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Full-field calibration and compensation of lateral chromatic aberration based on unwrapped phase

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ABSTRACT
Lateral chromatic aberration (CA) of color cameras has great effects on the imaging quality. This paper presents a novel method to full-field calibrate lateral CA between color channels by using unwrapped phase data. Closed circle sinusoidal fringe patterns having the optimum three-fringe numbers are generated and displayed on a liquid crystal screen consecutively through red, green and blue channels. These closed fringe patterns are captured by a color camera. The wrapped phase and unwrapped phase of each pixel can be calculated by using four-step phase shifting algorithm and optimum three-fringe number method, respectively. The pixel deviations produced by lateral CA are computed by comparing the obtained absolute phase data between red, blue, and green channels in polar coordinate system and calibration is accomplished in Cartesian coordinate system. Lateral CA between color channels of the color camera can be compensated by using the calibrated data. Simulated and experimental results show the validity of the proposed calibration and compensation method.

Keywords: chromatic aberration calibration, circle sinusoidal fringe patterns, unwrapped phase, color camera, polar coordinate system

1. INTRODUCTION
Optical three-dimensional (3D) measurement technique has been widely used in the fields of 3D topography measurement, reverse engineering, the protection of cultural relics, biometrics, machine vision¹,², and so on, due to its advantages of non-contact, high precision, rapid acquisition and non-destructive measurement. In the measurement system such as 3D fingerprint identification system³ and 3D human body scanning system⁴, fringe patterns are projected onto the tested surface by a projector, and the deformed fringes are captured by a camera. Moreover, whether it is a human topography measurement system or a machine vision system, cameras are the dispensable devices in these technique, especially color cameras can capture color texture and shape data simultaneously. However, images captured by color camera have the phenomenon of chromatic aberrations (CA) and lead to lower precision because of the intrinsic optical properties of lens itself⁵. CA can be divided into lateral chromatic aberration (LCA) and axial chromatic aberration (ACA). With the development of manufacturing process, color difference of lens mainly is caused by LCA. Therefore, it is vital and necessary to study how to calibrate and compensate for LCA of a color camera when higher precision and speed is required.

CA can be compensated for by using hardware-based and software-based methods. Dollond presented and invented the first achromatic lens by combining positive and negative lens together⁶. At present, apochromatic lens (APO Lens) is designed to focus three wavelengths (typically red, green, blue) on the same point in imaging plane⁷. However, expensive natural or synthetic low dispersion fluorite, and precise optical computing and lens assembly make not only the structure of APO lens more complex but also research cycle longer. Moreover, it cannot eliminate CA completely. A completely flat, ultra-thin lens was invented by Harvard School of Engineering and Applied Sciences. The lens can focus different wavelengths of light at the same point and achieve instant color correction in one extremely thin, miniaturized device⁸. The achievement is expected to be applied to optical element in the future, but time and price are unknown. Zhang et al. and Sterk et al. used calibration image with markers as a reference to calculate displacement differential of reference points in different color, then corrected CA based on certain correction ratios⁹,¹⁰. However, the accuracy relies on the number of markers. Zhang et al. proposed a novel linear compensation method to compensate for LCA resulting in...
from the refractive projection and imaging lenses in a color fringe projection system, but it is only applicable to the optimum three-fringe number selection algorithm\textsuperscript{11}. Willson et al. designed an active lens control system to calibrate CA\textsuperscript{12}. The best focus distance and relative magnification coefficients of red, green and blue channels are obtained. ACA and LCA are reduced by adjusting the distance between imaging plane and lens based on the obtained parameters. But the system is complicated and it is difficult to ensure the precision. Boult et al. used image warping to CA. Firstly, best focus distance and relative magnification coefficients are acquired by using the active lens control system. Then, these parameters are used to program the function of image warping to implement CA calibration in the horizontal and vertical direction\textsuperscript{13}. This method need external reference object as the standard of deformation: the more feature points, the better processing results. Kaufmann et al. established the relationship between the deviation of red, green and blue channels caused by LCA and pixel position with the help of black and white triangular mesh and then used least squares fitting to compensate for LCA effectively. Its precision is affected by frequency of the triangular mesh. Mallon et al. calibrated LCA between different color channels by using high-density checkerboard\textsuperscript{14}, but not realized full-field calibration. Chang et al. proposed a method of false color filtering to improve the phenomenon of image blurring and chromatic stripes produced by both ACA and LCA\textsuperscript{15}. Although the method can correct ghosting caused by color difference, its process is complicated and some parameters are set empirically. Therefore, all the existing methods cannot completely remove CA in a color image.

This paper presents a novel method to calibrate and compensate for LCA between color channels by using the unwrapped phase data. Different from the above mentioned correction methods, the full-field accurate pixel correspondence relationship between red, green and blue channels can be determined along the radial direction by comparing the unwrapped phase in the three color channels. In the following, Section 2 analyzes chromatic aberrations behavior and explains the principle of the proposed method. Some simulated and experimental results of compensation are shown in Section 3. Section 4 gives conclusions and remarks regarding future work.

## 2. PRINCIPLE

### 2.1 Analysis of chromatic aberration

In optics, color difference is mainly caused by the lens itself due to optical properties that different wavelengths have different refractive index in the same medium: the shorter the wavelength, the bigger refractive index and the refraction angle, on the contrary, the smaller. Therefore, different wavelengths of incident ray refract differently and focus on different positions, which called as CA. This makes image blur and prone to obvious color fringes in image corner areas far from the optical axis. As mentioned above, CA consists of ACA and LCA. ACA and LCA can also be called longitudinal chromatic aberration and radial chromatic aberration, respectively. ACA leads to different focal length for different color incident light, affects the sharpness of the image, as shown in Fig.1 (a). LCA changes the magnification of overall image of different color channel and causes radial-symmetric point displacements, as shown in Fig.1 (b). Therefore, when using color sinusoidal fringes to acquire data of texture and morphology of surface of objects, the number of fringes is enlarged or reduced to some degree for different color fringes. In addition, shape step is introduced when combining information of three channels together, and it affects the measurement accuracy seriously.

Reflected light rays and incident rays have the same direction when it go through the optical center, so different wavelengths of light can focus on the same position in image plane when the ray is parallel to the optical axis and goes through the optical center. That is, there is no color difference in the principal point. In view of the distribution characteristic of LCA, color difference far from the optical center is increased gradually and radial-symmetric regardless of the manufacturing error of lens. Therefore, closed circular sinusoidal fringe patterns are applied to calibrate and compensate for LCA since the radial displacement is symmetrical.

### 2.2 Calibration system

The proposed calibration system mainly includes a liquid crystal display (LCD) screen, a CCD color camera and a personal computer (PC), as illustrated in Fig.2. The computer is connected to the camera and the LCD screen by gigabit and HDMI, respectively. It will generate circular fringe pattern images, save and process the data captured by the camera. The camera is used to capture the displayed images on the LCD screen, and the LCD screen is used to display images generated by computer.
Before calibrating LCA of lens, the system needs to satisfy with two conditions. One is that the LCD is parallel to image plane, the other is the principal point of camera is in alignment with center of the circular sinusoidal fringes. It can be proceed as follows. 1) Intrinsic parameters of the CCD camera are calibrated based on a total of twenty images of a planar checkerboard by using the camera calibration toolbox for Matlab\textsuperscript{16}. 2) A picture of checkerboard is generated by software and displayed on the LCD screen, and captured by the CCD camera. 3) Extrinsic parameters of the checkerboard on LCD screen are computed. From step 1, the coordinates of principal point can be acquired. Step 2 and 3 are used to realize adjustment of the spatial parallel relationship between the camera and LCD through mechanical device. Besides, because unit pixel of the LCD screen is composed of red, green and blue color filter, it introduces system error inevitably when red, green and blue color fringes displayed on LCD. Therefore, the system error must be eliminated before calibrating LCA. However, this paper only displays the results of error caused by LCD, and the calculation and elimination method will be illustrated in detail in another paper.

![Diagram of chromatic aberration. (a) Axial chromatic aberration, and (b) Lateral chromatic aberration.](image)

**2.3 Unwrapped phase acquisition**

Red, green and blue closed circular sinusoidal fringes patterns having optimum fringe numbers are generated by software and sequentially displayed on the LCD screen. In front of the LCD screen, the color camera captures the fringes through red, green and blue channels to obtain twelve red fringes, twelve green fringes and twelve blue fringes, respectively. Four-step phase-shifting algorithm\textsuperscript{17} is applied to obtain three wrapped phase maps in red, green, and blue channels. Finally, three unwrapped phase maps are acquired by using the optimum three-fringe number section method in the three color channels\textsuperscript{18}.

The optimum fringe number section method defines the numbers of the projected fringes by Eq. (1). \(N_0\) and \(N_i\) is the maximum and \(i\)th number of fringes, respectively. \(n\) represents the number of fringe group. When the value of \(n\) is 3, this method is called the optimum three-fringe number section method. For example, if \(N_0 = 64\), the other two groups of fringe numbers are \(N_1 = 63\) and \(N_2 = 56\). A single fringe produced by beat frequency between \(N_0\) and \(N_i\) covers the whole field of view. Therefore, the problem of fringes classes can be resolved by this method and achieve better reliability.

\[
N_i = N_0 - (N_0)^{i-1/n} \quad i = 1, \ldots, n-1
\]

**2.4 Principal of calibrating and compensation**

In a color fringe reflection system or a color fringe projection system, when phase maps are determined on two or three color channels from displayed or projected patterns of the same fringes, the obtained phases are different due to LCA. Hence, when different color closed circular sinusoidal fringes are captured by a color camera, the unwrapped phase is...
different in the same pixel position in image plane between red, green and blue channels, but it is equal in the same radius in a single color channel due to the same refraction. As demonstrated in Fig.3 (a), points P1, P2 and P3 in red, green and blue fringes are in the same position on the LCD. Although they have the same absolute phase value, these points are at different positions in the captured image along radial direction due to LCA as shown in Fig.3 (b), while the absolute phase value of P0 and P1 is equal in the same radial position. Therefore, pixel deviations can be obtained by calculating phase difference in the radial direction $\Delta r_1$ between red and blue channel, and $\Delta r_3$ between green and blue channel, then decomposing them in $x$ and $y$ direction.

Three obtained absolute phase maps are used to build pixel-to-pixel correspondence between three color channels. Figure 4 is the flow diagram of LCA calibration. Firstly, Cartesian coordinate system is converted to polar coordinates system. Secondly, the absolute phase in same radius position is extract from three color channels. Thirdly, in order to avoid the error of extrapolation, blue channel is regarded as the benchmark, and the absolute phase in a certain radius is extracted. Fourth, the absolute phase of red and green channel in the same radius is calculated. Fifth, new radius of the same phase of blue channel is computed in red and green channels. The original value is replaced with it, and then polar coordinates system is converted back to Cartesian coordinate system. Finally, the pixel deviations can be acquired by Eq. (2), and full-field calibration and compensation is realized.

![Figure 3. Sketch of lateral chromatic aberration. (a) The original position in the circular fringe pattern on LCD screen. (b) Lateral chromatic aberration in the captured circular fringe pattern.](image)

\[
\begin{align*}
\Delta x_{RB} &= x_R - x_B \\
\Delta y_{RB} &= y_R - y_B \\
\Delta x_{GB} &= x_G - x_B \\
\Delta y_{GB} &= y_G - y_B
\end{align*}
\]  

(2)

Where $x_B, y_B$ are the original pixel coordinates of blue channel, $x_R, y_R$ are the actual pixel coordinates of red channel, and $x_G, y_G$ are the actual pixel coordinates of green channel. $\Delta x_{RB}, \Delta y_{RB}$ are pixel deviations in $x$ and $y$ direction between red and blue channel, respectively, and $\Delta x_{GB}, \Delta y_{GB}$ are pixel deviations in $x$ and $y$ direction between blue and green channel, respectively.

Accurate compensation for LCA between three channels can be realized by moving the deviated sub-pixels $\Delta x_{RB}$ and $\Delta y_{RB}$, $\Delta x_{GB}$ and $\Delta y_{GB}$ in red and green channel to the corresponding subpixel positions in the blue channel$^{19}$, as shown in Fig.3. Furthermore, two-dimensional interpolation method is applied to the whole corrected image to find the accurate position. Therefore, color information in the three channels coincides after full-field LCA compensation.
3. EXPERIMENTS AND RESULTS

In order to verify the effectiveness of the proposed method to calibrate and compensate for LCA of lens, simulations have operated firstly, and then an experimental system have been set up. The system mainly consists of off-the-shelf components: a SVCam-ECO655 color camera with resolution 2050×2448 and pixel pitch 3.45um×3.45um, a AZURE-1614MM lens with fixed focus length 16 mm and adjustable aperture, a LP097QX2 TFT-LCD display of LG company, whose physical resolution is 1536×2048, and pixel pitch is 0.096mm×0.096mm, and its color filter is horizontally arranged.

3.1 Simulations

Twelve closed circular sinusoidal fringes patterns with resolution of 768×1024 were generated and modulated into red, green, and blue channels of the LCD screen. The generated fringe numbers are 32, 31.5 and 28 respectively, phase shift step is 90 degree in between, same as green and blue fringes. In this way, the wrapped phase and unwrapped phase can be computed precisely by using four-step phase-shifting algorithm and optimum three-fringe number method since both have the character of anti-noise. Moreover, it is obviously known that 32, 31.5 and 28 are not the number of optimum three-fringe number, but because the fringes are circular, the unwrapped phase can be obtained by using the optimum three-fringe number 64, 63 and 56 in the process of simulation\(^8\). The average intensity, fringe contrast and

Unwrapped phase of red, green and blue closed circular sinusoidal fringes captured by camera

Converting Cartesian coordinate system to polar coordinates system

Extracting the absolute phase of three channels in the same radius position respectively

Regarding blue channel as a benchmark and extracting its absolute phase \(\Phi_b\) in position of \(R\) radius

Obtaining the absolute phase \(\Phi_r\) of red channel in position of \(R\) radius

Obtaining the absolute phase \(\Phi_g\) of green channel in position of \(R\) radius

Computing the position of \(\Phi_b\) in red and green channel \(R_{rb}, R_{gb}\)

\(R_{rb}\) and \(R_{gb}\) replace the original value of \(R\)

Converting polar coordinates system back to Cartesian coordinate system

Calculating pixel deviation \(\Delta x, \Delta y\) in \(X, Y\) direction between red and green, blue and green channel

Compensating for chromatic aberration by pixel deviation

Figure 4. Flow diagram of LCA calibration.
intensity noise of the fringes generated by computer are 128, 100 and 2.5%, respectively, and the principal point of the camera is (384, 512). To obtain fringes having different magnification, the phase per pixel in LCD screen of red, green and blue are 0.1971, 0.1963 and 0.1952 respectively.

The original unwrapped phase between red, green and blue channels is different, as shown in Fig.5 (a) and (d). Fig.5 (b) and (e) show the pixel deviation caused by LCA of lens between red and blue channel, green and blue channel respectively. Fig.5 (c) and (f) verified the effectiveness of the proposed method since the phase deviation is decreased and the color stripes are eliminated. The image affected by LCA as shown in Fig.5 (g), the color stripes are obvious far away from the principal point due to LCA.

The experimental system was set up, as shown in Fig.6. Table 1 lists the system errors in the horizontal and vertical direction introduced by LP097QX2 TFT-LCD display. The system errors in the vertical direction are much smaller than the horizontal direction, and it can be ignored. The normal vector of LCD display plane in the camera reference frame is (-0.001763, 0.006313, -0.999979), so the angle between the camera target and the LCD display is 0.3756 degrees. Figure 7 shows the unwrapped phase from the captured fringe patterns in blue channel of the color camera, and its distribution is circular. Figure 8 shows the closed circular sinusoidal fringe patterns affected by LCA. After compensation, the color stripes are not obvious, as illustrated in Fig.8 (d) and (b). Figure 9 demonstrates the unwrapped phase deviation before and after the LCA compensation between red, green and blue channel. It is observed that the phase deviations between red and blue channel, green and blue channel are reduced after LCA correction. Figure 10 shows the fringe patterns in...
the middle row of the captured image. The purple stripes in (b) are reduced. Besides, it can be seen that the intensity curve of three channels are coincided after compensation by using the proposed method.

Table 1. The system errors in the horizontal and vertical direction introduced by LP097QX2 TFT-LCD display.

<table>
<thead>
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<th>Position</th>
<th>Red and Green (pixel)</th>
<th>Green and Blue (pixel)</th>
<th>Position</th>
<th>Red and Green (pixel)</th>
<th>Green and Blue (pixel)</th>
</tr>
</thead>
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<td>0.0791</td>
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</table>

Figure 6. Experimental system. Figure 7. Unwrapped phase maps in blue channel.

(a) (b) (c) (d)

Figure 8. Closed circle sinusoidal fringe patterns. (a) The original fringe patterns affected by LCA; (b) The enlarged image of the red area in (a); (c) The fringe patterns after compensation of LCA; (d) The enlarged image of the red area in (c).

(a) (b) (c) (d)

Figure 9. Unwrapped phase deviation between red, green and blue channel. (a) Original phase deviation affected by LCA between red and blue channel; (b) Phase deviation after compensation of LCA between red and blue channel; (c) Original phase deviation affected by LCA between green and blue channel; (d) Phase deviation after compensation of LCA between green and blue channel.
4. CONCLUSIONS

This paper presents a novel method to full-field calibrate and compensate for LCA between red, green and blue channels in a color camera based on unwrapped phase map. Radial correspondence between the three channels is obtained by using closed circular sinusoidal fringe patterns in polar coordinate system, and pixel-to-pixel correspondence is acquired in Cartesian coordinate system. Then LCA is compensated for in vertical and horizontal direction with subpixel accuracy. Experimental results have shown the effectiveness of the proposed method.

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