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Smith, Andrew, Iwnicki, Simon, Kaushal, Aniruddha, Odolinski, Kristofer and Wheat, Phill

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# Estimating the relative cost of track damage mechanisms: combining economic and engineering approaches

Andrew S. J. Smith<sup>a</sup>, Simon Iwnicki<sup>b</sup> Aniruddha Kaushal<sup>b</sup>,  
Kristofer Odolinski<sup>c</sup>, and Phill Wheat<sup>a</sup>

## Abstract

This paper proposes a new, two-stage methodology to estimate the relative marginal cost of different vehicle types running on rail infrastructure. This information is important particularly where infrastructure managers wish to differentiate track access charges by vehicle type for the purpose of incentivising the development and use of more track friendly vehicles. EU legislation requires that European infrastructure managers set access charges based on the incremental (marginal) cost of running trains on their networks.

The novelty of the approach derives from the combination of: (1) engineering simulation methods that estimate the track damage caused by rail vehicles; and (2) econometric methods that estimate the relationship between actual maintenance costs and the different damage mechanisms. This two-stage approach fills an important gap in the literature, given the limitations of existing “single-stage” engineering or econometric approaches in obtaining relative marginal costs for different types of damage.

We demonstrate the feasibility of the method using 45 track sections from Sweden, for which we have data on maintenance costs together with relevant track and vehicle data for 2012 (supplied by the Swedish Transport Administration). We demonstrate the feasibility of producing summary, section-level damage measures for three damage mechanisms (wear, rolling contact fatigue and settlement) which can be taken forward into the second stage. The second stage econometric results indicate that it is possible to obtain sensible relationships between cost and the different damage types – and thus produce relative marginal costs by damage mechanism and in turn vehicle type. Based on this feasibility study, settlement is found to be the most costly (in terms of maintenance cost) of the three mechanisms, followed by rolling contact fatigue and then wear. Future applications should focus on larger datasets in order to produce the required degree of precision on the marginal cost estimates.

## Keywords

Track damage, marginal cost, econometric methods, engineering simulation, top-down methods, bottom-up methods, access charging, track settlement index, rail wear index, and RCF index

Corresponding author:

Dr Andrew Smith

Senior Lecturer in Transport Regulation and Economics

Institute for Transport Studies (ITS) and Leeds University Business School

Room G.05, Geography East, Institute for Transport Studies (ITS)

Tel : + 44 (0) 113 34 36654

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## Introduction

European policy since the mid-1990s has emphasised the promotion of competition as a way of revitalising the fortunes of Europe's railways. Progressively freight and international passenger rail services have been opened up to competition. The proposals contained in the European Commission's Fourth Railway package (European Commission<sup>1</sup>) envisage further liberalisation of domestic passenger services. Vertical separation of infrastructure and operations or, at least, fair access to infrastructure and transparent prices for access, is seen by the Commission as a key enabler of competition in the sector.

The above developments mean that understanding the cost, and in particular, the marginal (infrastructure) cost, of running an extra train service on the network has become more important than ever. Existing legislation requires that charges for access to the infrastructure must be based on "costs directly incurred as a result of operating the train service" (European Commission<sup>2</sup>). This can be interpreted as what economists would call the short-run incremental (marginal) cost imposed on the infrastructure by the service running on the network. This paper focuses on one element of short-run marginal cost, namely the additional maintenance and renewal cost required to rectify the incremental damage caused by a train service (the marginal wear and tear cost).

The need to estimate marginal cost of infrastructure use is not solely for the purpose of meeting EU legislation. Economic efficiency considerations mean that train operators should pay at least the short-run marginal cost of running an extra train service. Further, track access charges that vary according to the different damage and cost imposed by different vehicles should ensure that the "right" vehicles are run on the network and that new rolling stock designs are developed that reduce whole system costs.

The previous academic literature, and practice by bodies responsible for charging, contains three approaches for estimating marginal costs of traffic and also different types of traffic: cost allocation methods; econometric approaches; bottom-up engineering methods. From an engineering perspective, the main role of the track is to support and guide the railway vehicle. The forces between the vehicle and the track are carried through the wheel-rail interface and include vertical support, lateral guidance, acceleration and braking. These forces lead to different forms of damage, which in turn result in interventions and cost. In practice the approaches used by charging setting bodies can be a hybrid of engineering and econometric methods, such as that used by the British Office of Rail Regulation (ORR); see ORR<sup>3</sup>. However, each of the approaches has significant drawbacks. The contribution of this paper is to propose an alternative, two-stage approach that combines engineering and econometric methods in a way that seeks to exploit the best features of both; and thus overcome some of the weaknesses of previous approaches. This is the unique contribution of the paper.

The remainder of the paper is structured as follows. We first review the previous literature on the estimation of marginal rail infrastructure cost. We then define the relevant damage measures covered by our study and how these are measured, before providing an overview of the two-stage methodological approach. Following this, the engineering stage (stage 1 of the method) is explained via an example for one of the track sections used in the analysis, demonstrating how damage estimates are obtained in the form needed for the second stage. The second stage econometric results, which draw on the damage estimates from stage 1, are then set out and discussed. The final section concludes and suggests future directions for the research.

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## Previous approaches to measuring marginal rail infrastructure cost

There are two methods for producing estimates of marginal costs in rail and transport more generally (Wheat and Smith<sup>4</sup>). Top-down methods relate actual costs to traffic volumes, controlling for characteristics of the infrastructure. Bottom-up methods use engineering models to estimate the damage inflicted by different types of vehicles on the network. Then assumptions can be made about the intervention / remediation required to deal with that damage, combined with estimates of unit costs of that remediation activity, to give the marginal cost estimates. These approaches are summarised in Figure 1. A “third” method, the so-called cost allocation method, can be thought of as a hybrid that utilises engineering judgement and econometric evidence and other rules of thumb to establish the variability of different cost categories. This approach is therefore not discussed further.

- Method 1: engineering approach
    - Simulate damage done by traffic (engineering model)
    - Determine action need to remedy damage (e.g. tamping)
    - Activity volume \* Unit cost of activity = (marginal) Cost
  - Method 2: top down statistical approach
    - Relate actual costs to passenger and freight tonne-km (regression)
    - E.g.  $\text{Log Cost} = a + b * \text{Log Passtonne} + c * \text{Freight tonne}$
    - Compute marginal costs from the parameter estimates (the a, b and c) from that model
- Note: a, b and c are parameters to be estimated by the econometric model (a is a constant term; b and c are elasticities of cost with respect to the relevant variable)

*Figure 1: Alternative approaches for estimating marginal costs*

Both methods have strengths and weaknesses. The advantage of top-down methods is that they use actual cost data. Their weakness lies in the fact that it is likely to be very hard to capture the complexity of factors that will affect the relationship between traffic and cost, and in particular, it has proved difficult to get sensible estimates of the relative cost of passenger and freight vehicles. The bottom-up method is very good at capturing complexity and it is possible to model and gain estimates of the relative damage of different vehicle types. The problem is how then to translate these damage estimates into cost. It is important to note, more precisely, why there is a difficulty in getting from damage to cost in bottom-up approaches. First, traffic results in different types of damage. In practice, one vehicle may cause more of one type of damage and less of another, thus meaning that information is needed on the relative cost of the different types of damage to obtain estimates of relative marginal costs by vehicle type. This leads to a second problem, namely that assumptions are needed on what type of activity and how much of it, are needed to rectify the damage done. This potentially requires a very detailed model or alternatively simplifying assumptions are needed which might be wrong.

Added complexities include the fact that the mix of damage types will affect what activity should be undertaken (e.g. some traffic types might cause damage but at the same time alleviate the need for other forms of remediation, such as rail grinding) and the fact that some damage mechanisms lead to more maintenance activity, whilst others result in more renewal (and the

costs of these can be very different). Finally it is hard to estimate unit costs of activities as these will depend on, inter alia, the location, the nature of the job, the length of possession and the scale of the activity.

The research question therefore is as follows: how can we obtain better estimates of the relative cost of different damage mechanisms, which in turn can then help estimate the relative marginal cost of different vehicle types?

In terms of its relation to other approaches, in the past ORR has used a top-down cost allocation approach, based on engineering judgement, to determine the general level of cost variability, and then used an engineering formulae to allocate costs to vehicles based on their relative damage (vertical forces only). Since then Network Rail has developed its bottom-up cost modelling approach, which is now based on the Vehicle Track Interaction Strategic Model (VTISM; Mills et. al.<sup>5</sup>) to estimate marginal costs from the bottom up). This approach measures the overall variability of costs with respect to traffic. Engineering approaches, based on both vertical and horizontal forces, are then used to allocate that element of costs that is deemed variable down to individual vehicles (see, ORR<sup>3</sup>). This approach means that Britain, unlike other European countries, has highly differentiated charges, by vehicle, which should incentivise the use of more track friendly vehicles.

At the same time, top-down econometric methods relating actual costs to traffic volumes, controlling for other factors have been extensively used and the results used by the European Commission (see, for example, Johansson and Nilsson<sup>6</sup>, Wheat and Smith<sup>4</sup>, Wheat et. al.<sup>7</sup> for a summary, and Andersson et. al.<sup>8</sup> for subsequent developments in modelling of renewals costs). These studies have covered a range of European countries, and suggest that the marginal cost of rail infrastructure maintenance is in the region of 20-35% of maintenance costs (or up to 45% for heavily used sections). Wheat et. al.<sup>7</sup> found that the available evidence was much less strong for renewals, though suggested an indicative overall cost variability proportion of around 35% of renewal costs. More recent evidence has put this at a higher level; at approximately 55% (Andersson et. al.<sup>8</sup>). As noted above, these methods have been useful in determining the extent of cost variability with traffic in general, but less effective at allocating to types of traffic or vehicle. It is worth further noting that the engineering based bottom-up approach used by Network Rail puts the variability proportion at less than 10% which is out of line with the top-down econometric evidence from across Europe.

Our proposed approach is therefore positioned within the existing literature (academic and regulatory) and has the potential to enhance track access charging regimes in Europe by providing new evidence on the relative cost of different damage mechanisms, in turn leading to better estimates of the relative marginal cost of different vehicle types. The approach could also be used to determine absolute marginal cost and cost variability levels to compare against the results of top-down and other engineering models.

## **Overview of the two-stage methodology**

The method in this paper consists of two stages. The first stage involves an engineering simulation exercise in which traffic (of certain vehicles and mixes of vehicles) is run on a network of known characteristics, to produce estimates of the resulting damage (denote these D1, D2, and D3, to represent the three main damage mechanisms: settlement, wear and rolling contact fatigue respectively and discussed in more detail in the next section). For this exercise

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we choose 45 actual track sections from Sweden where we have data on the maintenance and renewal costs, the traffic volumes and the infrastructure characteristics. The second stage involves establishing a statistical relationship between actual costs (maintenance and / or renewal) for actual track sections on a network (in our case the Swedish network) and damage. The approach is summarised in Figure 2 below.

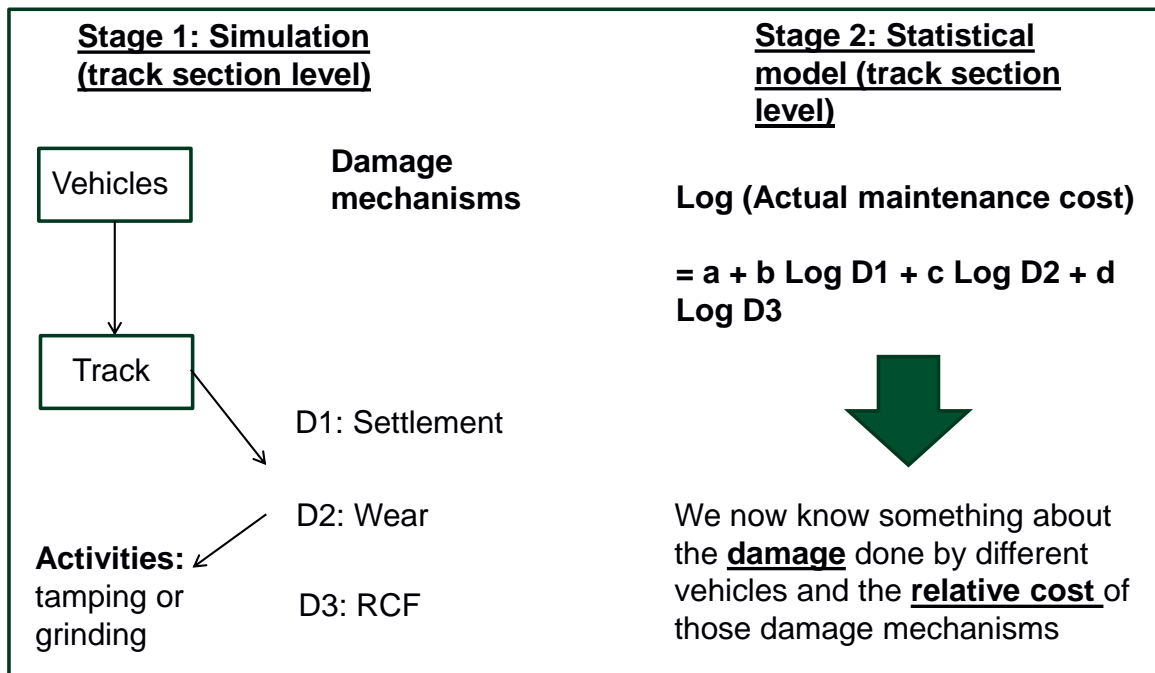


Figure 2: Overview of the methodology

Estimates of the relative cost of different damage mechanisms can be obtained from the parameters in this second stage regression (the b, c and d parameters in Figure 2), which in turn allows us to estimate the relative cost of different vehicles. In addition, a more complex relationship could be assumed in the second stage statistical model, for example to include interactions between damage types (e.g.  $D1 \cdot D2$ ). The detailed assumptions for stages 1 and of the approach, as they apply in practice, are set out below, after first defining the damage mechanisms and their units of measurement.

## Damage Mechanisms and modelling of damage

In this paper we consider the three main damage mechanisms: rail wear, rail rolling contact fatigue and track settlement. These are described in turn below.

### *Wear and rolling contact fatigue*

Wear of the rail is a natural process in which material is removed from the head and/or the inside (gauge) corner of the rail when railway vehicles run. The rate of removal is affected by the forces and contact conditions. Severe wear can change the cross sectional profile of the rail, resulting in a change of the running surface seen by the wheel. Rolling contact fatigue (RCF) occurs if the rail surface is subjected to repeated plastic deformation as is often caused by repeated wheel passages.

The ‘Tgamma’ value is the product of the tangential force (T) and the creepage or slip (gamma) in the contact patch between a wheel and rail. It is a measure of the energy dissipated in the contact patch and can be used to drive models of the wear and RCF likely to take place at each wheel as defined below. In this work the Tgamma value is averaged over 40m to produce an indication of the level of wear produced by each wheel.

Work by Burstow<sup>9</sup> and others has shown that Tgamma combined with a non-linear damage function produces a RCF damage index as shown in Figure 3. This recognises that:

- Below a Tgamma value of 15 J/m there is insufficient energy to initiate RCF cracks.
- Above 15 J/m, the probability of RCF initiation increase, to a maximum of 1 at a Tgamma value of 65 J/m.
- As Tgamma increase further from 65 to 175 J/m the level of energy is such that the dominant form of surface damage is wear (rather than crack initiation), therefore the probability of RCF damage decreases as wear increases.
- Tgamma values greater than 175 J/m, result in wear but no RCF initiation.
- The units of the RCF damage index are  $10^{-5}$  per axle. This indicates that for a damage index of 1, 100000 (One hundred thousand) axle passes would result in RCF initiation.

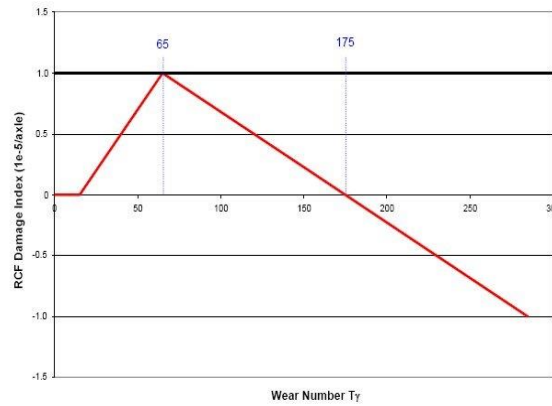


Figure 3: Relationship between wear number (Tgamma/ [J/m]) and RCF damage index [-] from Burstow<sup>9</sup>

### Track Settlement

Track settlement results in the track sinking vertically into the ballast under the passage of railway vehicles. Differential settlement results in increased track irregularities and requires maintenance action typically by tamping. A number of models have historically been used to predict track settlement (Iwnicki<sup>10</sup>). Initially in this work the model defined by the Technical University of Munich (TUM) and the Sato model were considered. However after investigation a simplified method was used. This method has been used earlier to estimate track settlement where the calculation of the ballast pressure has not been necessary. Instead of calculating the ballast pressure, this method uses peak vertical force at the wheelset to calculate the settlement.

The TUM model

$$\text{Ballast settlement (mm)} = A p \ln \Delta N + B p^{1.21} \ln N \quad (1)$$

The simplified model

$$\text{Ballast settlement (mm)} = A * Q^{1.21} \log N \quad (2)$$



Where:

$N$  = number of axles passes

$\Delta N$  = number of axle passes  $\leq 10,000$  after tamping (to represent more rapid deterioration rate immediately after tamping)

$p$  = ballast pressure [ $\text{N/mm}^2$ ]

$Q$  = peak vertical force at the wheelset [ $\text{N}$ ]

$A$ ,  $B$  and  $A^*$  are constants

*In summary, the following indices are used in this work:*

- Rail wear index – the Tgamma value calculated for each wheel over an average of 40m and then summed;
- Rail RCF index – the Tgamma value calculated for each wheel and weighted by the non-linear damage function and summed for each wheel;
- Track settlement index – the peak vertical force at a wheelset weighted according to equation (2).

## **Engineering method and case study results: method stage 1**

We now explain the stage 1 method and illustrate this for one case study section (denoted section 629 in the Swedish Transport Administration database).

### *Outline of the engineering method*

Engineering simulations of 45 selected track section and three generic vehicle types have been undertaken. The latter include two types of freight locomotives (with associated wagons) and one EMU (see Table 1). The simulations carried out provide an estimation of the relative damage for a variety of vehicle/track combinations which can then be used in an econometric model to estimate the relative costs of the different damage mechanisms.

A number of modelling techniques, in combination with computer simulation packages, can be used to estimate different damage mechanisms. As noted earlier, the damage mechanisms of interest are rail wear, rail rolling contact fatigue and track settlement. The determining factors behind the mechanisms are the vertical and lateral wheel-rail forces and the energy dissipation at the surface of the rail. In order to make an accurate prediction of damages a detailed description of the track and the vehicles is required. This includes details of the masses and geometries of the vehicle and its suspension and the track design and irregularities.

Track data was obtained from track measurement vehicles running on the infrastructure, provided by the Swedish Transport Administration; the latter also provided vehicle data.

### *Track section 629 case study: data*

We explain the method and results by focusing on one of the track sections (track section 629 in Sweden). This route is dominated by freight traffic which is 80% of the total traffic-km run. The length of the track is 5.1 km from the track geometry data.

The track quality data were provided by the Swedish Transport Administration as noted. The data were obtained using a track geometry recording coach which measures distance, curvature,

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cross level, vertical irregularity, lateral irregularity and gauge. This track data was then used as input data for the simulation and includes:

- Cross level versus Distance along the track;
- Curvature versus Distance along the track;
- Lateral Irregularity versus Distance along the track;
- Vertical Irregularity versus Distance along the track;
- Gauge Variation versus Distance along the track.

Details of the vehicles running on the section and their traffic volumes are shown in Table 1. This information was provided by the Swedish Transport Administration as noted and revealed the generic type of vehicles and their usage on the network. The vehicle models were described in terms of mass properties, geometry, axle load, unsprung mass, wheel radius and suspension characteristics. Representative worn wheel profiles were selected from a library of measured profiles for each vehicle type. The Vampire simulation package uses the model description to generate equations of motion for the vehicle which it then solves with the track input. Once the simulation is complete outputs are available for wheel-rail forces vehicle motions and can be used to estimate damage. It should be noted that there were some gaps in the precise details of the individual vehicle types, which meant that the data had to be supplemented with information from the EU funded research project, INNOTRACK<sup>1</sup>. The latter project provided information on typical freight and EMU vehicles running on the EU network.

Vehicle	Vehicle type	Freight train-km	Sum of axle load [t]	Axles per vehicle	Max Speed [km/h]
RC2	BoBo Loco	990.23	76.8	4	135
RC4	BoBo Loco	1455.51	78.0	4	135
Wagon 1	Bogie type 1	-	102.0	4	-
Wagon 2	Bogie type 2	-	45.0	2	-

*Table 1: Vehicle description*

### *Track section 629 case study: results*

The outputs from the simulations were the Tgamma value and peak force at each wheel. Tgamma was then used to calculate the rail wear and rail RCF damage indices as described above. The results are shown in appendix 1 and 2. The maximum vertical force over each 200m of simulation output was used to calculate the track settlement index. These values are

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<sup>1</sup> A link to the concluding technical report of this EU funded project can be found at: <http://publications.lib.chalmers.se/publication/129645><sup>11</sup>.

normalised by vehicle mass and this is shown in table 2 for the three damage mechanisms for track section 629 for each vehicle type.

Vehicle models for RC2 and RC4 locomotives were run on this track route and the results for track settlement for each vehicle are shown as a percentage distribution in Figure 4. As RC4 makes a larger proportion of traffic on this route, it causes more settlement as seen in Figure 4. From the data available it was estimated that in a fixed period of time 192 RC2 locos are operational on this track section whereas in this same period 282 RC4 locos carry freight on this route.

It should be noted that, as described in more detail in the description of the stage 2 modelling, since the vehicles that we have modelled do not quite represent 100% of the traffic, there is a need to scale up the damage measures in the above tables so that they represent an estimate of the total damage on the section in a given year.

Track Section	Vehicle	Loading condition	Damage index per tonne-km	Wear index per tonne-km	RCF index per tonne-km
629	RC2	Laden	103.81	3081	9.96
	RC4	Laden	104.98	3085	9.71
	Wagon 1	Tare	20.51	425533	166.6
		Laden	128.57	1998	15.3
	Wagon 2	Tare	24.12	21645	229.7
		Laden	124.29	940	10.69

Table 2: Results for different damage mechanisms

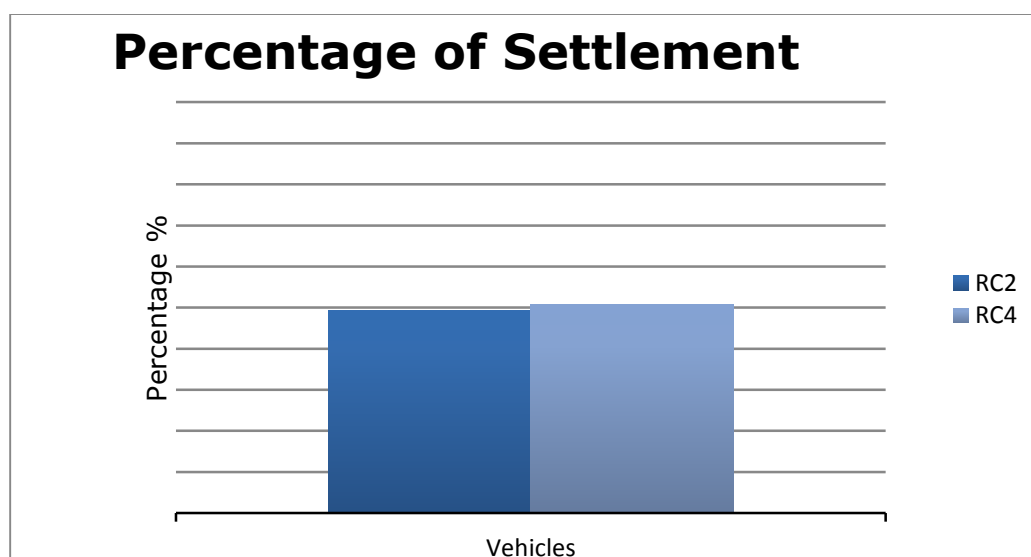
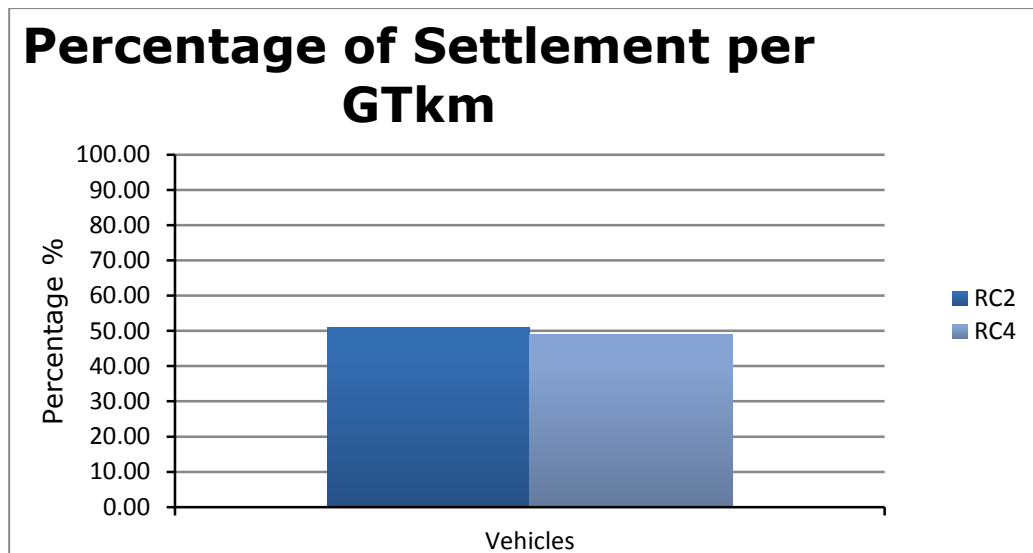


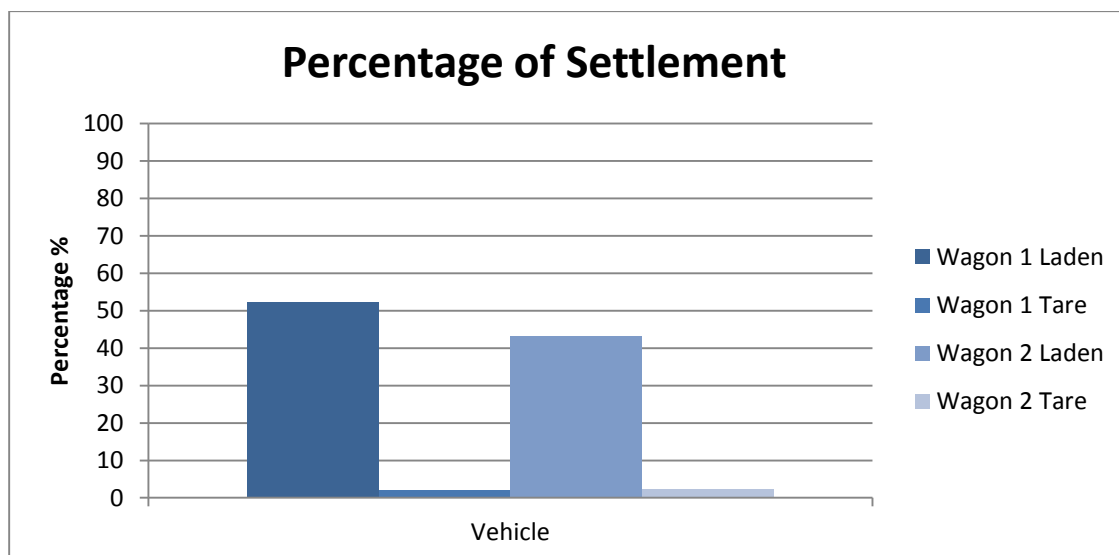
Figure 4: Percentage distribution of Settlement by actual damage by Locomotives

Figure 5 represents the percentage of settlement per tonne of axle load for the locos. RC4 is a later version of the RC2 loco and is slightly less damaging to the rail compared to the RC2 even though the RC2 has a lower axle load.



*Figure 5: Percentage distribution of Settlement per tonne of axle load by Locomotives*

A comparison of the damage caused two kinds of wagon was also carried out. Figure 6 and 7 show the settlement damage by the different wagons operating on this section of track. As can be expected a laden wagon causes more settlement compared to a tare wagon. There is no significant difference in the percentage of damage per GTkm caused by the wagons with different bogie types.



*Figure 6: Percentage distribution of Settlement by actual damage by Wagons*

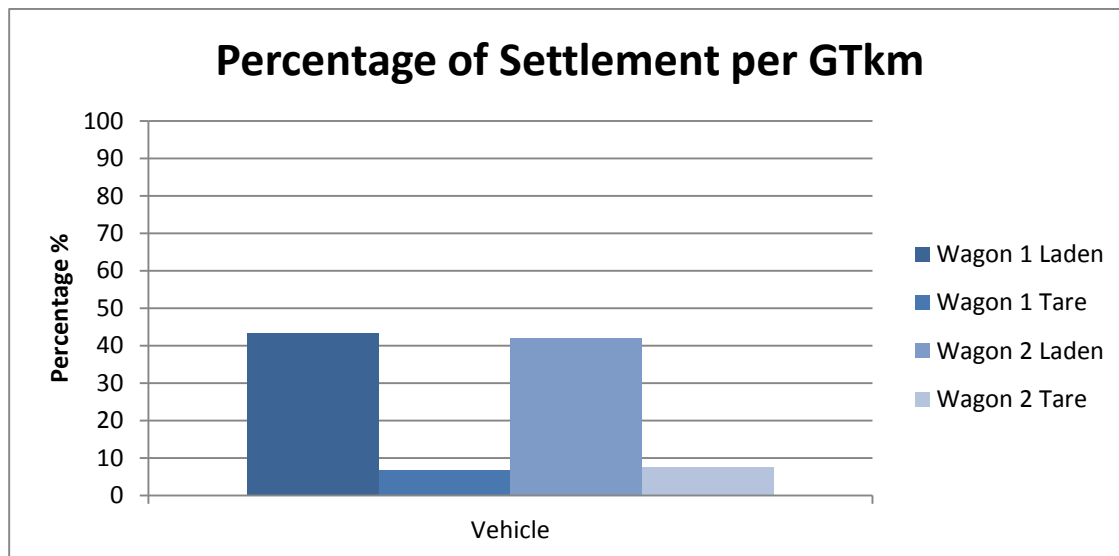


Figure 7: Percentage distribution of Settlement per ton of axle load by Wagons

Figure 8 shows the distribution of the Tgamma and RCF Indices along the track. Negative values of RCF Index imply a large amount of wear. This can be seen in the section of track between 4500m to 5000m where Tgamma values are high (meaning greater wear). Figure 9 shows the percentage of track where high wear, RCF or a combination of both occurs. Both RC2 and RC4 cause similar levels of wear and RCF damage. Figure 10 shows the percentage distribution of wear and RCF damage by the two-axle and the four-axle wagon.

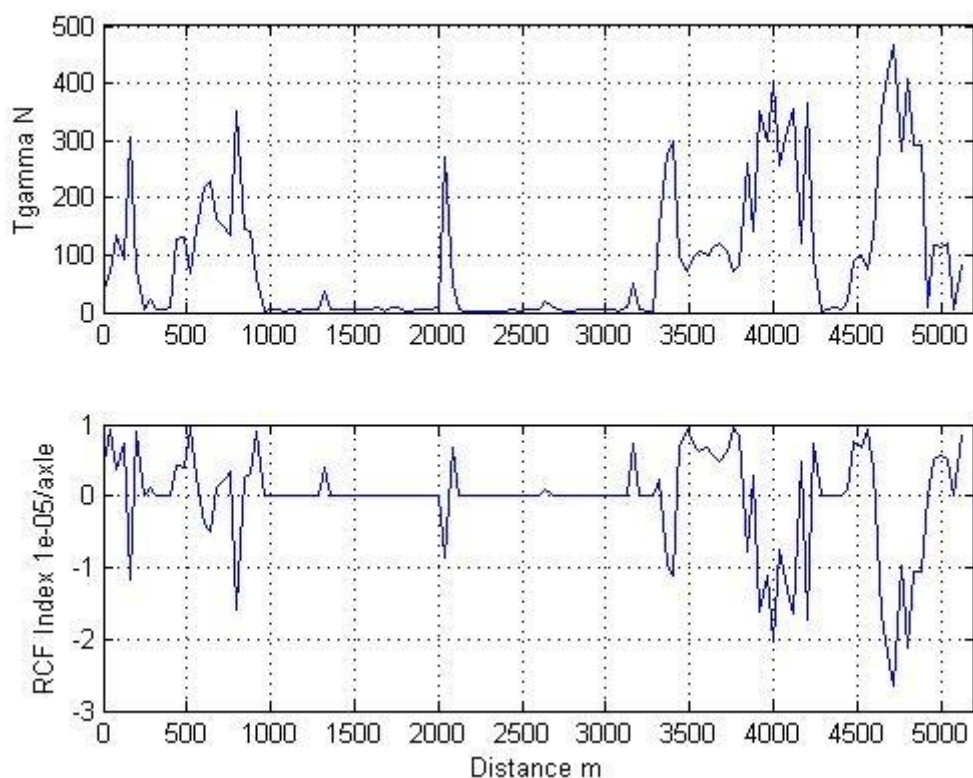


Figure 8: Tgamma and RCF Index for Section 629

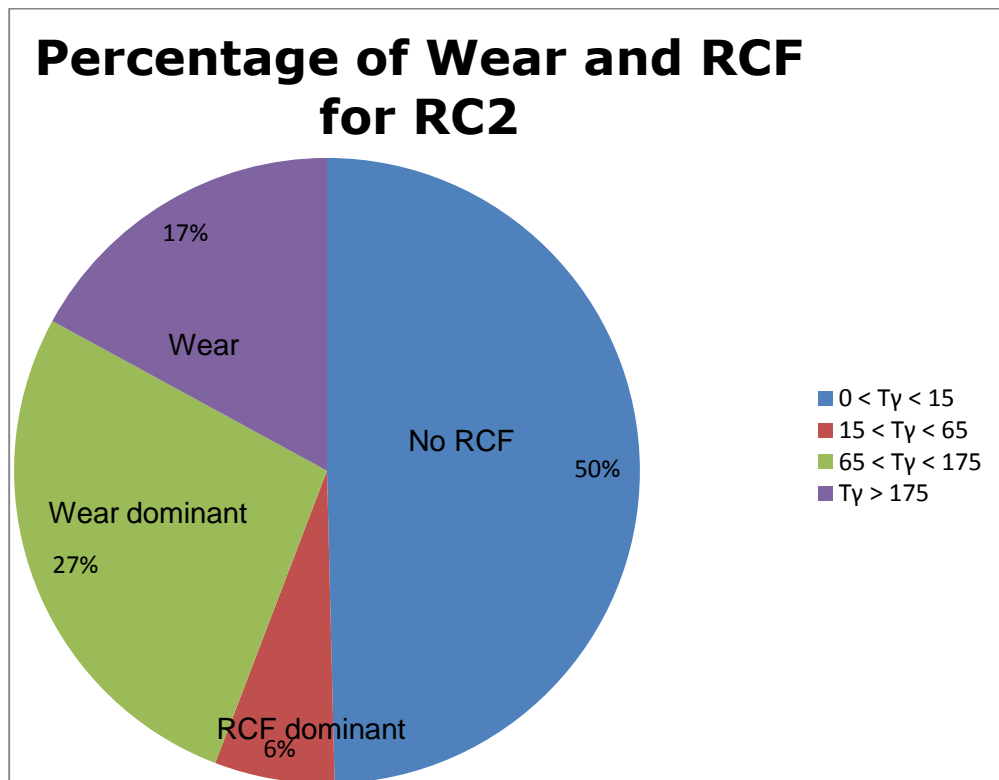


Figure 9: Percentage distribution of Wear and RCF by RC2

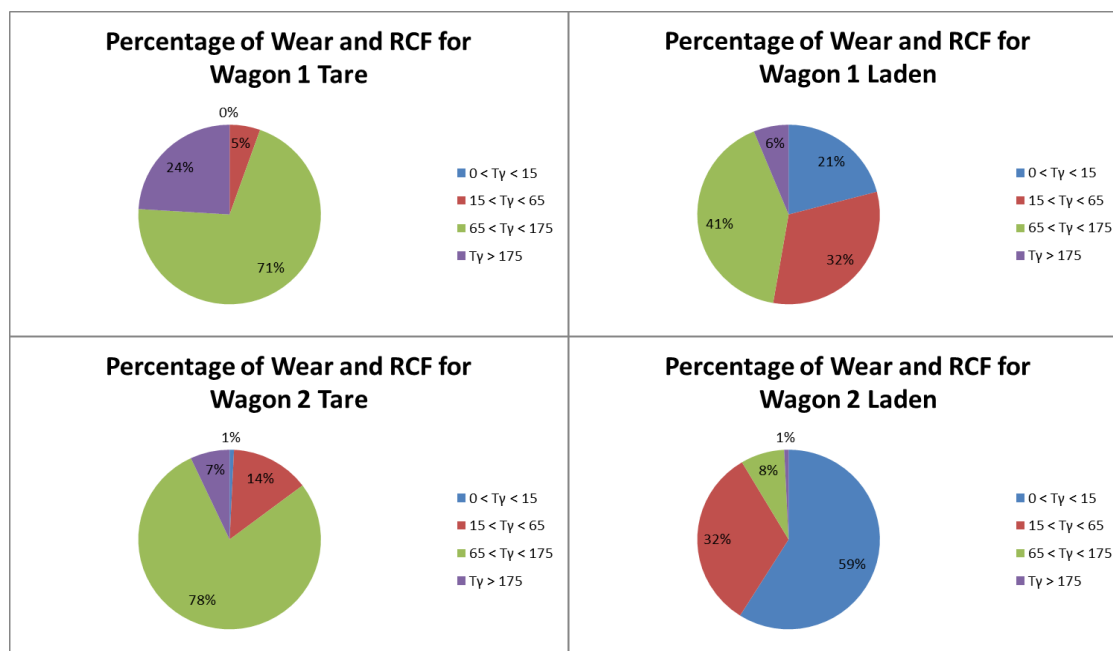


Figure 10: Percentage distribution of Wear and RCF by Wagons

### *Track section 629 case study: summary*

From the simulations carried out on track section 629 it is seen that the locomotives RC2 and RC4 cause a similar level of track settlement, rail wear and rail RCF damage per gross tonne-km. The two wagons considered lead to similar amount of track settlement. Track settlement however depends on the payload on these wagons. A tare wagon is less damaging than a laden wagon as the vertical forces at the wheelset are lower. Rail wear and rail RCF damages are however different for both wagons as they have dissimilar bogie configuration. Rail wear is the dominant form of damage for wagon 1 in tare and laden condition. Propensity for rail RCF is higher for a laden wagon when compared to a tare wagon.

Whilst this section only shows the results for one section, the same simulations were carried out for 45 track sections as noted above and the resulting damage measures used in the second stage analysis. This second stage, described below, explores the relationship between damage and cost, with the aim of better understanding the relative cost of the different damage mechanisms and in turn vehicle types.

## **Econometric modelling: method stage 2**

### *Introduction*

As set out in the overview of the methodology above, the first stage engineering simulation model produces damage estimates, which initially were reported for five track sections (see Smith et. al.<sup>12</sup>). It has now been possible to carry out engineering simulations for 45 sections. In stage 2 of the methodology, the objective is to relate the maintenance (and also renewal) costs to these damage estimates. In carrying out this exercise it is important to control for factors such as the length of section and the characteristics of the section, as this could affect the cost of rectifying damage, quite apart from the damage itself. This control is akin to including track length and traffic volumes, alongside track characteristics variables in the rail infrastructure marginal cost studies in the wider literature (for example, Wheat et. al.<sup>7</sup>). In terms of a priori expectations, there may be scale economies which, for a given level of damage, reduce the cost of rectification; likewise, rectification costs may be different depending on the track standards required for different levels of permitted linespeed.

### *Descriptive statistics*

The descriptive statistics for the available variables are shown in Table 3. It contains the data on costs (at section level), the damage measures resulting from the first stage of the analysis described above, together with some other variables that are important for inclusion in the second stage model, such as track quality (Qualave). The last column in the table indicates the number of observations with zero values, showing that a majority of the track sections did not have a renewal cost in 2012. Thus our analysis focuses on maintenance costs.

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	Obs.	Mean	Std. Dev	Min	Max	No. of zeros
MaintC	45	11 710 759	8 533 885	323 503	35 307 461	0
RenwC	45	11 604 054	67 459 659	0	453 142 803	25
M&R	45	23 314 812	70 368 666	323 503	480 456 207	0
Wear*	45	44 229 209	166 599 095	310	1 064 240 828	0
RCF*	45	6 603	9 706	6	45 474	0
Settlement*	45	47 902	64 058	45	322 782	0
Totdamage*	45	44 283 714	166 590 909	3 777	1 064 254 678	0
Ton_km	45	370 560 930	471 715 298	703 385	2 487 231 414	0
TgtDEN	45	6 665 663	6 162 461	46 839	22 719 570	0
Route_km	45	56	37	4	150	0
Track_km	45	71	48	8	248	0
RatioTLRL	45	1	1	1	5	0
Rail_age	45	21	11	2	48	0
Rail_w	45	52	5	42	60	0
Sleeper_age	45	21	12	2	49	0
Curv_km	45	17	11	1	46	0
Curvcl12_km	45	2	3	0	11	4
Curvcl13_km	45	5	6	0	23	2
Curvcl14_km	45	7	7	0	32	1
Bridge_km	45	1	1	0	8	0
Struct_km	45	1	3	0	22	0
Qualave	45	2	1	0	4	0
Switch_km	45	2	1	0	7	0
Switch_age	45	22	12	0	48	1
No switches	45	96	74	10	355	0
Ballast_age	45	22	13	0	49	1
D.Elect	45	1	0	0	1	0

Definition of variables:

MaintC = Maintenance costs, SEK (Swedish Krona)

RenwC = Renewal costs, SEK

M&R = Maintenance + renewal costs, SEK

Wear\* = Wear index/1 000 000

RCF\* = RCF index/1 000 000

Settlement\* = Settlement index/1 000 000

Totdam\* = (Wear index + RCF index + Damage index)/1 000 000

Ton\_km = Tonne-km

TgtDEN = Tonne-km/Route-km

Route\_km = Route-km

Track\_km = Track-km

RatioTLRL = Track-km/Route-km

Rail\_age = Average rail age

Rail\_w = Average rail weight kg

Sleeper\_age = Average sleeper age

Curv\_km = Track curvature km



Curvcl12\_km = Track curvature with absolute value of radius  $\in [0,450)$   
 Curvcl13\_km = Track curvature with absolute value of radius  $\in [0,600)$   
 Curvcl14\_km = Track curvature with absolute value of radius  $\in [0,800)$   
 Bridge\_km = Track length of bridges, km  
 Struct\_km = Track length of structures (tunnels and bridges), km  
 Qualave = Average quality class; a high value of average quality class implies a low speed line  
 Switch\_km = Track length of switches, km  
 Switch\_age = Average age of switches  
 No switches = Number of switches  
 Ballast\_age = Average age of ballast  
 D.Elect = Dummy variable indicating electrified track section

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*Table 3: Descriptive statistics*

### *Scaling up assumptions*

The simulation runs do not include all the different types of vehicles running on each track section. Hence, we have an additional share of tonne-km from other vehicles to consider when calculating the total damage on each track section. The damage caused by these vehicles is assumed to be proportionate to the damage caused by the vehicle types in the simulation runs. We therefore calculate the share of tonne-km for each vehicle type in the simulation and use their respective damage measure to estimate the damage caused by vehicles not included in the simulation runs.

### *Model specification and estimation results*

The literature contains a range of functional forms that may be estimated. The simplest model is the Cobb-Douglas model (this is a double log or log-linear model). The Cobb-Douglas model however is quite restrictive and the cost modelling literature typically favours the more general translog functional form (this model is a logarithmic model but includes squared and interaction terms; see equation 1 below). The translog offers greater flexibility in respect of the relationship between cost and the explanatory variables which may offer more intuitive economic interpretations. See, for example, Coelli et. al.<sup>13</sup> for more detail on functional forms used in cost function estimation.

We estimate three different models as set out in Table 4. The first model is the simplest model in terms of the variables and also the functional form. The second model attempts to add complexity in terms of variables included in the model, whilst also retaining some, but not all, of the second order and interacted terms (hence the term, restricted translog, used in Table 4). The final model, though estimated, is not reported. Each model is explained in more detail below. For all models, data is transformed by dividing by the sample mean prior to taking logs. We can therefore interpret the first order coefficients in the estimation results as elasticities at the sample mean.

Model	Functional form	Dependent Variable
1	Cobb-Douglas	Maintenance
2	Restricted Translog	Maintenance

Table 4: Models

In model 1 we only include the damage measures (Wear, RCF and Settlement) as explanatory variables. In line with the literature, we start with a translog model:

$$\ln C_i = \alpha + \beta_1 \ln Wear_i + \beta_{11} (\ln Wear_i)^2 + \beta_2 \ln RCF_i + \beta_{22} (\ln RCF_i)^2 + \beta_3 \ln Settlement_i + \beta_{33} (\ln Settlement_i)^2 + \beta_{12} (\ln WEAR_i \cdot \ln RCF_i) + \beta_{13} (\ln WEAR_i \cdot \ln Settlement_i) + \beta_{23} (\ln RCF_i \cdot \ln Settlement_i) + \varepsilon_i \quad (3)$$

where the variables are as defined earlier, expressed in natural logarithms and  $\alpha$  and  $\beta_1$  to  $\beta_{33}$  are parameters to be estimated and  $\varepsilon_i$  represents the standard random noise term. Note that the model includes squared and interaction terms as is standard.

We then test the Cobb-Douglas restrictions (the Cobb Douglas model does not have the squared or interaction terms):

$$\beta_{11} = \beta_{22} = \beta_{33} = \beta_{12} = \beta_{13} = \beta_{23} = 0, \quad (4)$$

Based on an F-test we cannot reject the following null hypothesis:

$$\beta_{11} = \beta_{12} = \beta_{13} = 0, \quad (5)$$

which means that we can exclude the squared wear variable and the cross products between wear and other variables. However, the first order coefficients for RCF and Settlement are not significant in this restricted translog model, so the elasticities are not significant at the sample mean. For this reason the remaining second order terms were dropped. Since we are seeking to illustrate the method, and are working with a relatively small sample size, we consider that the use of a Cobb-Douglas functional form is reasonable. The estimation results are presented in Table 5.

	Coefficient	Standard Error	[95% Confidence Interval]
Constant ( $\alpha$ )	16.4467***	0.1746	16.0940 16.7994
Wear ( $\beta_1$ )	0.0670	0.0441	-0.0221 0.1560
Rcf ( $\beta_2$ )	0.0023**	0.0930	-0.1856 0.1902
Settlement ( $\beta_3$ )	0.2401***	0.0881	0.0622 0.4181

$R^2=0.3787$ , Adjusted  $R^2=0.3333$ , Mean VIF=2.08. \*\*\*=statistically significant at the 1% level; \*\*=statistically significant at the 5% level; \*=statistically significant at the 10% level

Table 5: Estimation results, model 1

We test if the coefficients are significantly different from each other. The coefficients for wear and settlement are significantly different at the 10 per cent level, i.e. we reject the null hypothesis (however, the coefficient for wear is not significant). See results from the F-tests in Table 6.

Null hypothesis	F Test p value
Wear - RCF = 0	0.583
Wear - Settlement = 0	0.080*
RCF - Settlement = 0	0.164

\*=statistically significant at the 10% level;

Table 6: F-tests for difference between damage coefficients, model 1

In model 2 we include route length (Route tl) and average quality class (Qualave) as control variables. We start with a full translog model and test the Cobb-Douglas restrictions.

Most of the second order terms were insignificant and were dropped from the model. The parameter estimate for the interaction variable between RCF and route length is significant and negative, as is the first order coefficient for RCF. This suggests that an increasing RCF interacted with route length decreases the maintenance costs, which seems implausible. We therefore exclude this variable, and the first order coefficient for RCF is then no longer statistically significant (though remains negative). We also drop the interaction variable between Qualave and Wear on the basis that the first order coefficient for Qualave is not significant (p-value =0.79). This results in a first order coefficient for Qualave that is nearly significant at the 10 per cent level (p-value=0.112). The final model we estimate is:

$$\ln C_i = \alpha + \beta_1 \ln \text{Wear}_i + \beta_2 \ln \text{RCF}_i + \beta_3 \ln \text{Settlement}_i + \beta_4 \ln \text{Route tl}_i + \beta_5 \ln \text{Qualave}_i + \beta_{55} (\ln \text{Qualave}_i)^2 + \beta_{35} (\ln \text{Settlement}_i \cdot \ln \text{Qualave}_i) + \varepsilon_i \quad (6)$$

The estimations results from the final model are presented in Table 7. The parameter estimate for the interaction variable between Settlement and Qualave is significant and negative in model 2. This shows that the maintenance cost for settlement damage is lower when the requirements on track geometry standard are lower. Note that a higher value for the variable Qualave indicates lower track standard requirements and lower linespeeds, which also explains the negative first order coefficient for Qualave.

	Coefficient	Standard Error	[95% Confidence	Interval]
Constant ( $\alpha$ )	16.3307***	0.1669	15.9925	16.6689
Wear ( $\beta_1$ )	0.0434	0.0399	-0.0374	0.1242
Rcf ( $\beta_2$ )	-0.0509	0.0851	-0.2234	0.1216
Settlement ( $\beta_3$ )	0.1972**	0.0824	0.0301	0.3642
Route tl ( $\beta_4$ )	0.3193**	0.1546	0.0060	0.6327
Qualave ( $\beta_5$ )	-0.5089	0.3122	-1.1415	0.1237
Settlement*Qualave ( $\beta_{35}$ )	-0.3646***	0.1348	-0.6378	-0.0914
Qualave <sup>2</sup> ( $\beta_{55}$ )	-0.3712**	0.1803	-0.7365	-0.0058

Model 2:  $R^2=0.5560$ , Adjusted  $R^2=0.4720$ , Mean VIF=3.18. \*\*\*=statistically significant at the 1% level; \*\*=statistically significant at the 5% level; \*=statistically significant at the 10% level

Table 7: Estimation results, model 2

We test if the coefficients for the damage measures are statistically different in model 2. The results are presented in Table 8, showing that the coefficients for Wear and Settlement are significantly different at the 10 per cent level. This is almost the case for the difference between RCF and Settlement (p value = 0.106).

Null hypothesis	F test p value
Wear - RCF = 0	0.376
Wear - Settlement = 0	0.093*
RCF - Settlement = 0	0.106

\*=statistically significant at the 10% level

*Table 8: F-tests for difference between damage coefficients, model 2*

The conclusion from models 1 and 2 is that it is important to control for the size of the section and also the characteristics of the infrastructure (e.g. the quality class variable, which principally reflects the permitted linespeed on the section). As noted earlier, there may be scale economies which, for a given level of damage, reduce the cost of rectification; likewise, rectification costs may be different depending on the track standards required for different levels of permitted linespeed. The settlement elasticity is the largest in absolute terms and is statistically significant at the 1% and 5% levels in models 1 and 2 respectively. In model 1 the wear coefficient is close to being significant at the 10% level, but the RCF coefficient is highly insignificant. In model 2 both the wear and RCF coefficients are highly insignificant and the RCF coefficient is wrong sign (though statistically insignificant as noted). The coefficient on settlement is fairly stable between the two models.

Since the model includes only maintenance costs, it is not entirely surprising that only settlement damage is significant in the model, since tamping is the remediation action for settlement (and this cost is included in maintenance). By contrast, remediation action for wear and rolling contact fatigue includes rail replacement (which is a renewal cost), though grinding and re-profiling may also occur which is included in maintenance costs. The problem of not being able to obtain statistically significant estimates for all of the damage types may also result from only having 45 observations.

Whilst in principle renewal costs can also be included in this methodology, in practice, as we are working at track section level, many of the sections have zero renewals costs. We therefore do not report a model with the dependent variable constructed based on the sum of maintenance and renewal costs<sup>2</sup>.

It is worth noting the overall cost variability with respect to damage resulting from the models run. At the sample mean the sum of the elasticities on the three damage mechanisms is in the range of roughly 20-30% (models 1 and 2). The CATRIN project (Wheat et. al.<sup>7</sup>) found a range of (mean) maintenance elasticities with respect to traffic of broadly 20% to 35%. Whilst the elasticities in this paper are elasticities with respect to damage and the elasticities in the

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<sup>2</sup> We did estimate such a model and it produced results very similar to the maintenance only model. A corner solution approach, as set out in Andersson et. al.<sup>8</sup> would be more appropriate for renewals at section level, and such an approach is beyond the scope of the current study.

CATRIN project are elasticities with respect to traffic, clearly damage results from traffic running on the network. Therefore, in terms of the proportion of cost variable with traffic / usage our results are in line with previous estimates in the literature.

### *Computation of relative marginal costs*

The marginal cost is calculated by multiplying the average cost for each track section by the cost elasticity for that section:

$$MC_i^j = \widehat{AC}_i^j \cdot \hat{\gamma}_i^j \quad (7)$$

where  $j = \text{wear, RCF or settlement}$ ,  $i$  denotes the track section number and  $\hat{\gamma}_i^j$  is the cost elasticity with respect to the  $j$ th damage mechanism, evaluated for each section. We use the predicted average costs, which is the fitted cost divided by total Wear, RCF or Settlement:

$$\widehat{AC}_i^j = \hat{C}_i / \text{Total damage}_i^j \quad (8)$$

$\hat{C}_i$ , as specified in eq. (9), derives from the double-log specification of our model that assumes normally distributed residuals (see Munduch et al.<sup>14</sup>, and Wheat and Smith<sup>4</sup>).

$$\hat{C}_i = \exp(\ln(C_i) - \hat{\varepsilon}_i + 0.5\hat{\sigma}^2) \quad (9)$$

where  $\hat{\varepsilon}_i$  is the observed error term and is  $\hat{\sigma}^2$  is its variance.

	Variable	Observations	Mean	Standard Deviation	Min	Max
Model 1:	Wear	45	0.052	0.325	0.000	2.185
	RCF	45	3.885	17.140	0.043	114.358
	Settlement	45	0.270	0.765	0.008	4.561
Model 2:	Wear	45	0.080	0.517	0.000	3.471
	RCF	45	5.135	26.965	0.059	181.656
	Settlement	45	0.399	1.793	0.007	12.079

Table 9: Average costs, ÖRE (1 ÖRE = 1 SEK/100)

	Variable	Observations	Mean	Standard Deviation	Min	Max
Model 1:	Wear	45	0.003	0.022	0.000	0.146
	RCF	45	0.009	0.039	0.000	0.261
	Settlement	45	0.065	0.184	0.002	1.095
Model 2:	Wear	45	0.003	0.022	0.000	0.151
	RCF	45	-0.262	1.374	-9.254	-0.003
	Settlement	45	0.053	0.202	-0.008	1.333

Table 10: Average Marginal costs ÖRE (1 ÖRE = 1 SEK/100)

The average costs and the relative marginal costs are presented in Table 9 and 10, respectively, where the latter are based on the average of the marginal costs computed for each track section.

Since the elasticities in model 1 were all positive, we use model 1 to draw conclusions. The results suggest that settlement is the most costly damage mechanism, with a marginal cost that is just over seven times that of rolling contact fatigue and twenty-one times that of wear. RCF damage is in turn three times more costly, per unit of damage, than wear. As noted earlier, in interpreting these findings it is important to remember that the estimation is based on 45 sections and not all the parameter estimates were statistically significant. Further, as noted, the dependent variable only includes maintenance. If renewal had been included in the model, thus permitting a more complete comparison of relative costs, it is likely that the relative damage costs would be very different.

It is possible, in principle, to cross-validate these results against other estimates. For example, rough estimates of different deterioration mechanisms' share of track maintenance and track renewal costs were used in bottom up-approach to estimate costs for different vehicle characteristics in Öberg et al.<sup>15</sup>. The estimates were given by the Swedish Rail Administration and relate to the whole railway network. The Swedish Rail Administration states that 25 per cent of the costs are attributed to track settlement deterioration, 40 per cent to wear + RCF, and 35 per cent to deterioration of other components. These are not directly comparable to our study, which focuses on maintenance only. Since our intention in this work is merely to illustrate the approach, we do not take this comparison further. In subsequent work we would seek to expand the sample size and then make a fuller comparison of this method with alternative methods.

This is, to our knowledge, the first time that the relative cost of damage mechanisms has been estimated in this way, based on engineering simulations combined with actual cost data. As noted earlier, whilst engineering models are adept at estimating damage, the challenges is to convert damage into cost. The traditional "bottom-up" approaches faces many problems, since an assessment needs to be made of the remediation activity that will be done, and its timing, and then the unit cost of these activities. Depending on the nature of the network, and the combination of the damage types, these estimates could vary widely. The advantage of our top-down approach is that it uses actual cost data to derive relative marginal costs.

Table 11 provides a simple illustration of how this approach might be useful for track charging purposes. For three vehicle types, running on a given network, estimates of damage can be obtained. The problem noted in previous approaches is that whilst these relative damage measures can be predicted with reasonable accuracy based on simulation models, it is then not clear what the relative costs will be. The latter is important for infrastructure managers to know and, given European legislation, to support track access charging regimes differentiated by vehicle.

The different vehicles in Table 11 produce different amounts of the different types of damage mechanisms; some resulting in more of one type of damage and less of other types. This situation therefore creates a problem as it is therefore not clear which vehicle results in the highest incremental cost which, as noted, is important for charging purposes. One might be able to produce a summary measure indicating the total level of damage (the fourth column in the table). Since the units are not directly comparable, this measure is not terribly meaningful. The final column of the Table shows that, by using the relative marginal costs coming out of model 1 (reported above), we can construct a weighted damage measure, weighted according the relative marginal cost. Since settlement is the most damaging mechanism, according to model 1, then vehicle 2 has by far the highest cost weighted damage index, and thus would attract a higher track access charge. Of course, as noted, we are not claiming that this is a definitive

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result, for the reasons set out earlier, but put this forward to demonstrate the potential power of the approach.

	Wear	RCF	Settlement	Simple Sum of Damage Measures	Cost-weighted Damage Index
Vehicle 1**	10	20	15	45	385
Vehicle 2	5	25	25	55	605
Vehicle 3	20	5	10	35	245
Relative marginal cost (Wear = 1.00)*	1	3	21		

*Table 11: Illustration of the outputs of the methodology*

\* This is based on the results of model 1, with the marginal cost of wear normalised to unity.

\*\* The damage numbers are simply created for the purpose of the illustration for three hypothetical vehicle types.

## Conclusions

This paper fills an important gap in the literature concerning the translation of damage measures into measures of the cost of damage remediation. It does so by combining engineering simulation models (stage 1) with top-down econometric methods linking cost and damage (stage 2). This is the first time a direct relationship between actual cost and track damage has been estimated econometrically. The approach has been implemented using track section data provided by the Swedish Transport Administration. The combination of these two approaches provides a new methodology for comparing the relative cost of damage done by different vehicles on rail infrastructure, which in turn can be used to inform track access charges differentiated by vehicle-type.

The advantage of this two stage method over its single stage counterparts are as follows. As compared to bottom-up engineering approaches, our two stage method is grounded in engineering simulation models in the first stage, but uses actual costs on track sections to estimate the relative cost of the different damage mechanisms in the second stage. Current bottom-up engineering methods rely on assumptions about the remedial work required, and the unit costs of those activities, to convert damage measures into costs. These assumptions can be potentially hard to justify and are highly uncertain, which is a significant drawback. As compared to single stage econometric approaches, our approach permits a more precise estimation of the relationship between costs and the damage done by different vehicle types. Existing econometric approaches often struggle to obtain sensible relationships between costs and different types of traffic.

We consider that our research has demonstrated the feasibility of the approach. We find, as expected, that it is possible and relatively straightforward to model the damage resulting from running vehicles on the network at section level. We have further shown that it is possible to produce summary, section-level damage measures for each of the three damage mechanisms (settlement, wear and rolling contact fatigue) which can be taken forward into the second stage.



Whilst the time taken to undertake the engineering simulations is not trivial, it is also not prohibitive.

The econometric results indicate that it is possible to obtain sensible relationships between cost and the different damage mechanisms; though the statistical significance of some of the findings has been limited in this case by the relatively small sample size (45 sections). Bringing together then, information on damage and the relative cost of different damage mechanisms, is potentially a powerful means of obtaining cost information that can be used to produce vehicle-differentiated track access charges. In turn, more cost reflective access charges should incentivise the development and use of more track friendly vehicles. Such information is also highly valuable to the industry and policy makers as it will allow the cost implications of different technologies that reduce damage to be more clearly assessed.

In this work, established vehicle dynamics tools have been used to predict the forces at the wheel-rail interface. These forces are then used to predict the levels of damage that will develop with traffic. It would in principle be possible to take measurements from vehicles (e.g. of wheelset or bogie accelerations and/or displacements) and to use these with a simpler model to predict wheel rail forces.

We find that settlement is the most costly (with respect to maintenance cost) of the three damage mechanisms, followed by RCF and then wear, with settlement being approximately seven times more costly than RCF and RCF approximately three times more costly than wear. As noted we caveat this finding because of the relatively small sample and the fact that we have only been able to include maintenance costs in the approach. Taken at face value, this result suggests that vehicles resulting in more settlement and RCF damage should attract higher track access charges, though of course the results are only indicative. The overall variability of cost with respect to damage – of between roughly 20-30% is also in line with previous evidence on the variability of costs with respect to traffic for maintenance of around 20-35%. This finding is important for charging purposes as it establishes, in aggregate, the quantum of cost that variable access charges should be seeking to recover.

Whilst the approach has been shown to be feasible, we offer some comments about future research. Further work might focus on generating more observations (more sections and / or exploiting panel data). This approach should be feasible using Swedish data, though it would be a significant research project. The approach is in principle applicable to many European railways where track section data is available, though again we do not claim that the approach is straightforward or trivial. That said, given that previous approaches rely in any case on engineering simulations, which is the most time consuming aspect of the work, we consider that the econometric second stage has the potential to contribute greatly to better understanding of the relative marginal cost of different vehicle types (and indeed different vehicle types running on different types of infrastructure); particularly in view of the well-established problem with existing approaches to translating damage into cost.

There are also a number of aspects to the research where assumptions have been used, for example, concerning the precise nature of the vehicles running on the network (we could identify passenger versus freight vehicles, and locomotive versus EMU, but had to use generic models with appropriate parameters for these rather than detailed models of specific vehicle types). It has also been assumed that the damage caused by one vehicle is independent of the damage caused by other vehicles, and that the damage measures from one vehicle run can be scaled up in a simple manner. These are limiting but pragmatic and sensible assumptions; however, relaxing them would be interesting and useful areas for future research. Validation of

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the damage estimates, as compared to actual measurements would also be a useful addition if the necessary information can be obtained.

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## Appendix 1 Sample values of the Rail wear index

Start (m)	Left wheel 1 Tgamma [J/m]	Left wheel 2 Tgamma [J/m]	Left wheel 3 Tgamma [J/m]	Left wheel 4 Tgamma [J/m]	Right wheel 1 Tgamma [J/m]	Right wheel 2 Tgamma [J/m]	Right wheel 3 Tgamma [J/m]	Right wheel 4 Tgamma [J/m]	Total Tgamma [J/m]	Rail wear index [-]
0	19.48	7.16	15.56	8.31	35.10	5.74	26.43	5.88	123.69	0
40	26.68	5.96	26.92	7.03	71.31	5.45	72.85	5.40	221.63	8865
80	135.46	6.48	4.00	5.08	79.57	9.11	5.88	4.64	250.24	10009
120	75.93	9.26	73.65	11.43	92.58	12.05	91.18	14.01	380.11	15204
160	202.58	28.70	83.76	20.39	305.3	28.61	244.43	12.54	926.33	37053
200	2.58	5.49	75.63	5.32	2.14	7.95	63.95	7.28	170.38	6815
240	3.70	3.43	4.68	4.97	4.09	4.15	4.24	6.41	35.71	0
280	20.89	4.45	4.23	6.31	4.33	4.68	4.158	5.30	54.38	0
320	2.65	3.18	3.70	5.21	3.07	2.61	4.037	3.95	28.42	0
360	3.70	4.25	4.53	4.99	3.78	3.55	4.375	5.44	34.65	0
400	4.63	4.97	4.02	4.83	5.02	4.83	4.052	7.40	39.76	0
440	99.86	7.72	21.11	8.33	127.00	9.35	60.16	7.61	341.20	13648
480	89.50	8.43	3.06	2.68	131.44	11.18	3.05	3.69	253.05	10122
520	67.58	5.80	68.64	5.71	65.38	6.85	54.00	7.90	281.88	11275
560	50.39	5.509	86.644	5.858	144.236	5.843	58.138	7.608	364.23	14569
600	3.91	4.689	89.441	7.426	4.108	6.414	218.17	10.04	344.21	13768
640	229.41	4.79	14.31	5.68	17.29	5.90	87.99	7.65	373.06	14922
680	161.38	8.64	5.39	12.71	74.367	6.772	5.18	5.64	280.094	11203
720	149.44	13.43	72.87	15.15	95.178	10.208	77.325	10.931	444.55	17782
760	136.29	8.53	100.54	9.33	94.709	6.9	82.719	7.166	446.2	17848
800	339.07	8.72	350.91	10.06	144.623	11.042	150.253	12.151	1026.844	41073
840	105.41	12.33	144.53	13.41	69.832	30.975	146.201	15.326	538.036	21521
880	9.44	6.37	140.32	17.13	10.91	5.66	102.24	12.17	304.27	1217

## Appendix 2 Sample values of the Rail RCF index

Start (m)	Rail RCF WS1	Rail RCF WS2	Rail RCF WS3	Rail RCF WS4	Total
0	378.35	462.39	465.35	462.3	1768.41
200	204.32	202.26	204.17	209.15	819.92
400	350.57	352.93	350.44	358.36	1412.31
600	426.31	429.28	434.65	433.68	1723.92
800	352.32	311.01	343.47	314.95	1321.76
1000	469.32	467.66	469.85	464.40	1871.25
1200	485.95	490.17	485.76	491.88	1953.77
1400	494.81	492.21	493.40	496.24	1976.67
1600	355.26	352.88	352.96	350.06	1411.18
1800	316.77	324.88	317.40	324.89	1283.96
2000	222.30	302.62	307.98	301.51	1134.42
2200	438.84	440.78	436.99	441.02	1757.65
2400	504.56	503.61	503.08	495.92	2007.19
2200	438.84	440.78	436.99	441.02	1757.65
2400	504.56	503.61	503.08	495.92	2007.19