Exponential Log-Periodic Antenna Design Using Improved Particle Swarm Optimization with Velocity Mutation

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An improved particle swarm optimization (PSO) method applied to the design of a new wideband log-periodic antenna (LPA) geometry is introduced. This new PSO variant, called PSO with velocity mutation (PSOvm), induces mutation on the velocities of those particles that cannot improve their position. The proposed LPA consists of wire dipoles with lengths and distances varied according to an exponential rule, which is defined by two specific parameters called length factor and spacing factor. The LPA is optimized for operation in 790-6000MHz frequency range, in order to cover the most usual wireless services in practice, and also to provide in this range the highest possible forward gain, gain flatness below 2dB, secondary lobe level below –20dB with respect to the main lobe peak, and standing wave ratio below 2. To demonstrate its superiority in terms of performance, PSOvm is compared to well-known optimization methods. The comparison is performed by applying all the methods on several test functions and also on the LPA optimization problem defined by the above-mentioned requirements. Furthermore, the radiation characteristics of the PSOvm-based LPA give prominence to the effectiveness of the proposed exponential geometry compared to the traditional Carrel's geometry.

***Index Terms*—Antenna optimization, log-periodic antennas, log-periodic dipole arrays, particle swarm optimization**

I. Introduction

Log-periodic antennas (LPAs) are special structures used for signal reception in many practical applications and particularly in cases where a wideband behavior is required by the receiver [1]. By properly adjusting its geometric dimensions, an LPA may provide optimal values for several radiation characteristics inside the desired operating frequency range, such as the standing wave ratio (*SWR*), forward gain (*FG*), front-to-back ratio (*FBR*), side lobe level (*SLL*) and gain flatness (*GF*) (i.e., the maximum forward gain variation inside the operating bandwidth of the antenna). An attempt to optimize any of the above-mentioned characteristics is a hard non-linear design problem and its difficulty is due the fact that the radiation characteristic under discussion has to be optimized not only for a single frequency but for every frequency inside the operating bandwidth. The only exception is *GF*, which is calculated due to its definition for the entire bandwidth and not for every frequency. The problem becomes harder if the demand is to simultaneously optimize all the above characteristics over this bandwidth. Such a design problem is classified as multi-target, and few efforts have been made so far to solve it and only for limited bandwidths [2].

The most popular LPA design method, considered now as traditional one, has been proposed by Carrel [1]. Carrel's method is based on the assumption that the LPA consists of wire dipoles all located inside the same angular sector. According to this assumption, the LPA geometry can easily be described by using two parameters, known as *scale factor* τ and *relative spacing* σ. The optimal values of τ and σ are defined according to the desired average directivity by utilizing the constant directivity contour curves of the well-known Carrel's graph [1]. The whole LPA geometry (i.e., dipole lengths, radii and distances) is estimated by taking into account the boundaries of the required operating bandwidth as well as the optimal values of parameters τ and σ. The antenna derived from Carrel's method is usually able to achieve optimal *SWR* values over the desired bandwidth. Nevertheless, this method cannot provide optimization of *FG*, *GF*, *FBR* or *SLL* in this bandwidth.

The concurrent optimization of *SWR*, *FG*, *FBR* and *SLL* for every frequency inside a given bandwidth as well as *GF* for the same bandwidth is a multi-target non-linear design problem, as mentioned before, and can effectively be solved by employing evolutionary optimization algorithms. Such algorithms have already been used in the past but only for limited bandwidths and not to optimize all the above characteristics [3]. An effort to concurrently optimize *SWR*, *FG*, *FBR*, *SLL* and *GF* in a wide frequency range is made in the present study. Here, the LPA is required to operate in the frequency range 790-6000MHz, in order to cover the most usual wireless services in practice (2G, 3G, 4G and Wi-Fi), and also to achieve over this range (i) $SWR \leq 2$, (ii) the highest possible *FG*, (iii) $GF \leq 2$dB, and (iv) secondary lobe level ($SecLL$) ≤ -20dB with respect to the main lobe peak. Such an antenna can be used for spectrum monitoring, radiation measurements and signal reception from multiple wireless services. It is noted that the calculation of *SecLL* considers all the side lobes and the back lobe of the radiation pattern. Therefore, if the LPA achieves $SecLL \leq -20$dB, it satisfies two requirements at once concerning respectively *FBR* and *SLL* (i.e., $FBR \geq 20$dB and $SLL \leq -20$dB). To calculate the radiation characteristics, the LPA undergoes full wave analysis by applying CST Microwave Studio (CST MWS).

To help the LPA reach more easily the above-mentioned requirements, we propose an exponential LPA geometry. In this geometry, the lengths and distances of the wire dipoles that compose the LPA vary according to an exponential rule (Fig. 1). Thus, the dipoles are no longer considered as being inside the same angular sector as in Carrel's method. It is believed that this type of geometry in combination with an evolutionary optimization method induces a greater design freedom and therefore the concurrent satisfaction of the four requirements specified before may more easily be achieved.

To simultaneously satisfy these requirements, we introduce an improved particle swarm optimization (PSO) variant, called *PSO with velocity mutation* (PSOvm). In PSOvm, a mutation mechanism is applied on the velocities of those particles that are not able to find a better position. To demonstrate its superiority in terms of performance, PSOvm is compared to well-known evolutionary optimization methods, such as the conventional PSO [4], the differential evolution (DE) [5], the invasive weed optimization (IWO) [2], and a typical genetic algorithm (GA). The comparison is performed by applying them on several test functions and also to the exponential LPA design. Finally, to demonstrate the effectiveness of the proposed exponential LPA, the radiation characteristics of the PSOvm-based LPA are compared to those of a traditional LPA designed by Carrel's method.

II. Description of Exponential Geometry

The proposed exponential geometry of an M-dipole LPA is explicitly described in Fig. 1. The lengths L_m (m=1,...,M) of the wire dipoles and their distances S_m (m=1,...,M–1) from their next ((m+1)-th) dipole undergo an exponential variation as moving from larger dipoles to shorter ones (i.e., in ascending order of index m), as given by the expressions:

$$L_m = L_1 \exp\left[-a\left(m-1\right)\right], \quad m = 1,...,M \tag{1}$$

$$S_m = S_1 \exp\left[-b\left(m-1\right)\right], \quad m = 1,...,M-1 \tag{2}$$

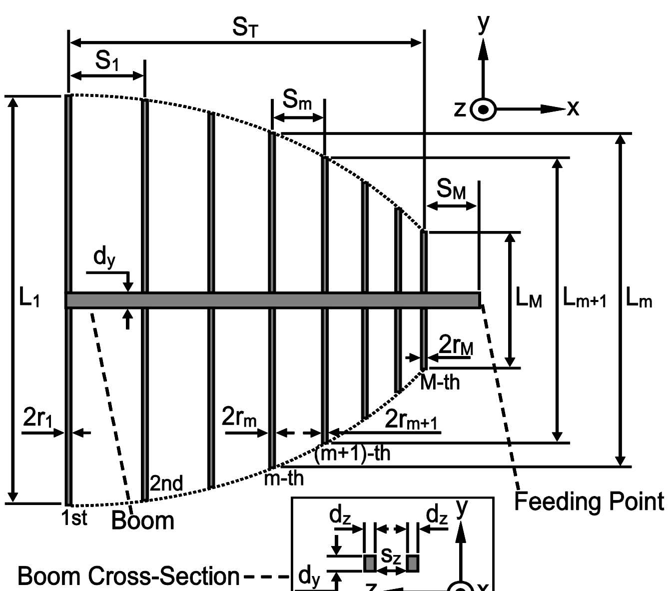

Fig. 1. Exponential LPA geometry with $a < b$.

where L_1 is the length of the largest (1st) dipole, S_1 is the distance between the 1st and 2nd dipole, a is the *length factor* used to describe the variation of L_m, and b is the *spacing factor* used to describe the variation of S_m. On the other hand, the dipole radii r_m (m=1,...,M) are all equal to the typical value of 2mm for easy practical fabrication. Fig. 1 displays the LPA layout when $a<b$. The exponential envelope (dotted line) that enfolds the dipoles is vertically flipped when $a>b$. Apparently, in the special case where $a=b$, the exponential geometry is transformed into the conventional (linear) one.

To satisfy the above four requirements, we must find proper values for the following seven parameters: L_1, S_1, a, b, the length S_M of the boom segment between the shortest dipole and the feeding point, the dimension d_y of the rectangular cross-section of each rod of the boom along y-direction, and finally the distance s_z between the closest surfaces of the rods along z-direction (Fig. 1). The dimension d_z of each rod of the boom along z-direction is considered fixed and equal to 4mm. This value was chosen since it was found after several trials that a decrease in d_z improves the radiation characteristics of the LPA. Thus, d_z was decided to have the lowest possible value that still provides the ability to attach the dipoles to the boom. This value is equal to the dipole diameter (4mm).

III. PSO with Velocity Mutation

Several PSO variants have been proposed so far with remarkable performance [6], [7]. PSOvm is a PSO variant that adopts the *gbest* model of the *constriction coefficient based PSO* (CCPSO) version [4]. By assuming that a swarm of N particles disperses in a D-dimensional search space (where D is actually the number of parameters to be optimized), the velocities and positions of the particles are respectively updated at the i-th iteration (i=1,…,I), as considered in CCPSO, by the following expressions:

$$\begin{aligned} v_{nd}\left(i+1\right) = k\Big\{ & v_{nd}\left(i\right) + \varphi_1 R\left[p_{nd}\left(i\right) - x_{nd}\left(i\right)\right] \\ & + \varphi_2 R\left[g_d\left(i\right) - x_{nd}\left(i\right)\right]\Big\} \end{aligned} \tag{3}$$

$$x_{nd}\left(i+1\right) = x_{nd}\left(i\right) + v_{nd}\left(i+1\right) \tag{4}$$

where v_{nd} and x_{nd} are respectively the d-th velocity component and the d-th position coordinate (d=1,…,D) of the n-th particle (n=1,…,N), p_{nd} and g_d are the d-th coordinates of the best positions found at the end of the i-th iteration respectively by the n-th particle and the whole swarm, and R represents random numbers uniformly distributed in the interval (0,1). Also, as explained in [4], φ_1 and φ_2 are respectively the cognitive coefficient and the social coefficient, both equal to 2.05, and finally k is the constriction coefficient equal to 0.73.

The basic idea for proposing a mutation process in PSOvm is the consideration that if a particle cannot improve its fitness, then its previous velocity vector should not directly affect its next velocity vector, as happens in (3), but it should undergo a slight random perturbation. So, if any n-th particle fails to achieve a better fitness at the end of any i-th iteration, then, its velocity components $v_{nd}(i)$ are mutated by multiplying them by a factor F_m=(0.6+0.1m)(2R–1), where m is the number of iterations passed in a row with no fitness improvement for this particle. In such a case, the velocity update is given by:

$$v_{nd}\left(i+1\right)=k\left\{\left(0.6+0.1m\right)\left(2R-1\right)v_{nd}\left(i\right)\right.$$
$$\left.+\varphi_1 R\left[p_{nd}\left(i\right)-x_{nd}\left(i\right)\right]+\varphi_2 R\left[g_d\left(i\right)-x_{nd}\left(i\right)\right]\right\},\ m=1,...,6 \tag{5}$$

The position update for such a particle is applied by using (4). The form of F_m has been derived from findings extracted after many trials on PSOvm. So, it was found that if only one failure of fitness improvement occurs (m=1), it is better to multiply $v_{nd}(i)$ by random numbers uniformly distributed in the interval (–0.7,+0.7). Also, the absolute values of both boundaries of this interval must be increased by 0.1, for every additional failure in a row. Finally, for the sake of convergence speed, the mutation process must be applied for up to six failures in a row (m=1,…,6 in (5)). Thus, the mutation process stops when either six failures of fitness improvement occur in a row or the mutation results in a better fitness. Then, in the next iteration, the particle's velocity is updated by using (3).

TABLE I
STATISTICAL MEAN AND STANDARD DEVIATION OF FINAL FITNESS VALUES

Test Function	PSOvm Mean Fit / Std Dev	CCPSO Mean Fit / Std Dev	DE Mean Fit / Std Dev	IWO Mean Fit / Std Dev
Ackley's	**0.4489** / **0.2753**	4.3439 / 2.7085	6.5223 / 1.8891	1.3135 / 1.5335
De Jong's N.1	**0.0000** / **0.0000**	**0.0000** / **0.0000**	0.6171 / 0.7936	0.0075 / 0.0019
De Jong's N.3	**0.0409** / **0.0389**	0.0614 / 0.3511	0.2938 / 0.3244	0.1812 / 0.2881
De Jong's N.4	**0.0000** / **0.0000**	**0.0000** / **0.0000**	0.0830 / 0.1905	**0.0000** / **0.0000**
Easom (2D)	**-1.0000** / **0.0000**	**-1.0000** / **0.0000**	**-1.0000** / **0.0000**	**-1.0000** / **0.0000**
Eggholder	**-16326.3** / 1475.5	-16232.9 / 1283.2	-14301.2 / 3549.5	-14184.4 / **984.5**
Holder Table (2D)	**-19.2315** / **0.3719**	-19.1084 / 0.5159	-18.8920 / 0.6507	-19.2085 / 0.4718
Levy	6.6263 / 5.7797	6.7984 / 4.9205	4.3218 / 2.6525	**1.0559** / **1.2247**
Michalewicz	**-22.4942** / **1.1055**	-22.3976 / 1.4193	-14.3558 / 2.0444	-21.9247 / 1.1057
Rana's	-10271.8 / 945.4	-10249.7 / 793.3	**-11408.4** / 1147.5	-8156.2 / **460.7**
ShiftedRotated Griewank	**-179.8591** / **0.0450**	-179.6796 / 0.3304	-123.3609 / 56.8099	-178.7402 / 0.1661
ShiftedRotated Rastrigin	**-275.3862** / 55.8026	-105.0404 / 77.1869	-99.0923 / 19.6590	-180.2995 / **10.6864**
Shifted Rosenbrock	**600.6099** / **260.5250**	600.6779 / 318.6767	1.15e8 / 1.57e8	5440.3 / 7186.2
Schwefel	**-328.1901** / **16.4253**	-326.8062 / 16.5886	-289.2526 / 31.1652	-295.1572 / 17.3701
Shubert (2D)	**-186.7309** / **0.0000**	**-186.7309** / **0.0000**	**-186.7309** / **0.0000**	**-186.7309** / **0.0000**
Sinewave	**8.8087** / 1.0246	9.0188 / 1.0503	11.8375 / **0.4725**	9.7411 / 0.6878

Restrictions on particle velocities and positions used in CCPSO are also used in PSOvm [4]. So, every velocity component is restricted in PSOvm by a maximum allowed value (vmax) defined to be equal to 15% of the width of the search space in the respective dimension. To confine the particles within the search space, the absorbing walls condition is adopted in PSOvm as well.

To demonstrate its superiority, PSOvm is compared to the conventional CCPSO [4], a DE algorithm based on the popular DE/rand/1/bin strategy [5], and the conventional IWO [2]. All the methods use populations of 20 particles (N=20). The comparison is made by applying the methods on 16 well-known mathematical test functions considered as fitness functions to be minimized. The use of such functions is typical for comparisons among evolutionary optimization methods. 30 variables (D=30) are used for all functions except for those indicated as 2D functions, which use only two variables (D=2) due to their structure. Each method is executed 500 times for each test function. Each execution terminates after 2×10^4 fitness evaluations and the final fitness value is recorded. Therefore, 500 final fitness values are recorded per function and per method. These values are used to calculate the mean final fitness (Mean Fit) and the respective standard deviation (Std Dev). As shown in Table I, the mean value and standard deviation of final fitness achieved by PSOvm are better than (or at least equal to) the respective values achieved by the other three methods for most test functions.

IV. LPA Optimization Results

PSOvm, CCPSO, DE and IWO that were used above and a typical GA are all employed here to solve a multi-target problem, which is to design a 15-dipole exponential LPA (M=15) that concurrently satisfies requirements for *SWR*, *FG*, *GF* and *SecLL*, as defined in Section I, over the 790-6000MHz frequency band. However, the aim of all these methods is to find the near global minimum of a single mathematical function known as fitness function. To deal with this, the above four requirements must be described by respective mathematical terms and these terms must merge into a fitness function *Fit* in such a way that, when *Fit* achieves its near global minimum, all the terms reach their respective minimum values and therefore all the requirements are satisfied. Thus, *Fit* is defined as a linear combination of four terms as follows:

$$Fit = k_1 \max\left(SWR_{\max},\ 2\right)+k_2\left(-FG_{\min}\right)$$
$$+k_3 \max\left(GF,\ 2\right)+k_4 \max\left(SecLL_{\max},\ -20\right) \tag{6}$$

where $SWR_{\max}$, $FG_{\min}$ and $SecLL_{\max}$ are respectively the maximum *SWR*, the minimum *FG* in dBi and the maximum *SecLL* in dB found over the 790-6000MHz frequency band. As $FG_{\min}$ increases, the 2nd term of (6) decreases and thus is used to maximize *FG* over the entire bandwidth. The rest three terms are built in such a way that values of $SWR_{\max}$, *GF* and $SecLL_{\max}$ respectively less than 2, 2dB and –20dB do not cause further minimization of *Fit*, since the respective requirements have already been satisfied. Also, k_i (i=1,…,4) are positive weights used to balance the minimization rates of the four terms of (6). To achieve balancing, the value of each k_i must be reversely proportional to the difficulty encountered by the optimization method in decreasing the respective term. If a term decreases with greater difficulty than another term, it must be multiplied by a greater weight and then its lower minimization rate is emphasized more than the higher rate of the other term. In this way, both rates are balanced. After many trials, we arrived at the following weights: k_1=45, k_2=8, k_3=12 and k_4=35.

To save computational time, *SWR*, *FG* and *SecLL* are not calculated for every frequency inside the above band. Thus, a set of equally spaced frequency samples at steps of 25MHz is chosen to represent this band. So, when a fitness calculation is required by any optimization algorithm, CST MWS is called to perform full wave analysis on the LPA and extract the values of *SWR*, *FG* and *SecLL* only for the frequency samples. These values are then used to find SWR_{max}, FG_{min}, *GF* and $SecLL_{max}$, which are utilized in (6) for the fitness calculation.

Each optimization method uses a population of 20 particles (N=20) and terminates after 500 fitness evaluations in total, while the best fitness value is recorded at the end of every iteration. The graphical representation of fitness variation with respect to the number of fitness evaluations is called *convergence graph* and is estimated for every optimization method. These graphs are given in Fig. 2. It is obvious that PSOvm outperforms CCPSO, DE, IWO and GA, since it is capable of achieving the lowest fitness value at the end of the process. This means that the optimized exponential LPA geometry found by PSOvm is closer to the requirements than the respective geometries found by CCPSO, DE, IWO and GA. The parameters that define the PSOvm-based geometry are: L_1=214.2mm, S_1=72mm, a=0.2079, b=0.2009, d_y=2.6mm, s_z=2mm, and S_M=2.4mm.

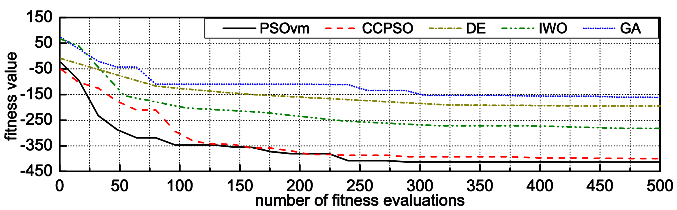

Fig. 2. Convergence graphs.

The optimized LPA derived by PSOvm is finally compared to a conventional LPA, which has the same length S_T=0.372m (see Fig. 1) and operates in the same frequency range (790-6000MHz) as the PSOvm-based LPA. The conventional LPA is derived by applying Carrel's method described explicitly in [1]. In particular, if we consider antenna length equal to 0.372m and operating bandwidth 790-6000MHz, then we get from Carrel's graph τ = 0.8517 and σ = 0.1554. These two parameters are used to calculate the dipole lengths and distances. Since Carrel's method imposes direct proportionality between dipole lengths and radii, the length values are used to calculate the radii, provided that we already know the radius value of the largest dipole. This value is set equal to 2mm, i.e., equal to the fixed radius value of all the dipoles of the PSOvm-based LPA. The evident drawback of the derived Carrel-based LPA is that it consists of 18 dipoles, i.e., three dipoles more than the 15-dipole PSOvm-based LPA. The two LPAs are also compared in terms of *SWR*, *FG*, *GF* and *SecLL*. From the results shown in Figs. 3-5, it is obvious that Carrel's geometry induces greater fluctuations in *SWR*, *FG* and *SecLL*, mainly in low frequencies, and thus it cannot come close to the requirements like the PSOvm-based LPA.

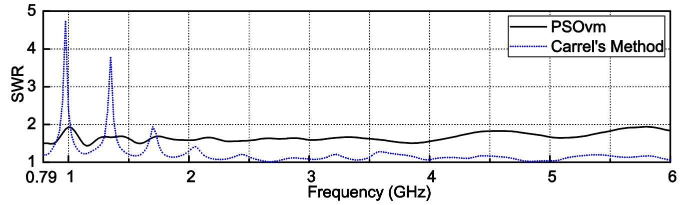

Fig. 3. Comparative graphs of SWR vs. frequency.

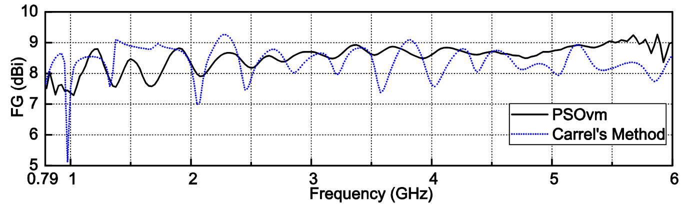

Fig. 4. Comparative graphs of FG vs. frequency.

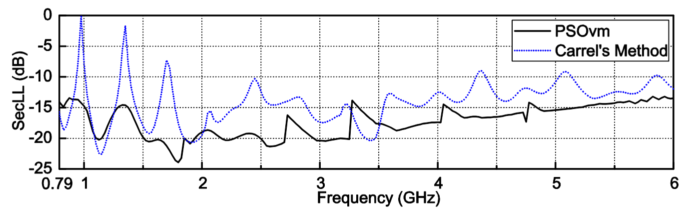

Fig. 5. Comparative graphs of SecLL vs. frequency.

V. CONCLUSION

The comparison among all optimization methods used in this work reveals that PSOvm provides the best results at exactly the same computational time (the same number of fitness function evaluations) for most test functions and, most importantly, for the antenna design problem. On the other hand, the PSOvm-based exponential LPA is proved to be superior compared to Carrel's geometry, since it comes closer to the requirements initially defined for *SWR*, *FG*, *GF* and *SecLL* over an ultra-wide frequency range.

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