Statistical modelling of wear and damage trajectories of railway wheelsets

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Abstract

This paper discusses the use of Linear Mixed Models (LMM) and Generalized Linear Mixed Models (GLMM) to predict the wear and damage trajectories of railway wheelsets for a fleet of modern multiple unit trains. The wear trajectory is described by the evolution of the wheel flange thickness, the flange height and the tread diameter; whereas the damage trajectory is assessed through the probabilities of various types of wheel tread damage such as rolling contact fatigue, wheel flats and cavities occurring. Different model specifications are compared based on an information criterion.

1. Introduction

Statistical modelling of degradation processes provides a quantitative basis to assess the maintenance and renewal needs of a given system. In the railway system, train operating companies spend a significant part of their maintenance budget on wheelsets. Railway wheelsets are crucial to ensure safety and passenger comfort and thus, their dimensions must comply with technical specifications regarding their shape and wheel diameter[1]. Wheel profiles evolve with use due to wear and damage phenomena. This paper discusses the use of statistical models to predict the evolution of wear and damage trajectories for a sample of railway wheels from a fleet of modern multiple units, using data which are routinely collected by maintenance depots and, therefore, can be applied to any fleet of vehicles.

Improved statistical modelling to predict wear and damage trajectories of railway wheelsets may provide a better estimate of the expected wheelset life-cycle, as well as supporting wheelset maintenance decision making during its life-cycle. Note that our aim in applying the following statistical models is not to be able to predict wear and damage per se, but to understand which factors/effects might contribute to explain the variability in these predictions. If we can control this variability better then we could think about making better decisions in wheel maintenance.

A wheelset is composed of two wheels linked with a rigid axle. The outer rim profile of the wheel has two main parts: the tread and the flange. A comprehensive degradation model for a wheel would assess the evolution of the following geometrical variables throughout the life cycle of the wheel:

i) the wheel diameter (D);
ii) the flange thickness (F_t);
iii) the flange height (F_h).

It would also need to account for other damage defects that may occur during the wheel's life cycle, such as rolling contact fatigue (RCF), wheel flats and cavities. These can significantly shorten a wheel's life as they often require a large reduction in the wheel diameter in order to remove the damaged material.

Figure 1 provides a schematic representation of the flange thickness and the flange height measures and the tread datum position (70 mm measured from the flange back and A = 13 mm for UK profiles or A = 10 mm for EN profiles).

So far, section 1 introduced the need to statistically model the wear and damage trajectories of railway wheelsets. The outline of the rest of this paper is as follows: section 2 briefly reviews past research on this topic, followed by an overview of the statistical methods used, namely Linear Mixed Model and Generalized Linear Mixed Model in section 3. Section 4 applies such models to the wear and damage trajectories of railway wheelsets and compares several model specifications. Finally, the main conclusions and some directions for further research are given in section 5.

2. Background

Statistical modelling of wear and damage trajectories of railway wheelsets is not a common approach when studying wear and damage of railway wheelsets. Most studies do not introduce probabilistic issues in their modelling. In fact, only some studies were found that use a statistical modelling approach[2,3,4], although none of these studies use the LMM and the GLMM discussed here. More recently, Wang et al[5] recently published a data-driven model to optimize the re-profiling strategy of metro wheel using as a case study the Guangzhou Metro Line, mainly focusing on the deterioration due to wear and without considering the damage trajectory/defects. Lin and Asplund[6] also used more complex Weibull frailty models to model lifetime data for a sample of locomotive wheels. They also conducted a statistical descriptive analysis of the impact wheel turning (also known as re-profiling) in the main wheel shape parameters.

Molyneux-Berry and Bevan[7] associated the observed damage types and wheel locations with the vehicle running conditions. Their study required a more detailed monitoring of the shape and length of RCF cracks in wheels than is currently done by most wheel maintainers. It also reports that leading wheelsets suffered more rapid RCF damage than other wheelsets and that smaller diameter wheels suffer more rapid RCF damage. Nevertheless, these observations are not supported by any statistical test and tend to be more of a 'feeling' rather than a hypothesis that is statistically tested with evidence from the data, controlling for other variables that influence the occurrence of such defects. Bevan et al.[8] have discussed the optimisation of wheelset maintenance taking into account not only the costs associated with wheelsets but also associated with the rail component, combining the perspective of train operating companies and the infrastructure manager, respectively. Our paper builds on this previous research and intends to understand the different behaviour in the wear and damage trajectories of different wheelsets using a statistical approach based on LMM and GLMM.

For GB mainline railways, Railway Group Standard GM/RT 2466 provides the wheel profile limits for the flange thickness (F_t^{lim}) and the flange height (F_h^{lim}) for a given tread profile. The standard specifies a minimum value for the flange thickness and a maximum value for the flange height. If a wheel infringes these limits, then wheel turning is needed in order to restore the full profile and both flange thickness and height to their initial values (or reasonably close to the initial values depending on the lathe's accuracy). Making the decision to turn a wheel imposes a loss in the wheel diameter (ΔD_T). Once the wheel reaches its scrap diameter (D_s), the vehicle must be removed from service and the wheelset replaced.

Regarding other international experiences, Pascual and Marcos[9] reported on the US experience of Talgo on wheel wear management of 'high-speed' passenger trains. Talgo developed a maintenance program called Total Logistic Care (TLC) that keeps the flange thickness within an 'optimal' range of operation instead of waiting until the wheel is out of the specifications. The French train operating company SNCF bases their maintenance rules on a risk management system (called REX[10]) that attempts to combine quantitative data with perceptions and experience of the wheel maintainers, (thus adding a subjective dimension to risk assessment) in order to tackle organisational issues with multiple decision-makers and multiple criteria. Nevertheless, no specific information on the statistical modelling of wear and damage trajectories were found.

Finally, Ferreira et al.[11] from a statistical perspective, in which the authors distinguish operating conditions (referring to wheel position, i.e. whether the axle is outer or inner, without giving information on whether it is a motored or a trailer) and Ekberg et al.[12], Pombo et al.[13] and Nia et al.[14] provide mechanical engineering models that consider physical quantities (e.g. vertical wheel load, diameter of the wheel, radius of the railhead, residual stresses, longitudinal and transverse contact stresses, etc), and which can be integrated in multibody dynamics simulations.

3. Linear Mixed Models and Generalized Linear Mixed Models

This section discusses the statistical methods used to model the wear and damage trajectories of railway wheelsets, namely Linear Mixed Models (LMM) and Generalized Linear Mixed Models (GLMM).

- Linear Mixed Models

Linear Mixed Models (LMM) are flexible linear models that can tackle the fixed effects of different controlling variables ($\boldsymbol{X}_i\boldsymbol{\beta}$) in the expected mean of the dependent variable, as well as the random effects associated with some factor or group ($\boldsymbol{Z_i}\boldsymbol{b_i}$). In mathematical terms, if one considers a single grouping level, LMMs can be formulated according to Galecki and Burzykowski[15] as:

$$\boldsymbol{y}_i = \boldsymbol{X}_i\boldsymbol{\beta} + \boldsymbol{Z}_i\boldsymbol{b_i} + \boldsymbol{\varepsilon_i}$$

In which: $\boldsymbol{y}_i$ is the dependent variable for group i, $\boldsymbol{X}_i$ is the design matrix for that group i, $\boldsymbol{\beta}$ is the slope parameter and $\boldsymbol{\varepsilon_i}$ is the residual error for group i. $\boldsymbol{Z_i}$ are the matrix of covariates and $\boldsymbol{b_i}$ is the corresponding random effects for each group i.

Some assumptions then have to be made on the random components:

$$\boldsymbol{b_i} \sim N(\boldsymbol{0}, \boldsymbol{\mathcal{D}}),\ \boldsymbol{\varepsilon_i} \sim N(\boldsymbol{0}, \boldsymbol{\mathcal{R}_i}),\ \text{with } \boldsymbol{b_i} \perp \boldsymbol{\varepsilon_i}$$

The random effects associated with a given group ($\boldsymbol{b_i}$) and the residual error for each group ($\boldsymbol{\varepsilon_i}$) are normally distributed with zero mean and co-variance matrices equal to $\boldsymbol{\mathcal{D}}$ and $\boldsymbol{\mathcal{R}_i}$ respectively. Both error terms are assumed to be independent between each other (for the same group i and between different groups). Additionally, the co-variance matrices are specified with an unknown scale parameter σ^2:

$$\boldsymbol{\mathcal{D}} = \sigma^2\boldsymbol{D} \text{ and } \boldsymbol{\mathcal{R}_i} = \sigma^2\boldsymbol{R_i}$$

Some additional constraints on the matrices $\boldsymbol{D}$ and $\boldsymbol{R_i}$ have to be made to guarantee identifiability[15]. These constraints are usually simplifications leading to choices of the matrices $\boldsymbol{D}$ and $\boldsymbol{R_i}$ that are multiples of the identity matrix.

- Generalized Linear Mixed Models

Generalized Linear Mixed Model (GLMM) allow the definition of probability distributions different from the Gaussian/Normal distribution, like Binomial or Poisson distributions, which are particular useful to model discrete dependent variables. GLMM requires the definition of a distribution from the exponential family and the definition of a link function that would relate the mean value of the dependent variable with the linear predictor. We will not provide more details on the GLMMs for the general case and instead refer the interested reader to some references[16,17,18], which discuss the specific GLMM used in this paper in more detail: the Mixed Logistic Model, defined through the Binomial distribution and the 'logit' link function. The choice of this particular model (and the 'logit' link function) had to do with the author's familiarity with it rather than a fundamental theoretical reason. Alternative link functions (e.g. 'probit' or 'cloglog') were tested but did not offer substantial improvement in the information criterion. For instance, the probability of a positive response for the dependent variable Y_{RCF}, i.e. the probability that an RCF defect (p_{RCF}) occurs can be computed as:

$$p_{RCF\,j} = \frac{1}{1 + e^{-(\sum_i \beta_i X_{ij})}}$$

Where: $p_{RCF\,j}$ is the probability of the occurrence of an RCF defect to wheelset j; β_i are the slope parameters associated with each covariate i and X_{ij} are the values for each covariate i and wheelset j.

The term in parenthesis (i.e. $\sum_i \beta_i X_{ij}$) can then be enhanced to account for random effects associated with some other factors, as was done previously for the Linear Mixed Models, though this time, it is no longer a normal distribution with an 'identity' link function, but a binomial distribution with a 'logit' link function.

LMMs and GLMMs are particularly useful because one might be interested in specifying a random effect associated with train unit or month of measurement, and identifying which one of them might be more influential in the variability of a particular geometrical measure of wheel degradation. Of course, one might also want to control for some fixed effects that do not vary for different groups and may contribute to the mean effect, like mileage since turning or wheelset type.

The next section will apply the models described above to the main dependent variables describing the wear and damage trajectories of a railway wheelset. All the following statistical models were estimated using the 'lme4' package for the R software code[19,20].

4. Statistical modelling

This section conducts an exploratory statistical analysis on wear and damage trajectories of different wheelsets from a fleet of modern multiple unit trains. Firstly, it provides a general description of the analysed sample and then applies different specifications of the LMM and the GLMM in order to find the most suitable statistical models to predict the main geometrical variables describing wear and damage trajectories of a wheelset.

The database analysed compiles wheel data from December 2006 up to July 2012 (i.e. a 7-year interval) from a single fleet of train (i.e. it only contains trains of one type or class). Each unit has 3 vehicles, and each vehicle has 8 wheels (i.e. 4 wheelsets). Figure 2 provides a schematic representation of a three car unit.

This wheel database contained the following information: unit number, unit running miles (cumulative mileage), vehicle type (DMC, DMS and MS), date, wheelset position (ws1, ws2, …, ws12), wheelset type (Motored, Internal or Leading trailer), event (i.e. brake disk skimming, initiation, post-turning, pre-turning, wheel set change), name of the technician performing the work, tread diameter (D) pre-turning and post-turning, flange thickness (F_t) and flange height (F_h) pre-turning and post-turning, and condition (i.e. cavities, flats, Ok, pitting/hardspots, RCF, rollover, small cavities, small flats, toe radius build up, turn for mileage).

Table 1 provides an overview of the variables in the database used in the LMM and the GLMM, their description and type, as well as some statistics.

The following statistical analysis was split into two parts:

A. On the wear trajectory, i.e. on the evolution of the geometrical measures of the wheel profile, such as the change in the flange thickness (ΔF_t), the change in the flange height (ΔF_h), the change in the tread diameter due to wear (ΔD) and the diameter loss due to turning (ΔD_T);
B. On the damage trajectory, i.e. on the occurrence of other wheel tread defects, such as rolling contact fatigue (Y_{RCF}), wheel flats (Y_{FLAT}) and cavities (Y_{CAV}). Only these defects were included in this study, because the other ones (mentioned above) are much less common than RCF, flats or cavities.

4.1. On the wear trajectory

Figure 3 exhibits several observations of the change in the flange height (ΔF_h) associated with a certain mileage since turning (M). If a cubic polynomial line is used to fit the data, i.e. $\beta_0 + \beta_1 M + \beta_2 M^2 + \beta_3 M^3$, there is still a lot of variability around this cubic line that remains unexplained. The natural research question would be identifying which factors contribute most to this variability and analysing whether there are other statistically significant covariates, apart from the mileage since turning (M), which exhibit a statistically significant fixed effect. This is the motivation for the following exploratory statistical analysis and actually for using LMMs.

Similarly, Figure 4 plots the change in the flange thickness (ΔF_t), with the associated mileage since turning (in miles). Both Figures 3 and 4 provide a cubic line fit with a lot of unexplained variability (R^2=0.3619 and R^2=0.119).

Linear Mixed Models (LMMs) were then specified for all of the following dependent variables in order to assess the wheelset life-cycle:

i) The change in the flange thickness – ΔF_t
ii) The change in the flange height – ΔF_h
iii) The change in the tread diameter due to wear – ΔD
iv) The diameter loss due to turning – ΔD_T

For each dependent variable (from i) to iv)), several model specifications were estimated using as explaining variables: the mileage since turning (M), the wheel diameter (D), the unit (U), the vehicle (V) identifications, the month of measurement (M_n), the wheelset type (W) and the technician (T) responsible for the maintenance decision.

Some of the models explored were specified using mileage since turning (M) as an explaining variable with three terms: a linear, a quadratic and a cubic term. The main reason to include this variable in a polynomial form resulted from the fact that its shape mimics a qualitative behaviour that is considered reasonable to quantify different stages of wear: i) an initial stage with high wear rate, ii) a normal stage with much lower wear rate and iii) a final stage with increasingly higher wear rate[1]. The other five nominal variables or factors (T, W, U, V, M_n) were used to specify random effects for different groups identified by those factors, except the wheelset type (W) that was used to specify fixed effects. For instance, the factor 'Wheelset type' (W) identifies three different groups: the group of motored wheelsets, the group of leading trailer wheelsets (non-motored wheelsets at the outer ends of the 3-vehicle set which are often thought to suffer more RCF damage) and the group of internal trailer wheelsets.

[1] The final stage with increasing higher rate (iii)) might not be a real effect, and in fact there are too few data points for mileage since turning higher than 200,000 miles. However, the estimates for all coefficients of the 3rd order polynomial are statistically significant.

In LMM with several factors, there are typically two ways of modelling random effects with multiple groups: i) crossed random effects and ii) nested random effects. The nested random effects take the assumption that particular random effects within a group are 'nested', i.e. are specific to a member of another group. For instance, say one is interested in modelling the wheelset degradation from a wheelset in a given vehicle. Then, one might consider random effects of wheelset position nested within each vehicle type, or one might not consider the random effect of wheelset position within each vehicle type and instead model these random effects in a crossed manner. In this paper, only the crossed random effects were considered because no statistically significant increase in information was found when nested random effects were considered. Different model specifications were assessed and compared based on a 'goodness of fit' measure: the -2 Restricted Log Likelihood. The reason why we conducted model comparison using the Restricted Maximum Likelihood (REML) criterion for model selection had to do mostly with the fact that *lme4* package fits the model using that same criterion[21]. Note that the Akaike Information Criterion (AIC) is used solely to compare models with different Fixed Effects (FE) and without Random Effects (RE), whereas the REML criterion is used to compare models with the same Fixed Effects but different random effects (RE). For a deeper discussion on the use of different criteria in model comparison in LMM, see Müller et al.[22], namely on the lack of consensus on how to approach model selection in LMM.

Table 2 identifies the Fixed Effects (FE) and Random Effects (RE) included in each model specification (M0-M5c). In terms of Variance structure for the RE, the variances for the different groups defined by a factor were assumed to be the same (i.e. the simpler Variance Component structure was assumed - VC - a multiple of the identity matrix). For instance, for the groups defined by the factor Vehicle (V), the variance for the group of DMC (Driving Motor Composite type) vehicles, the variance for the group of DMS (Driving Motor Second type) vehicles and the variance for the group of MS (Motor Second type) vehicles, are all considered the same.

Different model specifications were tested and only a few are included in the paper. It should be noted that all models with different combinations of factors for the random effects (all combinations of three/four factors) were analysed. However, only those that presented better results in terms of the information criterion (the -2 Restricted Log Likelihood values) are presented for an increasing number of factors (i.e. model M2a with one factor up to model M4a with three factors). In this way, one can identify which are the factors that add more variability around the expected mean (i.e. controlling for different values for the fixed effects). For the fixed effects, models M0 and M0c use the most important fixed effects (M and F_t respectively) and other models use all the statistically significant variables for each dependent variable.

For instance, the specification of model M3a would result in the following expression:

$$y_{mui} = \beta_0 + \beta_M . M_{mui} + \beta_{M^2} . M_{mui}^2 + \beta_{M^3} . M_{mui}^3 + \beta_W . W + b_{0m} + b_{0u} + \varepsilon_{mui}$$

Where m indexes the month of measurement ($m = 1,2,\dots,N$), u indexes the unit ($u = 1,2,\dots,n$) and i indexes the individual measurement of that wheel ($i = 1,2,\dots,N'$). Therefore, the model M3a considered for Fixed Effects the 3rd order polynomial on mileage since turning (M) with linear, quadratic, cubic and the intercept terms, as well as the wheelset type (W), and it also adds two crossed random effects: month of measurement (M_n) and unit (U). In terms of Variance assumptions for model M3a, the crossed random effects - b_{0m} and b_{0u} that are added to the intercept β_0, are assumed to be normally distributed with zero mean, i.e. $N(0, d_{Mn})$ and $N(0, d_U)$, respectively, and independent of each other. Finally, ε_{mui} is the traditional normally distributed random error, i.e. $N(0, \sigma^2)$, which is also independent from b_{0m} and b_{0u}.

Regarding the other models shown in Table 2: models M0 (M0 and M0c) are the simplest models only with an intercept and a slope parameter (for covariate M and covariate F_t respectively); models M1 are the reference models without any random effect considered; model M2 add solely a random effect term defined by the factor month of measurement (M_n); model M3a is described above, whereas model M3b excludes the cubic term of the fixed effect for the mileage since turning, when compared with M3a. Models M4 (a and b), when compared to models M3 (a and b), add a random effect term defined by the factor vehicle (V); whereas models (M1c-M5c) for the dependent variable diameter loss due to turning (ΔD_T) also consider the fixed effects associated with the binary variables (Y_{RCF}, Y_{FLAT} and Y_{CAV}) describing the occurrence of the associated damage defect and add a random effect term associated with the factor technician (T).

The decision to model wheel degradation at the wheelset level and not at the wheel level should be clarified at this point. This modelling approach was taken as the degradations of wheels from the same axle (i.e. wheelset) are very correlated and maintenance decisions due to parity, where wheels are turned solely to match diameters between axles on a bogie or vehicle, were not considered for this fleet.

The following discusses some results of the estimated models specified above in Table 2 for each dependent variable i) to iv):

i) The change in the flange thickness – ΔF_t

The first dependent variable to be analysed is the change in the flange thickness (ΔF_t) compared to the initial flange in a new profile (e.g. profile P8: $F_t = 28.5$ mm). As previously shown in Figure 3, there is a lot of variability around the 3rd order polynomial to describe the evolution of the change in the flange thickness with mileage since turning. This variability is then explored again through a LMM, comparing the different specifications discussed previously (models M0-M4a). Table 3 provides the Restricted Maximum Likelihood (REML) estimates for the parameters of the models tested. Model M4a-ΔF_t shows the minimum value for the information criterion (-2 Restricted Log Likelihood). Note that all coefficients are statistically significant at the 5% significance level for all fixed effects. The random effects associated with the factor month of measurement (M_n) exhibit a higher variance, followed by the factors unit (U) and vehicle (V). Comparing their variances with the total variance ($\sigma^2 + d_{M_n} + d_U + d_V = 0.06337$), we find out that the measurement noise still captures 65.7%, the factor month of measurement (M_n) captures 30.5%, the factor unit (U) captures 3.5% and finally the factor vehicle (V) captures 0.3% of the total variance.

ii) The change in the flange height – ΔF_h

The second dependent variable that needs to be statistically modelled is the change in flange height (ΔF_h) compared to the initial flange on a new profile (e.g. profile P8: $F_h = 30$ mm). As previously shown in Figure 4, there is a lot of variability around the 3rd order polynomial to describe the change in the flange height with mileage since turning. This variability is then explored again through a LMM, comparing the same specifications tested previously (models M0-M4a). Table 4 provides the REML estimates for the parameters of the models. Model M4a-ΔF_h shows the minimum value for the information criterion (-2 Restricted Log Likelihood). Note that all coefficients are statistically significant at the 5% significance level for all fixed effects. The random effects associated with the factor month of measurement (M_n) exhibit a higher variance, followed by the factors unit (U) and vehicle (V). Comparing their variances with the total variance ($\sigma^2 + d_{M_n} + d_U + d_V = 0.09217$), we find out that the measurement noise still captures 61.4%, the factor month of measurement (M_n) captures 31.4%, the factor unit (U) captures 5.8% and finally the factor vehicle (V) captures 1.4% of the total variance.

iii) The change in the tread diameter – ΔD

The third dependent variable that needs to be modelled is the change in the tread diameter due to wear (ΔD). Typically, in life-cycle calculations one tends to consider a wear rate, i.e. the change in the tread diameter over a certain time or mileage interval. Figure 5 plots the change in the tread diameter with the associated mileage since turning, as well as a 2nd order polynomial curve to fit this data. As Figure 5 shows, there is a lot of variability around this 2nd order polynomial curve, and thus LMMs were used in a similar manner to that discussed above. Model specifications contained in

Table 2 were explored (models M0-M4b), though this time, without the cubic term, as it did not provide any significant additional explaining power to the fixed effects terms. Table 5 provides the REML estimates for the parameters of the models explored. Model M4b-ΔD shows the minimum value for the information criterion (-2 Restricted Log Likelihood). Note that all coefficients are statistically significant at the 5% significance level for all fixed effects. The random effects associated with the factor month of measurement (M_n) exhibit a higher variance, followed by the factors unit (U) and vehicle (V). Comparing their variances with the total variance ($\sigma^2 + d_{M_n} + d_U + d_V = 0.81644$), we find out that the measurement noise still captures 82.6%, the factor month of measurement (M_n) captures 12.3%, the factor unit (U) captures 3.6% and finally the factor vehicle (V) captures 1.5% of the total variance.

iv) The diameter loss due to turning – ΔD_T

Finally, the last dependent variable on the wear trajectory is the diameter loss due to turning (ΔD_T). This variable together with the previous dependent variables - ΔF_h, ΔF_t and ΔD - complete the overall predictive model for the evolution of the main geometric variables to assess the wheelset life-cycle on the wear trajectory.

A discussion based on a geometrical argument may show that the diameter loss due to turning (ΔD_T) might be approximated reasonably well with a linear relation with the flange thickness (F_t)[15]. By assuming that the flange will wear with a constant flange angle (qR=68°-70°), a 1 mm reduction in flange thickness (i.e. a 1 mm horizontal shift) will translate into a 1mm×tan(qR) of reduction in radius (i.e. a 2.48-2.75 mm vertical shift), or approximately 5.00 mm of diameter loss. In fact, the wheel Tread Damage Guide[23] puts forward a 'rule of thumb' on the assessment of the loss of diameter: 'it is typically about five times the flange wear (depending on the flange angle and shape)'. Although it is possible to find particular wheels in which no other damage defects, like RCF or wheel flats, has occurred, and where this relation demonstrates a good agreement, in the general case there is a lot of variability around this linear relation.

Other covariates may contribute to explain the variability between the diameter loss due to turning (ΔD_T) and the flange thickness. For instance, the technicians' experience may contribute to explain this variability, and thus the random effects associated with factor technician (T) may capture a large amount of the variance. Moreover, other factors contribute to additional loss of diameter, such as the need to remove tread defects (e.g. RCF, wheel flats and cavities).

Model specifications contained in Table 2 were explored (models M0c-M4c), though this time, without the cubic and quadratic terms, as they did not provide any significant additional explaining effect. Table 6 provides the REML estimates for the parameters of the models explored. Model M5c-ΔD_T shows the minimum value for the information criterion (-2 Restricted Log Likelihood). Note that all coefficients are statistically significant at the 5% significance level for all fixed effects. The random effects associated with the factor technician (T) exhibit a higher variance, followed by the factors month of measurement (M_n), unit (U) and vehicle (V). Comparing their variances with the total variance ($\sigma^2 + d_T + d_{M_n} + d_U + d_V = 12.93047$), we find out that the factor technician (T) captures 48.9% and the measurement noise captures 39.1%, the factor month of measurement (M_n) captures 8.7%, the factor unit (U) captures 1.9% and finally the factor vehicle (V) captures 1.4% of the total variance.

It is important to note that the rule of thumb on the linear relation between diameter loss due to turning (ΔD_T) and the flange thickness (F_t) of -5.0 mm/mm of flange wear is quantified as -0.317 mm/mm in the model M0c and -1.367 in the final model M5c. This means that the estimated models are quantifying the effect of the flange wear only as -1.367 mm/mm of flange wear, and not as -5.0 mm/mm of flange wear as the rule of thumb would indicate. Moreover, the diameter loss due to turning is affected by the occurrence of damage defects (i.e. considering the quantities derived in for model M5c), an additional loss of 3.265 mm for the RCF defect, an additional loss of 1.517 mm for a wheel flat and an additional loss of 2.839 mm for cavities.

This LMM for the diameter loss due to turning (ΔD_T) needed further research, especially because it falls into another type of statistical model used to assess technical frontiers – the Stochastic Frontier Analysis, similar to the one provided by the rule of thumb discussed above. This research work is reported elsewhere[24], focusing on the variability between the different wheel lathe operators. This resulted from the observation above that the factor technician captured even more variance than the measurement noise.

4.2. On the damage trajectory

The previous models have focused on the prediction of the main geometrically dependent variables, i.e. the changes in the flange thickness (ΔF_t), flange height (ΔF_h) and tread diameter (ΔD) and the diameter loss due to turning (ΔD_T), without considering the occurrence of other type of defects in the wheel, namely defects associated with the damage trajectory. In order to have a comprehensive degradation model that couples other failure modes, such as the occurrence of rolling contact fatigue (RCF) defects or wheel flats, one would need to assess the probability of such defects occurring given other explaining variables that may influence them through a GLMM.

Four dependent variables were created to control for the occurrence of rolling contact fatigue (RCF) defects (Y_{RCF}), the occurrence of wheel flats (Y_{FLAT}) and the occurrence of cavities (Y_{CAV}). All these three variables are binary, i.e. they are equal to one if that defect is present for a given wheel and equal to zero otherwise.

Generalized Linear Mixed Models (GLMMs) were then specified for all of the following dependent variables in order to assess the wheelset life-cycle:

i) The occurrence of a rolling contact fatigue (RCF) defect – Y_{RCF}
ii) The occurrence of a wheel flat defect – Y_{FLAT}
iii) The occurrence of a cavity defect – Y_{CAV}

Table 7 summarises the Generalized Linear Mixed Models developed for each dependent variable (Y_{RCF}, Y_{FLAT} and Y_{CAV}) with the fixed effects (FE), the random effects (RE) and their respective variance structure. Models investigated included as fixed effects (FE): the mileage since turning (M), the tread diameter (D) and the wheelset type (W). Random effects investigated included terms associated with the factors month of measurement (M_n), unit (U) and vehicle (V). Regarding the variance structure associated with each random effect, the simpler structure was again defined (i.e. variance component - VC - a multiple of the identity matrix), not allowing different variances for different groups defined by that factor.

i) The occurrence of a rolling contact fatigue (RCF) defect – Y_{RCF}

Table 8 provides the Maximum Likelihood (ML) estimates for the parameters of the models explored for estimating the occurrence of a rolling contact fatigue defect (M0'-M4'a). In all the RCF defect probability models, the slope parameters associated with each fixed effect have the same sign, i.e. the mileage since turning (M) seems to have a mathematically positive effect in the occurrence of RCF defects, whereas the wheel diameter (D) seems to have a mathematically negative effect in the occurrence of RCF defects (the smaller the diameter, the more likely that RCF defects will occur). Nevertheless, there are some significant differences in the magnitude of the coefficients associated with each fixed effect in the model without random effects (M0) and the other models (M1'a-M4'a). The models show that the position of the wheelset in the train (i.e. the wheelset type – W) also has an effect on the probability of RCF occurring. The trailer wheelsets at the outer ends of the set (i.e. leading trailer) tend to have a higher probability of RCF occurring than other trailer or motored wheelsets. These have a lower RCF defect probability, controlling for the wheel diameter (D) and the mileage since turning (M). Model M4'a-p_{RCF} shows the minimum value for the information criterion (AIC). Note that all coefficients are statistically significant at the 5% significance level for all fixed effects. The random effects associated with the factor month of measurement (M_n) exhibit a higher variance, followed by the factors unit (U) and vehicle (V).

ii) The occurrence of a wheel flat defect – Y_{FLAT}

Table 9 provides the ML estimates for the parameters of the models explored for estimating the occurrence of a wheel flat defect (M0'-M4'a). It was found that the effect of the wheel diameter (D) was no longer statistically significant and it was left out of the set of variables included as fixed effects. The probability of wheel flats occurring seems to decrease with mileage since turning (M), and the trailer wheelsets at the outer ends of the set (i.e. leading trailer) seem to have higher probability of occurrence of a wheel flat when compared to other wheelsets (i.e. motor and internal trailer), controlling for the effect of mileage since turning (M). Model M4'a-p_{FLAT} shows the minimum value for the information criterion (AIC). Note that all coefficients are statistically significant at the 5% significance level for all fixed effects and again the random effects associated with the factor month of measurement (M_n) exhibit a higher variance, followed by the factors unit (U) and vehicle (V).

iii) The occurrence of a cavity defects – Y_{CAV}

Finally, Table 10 provides the ML estimates for the parameters of the models tested to estimate the occurrence of a cavities (M0'-M4'b). Both fixed effects associated with mileage since turning (M) and with wheel diameter (D) exhibit statistically significant estimates which are both negative, indicating that the probability of cavities occurring decreases with higher mileage since turning (M) and increases for smaller wheel diameters (D). Model M3'b-p_{CAV} shows the minimum value for the information criterion (AIC), with very similar coefficients to the model M4'b-p_{CAV}. The random effect associated with factor vehicle (V) does not capture any variability ($\sqrt{d_V} = 0.000$). Note that all coefficients are statistically significant at the 5% significance level for all fixed effects, except for the motored wheelsets (β_{Motor}), and again the random effects associated with the factor month of measurement (M_n) exhibit a higher variance, followed by the factors unit (U) and vehicle (V).

5. Conclusion and further research

This paper explored the use of Linear Mixed Models (LMM) and Generalized Linear Mixed Models (GLMM) to predict the wear and damage trajectories of railway wheelsets. For the modern multiple unit fleet investigated, the findings suggest that the changes in the flange thickness, the change in the flange height and the change in the wheel diameter due to wear are mainly dependent on the mileage since last turning (M). Mileage since turning is also a statistically significant variable to model the occurrence of rolling contact fatigue, wheel flats and cavities. The factor month of measurement (M_n) exhibits a high variance in every model, which is likely to be due to adhesion variations (i.e. lower in Autumn), whereas the technician operating the lathe (T) assumes particular importance in the modelling of diameter loss due to turning. Moreover, wheels with smaller diameters are more likely to develop rolling contact fatigue and cavity defects. It was also found that rolling contact fatigue defects and wheel flats are more likely to occur in wheelsets in the leading trailer position (i.e. the wheelsets at the extreme outer ends of the unit), controlling for other fixed effects, than on wheelsets elsewhere on the train.

The statistical models described in this paper provide a quantitative basis for future simulation exercises to optimise maintenance and renewal of train wheelsets. Applying the statistical methods described in this paper can provide significant insights into the effects of the various degradation modes on wheelset life. The data used is typically collected during normal maintenance and wheel turning and, therefore, our models can be applied to any fleet of vehicles.

The use of LMM and GLMM in statistical modelling of wheel degradation is particularly useful if one wants to conduct a life-cycle cost study as it provides a straightforward simulation mechanism to control for the variability within and between different groups, in this case different vehicles and units in a fleet. A further step on the use of these models has already been taken by applying them to examine the potential cost reductions associated with introducing a new maintenance strategy, called 'economic tyre turning'[25].

A future step which could provide a more comprehensive understanding on this topic is to apply the same statistical analysis to other fleets running in different routes and with other wear conditions (e.g. more flange wear than tread wear, freight vehicles).

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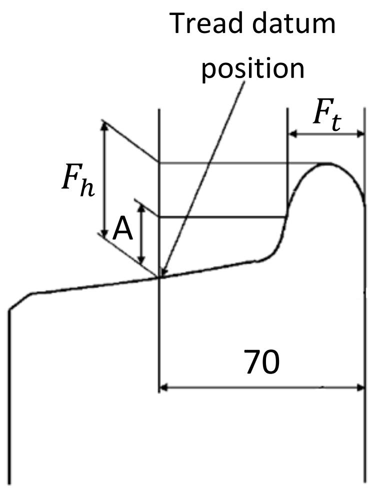

Figure 1 – Flange height (F_h) and flange thickness (F_t)

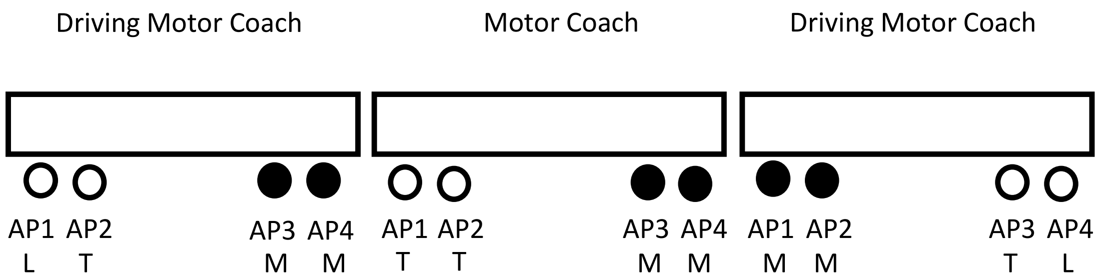

Figure 2 – Schematic representation of a three car unit (DMC, MS and DMS) and with four axle positions each (AP1-AP4).

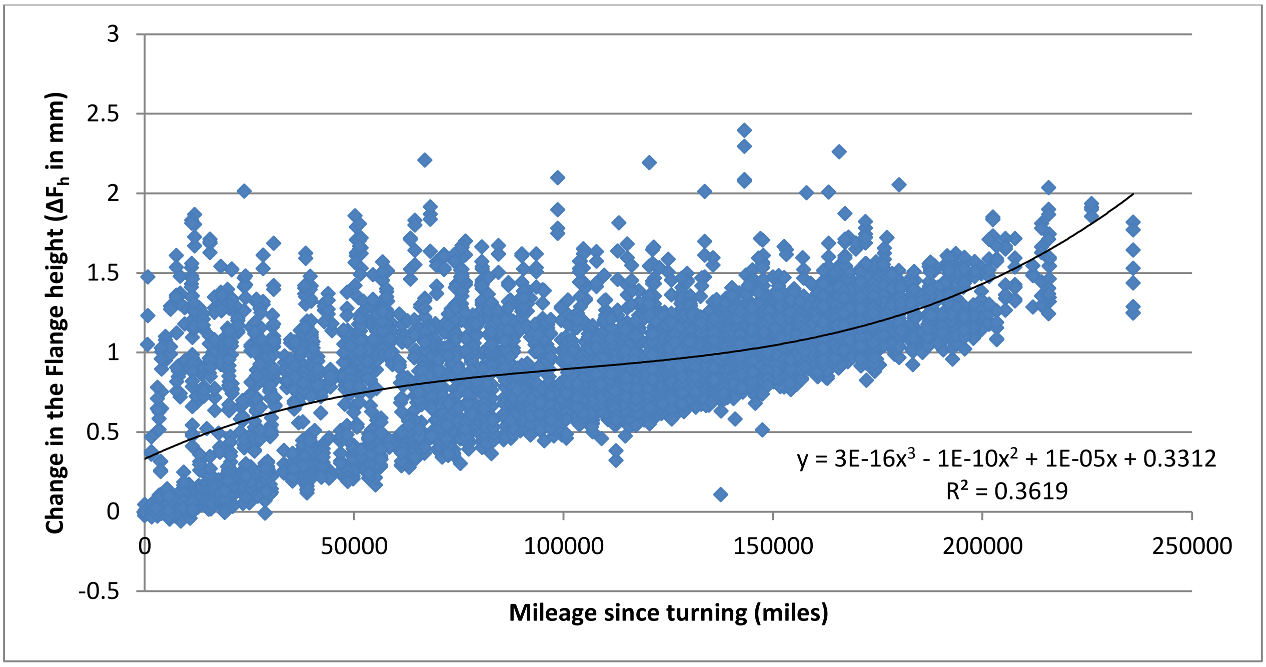

Figure 3 – Change in the flange height (ΔF_h in mm) with mileage since turning (in miles).

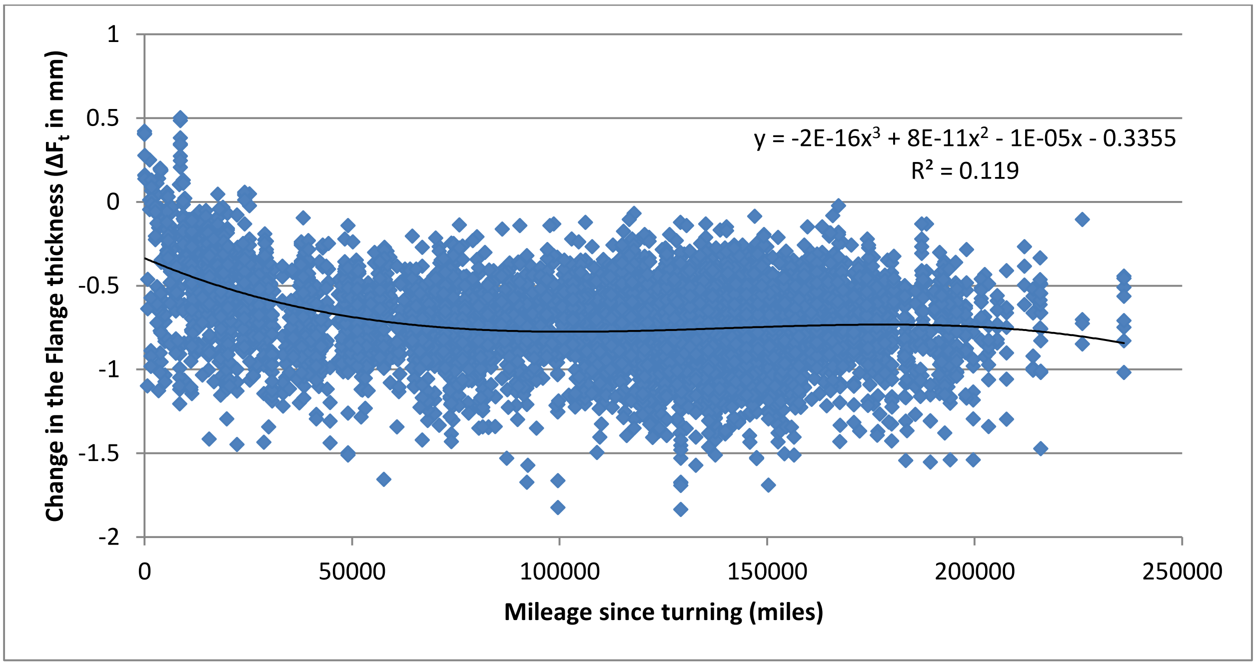

Figure 4 – Change in the flange thickness (ΔF_t in mm) with mileage since turning (in miles).

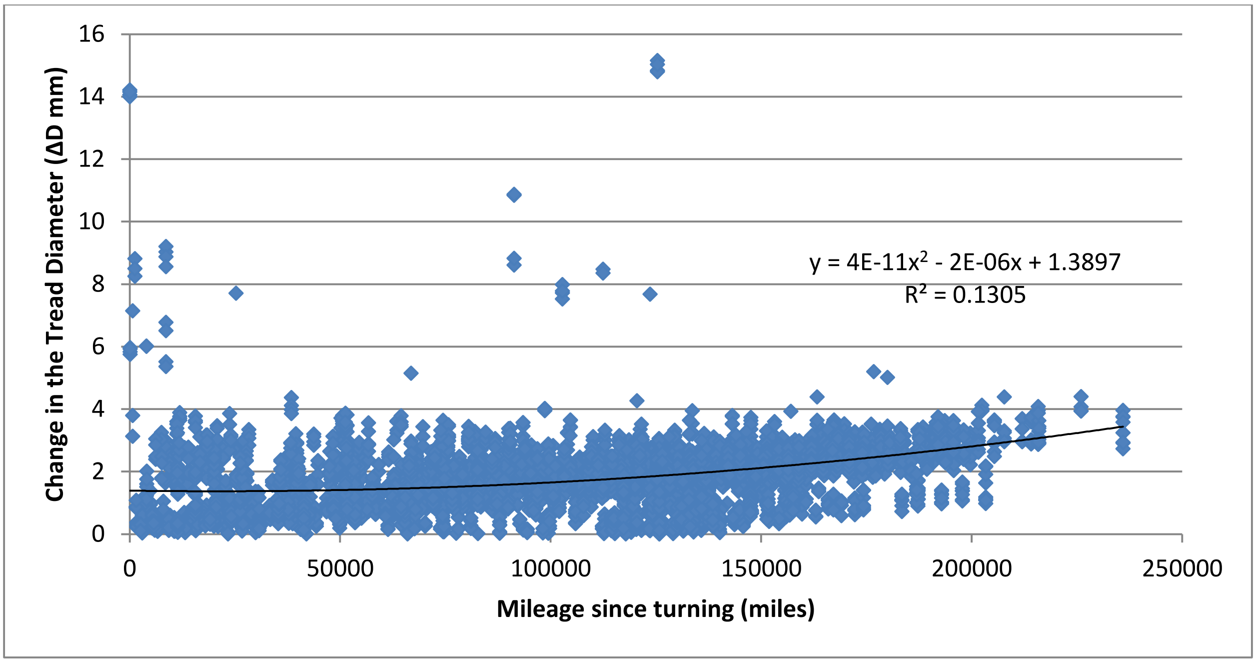

Figure 5 – Change in the Tread Diameter (ΔD in mm) with mileage since turning – reduction due to wear only, effect of wheel turning not shown.

Variables	Description	Type	Mean	Min	Max
ΔF_t	Change in the flange thickness (in mm)	Continuous	-0.7186	-1.836	0.484
ΔF_h	Change in the flange height (in mm)	Continuous	0.9268	-0.06	2.395
ΔD	Change in the wheel diameter due to wear (in mm)	Continuous	1.8390	0.002	15.151
ΔD_T	Wheel diameter loss due to turning (in mm)	Continuous	7.5253	0.037	27.443
Y_{RCF}	1 if a Rolling Contact Fatigue (RCF) defect occurred, 0 otherwise.	Binary	0.1002	0	1
Y_{CAV}	1 if a cavity defect occurred, 0 otherwise.	Binary	0.0195	0	1
Y_{FLAT}	1 if a wheel flat defect occurred, 0 otherwise.	Binary	0.1313	0	1
M	Mileage since turning (in 1000 miles)	Continuous	111.54	0.02	235.98
D	Tread diameter pre-turning (in mm)	Continuous	832.14	799.21	850.64
F_t	Flange thickness pre-turning (in mm)	Continuous	27.782	26.66	28.98
T	Technician (4 different technicians)	Nominal	-	-	-
W	Wheelset type (3 types: motored, internal or leading trailer)	Nominal	-	-	-
U	Unit number (51 units)	Nominal	-	-	-
V	Vehicle type (3 types: DMC, MS and DMS)	Nominal	-	-	-
M_n	Month of measurement (68 months)	Nominal	-	-	-

Table 1 – Variables, their description, type and some statistics.

Dependent Variable	Models	Fixed Effects (FE)	Random Effects (RE)	Variance Structure
ΔF_t	M0	$1, M$	-	-
	M1a	$1, M, M^2, M^3, W$	-	-
	M2a	$1, M, M^2, M^3, W$	M_n	VC
	M3a	$1, M, M^2, M^3, W$	M_n, U	VC, VC
	M4a	$1, M, M^2, M^3, W$	M_n, U, V	VC, VC, VC
ΔF_h	M0	$1, M$	-	-
	M1a	$1, M, M^2, M^3, W$	-	-
	M2a	$1, M, M^2, M^3, W$	M_n	VC
	M3a	$1, M, M^2, M^3, W$	M_n, U	VC, VC
	M4a	$1, M, M^2, M^3, W$	M_n, U, V	VC, VC, VC
ΔD	M0	$1, M$	-	-
	M1b	$1, M, M^2, W$	-	-
	M2b	$1, M, M^2, W$	M_n	VC
	M3b	$1, M, M^2, W$	M_n, U	VC, VC
	M4b	$1, M, M^2, W$	M_n, U, V	VC, VC, VC
ΔD_T	M0c	$1, F_t$	-	-
	M1c	$1, F_t, Y_{RCF}, Y_{FLAT}, Y_{CAV}$	-	-
	M2c	$1, F_t, Y_{RCF}, Y_{FLAT}, Y_{CAV}$	M_n	VC
	M3c	$1, F_t, Y_{RCF}, Y_{FLAT}, Y_{CAV}$	M_n, T	VC, VC
	M4c	$1, F_t, Y_{RCF}, Y_{FLAT}, Y_{CAV}$	M_n, T, U	VC, VC, VC
	M5c	$1, F_t, Y_{RCF}, Y_{FLAT}, Y_{CAV}$	M_n, T, U, V	VC, VC, VC, VC

Table 2 – Linear Mixed Models explored for each dependent variable with Fixed Effects (FE) and Random Effects (RE) and respective Variance Structure.

Model Label	Parameter	M0 - ΔF_t	M1a - ΔF_t	M2a - ΔF_t	M3a - ΔF_t	M4a - ΔF_t
Fixed Effects						
1	β_0	-0.5849 (0.007726)	-0.3993 (0.01636)	-0.4811 (0.02293)	-0.4902 (0.02387)	-0.4936 (0.02538)
M	β_M	-0.001200 (6.307×10^{-5})	-0.009800 (0.0005859)	-0.00764 (0.0005382)	-0.007419 (0.0005399)	-0.007469 (0.0005391)
M^2	β_{M^2}	-	7.410×10^{-5} (6.263×10^{-6})	6.006×10^{-5} (5.700×10^{-6})	5.792×10^{-5} (5.727×10^{-6})	5.778×10^{-5} (5.719×10^{-6})
M^3	β_{M^3}	-	-1.728×10^{-7} (1.94×10^{-8})	-1.460×10^{-7} (1.759×10^{-8})	-1.397×10^{-7} (1.768×10^{-8})	-1.366×10^{-7} (1.770×10^{-8})
W	β_{Motor}	-	0.02230 (0.008712)	0.03017 (0.007502)	0.03161 (0.007358)	0.03527 (0.007570)
	$\beta_{Trailer}$	-	0.08001 (0.00913)	0.08162 (0.007585)	0.08248 (0.007701)	0.08746 (0.008174)
	$\beta_{Leading}$	-	0[b]	0[b]	0[b]	0[b]
Random Effects						
M_n	$\sqrt{d_{Mn}}$	-	-	0.140	0.140	0.139
U	$\sqrt{d_U}$	-	-	-	0.048	0.047
V	$\sqrt{d_V}$	-	-	-	-	0.015
Scale	σ	0.253	0.244	0.209	0.204	0.204
-2 Restricted Log Likelihood		-	-	-1508.95	-1681.86	-1697.65
AIC value		538.42	99.48	-	-	-
Number of parameters		3	7	8	9	10

Table 3 – Restricted Maximum Likelihood (REML) estimates for the parameters of models M0-M4a for the dependent variable: Change in the flange thickness (ΔF_t).

[a] Approximate Standard Errors for Fixed Effects are included in parentheses. [b] This parameter is considered redundant.

Model Label	Parameter	M0 - ΔF_h	M1a - ΔF_h	M2a - ΔF_h	M3a- ΔF_h	M4a - ΔF_h
Fixed Effects						
1	β_0	0.4754 (0.008881)	0.3437 (0.01883)	0.4749 (0.02720)	0.4701 (0.02923)	0.4791 (0.03597)
M	β_M	0.004047 (7.249×10^{-5})	0.01193 (0.0006745)	0.008182 (0.0006406)	0.008148 (0.0006343)	0.008076 (0.0006305)
M^2	β_{M^2}	-	-9.597×10^{-5} (7.210×10^{-6})	-6.895×10^{-5} (6.785×10^{-6})	-6.872×10^{-5} (6.728×10^{-6})	-6.999×10^{-5} (6.689×10^{-6})
M^3	β_{M^3}	-	3.169×10^{-7} (2.236×10^{-8})	2.474×10^{-7} (2.094×10^{-8})	2.495×10^{-7} (2.078×10^{-8})	2.621×10^{-7} (2.070×10^{-8})
W	β_{Motor}	-	0.05269 (0.01003)	0.03952 (0.008930)	0.04157 (0.008629)	0.03175 (0.008855)
	$\beta_{Trailer}$	-	-0.06832 (0.01051)	-0.07586 (0.009354)	-0.07648 (0.009031)	-0.09271 (0.009582)
	$\beta_{Leading}$	-	0[b]	0[b]	0[b]	0[b]
Random Effects						
M_n	$\sqrt{d_{Mn}}$	-	-	0.166	0.170	0.170
U	$\sqrt{d_U}$	-	-	-	0.073	0.073
V	$\sqrt{d_V}$	-	-	-	-	0.036
Scale	σ	0.290	0.281	0.249	0.240	0.238
-2 Restricted Log Likelihood		-	-	665.98	332.65	264.31
AIC value		2278.83	1858.45	-	-	-
Number of parameters		3	7	8	9	10

Table 4 – Restricted Maximum Likelihood (REML) estimates for the parameters of models M0-M4a for the dependent variable change in the flange height (ΔF_h).

[a] Approximate Standard Errors for Fixed Effects are included in parentheses. [b] This parameter is redundant.

Model Label	Parameter	M0 - ΔD	M1b - ΔD	M2b - ΔD	M3b - ΔD	M4b - ΔD
Fixed Effects						
1	β_0	0.987 (0.02743)	1.217 (0.04667)	1.341 (0.06175)	1.336 (0.06614)	1.394 (0.09236)
M	β_M	0.007639 (0.000224)	0.001903 (0.0008393)	-0.0008263 (0.0008682)	-0.0006207 (0.0008736)	-0.0005329 (0.0008854)
M^2	β_{M^2}	-	2.890×10^{-5} (4.09×10^{-6})	2.609×10^{-5} (4.228×10^{-6})	3.967×10^{-5} (4.267×10^{-6})	3.560×10^{-5} (4.382×10^{-6})
W	β_{Motor}	-	0.06036 (0.03172)	0.02628 (0.03014)	0.02927 (0.02967)	-0.01270 (0.03050)
	$\beta_{Trailer}$	-	-0.1496 (0.03326)	-0.1580 (0.03159)	-0.1611 (0.03107)	-0.2277 (0.03302)
	$\beta_{Leading}$	-		0[b]	0[b]	0[b]
Random Effects						
M_n	$\sqrt{d_{Mn}}$	-	-	0.322	0.320	0.317
U	$\sqrt{d_U}$	-	-	-	0.170	0.172
V	$\sqrt{d_V}$	-	-	-	-	0.111
Scale	σ	0.897	0.888	0.840	0.825	0.821
-2 Restricted Log Likelihood		-	-	15766.06	15629.8	15576.67
AIC value		16367.6	16249.2	-	-	-
Number of parameters		3	6	7	8	9

Table 5 – Restricted Maximum Likelihood (REML) estimates for the parameters of models M0-M4b for the dependent variable change in the Tread Diameter (ΔD).

[a] Approximate Standard Errors for Fixed Effects are included in parentheses. [b] This parameter is redundant.

Model Label	Parameter	M0c - ΔD_T	M1c - ΔD_T	M2c - ΔD_T	M3c - ΔD_T	M4c - ΔD_T	M5c - ΔD_T
Fixed Effects							
1	β_0	16.8587	34.48553	39.58786	45.9905	44.50749	46.51300
		(3.7467)	(3.50410)	(3.64597)	(3.99044)	(3.97864)	(3.9704)
F_t	β_{F_t}	-0.3360	-0.98135	-1.12562	-1.35069	-1.29974	-1.36727
		(0.1349)	(0.12646)	(0.12487)	(0.13599)	(0.13629)	(0.13515)
Y_{RCF}	β_{RCF}	-	3.53981	3.61747	3.45374	3.38064	3.26511
			(0.10808)	(0.10705)	(0.10805)	(0.10723)	(0.10677)
Y_{FLAT}	β_{FLAT}	-	1.50080	1.43816	1.45948	1.47444	1.51732
			(0.09762)	(0.09617)	(0.10985)	(0.10990)	(0.10888)
Y_{CAV}	β_{CAV}	-	2.68745	2.90104	2.91918	2.87964	2.83966
			(0.23063)	(0.22731)	(0.22543)	(0.22390)	(0.22174)
W	β_{Motor}	-	-0.44570	-0.42377	-0.47616	-0.49889	-0.59099
			(0.09046)	(0.08894)	(0.08403)	(0.08260)	(0.08472)
	$\beta_{Trailer}$	-	-0.24949	-0.21310	-0.21884	-0.22938	-0.37037
			(0.09476)	(0.09322)	(0.08808)	(0.08652)	(0.09170)
	$\beta_{Leading}$	-	0[b]	0[b]	0[b]	0[b]	0[b]
Random Effects							
T	$\sqrt{d_T}$	-	-	2.264	2.558	2.422	2.514
M_n	$\sqrt{d_{Mn}}$	-	-	-	1.023	1.050	1.062
U	$\sqrt{d_U}$	-	-	-	-	0.489	0.500
V	$\sqrt{d_V}$	-	-	-	-	-	0.423
Scale	σ	2.769	2.510	2.467	2.318	2.273	2.248
-2 Restricted Log Likelihood		-	-	29041.38	28441.62	28289.20	28168.70
AIC value		30455.72	29233.51	-	-	-	-
Number of parameters		3	8	9	10	9	10

Table 6 – Restricted Maximum Likelihood (REML) estimates for the parameters of models M0c-M5c for the dependent variable Diameter loss due to turning (ΔD_T).

[a] Approximate Standard Errors for Fixed Effects are included in parentheses. [b] This parameter is redundant.

Dependent Variable	Models	Fixed Effects (FE)	Random Effects (RE)	Variance Structure
Y_{RCF}	M0'	1, M, D	-	-
	M1'a	1, M, D, W	-	-
	M2'a	1, M, D, W	Mn	VC
	M3'a	1, M, D, W	Mn, U	VC, VC
	M4'a	1, M, D, W	Mn, U, V	VC, VC, VC
Y_{FLAT}	M0'	1, M,	-	-
	M1'a	1, M, D	-	-
	M2'a	1, M, D	Mn	VC
	M3'a	1, M, D	Mn, U	VC, VC
	M4'a	1, M, D	Mn, U, V	VC, VC, VC
Y_{CAV}	M0'	1, M	-	-
	M1'b	1, M, D, W	-	-
	M2'b	1, M, D, W	Mn	VC
	M3'b	1, M, D, W	Mn, U	VC, VC
	M4'b	1, M, D, W	Mn, U, V	VC, VC, VC

Table 7 – Generalized Linear Mixed Models explored for each dependent variable with Fixed Effects (FE) and Random Effects (RE) and respective Variance Structure.

Model Label	Parameter	M0' - p_{RCF}	M1'a - p_{RCF}	M2'a - p_{RCF}	M3'a - p_{RCF}	M4'a - p_{RCF}
Fixed Effects						
1	β_0	13.1256 (2.7931)	14.4409938 (2.8364365)	32.943999 (4.23178)	32.552654 (4.311109)	31.769104 (4.287260)
M	β_M	0.00515 (0.00089)	0.0051057 (0.0008927)	0.009828 (0.001107)	0.009727 (0.001166)	0.012609 (0.001264)
D	β_D	-0.01916 (0.00337)	-0.0201701 (0.0034274)	-0.043435 (0.005127)	-0.043067 (0.005217)	-0.042256 (0.005168)
W	β_{Motor}	-	-1.0079563 (0.1146005)	-1.153113 (0.125025)	-1.212075 (0.127960)	-1.614640 (0.142675)
	$\beta_{Trailer}$	-	-0.1747294 (0.1076079)	-0.224323 (0.118458)	-0.265700 (0.120790)	-0.783396 (0.142534)
	$\beta_{Leading}$	-	0[b]	0[b]	0[b]	0[b]
Random Effects						
M_n	$\sqrt{d_{Mn}}$	-	-	1.435	1.450	1.486
U	$\sqrt{d_U}$	-	-	-	0.612	0.636
V	$\sqrt{d_V}$	-	-	-	-	0.695
Log Likelihood		-2002.272	-1949.875	-1684.64	-1648.79	-1589.64
AIC value		4010.54	3909.7	3381.27	3311.59	3195.27
Number of parameters		3	5	6	7	8

Table 8 – Maximum Likelihood (ML) estimates for the parameters of models M1-M6 for the probability of occurrence Rolling Contact Fatigue defects (p_{RCF}).

[a] Approximate Standard Errors for Fixed Effects are included in parentheses. [b] This parameter is redundant.

Model Label	Parameter	M0' - p_{FLAT}	M1'a - p_{FLAT}	M2'a - p_{FLAT}	M3'a - p_{FLAT}	M4'a - p_{FLAT}
Fixed Effects						
1	β_0	-0.1323385 (0.0730829)	0.216073 (0.110790)	-0.670410 (0.291599)	-0.785132 (0.338891)	-0.785229 (0.412802)
M	β_M	-0.0186813 (0.0007859)	-0.018777 (0.000788)	-0.019327 (0.001083)	-0.020149 (0.001187)	-0.021362 (0.001215)
W	β_{Motor}	-	-0.412531 (0.106822)	-0.535233 (0.133944)	-0.566112 (0.139108)	-0.444594 (0.143837)
	$\beta_{Trailer}$	-	-0.424209 (0.112708)	-0.496855 (0.139390)	-0.524672 (0.144890)	-0.353106 (0.155126)
	$\beta_{Leading}$	-	0[b]	0[b]	0[b]	0[b]
Random Effects						
M_n	$\sqrt{d_{Mn}}$	-	-	1.952	2.137	2.175
U	$\sqrt{d_U}$	-	-	-	0.844	0.818
V	$\sqrt{d_V}$	-	-	-	-	0.398
Log Likelihood		-2111.156	-2102.763	-1476.98	-1421.07	-1404.32
AIC value		4226.3	4213.5	2963.96	2854.13	2822.64
Number of parameters		2	4	5	6	7

Table 9 – Maximum Likelihood (ML) estimates for the parameters of models M1-M6 for the probability of occurrence of flat wheels (p_{FLAT}).

[a] Approximate Standard Errors for Fixed Effects are included in parentheses. [b] This parameter is redundant.

Model Label	Parameter	M0' - p_{CAV}	M1' - p_{CAV}	M2' - p_{CAV}	M3'b - p_{CAV}	M4'b - p_{CAV}
Fixed Effects						
1	β_0	10.5855	9.251591	34.908354	47.736476	47.728031
		(5.7617)	(5.746094)	(9.018025)	(11.006983)	(11.020602)
M	β_M	-0.01530	-0.015278	-0.012384	-0.011628	-0.011614
		(0.00179)	(0.001791)	(0.002209)	(0.002635)	(0.002637)
D	β_D	-0.01579	-0.014027	-0.046076	-0.062783	-0.062787
		(0.00697)	(0.006949)	(0.010906)	(0.013314)	(0.013330)
W	β_{Motor}	-	0.064226	-0.077100	0.061992	0.064044
			(0.244704)	(0.265408)	(0.282116)	(0.282268)
	$\beta_{Trailer}$	-	-0.574918	-0.695272	-0.789285	-0.788606
			(0.288857)	(0.310097)	(0.328704)	(0.328855)
	$\beta_{Leading}$	-	0[b]	0[b]	0[b]	0[b]
Random Effects						
M_n	$\sqrt{d_{Mn}}$	-	-	1.748	2.291	2.299
U	$\sqrt{d_U}$	-	-	-	1.327	1.334
V	$\sqrt{d_V}$	-	-	-	-	0.000
Log Likelihood		-557.189	-552.799	-490.859	-464.63	-468.63
AIC value		1120.40	1115.6	993.72	943.25	945.25
Number of parameters		3	5	6	7	8

Table 10 – Maximum Likelihood (ML) estimates for the parameters of models M0'-M4'b for the probability of occurrence of cavities (p_{CAV}).

[a] Approximate Standard Errors for Fixed Effects are included in parentheses. [b] This parameter is redundant.