}^{x-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{NR}(b, i) \cdot (x+1-k) \]

\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NR}(b, i) \cdot (n+1-k) \]

\[ P_{fN}(b, i) := P_{eNS}(b, i) + P_{eNR}(b, i) \]

\[ P_{efN}(b, i) := \sum_{x=1}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=1}^{x} (P_{fN}(b, i) \cdot (x+1-k)) \]

\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=1}^{n} (P_{fN}(b, i) \cdot (n+1-k)) \]

\[ + \sum_{x=2}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=2}^{x} (P_{fN}(b, i) \cdot (x+1-k)) \]

\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=2}^{n} (P_{fN}(b, i) \cdot (n+1-k)) \]
Total false alarm

\[ P_{ef}(b, i) := P_{efN}(b, i) + P_{efR}(b, i) \]

\[ b := 5 \cdot 10^3 \]

Total Error

\[ P_{eb}(b, i) := P_{er}(b, i) + P_{efR}(b, i) + P_{efN}(b, i) \]

\[ pc(b, i) := \log(P_{eb}(b, i)) + 9 \]

\[ a_i := \text{root}(pc(b, i), b) \]

\[ v_{off} \equiv 0.55 \]

\[ \text{minimum} := \min(a) \]

\[ \text{minimum} = 6.683 \cdot 10^3 \]

\[ \text{noise} = 2.294 \cdot 10^{-14} \]

\[ gu \equiv 2 \]

\[ dBm := 10 \cdot \log \left( \frac{\text{minimum} \cdot \text{photon\_energy}}{10^{-3}} \cdot \left( \frac{n+1}{2 \cdot (gu+2) \cdot n} \right) \cdot B \right) = -39.284 \]
C.7.5 Tuned B (0.4) di-code PPM receiver (with wireless channel effect)

Di-code PPM (PIN-BJT Butterworth 1 Gbit/s)

PPM Rx performance with PIN-BJT input configuration, Butterworth pre-detection filter. calculations are as follow

- Di-code PPM terms
- Rx terms (noise+TF)
- Noise optimisation
- Pulse shaping, volatges
- ISI/Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

**Di-code PPM terms**

\[
i := 0, 1 \ldots 100 \quad n := 10 \quad j := 0, 1 \ldots 10 \quad \text{Set up the scan limits}
\]

\[
v_i := v_{off} + \frac{i}{1000}
\]

\[
x := 0 \ldots n \quad \text{This gives the row of the matrix} \quad y := 0 \ldots n \quad \text{This gives the column of the matrix}
\]

\[
B := 1 \cdot 10^9 \quad \text{Bit rate}
\]

\[
T_b := \frac{1}{B} = 1 \cdot 10^{-9} \quad \text{PCM bit time}
\]

\[
T_s := \frac{T_b}{2 + gn} \quad \text{Slot time}
\]

\[
n := 10 \quad \text{Number of like symbols in PCM}
\]

\[
\eta q := 1.6 \cdot 10^{-19} \quad \text{Quantum energy}
\]

\[
\lambda := 1.55 \cdot 10^{-6} \quad \text{This is the wavelength of operation}
\]

\[
\text{photon\_energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda}
\]

\[
R_o := \frac{\eta q}{\text{photon\_energy}}
\]

\[
PPM_B := \frac{1}{T_s} = 4 \cdot 10^9 \quad \text{this PPM bit rate, used for filter bandwidth}
\]
Preamplifier terms

\[ ts := \frac{1}{\text{PPM}_B} = 2.5 \cdot 10^{-10} \]

Feedback value for (R) Tuned B

\[ \Delta_L := 1.9 \quad \Delta_R := 2.75 \]

Feedback for Tuned B

\[ Rf_B := \Delta_R \cdot \frac{A_v + 1}{2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \cdot C_T} = 1.6048 \cdot 10^3 \]

\[ \alpha_b := 0.4 \quad \text{splitting ratio} \]

\[ Lc := \frac{\left( \frac{Rf_B}{1 + A_v} \right)^2 \cdot C_T}{\Delta_L} = 1.68 \cdot 10^{-8} \]

\[ C1 := (1 - \alpha_b) \cdot (C_T) = 9 \cdot 10^{-13} \]

\[ C2 := \alpha_b \cdot (C_T) = 6 \cdot 10^{-13} \]

\[ Z_{TIA,B}(\omega) := \frac{1}{\left( (1 - \omega^2 \cdot Lc \cdot C1) + \frac{Rf_B}{1 + A_v} \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \right)} \]

\[ \omega_B := 2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \]

\[ \text{filter} (\omega) := \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \]

Receiver noise

\[ Z_{\text{tuned},B}(\omega) := Z_{TIA,B}(\omega) \cdot \text{filter}(\omega) \]

\[ \text{NEB}_B := \frac{1}{2 \cdot \pi} \int_0^{10^6} \left( \left| Z_{\text{tuned},B}(\omega) \right|^2 \right) d\omega = 1.636 \cdot 10^9 \]

\[ I_{2,B} := \frac{1}{2 \cdot \pi} \int_0^{10^6} \left( \left| \left( (1 - \omega^2 \cdot Lc \cdot C1) \right) \cdot \left| Z_{\text{tuned},B}(\omega) \right| \right|^2 \right) d\omega = 1.035 \cdot 10^9 \]
\[ I_{3\,B} := \frac{1}{2\pi} \int_{0}^{10^4} \left( \frac{\left( \left( 1 - \omega^2 \cdot Lc \cdot C1 \right) \right.}{\left. + Rf_B \cdot (\omega \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \cdot i1 \right)} \right)^2 \cdot Z_{\text{tuned}_B}(\omega) \cdot d\omega = 5.019 \cdot 10^4 \]

**noise-opt BJT**

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100 \]

\[ x_B(x) := \frac{4 \cdot k \cdot T}{Rf_B} \cdot I_{2\,B} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{2\,B} + 2 \cdot q \cdot \frac{x^2}{(25 \cdot 10^{-3})^2} \cdot I_{3\,B} \]

\[ I_{\text{opt}_B} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3\,B}}{I_{2\,B}}} = 1.7412 \cdot 10^{-3} \]

\[ I_{n\,B} := x_B(I_{\text{opt}_B}) = 2.214 \cdot 10^{-14} \]

**noise** := \[ I_{n\,B} = 2.214 \cdot 10^{-14} \]

**Pulse shape terms**

\[ \text{Pulse}(t) := \frac{1}{\pi} \cdot \int_{0}^{\frac{T}{T_s} \cdot 10^2} \left( \sin \left( \frac{\omega \cdot T_s}{2} \right) \right)^2 \cdot \frac{1}{\left( \frac{\omega \cdot T_s}{2} \right)^2} \cdot \left( \frac{\left( 1 - \omega^2 \cdot Lc \cdot C1 \right)}{\left( 1 + Av \right) \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2)} \right)^2 \cdot \exp \left( i \cdot \omega \cdot (t) \right) \cdot \omega_B^3 \cdot (1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3 \right) \]

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\[ I_1(t) := \frac{T_s}{\pi} \cdot \frac{\sin \left( \frac{\omega \cdot T_s}{2} \right)}{(\omega \cdot T_s)^2} \cdot \Re \left( \frac{1}{(1 - \omega^2 \cdot Lc \cdot C1)^{\frac{1}{2}}} \cdot \frac{1}{(1 + Av)} \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \cdot \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \cdot \exp(1i \cdot \omega \cdot t) \right) \]

\[ t := -1 \cdot T_s, -0.9 \cdot T_s, \ldots, 6 \cdot T_s \]

Convolution (non-directed)

\[ u(t) := \begin{cases} 1 & \text{if } t < 0, 0, 1 \\ 0 & \text{otherwise} \end{cases} \quad Drms := 10 \cdot 10^{-9} \quad ts = 2.5 \cdot 10^{-10} \]

\[ Dnor := Drms \]

Channel impulse response

\[ h(t) := \left( \frac{6 \cdot (0.1 \cdot Dnor)^6}{(t + 0.1 \cdot Dnor)^7} \right) \cdot u(t) \]

\[ I_0(t) := \int_0^t Pulse(\tau) \cdot h(t - \tau) \, d\tau \]

Pulse shape with Channel effect (OWC)
Peak voltage

\[ t := 2.05 \cdot T_s \]

\[ t_{pk} := \text{root} \left( I_s^2 \cdot I_1(t), t \right) \]

\[ t_{pk} = 5.125 \cdot 10^{-10} \]

\[ v_{pk} := I_0(t_{pk}) = 2.268 \cdot 10^9 \]

Erasure of pulse

\[ Q_{e_i} := \eta q \cdot \frac{v_{pk} - v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

\[ P_r(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{e_i} \cdot b}{\sqrt{2}} \right) \]

\[ P_{er}(b, i) := 2 \cdot \sum_{x=0}^{n-1} \left( \frac{1}{2} \cdot P_r(b, i) \cdot (x + 1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_r(b, i) \cdot (n + 1) \right) \]

False alarm

False alarm when pulse appears in slot R can spread into S-slot of following symbol or into previous S-slot of same symbol

\[ v_{oIS1\cdot Ts} := v_i \cdot I_0(t_{pk} - T_s) \]

\[ v_{oIS2\cdot Ts} := v_i \cdot I_0(t_{pk} + T_s) \]

\[ Q_{eIS1_i} := \eta q \cdot \frac{v_i \cdot v_{pk} - v_{oIS1\cdot Ts_i}}{\sqrt{\text{noise}}} \]

\[ Q_{eIS2_i} := \eta q \cdot \frac{v_i \cdot v_{pk} - v_{oIS2\cdot Ts_i}}{\sqrt{\text{noise}}} \]

\[ P_{eIS1}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{eIS1_i} \cdot b}{\sqrt{2}} \right) \]

\[ P_{eIS2}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{eIS2_i} \cdot b}{\sqrt{2}} \right) \]
\[ P_{eR}(b, i) := \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{eISI1}(b, i) \cdot (x) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{eISI1}(b, i) \cdot (n) \] 
\[ + \sum_{x=0}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{eISI2}(b, i) \cdot (x+1) + \left( \frac{1}{2} \right)^{n+2} \cdot P_{eISI2}(b, i) \cdot (n+1) \] 

False alarm no ISI occurs between S and R and the error appears within the run of \( N \)-symbols where \( k \) is the symbol position.

False alarm between R and S pulses - N to SET

\[ Q_{NS}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \] 
\[ P_{NS}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NS}(b, i)}{\sqrt{2}} \right) \] 
\[ P_{eNS}(b, i) := \sum_{y=3}^{n-1} \sum_{k=2}^{y-1} \left( \frac{1}{2} \right)^{y+3} \cdot P_{NS}(b, i) \cdot (y+1-k) \] 
\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NS}(b, i) \cdot (n+1-k) \] 

False alarm between S and R pulses - N to R

\[ Q_{NR}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \] 
\[ P_{NR}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NR}(b, i)}{\sqrt{2}} \right) \] 
\[ P_{eNR}(b, i) := \sum_{x=3}^{n-1} \sum_{k=2}^{x-1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{NR}(b, i) \cdot (x+1-k) \] 
\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NR}(b, i) \cdot (n+1-k) \] 
\[ P_{fN}(b, i) := P_{eNS}(b, i) + P_{eNR}(b, i) \] 
\[ P_{eF}(b, i) := \sum_{x=1}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=1}^{x} \left( P_{fN}(b, i) \cdot (x+1-k) \right) \] 
\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=1}^{n} \left( P_{fN}(b, i) \cdot (n+1-k) \right) \] 
\[ + \sum_{x=2}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=2}^{x} \left( P_{fN}(b, i) \cdot (x+1-k) \right) \] 
\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=2}^{n} \left( P_{fN}(b, i) \cdot (n+1-k) \right) \]
Total false alarm

\[ P_{ef}(b,i) := P_{efN}(b,i) + P_{efR}(b,i) \]

\[ b := 4 \cdot 10^3 \]

Total Error

\[ P_{eb}(b,i) := P_{er}(b,i) + P_{efR}(b,i) + P_{efN}(b,i) \]

\[ pc(b,i) := (\log(P_{eb}(b,i)) + 9) \]

\[ a_i := \text{root}(pc(b,i), b) \]

\[ v_{off} \equiv 0.55 \]

\[ \text{minimum} := \min(a) \]

\[ \text{minimum} = 6.096 \cdot 10^3 \]

\[ \text{noise} = 2.214 \cdot 10^{-14} \]

\[ gu \equiv 2 \]

\[ dBm := 10 \cdot \log\left(\frac{\text{minimum} \cdot \text{photon\_energy}}{10^{-3}} \cdot \left(\frac{n+1}{2 \cdot (gu+2) \cdot n}\right) \cdot B\right) = -39.683 \]
C.7.6 Tuned B (0.5) di-code PPM receiver (with wireless channel effect)

Di-code PPM (PIN-BJT Butterworth 1 Gbit/s)

PPM Rx performance with PIN-BJT input configuration, Butterworth pre-detection filter, calculations are as follow

- Di-code PPM terms
- Rx terms (noise+TF)
- Noise optimisation
- Pulse shaping, voltages
- ISI/Error bit rate
- Optimum threshold/minimum number of photons
- Receiver sensitivity

**Di-code PPM terms**

\[ i := 0, 1 \ldots 100 \quad n := 10 \quad j := 0 \ldots 10 \quad \text{Set up the scan limits} \]

\[ v_i := v_{off} + \frac{i}{1000} \]

\[ x := 0 \ldots n \quad \text{This gives the row of the matrix} \quad y := 0 \ldots n \quad \text{This gives the column of the matrix} \]

\[ B := 1 \cdot 10^9 \quad \text{Bit rate} \]

\[ T_b := \frac{1}{B} = 1 \cdot 10^{-9} \quad \text{PCM bit time} \]

\[ T_s := \frac{T_b}{2 + gu} \quad \text{Slot time} \]

\[ n := 10 \quad \text{Number of like symbols in PCM} \]

\[ \eta q := 1.6 \cdot 10^{-10} \quad \text{Quantum energy} \]

\[ \lambda := 1.55 \cdot 10^{-6} \quad \text{This is the wavelength of operation} \]

\[ \text{photon}_\text{energy} := \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^8}{\lambda} \]

\[ R_o := \frac{\eta q}{\text{photon}_\text{energy}} \]

\[ PPM_B := \frac{1}{T_s} = 4 \cdot 10^9 \quad \text{this PPM bit rate, used for filter bandwidth} \]
Preamplifier terms

\[ t_s := \frac{1}{PPM_B} = 2.5 \cdot 10^{-10} \]

Feedback value for (R) Tuned B

\[ \Delta_L := 1.5 \quad \Delta_R := 2.65 \]

feedback \( \Delta_R \), time constant ratio \( \Delta_L \)

\[ Rf_B := \Delta_R \cdot \frac{Av + 1}{2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \cdot C_T} = 1.5465 \cdot 10^3 \]

feedback for Tuned B

\[ \alpha_b := 0.5 \]

splitting ratio

\[ Lc := \left( \frac{Rf_B}{1 + Av} \right)^2 \cdot C_T \cdot \frac{1}{\Delta_L} = 1.976 \cdot 10^{-8} \]

\[ C1 := (1 - \alpha_b) \cdot (C_T) = 7.5 \cdot 10^{-13} \]

\[ C2 := \alpha_b \cdot (C_T) = 7.5 \cdot 10^{-13} \]

\[ Z_{TIA,B}(\omega) := \frac{1}{(1 - \omega^2 \cdot Lc \cdot C1) + \frac{Rf_B}{(1 + Av)} \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2)} \]

\[ \omega_B := 2 \cdot \pi \cdot 0.5 \cdot \frac{1}{T_s} \]

\[ \text{filter}(\omega) := \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \]

Receiver noise

\[ Z_{tuned,B}(\omega) := Z_{TIA,B}(\omega) \cdot \text{filter}(\omega) \]

\[ NEB_B := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^8} \left( \frac{1}{|Z_{tuned,B}(\omega)|^2} \right) d\omega \right) = 2.048 \cdot 10^9 \]

\[ I_{2,B} := \frac{1}{2 \cdot \pi} \cdot \left( \int_0^{10^8} \left( \frac{1}{|\left( 1 - \omega^2 \cdot Lc \cdot C1 \right) \cdot \left( Z_{tuned,B}(\omega) \right)|^2} \right) d\omega \right) = 1.068 \cdot 10^9 \]
\[ I_{3\_B} := \frac{1}{2 \cdot \pi} \int_{0}^{10^{10}} \left( \frac{\left( \frac{\left(1 - \omega^2 \cdot Lc \cdot C1\right)}{Rf\_B} + Rf\_B \cdot (\omega) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \cdot i \cdot 1 \right)}{Z\_tuned\_B(\omega)} \right)^2 d\omega = 5.239 \cdot 10^4 \]

**noise-opt BJT**

\[ q := 1.6 \cdot 10^{-19} \quad k := 1.38 \cdot 10^{-23} \quad T := 298 \quad hfe := 100 \]

\[ x_{\_B}(x) := \left( \frac{4 \cdot k \cdot T}{Rf\_B} \cdot I_{2\_B} + \left( 2 \cdot q \cdot \frac{x}{hfe} \right) \cdot I_{2\_B} + 2 \cdot q \cdot \frac{x}{(25 \cdot 10^{-3})} \right) \]

\[ I_{opt\_B} := 25 \cdot 10^{-3} \cdot \sqrt{hfe} \cdot \sqrt{\frac{I_{3\_B}}{I_{2\_B}}} = 1.7513 \cdot 10^{-3} \]

\[ I_{n\_B} := x_{\_B}(I_{opt\_B}) = 2.332 \cdot 10^{-14} \]

\[ noise := I_{n\_B} = 2.332 \cdot 10^{-14} \]

**Pulse shape terms**

\[ Pulse(t) := \frac{1}{T_s \cdot 10^3} \left( \int_{0}^{\frac{\omega \cdot T_s}{2}} \sin \left( \frac{\omega \cdot T_s}{2} - \left( \frac{\omega \cdot T_s}{2} \right) \right) d\omega \cdot \frac{1}{\left( (1 - \omega^2 \cdot Lc \cdot C1) + \frac{Rf\_B}{(1 + Av)} \cdot (\omega \cdot 1i) \cdot (C1 + C2 - (\omega)^2 \cdot Lc \cdot C1 \cdot C2) \right)} \left( \frac{\omega_B^3}{(1j \cdot \omega)^3 + 2 \cdot (1j \cdot \omega)^2 \cdot \omega_B^2 + 2 \cdot (1j \cdot \omega) \cdot \omega_B^2 + \omega_B^3} \right) \right) \]
\[ I_1(t) = \frac{T_s}{\pi} \int_0^{10^7} \sin \left( \frac{\omega \cdot T_s}{2} \right) \cdot \left( \frac{\omega \cdot T_s}{2} \right)^{\frac{1}{2}} \cdot \Re \left( 1 + \omega \cdot \left( 1 - \omega^2 \cdot Lc \cdot C1 \right) \right) \cdot \left( \frac{R_f \cdot B}{1 + Av} \right)^{\frac{1}{2}} \cdot \left( C1 + C2 - (\omega^2 \cdot Lc \cdot C1 \cdot C2) \right)^{\frac{1}{2}} \cdot \omega_B^3 \cdot \exp \left( 1i \cdot \omega \cdot (t) \right) \, d\omega \]

\[ t = -1 \cdot T_s, -0.9 \cdot T_s, \ldots, 6 \cdot T_s \]

Convolution (non-directed)

\[ u(t) = \text{if} \, (t < 0, 0, 1) \quad D_{rms} = 10 \cdot 10^{-9} \quad ts = 2.5 \cdot 10^{-10} \]

\[ \text{Dnor} = D_{rms} \]

Channel impulse response

\[ h(t) = \frac{6 \cdot (0.1 \cdot \text{Dnor})^6}{(t + 0.1 \cdot \text{Dnor})^7} \cdot u(t) \]

\[ I_0(t) = \int_0^t \text{Pulse}(\tau) \cdot h(t - \tau) \, d\tau \]

pulse shape with Channel effect (OWC)
Peak voltage

\[ t := 2.05 \cdot T_s \]

\[ t_{pk} := \text{root} \left( T_s^2 \cdot I_1(t), t \right) \]

\[ t_{pk} = 5.125 \cdot 10^{-10} \]

\[ v_{pk} := I_0(t_{pk}) = 2.43 \cdot 10^9 \]

Erasure of pulse

\[ Q_{ei} := \eta q \cdot \frac{v_{pk} - v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

\[ P_r(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{ei} \cdot b}{\sqrt{2}} \right) \]

\[ P_{er}(b, i) := 2 \cdot \sum_{x=0}^{n-1} \left( \frac{1}{2} \cdot P_r(b, i) \cdot (x+1) + \left( \frac{1}{2} \cdot P_r(b, i) \cdot (n+1) \right) \right) \]

False alarm

False alarm when pulse appears in slot R can spread into S-slot of following symbol or into previous S-slot of same symbol

\[ v_{olSI1\cdot Ts} := v_i \cdot I_0(t_{pk} - T_s) \]

\[ v_{olSI2\cdot Ts} := v_i \cdot I_0(t_{pk} + T_s) \]

\[ Q_{elSI1_i} := \eta q \cdot \frac{v_i \cdot v_{pk} - v_{olSI1\cdot Ts_i}}{\sqrt{\text{noise}}} \]

\[ Q_{elSI2_i} := \eta q \cdot \frac{v_i \cdot v_{pk} - v_{olSI2\cdot Ts_i}}{\sqrt{\text{noise}}} \]

\[ P_{effSI1}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{elSI1_i} \cdot b}{\sqrt{2}} \right) \]

\[ P_{effSI2}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{elSI2_i} \cdot b}{\sqrt{2}} \right) \]
False alarm no ISI occurs between S and R and the error appears within the run of N-symbols where k is the symbol position

False alarm between R and S pulses - N to SET

\[ Q_{NS}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

\[ P_{NS}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NS}(b, i)}{\sqrt{2}} \right) \]

\[ P_{eNS}(b, i) := \sum_{y=3}^{n-1} \sum_{k=2}^{y-1} \left( \frac{1}{2} \right)^{y+3} \cdot P_{NS}(b, i) \cdot (y + 1 - k) \]

\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NS}(b, i) \cdot (n + 1 - k) \]

False alarm between S and R pulses - N to R

\[ Q_{NR}(b, i) := b \cdot \eta q \cdot \frac{v_i \cdot v_{pk}}{\sqrt{\text{noise}}} \]

\[ P_{NR}(b, i) := \frac{1}{2} \cdot \text{erfc} \left( \frac{Q_{NR}(b, i)}{\sqrt{2}} \right) \]

\[ P_{eNR}(b, i) := \sum_{x=3}^{n-1} \sum_{k=2}^{x+1} \left( \frac{1}{2} \right)^{x+3} \cdot P_{NR}(b, i) \cdot (x + 1 - k) \]

\[ + \sum_{k=2}^{n-1} \left( \frac{1}{2} \right)^{n+2} \cdot P_{NR}(b, i) \cdot (n + 1 - k) \]

\[ P_{eF}(b, i) := P_{eNS}(b, i) + P_{eNR}(b, i) \]

\[ P_{eF}(b, i) := \sum_{x=1}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=1}^{x} (P_{eF}(b, i) \cdot (x + 1 - k)) \]

\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=1}^{n} (P_{eF}(b, i) \cdot (n + 1 - k)) \]

\[ + \sum_{x=2}^{n-1} \left( \frac{1}{2} \right)^{x+3} \cdot \sum_{k=2}^{x} (P_{eF}(b, i) \cdot (x + 1 - k)) \]

\[ + \left( \frac{1}{2} \right)^{n+2} \cdot \sum_{k=2}^{n} (P_{eF}(b, i) \cdot (n + 1 - k)) \]
Total false alarm

\[ P_{ef}(b, i) := P_{eFN}(b, i) + P_{eFR}(b, i) \]

\[ b := 5 \cdot 10^3 \]

Total Error

\[ P_{eb}(b, i) := P_{er}(b, i) + P_{eFR}(b, i) + P_{eFN}(b, i) \]

\[ pc(b, i) := (\log(P_{eb}(b, i)) + 9) \]

\[ a_i := \text{root}(pc(b, i), b) \]

\[ v_{off} \equiv 0.55 \]

\[ \text{minimum} := \min(a) \]

\[ \text{minimum} = 5.682 \cdot 10^3 \]

\[ \text{noise} = 2.332 \cdot 10^{-14} \]

\[ gu \equiv 2 \]

\[ dBm := 10 \cdot \log\left(\frac{\text{minimum} \cdot \text{photon\_energy}}{10^{-3}} \cdot \left(\frac{n+1}{2 \cdot (gu+2) \cdot n}\right) \cdot B\right) = -39.989 \]