Automated error analysis of serial manipulators and servo heads

J M Freeman* and **D G Ford**
The Ultra Precision Engineering Centre, University of Huddersfield, UK

Abstract: This paper presents a general mathematical treatment of serial manipulators, an important example of which is the servo head. The paper includes validation by application to the angle head via comparison with the previously known transformations and a new application to the error analysis of the angle head.

The usual approach to the error analysis of a servo head is to develop a geometrical model by elementary geometrical considerations using trigonometric relationships and various simplifying assumptions. This approach is very error prone, difficult to verify and extremely time consuming. The techniques described here constitute matrix methods that have been programmed in a general way to derive automatically the analytical equations relating the angles of rotation of the head and alignment errors in the head to the position of the tool and errors in that position. The approach is to use rotation and transformation matrices to evaluate the influence of the various errors such as offsets and angular errors. A general approach to the sign convention and notation for angular errors is presented in an attempt to reduce the possibility of errors of definition.

Keywords: servo head, geometric errors, error analysis, error compensation

NOTATION

A	front axis
A_{p} and C_{p}	angles of rotation about axes A and C
C	rear axis
N	total number of transformations to be performed
$\boldsymbol{q}$	arrray of structures defining the variables
$\mathbf{R}_A^x$	transformation representing rotation by A degrees about axis x
$\mathbf{T}(x', y', z')$	transformation representing translation by x', y' and z' along axes x, y and z
$\boldsymbol{X}$	vector of Cartesian coordinates x, y and z
$\boldsymbol{X}^0$	initial $\boldsymbol{X}$ vector
Y_{p}	fixed angle between the A and C axes
Ψ	general transformation matrix which may be a rotation or a translation

The MS was received on 1 October 2002 and was accepted after revision for publication on 20 June 2003.
* *Corresponding author: The Ultra Precision Engineering Centre, University of Huddersfield, Queensgate, Huddersfield HD1 3DH, UK.*

1 INTRODUCTION

A machine tool requires accurate orientation and positioning of a cutter in three-dimensional space, whereas a measuring machine requires accurate knowledge of only the position of its probe tip. Thus the essential difference between measurement machines and machine tools is that any errors due to the tilt of a measurement sensor can be compensated for in software, whereas the tilt of a cutting tool requires to be within tolerance, no software corrections being possible. Thus it is necessary in cutting machines to apply geometrical compensation in real time [**1–3**] in order to ensure accurately machined components. This can be achieved by introducing corrections into the feedback to the controller [**1, 2**]. The volumetric accuracy of the machine can be simulated off-line using an error simulation package [**4**].

In principle the location of a sensor or a cutter head exhibits six degrees of freedom. These may be specified in various ways, the usual constituting the x, y and z Cartesian coordinates of the head and the angular rotations of the components of the head about the various rotational axes of the head. The physical arrangement of these axes depends on the detailed nominal design of the head. It is impossible to manufacture heads to the exact nominal design and even small offsets of a few micrometres or angular errors of a few arcseconds in the alignment of the various components of the head would produce significant positioning and orientation errors of the tool tip. Thus it is essential to measure the true geometry of the head and to apply compensation in real time to correct for the small errors from the nominal design.

The usual approach to the error analysis of such a servo head is to develop a geometrical model by elementary geometrical considerations using trigonometric relationships and various simplifying assumptions [**5**]. This approach is very error prone, difficult to verify and extremely time consuming. The results of such an analysis [**5, 6**] are available for a fork head, which is considerably simpler than the angle head of interest here.

The techniques described in this paper constitute matrix transformation methods [**3**] that have been programmed in a general way to derive automatically the analytical equations relating the angles of rotation of the head and alignment errors in the head to the position of the tool and errors in that position. The approach is to use rotations and translations to evaluate the influence of the various errors such as angular errors and offsets in the components of the head. A general representation of the problem is possible and the problem for the user is reduced to defining the components of error and the order in which the rotations and translations should be applied. A general approach to the notation and sign convention for angular errors is presented in an attempt to reduce the possibility of errors of definition.

2 A REVISED NOTATION FOR ANGULAR ERRORS

The normal procedure is to define angles by mechanical terms such as yaw, pitch, roll, squareness to spindle, squareness to vertical, etc. [**5, 6**]. The signs of these angles must be specified by reference to diagrams. This is potentially confusing and leads to possible ambiguity. The approach here is to use a purely mathematical definition of angles. Thus an angle of rotation $\mathbf{R}_A^x$ is defined by reference to its axis of rotation x and its size A. The sign of the angle is defined as positive in an anticlockwise direction looking up the axis. Since matrix rotations can only be performed around axes passing through the origin, it is sometimes necessary to perform translations to ensure that this condition is satisfied before a rotation is attempted. If necessary, the translation can be reversed afterwards. A translation or offset is defined simply as the corresponding displacements along the three Cartesian axes $\mathbf{T}_{x,y,z} = (x, y, z)$.

3 MATHEMATICAL ANALYSIS

For the purpose of illustration of the method, assume that it is required to analyse a head with a tool of length G in a spindle having two main angles of rotation A_p and C_p about axes A and C, the front and rear axes. The terms 'front' and 'rear' are intended to be used as a generic description for heads of this type, describing the relationship between the axes. The front axis is carried by the back axis in order to orient the cutter in any desired direction. Any additional axes in a more complex system should be similarly ordered from front to back. This arrangement has many possible errors, including offsets and various angular errors of alignment of the various axes.

3.1 The basic transformations for the angle head

The angle head is used on some five axis machines to enable flexible orientation of the tool tip. It has two rotational axes and is illustrated in Fig. 1. The angle C_p defines the rotation of the head about the C axis, which is along the z axis in the figure. The A axis is at an arbitrary angle Y_p to the C axis and a rotation A_p about axis A allows the tool tip to be oriented in most desired directions (depending on the actual value of Y_p). If Y_p is 50° then the head can sweep through a total of 190°. During normal operation the orientation of the head is controlled by the computer numerically controlled (CNC) controller and is transparent to the programmer. However, if there are errors in the construction of the head from its nominal design, these will cause errors in the tool tip position which must be analysed and compensated for.

The first requirement is to analyse the relationship between the angles A_p and C_p and the Cartesian coordinates of the tool tip. This has already been investigated and the analytical solution obtained by geometrical methods is available [**7**]. Here description is confined to the new approach. The definition of angle C_p is positive in a clockwise direction in Fig. 1, this being retained for ease of comparison with the analytical solution.

The procedure involves assuming that the tool is initially in an easily defined configuration; then rotations about axes A and C are performed to bring the tool into its actual position. A slight complication arises because the C axis does not pass through the origin and the A axis is not in line with any of the Cartesian axes. Hence translations and additional rotations must accompany the rotations by angles A_p and C_p to ensure that they are executed correctly.

Assume that the origin is at the intersection of the A axis and the axis of the spindle and that the angles A and C are both zero. This can be achieved by a translation G, the length of the spindle, along the z axis. The tool is

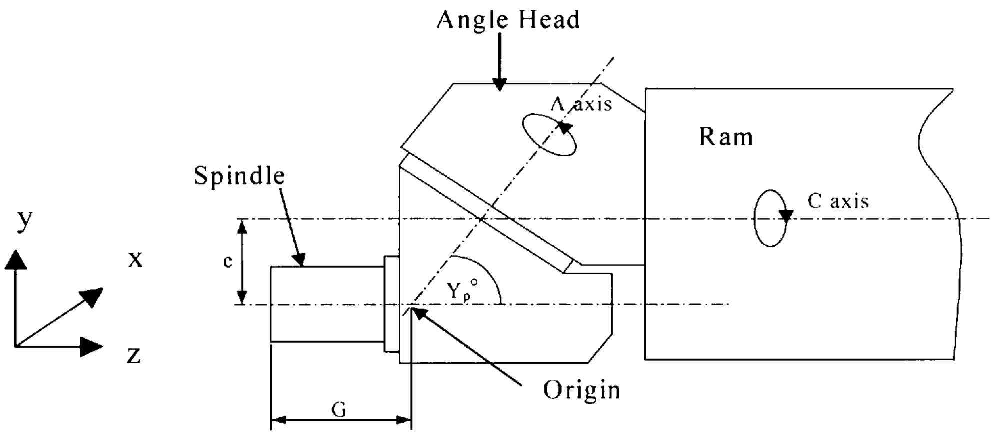

Fig. 1 Basic configuration of the angle-type servo head

then in the position $(0, 0, -G)$, as in Fig. 1. Let the angle Y_p generalize the fixed tilt for the head in Fig. 1. If the spindle is now rotated clockwise about the x axis by an angle Y_p (or $-Y_p$ in the anticlockwise direction, which is defined as positive) the axis A then lies along the z axis. The spindle can then be rotated by angle A_p about the z axis as shown in Fig. 2. The rotation by Y_p should now be reversed to bring the A axis back to its original position, leaving the tool tip effectively rotated by A_p about the A axis (see Fig. 2). This requires a rotation Y_p anticlockwise about the x axis. Before the rotation about C can be performed, the assembly requires shifting so that the C axis passes through the origin. This can be achieved by a translation of $(0, -e, -e/\tan Y)$, placing the origin at the intersection of the A and C axes. The rotation by angle C_p can then be implemented as a rotation $-C_p$ about the z axis. It is important to realize that the front rotation by A_p must be executed before the back rotation C_p. The effect on the angle head of all these transformations is shown in Fig. 2 to illustrate the method.

The result of the above rotations and translations is that the tool tip is moved from its initial position corresponding to $A = 0$ and $C = 0$ to its actual position for any given angles A_p and C_p. This assumes that there are no errors in the assembly. Obviously, the result would normally be numerical if performed on a computer using numerical matrix multiplication and addition routines.

The mathematical representation of the above procedure can be concisely represented by

$$\boldsymbol{X} = \mathbf{R}^{z}_{-C_p}\mathbf{T}_{(0,-e,-e/\tan Y_p)}\mathbf{R}^{x}_{Y_p}\mathbf{R}^{z}_{A_p}\mathbf{R}^{x}_{-Y_p}\mathbf{T}_{(0,0,-G)}\boldsymbol{X}^0$$

where $\boldsymbol{X}^0$ is chosen to represent the origin and the transformations are executed from the right by repeated application of succeeding transformations to $\boldsymbol{X}^0$. The result vector $\boldsymbol{X}$ contains the Cartesian coordinates of the tool tip, taking the intersection of the A and C axes as the origin. If required, a translation can be performed to return the origin to its original position at the tool tip.

3.2 The error analysis for the angle head

The procedure is similar to that used for the basic transformation, but all the sources of error must be included as transformations. The first step, as before, is to fix the tool along the z axis. The spindle can be out of alignment with the tool, having possible angular errors about the x and y axes only. One of these, the one about x, can be incorporated into errors in Y_p. A rotation ThetaA_s, as defined in Table 1, can be performed about y to bring the spindle in line with the z axis, thus displacing the tool tip to a new position that includes the effect of the angular error.

The next step is to apply a translation to bring the main rotational axis A so that it passes through the origin. This involves the offset between A and the spindle in the y direction. This must be followed by the rotation by $-Y_p$ about the x axis to bring the A axis nominally along the z axis as before, followed by a rotation by the error in Y_p, dY. Then, since the A axis may be out of alignment with the yz plane, a rotation ThetaA_{vp} will be necessary to bring the A axis exactly along the z axis. Then a rotation about A can be performed.

The procedure continues until all angular errors and both main rotational angles A_p and C_p have been accounted for. It is here assumed that all the errors are small and therefore any interactions between the rotations and translations can be ignored as second-order effects. The result is that the transformed tool tip will then be in its actual position, accounting for the angular rotations and all the errors but ignoring second-order effects.

Table 1 gives a complete definition of all the possible variables and errors in the angle head and a description of the meaning of the notation as well as an indication of which variables represent small errors and which are large

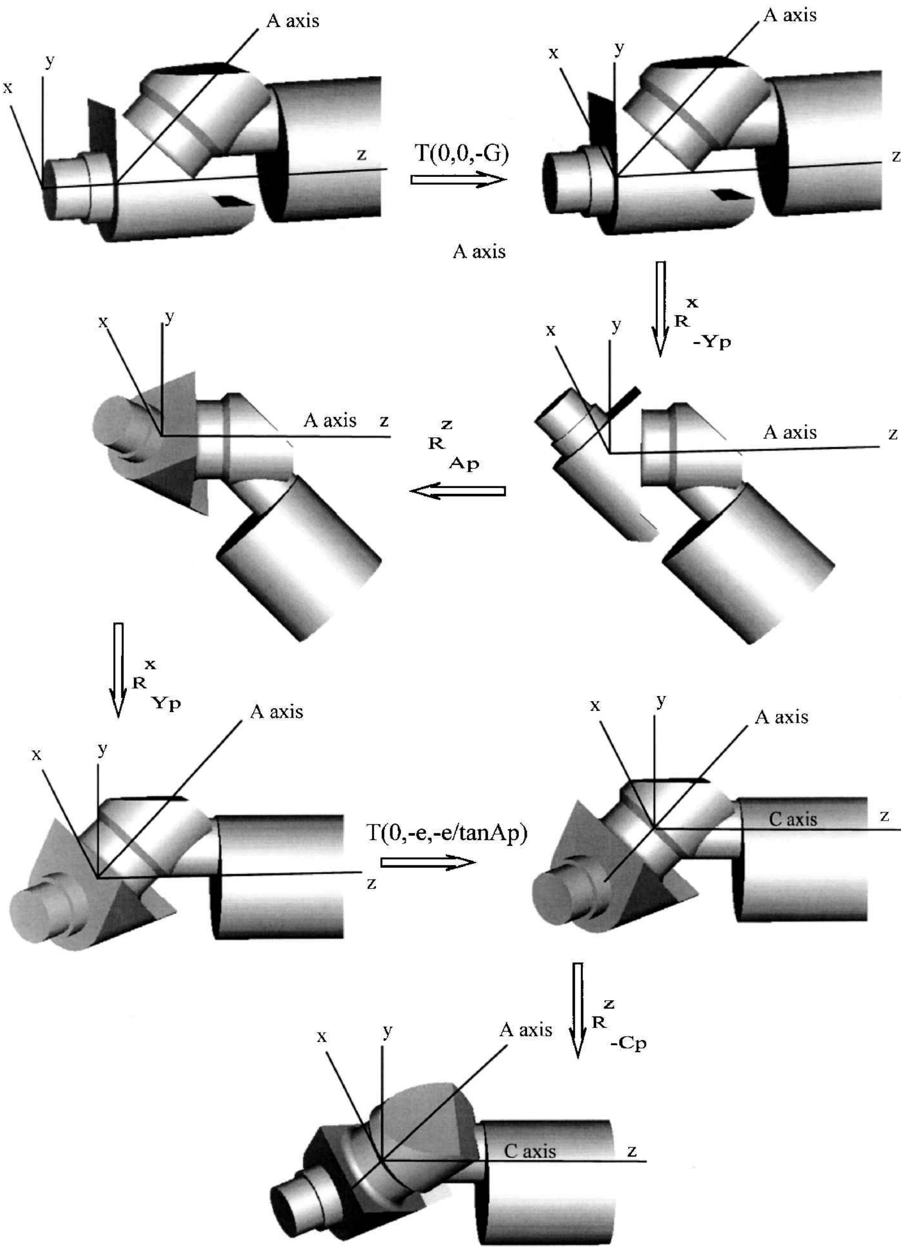

Fig. 2 The basic transformations on the angle head

quantities. Also indicated is the sign, which will be negative if it is defined in a clockwise direction to maintain consistency with previous work. The complete mathematical representation of the transformations required to perform the error analysis using all the variables in Table 1 assuming that $\boldsymbol{X}^0$ represents the origin is

$$\begin{aligned}\boldsymbol{X} = {} & \mathbf{R}^{x}_{\mathrm{Theta}C_{\mathrm{v}}}\mathbf{R}^{y}_{\mathrm{Theta}C_{\mathrm{h}}}\mathbf{R}^{z}_{-C_{\mathrm{p}}}\mathbf{T}_{(0,-e,-e/\tan Y)}\\ & \times \mathbf{T}_{(O_{C_x},O_{C_y},0)}\mathbf{R}^{x}_{Y_{\mathrm{p}}}\mathbf{R}^{z}_{A_{\mathrm{p}}}\mathbf{R}^{y}_{\mathrm{Theta}A_{\mathrm{vp}}}\mathbf{R}^{x}_{-\mathrm{d}Y}\mathbf{R}^{x}_{-Y\mathrm{p}}\\ & \times \mathbf{T}_{(0,O_{A_{\mathrm{s}}},0)}\mathbf{R}^{y}_{-\mathrm{Theta}A_{\mathrm{s}}}\mathbf{T}_{(0,0,-G)}\boldsymbol{X}^0\end{aligned}$$

4 ALGEBRAIC REPRESENTATION OF THE PROCESS

The derivation above gives numerical results if implemented directly on a digital computer. However, what is required is an algebraic representation of the position of the tool tip and algebraic matrix translations and rotations. This requires matrix multiplication and addition routines utilizing techniques of algebraic representation via specialized encoding methods. The general form of the process is discussed by Chen and

Table 1 Definition of variables in the angle head

Variable name	Size	Sign	Definition
1	Large		Constant = 1
G	Large		Spindle length
Y_{p}	Large	+ve	Fixed angle between the A and C axes
A_{p}	Large	+ve	Angular rotation A_{p}: front axis
E	Large		Displacement between the C and A axes with $A=0$ and $C=0$
C_{p}	Large	−ve	Angular rotation C_{p}: back axis
ThetaA_{s}	Small	−ve	Angle between A and spindle S
$O_{A_{\mathrm{s}}}$	Small		Offset A to S
ThetaA_{vp}	Small	+ve	Angle between A and the yz plane
DY	Small		Error in angle Y_{p}
O_{C_x}	Small		Offset of C in the x direction
O_{C_y}	Small		Offset of C in the y direction
ThetaC_{h}	Small	+ve	Angle between C and the xz plane
ThetaC_{v}	Small	+ve	Angle between C and the yz plane

Geddam [3] and can be represented by

$$\boldsymbol{X} = \left(\prod_{i=0}^{N-1} \Psi_i\right)\boldsymbol{X}^0$$

where vector $\boldsymbol{X}^0$ represents the Cartesian coordinates of the tool tip before any transformations are performed and vector $\boldsymbol{X}$ represents the Cartesian coordinates of the tool tip in its final position. For simplicity, each of the N transformations Ψ_i is considered to represent either a rotation or translation, rather than the usual combination of the two, depending on whether the error i is angular or is an offset.

It is necessary to develop a coding system to define algebraically the $\boldsymbol{X}$ and $\boldsymbol{X}^0$ vectors and the transformation matrices Ψ. In the type of problem of interest there are a limited number of possible elements that can occur in $\boldsymbol{X}$ and Ψ. These are either constants, angles, the sines or cosines of angles or simple offsets, so it is feasible to define a set of these variables in an array of structures $\boldsymbol{q}[i]$, where i is the variable number, which is sufficient to construct $\boldsymbol{X}$ and Ψ by coded reference to $\boldsymbol{q}$. The elements of the structures $\boldsymbol{q}$ are the actual names of the variables and an indication of the size of the variable, as found in Table 1, indicating whether it represents a main variable or a small error. The latter is used to aid simplification of the algebra by removing user-specified higher products of small quantities and replacing trigonometric relationships by first-order approximations. The matrix operations cause many additional algebraic terms to be generated at every step, so algebraic simplification is necessary, this being achieved by searching for trigonometric simplifications and any zero multipliers.

The required formulae can be obtained by interpreting the final vector $\boldsymbol{X}$ in an algebraic form. Also, it is possible to interpret the names contained in vector $\boldsymbol{q}$ in a form suitable for inclusion as declarations in other software. This reduces the likelihood of errors and speeds up the process of writing software using the formulae.

Table 2 gives the formulae generated by the software for the angle head in a form suitable for inclusion in a high-level language program. The results are quoted for the direct transformation with no errors included, represented by the formulae for X, Y and Z and for the inclusion of first-order error terms only represented by DX, DY and DZ. Thus X gives the x position of the tool tip and DX gives the error in the x position at the tool tip to a first-order approximation, etc.

4.1 Validation of the direct transformation

The software to derive the direct transformation was first successfully validated against the fork head, for which a manual analytical solution is available [5]. The formulae for the direct transformation of the angle head with no errors have been compared with a previous analysis [7], the equations of which are not of exactly the same form, by computing x, y and z for various typical A and C values. The assumed values for G, e and Y were 20 mm, 10 mm and 50° respectively. The results are quoted in Table 3, where the origin for the computer-generated results has been shifted to correspond to that previously assumed. The equivalence of the two sets of formulae is evident.

4.2 The transformation with errors included

The software to derive the first-order error equations was first successfully validated against the fork head, for which a manual analytical solution is available [5]. No previous results are available for the error analysis of the angle head for comparison with the present results since the equations are too laborious to derive manually. To visualize the implications of the formulae

Table 2 The computer-generated analytical equations for the angle head

X = G*sin(Yp)*sin(Ap)*cos(Cp) -G*sin(Yp)*cos(Ap)*cos(Yp)*sin(Cp) +G*cos(Yp)*sin(Yp)*sin(Cp) +e*sin(Cp);	Y = G*sin(Yp)*sin(Ap)*sin(Cp) +G*sin(Yp)*cos(Ap)*cos(Yp)*cos(Cp) -G*cos(Yp)*sin(Yp)*cos(Cp) -e*cos(Cp);	Z = -G*sin(Yp)*cos(Ap)*sin(Yp) -G*cos(Yp)*cos(Yp) -e/tan(Yp);

```
DX = G*ThetaAs*cos(Ap)*cos(Cp) -G*cos(Yp)*ThetaAv*cos(Ap)*cos(Cp) +Oas*cos(Yp)*sin(Ap)*cos(Cp)
-G*cos(Yp)*DY*sin(Ap)*cos(Cp) +Ocx*cos(Cp) +G*ThetaAs*sin(Ap)*cos(Yp)*sin(Cp)
-G*cos(Yp)*ThetaAv*sin(Ap)*cos(Yp)*sin(Cp) -Oas*cos(Yp)*cos(Ap)*cos(Yp)*sin(Cp)
+G*cos(Yp)*DY*cos(Ap)*cos(Yp)*sin(Cp) +G*sin(Yp)*DY*sin(Yp)*sin(Cp) -Oas*sin(Yp)*sin(Yp)*sin(Cp)
-Ocy*sin(Cp) -G*sin(Yp)*cos(Ap)*sin(Yp)*ThetaCh -G*cos(Yp)*cos(Yp)*ThetaCh -e/tan(Yp)*ThetaCh;
```

```
DY = G*ThetaAs*cos(Ap)*sin(Cp) -G*cos(Yp)*ThetaAv*cos(Ap)*sin(Cp) +Oas*cos(Yp)*sin(Ap)*sin(Cp)
-G*cos(Yp)*DY*sin(Ap)*sin(Cp) +Ocx*sin(Cp) -G*ThetaAs*sin(Ap)*cos(Yp)*cos(Cp)
+G*cos(Yp)*ThetaAv*sin(Ap)*cos(Yp)*cos(Cp) +Oas*cos(Yp)*cos(Ap)*cos(Yp)*cos(Cp)
-G*cos(Yp)*DY*cos(Ap)*cos(Yp)*cos(Cp) -G*sin(Yp)*DY*sin(Yp)*cos(Cp) +Oas*sin(Yp)*sin(Yp)*cos(Cp)
+Ocy*cos(Cp) -G*sin(Yp)*cos(Ap)*sin(Yp)*ThetaCv -G*cos(Yp)*cos(Yp)*ThetaCv -e/tan(Yp)*ThetaCv;
```

```
DZ = -G*sin(Yp)*sin(Ap)*sin(Cp)*ThetaCv -G*sin(Yp)*cos(Ap)*cos(Yp)*cos(Cp)*ThetaCv
+G*cos(Yp)*sin(Yp)*cos(Cp)*ThetaCv +e*cos(Cp)*ThetaCv -G*sin(Yp)*sin(Ap)*cos(Cp)*ThetaCh
+G*sin(Yp)*cos(Ap)*cos(Yp)*sin(Cp)*ThetaCh -G*cos(Yp)*sin(Yp)*sin(Cp)*ThetaCh -e*sin(Cp)*ThetaCh
+G*ThetaAs*sin(Ap)*sin(Yp) -G*cos(Yp)*ThetaAv*sin(Ap)*sin(Yp) -Oas*cos(Yp)*cos(Ap)*sin(Yp)
+G*cos(Yp)*DY*cos(Ap)*sin(Yp) -G*sin(Yp)*DY*cos(Yp);
```

Table 3 Numerical comparison of analytical and computer-generated equations for the angle head

Angle A (deg)	Angle C (deg)	Analytical X (mm)	Computer X (mm)	Analytical Y (mm)	Computer Y (mm)	Analytical Z (mm)	Computer Z (mm)
30	40	13.144206	13.144206	6.252881	6.252881	−18.427610	−18.427610
20	50	11.483648	11.483648	7.204478	7.204478	−19.292204	−19.292204
10	90	10.149614	10.149614	12.660444	12.660444	−19.821696	−19.821696

for DX, DY and DZ in Table 2, simulation software was written to use the computer-generated equations for both first- and second-order approximations of the error model. The second-order equations are obtained by ignoring only terms involving products of more than two small quantities and are very lengthy. They are therefore not quoted here. The angular errors were assumed to be small, which is not strictly true for the second-order solution. However, the results give a good indication of the magnitude of the second-order effects, which gave no significant change from the first-order approximations, as would be expected for the small angular errors assumed to be present in these structures.

The output of the simulation software consists of separate tables of errors for x, y and z. These tabulate the errors against various regularly spaced values of A_p and C_p. The tables can be plotted to form three-

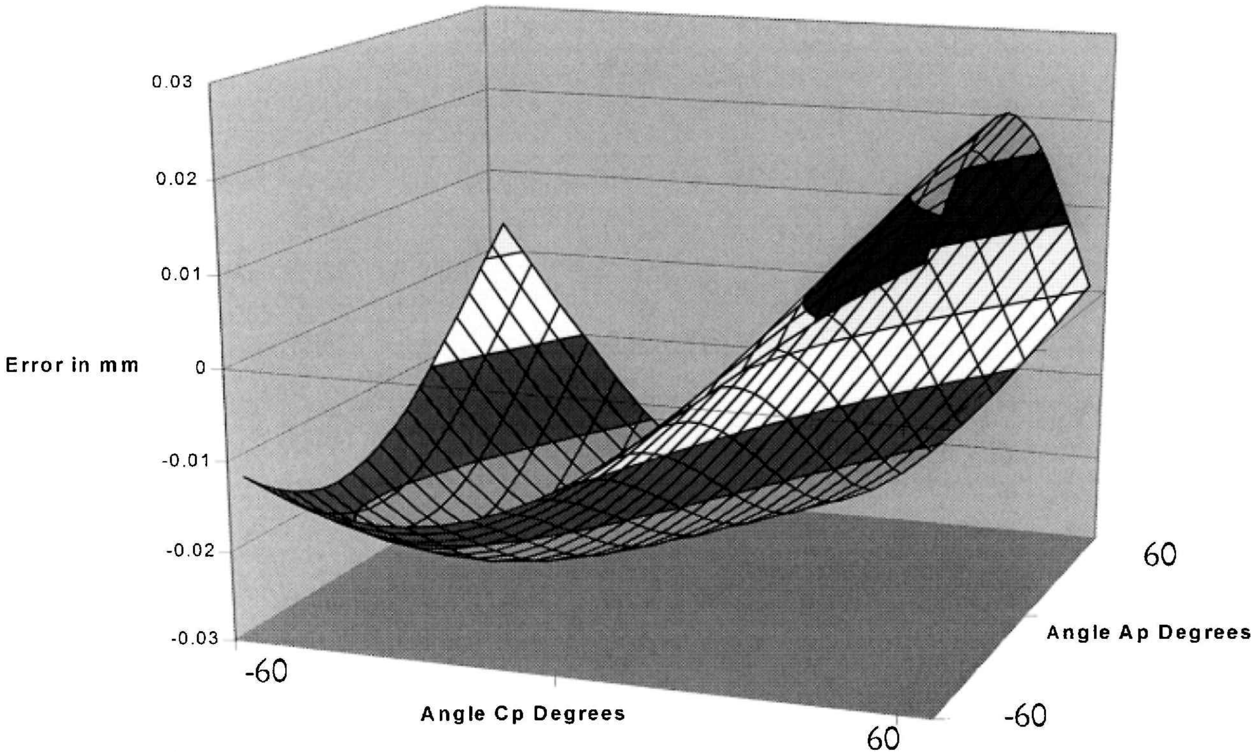

Fig. 3 Graphical representation of error in the Y axis versus A_p and C_p

dimensional representations of the error plotted on a rectangular grid against A_p and C_p. A typical result is given in Fig. 3 for the first-order errors for which it was assumed that $G = 20$ mm, $Y_p = 500$, $O_{C_x} = 0.005$ mm, $O_{C_y} = -0.015$ mm, $DY = 0$, Theta$A_s = 100$ arcsec, Theta$C_h = 150$ arcsec and Theta$C_v = -50$ arcsec. C_p and A_p vary from $-60°$ to $60°$ across the graph.

It is not possible to plot representative graphs for all possible angular errors because of the number of variables involved. However, the error simulation software ESP developed at the University of Huddersfield [**4**] could be adapted to include these sources of error.

5 THE EXPERIMENTAL MEASUREMENT OF THE ERRORS

In order to measure the errors with a view to including them in compensation software it is necessary to find measurement protocols that will allow the derivation of the individual errors from actual measurable quantities. Thus equations are required in as simple a form as possible, so that they can be used to evaluate the errors, ideally one by one. There are a very large number of possible measurement experiments that could be performed, but the resulting equations will vary in complexity for each. The requirement is for a measurement protocol that leads to the simplest possible analysis and is feasible practically. The approach is to allow A and C to have values of 0, 90, 180 and 270°, leading to 16 possible combinations.

If A and C are both set to zero the equations are simplified and errors can be extracted from one or more of the resulting equations. Also, as an illustration, if X is measured for $A = 90$, $C = 0$ and $A = 270$, $C = 180°$, the difference DX in X between the two measurements gives $2O_{Cx}$. Since there are 16 sets of equations it is too difficult to analyse and search for tractable equations manually.

The error analysis software includes a feature whereby all 16 sets of equations are generated and the known values substituted for A and C. These simultaneous equations are then algebraically added and subtracted to and from each other for all the possible combinations and the resulting equations simplified. The user can then choose to set any particular variable to zero, such as DY, which is reasonable since it represents the error in the fixed angle between the A and C axes. An automatic search is then made for the simplest equations and these are displayed on screen. If one or more equations involving only one unknown is available, the user chooses the most appropriate equation. The software then considers that the variable has been identified and that its value is known in equations generated subsequently. The user then chooses the next variable to be identified and the software continues to find the simplest equation

Table 4 Output of the software describing the measurement protocol

PROTOCOL EQUATIONS

Parameters assumed zero DY

Test 1	Ocx from Xdiff (90,0,-270,180) = Ocx*2
Test 2	Oas from Xdiff (90,0,-270,0) = Oas*cos(Yp)*2 +G*sin(Yp)*2
Test 3	Ocy from Xdiff (0,270,-0,90) = Oas*2 +Ocy*2 -e*2
Test 4	ThetaCh from Zdiff (0,270,-0,90) = e*ThetaCh*2
Test 5	ThetaCv from Zdiff (0,0,-0,180) = e*ThetaCv*2
Test 6	ThataAs from Xdiff (0,0,-0,180)= G*ThetaAs*2 -G*cos(Yp)*ThetaAv*2 +Ocx*2
Test 7	TheatAs from Ydiff (90,180,-270,180) = G*ThetaAs*cos(Yp)*2 - G*cos(Yp)*ThetaAv*cos(Yp)*2
Test 8	Theta Av from Xdiff (0,0-0,180) = G*ThetaAs*2 -G*cos(Yp)*ThetaAv*2 +Ocx*2

Notes - Xdiff is the difference in X at the tool tip for two measurements made with Ap and Cp as quoted in parenthesis in the order $(A_p(1),C_p(1),A_p(2),C_p(2))$.

- Tests 6 and 7 form simultaneous equations from which ThetaAs can be found.

involving that variable and the least possible number of other unknowns. When there is more than one unknown then the user must choose suitable simultaneous equations from which the variables can be determined. A report is produced by the software, which summarizes all the user choices and lists the necessary equations. This can be seen in Table 4 for the angle head.

6 CONCLUSIONS

A general solution to the geometrical analysis of serial manipulators, in particular the servo head, has been presented. It makes possible the automatic generation of analytical equations for the direct transformation and for all the Cartesian errors of any new head without resorting to manual geometrical derivations. A good understanding of the possible errors in the head and their representation as rotations or translations is all that is required to use the software.

The computer-generated equations can easily be built into compensation software, thus providing full real-time error compensation for multiple-axis machines. Simulation software has been written that can investigate the effect of the various errors in the head on the resultant error of the machine. Also, it is possible in the future to develop the ESP simulation software package [**4**] in order to estimate the volumetric accuracy of the machine.

REFERENCES

1 **Postlethwaite, S. R.** Electronic based accuracy enhancement of CNC machine tools. PhD thesis, The University of Huddersfield, 1992.

2 **Ford, D. G., Postlethwaite, S. R.** and **Morton, D.** Error compensation applied to high precision machinery. In Proceedings of the 1st International Conference on *Laser Metrology and Machine Performance—LAMDAMAP '93*, 1993, pp. 105–112 (Computational Mechanics Publications, Southampton and Boston).

3 **Chen, X. B.** and **Geddam, A.** Accuracy improvement of three-axis CNC machining centers by quasi-static error compensation. *J. Mfg Systems*, 1997, **16**(5), 323–336.

4 **Postlethwaite, S. R.** and **Ford, D. G.** Geometric error analysis software for CNC machine tools. In Proceedings of the 3rd International Conference on *Laser Metrology and Machine Performance—LAMDAMAP '97*, 1997, pp. 305–316 (Computational Mechanics Publications, Southampton and Boston).

5 **Postlethwaite, S. R.** Five axis error compensation training course. Internal Report, Ultra Precision Engineering Centre, The University of Huddersfield, 1999.

6 **Postlethwaite, S. R.** and **Ford, D. G.** A practical system for 5 axis volumetric compensation. In Proceedings of the 4th International Conference on *Laser Metrology and Machine Performance—LAMDAMAP '99*, Longhirst Hall, Northumberland, 1999, pp. 379–388.

7 **Postlethwaite, S. R.** Calculation of head offsets for angle-type servo head. Internal Report, Ultra Precision Engineering Centre, The University of Huddersfield, 1999.