Modeling of Two-Phase Flow with Deposition in Vertical Pipes

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Abstract*—Deposition is found in many engineering processes, such as the asphaltene deposition in oil pipelines/wellbores, and biological and chemical foulings in pipes or heat exchangers. These deposition processes usually occur in a two-phase flow environment. This study develops a model for two-phase flow with deposition in vertical pipes. The model consists of three modules: Fluid Transport, Particle Transport, and Particle Deposition. The Fluid Transport module predicts the fluids' velocities and pressure. The Particle Transport module calculates the particle distribution. The Particle Deposition module models the actual attachment of particles onto the wall. The model is verified against a few limiting cases with analytical solutions. Then, it is validated against experimental data for two-phase flow without deposition. Demonstration of the model for bubbly flow with deposition is performed.*

Keywords—deposition; two-phase flow; model

I. INTRODUCTION

Deposition is a common phenomenon found in many industrial processes. For instance, asphaltene, wax or hydrate deposits in oil pipelines or wellbores, biological and chemical foulings in pipes or heat exchangers, etc. These deposition processes are very complex since they usually occur in a two-phase flow environment, e.g. asphaltene deposition in oil-water or oil-gas flow and foulings in phase change liquid-vapor flow in the heat exchanger. For deposition to be enhanced, prevented or controlled, a good understanding of the deposition processes is required. Modeling work plays an important role in understanding and predicting these processes. Nevertheless, modeling is challenging because of the presence of moving interfaces between the two fluids and between the fluids and the deposit. The fully coupled nature of deposition process in two-phase flow has not been explored sufficiently in the existing modeling work [1].

In these deposition processes, the domain of interest contains three phases, i.e. two fluids and particles. Generally. the modeling framework consists of three modules: (1) Fluid Transport module, (2) Particle Transport module, and (3) Particle Deposition module. Fluid Transport module describes fluids flow, i.e. velocities and pressure. It is usually modeled as homogeneous [2-5] or separated [6-15]. Particle Transport module predicts the particle distribution in the pipe. It can be modeled by Lagrangian approach [16] and Eulerian approach [17-19]. Particle Deposition module models the actual attachment of the particles onto the fluid-deposit interface (as shown in Fig. 1). There are mainly three approaches by employing: (1) a critical length [20, 21], (2) a sticking probability theory [22-24] and (3) an m-th order deposition reaction [25, 26]. In the existing literature, modeling works on two-phase flow with deposition is not actively pursued. There are very few published studies in this area [27-30].

This article presents a model for two-phase flow with deposition. The model intends to integrate the above mentioned three modules together in a fully coupled manner.

II. PROBLEM DESCRIPTION

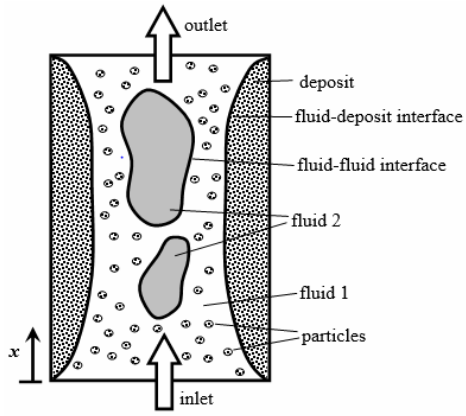


Fig. 1. Deposition in a two-phase flow in a vertical pipe.

Fig. 1 shows two immiscible fluids (i.e. fluid 1 and fluid 2) flowing up in a vertical pipe. Fluid 1 carries solid particles. These particles gradually deposit onto the fluid-deposit interface to form a deposit layer. As a result of deposition, the fluid-deposit interface evolves and the deposit layer grows reducing the flow area. The velocities and pressure change correspondingly. Since the particles are consumed during the deposition process, the particle concentration decreases.

III. Mathematical Formulation

A. Assumptions

(a) One-dimensional flow.
(b) The two fluids are immiscible.
(c) The two fluids share the same pressure.
(d) Only fluid 1 carries particles.
(e) Particles do not interact with each other.
(f) Deposit is rigid and immobile.
(g) Diffusive transport along the pipe is small compared to convection and therefore neglected.

B. Fluid Transport

The mass conservation equations for the particles, fluid 1 and fluid 2 are respectively

$$\frac{\partial}{\partial t}(\alpha_d \rho_d) = \dot{M}_d \alpha_1 \tag{1}$$

$$\frac{\partial}{\partial t}(\alpha_1 \rho_1) + \frac{\partial}{\partial x}(\alpha_1 \rho_1 u_1) = -\dot{M}_d \alpha_1 \tag{2}$$

$$\frac{\partial}{\partial t}(\alpha_2 \rho_2) + \frac{\partial}{\partial x}(\alpha_2 \rho_2 u_2) = 0 \tag{3}$$

where α, ρ, and u are the volume fraction, density, and velocity respectively. $\dot{M}_d$ is the deposition rate. The subscripts '*1*', '*2*', and '*d*' represent fluid 1, fluid 2, and deposit, respectively. Note that α_1, α_2 and α_d satisfy

$$\alpha_1 + \alpha_2 + \alpha_d = 1 \tag{4}$$

The momentum conservation equations for fluid 1 and fluid 2 are respectively

$$\frac{\partial}{\partial t}(\alpha_1 \rho_1 u_1) + \frac{\partial}{\partial x}(\alpha_1 \rho_1 u_1^2) = -\alpha_1 \frac{\partial p}{\partial x} - F_{w1} + F_{12} - \alpha_1 \rho_1 g - \alpha_1 \dot{M}_d u_1 \tag{5}$$

$$\frac{\partial}{\partial t}(\alpha_2 \rho_2 u_2) + \frac{\partial}{\partial x}(\alpha_2 \rho_2 u_2^2) = -\alpha_2 \frac{\partial p}{\partial x} - F_{w2} - F_{12} - \alpha_2 \rho_2 g \tag{6}$$

where p and g are respectively the pressure and gravitational acceleration. F_{w1}, F_{w2}, and F_{12} are the interfacial forces between the wall and fluid 1, the wall and fluid 2, and between two fluids, respectively. These are flow patterns specific [31].

C. Particle Transport

The species conservation equation is

$$\frac{\partial}{\partial t}(\alpha_1 C) + \frac{\partial}{\partial x}(\alpha_1 u_1 C) = -\dot{M}_d \alpha_1 \tag{7}$$

where C is the particle concentration.

D. Particle Deposition

The deposition process is modeled as an m-th order deposition reaction [32] with the deposition rate expressed as

$$\dot{M}_d = kC^m \tag{8}$$

where k is the deposition rate constant and m is the deposition reaction order. In this article, m is set to 1.

IV. Numerical Method

Equations (1), (2), (3), (5), (6) and (7) can be written in the form of a general transient convection equation as

$$\frac{\partial}{\partial t}(\tilde{\rho}\varphi) + \nabla \cdot (\tilde{\rho}\vec{u}\varphi) = S_\varphi \tag{9}$$

where $\tilde{\rho}$ is the appropriate 'density', φ is the variable of interest, $\vec{u}$ is the velocity vector and S_φ is the source term. This equation is solved using a finite volume method on mesh shown in Fig. 2 (the pipe is drawn horizontally for presentation). The velocity-pressure coupling is solved via a two-phase SIMPLER algorithm [33] with modification to account for the growing deposit layer.

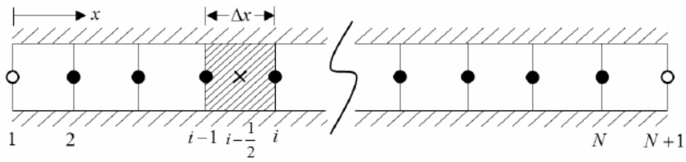


Fig. 2. The discretized computational domain

D. Solution Algorithm

(1) Specify the initial and inlet boundary conditions.
(2) Advance the time step to $t + \Delta t$.
(3) Calculate p, u_1 and u_2 (two-phase SIMPLER [33]).
(4) Calculate α_d (Eq. 1).
(5) Calculate α_1 (Eq. 2).
(6) Calculate α_2 (Eq. 4).
(7) Calculate C (Eq. 7).
(8) Repeat steps (4) to (7) until the solution converges.
(9) Repeat steps (2) to (8) for all subsequent time steps.

V. Results and Discussions

A. Verifications

1) Single-phase "bubbly" flow with deposition

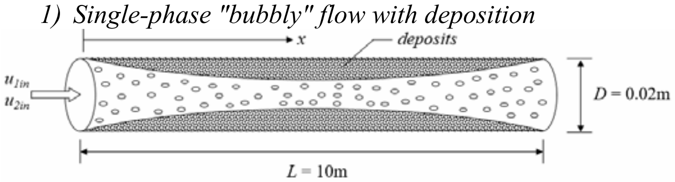


Fig. 3. A bubbly flow with deposition in a pipe.

Fig. 3 shows the schematic of a bubbly flow with particles in a circular pipe. The properties of the two fluids are identical (TABLE I). The two fluids have the same inlet velocities of 0.2 m/s. The fluid 1 and fluid 2 inlet volume fractions are 0.8 and 0.2 respectively. In this hypothetical "bubbly" flow, there is no interfacial force between the two fluids, i.e. $F_{12} = 0$. The deposition rate $\dot{M}_d$ is prescribed in this hypothetical case as

$$\dot{M}_d = 100x / L \tag{10}$$

With these, this special "bubbly" flow can be treated as a single-phase flow with deposition. The following initial and boundary conditions apply.

Initial conditions,

$$\begin{aligned} &\alpha_1 = 0.8, \alpha_2 = 0.2, \alpha_d = 0, u_1 = u_2 = 0.2 \text{ m/s}, \\ &p = 0 \ \text{ for } 0 \le x \le L \end{aligned} \tag{11a}$$

Boundary conditions,

$$\alpha_1 = 0.8, \alpha_2 = 0.2, \alpha_d = 0, u_1 = u_2 = 0.2 \text{ m/s}, \\ p = 0 \text{ for } x = 0 \tag{11b}$$

The analytical solutions of α, u and p are given respectively by

$$\alpha(x,t) = 1 - \frac{1}{82}xt \tag{12}$$

$$u(x,t) = \frac{1}{5\alpha} \tag{13}$$

$$p(x,t) = \frac{16.4x(xt^2 + 316xt - 164t - 25912)}{(xt - 82)^2} \tag{14}$$

For the purpose of comparison, a total fluid volume fraction and a mean fluid velocity are respectively are defined.

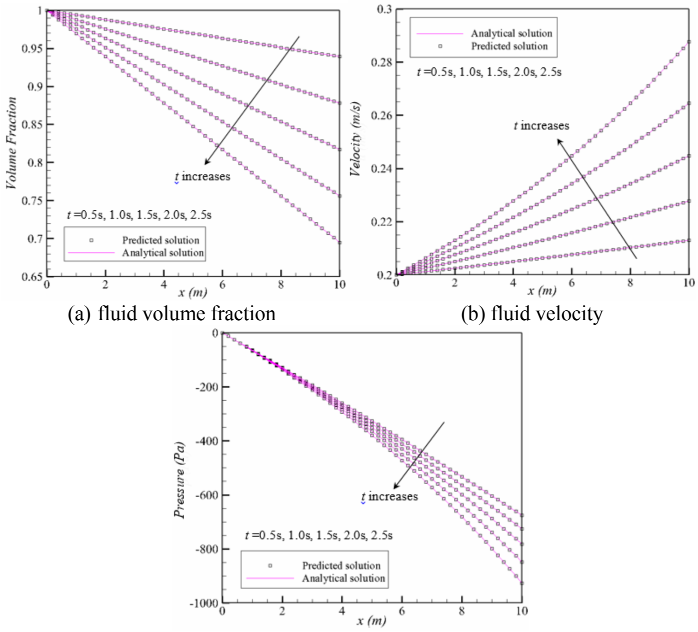
$$\alpha = \alpha_1 + \alpha_2 \tag{15}$$

$$u = \frac{\alpha_1 u_1 + \alpha_2 u_2}{\alpha_1 + \alpha_2} \tag{16}$$

By setting C=0, the current model can be used to predict this limiting case. Fig. 4 shows the predicted solutions obtained by the current model at five different t. The predicted solutions agree well with the analytical solutions. This partially verifies the developed model.

TABLE I. PARAMETERS USED FOR SINGLE-PHASE FLOW WITH DEPOSITION IN A PIPE

Fluid Density (ρ, kg/m^3)	**Deposit Density (ρ_d, kg/m^3)**	**Fluid Dynamic Viscosity (μ, mPa·s)**	**Inlet Velocity (u_{in}, m/s)**
820	820	3.95	0.2



(a) fluid volume fraction (b) fluid velocity

(c) pressure

Fig. 4. Distribution of α, u and p for single-phase flow with deposition.

B. *Validations*

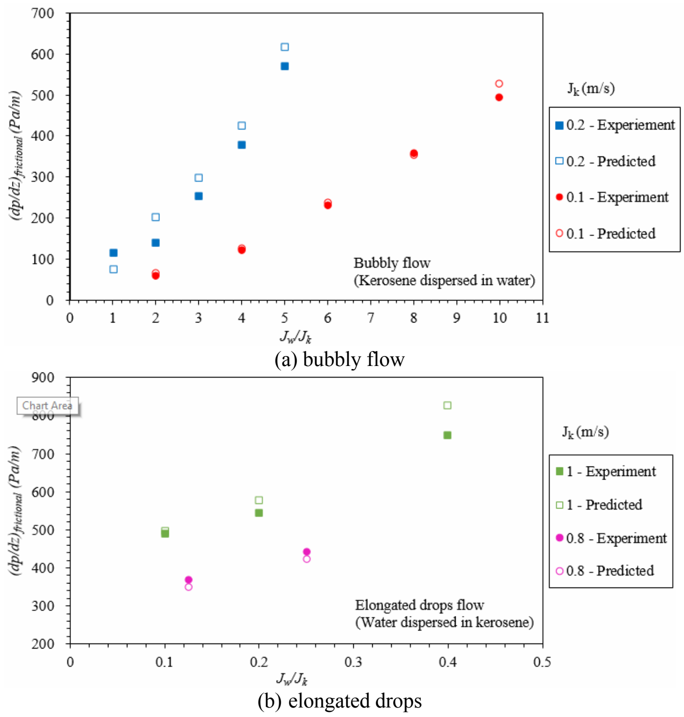
1) Two-Phase Water-Kerosene Bubbly Flow

This section validates the capability of the model against the experimental results for two-phase water-kerosene flow of Suguimoto and Mazza [34]. Only the data for bubbly flow (kerosene dispersed in water) and elongated drops flow (water dispersed in kerosene) are extracted and compared with the prediction of the current model. Properties for water and kerosene are tabulated in TABLE II.

TABLE II. FLUIDS' PROPERTIES FOR WATER AND KEROSENE

Water Density (ρ_w, kg/m^3)	**Kerosene Density (ρ_k, kg/m^3)**	**Water Dynamic Viscosity (μ_w, mPa·s)**	**Kerosene Dynamic Viscosity (μ_k, mPa·s)**	**Surface Tension (σ, mN/m)**
998	793	1.0	1.1	48.0

By setting C = 0 and $\dot{M}_d = 0$ (no deposition), the model reduces to a limiting case of two-phase flow without deposition. It predicts the frictional pressure drop for the cases considered in the experiment. The mesh-independent frictional pressure drop is presented in Fig. 5 as a function of the water-kerosene input ratio (J_w/J_k) together with the experimental data. The predicted frictional pressure drop agrees well with the experimental data. This validates the two-phase flow modeling capability of the model.



(a) bubbly flow

(b) elongated drops

Fig. 5. The experimental and predicted frictional pressure drop.

2) Two-Phase Crude Oil-Water Annular Flow

This section validates the developed model against experimental data of a crude oil-water annular flow [35]. The properties for crude-oil and water are listed in TABLE III. Again, by setting C=0 and $\dot{M}_d = 0$, the current model reduces to a limiting case of two-phase flow without deposition. The mesh-independent frictional pressure drop obtained from the model is compared against the experimental data in Fig. 6.

The current model predicts well the general trend of the pressure drop, in particular, those of high oil superficial velocity. Note that the uncertainties of the pressure drop measurement in the experiment are approximately ±25% for the lowest oil superficial velocities and ±6% for the highest oil superficial velocities [35]. This again validates the capability of the developed model in predicting two-phase flow.

TABLE III. FLUIDS' PROPERTIES FOR CRUDE OIL AND WATER

Water Density (ρ_w, kg/m^3)	Crude Oil Density (ρ_c, kg/m^3)	Water Dynamic Viscosity (μ_w, mPa·s)	Crude Oil Dynamic Viscosity (μ_c, mPa·s)	Surface Tension (σ, mN/m)
997	925	0.89	500	26.3

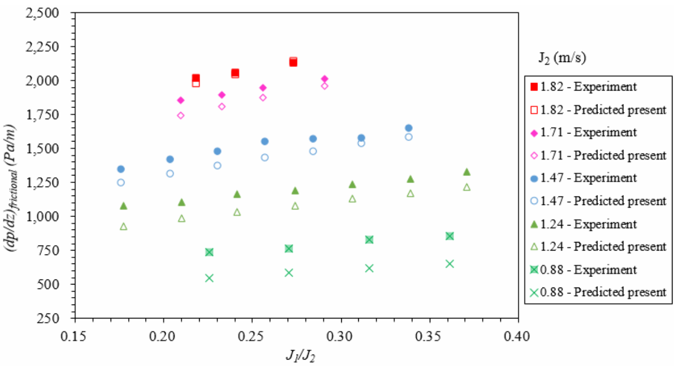


Fig. 6. The experimental and predicted frictional pressure drop.

C. *Case Study for Bubbly Flow with Deposition*

The model can be used for bubbly, transitional and annular two-phase flow with deposition. Here, bubbly flow with deposition is presented. Studies on other flow patterns with deposition can be found in [31].

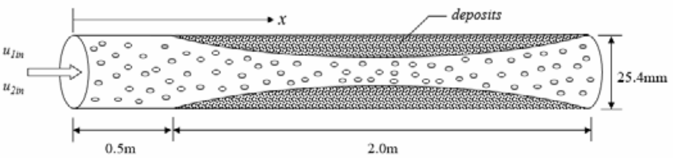


Fig. 7. Two-phase water-kerosene bubbly flow with deposition.

Fig. 7 shows the schematic of a two-phase bubbly flow of water (fluid 1) and kerosene (fluid 2) in a vertical pipe (drawn horizontally for presentation purpose). The pipe has an inner diameter of 25.4mm and a length of 2.5m. Water and kerosene flow with inlet velocities of 0.2m/s and 0.25m/s respectively. The water and kerosene inlet volume fractions are respectively 0.85 and 0.15. Water carries particles. The particles deposit on the wall and form a deposit layer. Assumed the pipe is treated with an anti-deposition coating for $x \le 0.5$m, i.e. particles do not deposit in this section. With these, the particle deposition rate constant (k) (Eq. 8) used is set to

$$k = \begin{cases} 0 \quad , & \text{for } x \le 0.5\,\text{m} \\ 0.1\,\text{s}^{-1}, & \text{for } x > 0.5\,\text{m} \end{cases} \tag{18}$$

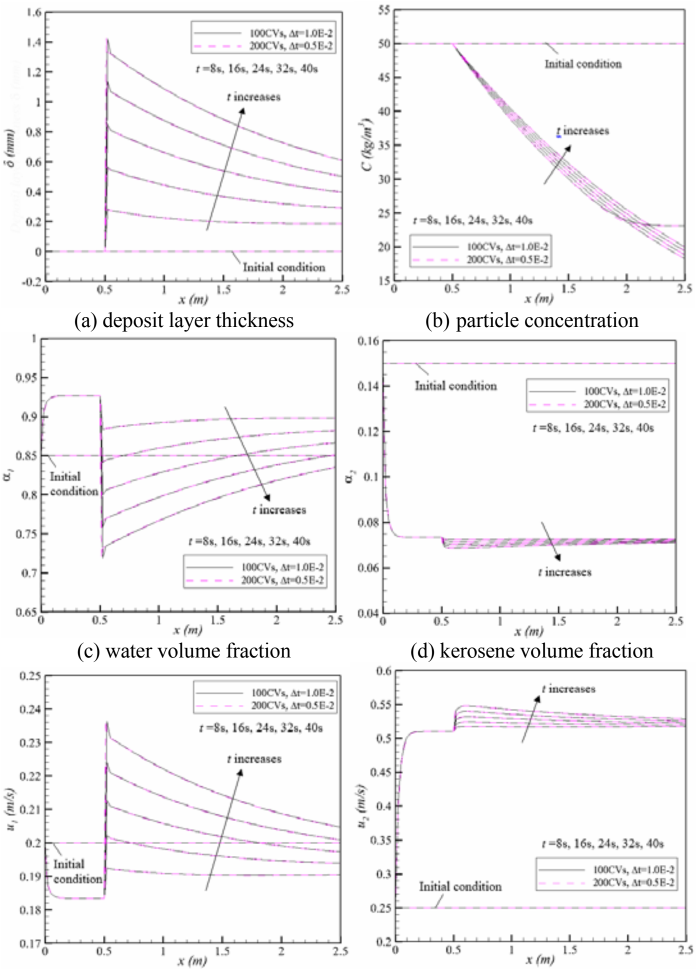
The following initial and boundary conditions are enforced.

Initial conditions,

$$\begin{aligned} &\alpha_1 = 0.85, \alpha_2 = 0.15, \alpha_d = 0, u_1 = 0.2\,\text{m/s}, \\ &u_2 = 0.25\,\text{m/s}, C = 50\,\text{kg/m}^3, p = 0 \quad \text{for } 0 \le x \le L \end{aligned} \tag{19a}$$

Boundary conditions,

$$\begin{aligned} &\alpha_1 = 0.85, \alpha_2 = 0.15, \alpha_d = 0, u_1 = 0.2\,\text{m/s}, \\ &u_2 = 0.25\,\text{m/s}, C = 50\,\text{kg/m}^3, p = 0 \quad \text{for } x = 0 \end{aligned} \tag{19b}$$



(a) deposit layer thickness (b) particle concentration

(c) water volume fraction (d) kerosene volume fraction

(e) water velocity (f) kerosene velocity

Fig. 8. Simulation results of bubbly flow with deposition in a vertical pipe.

Fig. 8 shows the simulation results at different times. There is not deposit form for $x \le 0.5$m (Fig. 8a). From x = 0.5m onwards, the particles start to stick onto the wall to form a deposit layer. As the particles are continuously consumed in the deposition process, i.e. become deposit, the amount of particles flowing downstream decreases. As the particle concentration decreases along the pipe (Fig. 8b), the particle deposition rate ($\dot{M}_d = kC$) also decreases along the pipe. The deposit layer becomes thinner along the pipe. Besides, the deposit layer becomes thicker over time.

With the formation of the deposit for $x > 0.5$m, the available flow area suddenly decreases. Thus, there is a sharp increase in the water and kerosene velocities, i.e. u_1 and u_2 (Fig. 8e and 8f). Then, u_1 and u_2 decrease gradually downstream for the deposit layer becomes gradually thinner. Near the inlet where there is not deposit ($x < 0.5$m), u_1 decreases quickly whereas u_2 increases. This is because water is denser than kerosene, therefore experiencing a larger gravitational effect in an upward flow. As a result, there is more water (higher volume fraction α_1, Fig. 8c) and less kerosene (lower volume fraction α_2, Fig. 8d) in this region.

Note that the deposit layer surprisingly does not affect the pressure much as the flow is dominantly determined by the gravitational force. In a short period of the time ($t \leq 8$s), the particle concentration (C) decreases because of deposition. Over time however, particles in the pipe are replenished by those from the inlet.

VI. CONCLUSIONS

This study presents a model for two-phase flow with deposition. The model consists of three modules: Fluid Transport, Particle Transport and Particle Deposition. The model is partially verified against existing analytical solutions and validated against experimental data. It is then demonstrated for bubbly flow with deposition.

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