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Condition Monitoring of Helical Gearboxes based on the Advanced Analysis of Vibration Signals

Fathalla J.M. Elbarghathi

A thesis submitted to the University of Huddersfield in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

University of Huddersfield

August 2015
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<td>Ampere</td>
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<td>Analogy digital convertor</td>
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<td>Data acquisition card</td>
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<td>Discrete Fourier Transformer</td>
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<td>SCC</td>
<td>Shape Correlation Shape</td>
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<tr>
<td>SDC</td>
<td>Slope Deviation Curve</td>
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<tr>
<td>SDOF</td>
<td>Single degree of freedom</td>
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<td>SG</td>
<td>Scalogram</td>
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<tr>
<td>SPD</td>
<td>Shape Percentage Different</td>
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<tr>
<td>STFT</td>
<td>Short-Time Fourier Transformer</td>
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<tr>
<td>SVAF</td>
<td>Space vector Angular Fluctuation</td>
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<tr>
<td>$S_{xx}$</td>
<td>Power Density</td>
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<tr>
<td>TSA</td>
<td>Time Synchronous Averaging</td>
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<td>V</td>
<td>Volt</td>
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<td>WPT</td>
<td>Wavelet Packet Transformer</td>
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<td>WTs</td>
<td>Wavelets transform</td>
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<td>WVD</td>
<td>Winger-Vill Distribution</td>
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<td>$y$</td>
<td>The sampled time series</td>
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<td>$\mu V$</td>
<td>micro-Volt</td>
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<tr>
<td>$\ddot{d}(k)$</td>
<td>Normalized wavelet coefficients</td>
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<td>1X</td>
<td>First harmonic Amplitude</td>
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### List of Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>a</td>
<td>Wavelet Scale Coefficient</td>
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<td>b</td>
<td>Wavelet Space (time) Coefficient</td>
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<tr>
<td>t</td>
<td>Time Shift</td>
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<tr>
<td>dt</td>
<td>Time Derivative</td>
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<tr>
<td>d\omega</td>
<td>Frequency Derivative</td>
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<tr>
<td>e</td>
<td>Eccentricity</td>
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<td>f</td>
<td>Frequency in Hz</td>
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<tr>
<td>Fs</td>
<td>Sampling Frequency</td>
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<td>w</td>
<td>Rotational Frequency in Radians/Seconds</td>
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<tr>
<td>g(k)</td>
<td>Quadrature Mirror Filter</td>
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<tr>
<td>g(t)</td>
<td>Window Function</td>
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<td>H</td>
<td>High pass Filter</td>
</tr>
<tr>
<td>h(k)</td>
<td>Quadrature Mirror Filter</td>
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<td>J</td>
<td>Wavelet transformation level</td>
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<td>K</td>
<td>Stiffness Matrix</td>
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<td>ks</td>
<td>Stator radial Stiffness</td>
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<td>m</td>
<td>Unbalanced Mass</td>
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<td>sec</td>
<td>Seconds</td>
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<td>t</td>
<td>Time in Seconds</td>
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<td>w</td>
<td>Wavelet packet Function</td>
</tr>
<tr>
<td>d</td>
<td>Blade Tip Deflection During Rubbing</td>
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</tbody>
</table>
Δf  Frequency Window Size
Δt  Time Window Size
ψ(t)  Wavelet Function
Φ(t)  Wavelet Scaling Function
Z1  Number of teeth on the pinion gear at the 1st stage
Z2  Number of teeth on the driven gear at the 1st stage
Z3  Number of teeth on the pinion gear at 2nd stage
Z4  Number of teeth of the driven gear at 2nd stage
f_m  Tooth meshing frequency
x_{vib}(t)  Gear mesh vibration
k  Number of mesh frequency harmonics
A_k  Amplitude of harmonics
Φ_k  Phase of Harmonics
W(t)  Noise signal
a_w(Θ)  Amplitude modulating function
b_k(Θ)  Phase Modulating Function
y(t)  Average vibration signal
f_r  Shaft rotation frequency
N_t  Number of teeth on the gear
m  Meshing harmonic number
k  Integer number
N  Number of samples taken in the signal
\bar{x}  Mean Value of the N amplitudes
$x(n)$  
Amplitude of the signal for the $n$th sample

$x_i$  
Amplitude of the signal for the $i$th sample

$f_{m11}$  
Mesh frequency of the 1st gear pair

$f_{m12}$  
2nd harmonic of mesh frequency of the 1st gear pair

$f_{m21}$  
Mesh frequency of the 2nd Gear Pair

$f_{m22}$  
2nd harmonic of mesh frequency of the 2nd gear pair

$g(t)$  
Sliding window function

$W(t,f)$  
Wigner-Ville Distribution function

$W_{12}(t,f)$  
Cross-Wigner distribution

$A_1$, $A_2$  
Amplitudes of the signals $f_1$ and $f_2$

$\tau$  
Time to perform a “time-localized” Fourier transform

*  
Complex conjugate

$W^*$  
Complex conjugate of the analyzing wavelet (tw)

$s$  
Scale or dilation factor

$\varepsilon_c$  
Contact ratio

$\varepsilon_a$  
Overlap ratio

$I_{p1}, I_{p2}, I_{g1}, I_{g2}$  
Pinions and gears moment of inertia

$F_1$, $F_2$  
Gearing stiffness forces

$F_{1t}, F_{2t}$  
Gearing damping forces

$M_1$, $M_{1t}$  
Internal moment stiffness damping first stage

$M_2$, $M_{2t}$  
Internal moment stiffness damping second stage
Condition Monitoring of Helical Gearboxes based on the Advanced Analysis of Vibration Signals

\( M_3, M_{3t} \)  
Internal moment stiffness damping third stage

\( \varphi_1 \)  
Motor angular displacement

\( \varphi_2 \)  
First pinion angular displacement

\( \varphi_3 \)  
First gear angular displacement

\( \varphi_4 \)  
second pinion angular displacement

\( \varphi_5 \)  
second gear angular displacement

\( \varphi_6 \)  
Load angular displacement

\( \varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6 \)  
Angular velocities

\( \dot{\varphi}_1, \dot{\varphi}_2, \dot{\varphi}_3, \dot{\varphi}_4, \dot{\varphi}_5, \dot{\varphi}_6 \)  
Angular Acceleration

\( r_1, r_2, r_3 \)  
Pinions radius

\( r_{p1}, r_{p2} \)  
Pinions radius

\( r_{g1}, r_{g2} \)  
Gear radius

\( K_1, K_2, K_3 \)  
Stiffness

\( C_1, C_2, C_3 \)  
Damping

\( K_{t1}, K_{t2}, K_{t3} \)  
constant parameters

\( C_{t1}, C_{t2}, C_{t3} \)  
constant parameters

\( K_{z1} \)  
First stage gearing stiffness (meshing stiffness)
$K_{z2}$  
Second stage gearing stiffness (meshing stiffness)

$C_{z1}, C_{z2}$  
Damping coefficient
Abstract

Condition monitoring of rotating machinery and machine systems has attracted extensive researches, particularly the detection and diagnosis of machine faults in their early stages to minimise maintenance cost and avoid catastrophic breakdowns and human injuries.

As an efficient mechanical system, helical gearbox has been widely used in rotating machinery such as wind turbines, helicopters, compressors and internal combustion engines and hence its vibration condition monitoring is attracting substantial research attention worldwide. However, the vibration signals from a gearbox are usually contaminated by background noise and influenced by operating conditions. It is usually difficult to obtain symptoms of faults at the early stage of a fault.

This study focus on developing effective approaches to the detection of early stage faults in an industrial helical gearbox. In particular, continuous wavelet transformation (CWT) has been investigated in order to select an optimal wavelet to effectively represent the vibration signals for both noise reduction and fault signature extraction. To achieve this aim, time synchronous average (TSA) is used as a tool for preliminary noise reduction and mathematical models of a gearbox transmission system is developed for characterising fault signatures.

The performance of three different wavelet families was compared and henceforth a criterion and method for the selection of the most discerning has been established. It has been found that the wavelet that gives the highest RMS value for the baseline vibration signal will show the greatest difference between baseline and gearbox vibration with a fault presence. Comparison of the three wavelets families shows that the Daubechies order 1 can give best performance for feature extraction and fault detection and fault quantification.

However, there are limitations that undermine CWT application to fault detection, in particular the difficulty in selecting a suitable wavelet function. A major contribution of this research programme is to demonstrate a possible route on how to overcome this deficiency. An adaptive Morlet wavelet transform method has been proposed based on information entropy optimization for analysing the vibration signal and detecting and quantifying the faults seeded into the helical gearbox.

This research has also developed a nonlinear dynamic model of the two-stage helical gearbox involving time–varying mesh stiffness and transmission error. Based on experimental data collected with different operating loads and the simulating results vibration signatures for gear faults are fully understood and hence confirms the CWT based scheme for signal enhancement. These results also indicate that the dynamic model can be used in studying gear faults and would be useful in developing gear fault monitoring techniques.
No portion of the work presented in this thesis has been submitted in support of an application for another degree or qualification of this or any other University or other institute of learning.
Dedication

I dedicate this work in memory of my beloved spirit of my parents and in memory of my beloved brother Zaid.
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The Author

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The author, during his work with AGOCO, has attending many technical courses relevant to machine maintenance, and was sponsored by AGOCO to attend a six-month training course in Berlin, Germany, on rotating machinery such as centrifugal pumps and gas compressors.

During 2000-2002, the author attended The University of Manchester, and graduated with a Master’s degree in Mechanical Engineering. During 2003-2007, the author was a part-time lecture at Benghazi University and the Higher Education Institute in Benghazi (Libya). For thirteen years, 1994-2007, the author organized and taught technical seminars on rotating machine maintenance, sponsored by the Central Cement Training Centre, Benghazi, and AGOCO. The courses were for staff from many Libyan industrial companies including Sirte Oil, AGOCO and Ras Lanauf. In total the author spent around 3000 hours teaching on these courses.

In April 2007 the author jointed the academic staff members as a full-time lecture at the Omar Al-Mukhtar University, (Libya) in the Engineering Faculty where he was an academic with numerous teaching and technical responsibilities.

In 2011, the author joined the research group in the School of Engineering and Computing at The University of Huddersfield as a full-time research student sponsored by the Libyan Ministry of Higher Education. His research activities focused on the condition monitoring of rotating machines. During his PhD study the author attending a number of many academic events offers by The University of Huddersfield Engineering and Computing School related to rotating Machines Condition Monitoring CM, during his time as a researcher the author has produced several technical and professional publications: two presentations at international conferences. The first was Two stage helical gearbox fault detection and diagnosis based on continuous wavelet transformation of time synchronous averaged vibration signals at COMADEM 2012, Huddersfield, UK, and the
second was *Multi-stage helical gearbox fault detection using vibration signal and Morlet wavelet transform adapted by information entropy difference* at COMADEM 2013, Helsinki Finland.

The author also organized two poster sessions at The University of Huddersfield and was awarded the prize for the best quality poster presentation at the Diamond Jubilee Research Conference in the School of Computing and Engineering, University of Huddersfield March 2012.
Finally, the author has been able to publish number of papers:


- Elbarghathi, F., et al., Multi-stage helical gearbox fault detection using vibration signal and Morlet wavelet transform adapted by information entropy difference, Comadem 2013, Helsinki.

Reward

I was awarded the prize for the best quality poster presentation at the Diamond Jubilee Research Conference (CEARC12) in the School of Computing and Engineering, University of Huddersfield 12 March /2012.
CHAPTER ONE

INTRODUCTION

This chapter begins by presenting the relevant background and motivation for this research project with a brief description of the condition monitoring of rotating machinery. This is followed by the importance of maintenance and fault detection with the emphasis on gearbox condition monitoring. Next, the chapter lists the aim and objectives of the project, the tools used, the scientific approach followed and the contribution to knowledge of this research. The chapter ends by describing the thesis content, structure and layout.
1.1 Background

The application of wavelet transforms to the condition monitoring (CM) of industrial machines for predictive maintenance is an area of great interest and rapid progress due to high demand (Davies.A, 1998). It is commonplace today for industries to insist on reliable and efficient machines that run at high operating speeds and loads. An important method of increasing reliability and performance of machines is the application of CM for the early detection of faults which allows appropriate maintenance strategies to be implemented in time to resolve the problem before the fault causes plant shut down and, possibly, a severe accident which could result in human injury. The application of CM systems has mushroomed with more and more industrial companies attempting to reduce machine faults and eliminate consequent negative effects. Such an approach has necessitated alternative uses of resources with significant investment in CM.

Benefits of using CM include principally (Davies.A, 1998):

- Reducing maintenance costs and avoids unnecessary replacement of parts.
- Developing safety and reliability, and decrease human injuries.
- Upgrading production quality at low cost.
- Improving industrial profitability and product competitiveness.
- Avoiding and even eliminating catastrophic faults.
- CM can offer clear information of the nature of a fault in advance of failure, so that arrangements can be made during the interval between identification and failure so that loss of production minimised.
- CM can provide full details of the fault to be repaired so that the correct replacement parts may be ordered, the appropriate staff scheduled so that disruption to the production process is minimised.
- Effective use of CM can reduce maintenance costs and avoid the unnecessary replacement of part-worn machine components as could happen with some planned maintenance methods.

Nowadays, run-to-failure is an unacceptable strategy for many process and industrial operations. The more desirable alternative is a CM system which collects and analyses various machine data because it provides early detection of machinery faults.

Commonly used condition monitoring techniques are (Davies.A, 1998):

- Vibration signal analysis.
- Oil-debris analysis.
Vibration analysis is a powerful technique for CM of rotating machinery. Vibration analysis is based on well-known and widely-used techniques, such as time-domain and frequency-domain analysis. However, these relatively simple methods have the common drawback that, strictly, they are applicable only to stationary vibration signals, whereas most rotating machinery produces non-stationary vibration signals. Joint time-frequency analysis and has been developed and used to overcome this difficulty (Davies.A, 1998).

The health condition of most dynamic rotating machine systems changes with time, thus its vibration signal will change with time and the signals features can provide important information which can be used to explore the condition of the rotating machinery. Nowadays, most research studies concentrate on using a signal processing technique to analyse the signal because this technique provides more useful information for the prediction and diagnosis of faults at an early stage (Kelly, 1997; R. B. Randall, 2010; B. K. N. Rao, 1996).

1.2 Research Motivation

Vibration analysis has been shown to be a powerful tool in machines CM and many researchers have spent considerable efforts developing this technique and a huge amount of research activity is being carried out into its application to the CM of gearboxes (Behbahanifard, Karshenas, & Sadoughi, 2008; F. Gu & Ball, 1995). However, the challenge of producing a CM method capable of detecting the existence of, and diagnosing the cause of a fault at an ever earlier stage is, by definition, never ending. The goal of detecting gearbox faults as early as possible will provide maximum time for remedial action, prevent unnecessary damage and extend gearbox life.

Vibration-based CM provides non-stationary signals from which can be extracted valuable information concerning the condition of the machine. Traditionally this has been used for machine maintenance scheduling with machine health continuously monitored online. Vibration-based CM has also been applied in many other applications where signature pattern analysis can determine a fault.
Moreover, many gearbox CM studies which have been carried out under different operating condition such as varying load and varying speed, were been carried out in depth (Bebbahanifard et al., 2008; F. Gu & Ball, 1995). In addition, the majority of past studies have investigated gearbox CM with the sensor mounting located on the gearbox casing.

In this research project, initial activities will investigate whether vibration signatures generated by the gearboxes are able to provide useful information for gearbox CM for a gearbox operating under different loads. The rest of thesis will investigate a novel approach to pattern recognition and fault detection using advanced signal processing.

The present work uses the signature (pattern) analysis of the vibrational signal. To investigate gearbox CM we collect the vibration signal from sensors mounted on the exterior of the gearbox casing under full shaft speed 1465 RPM and different loads ranging from no load to full load 0 to 100% of rated load.

The detailed motivation of the research topic for gearbox CM based on vibration signal analysis will be discussed in Chapter 2 of this thesis.

1.3 The Importance of Maintenance and Fault Detection

In the industrial field, maintenance costs are an important factor and so researchers have investigated the relationship between best quality productions at minimum maintenance cost. The increasing cost of maintenance, considered as the total operating cost of production, is 15% and 40% of the cost of manufactured goods (P. McFadden & Smith, 1985). Maintenance cost is mainly the cost of spare parts needed to correct the faults developed by the machines (Alliaz, 1978). The consequences of machine faults are loss of production, reduction of product quality, reduction of operator safety and increased maintenance and operating costs. The most common maintenance systems in industrial plants were reactive: maintenance was carried out and repairs made according to a schedule. This was, and is, wasteful because parts may be replace before their useful life is ended or, as sometimes happens, a part does not complete its expected life cycle and prematurely fails. Nowadays, advanced computer techniques are used to manage industrial preventive maintenance (PM) strategies which identify machines failures by monitoring and assessing the condition of the machine components and predicting maintenance demands (Morris, 1994).

By facilitating using the time-domain, frequency-domain and joint time-frequency domain for machine fault detection and preventing failure generally reduces both maintenance costs and risk to the operator. Currently, PM applications based on employing advanced computer techniques to
process the acquired vibration signals gives the opportunity to identify the machine faults at an early stage, with continuous fault detection and diagnosis leading to an upgrading of machine performance (Behbahanifard et al., 2008). The majority of past research studies were focused on improving the sensitivity of vibration monitoring using time-domain and frequency-domain analysis (Ball. A. D. & Gu, 1995; R. B. Randall, 2004a). This is because while most chronic faults are tiny and invisible, they occur much greater frequency than catastrophic failure of a part and, in total, are much more costly.

1.4 Gearbox Condition Monitoring and Project Motivation

Gearboxes are important components in many critical industrial applications, e.g. wind turbines and helicopters, and are crucial in power transmission where a change of drive speed and/or working direction is required. This is the reason why there is constant pressure to upgrade and enhance the measuring techniques and analytical tools for diagnosis of gearbox faults at the earliest possible stage.

In a power transmission system the gearbox normally operates at high speeds under fluctuating loads. There are a number of different types of defect that may be caused by installation or manufacturing errors that can lead to gearbox failure. Installation, or mounting, errors would include misaligned shafts, lack of adequate clearance, etc. Manufacturing defects would include errors in tooth profile and inadequate strength of material used. Defects would appear as tooth breakage, cracks, uneven wear and misalignment, etc. (P. McFadden & Smith, 1985). As a result, gears are prone to premature failure due to wear and material fatigue.

However, physical parameters such as sound, temperature, motor current and vibration can be used to monitor the condition of gearboxes (Blakeley, 2001; Chorafas, 1990; R.B Randall, 1980; Reintjes, 1997). Over the past years many techniques have been developed for processing vibration signals to detect gear failure, based on the assumption that any change in the gearbox condition may be detected by change in the measured vibration signal. Earlier reports on gear faults detection and diagnoses were focused on the time-domain and frequency-domain analysis of the vibration signal; spectrum, cestrum, amplitude and phase modulation techniques were also used to detect different types of gear failure. Most of these techniques may help to detect and indicate faults but could not provide much information about location and severity of the fault and were not eligible for use with non-stationary signals (Francois Combet & Gelman, 2007; Fakhfakh, Chaari, & Haddar, 2005; R. Randall, 1982; P. W. Stevens, Hall, & Smith, 1996).
As most measured vibration signals from gearboxes show non-stationary properties in the last 20 years joint time-frequency domain methods have been focused more by researchers as a more effective and reliable method for developing CM techniques of rotating machinery (Fakhfakh et al., 2005). Of many different types of time-frequency transforms, continues wavelet transforms are of more appealing as they can highlight more on the localised vibration signatures for accurately capture the weak signatures. However, the large family of wavelets and the wide options of wavelet parameters make it difficult to select one for best diagnosis. Therefore, this research focuses on the investigation of optimal use of wavelets.

In addition, most previous researches were carried out based on specially designed spur gearboxes for evaluating CM techniques developed. As such test system usually has large differences from real applications, current research employed an industrial gearbox obtained from the production line of a major company. Therefore, vibration signals will be more close to applications and corresponding signal analysis techniques developed will be more representative and reliable for wide applications.

1.5 Research Aim and Objectives

Most modern technique used for the CM of gearboxes are based on the analysis of vibration signals collected by accelerometer sensors fixed on the gearbox casing (Fakhfakh et al., 2005). An important goal is to detect the existence and type of fault at an early a stage as possible not only to avoid breakdown but also to assess the machine’s residual life and to schedule repair and maintenance. With gear vibration, the most important components are well known to be the tooth meshing frequencies and their harmonics, there will also be sidebands due to modulation phenomena. Importantly, any change in the number and amplitude of the sidebands is taken as indicating a faulty condition. The aim of this research investigation is to detect simulated gearbox faults by comparing the measured vibration signal obtained for baseline conditions with that obtained when known and qualified faults were seeded into the gearbox.

Many researchers have attempted to use spectrum analysis of the vibration signal to detect gear faults at an early stage. However, Cepstral analysis has also been used and results show that it is applicable to both detection and diagnosis of gear faults. On the other hand the application of the wavelet transform (WT), which is time-frequency domain, is highly recommended as a method to detect and localise faults such as a crack in a gear tooth (Stander, Heyns, & Schoombie, 2002; Zhan, Makis, & Jardine, 2006).
In this research into the CM of a two stage helical gearbox transmission system, vibration analysis has been used to investigate signals generated under different operating conditions and which were collected by a sensor mounted on the gearbox casing. These signals were processed and analysed using joint time–frequency domain signal processing wavelet transforms (WT). The outcome produced features which were employed to detect rotating machine faults.

The main objectives of this thesis are as follows:

**Objective 1**: To review and study existing condition monitoring techniques of rotating machinery.

**Objective 2**: To describe the principles of gearbox transmission systems, including gear type and classification, gear vibration characteristics and gear failures.

**Objective 3**: To review the literature describing conventional signal processing methods of gearbox fault detection and diagnosis based on analysis of the vibration signals.

**Objective 4**: To become familiar with a two stage helical gearbox and experimental procedures for condition monitoring of the transmission system.

**Objective 5**: To introduce a specific fault into one of the gearboxes; e.g. a broken tooth located on the gear (pinion) in the first stage of the two stage helical gearbox transmission system. To investigate the effect of different levels of fault severity on the gearbox vibration signal by introducing a series of quantified faults simulating different gear fault severities; e.g. tooth breakage of 25%, 50%, 75% and 100%. The last being complete removal of one tooth on the first stage pinion.

**Objective 6**: To continuously review the current literature on the use of vibration techniques for gearbox CM, especially gearbox fault detection methods based on using advanced signal processing technique such as the continuous wavelet transform (CWT).

**Objective 7**: To collect and analyse the gearbox vibration signal data from healthy and faulty gears at different operating conditions using advanced signal processing such as the Morlet wavelet transform adapted by information entropy difference and thus obtain an effective set of features for detecting and diagnosing the seeded gear tooth faults.

**Objective 8**: To create and develop a mathematical model to simulate two stages helical gearbox vibration signals for healthy and faulty gears operating under different conditions. This will help to
characterise changes of dynamic properties due to various types of gear faults, which assists measurement setup, data processing selection and diagnosis rule development.

**Objective 9:** To suggest and recommended a guideline for further research work activities in this field.

### 1.6 Tools and Approaches

In order achieve the aim of this research the following methods have been used:

- Introducing the joined technique to detect and diagnose the simulated fault of the gear in the first stage gear (pinion) in the helical gearbox.
- Applying time synchronous averaging (TSA) in the time domain to minimise random noise in a repetitive signal.
- In the present study, the following wavelets were selected to analyse the TSA baseline vibration signals and those obtained under different fault conditions: Daubiechies order 1 (db1), Symlets order 2 (sym2) and Coiflets order 3 (coif3). These wavelets are all orthogonal wavelets with faster, perfect reconstruction and non-redundant decomposition and have been used in many applications (reference?)
- In this study, a denoising method has been used for fault diagnosis of the gearbox. This was based on the adaptive Morlet wavelet transform WT where the parameters were optimized using maximum Shannon entropy difference.

### 1.7 Scientific Contribution of This Research

This thesis studies and investigates the fault diagnosis of helical gearbox transmission system using vibration signal analysis, and the contributions to knowledge are summarized below:

- The ability to obtain a distinguishing feature for fault indication using three types of wavelet families: db1, sym2 and coif3 were explored to find the optimal wavelet for identifying a small fault. It demonstrates that different wavelets produce different signal separation results with wavelet db1 producing the best fault separation whereas the coif3 wavelet failed to identify the fault.
- To obtain the best fault separation, a careful selection of wavelets were carried out. Based on this study, it is suggested that the wavelet which produces the highest RMS value of wavelet coefficients when applied to the baseline data should be selected.
The review of the literature carried out as part of this research indicates that there has been only limited investigation of gearbox fault detection using adaptive Morlet wavelet transforms. This research has developed the Morlet wavelet transform by optimising its parameters using maximum Shannon entropy difference.

A mathematical model has been developed for simulating the gearbox vibration signal operation under different operating conditions for machine condition prognosis.

The research can be applied and extended into other fields such as condition monitoring of wind turbine planetary gearboxes, helicopter gearboxes, marine vehicles and automobiles. The developed fault detection techniques will contribute to reducing maintenance costs and will improve machine performance.

1.8 Thesis Outline Organization
This thesis is organized into seven chapters. This first chapter serves as an introduction to the work, stating the background of this study and giving its aim and objectives.

Chapter 2
This chapter presents a literature review of machine maintenance procedures; this is followed by a brief discussion of CM with an overview of gear types and their operation, including common features of gear vibration. In addition, this chapter presents a review of signal processing techniques which are used to monitor the condition of geared transmission systems based on vibration signals. The three basic vibration techniques used for fault detection and diagnosis; time, frequency and time–frequency analyses are briefly reviewed. It is shown that time and frequency-domains alone are not adequate to identify symptoms of a fault in a helical gearbox in the early stage of development and that, for reliable diagnosis, an effective and powerful technique needs to be developed using the joint time-frequency domain, such as wavelet transformation. The wavelet transform is briefly reviewed, as an appropriate signal processing method for early fault detection. Some of its unique characteristics are presented: that it can analyse both stationary and non-stationary signals, and can transform any signal directly into time/space and frequency/scale domains, which can provide detailed information about signal evolution.
Chapter 3

This chapter describes the test rig and the parameters associated with incipient defect detection in a gearbox. This test rig platform was used to simulate a real gearbox fault as found in industry and for diagnosing that fault. This chapter describes the procedures carried out to investigate the vibration signals collecting from the gearbox under healthy and different levels of fault. It starts by describing the experimental procedure and the equipment’s and instrumentations used to collect the vibration signals, including non-stationary and nonlinear. These were processed using traditional methods and joint time-frequency domain.

Chapter 4

This chapter reviews the theoretical background of applying Continuous Wavelet Transform (CWT) techniques to TSA signals for the detection and diagnosis of incipient gearbox faults using vibration signals. The vibration signals for different gear condition (healthy and faulty) under different operating condition were analysed using joint time–frequency CWT. The chapter describes how the experiments were performed, and how the vibration signals were recorded and analysed using three commonly-used wavelet families: Daubechies order 1 (db1), Symlets order 2 (sym2) and Coiflets order 3 (coif3). The usefulness of these three techniques for incipient fault detection was investigated.

Chapter 5

This chapter present and proposes a new adaptive Morlet wavelet method for signal analysis based on maximum wavelet entropy difference. It has been shown to be an effective tool for rotating machinery fault detection and diagnosis. In this study, the fault detection was carried out using an adaptive Morlet wavelet combined with TSA. The TSA reduced signal noise and revealed the fault contained an related impulse component which provided the basis for accurate feature extraction. Next, the maximum Shannon entropy difference was used to optimize the central frequency and bandwidth parameter of the Morlet wavelet in order to achieve optimal match with the impulsive components and to extract the features of the gear faults. The results show that the proposed method is much better than the method of adaptive Morlet based on the kurtosis maximisation.

Chapter 6

This chapter presents a mathematical model that simulate vibration signal from a helical gearbox. A nonlinear dynamic model of a two-stage helical gearbox involving time–varying mesh stiffness,
transmission error and backlash has been established. Then, the governing equations are solved using a Runge-Kutta method. Based on experimental data which is collected from different operating torque loads and the simulating results, gear fault severities are identified. These results also indicate that the dynamic model can be used in studying gear faults and would be useful in developing gear fault monitoring techniques.

Chapter 7

This chapter summarizes the achievements of the research and explains how the aim and objectives stated in Chapter 1 were achieved. It includes a summary of the author's contribution to knowledge and the novel aspects of the research work. Finally, recommendations for future work for remote condition monitoring of gear transmission systems are provided and conclusion made.
CHAPTER TWO

LITERATURE REVIEW

This chapter presents a literature review of maintenance procedures for rotating machines, followed by a brief overview of condition monitoring of machinery, an introduction to gear types and their operation, and gear vibration features. The chapter also presents a very brief review of signal processing techniques used to monitor the condition of geared transmission systems using measured vibration signals. The three vibration analysis techniques: time, frequency and joint time–frequency used for fault detection and diagnosis are briefly reviewed.

Time-domain and frequency-domain analyses are shown not to be adequate to detect the symptoms of early stage faults introduced into a helical gearbox system. For reliable diagnosis of non-stationary signals, an effective and powerful technique needs to be developed for processing the measured signals using a joint time-frequency domain analysis such as wavelet transformation. The wavelet transform is briefly reviewed, as an appropriate signal processing method capable of early fault detection because it can cope with both stationary and non-stationary signals, and can transform any signal directly into time/space and frequency/scale domains, which can provide detailed information about signal evolution. CWT is defined as the sum over all time of the signal, and is one of the best transforms for singularity detection.
2.1 Introduction

Effective maintenance of rotating machinery is one of huge economic benefit to industrial field plant. In any medium and long term maintenance schedules for effective operation of such as plant and machinery there are many factors to consider, including investment, effective plant operation and continuity of production. These factors, and others, can be successfully married only if there is an effective maintenance strategy to detect and diagnose machine faults and predict time to failure so that the necessary remedial actions can be carried out before machine break-down (M. N. Regeai, 2007).

A gearbox is a major and essential component in many industrial arrangements such as electric power plant utilities. Gear transmission systems transfers power from one rotating shaft to another with high efficiency provided there is no failure in gear transmission. Whenever a faults exists in a gear transmission system (wear, cracking, scoring, etc.) the performance of the gears declines with a consequent drop in the efficiency with which the power is transmitted. Due to these reasons gearbox CM is of great importance to prevent failure and continued operation of the industrial plant (M. N. Regeai, 2007).

2.2 Maintenance Procedures

In industrial plant, to protect the continuity of production with minimum expense and to preserve invested capital, proper and efficient maintenance to predict faults and initiate the scheduling of repair handwork measure necessary action is required. Maintenance plans and strategies can be classified into three main groups as follows:

2.3 Reactive Maintenance

With Reactive Maintenance (RM) the machines are kept running until a failure appears, no prior action is taken to stop the failure. RM can be convenient depending on the number of maintenance staff and the size of operating systems. RM can be used if the cost of replacing machines is not high or where machine breakdown has no other consequences. In more complex systems RM should be avoided as it can cause unscheduled large-scale loss of production and may require the on-site storage of expensive spare parts to reduce plant downtime (M. N. Regeai, 2007).
2.3.1 **Regular Preventive Maintenance (RPM)**

Regular Preventive Maintenance (RPM) is based on a prescribed time interval between scheduled maintenance visits. The time intervals and the maintenance processes are based on the recommendation of the manufacturers and statistical analysis of the history of machines. RPM is commonly used in industrial fields as a primary level of maintenance above RM.

RPM is replacement of worn parts with new or reconditioned parts according to a set schedule. Maintenance costs can be divided into two, the cost of new/spare parts plus staff time required to replace the parts, and the production downtime required to replace the old part. The main objectives of RPM are to avoid catastrophic faults and personal injuries.

In 1994 Morris showed that RPM could offer a 30% reduction in maintenance costs compared to RM (Morris, 1994).

2.3.2 **Condition-Based Maintenance (CBM)**

Condition-Based Maintenance (CBM) is used as an industrial online monitoring system where operational security is the main concern. CBM minimises machine breakdowns by evaluating the machine condition, identifying machine component faults and predicting time to breakdown. The information is gathering from specific monitored parameters such as vibration, temperature and other process parameters. CBM allows maintenance staff to perform the necessary corrective actions at an optimum time before failure occurs. In addition, on line-machine monitoring can be carried out while the machine is in operation, which not only reduces machine downtime, but also best utilises staff time by enabling spare parts to be ordered in advance and the maintenance to be carried out at the most convenient time. All of these are important factors in reducing production costs (Morris, 1994).

2.4 **Machine Condition Monitoring and Fault Diagnosis**

The main objective of monitoring of industrial machinery is to predict failure at an early stage and avoid unnecessary loss of production and personal injuries. Monitoring can also allow concerned maintenance staff to carry out necessary corrective measures such as replacement of old parts with new spare parts for the machinery. Correct monitoring will prevent loss of production and reduce maintenance costs. Diagnosis is analysis of machines symptoms to define the fault and predict failure of a machine. The application of online monitoring and diagnosis of machine faults is
Condition Monitoring of Helical Gearboxes based on the Advanced Analysis of Vibration Signals

growing rapidly and is now considered a branch of computer integrated manufacturing (CIM) (Leonds, 2000), as with the CM of rotating machinery such as gearboxes. A CIM system uses the new high speed data processing facilities of the modern computer to process signals collected from the machinery during its operation and extract useful information about the machine’s condition. This allows informed decisions to be made regarding possible repair and maintenance. Most modern industrial engineering CM follows the same procedures, see Figure 2.1.

The condition of a machine can be considered as the “input” of system and the signal collected by the sensors as the output of system. The signal processing then captures the features of interest contained in the signal (Leonds, 2000) and the machine condition is evaluated. The signal processing element is crucial because the signal contains the information on the machine’s condition. Parameters that can be used perform machine CM include motor current, surface vibration signal, acoustic emissions (Leonds, 2000). Unfortunately, the signals are always contaminated by background noise due to the working process and this noise may initially hide the information within the signal. Signal processing removes this unwanted noise and accesses the information “hidden” in the raw signal allowing accurate evaluation of the machine’s condition.

![Figure 2-1 Engineering Monitoring and Diagnosis Procedure](image)

**2.4.1 Condition Monitoring Techniques**

Condition monitoring to detect the machine malfunction and predict failure includes many measurement and analysis techniques; including acoustic emission (AE), wear debris, oil and vibration analysis all of which are widely used in industry.

However, these techniques are usually used to detect machine malfunction rather than diagnose faults. Oil analysis can provide information about the concentration of wear metal in an oil sample, which indicates the degree of friction between component parts during machine operation. Wear debris analysis provides only information about the size and distribution of wear debris. AE can detect only changes in material due to defects such as cracks in a shaft. Vibration analysis is the
most universal method used to monitoring rotating machinery; the vibration analysis technique is able to gather the signal data from the operational machine and display them in vibration curve which can reveal the useful signal features indicating the machine condition (Davies.A, 1998).

### 2.4.1.1 Visual and Aural Inspections

Visual inspection should be a routine daily check performed by the technicians on industrial sites. Visual inspection was the first method used for predictive maintenance, and experienced technicians using only their visual sense may immediately identify the condition of a machine without any further analysis. For example, human vision can check for and identify oil leaks or cracks occurring in the machine.

Aural inspection or touch is other techniques based in the use of a human sense. An experienced technician will be able to detect abnormal noise and by touching a machine detect excessive vibration or elevated temperature (Mobley, 2002).

Such inspections may be supported by simple instrumentation such as stroboscopes or infra-red detectors (S. S. Rao, 1995; Yesilyurt, 1997). However the main limitation of these method are that they are not useful for early stage fault detection (Mobley, 2002).

### 2.4.1.2 Performance Monitoring

Performance parameter monitoring: there are many parameters influencing the performance of machine (e.g. torque, speed, etc.) that can be used to identify faults. Any noticeable change in the operational parameters will gives indication of the machine’s condition (S. S. Rao, 1995; Yesilyurt, 1997).

### 2.4.1.3 Thermal Monitoring

Thermal monitoring to identify excessive heat generation is very useful in identifying machine faults, both mechanical and electrical. This type of monitoring is performed using different types of thermal sensors such as thermocouples, thermometers, thermographic and thermal cameras (S. S. Rao, 1995; Yesilyurt, 1997).

### 2.4.1.4 Wear Debris Monitoring

Wear debris monitoring is the most important traditional technique used in industrial CM. It involves determining the type of contamination and size of wear particles in the oil (Ebersbach, Peng, & Kessissoglou, 2006; Lebold, McClintic, Campbell, Byington, & Maynard, 2000). Basically
wear exists if two surfaces (usually metal) are in contact and move against each other under a normally acting force (S. S. Rao, 1995; Yesilyurt, 1997). Lubricant is used to minimise friction and wear between the two surfaces in contact, but these will increase substantially with excessive load, too high a temperature and incorrectly selected lubricant, causing contact friction with material removed from surfaces in direct contact. However, wear debris can be controlled by lubricant and/or debris monitoring. A sample of lubricant is removed at regular intervals and analysed in an appropriate laboratory using spectrometric and ferrographic oil analysis. Debris can be monitored by having a magnetic plug at a low point in the system to give an indication of ferrous debris build-up.

Debris is formed from accumulated material damage due to surfaces in contact (Dempsey, Morales, & Afjeh, 2002; Stachowiak, Stachowiak, & Podsiadlo, 2008). The limitation of wear debris monitoring is that it needs costly laboratory equipment and is time consuming. However, a serious disadvantage of this technique is that it is not useful for detecting a failure such as component fatigue unless that failure causes shedding of metallic particles (Flodin, 2000; Roylance, Williams, & Dwyer-Joyce, 2000).

### 2.4.1.5 Acoustic Emission Monitoring

Over the past two decades, the use of Acoustic Emission (AE) as an effective tool for monitoring gearboxes, has been increased because of the existence of microscopic sources of AE activity in rotating machines (Eftekharnejad & Mba, 2009) (Mohamed Elforjani & Mba, 2009; M Elforjani & Mba, 2010). AE is generated when mechanical components of a machine are structurally damaged (Al-Dossary, Hamzah, & Mba, 2009; N Baydar & Ball, 2003; Raja Hamzah & Mba, 2009). A fault is detected (and may be identified) when the amplitude or spectrum of the AE signal change (Toutountzakis & Mba, 2003; Q. W. Wang, 2002). The major limitation of AE monitoring is that the signal-to-noise ratio is low, and often the level of the noise increases with time, which makes the signal difficult to interpret (Tan & Mba, 2005; Toutountzakis, Tan, & Mba, 2005).

### 2.4.1.6 Motor Current Analysis (MCSA)

MCSA is another monitoring technique used for machine monitoring. This technique has advantages over other monitoring techniques. MCSA is based on the principle that any torsional vibration in a system will affect the frequency spectrum of the current signature (Benbouzid, Vieira, & Theys, 1999; Kar & Mohanty, 2006).
2.4.1.7 **Vibration Monitoring**

Vibration technique is a powerful technique that can be used CM of all industrial machinery. During the machine’s operation it generates forces and motion that create structural vibrations which are transmitted from one part of the machine to another. The change in vibration signal can be used to detect and identify the occurrence of a fault. Vibration monitoring can be introduced to detect different types of faults such as faulty gears and bearings, misaligned components, eccentric shafts and incorrect clearance. The most used vibration-based techniques are time- and frequency-domain analyses, Cepstrum analysis and joint time-frequency domain analysis (S. S. Rao, 1995; Yesilyurt, 1997). The focus of this research project is to assess the diagnostic capability of vibration analysis for gearbox condition monitoring.

Unfortunately, most of the above techniques such as wear debris analysis, AE and MCSA are subject to limitations when used to detect a machine fault signal (Serridge 1989). Currently vibration signal analysis is the most popular method used for CM, because it has demonstrated an ability to detect faults at an early stage. Previous research studies have shown that vibration signal analysis using online CM can identify the existence of a fault in 60% to 70% of cases (Serridge 1989).

2.5 **Vibration Analysis for Gearbox Condition Monitoring**

The function of CM is to choose a set of measurable parameters indicating the health or condition of a machine. Once a change is detected it can help the maintenance engineer to make detailed analysis of the measured data to localise and determine the abnormalities present in a machine at an early stage before failure occurs (Davies.A, 1998).

Mechanical vibration measurement for fault detection is widely used for rotating machinery because the vibration signal reflects very accurately any change of machine parameters though the technique is usually limited to machine speeds greater than 600 RPM. The changes in signal will include changes in amplitude and spectrum and, in particular, the appearance of harmonics and sub-harmonics in response to modulation of machine speed (Davies.A, 1998).

As stated above, vibration analysis is an immensely useful tool for diagnostic applications and predictive maintenance for mechanical plant and equipment used in product manufacturing. Vibration analysis will provide the information necessary to determine whether the machine is in a good condition and operating efficiently. It can also be used for non-destructive testing and
improving the performance of plant. Vibration analysis has some limitations such as the relatively high cost of the sensors used to capture the signal from rotating machinery. Other limitations include the case where a microprocessor-based vibration monitoring analyser cannot accurately capture low frequency vibration data generated by low speed machinery, or when the database is not properly configured, which affects to the performance of the automated analysis and can give a faulty diagnosis (Mobley, 2002).

In the past two decades joint time-frequency signal processing has become the most powerful tool for analysis of the vibration signal generated by rotating machinery. This is now a common and important tool for condition monitoring (Al-Balushi & Samanta, 2002; N Baydar & Ball, 2003).

Various different approaches can be taken when using joint time-frequency analysis for signal processing. With CM of industrial machinery these commonly include the Short-Time Fourier Transform (STFT) (Naim Baydar & Ball, 2001; W. Wang & McFadden, 1993), Wigner-Ville Distribution (WVD) (Lee, Kim, & Kim, 2001; W. Staszewski, Worden, & Tomlinson, 1997; Tang, Liu, & Song, 2010), and wavelet transforms (WTs) (W. Staszewski & Tomlinson, 1994; J.-D. Wu & Chen, 2006; Zheng, Li, & Chen, 2002). However, WTs are generally considered a very useful technique for analysing the vibration signals due to factors other than the signal processing involved. WT is useful for non-stationery signals, and can decompose a vibration signal into various frequencies with several different resolutions, it can also display the features of a vibration signal in time-scale (frequency), detailing the vibration signals generating from, for example, a machinery gearbox (Jardine, Lin, & Banjervic, 2006; Lu, Li, Wu, & Yang, 2009; Z. Peng & Chu, 2004; Rubini & Meneghetti, 2001; C. Sung, Tai, & Chen, 2000).

Generally, gear defects are classified into three groups: gearbox operation defects (e.g. gear breakage, gear cracking), mechanical defects in gear mountings (e.g. incorrect gear clearance adjustment), and manufacturing production defects (e.g. error in the gear material selection , improper tooth profile geometry) (Fakhfakh et al., 2005; R. Randall, 1982). The worst operation defect is gear tooth breakage, which usually begins as a crack at the base of the tooth, and then progresses until the tooth fractures which can lead to failure of the gearbox transmission system. If the crack can be discovered early enough the gear can be replaced before the tooth breaks and a catastrophic failure occurs (Hongkai, Zhengjia, Chendon, & Peng, 2006; Lin & Zuo, 2003).
Considering gearboxes most researchers are focusing their attention on upgrading the monitoring techniques into powerful tools for early detection and diagnosis of gearbox failures in the primary stages (Ma & Li, 1995). Obviously, the gears are the vital components in gearboxes, and are subjected to high stresses over a range of operating conditions, which lead to such common failures as gear wear and breakage (Ma & Li, 1995). The development of these new analytical techniques has meant CM of gearboxes is has become an area of attention for research (Fakhfakh et al., 2005).

As a gear rotates it experiences varying torque and external force, in particular as the gear teeth mesh they produce a vibration signal at the a tooth meshing frequency. The frictional force generated as a result of gear tooth contact and their rubbing against each other generate an episodic random vibration signal (Martin, 1989). Obviously, the vibration signal amplitude depends on many factors such as gear geometry, frictional force, applied load on the gear, properties of the gear material including tooth stiffness, gear tooth profile, gear eccentricity, shaft clearance adjustment and alignment, and gear angle of rotation. However, there will be a vibration signal generated at the teeth meshing frequency which will be transmitted from the gears to the shaft, then to the bearings finally to the gear casing where is detected and captured by an accelerometer mounting on the casing. As the vibration travels outwards from its point of origin to the accelerometer, the signal will subject to interference and contamination by other vibration signals present in the environment around the gearbox. This is an important reason why the detected signal must be processed (Martin, 1989).

Most gearbox vibration signals contain non-linear features such as impulsive transients generating by two gear surfaces coming into contact with each other. There are three main elements to the signal; 1- a periodical element created by gear meshing, 2- a transient element which is the result of impact damage to the gear tooth or teeth, and 3- background noise (Stander et al., 2002; W. Wang & McFadden, 1995a).

Vibration analysis is a unique technique for gearbox CM performance activities; because any change in the gearbox condition can be lead to change in the vibration signature. Thus a gear fault can affect both amplitude and phase modulation of the gearbox vibration signal and conversely changes in the vibration signal signature is a possible fault indication (Bartelmus & Zimroz, 2009a, 2009b; Stander & Heyns, 2005; W. Wang & McFadden, 1995a). However, because the amplitude of the vibration signals at the primary stages of fault damage initiation is very small it will be much
less than the signals generated by other sources of vibration in the vicinity of the gearbox. Thus ever more powerful signal processing methods are required to achieve better analysis of the measured vibrational signal and more effective CM of gearboxes.

Elementary analysis of the vibration signal for gearbox CM uses time-domain analysis to obtain such statistical parameters as peak value, kurtosis, root mean square (RMS) value and crest factor to evaluate the condition of the gearbox (Lebold et al., 2000; W. Wang & McFadden, 1995a). Stevens et al., have demonstrated that statistical parameters are useful for detection of mechanical component faults (P. W. Stevens et al., 1996). The second important method used for detection of gearboxes faults is spectral analysis of the vibration signal, which contains much useful information. Each measurement point has to be at a specific location and orientation to correctly identify incipient problem (J. J. Zakrajsek, Townsend, & Decker, 1993).

The Fast Fourier Transform (FFT) signature is an accurate image of the mechanical motion of a machine-train in a specific direction at a specific point and time. However, one basic function of a gear is to change the speed of the primary driver. The basic parameters for the CM of gearboxes using vibration technique include gear-mesh frequency and running speed and bearing. However, gearbox CM requires that the profile of the frequency components for each gear should be monitored.

The fundamental gear-mesh frequency is equal to the number of teeth on the pinion or gear multiplied by the shaft rotation speed. In addition, each gear generates a series of modulations or sidebands that surround the basic gear-mesh frequency. The spectral contents of the measured sideband signal are much more useful than vibration amplitude for determining gear condition. Most gearboxes maintenance programs depend on frequency-domain vibration data. Therefore, the vibration signal is gathered in the time-domain and automatically converted to frequency-domain by a microprocessor-based analyser using the FFT (J. J. Zakrajsek et al., 1993). Randall claims to have found that there is enough information in the primary three gear meshing harmonics and their sidebands to identifying gear faults, and has reported that a change of amplitude of the sidebands is a good indication of the presence of a gear fault (R. Randall, 1982). The FFT is one of the most commonly used methods for signal processing frequency-domain signals for fault diagnosis, but it can be very difficult to extract useful information from it. However, FFT-based methods are not suitable for non-stationary signal analysis and are not able to give information on non-stationary
components. Because of this disadvantage of the FFT, it is necessary to find an alternative method for the analysis of non-stationary signals. It is for this reason that time-domain and frequency-domain analyses are not sufficient to detect fault symptoms in a helical gearbox at an early stage of development. For reliable diagnosis of non-stationary signals, an effective and powerful signal processing technique needs to be developed using the joint time-frequency domain, this could be the WT (R. Randall, 1982; W. Wang & McFadden, 1993)

2.6 Gear Overview, Types and their Operation

The main functions of gears in industry are power transmission and change of rotational speed (R. S. G. Khurmi, J.K., 2005). The construction of gearboxes consists of a set of gears mounted on shaft, which are for input and output speeds; supported by bearings. All these components are fitted and mounted within housing or gearbox casing which contains lubricant. The gearbox is driven by an electric motor via the gearbox input shaft which, mostly, rotates at high speeds. The speed is reduced and transmitted to the output shaft by the gear transmission inside the gearbox, as the speed is reduced the output torque will increase (R. S. G. Khurmi, J.K., 2005). The gears transmit the power by the teeth of one gear engaging with the teeth of another, the teeth of two gears mesh and rotate together, the one gear driving the other. The gear with the lower number of teeth is called the pinion; the gear with the higher number of teeth is called the gear. The output speed increases when the gear drives the pinion and is decreased when the pinion drives the gear. The speed reduction ratio of the arrangement is equal to the number of pinion teeth divided by the number of gear teeth. However, in industry, depending on the power to be transmitted and the direction in which it is to be transmitted, gears will vary in shape, size and arrangement (Shigley, 1989).

Spur and helical are the most common types of gears used in industry today (R. S. G. Khurmi, J.K., 2005). Other types, such as bevel and worm, are also in common use. But no matter what the shape and arrangement, all have the same function, which is power transmission and all have a driven wheel and driving wheel.
2.6.1 Spur Gears

Spur gears have straight cut teeth as shown in Figure 2.2. Spur gears are mostly used at moderate speeds, when the input and output shaft run parallel to each other, with no axial thrust. Spur gears are widely used in most machine application due to their low cost. The disadvantage of spur gear is the production of audible noise in high speed applications (Dudely, 1962; R. S. G. Khurmi, J.K., 2005).

![Spur Gear Image](StuffWorks, Monday 19 August 2013 17:30)

Figure 2-2 Spur Gear (StuffWorks, Monday 19 August 2013 17:30)
### Table 2.1 Spur gears Types and Functions

<table>
<thead>
<tr>
<th>Gear type</th>
<th>Subtype</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spur gear</td>
<td>Normal Spur Gears</td>
<td>• Generates less noise with optimum design</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Lubrication required for steel pinion but not for plastic</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• High efficiency</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Used for parallel shafts</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Easy to design and produce</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Single ratio of up to 1:10</td>
</tr>
<tr>
<td></td>
<td>Internal Spur Gears</td>
<td>• Has compact geometry profile</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Used in planetary gear production</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Has the same function as normal spur gears</td>
</tr>
</tbody>
</table>

#### 2.6.2 Helical Gears

Figure 2.3 shows a helical gear can have much the same general shape as a spur gear but with one very important difference. This is that the teeth on helical gears are cut at an angle to the face of the gear. When two teeth on a helical gear system engage, the contact starts at one end of the tooth and gradually spreads as the gear rotate until the two teeth are fully engaged (Dudely, 1962; R. S. G. Khurmi, J.K., 2005). Hence, the teeth enter the meshing zone smoothly and progressively making a helical gear less noisy with less vibration than a spur gear. Another advantage of helical gears is that during operation the helical gear teeth are not parallel to the shaft; hence the gear shaft will retain gear alignment and absorb any thrust load, a disadvantage is that there is a need for axial constraint bearings (Dudely, 1962; Shigley, 1989).

The helix angle creates an extended length of contact between the teeth (Frost, 2005) which means helical gear teeth are much longer than the teeth on a spur gear of equivalent width so the load is distributed over a greater tooth area so helical gears tend to wear less than spur gears. It also means a greater load can be transmitted using helical gears.
Table 2.2 Helical gears Types and Functions

<table>
<thead>
<tr>
<th>Gear type</th>
<th>Subtype</th>
<th>Remarks</th>
</tr>
</thead>
</table>
| Helical gears        | Single Helical Gears  | • Higher torque and life comparing to spur gears  
                       |                       | • Same properties as spur gears Can run at high speed with larger diameter  
                       |                       | • Smooth and quite running                                                  |
|                      | Double Helical Gears  | • Higher efficiency than single helical  
                       |                       | • Same performance as single helical but without producing side thrust     |
|                      | Crossed Helical gears | • Quiet driving  
                       |                       | • Shaft angle at 90°  
                       |                       | • Not easy to mount accurately                                             |

2.6.3 Bevel Gears

Bevel gears, Figure 2.4, tend to be used for power transmission between two mutually perpendicular shafts. The teeth of these gears are straight cut and each forms the base of a right circular cone. The teeth all point to the apex of the cone of which they form the base. The two shafts “intersect” at the apex of the cones. However, bevel gears cannot be used for parallel shafts and can be noisy at high speeds (Dudely, 1962; Shigley, 1989).
2.6.4 Worm Gears

Figure 2.5 shows that worm gears are gears that resemble screws and can be used to drive spur, helical or bevel gears. Worm gears can be used for large loads, and to transfer rotational movement to translation movement. Worm gears can be used for large pitch diameters and mesh two mutually perpendicular, non-intersecting shafts. An important feature of worm gears meshes is that when a worm gear is turned, the meshing spur will turn, but turning the spur will not turn the worm gear (Dudely, 1962; Shigley, 1989).
2.6.5 Gear Failures

Gear failures can be due to many different causes including maintenance errors (e.g. lack of lubrication, incorrect re-assembly after maintenance) or manufacturing errors (e.g. inadequate strength of materials used in the construction of the gears) (R. S. G. Khurmi, J.K. , 2005).

The two important classifications of gear failures are distributed and local. A distributed gear fault could be corrosion affecting a number of gears or gear teeth. Local faults are more dangerous because, although they start small they can increase rapidly, and usually have a dramatic and adverse effect on power transmission. Pitting, scoring and tooth breakage are the most important local faults (Merritt, 1954). It is important to detect this kind of fault at as early a stage as possible.

However, failure may be caused by unexpected transmission system interfaces or by operation aspect and failures in the gear itself.

During its operational life the movement (rotation) of the gear itself will generate wear, see Figure 2.6 which shows the possible wear action which may create gear failure, particularly if the gear movement includes sliding movement above or below the gear pitch line.

![Gear Pitch Line](image)

**Figure 2-6 Gear Pitch Line (M. N. Regeai, 2007)**

Gears faults can be classified into four groups as follows:

- Gears wear failures.
- Tooth breakage failures.
- Gears surface fatigue failures.
- Plastic flow failures.

Each group is described according to the physical nature of the fault and its causes (Boyer, 1975; Smith 2003 ).
2.7 Gear Wear Failures

Wear of gears may be considered as the unwanted removal of surface material. This kind of failure usually occurs as a result of gear teeth surfaces coming into direct contact with one another; causing removal or shedding of surface material (Smith 2003). Obviously, wear is of many kinds, the most important of which are abrasive wear, adhesive wear, scoring and corrosive wear. The effects will depend upon the gear’s operating condition. All wear damages the surface of the gear which reduces the life of the gear. Wear can be accelerated by, for example, having an inadequate supply of lubricant and such excessive wear will generate noise during operation, may cause pitting or even gear tooth breakage and reduce the expected working life of the gear (R. Khurmi & Gupta, 2005). Correct lubrication is essential for ensuring maximum life of gears. Of course changes in operating conditions, such as increasing rotation speed or load, will give a corresponding increase in lubricant temperature, but an elevated oil temperature is considered an important indicator that there is a possible problem with the lubrication system. Not only is the elevated temperature an indication of a possible problem but any noticeable temperature increase will reduce lubricant viscosity which will increase the likelihood of e.g. direct metal-to-metal contact and gear teeth breakage.

2.7.1 Corrosive Wear

Corrosion is a defect that occurs when gears operate within an adverse chemical environment, such as contaminated lubricant, such as when acid attacks damage the surface of the gear teeth, see Figure 2-7. Rust is a common form of chemical attack and occurs when moisture (water) enters the bearing and the lubricant fails to provide an adequate protection for the surfaces of the bearings. The rust “eats” into the surface very quickly and causes pitting. Corrosion is usually accompanied by an increase in vibration and rapid wear (R. S. G. Khurmi, J.K., 2005).

Figure 2-7 Photo of Gear Corrosive Wear (Inlines, 21 August 2013 16:00 PM)
2.7.2 Adhesive Wear

Adhesive wear occurs where the force of one gear tooth on another is sufficient to cause a weld at the contact point. As the gears rotate and the weld breaks, metal particles are released from the gear surfaces (see Figure 2.8) which contaminate the system (W. Wang & McFadden, 1995a).

![Photo of Gear Adhesive Wear](image)

Figure 2-8 Photo of Gear Adhesive Wear (Lubrication, 21 August 2013 17:00 PM)

2.7.3 Abrasive Wear

Abrasive wear occurs as a result of the existence of foreign particles in the gear lubricant, these may be metal particles shed due to wear, or sand entering through worn or damages seals. Such particles cause excessive wear (R. Khurmi & Gupta, 2005).

![Photo of Gear Abrasive Wear](image)

Figure 2-9 Photo of Gear Abrasive Wear (Lubrication, 21 August 2013 17:00 PM)

2.7.4 Scoring

Scoring (damage to gear tooth surface) occurs as a result of incorrect gear lubrication which can be exacerbated by overloading and gear misalignment. As scoring progresses gear teeth may become distorted or even break, which will affect gear life and reduce power transmission performance (R. Khurmi & Gupta, 2005).
2.7.5 Tooth Breakage Failure

Tooth breakage is considered the most dangerous gear fault because pieces of fractured tooth can cause additional serious problems with other gearbox components such as bearings, shafts, etc. and can lead to failure of the power transmission system. Tooth breakage invariably starts with a small crack which progresses across the entire tooth (Dudely, 1962). This research concentrates on the incremental progress of the fault across a gear tooth.

2.7.6 Gears Surface Fatigue Failures

This type of fault generally occurs as result of poor quality lubricant or lack of lubrication, though another important cause of fatigue failure is repeated over-stressing of the gears. This gear fault it is classified by the degree of pitting and spalling to the surface of the gear’s teeth (R. Khurmi & Gupta, 2005).
2.7.7 **Pitting**

Pitting occurs when the gears are subject to contact stresses greater than the allowable limit under varying load. Initially, most pitting occurs in small areas on the tooth surface and then progresses rapidly and leads to gear damage during normal operating conditions (R. Khurmi & Gupta, 2005).

![Figure 2-12 Photo of Gear Pitting (Services, 21 August 2013 20:00 PM )](image)

2.7.7.1 **Plastic Flow Failures**

Plastic flow is a defect that is linked with the material properties of the gear tooth surface. Any metal subjected to a sufficient load will deform, and gear teeth are no exception. The surface of the tooth is deformed as a result of the tooth yielding to an extreme load, greater than the yielding stress of the gear. Plastic flow failure is a property of the hardness of the metal used for the gears. Plastic flow of the gear tooth surface invariably leads to catastrophic failure.

There are three main types of gear plastic flow defects; ridging, rippling and rolling, and peening (R. Khurmi & Gupta, 2005).

2.8 **Failure Types: Statistical Survey**

Figure 2.13 presents the results of a survey by gear manufacturers which classified failures in gears into four categories. It is concluded that the largest proportion gear failures (61.2%) in industrial power transmission systems are due to breakages.
2.9 Gear Vibration Features

The more important component of the vibration signal produced by a multi-stage gearbox is the periodic signal due to tooth meshing (R. Randall, 1982). However, in a real gearbox the teeth are not perfectly rigid, which leads to small variations in the force transmitted between the teeth, and consequently small variations in the speed of rotation. In addition the profiles of the teeth will not be perfect which means that the point of application of the forces between the teeth will not follow the gear pitch line shown in Figure 2.6, but will deviate slightly. The effects of these imperfections on the vibration signal will include:

- A periodic signal at the tooth contact meshing frequency due to deviation from the ideal tooth profile,
- Amplitude modulation of the signal due to variation in tooth loading,
- Frequency modulation due to changes in speed of rotational (there will also be frequency modulation due to geometric factors such as imperfections in tooth spacing),
- Impulsive components due to local tooth defects (R. Randall, 1982).

2.9.1 Deviation from Ideal Tooth Conditions

This section considers in more detail the results of non-ideal behaviour of the gear teeth. These considerations include deflection of the gear tooth due to variation in load, wear and geometric errors which are the result of the manufacturing process (R. Randall, 1982).
2.9.2 **Tooth Deflection under Load**

As a result of tooth deflection under load the vibration signal waveform will have a stepped nature based on the periodic signal varying due the load affecting different numbers of teeth. This is a cyclic process with a periodicity of its own which depends on the geometry of the gears and thus will produce peaks in the vibration signal not only at the tooth meshing frequency but its harmonics. These general characteristic of the periodic signal are present for different kinds of gears such as worm gears, spur gears and helical gears but the specifics will depend on the gear type. It has been found that load fluctuations both below and above the design limit load will be produce a much larger vibration signal than that produced at the design load (R. Randall, 1982).

2.9.3 **Uniform Wear**

The sliding motion between gear teeth in contact causes wear along the line of the pitch circle. This wear will alter the gear profile. The best that can be hoped is that the wear is the same for all teeth, in which case the vibration signal will contain added components which are harmonics of the tooth meshing frequency. Because the wear will, initially, be minimal these added vibration components will not be noticed until the wear has advanced significantly. Care must be taken to differentiate the changes in the vibration signal due to uniform wear and tooth deflection. It has been suggested that wear will produce more energy in the higher harmonics than does tooth deflection. This effect will become more pronounced the greater the wear (R. Randall, 1982).

2.9.4 **Manufacturing Errors**

Manufacturing errors usually occur during the machining process which inevitably introduces minute profile errors on the gear teeth. It is usually assumed that these errors are the same for all teeth and introduce additional vibrations at the tooth meshing frequency and its harmonics. However, if it also assumed that there are additional random errors introduced into the tooth profiles, such errors will generate a series of small impulses which vary from tooth to tooth. However, each tooth will engage once per rotation so these impulses will have a periodicity imposed on them. An impulse has a wide frequency bandwidth, so the random errors on the tooth profiles will generate a low amplitude vibration signal with a frequency bandwidth ranging over a number of harmonics of the gear shaft rotation frequency. It would be expected that with time these random fluctuations in tooth profile would wear away and hence the signal they produced would
become smaller. Typically the amplitude of the vibration signal due to the random errors in the tooth profile is much less than that due to tooth deflection (R. Randall, 1982).

2.9.5 **Ghost Components**

Ghost components refer to faults introduced into gear transmission systems due to manufacturing errors in the production machining processes. These errors generate a vibration signal at a ghost component frequency and its harmonics. As a fixed geometric error the ghost component should not depend on the load. If necessary, comparison of the vibration signal obtained under different loads should enable identification of ghost components. However, it would be expected that the vibration signal for a ghost component would reduce with wear of the gear transmission system (R. Randall, 1982).

2.10 **Modulation Effects**

2.10.1 **Amplitude Modulation**

The amplitude of the vibration signal generated by gear meshing is affected by load. Different defects can generate an amplitude modulated vibration signal; these faults can be classified into two categories; ‘localized’ and ‘distributed’. A local fault could be pitting on a gear tooth at the pitch line. Such a fault would, once per revolution, modulate the amplitude of the vibration signal. A distributed fault would be one that created a nonstop continuous modulation of the vibration signal at a frequency corresponding to the rotational speed of the gear transmission system, e.g. an example of a distributed fault is gear eccentricity (R. Randall, 1982; R. B. Randall, 1987).

2.10.2 **Frequency Modulation**

Frequency modulation of the tooth meshing frequency will occur due to variation of both the gear tooth profile and speed of rotation. Variations in rate of gear tooth contact will cause modulation of the amplitude of the vibration signal, and changes in the applied torque will give a periodic variation in the angular speed. The greater the inertia of the rotating components (greater mass) the greater the amplitude modulation compared to the frequency modulation (R. Randall, 1982; R. B. Randall, 1987).
2.10.3 Additive Impulses

Regarding amplitude and frequency modulation; the majority of localized faults associated with tooth meshing will be impulsive, of short time duration, produce wide band signals creating additive impulses. These are so-called because, unlike modulation effects which are symmetrical about the zero line, an additive impulse moves the vibration signal so it is no longer symmetrical about the zero line.

It is common for the periodic impulses associated with local tooth faults to excite system resonances, amplifying component in the signal spectrum near the resonant frequencies (R. Randall, 1982; R. B. Randall, 1987).

2.11 Vibration Based Gear Fault Detection and Diagnosis Techniques

The use of vibration analysis is an important tool in predictive maintenance for diagnostic application. There are different options for the gathering of vibration data for machine trains and many techniques used to process the vibration signals that can be used for gear fault detection and diagnosis. Here, we consider the three basic data-type classifications; time-domain, frequency-domain and joint time-frequency-domain analysis (Mobley, 2002). The success of any CM programmer depends largely on the accuracy of the measuring equipment, measurement techniques and data handling. With a sensor that has an appropriate frequency and dynamic range, is properly calibrated, whose presence does not affect the system, whose mounting does not limit its frequency and dynamic ranges, and when all measurements are made at the same load and always collected at the same locations, then the measurements may be considered reliable, comparable and accurate (Alguindigue, 1993). In particular, the failure diagnosis is influenced by the signal processing techniques used. The vibration signals collected by sensors will be a mixture of signals different sources; they will be non-stationary and polluted by background noise. The processing of the vibration signal is pivotal to extracting information relevant to detecting and identifying a component fault. The time-domain, frequency-domain and joint time-frequency–domain techniques are reviewed below (Norton, 2003).

2.11.1 Time–Domain Analysis

Time-domain vibration signal analysis is a traditional, widely used technique for gearbox fault detection. It is a relatively low-cost and easy to use and understand method that extracts information
from the gearbox vibration signal in the time-domain to detect and identify gearbox faults (P. Stevens, Hall, D., Smith, E., 1996 ). The time-domain technique is most reliable when it contains sharp peaks generated by periodic impulses (P. Stevens, Hall, D., Smith, E., 1996 ). Changes in the vibration spectrum from the baseline signature can be used to detect the presence and source of faults.

Fault detection using the time-domain signal is based on statistical analysis. The most commonly used statistical measures are peak value, root mean square (RMS), crest factor (CF) and kurtosis (K) (D.G, 2003).

However, here an accelerometer converts the mechanical vibration of a gearbox surface into an electrical signal on which the analysis is performed. The raw signal will require conditioning (e.g. amplification, filtering, and impedance matching) to make it suitable for further processing and these functions will usually be performed by the pre-amplifier of the accelerometer. It is to be expected that the measured levels of vibration for a damaged gear will tend to be greater than the values for a normal gear. Thus, comparison of peak and RMS values of the vibration signals for a healthy gear and the actual gear will allow the presence and severity of a defect to be detected.

In the early stages of gear damage, the signal will be of a more impulsive nature and kurtosis and crest factor which measure the “spikiness” of a signal may be of use in detecting such faults. However, as the damage increases, the vibration becomes less impulsive and less spikey and more random so the values for crest factor and kurtosis fall to more typical levels. Kurtosis and crest factor lack the ability to detect gear defects in their later stages.

2.11.1.1 The Root Mean Square (RMS)

The root mean square (RMS) is defined to be the square root of the average of the sum of the squares of N samples of a signal, of course the larger the value of N the more accurate the RMS. Peak value and RMS value for sinusoidal signals are related. The RMS value is approximately 0.707 x Peak value.

RMS is a useful aspect of the vibration signal and has often been used for fault detection, based on the observation that an increase in the RMS value of the gear vibration signal can be due to the development of a fault. RMS value of the amplitude of the acceleration from a given signal is expressed, as (D.G, 2003) :
\[ \text{RMS}_x = \sqrt{\frac{1}{N} \sum_{n=1}^{N} (x(n) - \bar{x})^2} \]  
\[ \bar{x} = \frac{1}{N} \sum_{n=1}^{N} x(n) \]
\[ \bar{x} = \frac{1}{N} \sum_{n=1}^{N} x(n) \]

Where: \( N \) is the number of samples taken, \( x(n) \) is the amplitude of the \( n \)th sample, and \( \bar{x} \) is the mean value of the \( N \) samples.

RMS is considered a reliable measure for monitoring the vibration of a gearbox. Its effectiveness is best when applied to wide-band vibration, and it calculates an average which means it is less sensitive to incidental vibration impulses (D.G. 2003).

### 2.11.1.2 The Crest Factor

The Crest factor (CF) is calculated by dividing the maximum positive peak value of a signal by its RMS value. The CF is typically used on the raw vibration signal. For typical operation, the CF are likely to be between 2 and 6, with values above 6 associated with machinery problems (Swansson, 1980).

\[ \text{CF} = \frac{\text{Peak.value}}{\text{RMS}_x} = \frac{\text{sup} \left| x(n) \right|}{\sqrt{\frac{1}{N} \sum_{n=1}^{N} [x(n)]^2}} \]  
\[ \text{sup} \left| x(n) \right| \]

Where: \( N \) is the number of samples taken, \( x(n) \) is the amplitude of the \( n \)th sample, and \( \text{sup} \left| x(n) \right| \) is the maximum absolute value of the signal (Swansson, 1980).

### 2.11.1.3 Kurtosis

Kurtosis (K) is a statistical calculation which gives a measure of the number and amplitudes of the peaks in a given signal (Stewart, 1982). K is the normalized fourth moment of the signal and is commonly termed a measure of the “peakedness” of the signal, that is a signal that has more and sharper peaks will have a larger K value (Stewart, 1982).
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\[ kurtosis = \frac{1}{N} \sum_{n=1}^{N} \left( x(n) - \bar{x} \right)^4 \left( \frac{1}{N} \sum_{n=1}^{N} \left( x(n) - \bar{x} \right)^2 \right)^2 \]  

\[ (2.3) \]

2.11.1.4 Time Synchronous Averaging (TSA)

The main purpose of this technique in the processing of vibration signals is to remove noisy random background contamination of non-stationery signals which is not synchronous with the gear signal (Eric B., 2009; Liu, 2000). Gearbox operation is, essentially, a periodical rotational motion. TSA divides the vibration signal into contiguous segments of the same length, usually the period of the shaft rotation, and by averaging these segments, the vibration signal synchronous with periodical shaft movement will be enhanced and random non-synchronous elements will tend to cancel out (P. D. McFadden, 1987).

Gear vibration is mostly due to the meshing of the gear teeth, and the vibration signal for each complete revolution represents the features in the time–domain (Ahamed, Pandya, & Parey, 2014; McFadden P.D., 1989). TSA is used most advantageously where there is a continuous deterministic element in the signal. Usually the gearbox is generating a vibration signal under different operating condition such as variable applied load and constant speed which produces tiny frequency variations(Ahamed et al., 2014).

2.11.2 Frequency Domain Analysis

Although methods for analytically converting time-domain data to the frequency-domain have been known for a long time it was not until the microprocessor became widely available that the technique came into common usage. The technique is based on the Fast Fourier Transform (FFT) first developed by Gabor, D in 1946, (Gabor, 1946) to transforming the time–domain signal into its frequency-domain (Mobley, 2002). The frequency domain, obtained for each measurement point on the gearbox will contain much useful information, and gearbox CM will be enhanced if the frequency spectrum is monitored. The measurement points must be at specific locations and orientations to correctly identify incipient problems (Mobley, 2002). The FFT signature is a measure of the mechanical motion of a machine-train in specific direction at a specific point and time (J. J. Zakrajsek, Townsend D.P. and Decker H. J. , 1993). The fundamental gear-mesh frequency will be accompanied by a series of sidebands (modulation), located on both sides of the
gear tooth meshing frequency and its harmonics, and separated by integer multiples of the gear rotation frequency. It has been found that changes in the number and amplitudes of these sidebands can be used to identify defects such as eccentricity, misalignment, fatigue cracks, or local tooth damage, and are considered more useful in determining gear condition than vibration amplitude (R.B., 1982). Most modern gearboxes maintenance programmers depend on frequency-domain vibration data.

It is claimed that frequency domain data is often more reliable and easier to understand than time-domain data and better at identifying incipient problems within the machine to provide a complete diagnostic picture (J. J. Zakrajsek, Townsend D.P. and Decker H. J., 1993).

Here, the signal is gathered in the time-domain, and then automatically transformed to the frequency-domain using the Fast Fourier Transform (FFT)(J. J. Zakrajsek et al., 1993).

2.11.3 The Joint Time-Frequency (Wavelet Transform) Analysis

Signal analysis is the most important method used for condition monitoring and fault diagnosis (Davies.A, 1998). Identifying features within the vibration signal is very important for dependable, regular CM of machines. Essentially, signals collected from machines are classified into one of two categories, stationary or non-stationary signals (Julius, 1986, ). For stationary processes, the well-known FFT technique is used to transform the time-domain signal into its frequency-domain. But the weakness of the FFT method is that the signal should be stationary with the statistical parameters time–invariant.

However, signals from non-stationary processes have statistical parameters which vary with time so, strictly, FFT should not be used to transform these into the frequency domain for feature analysis (Julius, 1986, ). In the industrial field, rotating machinery tends to produce non-stationary vibration signals created by frictional forces and impulsive interactions within gears and bearings (S. Gu, Ni, J., and Yuan, J., 2002, ). However, these signals contain significant information which can be extracted for fault diagnosis. Because the FFT is unsuitable for the analysis of non-stationary signals new time-frequency methods have been developed. These include the wavelet transform (WT); a new and powerful tool in CM that can be used for analysis of non-stationary vibration signals and on-line fault detection.

The most important characteristics of the WT compared to the FFT are two-dimensional frequency-scale and time-space. The WT can also provide frequency-time or scale-time localization
Vibration signal classification requires windowing of the time series vibration signals to create signals on which linear or quadratic transformation can be applied. The difference between the two methods is that the linear transform can be inverted to reconstruct the time signal. For this reason the linear transform is useful for signal processing like noise reduction and time-varying filtering. The quadratic method describes the energy distribution of a signal in the joint time-frequency domain, which is useful for analyses, to classify and to detect features of the signal. However, quadratic time-frequency analysis loses the phase information so the signal cannot be reconstructed.

Several time-frequency techniques have been developed for analyzing non-stationary signals. Amongst these the Short-Time Fourier Transform (STFT), the Wavelet Transform (WT) and the Wigner-Ville Distribution (WVD) are widely used. For complex classes of rotating machines where the S/N ratio is low and a large number of frequency components are present, these techniques have the potential for easier detection and diagnosis of gear faults (L Cohen, 1989; Hubbard, 1998; Ville, 1948).
CHAPTER THREE

EXPERIMENTAL PROCEDURE AND FAULT SIMULATION

This chapter starts by describing the test rig, experimental procedure, the equipment and instrumentation used to produce and collect the vibration signals. The test rig platform has been designed and built to simulate a real fault in a gearbox and its detection.

Vibration signals were collected from the gearbox under healthy conditions and with a seeded fault of increasing magnitude. The instrumentation and methods used to collect the gearbox vibration measurements are described.
3.1 Introduction

The test rig platform consisted of a two stage gearbox as shown in Figures 3.1 and 3.2. It was designed and built to simulate a real helical gearbox system commonly used in industry with the added condition of easy seeding of simulated gear faults. In addition, the machine condition monitoring methods are used to collect and analyse the gearbox vibration measurements. This chapter describes the procedures were carried out to investigate the vibration signals collecting from the gearbox under healthy and faulty gears. It starts by illustrating the experimental procedure and the equipment’s and instrumentation used to collect the vibration signals. However, the signals collected from gearbox can be non-stationary and nonlinear in nature. They are processed using traditional method joint time-frequency domain. This experiment work was done on a gearbox platform test rig established and developed at the University of Huddersfield. However, the selection of this test rig experiment for this research work activity is based on many reasons such as it is commonly used in industry field, and it’s more easily allowing for gear faults simulate and different CM techniques to be widely estimated. The gearbox test as shown in Figure (3.1) consists of a two stage helical gearbox, 3-Phase induction Motor, measurement instruments, shaft, flexible coupling and a load device.

3.2 Test Facilities

3.2.1 Test Rig Description and Components

The gearbox test rig used in this study consists of a three phase induction motor (11kW, 1465rpm and four poles) produced by the Electro-drive Company, which is rigidly connected by a short input shaft to the first pinion of a two stage helical gearbox manufactured by David Brown Radicon Limited. The two stage helical gearbox has a contact ratio of 1.45 contact ratios is used in the test with the driving gear having 58 teeth, and the driven gear 47 teeth.

The motor speed is controlled by a speed controller with the maximum speed of 1465 rpm Vibration of the test gearbox is measured by an accelerometer (Type PCB 338C04) with sensitivity of 100 mv/g, and frequency response range from 1 Hz to 20 kHz. It was mounted on the gearbox housing casing. An incremental optical encoder able to measure instantaneous angular speed (IAS) over the range of 10 Hz to 2 kHz was installed to the end of the induction motor shaft. The encoder provided one pulse for each complete revolution and was connected to a data acquisition system directly.
A schematic diagram of the test rig is shown in Figure 3.2 and generator specifications are given in Table 3.1. Figure 3.2 presents the main features of the construction of the rig. Mechanically, the output shaft from the motor is directly connected to the gearbox input shaft via a cantilever type coupling arrangement (see Figure 3-3) and drives a load system consisting of two flexible couplings and a DC generator. The gearbox and AC motor casings are directly bolted together as shown in Figures 3.2 and 3.3 shown a photo of gearbox and Ac motor cantilever arrangement. The gears inside the box are easily accessible by unbolting both gearbox and AC motor flanges and it is a relatively easy matter to replace a gear set inside the box.

The shaft from the gearbox is supported by two ball bearings and transmits the drive power to a DC generator. The electricity generated is received by the resistor bank. Hence, a different load can be applied to the test rig system by changing the resistance value. Figure 3.4 shows the schematic diagram of the two-stage helical gearbox and Figure 3.5 is a photograph of an exterior of two-stage helical gearbox showing accelerometer.

Figure 3-1 Photo of the experimental gearbox test rig
Figure 3-2 Test rig schematic diagram illustrating the location of accelerometer and encoder

Figure 3-3 Photo of gearbox and AC motor cantilever arrangement
Figure 3-4 Two – stage helical gearbox schematic

Figure 3-5 Photo of exterior of two –stage helical gearbox showing accelerometer
### Table 3.1 Specifications of two-stage helical gearbox

<table>
<thead>
<tr>
<th>Description</th>
<th>1st stage</th>
<th>2nd Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teeth number</td>
<td>$\text{PG0740.8/M07E}$</td>
<td>$\text{M07-24.5B-C}$</td>
</tr>
<tr>
<td>Speeds of Shaft</td>
<td>$\omega_{r1}$</td>
<td>$\omega_{r2}$</td>
</tr>
<tr>
<td></td>
<td>24.42 Hz (input) at full load</td>
<td>6.64 Hz (output) at full load</td>
</tr>
<tr>
<td>Meshing frequency</td>
<td>$f_{mesh} = n_p \times \omega_{r1}$</td>
<td>$f_{mesh} = N_g \times \omega_{r1}$</td>
</tr>
<tr>
<td></td>
<td>1416.36 Hz</td>
<td>391.76 Hz</td>
</tr>
<tr>
<td>Contact ratio</td>
<td>1.450</td>
<td>1.469</td>
</tr>
<tr>
<td>Reduction ratio</td>
<td>$\frac{\omega_{r1}}{\omega_{r2}} = \left(\frac{n_p}{n_g}\right) \left(\frac{N_p}{N_g}\right)$</td>
<td>= 0.272</td>
</tr>
</tbody>
</table>

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Figure 3.6 is a photo of the AC motor; Table 3.2 shows the manufacture’s design specifications. AC motor parameters such as number of pole pair and slip are adjusted during manufacture. However, AC motor speeds are controlled by varying the supply frequency ($f_{sup}$) which is how experimental test rig speed was adjusted.

**Figure 3-6 Photo of the three-phase 11Kw induction motor**

**Table 3.2 Induction motor specifications**

<table>
<thead>
<tr>
<th>Classification</th>
<th>Induction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase No.</td>
<td>3</td>
</tr>
<tr>
<td>Poles No.</td>
<td>2-Pair</td>
</tr>
<tr>
<td>Rated power</td>
<td>11 kW</td>
</tr>
<tr>
<td>Rated voltage</td>
<td>415 V</td>
</tr>
<tr>
<td>Current</td>
<td>22A @Full Load</td>
</tr>
<tr>
<td>Rated speed</td>
<td>1465 rpm @full Load</td>
</tr>
<tr>
<td>Winding</td>
<td>Y Star To Δ delta</td>
</tr>
<tr>
<td>Stator slots Number</td>
<td>48</td>
</tr>
<tr>
<td>Rotor slots Number</td>
<td>40</td>
</tr>
</tbody>
</table>
Figure 3.7 is a photograph of the DC generator used in this test rig system to generate electricity and acts as a load on the gear transmission system. It converts mechanical energy into electricity, which is dissipated in the resistor bank. Table 3.3 gives the manufacturer’s specification for the DC generator used in this test rig system.

![DC Generator](image)

**Figure 3-7 Photo of DC-generator (mechanical load)**

**Table 3.3 DC-generator, manufacturer’s specification**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>G63801N</td>
</tr>
<tr>
<td>Size</td>
<td>SD</td>
</tr>
<tr>
<td></td>
<td>200XLC</td>
</tr>
<tr>
<td>Power</td>
<td>85 kW</td>
</tr>
<tr>
<td>Speed</td>
<td>1750 rpm</td>
</tr>
<tr>
<td>Duty type</td>
<td>S1</td>
</tr>
<tr>
<td>Ins Class</td>
<td>F</td>
</tr>
<tr>
<td>Mass</td>
<td>482 Kg</td>
</tr>
</tbody>
</table>
The DC generator consists of two parts: a stationery part (stator) and a rotating part (rotor or armature) as shown in Figure 3.8. Due to an applied field current the windings on the stator generate a magnetic flux and as the torque applied via the shaft from the gearbox rotates the armature a current is induced. Increasing either the field current or the speed of rotation causes an increase in DC output current.

![Cross-section illustrating the DC motor field/armature circuit; Re-drawn by ML(M. N. Regeai, 2007)](image)

**Figure 3-8 Cross-section illustrating the DC motor field/armature circuit; Re-drawn by ML(M. N. Regeai, 2007)**

### 3.2.2 Test Rig Experiment System Electrical Control panel

Speed and load were the main test parameters. The test rig control panel used to control the motor speed in percentage setting from 0% to 100% The DC motor load was also changed in percentage settings from 0% to 100%. Tests to detect and diagnose the fault introduced into the gearbox were carried out at constant rotational speed and the load varied. Figure 3.9 shows the front view of test rig panel of the following control elements and their functions.
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Figure 3-9 Test rig speed and load control panel

**Armature current:** This shows the DC motor armature current which ranges from 0 to 200A maximum. This is a measure of the output current of the DC transformer.

**Load control:** This is used to adjust and control the DC motor field current between 0% and 100%. This potentiometer is installed directly into the DC field controller.

**Field current:** Shows the DC motor current range from 0% to 50% of its maximum value this signal is received from the buffered output DC field controller.

**Switch on/off speed:** Are push buttons used to start and stop the AC drive; the green push button is to start and the red push button is to stop.

**Local control keys:** These are on a keypad mounted on the front of the control panel of the test rig and are connected directly to the inverter drive using a serial communication.

**Programming keys:** The keypad is used to increase or decrease the motor speed.
3.2.2.1 Speed Control

Figure 3.10 shows the main features of the speed control system, the main three-phase supply (415 VAC) is directly linked to the inverter which controls the speed of the AC motor. The speed set point generates the required supply voltage to the AC motor to allow the inverter to modify the supply frequency. This combination operates the AC motor and system at variable speeds.

![Experimental test rig electrical block diagram](image)

**Figure 3-10 Experimental test rig electrical block diagram**

3.2.2.2 Load Control

The DC generator motor provides the load for the AC motor. The shaft drive speed controls the generation of the DC motor field. The DC motor is connected to a two line supply phase, a potentiometer is configured to supply a current pre-set to vary the load on the AC motor (and hence the gearbox) over the range from 0% to 100% of the full load.

3.2.3 Data Acquisition System (DQS)

The data acquisition system (DQS) is a crucial part of the instrumentation used on the test rig system. The DQS was designed to monitor such different parameters such as vibration, temperature and sound, and the analog data produced by the sensors was stored as digital data. The DQS was used to record the measured data at a sample rate of 500k samples/s.

A DAQ consists of two parts: hardware and software. The software included most basic data analysis tools such as spectrum analysis. Other facilities are able to access and analyses the stored data for the purpose of online condition monitoring and data investigation. The hardware consisted of data...
storage space, a DQS card and PC computer with control software. The hardware also consisted of
an accelerometer connected to a charge amplifier, then to the DQS and then to the PC. The main
function of amplifier is amplifying the vibration signal because it is often very weak.

The data acquisition system used a type PD2-MF-16-500/16L. This has five channels used to
measure the accelerometer signals obtained from gearbox, motor and speed encoder. The
accelerometers were fitted directly onto the test rig and each accelerometer was connected to the
DQS by coaxial BNC cable to reduce signal noise. Usually piezo-electric accelerometers produce a
voltage output proportional to the amplitude of the vibration signal. The DQS received all the
signals from the accelerometers and transmitted them to the PC. The data acquisition control
program was developed in LabView. The important DQS specifications are presented in Table 3.4.

<table>
<thead>
<tr>
<th>Table 3.4 Data acquisition system specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>No. of Channels</td>
</tr>
<tr>
<td>resolution</td>
</tr>
<tr>
<td>Sampling rate (maximum)</td>
</tr>
<tr>
<td>Max working voltage</td>
</tr>
</tbody>
</table>

The shaft speed and angular speed from the test rig were connected to channels one and two of the
DQS. The third and fourth channels were for vibration signals from different locations on the
gearbox. The fifth channel was for the current signal from the motor. See Figure 3.11 for the signal
measurement schematic diagram.

The DQS was connected to a software package allowing the display of virtual instruments on the
computer screen, see Figure 3.12. The software also allowed the display of operating conditions.
3.2.4 Instrumentation

3.2.4.1 Accelerometers

In this work the vibration signal was captured using a piezo-electric accelerometer. This is considered the most important sensor and had to be capable of measuring both low and high frequencies from the different source generated within the system, such as shaft rotation and gear
meshing. It must also have a high sensitivity to detect low amplitude signals generated from a gear fault in its initial stages. The piezo-electric accelerometer is constructed from three main elements; a piezo-electric material, a seismic mass and the base. The piezo-electric crystal functions as a spring between the accelerometer base and seismic mass. When a vibration force acts on an accelerometer, the seismic mass applies a force on the piezo-electric material which, according to Newton’s second law, is equal to the product of its mass and acceleration (F=ma). This force acting on piezo-electric material produces a charge which is proportional to the acceleration. This charge will be detected by either an internal or external charge amplifier and amplified so that it is suitable for signal recording. The use of an internal charge amplifier (compared with a transducer using an external charge amplifier) will allow for a longer distance of data transfer. Here one accelerometer model PCB (336C04) were installed to measure the dynamic excitation. The accelerometer frequency range is 1Hz to 20 kHz with sensitivity of 10 mV/g and temperature limit about 90°C. One was fixed directly on the side casing of the gearbox using a screw-thread brass base, with the accelerometer bonded to the base with ceramic cement, which has the advantage of helping to avoid accelerometer overheating. Figure 3.13 shows a photo of the accelerometers used in this test work, its specifications are summarized in Appendix (E).

3.2.4.2 Shaft Encoder

The test rig was fitted with an incremental optical encoder type RI32 to measure instantaneous angular speed (IAS) over the range of 10 Hz to 2 kHz. It was installed at the end of the induction motor shaft, see Figure 3.14. The encoder has 360 opaque segments equally spaced around its perimeter to form an encoder of sufficient accuracy for small changes (1°) in shaft position to be recorded. No amplification of measured IAS signal was required. The encoder was directly connected to the computer via the DQS system, to produce a one–pulse per revolution synchronizing pulse which was applied to trigger the beginning of data capture. This guaranteed
that signal averaging is synchronized with the shaft position so that the location of the broken tooth can be defined (M.N. Regeai, 2007).

![Figure 3-14 Motor encoder in its mounted position](image)

3.3 Experimental Test Procedure (Variable Load at Full Speed)

Previous research studies investigating the CM of gearboxes did not consider that gearboxes regularly operate under variable condition such as different loads and speeds. Instead most studies into gearbox CM were for constant loads and rotation speeds. Here, in this research, the investigation of CM for a multi-stage helical gearbox will include vibration signals generated under different operating condition for different loads at constant (full) speed.

This research work test experiment has been carried out in the diagnostic engineering research group at the University of Huddersfield Labs. In this part of the research a single type of fault was investigated - tooth breakage - but at two levels of damage to the first pinion in the two-stage gearbox. In two separate and sequential experiments 20% and 100%, of a tooth (Figure 3.15) were removed from pinion 1 in Figure 3.4, to investigate gearbox vibration response. These types of faults are common in gear transmission systems and have been studied extensively.
In this study, two sets of tooth breakages were simulated: the first was 20% tooth removal and 100% tooth removal (Figure 3.15). The second was 30%, 60%, and 100% tooth removal (Figure 3.16). The fault is produced by removing the specified percentage of the tooth face on the pinion gear along the width of the tooth, as shown in the figures. Each tooth was inserted as the first pinion and the vibration signals collected.

Because of the high overlap ratio the power transmission is little influenced by small tooth breakage. These faulty gears sets are labelled Gear no. 3, Gear no. 10, and Gear no. 12 is the healthy which is considered as baseline. Vibration experiments were carried out for three different cases; A) healthy condition, B) 20%, and C) 100% tooth removal. The data was collected using one accelerometer (PCB Model 338CO4) mounted on the gearbox casing. The analogue vibration signals were collected and sampled at 50 kHz. For each measurement, $10^6$ data points were taken, a recorded time of 20 second which meant a frequency resolution of 0.05 Hz. The DQS Card PD2-MF-16-

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**Figure 3-15** Gear faults: (a) 20% of one tooth removed (b) Complete tooth removed

**Figure 3-16** Simulated broken tooth for first pinion in gearbox
500/16L PCI was used throughout and the data recorded on the PC for further processing using MATLAB. In the following sub-sections three different cases would be described, namely; healthy and two gear tooth breakage cases. These measurements will be conditioned using TSA and analysed using CWT.

### 3.3.1 Experimental Procedure Steps

- For all vibration signals the raw data were collected using an accelerometer mounted on the gearbox casing and then passed to a data acquisition system. Normal test rig operation was under full speed and with different load conditions.
- The analogue vibration data was collected in the time domain, and stored on the computer under different identifying file names.
- Vibration signals will also be studied in real time for a better understanding of the characteristics of vibration signal.
- All vibration signal data were processed in the joint time-frequency domain using software written in MATLAB code to investigate and study the vibration signals for features which could be used to evaluate relative performance and differentiate the simulated faults from the healthy gear.
CHAPTER FOUR

MONITORING AND ANALYSIS OF GEARBOX SYSTEM VIBRATION SIGNAL USING CONTINUOUS WAVELET TRANSFORM

This chapter reviews the theoretical background of the Time Synchronous Averaging (TSA) technique and applies the Continuous Wavelet Transformer (CWT) to the vibration signal from the gearbox system for fault detection and diagnosis.

The vibration signals for different gear conditions (healthy and with different degrees of a specific fault inserted) under different operating conditions (constant speed but varying load) were recorded from a sensor mounted on the gearbox casing. These signals were analysed using the joint time – frequency CWT.

The vibration signals were recorded for a healthy gearbox and a gearbox with two levels of tooth breakage on the first pinion. The time domain signals were obtained using TSA and the RMS and kurtosis values determined. After that, three commonly used wavelet transform families; Daubechies order 1 (db1), Symlets order 2 (sym2) and Coiflets order 3 (coif3) are applied to the TSA signal and their usefulness for incipient fault detection investigated, using the fault feature indicators kurtosis and RMS.
4.1 Introduction

The use of the Continuous Wavelet Transform (CWT) for fault diagnosis of rotating machinery has been increasing dramatically. However, the selection of an appropriate signal processing technique for gathering hidden information in the vibration (or other) signals from a multistage gearbox it is not an easy task.

Tradition methods such as time-domain and conversion of the signal into the frequency-domain using the Fast Fourier Transformer (FFT) are still widely and usefully used (Mobley, 2002). But, as explained in the previous chapter, FFT-based methods are not suitable for non-stationary signal analysis and thus unsuitable for early stage fault detection and diagnosis. For reliable detection and diagnosis of non-stationary signals, an effective and powerful signal processing technique needs to be developed. Time-frequency analysis using wavelet transforms is a suitable powerful tool for CM using the vibration signal from the rotating component. Wavelet analysis is a very useful tool for on-line detection because it can reveal more features of vibration signals than FFT techniques (Mobley, 2002).

The wavelet transform is briefly reviewed, as an appropriate signal processing method which can be used effectively for fault detection. Important considerations are that it can cope with both stationary and non-stationary signals, and can transform any signal directly into time/space and frequency/scale domains, which can provide detailed information about signal evolution. CWT is defined as the sum over all time of the signal, and is one of the best transformers for singularity detection.

The aim of this chapter is to present and discuss the results obtained using the CWT to analyses the signals obtained from the helical gearbox. The original vibration signals obtained from the gearbox were averaged for one rotation of the gear, using Time Synchronous Averaging (TSA). Next each of three different wavelet transform families; Daubechies order 1(db1), Symlets order 2 (sym2) and Coiflets order 3 (coif3) were used to analyses the TSA averaged signal. Fault feature indicators kurtosis and RMS were used to extract useful information from the signals and investigated with the out covariant of maximal energy wavelet coefficient to select the best wavelet analysis for signal processing.
4.2 Joint Time-Frequency (Wavelet Transform) Analysis

The most important characteristics of wavelet analysis compared to the FFT are that wavelet analysis can provide two-dimensional frequency/scale and time information. To analyze a time-domain vibration signal using wavelets requires the application of windowing to the time-domain vibration signals to create signals on which linear and quadratic transformation can be applied. The time – frequency approach classifies the signal according to either the linear method or the quadratic method, see Section 2.11.3.

Several time-frequency techniques have been developed for analyzing non-stationary signals. Among those, the Short-Time Fourier Transform (STFT), Wigner-Ville Distribution (WVD) and Wavelet Transform (WT) are widely used. For complex classes of rotating machines where the S/N ratio is low and a large number of frequency components are present, these techniques have the potential for detecting and diagnosing gear faults (L Cohen, 1989; Hubbard, 1998; Ville, 1948). Of these the STFT has significant drawbacks, for example the window duration is fixed which is not suitable where there is a wide range of frequency peaks in the signal and predetermined scaling is inappropriate (Merzoug, Ait-Sghir, Miloudi, Dron, & Bolaers, 2015; W. Wang, and McFadden, P, 1993). The most used techniques for joint time-frequency signal analysis is the WT (Z. Peng & Chu, 2004; W. Staszewski & Tomlinson, 1994; W. Wang & McFadden, 1996) and the WVD (Lee et al., 2001) because they overcome the disadvantages of separate time-domain and frequency-domain analyses of vibration signals from rotating machinery (Atlas, Bernard, & Narayanan, 1996; Jardine et al., 2006; Zhu, San Wong, & Hong, 2009).

The WT function is based on a number of windows of different durations for analyzing different frequency components within the signal. The wavelet transform may thus be considered as an optimum and powerful tool for adaptive, non-stationary signal processing. However, results previously obtained when wavelet transforms were applied to the test signal show the importance of selecting a suitable base wavelet for the success of the signal processing of the signal for the given application (Calderbank, Daubechies, Sweldens, & Yeo, 1998; Strang & Nguyen, 1996). A description of the WTs used is given in the following sections.

4.2.1 Linear Time-Frequency Analysis Using STFT

The STFT was introduced by Gabor in 1946 (Meyer, 1993):
\[ STFT(\omega, t) = \int f(t)g(t - \tau)e^{-j\omega \tau}d\tau \]  \hspace{1cm} (4.1)

Mathematically the STFT maps a time domain signal into a 2-D function of time and frequency but, practically, it is difficult to find a fast and effective algorithm to calculate the STFT.

The STFT (sometimes known as the windowed Fourier transform) was developed to overcome the inability of the FT to analyze non-stationary signals. The STFT is based on a sliding window function \( g(t) \) centered at time \( \tau \) to achieve a “time–localised” Fourier Transform of the signal \( x(t) \). Time localization can be achieved by performing a frequency transform on the signal content contained within the window function, as shown in Figure 4.1. Applying the STFT transforms one-dimensional data into two-dimensional data, the first dimension is frequency and the second dimension is the location of the window in the data. This method was applied to the CM of gearboxes as an early form of time-frequency analysis (B. D. Forrester, 1989). This method has been shown to have limitations for gear fault detection and analysis because the window width is not variable but remains constant and this lead to poor frequency resolution.

The uncertainly principle imposes the restriction that signal analysis in both time and frequency domains cannot give precise results for both instantaneously. This arises due to the introduction of the windowing function and will affect the signal resolution in the frequency domain (Lee et al., 2001; Ville, 1948). A contradiction arises, a small window width must be chosen for better time resolution of high frequencies of short periods, but this will result in poor frequency resolution for low frequencies. However, selection of a wide window to give accuracy at low frequencies will have poor time resolution at high frequencies (Z. Peng & Chu, 2004).
4.2.2 Quadratic Time-Frequency Analysis

The Wigner-Ville distribution (WVD) was one of the first time-frequency methods used and has been widely studied [19]. The WVD is a bilinear convolution integral (L Cohen, 1989) and has been used to represent the vibration signal in time–frequency resolution representation plane (P. Rao, Taylor, & Harrison, 1990). Unfortunately, interpretation becomes difficult if the signal is even moderately complicated due to crossing terms which could mislead in signal interpretation. Many methods have been proposed to overcome the disadvantages, one of the earliest was the Choi-Williams distribution (CWD), but others are available (Z. Peng & Chu, 2004).

Nevertheless, many authors claim to have successfully used the WVD in the CM of rotating machinery, specifically in detecting the initial stages of a broken gear tooth failure (P., May 1999). Baydar and Ball (Naim Baydar & Ball, 2001; N Baydar, Gu, Ball, & Li, 1999), Forrester B.D (B. Forrester, 1989), and Wang and McFadden (P. McFadden & Wang, 1991) have all used this method to detect gear faults including cracked and pitted teeth.

The vibration signal from a gearbox with a simulated broken tooth was analysed using WVD and presented as a contour plot to in order to detect any significant changes in the statistical characteristics. Due to the advanced facilities on modern computers, WVD can be performed with both good time-frequency and good time resolution (W. Staszewski et al., 1997).
The WVD algorithm, \( W(t, f) \) can be found in reference (W. Staszewski & Tomlinson, 1994) expressed in term of time as follows:

\[
W(t, f) = \int s(t + \frac{\tau}{2})s^*(t - \frac{\tau}{2})e^{-j2\pi f \tau d\tau}
\]  

(4.2)

Where: \( W(t, f) \) denotes the WVD of the signal

\( S^* = \) The complex Conjugate of \( S(t) \)

\( W(t, f) \) can be expressed in term of frequency as follows:

\[
W(t, f) = \int s(f + \frac{\nu}{2})s^*(f - \frac{\nu}{2})e^{j2\pi \nu t} d\nu
\]  

(4.3)

By integrating \( W(t, f) \), with respect to frequency at a certain time, the energy density at that time can be defined as (W. Staszewski et al., 1997):

\[
\int_{-\infty}^{\infty} W(t, f) df = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s(t + \frac{\tau}{2})s^*(t - \frac{\tau}{2})e^{-j2\pi ft} d\tau df
\]  

(4.4)

\[
= |s(t)|^2
\]  

(4.5)

The integral of \( W(t, f) \) with respect to time at a specific frequency will give the spectral energy density at that frequency as follows:

\[
\int (t, f) dt = s(f)|^2
\]  

(4.6)

Both equations (4.5) and (4.6) define minimal conditions and are both presented as requisite for time–frequency energy distribution (Leon Cohen, 1966).

### 4.2.3 Wavelet Transform (WT)

The wavelet transform (WT) has been developed and used for vibration signal analysis for more than two decades (Rioul & Duhamel, 1992). However, the WT depend on its dilation parameters where the mother wavelet is essentially adapted by either expansion or contraction where this process is controlled by dilation parameters unlike multiplication by sines and cosines as in the method used by the STFT. Different wavelets are produced through the mother wavelet. Basically, the main wavelet function is acquired from a signal prototype wavelet and adapted by either scaling (dilating) or shifting (translation). The essential task of the wavelet transform in signal analysis is decomposing the signal into its main function, called wavelet (Rioul & Duhamel, 1992).
In recent years the WT has attracted the attention of many researchers who have attempted to enhanced WT as a useful and powerful signal processing tool for use in CM of rotating machinery (C. Sung et al., 2000). Above all, the WT is used to represent the vibration signal in the time-frequency domain to reveal features of the signal frequencies and durations of each frequency in the time domain (C. Sung et al., 2000). It has been shown that WT has the capabilities to distinguish between local and distributed gear faults by investigating scale changes in the vibration signal collected from the rotating machinery (W. Wang & McFadden, 1995a). In the CM of rotating machines, the WT is used to locate and examine any transient vibration signal creating by gear faults. WT is a powerful tool with multiple resolution features to localise short time components, and also has the potential that it can identify gears faults by representing the results of the signal transform through a time-scale display of signal distribution (N Baydar & Ball, 2003).

Table 4.1 summarises similarities and differences of several joint time–frequency techniques. This table is based on varying such parameters as dilation of wavelet size within the transform, such factors such as scaling local characteristics and displaying them in the time-scale plane. The efficiency of WT technique depends on the window function; whether a narrower or wider window will be based on whether the signal frequency is high or low.

Table 4.1 Comparison of performances between different types of time-frequency analysis techniques (Suh, 2002)

<table>
<thead>
<tr>
<th>Method</th>
<th>Resolution</th>
<th>Interference Term</th>
<th>Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>CWT</td>
<td>Good frequency resolution and low timeresolution for low-frequency components; lowfrequency resolution and good time resolution for high-frequency components</td>
<td>No</td>
<td>Fast</td>
</tr>
<tr>
<td>STFT</td>
<td>Dependent on window function, good time or frequency resolution</td>
<td>No</td>
<td>Slower than CWT</td>
</tr>
<tr>
<td>WVD</td>
<td>Good time and frequency resolution</td>
<td>Severe interference terms</td>
<td>Slower than STFT</td>
</tr>
<tr>
<td>CWD</td>
<td>Good time and frequency resolution</td>
<td>Less interference terms than WVD</td>
<td>Very slow</td>
</tr>
<tr>
<td>CSD</td>
<td>Good time and frequency resolution</td>
<td>Fewer interference terms than with CWD</td>
<td>Very slow</td>
</tr>
</tbody>
</table>
Researchers have been struggling with the challenge of selection of the base wavelet which has remained largely an ad-hoc process as shown Figure 4.2 shown a different types of wavelet. Matching the wavelet to the signal to be analysed is critical for best WT results.

**Figure 4-2 Different types of wavelets (Yan, 2007)**

4.2.4 Continuous Wavelet Transform

The CWT is a time-frequency method that builds on the STFT, but whereas the fixed window size of the STFT limits it’s in resolution, the CWT uses a variable window size with short windows for high frequencies and long windows for low frequencies. This allows the CWT to be a powerful tool in representing local features of a signal that the STFT may miss entirely.

In the field of machinery CM, the CWT is recognized as a powerful and effective tool for feature extraction from a non-stationary signal. The details of the wavelet algorithm used in this study can be found in reference (Zheng et al., 2002).

If \( \psi \), a mother wavelet function, and its Fourier transform satisfy the \( \psi \) admissibility condition
\[
C_{\psi} = \int_{-\infty}^{\infty} \left| \hat{\psi}(\omega) \right|^2 \frac{1}{|\omega|} d\omega < \infty 
\] (4.7)

Where \( L^2 \) is the space of square integral complex functions the corresponding family of wavelets consists of daughter wavelets shown as Equation (4.8).

\[
\psi_{a,b}(t) = |a|^{-1/2} \psi\left(\frac{t-b}{a}\right) 
\] (4.8)

Where \( a \) is scale (dilation) factor and \( b \) is time location (translation) factor, \( |a|^{-1/2} \) is used to ensure energy preservation. The daughter wavelets are the translated and scaled versions of the mother wavelet where the scale factor \( a \) and time location \( b \) vary continuously.

The CWT of a signal \( x(t) \) is defined as the inner product between signal \( x(t) \) and the wavelet family.

\[
WT_x(a,b) \leq \psi_{a,b}(t), x(t) \geq |a|^{-1/2} \int x(t)\psi_{a,b}^*(t)dt 
\] (4.9)

Where \( WT_x(a,b) \) denotes the wavelet transforming coefficient, \( \psi_{a,b}^*(t) \) representing the complex conjugate of the wavelet function. \( a \) is known as a dilation parameter and \( b \) gives the location of the wavelet which is known as translation parameter (Zheng et al., 2002).

For a discrete sequence \( x_m \), let \( a = m\delta \) and \( b = n\delta \) where \( m, n = 0, 1, 2, \ldots (N-1), N \) is the number of sampling points and \( \delta \) is the sampling interval. The CWT of \( x_m \) can be defined as:

\[
WT_x(a_j) = \sum_{m=0}^{N-1} x_m \psi^*\left[\frac{(m-n)}{a_j}\right] 
\] (4.10)

The amplitude of the feature corresponding to the scale and how this amplitude varies with time can be presented through varying the index \( j \) and \( n \) corresponding to the scale factor \( a \) and time location \( b \), respectively (Zheng et al., 2002).

Orthogonal and non-orthogonal are two types of wavelet functions commonly used in signal processing. The orthogonal wavelet is a wavelet whose associated wavelet transform is orthogonal, such as Haar, Daubechies, Coiflets, Symlets and Meyer, while the non-orthogonal wavelet functions include Morlet, Mexican hat and DOG, see Figure 4.2 (Zheng et al., 2002). The properties of the wavelet functions are different and can be selected as appropriate for different applications.
Wavelet coefficients obtained from a WT measure the similarity between the signal of interest and daughter wavelets which are dilated and translated from a particular mother wavelet. The greater the similarity between daughter wavelet and the feature component of the signal, the larger is the corresponding wavelet coefficient (Riou & Duhamel, 1992). Moreover, because a wavelet possesses oscillating and wave-like characteristics it has the ability to allow simultaneous time and frequency analysis. Fourier analysis requires a combination of waves of infinite duration and thus infinite energy; wavelets, on the other hand, have finite energy concentrated in a limited time interval (Niu et al., 2008). In addition, wavelets can also represent sharp peaks in a signal so an original signal can be completely reconstructed or recomposed (Burrus, Gopinath, & Guo, 1998; Qian & Chen, 1999). This allows transient signals such as occur with gearbox vibration to be represented accurately and efficiently.

It is necessary to choose an appropriate wavelet function for processing a gearbox vibration signal for accurate fault diagnosis. However, there are large numbers of different types of wavelets, each having its own particular characteristics and because the understanding of gearbox vibration is not perfect it is not yet determined which transform is best.

4.3 Continuous Wavelet Transform (CWT) Analysis of the TSA Signal

In this study, the vibration signals measured by the accelerometer were subject to TSA and then analyzed using the joint time-frequency domain wavelet technique. To obtain the best fault separation, a careful selection of wavelets needs to be carried out. The results obtained here have shown that wavelet db1 produces the best fault separation whereas the coif3 wavelet fails to produce any significant separation. Based on this study, it is suggested selecting the wavelet that produces the highest RMS value of wavelet coefficients when applied to baseline data.

4.3.1 Experimental Procedure

Vibration experiments were carried out to investigate two levels of pinion tooth breakage: 20% and 100% of the tooth are removed from two pinion gears (see Figure 3.15) of the helical gearbox shown schematically in Figure 3.4, and gearbox vibration response investigated. These faulty gears sets are labelled Gear no. 3, Gear no. 10, and Gear no. 12 was the healthy gear which is considered as baseline. In the following sub-sections the results obtained using TSA and CWT from the three different cases is described.
For each fault case the data was collected at full speed under 5 different loads: 12.4%, 21.2%, 30.8%, 39.3% and 42.9% of the full load.

4.3.2 TSA Pre-processing of Time-Domain Vibration Signals

During the tests, both the vibration and the encoder signals are collected simultaneously by the DQS at a sampling rate of 100 kHz. Each 16 seconds of data collection thus contains 1,600,000 points for 390 revolutions at full speed (1465rpm). This data length is sufficient for random noise suppression using the TSA process. The encoder signal measured the shaft speed and provided the reference point for synchronous averaging of the vibration signal. However, the time interval of the pulses provided by the encoder signal is not constant due to small oscillations of the shaft speed. The angular acceleration is calculated based on arrival times of three adjacent pulses obtained from sampling the encoder signal. Then, the correct placement of the next sample on the time axis (re-sample) is carried out based on this calculated angular acceleration.

In this study, the time axis is processed in sections of 1000 points. Five sections were selected. Once the re-sample times are calculated, the vibration signal is reset according to the re-sampled time axis to obtain the synchronous average.

Figure 4.3 shows the TSA averaged vibration signal for three revolutions of the input shaft (three revolutions of the drive shaft is about 0.12 seconds) when the gearbox operates under different loads and fault levels, and where the healthy case is the baseline. It can be seen that the amplitude of the vibration signals increases with the increasing load for both faults. Impulsive components within the vibration signals are seen for all test conditions, but especially for 100% tooth damage. The TSA signals show a clear difference between the signals obtained for 100% tooth damage compared with the baseline, for all load levels. However, the difference in signals between the baseline and 20% tooth damage do not appear significant at any load (Elbarghathi, Wang, Zhen, Gu, & Ball, 2012).
For a more detailed comparison, common feature parameters such as RMS and kurtosis were calculated from the TSA vibration signals. Figure 4.4 shows the RMS and kurtosis values obtained. It can be seen that compared with the baseline, the RMS values for 100% tooth damage are clearly separated from the other two cases under all loads. The RMS values for the 20% tooth fault do not show significant differences with the baseline. For the kurtosis values, the difference between the baseline, 20% tooth damage and 100% tooth damage are not significant (except at the lowest load) and the signals cannot be separated and used for fault separation.
Feature Extraction Based on CWT Analysis

In the present study, Daubechies order 1 (db1), Symlets order 2 (sym2) and Coiflets order 3 (coif3) wavelets were selected to analyse the TSA vibration signals obtained. These wavelets are all the orthogonal wavelet with fast, perfect reconstruction and non-redundant decomposition and have been used in many applications (Leonds, 2000; S. S. Rao, 1995).

Scales ranges from 0.5 to 20 were selected for all the wavelet applications in this study. TSA vibration signals of 5,000 data points were selected for CWT analysis, for all the test conditions. Figures 4.5, 4.6 and 4.7 show the db1, sym2 and coif3 wavelet coefficients respectively. To demonstrate joint time and scale characteristics, the wavelet coefficients are presented as contour plots for the different loads. It can be seen that there are clear differences between the baseline and 20% tooth damage, and between 20% and 100% damage for all the wavelets and load conditions. In particular, the wavelet coefficients for db1 had greater amplitudes with larger visible areas.
Figure 4-5 Contour plots of db1 wavelet coefficients for different gear operating conditions (full speed; healthy, 20% and 100% tooth removal; 21.2%, 10.8%. 39.3% and 42.9% full load)

Figure 4-6 Contour plots of sym2 wavelet coefficients for different gear operating conditions (full speed; healthy, 20% and 100% tooth removal; 21.2%, 10.8%. 39.3% and 42.9% full load)
From comparison of Figures 4.5, 4.6 and 4.7, it can be seen that the three wavelets give similar result but to different degrees. The same two statistical parameters applied to the time-domain vibration signals, RMS and kurtosis, were applied to the wavelet results to compare relative performance. Figure 4.8 shows significant differences between RMS values at 100% tooth removal and the RMS values for 20% tooth removal and healthy condition. The figure also shows a difference between baseline RMS and RMS for load index 4 or 20% tooth damage.

The results for kurtosis were similar, with significant differences in kurtosis value between 100% tooth removal and 20% removal and baseline, but no perceived difference between 20% tooth removal and baseline (F Elbarghathi et al., 2012).
Comparing the results presented in Figures 4.4 and 4.8, RMS and kurtosis values of TSA time-domain vibration signals and wavelet coefficients it can be concluded that CWT gives clearer fault separation than the TSA time-domain signal for 100% tooth removal, for both RMS and kurtosis. For the 20% tooth damage, the TSA time domain signal shows no significant difference with the baseline when this level of fault is introduced. For the wavelet analysis there appears to be a difference between RMS values at load index 4 only for 20% tooth damage.

Although all three types of the wavelets selected for our study give acceptable performance for faulty case separation for 100% tooth damage, and for 20% tooth breakage at higher loads, the responses were different in degree. In order to analyse which wavelet is more effective and suitable for gearbox CM and fault diagnosis, RMS values between the three wavelets were compared for the baseline and 20% tooth damage. Figure 4.9 presents the details of the comparison.
The wavelet db1 has highest differences in RMS values between baseline and 20% tooth fault, and is thus best for separating the signals for baseline and 20% tooth damage. On the other hand, coif3 did not separate the baseline and 20% tooth breakage signals clearly. The evaluation of Figure 4.9 indicates the wavelet db1 is best to classify the different faulty cases among the three wavelets.

Moreover, it was observed that higher RMS values in the case of baseline will produce a higher difference for the smaller damage. Based on this, an optimal wavelet can be determined without having to use data sets obtained from measurements where damage is present which are normally not available in CM practice.

### 4.4 Summary

This chapter has briefly reviewed the wavelet transform as an appropriate and important signal processing method for fault detection. The CWT, defined as the sum over all time of the signal, is considered one of the best transforms for singularity detection and in this research has been used as an integrated platform for analyzing the vibration signals for simulated gear faults in a two-stage helical gearbox under different loads while operating at maximum speed. The measuring accelerometer was mounted on a gearbox casing.
Time-frequency patterns were obtained by wavelet analysis of the TSA signal using three types of wavelets: Daubechies order 1, Symlets order 2 and Coiflets order 3. The purpose was to determine the optimal wavelet that would exhibit distinguishing features capable of detecting the presence of a small seeded fault (broken pinion tooth) and also indicate increased severity of the fault. The results have shown that wavelet Daubechies order 1 produces the best fault separation whereas the Coiflets order 3 wavelet fails to separate the signals properly. It means that different wavelets produce different separation results. Based on this study, it is suggested that wavelet which produces the highest RMS value of wavelet coefficients when applied to baseline data be selected for fault detection.
CHAPTER FIVE

ANALYSIS OF VIBRATION SIGNAL USING MORLET WAVELET TRANSFORM WITH INFORMATION ENTROPY

This chapter presents the application of a new method of vibration signal analysis: the Morlet Wavelet based on maximum wavelet entropy difference. It is shown to be an effective tool for fault detection and diagnosis in a multistage helical gearbox when used with time synchronous averaging.

The chapter describes the use of maximum Shannon entropy difference to optimize the central frequency and bandwidth parameter of the Morlet wavelet in order to achieve optimal match with impulsive components, and to extract the features of the gear faults in the multistage of a helical gearbox. The results show that the proposed method is much better than the use of adaptive Morlet transform based on kurtosis maximisation.
5.1 Introduction
The aim of this research investigation is to detect simulated gearbox faults by comparing the measured vibration signal obtained for baseline conditions with that obtained when known and qualified faults were seeded into the gearbox.

Wavelet transform analysis is a powerful tool commonly used for vibration signal analysis and fault diagnosis. However, there are drawbacks that challenge its widespread application. Wavelet transforms do not retain time invariance and that may result in the loss of beneficial information and decrease the correctness of any fault diagnosis. Other limitations on wavelet transforms are the choice of the appropriate threshold and the wavelet function. A main challenge for wavelet analysis is the adaptability of the parameters of the mother wavelet to the time variance of the given signal. To overcome this deficiency, this study suggests using an adaptive Morlet wavelet transform method based on the information entropy optimization.

The proposed wavelet transform method is used for analysing the measured vibration signals to detect and diagnose faults in a helical gearbox. A comparative study of the proposed method and kurtosis maximization has also been carried out to assess the suggested method. The results of this investigational study show that the proposed approach is very suitable for gearbox fault diagnosis.

5.2 Morlet Wavelet transform adapted by information entropy-Literature Review:
For monitoring the health of gearboxes statistical parameters associated with the time and frequency domains of measured vibration signals have been used, but these are usually very noisy and non-stationary (F. Elbarghathi, et al., 2013). To overcome these problems, time-frequency domain approaches have been used, such as the Short Time Fourier Transform and wavelet transform (WT) (Francois Combet & Gelman, 2007; F Combet & Gelman, 2009; García Márquez, Tobias, Pinar Pérez, & Papaelias, 2012; W. J. Staszewski & Robertson, 2007; W. Wang, and McFadden, P., 1993; Zimroz et al., 2011), and Wigner-Ville distribution (WVD) (W. Staszewski et al., 1997; W. Wang & McFadden, 1995b).

Bayder and Ball (N.Baydar, 2000) carried out an experimental investigation to detect gearbox local faults under different loads. They introduced a time-frequency distribution technique called the instantaneous power spectrum (IPS) and showed that the technique is could detect a local gear fault
under constant speed and load. However, they also found that this method of spectrum analysis did not detect the local gear fault under varying load (N. Baydar, 2000).

The WT reveals the signal features in the time and frequency domains simultaneously, which can be used to successfully analyse the non-stationary vibration signal for a local gear fault (J. Lin, 2003; Z. K. Peng, 2004). WTs are commonly used for rotating machine where the signal-to-noise ratio (SNR) is low and a large number of frequency components are present (Z. K. Peng, And Chu, F.L., 2004; K. C. Sung, Tai, H.M., and Chen, C.W.-, 2000; Zheng et al., 2002). However, as indicated above, there are some disadvantages in the use of WT, these include the selection of the threshold and the optimization of the wavelet mother functions. A study of the optimization of wavelet mother functions can be found in Younghua (Younghua, 2011), who used the modified Shannon wavelet entropy to optimize central frequency and bandwidth parameters of a Morlet wave function. The results of experiment analysis indicated that this method had better denoising performance than other traditional approaches. Chen et al. (Chen, 2005) proposed another method for the fault diagnosis of a hydraulic motor using a GA to optimise wavelet parameters. The results of the experiment showed details of the characteristic vibration signal with fine resolution.

In this study, a denoising method based on an adaptive Morlet WT where its parameters are optimized by using maximum Shannon entropy difference is proposed. This method is used for extracting features of vibration signals were collected from a two stage helical gearboxes with seeded faulty condition of 30%, 60% and 100% tooth breakage on the first pinion.

Normally, vibration signals from rotating machinery are a combination of periodic signals with random noise and here TSA was used to reduce the noise from the repeated vibration signal. The SNR of a vibration signal can be improved significantly by suppressing components which are asynchronous with the component of interest. This requirement is met by using an external trigger signal provided by a shaft encoder, then, the vibration signal is divided into small segments according to the period of revolution of the rotating part, and all the segments are summed up together so that non-coherent components and asynchronous components cancel out.

Next, maximum Shannon entropy difference was introduced to optimize central frequency and bandwidth parameters of the Morlet wavelet functions so as to obtain optimal match with the impulsive components. The results of this technique will be compared to those obtained using the maximum kurtosis principle.
5.3 Theoretical Background

5.3.1 Time Synchronous Averaging

TSA (Francois Combet & Gelman, 2007; W. Wu, Lin, Han, & Ding, 2009) assumes that the vibration signal \( x(t) \) consists of a periodic signal \( x_T(t) \) and a noisy component \( n(t) \), the period of \( x_T(t) \) is \( T_0 \) whose corresponding frequency is \( f_0 \), thus the signal can be expressed.

\[
x(t) = x_T(t) + n(t)
\]  

The synchronous average of the signal \( x(t) \) by using TSA can be expressed as

\[
y(t) = \frac{1}{M} \sum_{i=0}^{M-1} x(t + iT_0)
\]

Where \( M \) the number of the average segments is \( y(t) \) is the averaged result. Thus

\[
y(t) = \frac{1}{M} \sum_{i=0}^{M-1} x(t) + \frac{1}{M} \sum_{i=0}^{M-1} (iT_0)
\]

The second term on the right hand side is the sum of \( M \) successive “noise” elements of the time domain signal, each of duration, \( T_0 \). Because these will have random relative phase but be of much the same amplitude its value will get tend closer to zero the larger \( M \). With machinery rotating at several hundred rpm the value of \( M \) is often in the thousands. Note that TSA is a method of reducing random noise and that for maximum effect the segments should all be of the same length and correspond to one period of the relevant motion.

5.3.2 Definition of Morlet wavelet

The adaptive wavelet algorithm used in this study can be found in Yao (Yao et al., 2009) in detail. The wavelet transform of a signal \( x(t) \) is defined as:

\[
WT_x(a, b) = \frac{1}{\sqrt{a}} \int x(t) \psi^* \left( \frac{t-b}{a} \right) dt
\]

Where \( x(t) \) is the signal, “*” denotes the complex conjugation, \( a \) is scale factor and \( b \) is a shift factor. The factor \( \frac{1}{\sqrt{a}} \) is used to ensure energy preservation. \( \psi(t) \) is the mother wavelet. The wavelet coefficient \( WT_x(a, b) \) represent the similarity between the signal \( x(t) \) and a wavelet (Jiang, Tang, Qin, & Liu, 2011).

There are different types of wavelet functions for different uses. It is important to select the most appropriate wavelet for this specific vibration signal. In this work the Morlet mother wavelet function is used because faults which occur in rotary components such as gearboxes are periodical impulses tend to be similar to the Morlet wavelet, see Figure 4.2.
A Morlet wavelet and its Fourier transform are as follows:

\[ \psi_r(t) = \frac{1}{\sqrt{f_b \pi}} \exp \left( -\frac{t^2}{f_b} \right) \cos(2\pi f_c t) \]  
(5.4)

\[ \psi(a f) = e^{-\pi^2 f_c (af - f_c)^2} \]  
(5.5)

Where \( f_b \) is the bandwidth parameter, \( f_c \) is the central wavelet frequency (Jiang et al., 2011).

### 5.3.3 The procedure for optimising Morlet wavelet parameters based on maximum entropy difference

It is obvious from Equation (5.4) that the shape of the Morlet wavelet can be controlled by parameters \( f_b \) and \( f_c \) to balance time-frequency resolution. In this work, the wavelet Shannon entropy is applied to choose optimum values. Assume \( d(k), k = 1,2, \ldots N \) are wavelet coefficients of the signal \( x(i), i = 1,2, \ldots N \). After normalization the wavelet entropy can be defined as (Houxi Cui, 2009):

\[ H = -\sum_{k=1}^{N/2} \bar{d}(k) \log(\bar{d}(k)) \]  
(5.6)

Where \( \bar{d}(k) \) are the normalized wavelet coefficients:

\[ \bar{d}(k) = \frac{d(k)}{\sum_{k=1}^{N} d(k)} \]  
(5.7)

Thus, all normalised coefficients, \( \bar{d}(k) \), are less than unity. The procedure of using maximum entropy for optimizing the values of \( f_c \) and \( f_b \) is expressed as follows:

(i) Vary the parameters \( f_c \) and \( f_b \) within preselected intervals to produce different mother wavelets.

(ii) Perform wavelet transform for baseline and fault signal using each mother wavelet and calculate the entropy difference between the two condition signals.

(iii) Compare the maximum entropy difference value. The parameters \( f_c \) and \( f_b \) that correspond to the maximum value are the best parameters to use to reveal the fault features.

(iv) Repeat step (i)-(iii) to choose parameters for the three faulty cases 30%, 60% and 100% tooth breakage at different operating load conditions.

(v) Finally, perform fault diagnosis with the maximum entropy difference value.
5.3.4 The procedure of optimising Morlet wavelet parameters based on maximum kurtosis principle

Kurtosis can be used as the performance measure of a Morlet wavelet filter (Lin & Zuo, 2003). The definition of kurtosis is:

\[ \text{Kur}(y) = E(y^4) - 3[E(y^2)]^2 \]  

(5.8)

Where \( y \) is the sampled time series and \( E \) represents the mechanical expectation of the series. The procedure to perform the adaptive wavelet filtering is as follows:

(i) Varying the parameters \( f_c \) and \( f_b \) within preselected intervals to produce different mother wavelets.

(ii) Perform wavelet transform using each mother wavelet and calculate the maximum kurtosis of each outcome.

(iii) Compare the kurtosis values. The parameters \( f_c \) and \( f_b \) that correspond to the largest kurtosis are the best parameters to use to reveal hidden fault features.

5.3.4.1 TSA Pre-processing

Vibration and the encoder signals are collected simultaneously by the DAS for four cases; three faulty conditions with 30%, 60% and 100% tooth breakage and one healthy, baseline condition sampled at a rate of 100 kHz.

For each fault, the data was collected at five different loads: 9.1%, 18.2%, 31.8%, 54.5% and 77.3% of full load. However, in all cases full speed (1465rpm) was used, the duration of each test was 16 seconds which gave 1.6 x 10^6 points. This data length is sufficient for random noise suppression using the TSA.

As described in Section 4.3.2 there was a small oscillation in the shaft speed due to varying torque on the gear which meant that the time interval of the pulses in the encoder signal were not precisely constant. In this work, the time axis re-sampling was processed in sections of 1000 points, with 5 sections selected. As the re-sample times were computed, the vibration signal was sampled according to the re-sample time to ensure synchronous averaging.

Figure 5.1 shows TSA signals for the baseline condition and the three levels of tooth breakage - 30%, 60% and 100% - for the gearbox operating under 9.1%, 18.2%, 31.8%, 54.5% and 77.3% of full load. From Figure 5.1, it can be seen that the amplitude of the vibration signals increases with
the increase of load for baseline and three faulty cases. For the cases of 60% and 100% tooth damage the impulsive components are obvious and a clear indications of the presence of a fault. On the other hand, the TSA signals between baseline and 30% tooth damage do not show obvious differences for gear fault indication.

Figure 5-1 TSA vibration signals for baseline and three damaged gears under five different loads

5.3.4.2 Feature extraction of adaptive Morlet wavelet analysis based on maximum entropy difference and maximum kurtosis

In this work, an adaptive Morlet transform was selected to analysis the TSA vibration signals. This new method is compared with adaptive Morlet parameter optimization based on the maximum kurtosis principle. The TSA vibration signals of 5000 data points are selected to perform adaptive Morlet wavelet analysis for all the test condition to explore features in the joint time-frequency scale. Adaptive Morlet wavelet coefficients for both methods are presented with contour plots of maximum entropy difference and maximum kurtosis adaptive wavelet for baseline, 30%, 60% and 100% tooth breakages, operating under different loads and full speed (1465rpm).
From Figures 5.2 and 5.3, it can be seen that the adaptive wavelet coefficient image pattern are clearly different for different cases healthy, 30%, 60% and 100% tooth breakage operating under different five different loads. The periodic feature can be clearly seen.

The wavelet coefficient shown in Figure 5.2 gives good frequency resolution both low and high frequency which indicates the presence of a fault for all three cases for all operating loads compared with the baseline. In Figure 5.3, the meshing frequencies are clearly visible at scale 5 and 20. The varying resolution on the time-frequency plane is due to the change in the size of the wavelet during the analysis (Fathalla Elbarghathi, Tian, Tung Tran, Gu, & Ball, 2013).

Figure 5-2 Contour plots of maximum entropy adaptive wavelet coefficient for different baseline gears under five different loads
Figure 5-3 Contour plots of maximum entropy adaptive wavelet coefficient for fault different gear cases under five different loads
Figure 5-4 Contour plots of maximum kurtosis adaptive wavelet coefficient for baseline and three damaged gears under five different loads

The gearbox signal at different condition cause the distribution of peak amplitude value to stretch towards the higher frequency region and appear at different time unit and frequencies in the contour. Compared with maximum kurtosis shown in Figure 5.4, it can be seen that there are clear differences between the baseline and 60 and 100% tooth damage but is not clear for 30% for all loads as follow 9.09%, 18.2%, 31.8%, 54.5% and 77.3% compare with minimum entropy adaptive wavelet. In particular, the wavelet coefficients of minimum entropy were much higher with larger visible area.

For more detailed comparison as shown in Figure 5.5, it can be seen the value maximum entropy difference are clearly difference for four cases as healthy, 30%, 60% and 100% tooth damage operating under different load operating condition as follow 9.09%, 18.2%, 31.8%, 54.5% and 77.3%. Moreover, from Figure 5.5 it can be seen that the maximum kurtosis cannot show clear difference for all four cases. That are clear difference between baseline with 60% and 100% tooth damage but is not clear for 30% for all loads, compare with the maximum entropy adaptive wavelet, from the results as shown in Figure 5.5 shown that the proposed method based on adaptive Morlet
wavelet using parameters optimization is much better than the method of adaptive Morlet based on the kurtosis maximisation (Fathalla Elbarghathi et al., 2013).

![Graph showing maximum entropy difference and maximum kurtosis value of TSA vibration signal for baseline and three damaged gears under five different loads.](image)

**Figure 5-5 Maximum entropy difference and maximum kurtosis value of TSA vibration signal for baseline and three damaged gears under five different loads**

### 5.4 Summary

A main challenge of wavelet analysis is the adaptability of the parameters of the mother wavelet to the time variance of the given signal. In this work an attempt has been made to overcome this deficiency by using an adaptive Morlet wavelet transform method based on the information entropy optimization. The proposed wavelet transform method has been applied to the analysis the vibration signals obtained from a two-stage helical gearbox to detect the presence of seeded faults. Comparison of the results obtained by the proposed method and those of a previous study which used kurtosis maximization has been made and shown to be an improvement.
MATHEMATICAL MODEL

In this chapter, a nonlinear dynamic model of the two-stage helical gearbox involving time-varying mesh stiffness, and transmission error is established. Then, the governing nonlinear equations are solved by Runge-Kutta method to obtain vibration responses under different gear conditions. Based on experimental data which is collected from different operating torque loads and the simulated faults, the modelled is calibrated and allows predicting fault features with acceptable accuracy. Moreover, the results show that the dynamic modelling is necessary in understanding vibration responses upon different excitations such as manufacturing errors, fault impacts and uniform wearing that would be critical in developing gear fault monitoring techniques.
6.1 Introduction

Power transmission using helical gears is widely utilized in industrial applications. Such as gear systems are highly efficient and work under high applied loads and widely ranging operating speeds. Fault detection at early stage to avoid the further failure consequences as well as identification the gear damage severity is a challenging task in gear fault diagnosis, condition monitoring of rotating machinery components such as helical gearboxes is becoming ever more important, and as a consequence researchers are always striving to develop more capable tools for gearbox fault detection at earliest possible stages.

The main focus of this research is to developed mathematical model of a two stage helical gearbox system, and a comparison of its model-predicted vibratory behaviour with observed experimental behaviour. The dynamic modelling is aiming to simulate the faults of two stage helical gearbox vibration signals reveals in depth exploring the mechanisms phenomena which creates a signal during the contacting forces between gears in gears transmission systems. In addition gears modelling is mostly useful to classified any vibration signals for different sorts of gears faults dynamics possessions.

The model represents the two-stage helical gear system using a stiffness function to represent the forces acting between two pair of gears, and rotational stiffness and damping are also used to simulate the angular motion of the gears and shafts. The governing equations are solved by Runge-Kutta method. Based on experimental data which is collected from different operating torque loads and the simulated results, gear fault severities are identified. Furthermore, the model outcome results have revealed that the acceleration spectrum as it anticipated peaks of existing meshing frequencies and harmonics. Moreover, to simulate the gears transmission systems based on stiffness function of two pair contacting gears with different tooth broken simulated faults at different variation of loads and speeds, the model results shows the similarities with results produced by experimental investigation.

Matlab has been used to solve the model equations by iteratively adapting the parameters, and the approach illustrates that such a model, although fairly simple, can replicate accurately the dynamics of a real gearbox.

The simulation results show similar vibration signal sideband behaviour compared with the experimental signal investigation, providing a sound basis for this approach. The results also
indicate that the dynamic model can be used in studying gear faults and would be useful in developing gear fault monitoring techniques.

6.2 Modelling literature review

Gearboxes are playing a very important role in the industrial field, due to their ability to processing a large torque load(D.N.Chorafas, 1990). Helical gearboxes are common utilizing in several a numbers of mechanics machine components in general for power transmission in rotary machinery such as wind turbine, helicopters, automatic vehicle transmission, power generators and aircrafts(D.N.Chorafas, 1990; F Elbarghathi et al., 2012) Condition Monitoring CM of such transmission is an essential task activity for much industrial field applications (Lou & Loparo, 2004). A gearbox faults are the precise cause faults in rotary machines and may also causes the personal injury accident and major economic losses(Fathalla Elbarghathi et al., 2013; R.B., 1980). On-line condition monitoring of transmission systems and discovered the failures at early stage this will leads to the safe operation for rotary machines and avoidance the spectalation of any further risk accidents(Dalpiaz, Rivola, & Rubini, 2000).

Dynamic modelling of gearboxes signal vibration has recently increased the attention of researchers to develop more an analytical modelling analysis in the field of gearbox condition monitoring(Bartelmus, Chaari, Zimroz, & Haddar, 2010). In addition gears vibration modelling reveals in depth the sympathetic concept principle of mechanisms which creates vibration signal in the gearbox transmission system. Moreover, gears modelling could assist to exemplify any change of the dynamics features due to several types of gears faults(Khabou, Bouchaala, Chaari, Fakhfakh, & Haddar, 2011). With rapid growing concern concentration on gears transmission systems in industrial machines components, a high attention studies has been made to understand the gears dynamics modelling. Thus, too many papers have been published to study the behaviours of gear system dynamics.

Some studies have focused in the effects of fluctuating loads on the assessment of a planetary gearbox condition (Bartelmus, 2011; Bartelmus et al., 2010; Bartelmus & Zimroz, 2009a). Another paper studies have considering the gearbox dynamics parameters such as the varying in tooth stiffness and the coupling influencing both torsional and lateral vibration from gears and shaft to developed a classified gearbox models(Bartelmus, 2001; Howard, Jia, & Wang, 2001; Huang & Liu, 2000; Jia & Howard, 2006). Moreover, other investigation research concentrated on gears tooth
Condition Monitoring of Helical Gearboxes based on the Advanced Analysis of Vibration Signals

meshing which generates vibration in the transmission system (Parker, Vijayakar, & Imajo, 2000; Theodossiades & Natsiavas, 2000; Velex & Maatar, 1996). Furthermore, an interesting study has found that the vibration generated by gears meshing is modulated by variable of torque load (R. Randall, 1982). This work intends to be improves a modelling of a two stage helical gearbox transmission system. To study natural frequencies of stress contact gear of the transmission system, gears vibration features such as spectral components amplitude using mass-spring-damper SDOF/MDOF applied on gears system using appropriate stiffness function to signify the acting forces between two gears pair, and rotational stiffness and damping are also introduced to simulate the angular motion of gears and shafts.

6.3 Mesh Stiffness for Helical Gear Tooth Contacting

There are many Parameters that should be considered in mind for building up the mathematical model of meshing forces of two direct contact rotating gears, these parameters are such as gear geometry, tooth’s number, moment of friction and mesh stiffness (Bartelmus, 2001). In other hand, for signifying the helical gears vibration signal, the essential factors which has to be consider to establish and easy manage the model construction is mesh stiffness \( k(t) \) The model has simulating a gears failure by introducing broken tooth with different percentage of severity. Moreover, for any two direct contacting gears the meshing period which is defined as a function of time can be express as \( T/z_1 \), the element \( z_1 \) is the pinion teeth number and \( T \) is the shaft and pinion period time (s) The angular displacement is defined as the mathematical function of time for both pinion and shaft is approximately various between \( 0 - 2\pi/z_1 \). To process the numerical simulation for gears model the mesh stiffness concept should be consider in account as well. The mesh stiffness is known as a ratio between the following the acting force on the line of action and the displacement of tooth on the line of action at the moment. Mostly, there are a mixture of several parameters are regulating either increase or decrease the stiffness amplitude of a pair of contact meshing gears, these factors such as two teeth bending stiffness, Hertzain contacts stiffness and two direct contact teeth stiffness (Amabili & Rivola, 1997). To compute \( k(t) \) for gears model numerical simulation these factors should be consider and studies. Figure 6.1 shows the similarities of the tooth mesh stiffness to this study there for the tooth mesh stiffness shape almost same to Figure 6.1. The mesh stiffness shape is based on used of piecewise-linear foundation to characterize the transient rate stiffness through the two gears mesh in and mesh out, As can we noticed from Figure 6.1 that when pair of gears meshing the stiffness amplitude exciting to increase rapidly then remains constant in the middle part.
of gear tooth the highest rate of signal gear pair is about $10^7$ N/m. In addition the mesh stiffness stays constant at any two gears in contacting the reason for any lack of stiffness located in the contact gear tooth is increasing as result of rotating contacting point along each tooth either move up or move down. Mainly through the gears rotation motion are based on as the gears are rotating which lead to variation of gears teeth contact point line that causes the effect on the variation of contact line. Consequence will be creating a variation in mesh stiffness. In addition , in the application of spur gears are the contact ratio is small , mainly that lead that a mesh stiffness is highly varying as a result of applied carrying load on contact tooth of pair gears which is depend might be applied on the signal tooth or double tooth for each gears pair . On other hand compare with helical gears the variation in mesh stiffness is small at any gears pair meshes for the reason that a high contact ratio between the teeth gear pair meshing.

In this research study the meshing stiffness has been introduced as a function of an angular displacement which is computed through the gear pair meshing for each adjacent tooth, to compute from the two main factors such as contact ratio $\varepsilon_c$ factor and overlap factor $\varepsilon_a$. Moreover, $\varepsilon_c$ factor is usually scales the contact between gear pair teeth in transvers direction, the other $\varepsilon_a$ overlap ratio is functioning to scales in an axial direction the overlap of adjacent of gear pair contact teeth.

![Figure 6-1 Stiffness function of a single pair of teeth](image)

In this study a gears pair are meshing in the variable number of gears teeth in contact either they are in maximum or minimum gear teeth in contact this will be essential source of gears signal vibration due of this variable is about $2\pi/\tau_1$, the vibration amplitude shape is depended on the stiffness amplitude that control by the amount of gear pair teeth contacting on minimum or maximum of gear teeth in contact will be varying with an angular position. Mostly, in the industrial application the
major failure for gear pair is the broken teeth with different percentage of severity, various from small tooth damage to large which cause a variable in meshing stiffness.

In this study a model of helical with a simulated broken tooth with different percentage of tooth broken severity as follow 25%, 50%, 75% and 100% respectively. Figure 6.2 has revealed the effect of tooth damage on the meshing stiffness. From the Figure 6.2 shows that a simulated broken tooth with different severity is an essential causes to decrease in the stiffness amplitude. Furthermore, as an increment of broken tooth fault damage severity percentage this will be lead to reduce stiffness value with low amplitude, consequently is a dramatic decrease in stiffness which this will be a main creating in increasing of the gear vibration signal amplitude.
Figure 6-2 The meshing stiffness function $k_1$ and $k_2$ for the pinion and the gear of the first pair of gears (within one shaft period) shown in blue and red as a function of angular displacement $\theta$. Here (a) is the healthy case and (b), (c), (d) and (e) show various severity of faults 25%, 50%, 75% and 100% due to a broken tooth.
6.4 Dynamic Model of a Helical Gearbox

Gearboxes play an important role in industrial field applications for power transmission. Therefore, fault detection at early stage to avoid the further failure consequences as well as identification the gear damage severity is a challenging task in gear fault diagnosis. In this research study, a nonlinear dynamic model of two-stage helical involving time-varying mesh stiffness, transmission error, and backlash is established. The key object of an application a mathematical model on two stages helical gearbox in this study is to exploring the gears vibration signals features and presents them in both frequency domain and time domain. Moreover, in the past two decades this area of dynamic gears model has attracted too many researchers whom interesting to design and developed a systems for investigating a gears faults diagnosis based on applying using of a torsional and lateral vibration on gear dynamics model. (Bartelmus, 2001; Howard et al., 2001; Huang & Liu, 2000; Vinayak & Singh, 1998).

In this study it can be seen from the flow-chart shows of the gearbox dynamic modelling procedures in Figure 6.3 and in schematic diagram is shown in both following Figures 6.4 represents a model for two stage gearbox that molded a shaft and gear pair for torsional vibration. To simplified the model for considering a vertical vibration where the two gears are directly attached to the casing represented by two mass/stiffness feature. In addition, this model is introduced a Multiple Degree Of Freedom (MDOF), based on employing a mass-spring – damper system referred on the past study of Bartelmus (Bartelmus, 2001). Where in this study has considered mesh stiffness function $k_1$ and $k_2$ between two gear pair, and two gear broken tooth fault with several percentage severity has been introduced to the model. Damping is function in this model as a most relative to mesh stiffness to denote the acting forces between the gear pairs.

The mathematical model presents and simulates vibration signal to investigate vibration features of gearbox and present them in time and frequency domain. The model simulate at two stage gear test rig system using a suitable stiffness function to represent forces acting each contact of gears using Mass-Spring-Damping system. It is mainly is based on newton’s second law ($F=ma$) and for rotational movement $I\ddot{\theta} = \sum M$ and transitional movement $m\ddot{x} = \sum F$. The multiple degree of Freedom (MDOF) which using Mass-Spring-Damper is used.
The torsional equation of motion may be expressed by statement according to Newton’s second law.

\[ I_m \ddot{\phi}_1 + K_1 (\phi_1 - \phi_2) + C_1 (\dot{\phi}_1 - \dot{\phi}_2) = T_M \]  

(6.1)
\[ l_{p1}\dot{\phi}_2 = M_1 + M_{1t} - r_1(F_1 + F_{1t}) + M \]  
\[ (6.2) \]

\[ l_{g1}\dot{\phi}_3 = r_2(F_1 + F_{1t}) - M_2 - M_{2t} \]  
\[ (6.3) \]

\[ l_{p2}\dot{\phi}_4 = r_{g1}(F_1 + F_{1t}) - M_2 - M_{2t} \]  
\[ (6.4) \]

\[ l_{g2}\dot{\phi}_5 = M_3 + M_{3t} - r_3(F_2 + F_{2t}) - M_L \]  
\[ (6.5) \]

\[ l_t\dot{\phi}_6 = K_3(\phi_5 - \phi_6) + C_3(\dot{\phi}_5 - \dot{\phi}_6) - T_L \]  
\[ (6.6) \]

\[ m_1\ddot{x}_1 + k_{t1}x_1 + C_{t1}\dot{x}_1 = F_1 + F_{1t} \]  
\[ (6.7) \]

\[ m_2\ddot{x}_2 + k_{t2}x_2 + C_{t2}\dot{x}_2 = -(F_1 + F_{1t}) \]  
\[ (6.8) \]

\[ m_3\ddot{x}_3 + k_{t3}x_3 + C_{t3}\dot{x}_3 = F_2 + F_{2t} \]  
\[ (6.9) \]

\[ m_4\ddot{x}_4 + k_{t4}x_4 + C_{t4}\dot{x}_4 = -(F_2 + F_{2t}) \]  
\[ (6.10) \]

Where the values of forces and moments are given by,

\[ M_1 = K_1(\phi_1 - \phi_2) \]

\[ M_{1t} = C_1(\dot{\phi}_1 - \dot{\phi}_2) \]

\[ M_2 = K_2(\phi_3 - \phi_4) \]

\[ M_{2t} = C_2(\dot{\phi}_3 - \dot{\phi}_4) \]

\[ M_3 = K_3(\phi_5 - \phi_6) \]

\[ M_{3t} = C_3(\dot{\phi}_5 - \dot{\phi}_6) \]

\[ F_1 = K_{z1}(r_{p1}\phi_2 - r_{g1}\phi_3) \]

\[ F_2 = K_{z2}(r_{p2}\phi_4 - r_{g2}\phi_5) \]

\[ F_{1t} = C_{z1}(r_{p1}\dot{\phi}_2 - r_{g1}\dot{\phi}_3) \]

\[ F_{2t} = C_{z2}(r_{p2}\dot{\phi}_4 - r_{g2}\dot{\phi}_5) \]
Where:

\( I_{p1}, I_{p2}, I_{g1}, I_{g2} \)  
Pinions and gears moment of inertia

\( F_1, F_2 \)  
Gearing stiffness forces

\( F_{1t}, F_{2t} \)  
Gearing damping forces

\( M_1, M_{1t} \)  
Internal moment stiffness damping first stage

\( M_2, M_{2t} \)  
Internal moment stiffness damping second stage

\( M_3, M_{3t} \)  
Internal moment stiffness damping third stage

\( \varphi_1 \)  
Motor angular displacement

\( \varphi_2 \)  
First pinion angular displacement

\( \varphi_3 \)  
First gear angular displacement

\( \varphi_4 \)  
Second pinion angular displacement

\( \varphi_5 \)  
Second gear angular displacement

\( \varphi_6 \)  
Load angular displacement

\( \dot{\varphi}_1, \dot{\varphi}_2, \dot{\varphi}_3, \dot{\varphi}_4, \dot{\varphi}_5, \dot{\varphi}_6 \)  
Angular velocities

\( \ddot{\varphi}_1, \ddot{\varphi}_2, \ddot{\varphi}_3, \ddot{\varphi}_4, \ddot{\varphi}_5, \ddot{\varphi}_6 \)  
Angular Acceleration

\( r_1, r_2, r_3 \)  
Pinions radius

\( r_{p1}, r_{p2} \)  
Pinions radius

\( r_{g1}, r_{g2} \)  
Gear radius

\( K_1, K_2, K_3 \)  
Stiffness
$C_1, C_2, C_3$ Damping

$K_{t1}, K_{t2}, K_{t3}$ constant parameters

$C_{t1}, C_{t2}, C_{t3}$ constant parameters

$K_{z1}$ First stage gearing stiffness (meshing stiffness)

$K_{z2}$ Second stage gearing stiffness (meshing stiffness)

$C_{z1}, C_{z2}$ Damping coefficient

The meshing stiffnesses can be computed by $k_{z1} = k_1 k_1(\theta)$ and $k_{z2} = k_2 k_2(\theta)$ where $k_1$ and $k_2$ are the piecewise linear functions for the angle of shaft rotation $\theta$, as mentioned in the previous section and $k_{t1}$ and $k_{t2}$ are constant parameters (see Table 6.1). Damping coefficients are assumed to be proportional to the stiffnesses, i.e. $c_{z1} = \mu_1 k_{z1}$ and $c_{z2} = \mu_2 k_{z2}$, where $\mu_1$ and $\mu_2$ are shown in Table 6.1

<table>
<thead>
<tr>
<th>Parameters</th>
<th>values</th>
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</thead>
<tbody>
<tr>
<td>Shaft one stiffness ($k_1$)</td>
<td>$0.3 \times 10^8$ N/m</td>
</tr>
<tr>
<td>Shaft two stiffness ($k_2$)</td>
<td>$0.4 \times 10^8$ N/m</td>
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<tr>
<td>Shaft three stiffness ($k_3$)</td>
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<td>Shaft one damping coefficient ($C_1$)</td>
<td>$7.7227 \times 10^3$ Ns/m</td>
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<td>Shaft two damping coefficient ($C_2$)</td>
<td>$7.2938 \times 10^3$ Ns/m</td>
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<tr>
<td>Shaft three damping coefficient ($C_3$)</td>
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<td>First stage gearing stiffness ($K_{z1}$)*</td>
<td>$1.0006 \times 10^7$ N/m</td>
</tr>
<tr>
<td>Second stage gearing stiffness ($K_{z2}$)*</td>
<td>$1 \times 10^7$ N/m</td>
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<td>Input motor momentum ($M_m$)</td>
<td>71.75 N.m</td>
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<tr>
<td>output generator momentum ($M_L$)</td>
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<td>First shaft mass ($m_{s1}$)</td>
<td>1.42 Kg</td>
</tr>
<tr>
<td>Parameters</td>
<td>values</td>
</tr>
<tr>
<td>------------</td>
<td>--------</td>
</tr>
<tr>
<td>Second shaft mass ((m_{s2}))</td>
<td>0.95 Kg</td>
</tr>
<tr>
<td>Third shaft mass ((m_{s3}))</td>
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<tr>
<td>Upper bearing mass ((m_{bu}))</td>
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<tr>
<td>Lower bearing mass ((m_{bl}))</td>
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<td>First stage gear 1 pinion mass ((m_{p1}))</td>
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<td>First stage gear 2 mass ((m_{g1}))</td>
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<tr>
<td>Second stage gear 3 pinion mass ((m_{p2}))</td>
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<td>Second stage gear 4 mass ((m_{g2}))</td>
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<td>Overlap ration ((\varepsilon_a))</td>
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<td>Base radius of shaft 1 ((r_{s1}))</td>
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<tr>
<td>Base radius of shaft 2 ((r_{s2}))</td>
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<tr>
<td>Base radius of shaft 3 ((r_{s3}))</td>
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<td>Base radius of pinion 1 ((r_1))</td>
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<tr>
<td>Base radius of pinion 2 ((r_2))</td>
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</tr>
<tr>
<td>Base radius of pinion 3 ((r_3))</td>
<td>0.01525m</td>
</tr>
<tr>
<td>Base radius of gear 1 ((r_{g1}))</td>
<td>0.0117m</td>
</tr>
<tr>
<td>Base radius of gear 2 ((r_{g2}))</td>
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</tr>
<tr>
<td>Moment of motor ((I_m))</td>
<td>8.4759x10^{-4}</td>
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<tr>
<td>Moment of load system ((I_l))</td>
<td>33.88x10^{-4}</td>
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<tr>
<td>Pinion shaft stiffness ((k_p))</td>
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<tr>
<td>Gear shaft stiffness ((k_g))</td>
<td>2.0x10^6 N/m</td>
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<tr>
<td>Stiffness of the upper &amp; lower bearing ((k_{bu}) &amp; (k_{bl}))</td>
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</tr>
<tr>
<td>Damping coefficient of the upper &amp; lower bearing ((C_{bu}) &amp; (C_{bl}))</td>
<td>1.8x10^5 Ns/m</td>
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### Parameters and values

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>First stage gearing damping ($c_{z1}$)</td>
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</tr>
<tr>
<td>Second stage gearing damping ($c_{z2}$)</td>
<td>831.1938</td>
</tr>
<tr>
<td>First stage gearing damping coefficient ($\mu_1$)</td>
<td>$4.357 \times 10^{-5}$</td>
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<tr>
<td>Second stage gearing damping coefficient ($\mu_2$)</td>
<td>$8.3119 \times 10^{-5}$</td>
</tr>
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<td>Scaling factor for damping in the calculation of $c_p$, $c_g$, $c_{z1}$ ($\zeta$)</td>
<td>4</td>
</tr>
<tr>
<td>Moment of inertia of shaft one ($M_1$)</td>
<td>0.0032 Kg.m$^2$</td>
</tr>
<tr>
<td>Moment of inertia of shaft two ($M_2$)</td>
<td>0.00013 Kg.m$^2$</td>
</tr>
<tr>
<td>Moment of inertia of shaft three ($M_3$)</td>
<td>0.00078 Kg.m$^2$</td>
</tr>
<tr>
<td>Moment of inertia of gear one ($I_{p1}$) stage one</td>
<td>0.000042 Kg.m$^2$</td>
</tr>
<tr>
<td>Moment of inertia of gear two ($I_{g1}$) stage one</td>
<td>0.00073 Kg.m$^2$</td>
</tr>
<tr>
<td>Moment of inertia of gear three ($I_{p2}$) stage two</td>
<td>0.000154 Kg.m$^2$</td>
</tr>
<tr>
<td>Moment of inertia of gear four ($I_{g2}$) stage two</td>
<td>0.74 Kg.m$^2$</td>
</tr>
</tbody>
</table>

#### 6.5 Numerical Implementation of the Model

The main focus of this research work is to develop a mathematical model of a two stage helical gearbox system, and a comparison of its model-predicted vibratory behaviour with observed experimental behaviour. The model represents the two-stage helical gear system using a stiffness function to represent the forces acting between two pair of gears, and rotational stiffness and damping are also used to simulate the angular motion of the gears and shafts.

In this study, a nonlinear dynamic model of two-stage helical involving time-varying mesh stiffness, transmission error, backlash is established. Then, the governing equations are solved by Runge-Kutta method. Moreover, the widespread strong numerical method which has introduced in this mathematical model to solve a differential equation is Runge-Kutta method. The basic theory principle for Runge-Kutta method it is in form of first order differential equation, there for all system second order differential equation have to be converted as a system of first order differential equations based on using of state-space expansion (Dormand & Prince, 1980). Matlab has been used to solve the model equations by iteratively adapting the parameters, and the approach...
illustrates that such a model, although fairly simple, can replicate accurately the dynamics of a real gearbox. In addition, according the MATLAB has exhibited several function Ode45-Ode115s as solver to solve differential equations and complete a signal run. However, Ode45 function is delay in time to complete the signal run about an hour; in this case Runge-Kutta method is not appropriate to carried out a solution for these equations. Therefore, as per MATLAB catalogue instruction the best function with reliable period of time to solve all differential equation is Ode115s is selected for the reason that is more faster to solve all differential equations to complete the signal run with less than 10 minutes, so according to this reason the Ode115s function has been employed to use in this study simulation study. However, in this study simulation is considering the rotational movement which is including the rotational for both input and out shafts and gears. Moreover, to achieve a convenient vibration signal spectral analysis with enough data size the simulation has to run an approximately of input shaft is around 15 turns and for output shaft greater than 4 turns. Therefore, the amount of this data size is suitably for get a logical spectrum with reasonable resolution for high frequency to classify between their different side band constituent. The following flow-chart shows the implementation of the model Figure 6.5.
Condition Monitoring of Helical Gearboxes based on the Advanced Analysis of Vibration Signals

Figure 6-5 Dynamic model flowchart.

Input
Gearbox Parameters

Computing the gearbox model parameters
- Stiffness, K
- Damping Coefficient, C
- Moment of Inertia, I

Creating the gearbox model in different equations based on the number of DOF

Computing matrix equations based on state space
\[ \dot{X}(t) = [A]X(t) + [B] u(t) \]

Solving problem Outcome
- to predict natural frequency
- to assess the gearbox

Matching the predict and measured values

Nonlinear

Tooth breakage effect
- Representing two stage gear system using suitable stiffness function
- Representing the force action between two pairs of gears
- Rotational stiffness and damping are introduced to simulate angular motion of gear and

Adapting odds functions to solve problem numerically

Time and frequency domains of different DOF

Assess the effect of simulated tooth breakage on vibration signals
Figure 6.6 shows a characteristic result of the input shaft angular speed which is running under different operating condition.

The Figure 6.6 has revealed a remarkable simulation operation transient response at the commencement of the simulation, which required to be cancelled for spectral calculation and waveform extraction. In other hand this Figure 6.6 result for steady state is reasonably constant variable characteristic for rotating input shaft output shaft. Therefore, this result has given evidence that the employed function Ode115s solver can perform a promise result solution for this current model simulation.

![Figure 6-6 Angular Speed of the Input shaft](image)

6.6 Model Parameter Tuning and Calibration

The gearbox manufacturing specifications are usually employed to measure the system model factors such as masses and shaft stiffness. Moreover, due to the difficulties of gears tooth shape this will be also create another difficulty to be getting the model mesh stiffness. In addition, based on an advanced software to analysis gear tooth stiffness named Finite Element Analysis, the estimated stiffness amplitude is order of $10^8$ N/m (Du, Randall, & Kelly, 1998). Furthermore, the exact value can be estimated by validate the simulation results with measured result in the frequency domain.
However, all model factors have been calculated based on the validation between both the predicted and measured waveform and spectrum.
As it can been seen the following two Figures respectively Figure 6.7 and Figure 6.8 shows the results for model running under high operating condition (50% load and 100% of speed). Both results have revealed in Figures 6.7(a)-6.8(a) shows the results for predict and measured vibration waveform presented in time domain through a period between 0-1.5 s both Figures 6.7(a)-6.8(a) have a closed competitive waveform signal at same operating condition. And because some contaminated noise background during the measuring signal from the real test rig this can be realised the reason that why the amplitude of measured signal is more greater than the predict signal.

Figure 6.7(b)-6.8(b) both shows spectra signature results for predicted and measured signal results of gear model system running under same operation condition, both Figures have express of spectrum signature of pinion simulation with no simulated fault as a healthy case. These Figures component consist of fundamentals frequencies $f_{m1}$ and $f_{m2}$ and their harmonics which are physically can been seen in both figures as follow $f_{m11}, f_{m12}, f_{m13}, f_{m21}, f_{m22}, f_{m23}$. These predicted and measured vibration spectrum signatures were collected from one position. From these it can be seen a huge harmonics round the sideband harmonics of measured signal compare to the predicted simulated spectrum signature.

This is because of the same cause has been mentioned earlier, due to contaminated test rig background noise during the collected real measured data. Moreover, Both Figures 6.7(b)-6.8(b) their spectrum signature results shows a close similarities for the model and real vibration signal are presented in time and frequency domain for gear model without any simulated fault a healthy case as a baseline case.

### 6.7 Simulation Study of Faulty Gears

Following Figures shows the outcome results of model gear transmission systems simulation with simulated broken tooth 25%, 50%, 75% and 100% compared with the baseline at different operating condition. This gear transmission system construction and equation has been defined in details in section 6.4. However, these gained results are much useful for understanding a proposed gear system model consistent a vibration features for the model with simulated fault compared with a baseline with a suitable precision. Figures 6.9(a) – 6.9(b) have revealed the model gear transmission system a predicted vibration signal spectra signature in frequency domain for both healthy and proposed simulated gear broken tooth 25% and 50% were located on the pinion gear number 1.
However, these spectral signature results were collected under proposed different operating condition such as a full speed and full load. From 6.10 zoomed of the spectral signature has display a signal of the pinion 1 which direct connected with a motor shaft rotor, this Figure 6.10 shows the fundamental frequency with up to third harmonic it has showed a clear indication of simulated fault 25% compare with basilen and Figure 6.11 shows a sidebands around the first meshing frequency.

Figure 6-9 Predicted spectrum of the vertical acceleration of position one and two of a faulty gear

Figure 6-10 Zoomed predicted spectrum of the vertical acceleration of position one and two of a faulty gear
Condition Monitoring of Helical Gearboxes based on the Advanced Analysis of Vibration Signals

**Figure 6-11 Sidebands around the first meshing frequency (fm11)**

Figures 6.12(a)-6.12(b) displayed the same features for a vibration signal signature of pinion number 1 and gear number 1, moreover. It can be seen from zoomed of Figure 6.13(a) the fundamental frequency with it’s with a third harmonic for 25% compare with baseline. I addition, in Figure 6.13(b) has the same vibration features in frequency domain it exhibit a clear fundamental frequency with a multiple harmonics and Figure 6.14 shows a sidebands around the first meshing frequency, this has given an evidence the proposed model is worked with a simulated faults compare with a baseline can be work under different operating condition.
Figure 6-12 Predicted spectrum of the vertical acceleration of position one faulty gear

Figure 6-13 Zoomed predicted spectrum of the vertical acceleration of position one faulty gear
Figure 6-14 Sidebands around the first meshing frequency (fm11)

Figure 6.15(a) and Figure 6.15(b) shown the same features with different simulated broken tooth 50% and 100 % at proposed different operating condition the vibration spectra signature shows the fundamental frequency with up to third harmonics, in zoomed Figure 6.15(a) – Figure 6.15(b). Furthermore, from zoomed Figure 6.16 it can be seen the clear different between simulated 50% and 100 % with the basilen it exhibit the difference in the frequency sideband and Figure 6.17 shows a sidebands around the first meshing frequency.

Figure 6-15 Predicted spectrum of the vertical acceleration of position two faulty gear.
Summary

In this study a mathematical model has been introduced to simulate gears that generates vibration signal. A model is constructed to simulate a two gear transmission system based on using a stiffness function to represent the acting forces between each gears pair. The model has been applied for
different severities whereas in previous researches consideration, one complete gear tooth removing. In addition, the model has properties of two stage helical gearbox which includes all systems with motor and generator. The developed Two–stage gearbox mathematical model has been examined under a number of different operating conditions, which is the novelty of this work. The mathematical simulation showed the meshing stiffness function \( k_1 \) and \( k_2 \) for both gear and pinion and the gear were affected by the increased gear fault severities.

Rotational stiffness and damping are also used to simulate the angular motion of gears and shafts. Then the governing equations are solved by Runge-Kutta method. The results show that the dynamic model can be utilized in investigation gear faults and would be useful in developing gear fault monitoring techniques.
CHAPTER SEVEN

CONCLUSION AND RECOMMENDATIONS FOR FUTURE WORK

This chapter summarises the achievements of the research and explains how the objectives stated in chapter one were achieved. It includes a summary of the author’s contributions to knowledge and the novel aspects of the research work, and conclusion made. Finally, recommendations for future work in technologically advanced condition monitoring of gear transmission systems are provided.
7.1 Introduction

The main aim of this study is to develop gearbox condition monitoring using a new vibration analysis technique based on the continuous wavelet transform (CWT). This has been achieved, tested and shown to be an improvement on other methods.

As a first step several types of wavelets were studied, with a focus on wavelet properties to assist in the selection of a suitable wavelet for gearbox fault diagnosis. In particular, a major contribution of this study, presented in Chapter 5, has been to address a significant challenge facing wavelet analysis: adapting the parameters of the mother wavelet to the time variance of the given signal. This study has proposed an efficient approach to gearbox diagnosis, using an adaptive Morlet wavelet transform method based on the information entropy optimization. A comparative study of the proposed method and a previous study which used kurtosis maximization to adapt the wavelet parameters were also carried out to further assess the suggested method. The results of this investigative study showed that the proposed approach is very suitable for gearbox fault diagnosis.

A mathematical model of a two stage helical gearbox system has also been developed and a comparison of predicted vibratory behaviour with observed experimental behaviour carried out. This dynamic modelling is aimed at stimulating faults in the two stage helical gearbox and predicting the resulting vibration signals. The model predicts the vibration signal produced by mechanical contact forces between gears in a gear transmission system. Such modelling is useful in classifying vibration signals produced by different gear faults.

This thesis presents extensive results to demonstrate the effectiveness of the proposed methods.

7.2 Review of Aim, Objectives and Achievements

The research was concentrated on the condition monitoring of a gearbox transmission system. Both experimental and theoretical investigations were performed on the vibration signals obtained from the gearbox. As stated above this research programme was successful in achieving its main aim of developing a new vibration analysis technique based on applying CWT to vibration signals for gearbox condition monitoring.

The achievements of this research correlated with the objectives as presented in Chapter 1 are detailed below.

Objective 1: To review and study the condition monitoring of rotating machinery.
Achievement 1: The understudying of CM of rotating machinery has been discussed in Chapter 1, Section 1.1. Section 1.3 discussed the importance of maintenance and fault detection. Section 1.4 discussed gearbox CM, and Chapter 2 summarised maintenance strategies in Sections 2.1 and 2.2. The aim and objectives listed in Section 1.5 provided the structure for this research.

Objective 2: To describe the principles of gearbox transmission systems, including gear type and classification, gear vibration characteristics and common gear failure modes.

Achievement 2: The detail of a two stage helical gear transmission system has been presented clearly. A summary overview of gears is given in Chapter 2. Section 2.6 described gear classification and types. Section 2.7 discussed gear failure modes and Section 2.8 reviewed gear failure types and a statistical survey of relative occurrence. Section 2.9 described, in brief, gear vibration features.

Objective 3: To review the literature describing conventional signal processing methods of gearbox fault detection and diagnosis based on analysis of the vibration signals.

Achievement 3: The analysis of vibration based gear fault detection and diagnosis techniques was reviewed in Chapter 2, Section 2.11: time domain, frequency domain and wavelet transform were reviewed in Sections 2.11.1, 2.11.2 and 2.11.3 respectively.

Objective 4: To become familiar with a two stage helical gearbox and experimental procedures for condition monitoring of the transmission system.

Achievement 4: The test rig facility and data acquisition software were presented in detail in Chapter 3. A two stage helical reduction gearbox manufacturing by David Brown Radicon Limited, see Figure 3.1, was chosen for this research project. A number of measurement transducers were used on the test rig to monitor the gearbox for CM purposes. In addition, the test rig was used to provide experimental data for testing the mathematical modelling of a two-stage helical gearbox (Chapter 6). As the rig has a typical drive system, datasets are considering representative for CM practices.

Objective 5: To introduce a specific fault into one of the gearboxes; e.g. a broken tooth located on the gear (pinion) in the first stage of the two stage helical gearbox transmission system. To investigation the effect of different levels of fault severity on the gearbox vibration signal by
introducing a series of quantified faults simulating different gear fault severities; e.g. tooth breakage of 25%, 50%, 75% and 100%, the last being complete removal of one tooth on the first stage pinion.

**Achievement 5:** Gear tooth breakage was the fault simulated in this study test rig. The seeded fault was achieved by removing an increasing percentage of one tooth from a first stage pinion, see Section 3.3. The experimental tests were carried out under different operating conditions. The experiments were repeated with different levels of fault severity and the data collected both locally and remotely. All data were collected during repeated experiments with and without simulation faults.

**Objective 6:** To continuously review the current literature on the use of vibration techniques for gearbox CM, especially gearbox fault detection methods based on using advanced signal processing technique such as the CWT.

**Achievement 6:** Relevant monitoring and analysis techniques are summarised and discussed in Chapter 4 together with a description and discussion of Time Synchronous Averaging (TSA) as applied to the vibration signal from the gearbox. The TSA signal was then processed in the joint time–frequency domain using a CWT for incipient fault detection and diagnosis.

The vibration signals were from an accelerometer mounted on the gearbox casing and were recorded for different gear conditions (healthy and faulty) under different operating conditions. Three commonly used wavelet transform families; Daubechies order 1, Symlets order 2 and Coflets order 3 were assessed for their usefulness in incipient fault detection based on the TSA signals. The statistical measures kurtosis and RMS were applied to the CTW to determine a method by which the optimal transform could be selected.

**Objective 7:** To collect and analyse the gearbox vibration signal data from healthy and faulty gears at different operating conditions using advanced signal processing techniques such as the Morlet wavelet transform adapted by information entropy difference, and thus obtain an effective set of features for detecting and diagnosing the seeded gear tooth faults.

**Achievement 7:** Relevant wavelet analysis techniques are summarised and discussed in Chapter 5. The chapter presents and proposes a new adaptive Morlet wavelet transform method based on maximum wavelet entropy difference. It is shown to be an effective tool for fault detection of faults seeded into the two-stage helical gearbox. The TSA reduced background, random noise and
revealed the fault related impulse components and provided the basis for accurate feature extraction. Next, maximum Shannon entropy difference was used to optimize the central frequency and bandwidth parameters of the Morlet wavelet in order to achieve optimal match with the impulsive components, and to extract the features of the gear faults. The results show that the proposed method is better than adapting the Morlet based on kurtosis maximisation.

**Objective 8:** To create and develop a mathematical model to simulate two stages helical gearbox vibration signals for healthy and faulty gears operating under different conditions. This will help to characterise changes of dynamic properties due to various types of gear faults, which assists measurement setup, data processing selection and diagnosis rule development.

**Achievement 8:** A major focus of this research was the development of a comprehensive mathematical model of a two-stage helical gearbox system, and to compare model-predicted vibratory behaviour with observed experimental behaviour. This has been achieved.

The model, which represents the two-stage helical gear system uses a stiffness function to represent the forces acting between pairs of gears in contact, and rotational stiffness and damping to simulate the angular motion of the gears and shafts. The governing nonlinear equations were solved using a Runge-Kutta method. The model predicts an acceleration spectrum with peaks at existing meshing frequencies and their harmonics. Moreover, the simulated gear transmission system with seeded faults (different levels of tooth breakage on the first pinion) at different loads and speeds, produced results which show strong similarities with the results obtained during the experimental investigation. The results suggest that the dynamic model could be used in studying gear faults and would be useful in developing gear fault monitoring techniques, see Chapter 6.

**Objective 9:** To suggest and recommended a guideline for further research work activities in this field.

**Achievement 9:** A number of suggestions are provided for future work, see Section 7.6 below. In particular, adapting the techniques developed in this research to study the CM of gearboxes working under different operating condition with a range of simulated faults. In addition, the model developed for a two-stage helical gearbox is now ready to be extended for use with multi-stage gearboxes.
7.3 Conclusion on the Gear Transmission System Using vibration Analysis

It is possible to summarize the work presented in this thesis as:

7.3.1 Experimental Study

The vibration signal analyses was carried out for the gear transmission system experiments have concluded in the following:

1. The analytical investigation of gear transmission vibration signals under different operating conditions has shown evidence that the signal is a bank of unknown information associated with the waveform of the signal. In addition, the vibration signal amplitude varies with the operating conditions, which means limited information could be gained from the vibration waveform.

2. Joint time–frequency analysis is a more powerful sensitive tool for exploring and identifying the gear faults symptoms and upgrading the capabilities of this technique means it can be used for the early detection of incipient gear faults. In addition, for better analysis of the measured gear vibration signal, it is recommended that random noise is suppressed by using such methods as TSA.

3. Using TSA suppresses asynchronous vibration leaving only the repeated vibration generated by the rotation of the gear.

4. The combined TSA-CWT provided an analysis of the gearbox vibration signal which was very suitable for CM.

5. Although the wavelet analysis is a powerful tool and has been widely used for vibration signal based gearbox fault diagnosis it does not possess time invariance, which may result in decreased accuracy of fault diagnosis. (Repetition removed.) To overcome this limitation on the wavelet transform (choice of suitable threshold and wavelet function) an adaptive Morlet wavelet transform method based on the information entropy optimization was introduced. A comparison of this method with a previous study which used kurtosis maximization to adapt the wavelet parameters was carried out and showed entropy optimization to be the better method.

7.3.2 Conclusion on the Gear Transmission System Model

1. The two–stage helical gearbox simple mathematical model developed as a part of this research has been examined under a number of different operating conditions.
2. A nonlinear dynamic model of a two-stage helical gearbox involving time–varying mesh stiffness and transmission error has been established. Then, the governing equations are solved by Runge-Kutta method. Based on experimental data collected from different operating torque loads and the simulating results, gear fault severities have been identified.

3. The mathematical simulation showed the meshing stiffness functions \( k_1 \) and \( k_2 \) for both gear and pinion and the gear were affected by increased gear fault severities.

4. The model was validated against a set of experimentally obtained results. It was found that predicted results, based on the model, were very similar to the experimental results.

5. The modelling included different levels of fault severity whereas previous researchers considering only the case of one complete gear tooth removed. In addition, the model of the two – stage helical gearbox included a complete system with motor and load.

7.4 Novel Feature Summary and Contribution to Knowledge

- **First Novel Feature:**
  The author of this thesis believes that the way he developed the mathematical model of a two-stage helical gearbox system, and compared model-predicted vibratory behaviour with observed experimental behaviour was novel.

  The model represents a two-stage helical gear system using a stiffness function to represent the forces acting between two pair of gears, and rotational stiffness and damping to simulate the angular motion of the gears and shafts.

- **Second Novel Feature:**
  This research is novel in presenting an integrated platform for performing gear fault simulation of a two stage helical gearbox under different loads, though at constant speed.

  It is a novel feature of this research that it suggests a method of determining which wavelet is optimal for comparison of vibration signals for the detection of a fault. The wavelet producing the highest RMS value of wavelet coefficients when applied to baseline data should be the wavelet used for fault detection and diagnosis. Based on this criterion the time-frequency pattern obtained by wavelet analysis was able to distinguish fault features on the basis of severity. This selection criterion does not needs any faulty data sets, which is more realistic for condition monitoring practices.
• **Third Novel Feature:**
This research has uniquely proposed a new adaptive Morlet wavelet method based on maximum wavelet entropy difference as an effective tool for fault detection and diagnosis in rotating machinery.

In this study, the fault detection of a two-stage helical gearbox has been successfully carried out by using a combination of TSA and adaptive Morlet wavelet. Maximum Shannon entropy difference is used to optimize the central frequency and bandwidth parameter of the Morlet wavelet in order to achieve optimal match with the impulsive components and extract the features of the gear faults. The author believes the methodology is novel as no work has been found regarding this advanced analysis of gearbox vibration signal under different gearbox operation conditions.

### 7.5 Contributions to Knowledge

**The First Contribution:**
The application of vibration analysis for early detection and diagnosis of tooth breakages in a two-stage industrial helical gearbox has not been undertaken previously, especially for varying load conditions.

**The Second Contribution:**
This thesis develops gearbox CM methodology by combining the use of TSA to suppress the random noise in a vibration signal and CWT to enhance the non-stationary nature of the fault signal for more accurate and robust fault diagnosis. Uniquely, it is suggested that the wavelet that produces the highest RMS value of the wavelet coefficient when it is applied to baselines data be selected as the optimal one for diagnosing faults.

**The Third Contribution:**
The third contribution to knowledge has been summarised above as the “Third Novel Feature”; that is, successful fault detection of a two-stage helical gearbox using a combination of TSA and adaptive Morlet wavelet with maximum Shannon entropy difference to optimize the central frequency and bandwidth parameters of the Morlet wavelet in order to achieve optimal match with the impulsive components and extract the features of the gear faults.
The Fourth Contribution:
The model represents the two-stage helical gear system using a time-varying stiffness function to represent the dynamic forces acting between two pair of gears, and rotational stiffness and associated damping coefficient are also used to simulate the angular and translational motions of the gears and shafts. Based on experimental data collected for different operating loads and the simulated results, gear fault severities can be identified.

The two–stage gearbox mathematical model developed as a part of this research has been validated under a wide number of different operating conditions and different fault severities, which subsequently used to identify fault severity by matching key vibration features between the measured and predicted one. In the opinion of the author this is the first time this has been done.

As stated previously, the model of the two–stage helical gearbox was incorporated with a complete transmission system that includes dynamics of both the driving motor and system load. Especially, the driving torque is considered to be ‘soft’ to simulate the torque and speed dynamics of an induction motor and result in more accurate vibration response of the gearbox.

The Fifth Contribution:
The author believe the research work in this thesis is the first work to investigate mathematical simulation, showing that the meshing stiffness function $k_1$ and $k_2$ for both pinion and the gear respectively can be modified mainly to represent the increased gear fault severities.

7.6 Recommendation for Future Work on Condition Monitoring of Gearboxes
The author now makes a number of recommendations for research to improve the CM of gearboxes using vibration analysis, by developing and applying more advanced analysis techniques for fault detection and diagnosis to more complicated datasets. These include:

- **Recommendation 1:** More experimental work be carried out on several gearboxes to further investigate the sensitivity and reliability of the suggest diagnosis method, including a single–stage gearbox and back-to-back gearbox. This will allow a more precise consideration of the effect of the path transmission on the vibration signal.

- **Recommendation 2:** In this thesis experiments were carried out using a helical gearbox which reduced variation in the outcome from the fault due the high contact ratio, distribution of the load over a greater area, which thus reduced the induced vibration. More experimental work tests need to be carried out using different types of gears such spur gear with low
Condition Monitoring of Helical Gearboxes based on the Advanced Analysis of Vibration Signals

contact ratio so that load fluctuation created by a gear fault such as a broken tooth can be monitored.

- **Recommendation 3:** This research investigation study should be spread to contain different classifications of gearbox – e.g. Worm, Bevel and Planetary under different operating conditions with different simulated faults at different levels of severity.

- **Recommendation 4:** A data acquisition system that can rapidly gather and analyse huge amounts of data, and at the same time give a finer resolution, is required to enhance analysis of the vibration signature spectrums around the meshing frequency.

- **Recommendation 5:** This investigation should be expanded to introduce faults into the second stage for both pinion and driven gear, including pitting, tooth breakage, wear and fatigue for different operational condition with different levels of severity.

- **Recommendation 6:** The research should be expanded to gearboxes for other industrial rotating machinery; these might include gas and steam turbines, centrifugal pumps, reciprocating gas compressors, and diesel engines.

- **Recommendation 7:** This thesis had as an objective choosing the optimum base wavelet from standard wavelet families for gearbox vibration signal analysis. But gearbox vibration signals are usually affected by the surrounding background environment. For future research it is suggest designing a new base wavelet by taking the surrounding background environment of a gearbox with simulated defects into account and using the CWT for pattern recognition.

- **Recommendation 8:** After a classifying the gearbox fault severities, future research work should be expanded to establish a health index model that typifies failures and severities for a given level of gearbox. First diagnosing and then establishing the severity of the fault will assist in establishing likely remaining service life, so that the gearbox user can plan a timetable for any required maintenance work and to avoid any unexpected gearbox breakdowns.
References


Emeral. (21 August 2013 20:00 PM ). *Gear Tooth Breakage*


McFadden, P. D. (1987). A revised model for the extraction of periodic waveforms by time domain averaging,. *Mechanical Systems and Signal processing,, vol. 1*(1),, 83-95..


APPENDIX: A

Gearbox Specification

<table>
<thead>
<tr>
<th>Description</th>
<th>First Stage PG0740.8/M07E</th>
<th>Second Stage M07-24.5B-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reduction ratio</td>
<td>0.810</td>
<td>4.539</td>
</tr>
<tr>
<td>No. of teeth</td>
<td>58/47</td>
<td>13/59</td>
</tr>
<tr>
<td>Contact ratio</td>
<td>1.450</td>
<td>1.469</td>
</tr>
<tr>
<td>Overlap ratio</td>
<td>2.890</td>
<td>1.289</td>
</tr>
<tr>
<td>Helix angle</td>
<td>27°</td>
<td>13°</td>
</tr>
<tr>
<td>Circular pitch at reference diameter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal</td>
<td>3.927</td>
<td>6.283</td>
</tr>
<tr>
<td>Transverse</td>
<td>4.407</td>
<td>6.448</td>
</tr>
<tr>
<td>Circular pitch at running diameter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal</td>
<td>3.942</td>
<td>6.292</td>
</tr>
<tr>
<td>Transverse</td>
<td>4.428</td>
<td>6.485</td>
</tr>
<tr>
<td>Circular pitch at base circle diameter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal</td>
<td>3.690</td>
<td>5.904</td>
</tr>
<tr>
<td>Transverse</td>
<td>4.080</td>
<td>6.041</td>
</tr>
</tbody>
</table>

Two Stage Helical Gearbox

Manufacturer: David Brown, series M Radicon m.

Unit TYPE: M07202-5BMG1A, 11.4 AD
INPUT KW: 11KW
RATE: 2.5 A
ORDER No: 5032382
OIL GRADE: 320
ASSEMBLY: 1A

The gearbox is containing of two stage helical gear. The first – or primary – stage has two gears and the second stage involves of two gears as well.
## APPENDIX: B

### Gearbox Condition Monitoring Using Vibration Analysis

<table>
<thead>
<tr>
<th>Description</th>
<th>First Stage (Input Shaft)</th>
<th>Second Stage (Output Shaft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of teeth</td>
<td>$Z_1/Z_2=58/47$</td>
<td>$Z_3/Z_4=13/59$</td>
</tr>
<tr>
<td>Meshing frequency(Hz)</td>
<td>$F_{m12}=1416.36$</td>
<td>$F_{m23}=391.76$</td>
</tr>
<tr>
<td>Contact ratio</td>
<td>1.45</td>
<td>1.469</td>
</tr>
<tr>
<td>Transmission ratio</td>
<td>0.8103</td>
<td>4.5385</td>
</tr>
<tr>
<td>Shaft speeds(Hz)</td>
<td>Fr1 24.42 (input) at full load</td>
<td>Fr2 30.13 (middle) at full load</td>
</tr>
</tbody>
</table>
Characteristic Frequencies

Characteristic frequencies of the vibrations signal of the gear box can be calculated through the following Equations. And the first, second and third values of the shaft rotational frequencies: \( f_{r1}, f_{r2}, f_{r3} \) can be determined as

\[
f_{r1} = \frac{\text{Rotor Rotational Speed}}{60}
\]

\[
f_{r1} = \frac{1500}{60} \approx 25\text{HZ}
\]

\[
f_{r2} = (\frac{Z_1}{Z_2}) f_{r1}
\]

\[
f_{r2} = (\frac{58}{47}) 25 = 30.85\text{HZ}
\]

\[
f_{r3} = (\frac{Z_1}{Z_2} \cdot \frac{Z_3}{Z_4}) f_{r}
\]

\[
f_{r3} = (\frac{58}{47} \cdot \frac{13}{59}) 25 = 6.79\text{HZ}
\]

The meshing frequencies for both the first and second stage are computed as following equations

\[
f_{m1} = f_{r1} \cdot Z_1 = 25 \times 58 = 1450\text{HZ}
\]

\[
f_{m2} = f_{r2} \cdot Z_3 = 30.85 \times 13 = 401\text{HZ}
\]

Where

- \( f_{r1}, f_{r2}, f_{r3} \) Three first values of the rotational speed in Hz
- \( Z_1 \) Number of pinion teeth gear at first stage
- \( Z_2 \) Number of driven gear teeth at first stage
- \( Z_3 \) Number of pinion teeth gear at second stage
- \( Z_4 \) Number of driven gear teeth at second stage
Data acquisition card, type PCI 6221 Parameters technical specifications

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Channels</td>
<td>8 differential, 16 single ended</td>
</tr>
<tr>
<td>ADC resolution</td>
<td>16 bits</td>
</tr>
<tr>
<td>Sampling rate (maximum)</td>
<td>250 kS/sec</td>
</tr>
<tr>
<td>Timing accuracy</td>
<td>50 ppm of sample rate</td>
</tr>
<tr>
<td>Timing resolution</td>
<td>50 ns</td>
</tr>
<tr>
<td>Input voltage range</td>
<td>+/- 10V</td>
</tr>
<tr>
<td>Max working voltage</td>
<td>11 V</td>
</tr>
</tbody>
</table>
### Table C.1 ICP-type Accelerometer Specifications

<table>
<thead>
<tr>
<th>Model &amp; Serial Number</th>
<th>PCB Model no. 338C04 Accelerometer</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Frequency range</strong></td>
<td>0.5Hz to 10 kHz (± 5%)</td>
</tr>
<tr>
<td></td>
<td>0.3Hz to 12kHz (± 10%)</td>
</tr>
<tr>
<td></td>
<td>0.2Hz to 20kHz (± 3dB)</td>
</tr>
<tr>
<td><strong>Sensitivity</strong></td>
<td>100mV/g (± 10%)</td>
</tr>
<tr>
<td><strong>Temperature Range</strong></td>
<td>-53 to 93°C</td>
</tr>
<tr>
<td><strong>Excitation Voltage</strong></td>
<td>18-30 VDC</td>
</tr>
<tr>
<td><strong>Resonant Frequency</strong></td>
<td>≥ 35kHz</td>
</tr>
<tr>
<td><strong>Output Bias voltage Level</strong></td>
<td>8 to 12 VDC</td>
</tr>
<tr>
<td><strong>Broadband Resolution (1 to 10kHz)</strong></td>
<td>0.0018 m/s² RMS</td>
</tr>
<tr>
<td><strong>Discharge Time Constant</strong></td>
<td>0.8-2.4 sec</td>
</tr>
<tr>
<td><strong>Measurement Range</strong></td>
<td>±50g pk</td>
</tr>
</tbody>
</table>
APPENDIX: F

Photograph of Gear Fault Simulation with different severities

25% tooth breakage

50% tooth breakage

75% tooth breakage
APPENDIX: G

AC Motor

Model: V-DA160MJ, 0298659 (Electro-drives Company)

| Power = 11 kW (With a no-load motor base speed of 1492 RPM) | Speed = 1470 RPM |
| Voltage = 380 – 415 V | Winding: Y start |
| Current= 21.5A (at full load) | Number of Stator Slots: 48 |
| Phase: 3 | Number of Rotor Slots: 40 |

Two Stage Helical Gearbox

TYPE: M07202-5BMG1A, 11.4 AD (David Brown, series M Radicon)

| Input: 11KW | Rate: 2.5 A |
| Assembly: 1A | Order No: 5032382 |
| Oil Grade: 320 |

The gearbox is consisting of two stage helical gear in which in each stage there are two gear.

Generator (DC Motor)

| No: G63801N | DE. 6314 N.D.E.6309 |
| Size: SD 200XLC | Rated speed: 1750 rpm |
| Duty type: S1 | Rated power: 85kW |
| Ins Class: F | Armature V: 460, A: 200 |
| Enclosure: IP22 | Voltage (excitation): 360V, |
| Mass: 48.2Kg | Current: 4.7A |
ACCELEROMETER SPECIFICATIONS (TYPE- ICP)

Model & Serial Number:

PCB / 338C04

Frequency range:

- 0.5Hz to 10 kHz (± 5%)
- 0.3Hz to 12kHz (± 10%)
- 0.2Hz to 20kHz (± 3dB)

Excitation Voltage:

18-30 VDC

Resonant Frequency:

≥ 35kHz

Sensitivity:

100mV/g (± 10%)

Temperature Range:

-53 to 93°C

Output Bias voltage Level:

8 to 12 VDC

Broadband Resolution (1 to 10kHz):

0.0018m/s² RMS

Measurement Range:

±50g pk

Discharge Time Constant:

0.8-2.4 sec

DATA ACQUISITION CARD SPECIFICATIONS (Model - 6221 DAQ)

Model:

PCI- 6221
Condition Monitoring of Helical Gearboxes based on the Advanced Analysis of Vibration Signals

Channels:
- 8 differentials - 16 single ended

ADC resolution:
- 16 bits

Sampling rate:
- 250 kS/sec (maximum)

Input voltage range:
- +/- 10V

Max working voltage:
- 11 V

Timing resolution:
- 50 ns

Timing accuracy
- 50 ppm of sample rate
Parameter calculation for gearbox

Shafts moment of inertia

Basic principle:

\[ j = m \cdot r_g^2 \]

Radius of gyration of solid cylinder:

\[ r_g^2 = \frac{d^2}{8} \]

\[ r_g^2 = \frac{(2r)^2}{8} = \frac{r^2}{2} \]

\[ r_g = \frac{r}{\sqrt{2}} = 0.707r \]

Dimension of shaft one:

Density of the steel = 7.8 kg/dm³

Radius of the gyration of shaft one (\(r_{g1}\))
Condition Monitoring of Helical Gearboxes based on the Advanced Analysis of Vibration Signals

\[ r_{g1} = 0.707 \left( \frac{r_1 + r_2 + r_3}{3} \right) = 0.707 \left( \frac{14 + 20 + 30}{3} \right) \]

\[ r_{g1} = 15.06mm = 0.015m \]

Moment of inertia of shaft one:

\[ j_{s1} = m_{s1} \cdot r_{g1}^2 \]

\[ j_{s1} = 1.42(0.015)^2 = 0.00032 \text{ kg.m}^2 \]

![Figure G.2 Shaft one](image)

Dimension of shaft two

\[ r_{g2} = 0.707 \left( \frac{r_1 + r_2 + r_3}{3} \right) = 0.707 \left( \frac{12 + 17.5 + 12}{3} \right) = 11.66mm \]

\[ r_{g2} = 0.707m \]

Total mass \( m_{s2} = 0.95 \text{ g} \)

Moment of inertia of shaft two:

\[ j_{s2} = m_{s2} \cdot r_{g2}^2 \]

\[ j_{s1} = 0.95(0.0117)^2 = 0.00013 \text{ kg.m}^2 \]
Dimension of shaft three

\[ r_{g3} = 0.707 \left( \frac{r_1 + r_2 + r_3}{3} \right) = 0.707 \left( \frac{17.5 + 27.5 + 20}{3} \right) = 15.318 \text{mm} \]

\[ r_{g3} = 0.0153 \text{m} \]

Total mass \( m_{s3} = 3.34 \text{ g} \)

Moment of inertia of shaft tree:

\[ j_{s3} = m_{s3} \cdot r_{g3}^2 \]

\[ j_{s3} = 3.34(0.0153)^2 = 0.00078 \text{ kg.m}^2 \]
Moment of inertia of gears

Gears in Stage one:

Gear one (pinion):

Gear one (pinion) dimensions

Number of teeth (Z₁) = 58

Mass of gear using the weigh scale (m₁₁) = 0.23 kg

Calculation of pitch diameter of gear one (pinion):

Number of teeth (Z₁) = 58

The distance between the two gears, a = 76mm (measured).

Outer gear diameter, D₁₁ = 51.62 mm (measured)

Outer gear radius, R₁₁ = 25.8 mm

Gear thickness, t₁ = 25.15mm (measured)

Tooth gear head, h = 1.25 m from evolutes gear standard,
Calculation of the module (m):

\[ D_{p1} = Z_1 \cdot m \]
\[ D_{o1} = D_{p1} + 2h \]
\[ D_{o1} = Z_1 \cdot m + 2(1.25)m \]
\[ D_{o1} = (Z_1 + 2.5)m \]
\[ 52.2 = (3 + 2.5)m \]
\[ m = \frac{51.62}{36.5} = 1.41 mm \]
\[ D_{p1} = Z_1 \cdot m \]
\[ D_{p1} = 48.08 mm \]
\[ R_{p1} = 24.04 mm \]

Calculation of the total volume (complete gear as solid gear):

\[ V_{1T} = \pi R_{p1}^2 t_1 \]
\[ V_{1T} = \pi (24.04)^2 (25.15) \]
\[ V_{1T} = 45639.04 mm^3 \]

Calculation of the volume of shaft hole:

Diameter of the shaft hole, \( D_{1\text{hole}} \) = 28mm
Radius of the shaft hole \( R_{1\text{hole}} \) = 14mm
Gear thickness, \( t_1 \) = 25.15mm

\[ V_{1\text{hole}} = \pi R_{1\text{hole}}^2 t_1 \]
\[ V_{1\text{hole}} = \pi (14)^2 (25.15) \]
\[ V_{1\text{hole}} = 15386. mm^3 \]

Calculation of the volume of the keyway:

Width of the keyway, \( a_1 \) = 8 mm
Height of the keyway, \( b_1 \) = 4 mm
Thickness of the keyway, \( t_1 \) = 25 mm
Condition Monitoring of Helical Gearboxes based on the Advanced Analysis of Vibration Signals

\[ V_{\text{keyway}} = (a)(b)(t_1) \]

\[ V_{\text{keyway}} = (8)(4)(25) \]

\[ V_{\text{gear1}} = V_T - V_{\text{hole}} - V_{\text{keyway}} \]

\[ V_{\text{gear1}} = 45639.04 - 15386.0 - 800 \]

\[ V_{\text{gear1}} = 29453.07 \text{ mm}^3 \]

\[ V_{\text{gear1}} = 0.29453 \text{ dm}^3 \]

**Calculation of gear one (pinion) mass:**

\[ m_{\text{gear1}} = V_{\text{gear1}} \cdot \rho \text{ (density)} \]

\[ m_{\text{gear1}} = (0.029453)(7.8) \]

\[ m_{\text{gear1}} = 0.229734 \text{ kg} \]

**Calculation of the gear one (pinion) moment of inertia:**

Moment of inertia of gear 1:

\[ J_1 = m_1 \cdot R_{g1}^2 \]

where:

\[ m_1 = \text{mass of gear 1}, \ R_{g1} = \text{radius of the gyration of gear 1} \]

Radius of the gyration of gear one (pinion):

\[ R_{g1} = 0.707 \left[ R_{i1} + \left( \frac{R_{p1} - R_{i1}}{2} \right) \right] \]

\[ R_{g1} = 0.707 \left[ \left( \frac{R_{p1} + R_{i1}}{2} \right) \right] \]

\[ R_{p1} = \text{radius of pitch} \]

\[ R_{p1} = 24.2 \text{ mm} \]

\[ R_{i1} = \text{radius of the shaft hole of gear} \]

\[ R_{i1} = 14 \text{ mm} \]

Radius of gyration of gear one (pinion):

\[ R_{g1} = 0.707 \left[ \left( \frac{24.2 + 14}{2} \right) \right] \]
Rₐ₁ = 13.5 mm
Rₐ₁ = 0.0135 m

Mass of gear 1, m₁ = 0.23 kg

Moment inertia of gear 1:

\[ J_{p1} = 0.23 \times (0.0135)^2 = 0.000042 \text{ kg.m}^2 \]

**Gear two calculations**

**Figure G.5-Gear (pinion) one in first stage**

**Gear two dimensions**

Number of teeth, \( Z_2 \) = 47

Mass of gear using the weigh scale = 1.52 kg

The distance between the two gears, \( a = 76 \text{ mm} \) (measured).

Outer gear diameter, \( D_{og2} = 101.41 \text{ mm} \) (measured)

Outer gear radius, \( R_{og2} = 50.7 \text{ mm} \)

Gear cylindrical thickness, \( t_2 = 25.15 \text{ mm} \) (measured)

Gear total thickness, \( t_{g2} = 37 \text{ mm} \) (measured)
Tooth gear head, h = 1.25 m from evolutes gear standard,

**Calculation of the module (m):**

\[ D_{p2} = Z_2 \cdot m \]

Module of first gear (pinion) and second gear:

\[ m = 1.41 \text{ mm} \]

\[ D_{p2} = 70 \times (1.41) = 98.7 \text{ mm, therefore } R_{p2} = 49.35 \text{ mm} \]

Mass calculation of the Gear two:

Gear two profile consist of cylindrical and taper or cone formation.

To determine the high of full cone can apply similarity of triangle law.

Let \( L_2 \) indicate total high of cone,

\( L_{22} \): high of the small cone,

\( L_{12} \): high of a part of cone,

\( D_{21} \): diameter of cone base,

\( D_{22} \): diameter of upside of part of cone.

\( R_{21} \): radius of cone base, and \( R_{22} \): radius of upside of part of cone.

Similarity of triangle law

\[
\frac{L_2}{R_{21}} = \frac{L_{22}}{R_{22}}
\]

\[
\frac{L_2}{31} = \frac{(L_2 - L_{21})}{20}
\]

\[
\frac{L_2}{31} = \frac{(L_2 - 1)}{20}
\]

\[ 20L_2 = 31 (L_2-12) = 31L_2-72 \]

\[ L_2 = 33.8 \]

\[ L_{22} = L_2 - L_{21} \]

\[ L_{22} = 33.8 - 12 = 21.8 \text{ mm} \]
Calculate the total volume of the cone:

\[ V_{\text{cone}} = \pi R_{21}^2 \frac{L_2}{3} \]

\[ V_{\text{cone}} = \pi (31)^2 \frac{33.8}{3} \]

\[ V_{\text{cone}} = 33997.6 \, mm^3 \]

Calculate the small cone, \( V_{\text{cone1}} \)

\[ V_{\text{cone}} = \pi R_{22}^2 \frac{L_{22}}{3} \]

\[ V_{\text{cone}} = \pi (20)^2 \frac{21.8}{3} \]

\[ V_{\text{cone}} = 9126.93 \, mm^3 \]

Calculate the small cone, \( V_{\text{cone2}} \)

\[ V_{\text{cone2}} = V_{\text{cone}} - V_{\text{cone1}} \]

\[ V_{\text{cone2}} = 3997.6 - 9126.93 = 24870.66 \, mm^3 \]

Volume of the cylinder (e.g. the location of the shaft and the location of the keyway must be subtracted)

Volume calculation of cylindrical gear hole \( V_{\text{hole2}} \)

Radius of gear hole, \( r_{g2} = 12 \, mm \)

Thickness of gear 2, \( t_{g2} = 37 \, mm \)
\[ V_{\text{hole}2} = \pi r_{g2}^2 t_2 \]

\[ V_{\text{hole}2} = \pi (12)^2 (37) \]

\[ V_{\text{hole}2} = 16729.92 \text{ mm}^3 \]

**Calculation of the keyway volume of gear two, \( V_{\text{kw}2} \):**

\[ V_{\text{kw}2} = a_2 \cdot b_2 \cdot t_2 \]

\[ V_{\text{kw}2} = (8) \cdot (4) \cdot (37)^3 \]

\[ V_{\text{kw}2} = 1184 \text{ mm}^3 \]

Volume of the cylindrical gear part, \( V_{\text{cg}2} \)

Pitch radius of gear 2, \( R_{p2} = 49.35 \text{ mm}, t_2 = 25 \text{ mm} \)

\[ V_{\text{cg}2} = \pi r_{p2}^2 t_2^2 \]

\[ V_{\text{cg}2} = \pi (49.35)^2 (2) \]

\[ V_{\text{cg}2} = 191180.666 \text{ mm}^3 \]

Volume of gear two the total volume of gear 2, \( V_{g2} \)

\[ V_{g2} = V_{\text{cone}2} - V_{\text{hole}2} - V_{\text{kw}2} + V_{\text{cg}2} \]

\[ V_{g2} = 24870.66 - 16729.92 - 1184 + 11180.66 \]

\[ V_{g2} = 198137.4 \text{ mm}^3 \]

\[ V_{g2} = 0.1981374 \text{ dm}^3 \]

Mass of gear 2, \( m_{g2} \)

\[ m_{g2} = \rho V_{g2} \]

\[ m_{g2} = 1.546 \text{ kg} \]

Moment of inertia of the gear 2:

\[ J_2 = m_{g2} \cdot R_{g2}^2 \]

\( R_{g2} \): radius of the Gyration of gear 2.
\[ R_{g2} = 0.707 \left( R_{i2} + \left( \frac{R_{p2} - R_{i2}}{2} \right) \right) \]

\[ R_{g2} = 0.707 \left( \left( \frac{R_{p2} + R_{i2}}{2} \right) \right) \]

\[ m = 1.546 \text{ kg} \]

\[ R_{i2} = 12 \text{ mm} \]

\[ R_{g2} = 0.707 \left( \left( \frac{49.35 + 12}{2} \right) \right) \]

\[ R_{g2} = 21.687 \text{ mm} = 0.0217 \]

\[ J_2 = m_{g2}. R_{g2}^2 \]

\[ J_2 = (1.546)(0.217)^2 \]

\[ J_2 = 0.00073 \text{ kg. m}^2 \]

**Gears in Stage two**

Determine gear 3 and 4 pitch radiuses

\[ R_{p3} = \text{pitch radius of gear 3} \]

\[ R_{p4} = \text{pitch radius of gear 4} \]

Distance between the centre of gear 3 and centre of gear 4, \( a = 76 \text{ mm} \) (measurement result)
Gear ratio:

\[
\frac{R_{p4}}{R_{p3}} = i
\]

Second stage contact gear ratio (between gear 3 and 4), = 1.48 (gearbox design data)

\[
R_{p4} = R_{p3} \cdot i
\]

\[
a = R_{p3} + R_{p4}
\]

\[
a = R_{p3} + iR_{p3}
\]

\[
a = (1 + i)R_{p3}
\]

\[
R_{p3} = \left(\frac{a}{1 + i}\right)
\]

\[
R_{p3} = \left(\frac{76}{1 + 1.48}\right)
\]

\[
R_{p3} = 30.5 \text{ mm}
\]

And,

\[
R_{p4} = R_{p3} \cdot i
\]

\[
R_{p4} = (30.65)(1.48)
\]
\( R_{p4} = 45.36 \text{ mm} \)

**Gear 3 mass calculations:**

Data of gear 3:

- Number of teeth \((Z_3) = 13\)
- \(D_{p3}\) Pitch diameter, = 61.3 mm
- \(R_{p3}\) Pitch radius, = 30.65 mm (calculated)
- \(t_3\) Gear thickness, = 35mm (measured)
- \(D_{i3}\) Hole diameter, = 25mm (measured)
- \(R_{i3}\) Hole radius, = 12.5mm
- \(a_3\) Width of the keyway = 8mm
- \(b_3\) Thickness of the keyway = 4mm

Volume of the gear three, \(V_{g3}\)

\[
V_{g3} = \pi R_{p3}^2 t - \pi R_{i3}^2 t - a_3 b_3 t_3
\]

\[
V_{g3} = \pi(30.65)^2(35) - \pi(12.5)^2(35) - (8)4(35)
\]

\[
V_{g3} = 84950.58 \text{ mm}^3
\]

\[
V_{g3} = 0.08495 \text{ dm}^3
\]

Since \( m_{g3} = \rho V_{g3} \)

\[
m_{g3} = (0.08495)(7.8) = 0.663 \text{ kg}
\]

**Calculation of gear 3 moment of inertia \((J_3)\):**

\[
J_3 = m_{g3} R_{g3}^2
\]

\[
m_{g3} = \text{mass of gear 3}, 0.663 \text{ kg}
\]

\[
R_{g3} = \text{radius of gyration of gear 3}
\]
\[ R_{g3} = 0.707 \left( \frac{R_{p3} + R_{i3}}{2} \right) \]

\[ R_{g3} = 0.707 \left( \frac{30.65 + 12.5}{2} \right) \]

\[ R_{g3} = 15.25 \text{ mm} = 0.01525 \text{ m} \]

\[ J_3 = (0.663) \cdot (0.01525)^2 \]

\[ J_3 = 0.00014 \text{ kg.m}^2 \]

**Gear 4 mass calculations:**

Data of gear 4

Number of teeth \( Z_4 = 59 \)

Pitch diameter.

\( D_{p4} = 90.72 \text{ mm} \)

Pitch radius, \( R_{p4} = 45.36 \text{ mm} \)

Gear thickness, \( t_4 = 35 \text{mm} \)

Hole diameter, \( D_{i4} = 35 \text{mm} \)

Hole radius, \( R_{i4} = 17.5 \text{mm} \)

Width of the keyway, \( a_4 = 8 \text{mm} \)

Thickness of the keyway, \( b_4 = 4 \text{mm} \)

Volume of the gear four, \( V_{g4} \)

\[ V_{g4} = \pi \cdot R_{p4}^2 \cdot t_4 - \pi \cdot R_{i3}^2 \cdot t_4 - a_3 \cdot b_4 \cdot t_4 \]

\[ V_{g4} = \pi(45.36)^2(35) - \pi(17.5)^2(35) - (8)(4)(35) \]

\[ V_{g4} = 191345.625 \text{ mm}^3 = 0.191345628 \text{ dm}^3 \]
Since \( m_{g4} = \rho V_{g4} \)

\[ m_{g4} = (0.1913456)(7.8) = 1.493 \, kg \]

**Calculation of gear 4 moment of inertia (J₄):**

\[ J_4 = m_{g4} \cdot R_{g4}^2 \]

\[ m_{g4} = \text{mass of gear 3}, \, 1.493 \, kg \]

\[ R_{g4} = \text{radius of gyration of gear 4} \, m \]

\[ R_{g4} = 0.707 \left[ \left( \frac{R_{p4} + R_{i4}}{2} \right) \right] \]

\[ R_{g4} = 0.707 \left[ \left( \frac{45.36 + 17.5}{2} \right) \right] \]

\[ R_{g4} = 0.0222 \, m \]

\[ J_4 = (1493) \cdot (0.0222)^2 \]

\[ J_4 = 0.00074 \, kgm^2 \]