An Assessment Method for Highway Network Vulnerability

Abstract

This paper introduces a methodology to assess the level of vulnerability of road transport networks. A new technique based on fuzzy logic and exhaustive search optimisation is used to combine vulnerability attributes with different weights into a single vulnerability index for network links, which may be used to measure the impact of disruptive events. The network vulnerability index is then calculated using two different aggregations: an aggregated vulnerability index based on physical characteristics and an aggregated vulnerability index based on operational characteristics. The former uses link physical properties such as its length and the number of lanes, whilst the latter reflects aspects of the network flow. The application of the methodology on a synthetic network (based on Delft city, Netherland) demonstrates the ability of the technique to estimate variation in the level of vulnerability under different scenarios. The method also allows exploration of how variation in demand and supply impact on overall network vulnerability, providing a new tool for decision makers to understand the dynamic nature of vulnerability under various events. The method could also be used as an evaluation tool to gauge the impact of particular policies on the level of vulnerability for the highway network and highlight weaknesses in the network.

Keywords: Highway network; vulnerability; fuzzy logic; optimisation; vulnerability attributes; policy.

1 Introduction

According to Gaillard (2010) the concept of vulnerability was first introduced in the disaster literature as early as the 1970s and spread quickly in the 1980s to other disciplines. However vulnerability does not have a widely accepted definition based on the context (Jenelius et al. 2006). For example in the context of transport research, vulnerability is normally used to express the “susceptibility” or “sensitivity” of the transport network to threats or hazards (Berdica, 2002) that can lead to significant effects on road network performance. Jenelius et al. (2006) related the concept of vulnerability to risk theory. As a consequence they defined vulnerability using two components of risk assessment i.e. the probability of disruption and its consequences - in similar vein to risk evaluation. However, the probability of certain events could be very low in some geographic areas or not identified, which limits the potential of this approach. In contrast, Taylor and D’Este (2007) and Maltinti et al., (2011) suggested that the concept of vulnerability is more strongly related to the consequence of link failure, regardless of the probability of failure and the event itself.

This paper therefore presents a method to quantify the vulnerability of the highway network. The main advantage of the proposed method is the ability to take into account link attributes such as link flow, free flow speed and capacity in estimating a link vulnerability index. A new method based on fuzzification and an exhaustive search optimisation technique is employed to combine a set of defined attributes with different weights into a single vulnerability index. The proposed methodology can be extended in principle to include further attributes to reflect a wider set of vulnerability related issues.

2 Vulnerability assessment methods and indicators

A number of different vulnerability assessment methods and indicators are available in the literature, e.g. (Jenelius, 2009, Jenelius, 2010, Berdica, 2002, Rashed and Weeks, 2003, Taylor and Susilawati, 2012, Brenkert and Malone, 2005), arising from different interpretations of the concept of vulnerability and the scope of analysis. In general there are two main methods; use of a network wide screen (Jenelius et al., 2006) and techniques based on pre-selection of potentially vulnerable links according to a set of of criteria (Knoop et al., 2012). The network wide screen approach gives a full analysis of the transport network by investigating the impact of the closure of each link on the overall network performance, measured by the total travel time. However, the high computional time of this approach is considered to be something of a disadvantage. To address this issue, Murray-Tuite and Mahmassani (2004) introduced a bi-level approach based on game theory in order to identify the most critical links in the road transport network. They defined a vulnerability link index to measure the importance of a particular link to the connectivity of an origin-destination (OD) pair, then aggregated over all OD pairs to obtain a disruption link index. They did not demonstrate the application of the technique with an authentic road network however. Meanwhile Knoop et al. (2012) reviewed the link vulnerability attributes proposed by Tampère et al. (2007) and found that different criteria identified different links as the most vulnerable. Their conclusion was that attributes should be seen as a complementary set rather than singularly.

Different approaches in the literature could also be classified according to the indicators used to assess vulnerability. For example Taylor and D'Este (2007) and Chen et al. (2012) used accessibility and network efficiency indices as metrics of vulnerability to identify the wider socioeconomic consequences of link closure. Meanwhile Scott et al. (2006) employed transport network perfomance indicators to identify the most "critical" or "important" link in the road network. Overall, the use and applicability of each approach appears to be heavily dependent on the scope of the research.

Most of the previous research on vulnerability measures and methodologies has focused on assessing the impact of link closure for a particular origin-destination or at link level, but has not referred to the link characteristics that lead to vulnerability. This paper extends the work of Tampère et al. (2007) by introducing a new link vulnerability index developed on the basis of link vulnerability attributes. The vulnerability index could be used to measure the impact of disruptive events (e.g. manmade events such as accidents or natural events such as adverse weather conditions) on road transport network functionality. The network vulnerability index is then calculated using two different aggregations: an aggregated vulnerability index based on physical characteristics and an aggregated vulnerability index based on operational characteristics.

3 Modelling the vulnerability of the road transport network

According to Srinivasan (2002), a vulnerability assessment may include deterministic factors (such as network capacity), quantitative time-varying factors (such as traffic flow and speed), some qualitative measures (for example event type and expected consequences), plus some random factors. There is therefore a need to develop an index in such a way that it can take into account various attributes of vulnerability. In the vulnerability model described in this paper, a number of vulnerability attributes are selected from the literature (e.g. Srinivasan 2000; Tampère et al. 2007) and combined with relative weights to assess the vulnerability of the road transport network. The calculated vulnerability index value is then compared with the generalized travel cost to test the ability of the method to identify the most critical links in a case study (see section 4). Section 3.1 below presents the vulnerability attributes adopted to develop the index, whilst section 3.2 introduces the fuzzification and exhaustive search optimisation techniques used to develop the link vulnerability index.

3.1 Vulnerability attributes

Ideally, the set of vulnerability attributes should be as complete as possible, capturing as many features as possible of the impact of link closures in reality. It should also be as orthogonal as possible, capturing different aspects with a minimum degree of duplication. According to Srinivasan (2002), several types of attributes may have a significant effect on link vulnerability and these could be classified into four main categories, namely; network characteristics, traffic flow, threats and neighbourhood attributes. Network attributes could include characteristics such as road types and physical configuration, whilst traffic attributes could cover link capacity, flow and speed. Attributes concerning 'threats' may include event types and their expected consequences, with neighbourhood attributes capturing the influence of adjacent subsystems such as land use and population. Whilst the traffic and network related attributes are the main focus in the current research, the methodology developed here allows the addition of further attributes to cover each of the four categories.

A number of vulnerability attributes (VAs) were therefore selected from the literature in order to estimate a vulnerability index for each link of the network. The first three attributes (VA_1, VA_2 and VA_3) adopted here from Tampère et al. (2007) and Knoop et al. (2012), are dependent on link capacity, flow, length, free flow and traffic congestion density. VA_1 reflects the link traffic flow in relation to link capacity and is estimated by:

$$VA_1 = f_{am}^i / (1 - f_{am}^i / C_{am}) \tag{1}$$

where f_{am}^i is the flow on link a during period time i for a travel mode m, C_{am} is the capacity of link a for a travel mode m. As the flow f_{am}^i increases with respect to capacity C_{am}, the number of vehicles experiencing higher levels of delay will increase.

The second attribute VA_2 identifies the direct impact of link flow with respect to link capacity as defined below.

$$VA_2 = f_{am}^i / C_{am} \tag{2}$$

The main difference between VA_1 and VA_2 is that the calculated value of VA_1 from Eq. (1) is scaled with respect to the highest and lowest VA_1values for all links in the road network considered (see Eq. (7) below). This normalisation is not applied in the calculation of VA_2. Therefore, VA_1 measures the relationship between f_{am} and C_{am} for each link with respect to the whole network. VA_2 however is intended to reflect local values of f_{am} and C_{am} for each link.

VA_3 represents the inverse of the time needed for the tail of the queue to reach the upstream junction and is estimated by:

$$VA_3 = f_{am}^i (n_a \, kj_{am} - f_{am}^i / V_{am}) / l_a \tag{3}$$

where n_a is the number of lanes of link a that have been used by travel mode m, kj_{am} reflects congestion density for link a, V_{am} is the free flow speed of link a for a travel mode m, and l_a is the length of link a.

All the above attributes were derived based on accident scenarios (see Tampère et al. 2007 and Knoop et al. 2012). A number of other attributes were therefore also added to capture the significance of network characteristics (such as link capacity and length) on vulnerability. As a result two further attributes, VA_4 and VA_5, have been formulated and included in the vulnerability index.

The fourth attribute, VA_4, is calculated from the capacity of link a relative to the maximum capacity of all network links in order to reflect relative link importance, as presented in Eq. (4).

$$VA_4 = \frac{C_{am}}{C_{max}} \tag{4}$$

where C_{max} is the maximum capacity of all network links.

The fifth attribute, VA_5, simply uses the link length as a physical property representing the level of importance of the link, as given in Eq. (5).

$$VA_5 = l_a \tag{5}$$

Finally, the number of shortest paths that use the link is also considered due to the importance of this feature in link vulnerability analysis (Srinivasan, 2002), leading to the definition of attribute VA_6. This sixth attribute is calculated by Eq. (6) below reflecting the number of times the link is a component of the shortest path between different OD pairs.

$$VA_6 = \sum_{ij} s_{ij} \tag{6}$$

where s_{ij} is given a value of one if link a is a component of the shortest path between origin i and destination j and a value of zero otherwise. Expert opinion may also be used to allocate a higher weight to the value of VA_6 for a particular link if the link is part of a strategic route.

3.2 Link vulnerability index

To develop a single measure for vulnerability based on more than one attribute, three approaches have been proposed in the literature (Srinivasan, 2002). The first approach is based on experts' opinions in ranking or weighting each attribute and then combining these attributes using a simple linear regression model. This model can be calibrated using observed or reported vulnerability ratings for various levels of the contributing factors. In the second approach, a continuous vulnerability index is represented by a function that includes all the proposed attributes. The relative weights are derived according to the best fit between the model prediction and actual ratings. The vulnerability index is then compared against a set of ordered thresholds that are estimated from empirical models. For example if the vulnerability index is below the first threshold then the vulnerability rate will be 1 or if it falls in the range between the first and second thresholds then the vulnerability rate will be 2. However, the determining these thresholds in an accurate way is a significant challenge and much further research would be needed in order to establish the threshold values. The third approach is based on operational experience whereby experts choose a set of weights for some attributes (such as spare capacity and flows) in order to evaluate vulnerability if a particular scheme is implemented. The main advantages of this approach compared with the previous two methods are simplicity and flexibility (Srinivasan, 2002), however it may be difficult to obtain the necessary data in practice.

In the current research therefore, a new method based on fuzzification and an exhaustive search optimisation technique is employed to combine the various attributes (defined above) into a vulnerability index. Fuzzification is the process of converting a crisp quantity to a fuzzy one (Ross, 2005). It is adopted here to accommodate the complexity and uncertainty in traffic behaviour alongside randomised elements in both traffic data and the simulation process. Each attribute is evaluated according to four assessment levels represented by four fuzzy membership functions. An exhaustive search technique is then employed to identify the optimal weight contribution of each fuzzified attribute. This is determined by the level of weights at which the correlation between the vulnerability index (obtained from the weighted attributes) and the given total travel cost is the strongest. Travel cost could be estimated based on different factors such as travel time, distance or toll. In this research travel time is used as an estimate of travel cost, however, the method is flexible and could accommodate other cost measures. The full details of the technique are presented in the following sub sections.

3.2.1 Data normalization

A normalization process is firstly applied so that a standard method can then be used to allocate a membership grade value for each of the link attributes in the fuzzification process. Each calculated VA for each link is therefore normalized using the following equation:

$$(VA_{x,a})_{\mathrm{n}} = \frac{VA_{x,a} - VA_{x,min}}{VA_{x,max} - VA_{x,min}} \tag{7}$$

where $(VA_{\boldsymbol{x},\boldsymbol{a}})_{\mathrm{n}}$ and $VA_{x,a}$ are the normalized and non-normalized values of the vulnerability attribute x of link a. $VA_{x,max}$ and $VA_{x,min}$ are the maximum and minimum values of the vulnerability attribute set following normalization respectively. The normalisation process maps the value of each attribute into a closed interval [0, 1]. However given that the two vulnerability attributes, VA_2 and VA_4, are already scaled between [0, 1], these are not subject to the normalisation procedure using Eq. 7.

3.2.2 Fuzzy membership of vulnerability attributes

Four assessment levels are proposed to evaluate each VA, where each level is defined by a fuzzy function having membership grades varying from 0 to 1. Various membership functions have been proposed in the literature (Ross, 2005). However, triangular and trapezoid membership functions were adopted to fuzzify the four normalized vulnerability attributes. The rationale was twofold: these functions are by far the most common forms encountered in practice and are relatively simply in terms of calculating membership grades (Torlak et al., 2011; Ross, 2005). Other membership functions such as a Gaussian distribution may also be used. However previous research (e.g. Shepard, 2005) has indicated that real world systems are relatively insensitive to the shape of the membership function. The membership grade value μ of each normalised attribute $(VA_{\boldsymbol{x},\boldsymbol{a}})_{\mathrm{n}}$ for link a is obtained from the following fuzzy triangular and trapezoidal functions:

$$\mu_{low} = \begin{cases} 1 & 0 \leq (VA_{\boldsymbol{x},\boldsymbol{a}})_{\mathrm{n}} \leq 0.25 \\ \dfrac{0.5 - (VA_{\boldsymbol{x},\boldsymbol{a}})_{\mathrm{n}}}{0.5 - 0.25} & 0.25 < (VA_{\boldsymbol{x},\boldsymbol{a}})_{\mathrm{n}} < 0.5 \\ 0 & (VA_{\boldsymbol{x},\boldsymbol{a}})_{\mathrm{n}} \geq 0.5 \end{cases}$$

$$\mu_{Medium} = \begin{cases} 0 & (VA_{\boldsymbol{x},\boldsymbol{a}})_{\mathrm{n}} \leq 0.25 \\ \dfrac{(VA_{\boldsymbol{x},\boldsymbol{a}})_{\mathrm{n}} - 0.25}{0.5 - 0.25} & 0.25 < (VA_{\boldsymbol{x},\boldsymbol{a}})_{\mathrm{n}} \leq 0.5 \\ \dfrac{0.75 - (VA_{\boldsymbol{x},\boldsymbol{a}})_{\mathrm{n}}}{0.75 - 0.50} & 0.5 < (VA_{\boldsymbol{x},\boldsymbol{a}})_{\mathrm{n}} < 0.75 \\ 0 & (VA_{\boldsymbol{x},\boldsymbol{a}})_{\mathrm{n}} \geq 0.75 \end{cases}$$

$$\mu_{high} = \begin{cases} 0 & (VA_{\boldsymbol{x,a}})_{\mathrm{n}} \leq 0.5 \\ \dfrac{(VA_{\boldsymbol{x,a}})_{\mathrm{n}} - 0.5}{0.75 - 0.5} & 0.5 < (VA_{\boldsymbol{x,a}})_{\mathrm{n}} \leq 0.75 \\ \dfrac{1 - (VA_{\boldsymbol{x,a}})_{\mathrm{n}}}{1.0 - 0.75} & 0.75 < (VA_{\boldsymbol{x,a}})_{\mathrm{n}} \leq 1.0 \end{cases}$$

$$\mu_{very\ high} = \begin{cases} 0 & (VA_{\boldsymbol{x,a}})_{\mathrm{n}} \leq 0.75 \\ \dfrac{(VA_{\boldsymbol{x,a}})_{\mathrm{n}} - 0.75}{1 - 0.75} & 0.75 < (VA_{\boldsymbol{x,a}})_{\mathrm{n}} \leq 1.0 \\ 1 & (VA_{\boldsymbol{x,a}})_{\mathrm{n}} > 1.0 \end{cases}$$

The membership grade function outlined above can be adjusted or re-scaled to reflect real life conditions and expertise opinion. However a single membership grade function is assumed for each of the attributes in this paper.

Membership grades for link a represented by a fuzzy relationship $R(a)$ for different VA for link a in the network are calculated based on the equations above and are shown below:

$$R(a) = \begin{bmatrix} \mu(VA_1)_{low} & \mu(VA_1)_{medium} & \mu(VA_1)_{high} & \mu(VA_1)_{very\ high} \\ \mu(VA_2)_{low} & \mu(VA_2)_{medium} & \mu(VA_2)_{high} & \mu(VA_2)_{very\ high} \\ \mu(VA_3)_{low} & \mu(VA_3)_{medium} & \mu(VA_3)_{high} & \mu(VA_3)_{very\ high} \\ \mu(VA_4)_{low} & \mu(VA_4)_{medium} & \mu(VA_4)_{high} & \mu(VA_4)_{very\ high} \\ \mu(VA_5)_{low} & \mu(VA_5)_{medium} & \mu(VA_5)_{high} & \mu(VA_5)_{very\ high} \\ \mu(VA_6)_{low} & \mu(VA_6)_{medium} & \mu(VA_6)_{high} & \mu(VA_6)_{very\ high} \end{bmatrix}$$

Each row of the matrix above represents attribute membership grades, whilst the columns show the memberships grades for the four attributes for a particular assessment level.

To obtain a single vulnerability index $VI(a)$ for link a, based on VAs, the above matrix is modified by two vectors. First, a weighting vector w_i is introduced to reflect the importance of each VA in the vulnerability assessment as expressed in Eq. (8) below.

$$VI(a) = \mathrm{R(a)}w_i$$

$$\mathrm{VI(a)} = \sum_{i=1}^{6} w_i VA_i(a) \tag{8}$$

An optimization technique is used to identify the relative weight for each VA as described in section 3.2.3. The outcome of this step is a fuzzy vector containing the membership values for each link at each assessment level. There are then two possible approaches to calculate a single value for $VI(a)$ from the fuzzy vector. The first considers the maximum membership grade value whilst the second approach involves multiplying the fuzzy vector by a standardising vector to take into account the effect of each assessment level (Ross, 2005). In this research, the second method is used as it allows for the accumulating effect of each assessment level on the calculated $VI(a)$. The standardising vector (s) shown in Eq. (9) is therefore proposed in order to obtain a single value, adjusted from 0 to 1.

$$s = [0.25 \;\; 0.5 \;\; 0.75 \;\; 1] \tag{9}$$

The values of the standardising vector (s) are equal to those for $VA_{\boldsymbol{x}}$ when $\mu(VA_{\boldsymbol{x}}) = 1$ for low, medium, high and very high, as obtained from the membership grade function previously defined.

3.2.3 Attribute weight identification

The weight vector w_i for each attribute could be proposed by traffic experts and policy makers. It could also vary according to the modelled scenario. However in the current research, the weight value for each attribute is estimated by comparing the vulnerability index, $VI(a)$, for link a against the relative travel time per trip, $RTTpT(a)$, with the closure of link a – a similar approach to that used by Knoop et al. (2012). The relative travel time per trip, $RTTpT(a)$, is defined as the difference between the total network travel time during link closure and the total network travel time under normal conditions, with respect to the total network travel time under normal conditions.

A linear regression analysis between $VI(a)$ and $RTTpT(a)$ for the road network is then calculated and the weight vector is obtained when the coefficient of determination R^2 is maximised: i.e. maximise R^2 for the linear regression between $VI(a)$ and $RTTpT(a)$ subject to the following constraint:

$$\sum_i w_i = 1$$

In the above formulation w_i is implicitly included in $VI(a)$ and is the only design variable. An exhaustive search is employed to find the weight vector w_i for each attribute, where each weight W_i is increased from 0.0 to 1.0 with an increment of 0.01. For each weight combination, the vulnerability index, $VI(a)$, is calculated using

Eq. (8). A linear regression analysis is performed between $VI(a)$ for each weight combination and $RTTpT(a)$, with the coefficient of determination R^2 estimated by:

$$R^2 = 1 - \frac{ss_{resid}}{ss_{total}}$$

where ss_{resid} is the sum of the squared residuals from the regression and ss_{total} is the sum of the squared differences from the mean of the $RTTpT(a)$.

The above approach is repeated for various combinations of W_i considering the weight constraint and re-calculating R^2 for each combination. The weight combination achieving the highest R^2 is then selected as the optimum weight set for the attributes. The flow chart in Fig. 1 illustrates the procedure for obtaining the optimum weight combination for the attributes on the basis of the strongest correlation between $VI(a)$ and $RTTpT(a)$. A constrained linear least squares approach could also be used to find the weights that achieving the best fit between $VI(a)$ and $RTTpT(a)$. However, no particular advantage would be anticipated through this alternative method as the exhaustive search optimisation was a straightforward and low resource task with the search space limited between [0, 1].

3.3 Network vulnerability index

Based on the steps described above a vulnerability index for each link can then be calculated. Despite the importance of this link based index in identifying the most critical links, there is still a need however for an aggregated vulnerability index in order to evaluate the vulnerability of the overall network under different conditions. Two aggregated vulnerability indices are proposed i.e. a physically-based aggregated vulnerability index and an operational based aggregated vulnerability index. The physical based aggregated vulnerability index VI_{Ph} is calculated using the length and number of lanes of each link as follows:

$$VI_{Ph} = \frac{\sum_a^e VI_a l_a n_a}{\sum_a^e l_a n_a} \quad (10)$$

where e is the number of links in the road network, n_a is the number of lanes in link a and l_a is the length of link a. The operational based aggregated vulnerability index VI_{op} is calculated based on link capacity as follows:

$$VI_{op} = \frac{\sum_a^e VI_a f_a}{\sum_a^e f_a} \quad (11)$$

where f_a is the traffic flow using link a.

4 Case study

A synthetic road transport network of Delft city is used to illustrate the vulnerability of road network under different scenarios using the proposed methodology. Delft is a city and municipality in the province of South Holland in the Netherlands. The total population of the Delft city is 98675 with 4,324.1 per km^2 density (Statistics Netherlands 2012). In general, cars are widely used in the Netherlands where people use this mode for almost half their trips (Statistics Netherlands, 2012). The synthetic Delft road network model is supplied with the OmniTrans modelling software (6.022). The network used for the case study here is based on the authentic city network features, but has undergone some simplification and modification and as a result may deviate from the current network for the city of Delft. An relatively uncomplicated but still representative network was needed to demonstrate the method and, for the Delft study case, data were readily available. The main purpose here is not to carry out an empirical study of Delft, but rather to demonstrate and test the approach using sufficiently realistic data. It should be emphasised that at this stage the research is focused on the development of the methodology and in principle, it could be applied with any road transport network.

The Delft road transport network consists of 25 zones, two of which are under development (24 & 25) and 1142 links. 483 links are bi-directional and 176 are one-way including connectors and different road types as shown in Figure 2.

In the case study undertaken here, user equilibrium assignment (UE) was chosen to obtain the spatial distribution of the traffic volume. UE is based on Wardrop's first principle, whereby no individual trip maker can reduce his/her path cost by switching routes. This principle is also known as the user optimum (Wardrop, 1952). The suitability of the UE method for identifying the most critical link is based on two issues (Scott 2006). Firstly, the ability of the method to take into account the level of link functionality by allocating the user to the best route in terms of travel time, i.e. users can not improve their travel time by changing their route. Secondly, the use of user equilibrium assignment allows the impact of removing the link to be calculated for both the link user and non-users (due to rerouting the link user). The mathematical formulation of UE is explained in detail in Ortúzar and Willumsen (2011).

However, traffic data obtained from simulation based on static UE assignment without any junction modelling (as opposed to ‘real-world’ observations) cannot capture the full effects of unexpected link closures, as this process is not able to capture queuing, imperfect information, etc. As a result the optimum attribute weights arising from the highest R^2 criteria may be different from the weights that may arise from the best fit against observed data. However, real world measurements may also vary, for example according to individual traveller behaviour and this is not covered in the scope of the model presented in this paper. In order to examine the effect of queuing on the travel time, junction modelling was undertaken using the OmniTrans software for a case involving the closure of a small number of links. Junction modelling with OmniTrans generates outputs including queue lengths alongside a number of performance measures for the junction as a whole. The results indicated that travel time increased slightly and by a maximum of 1%.

For the case study as a whole, three different scenarios were considered. The first calculated VAs for each link in the network and estimated VI for each link. In the second scenario, the impact of demand variations on VI_{Ph} and VI_{op} were investigated using different departure rates during the morning peak. The impact of network capacity reduction under the same demand variations were then studied in the third scenario.

4.1 Results and discussion

4.1.1 Group one scenarios

All VAs were calculated for each link in the network based on the steps described in section 3, using a static assignment model for the morning peak. 1068 simulations (equivalent to the number of links in the network) were carried out to check the impact of each individual link closure on the network travel time. In each case, only one link was blocked, i.e. to represent a closed link due to a road accident or roadworks.

As OmniTrans does not allow “en-route” route-choice modelling, closure of the link is implemented at the start of simulation, resulting in a subsequent new equilibrium state. This implies that drivers would need to be aware of the link closure and of alternative routes. To overcome this shortcoming, a deterministic user-equilibrium (UE) assignment was used for the base condition scenario, assuming drivers have previous experience and knowledge of their shortest paths. A stochastic 'randomising' term (ε) was also added to the generalised cost in order to reflect the uncertainty associated with traveller behaviour under a link closure scenario. However, the use of this stochastic 'randomising' term (ε) leads to instability in link flows even with large number of iterations (up to 1000). Consequently, the stochastic 'randomising' term (ε) was abandoned and a deterministic UE assignment used for all scenarios instead. This implies that the perceived travel times are very accurate and therefore all vehicles on each link would experience the same travel time. In this case, the simulation results may underestimate the impact of each link closure in the new equilibrium state. To obtain more realistic impact results two issues should be considered; traveller behaviour (e.g. the proportion of travellers who will change their route with a link closure) and the availability of an en-route choice model implemented within the traffic assignment software. However, the main aim of the analysis reported here was to investigate the ability of the attributes to reflect link importance under different conditions. The results obtained and reported therefore assume that all drivers have good knowledge about the link closure and the availability of alternative routes. As the modelled period is the morning peak it would be quite reasonable to assume that a high proportion of the road users are regular commuters/travellers and nearly all the users have a high level of knowledge about route availability and traffic conditions. Alternatively, in practice a variable massage sign or in-vehicle intelligent transport system may update travellers knowledge of the link closure and alternative routes.

Figure 3 introduces the variation in VAs for each link for the base condition, i.e. no link closure. It should be noted that each VA highlighted a different set of critical links (in terms of highest values) in line with the findings of Knoop et al. (2012). Figure 4 shows the correlation of each attribute with relative travel time per trip, $RTTpT(a)$ arising from individual link closure. The coefficient of determination, R^2, for each attribute reflects its strength of association with $RTTpT(a)$. As an example, VA_1 has the highest R^2 (=0.5447) followed by VA_3 (=0.4403), then VA_4 (=0.4206). Meanwhile, VA_2 has a low R^2 (=0.191). Both VA_5 and VA_6 have a negligible correlation, with R^2 equal to 0.0039 and 0.0148, respectively. These findings highlight the need to develop a single vulnerability index taking into account all the four main attributes proposed in this research, whilst VA_5 and VA_6 would contribute little to the index.

The set of weights calculated above are not universal but network dependent. However, they can be used for the same network to consider different scenarios, for example to test the effectiveness of different policy or the impact of implementing new technology.

Figure 5 shows the correlation between the calculated vulnerability index, VI, for each link based on the combined weights of the four vulnerability attributes VA_1 to VA_4 and the relative travel time per trip. VA_5 and VA_6 are not considered in the derivation of VI as their correlation with $RTTpT$ is very weak, as described above. The relatively low value of R^2 presented in Figure 5 reflects the fact that the increase in the total travel time may not be the only consequence arising from link closure. For example, the closure of some links is likely to lead to the disconnection of some zones creating unsatisfied demand and a misleading value of reduced total travel time as a result of a lower overall load on the network. However this is a feature of the physical layout of the network and would therefore vary in magnitude for different links and with the application of the method in different cities. Figure 6 further illustrates the relationship between the relative travel time for different link closure scenarios with associated unsatisfied demand and the vulnerability index. Links with high VI and low $RTTpT$ are associated with unsatisfied demand.

When the results of the ‘cut’ links (i.e. links that when closed result in zone disconnection, creating unsatisfied demand) are removed from the data regression analysis, the coefficient of determination R^2 increases to 0.8667 as depicted in Figure 7.

However, excluding cut links from the estimation of VI could also be undesirable due to their importance in the vulnerability of the overall network - cut links create unsatisfied demand which in turn (intuitively) increases network vulnerability. As a result, modelling the impact of unsatisfied demand is essential to give a more realistic VI. From the literature there are two possible ways to overcome this issue, the first is to quantify the impact of link closure by two indicators; one for the cut links and the other for the remaining links (Jenelius et al., 2006). The other approach is to estimate the cost of time due to a particular link closure (Jenelius, 2009). In the current research, the second approach is adopted to obtain the total impact for all links in the network. The increase in total travel time due to the closure of links (cut links) is then modelled by adding the proposed unsatisfied demand impact (UnSDI), calculated by Eq. (12) below, to the total travel time.

$$UnSDI = pf_a(\tau + \frac{TTpT_a}{L_a} * l_a) \tag{12}$$

where p is the percentage of unsatisfied demand, f_a is the link flow, τ is the closure period, $TTpT_a$ is the total travel time per trip during the closure of link a, l_a is the length of link a and L_a is the total network length without link a.

The inclusion of the UnSDI in the total travel time calculation leads to an improvement in the correlation between VI and the modified relative travel time, increasing R^2 to 0.9125 as shown in Figure 8.

The influence of network configuration is implicitly included by considering unsatisfied demand, as the percentage of unsatisfied demand reflects the ability of the network to offer alternative routes during a certain link closure. For example, zero unsatisfied demand highlights the ability of the network to offer alternative routes for all OD pairs during a link closure.

4.1.2 Group two scenarios

Here the impact of variations in demand on VI_{Ph} and VI_{op} is investigated using different departure rates during the morning peak. VI_{Ph} and VI_{op} are calculated using Eqs. (10) and (11). Figure 9 shows both VI_{Ph} and VI_{op} under uniformly distributed departure rates, whilst Figure 10 plots the variations of VI_{Ph} and VI_{op} under different departure rates, with and without UnSDI. The vulnerability level is measured by both indices (VI_{Ph} and VI_{op}) and increases in line with the rate of increase in the departure rate, as depicted in Figure 10. It is also apparent that the inclusion of UnSDI increases the vulnerability level. This leads to the conclusion that both indices are able to reflect the impact of increases in demand on the level of vulnerability.

4.1.3 Group three scenarios

In this analysis the ability of VI to capture the impact of reductions in network capacity under the same variations in demand is investigated. Overall network capacity could be reduced in practice due to the effects of network wide events such as heavy rain or snowfall. The level of reduction in network capacity and speed were assumed based on evidence in the literature (Enei et al., 2011; Pisano and Goodwin, 2004; Koetse and Rietveld, 2009). This group of scenarios was undertaken using reduced capacity in addition to a reduction in saturation flow or free flow speed by 10%, in order to model the impact of a weather related event. Figure 11 shows the variations of VI_{Ph} and VI_{op} under different departure rates and variations in supply. The vulnerability level measured by both indices, VI_{Ph} and VI_{op}, increases in the case of reduced capacity compared with full network capacity. Furthermore the difference between the vulnerability indices (i.e. full network capacity and reduced capacity) increases with increased in demand and diminishes at low demand. This leads to the conclusion that the VI_{Ph} and VI_{op} indices are both able to reflect the impact of varying reductions in supply and demand on the level of vulnerability.

5 Conclusions

A new methodology for assessing the level of vulnerability of road transport networks has been introduced which is able to reflect the importance of network links. The proposed technique is a two-stage process where a link vulnerability index is first developed and subsequently network vulnerability indices are estimated. The development of the link vulnerability index is based on a fuzzy membership grade and exhaustive optimisation search. It allows the identification of the relative weights of vulnerability attributes when combined in a single vulnerability index for each link in the network. The proposed methodology is able to accomodate further attributes in order to reflect wider vulnerability related issues, such as road type and the economic value of the traffic flow. Two overall network vulnerability indices, namely physical and operational vulnerability indices, are then developed. The technique has been successfully demonstrated on a representative road transport network.

Correlations between each attribute and the total travel time due to link closure in a synthetic Delft city network are investigated. It was found that none of the attributes on its own is able to justify the full impact of link closure. These findings reveal the need to develop a single vulnerability index that is able to take into account a number of attributes. A term to reflect the impacts of unsatisfied demand has also been proposed to model the decrease in the total travel time that arises when particular cut links result in unsatisfied demand. An exhaustive search optimisation technique for attribute weight identification produced a high correlation between the single vulnerability index and the total travel time, with an R^2 value of 0.9125. Two attributes (related to link length and the shortest paths) yielded a low contribution to the single vulnerability index as they are heavily dependent on the network configuration and infrastructure characteristics. It is therefore suggested that the number of link lanes may be combined with the link length in order to enhance their overall contribution to the vulnerability index.

It should be noted that the relative weights of the vulnerability attributes are not universal but network dependent. However, the weights calculated for each attribute can be used with a particular network in order to consider the impacts of different scenarios - for example to test the effectiveness of different policies or the impact of introducing new technology.

Finally, the estimated network physical and operational vulnerability indices show a good correlation with variations in both supply and demand. These indices represent a potential tool that could be used to gauge the total network vulnerability under different scenarios. It can also be used to assess the effectiveness of different policies or technologies to improve the overall network vulnerability.

References

BERDICA, K. 2002. An introduction to road vulnerability: what has been done, and should be done. *Transport policy,* 9**,** 117-127.

BRENKERT, A. & MALONE, E. 2005. Modeling Vulnerability and Resilience to Climate Change: A Case Study of India and Indian States. *Climatic Change,* 72**,** 57-102.

CHEN, B. Y., LAM, W. H. K., SUMALEE, A., LI, Q. & LI, Z. C. 2012. Vulnerability analysis for large-scale and congested road networks with demand uncertainty. *Transportation Research Part A: Policy and Practice,* 46**,** 501-516.

ENEI, R., DOLL, C., KLUG, S., PARTZSCH, I., SEDLACEK, N., KIEL, J., NESTEROVA, N., RUDZIKAITE, L. & PAPANIKOLAOU, A. 2011. Vulnerability of transport systems- Main report Transport Sector Vulnerabilities within the research project WEATHER (Weather Extremes: Impacts on Transport Systems and Hazards for European Regions) funded under the 7th framework program of the European Commission, Project co-ordinator: Fraunhofer-ISI. Karlsruhe, 30.9.2010.

GAILLARD, J. C. 2010. Vulnerability, capacity and resilience: Perspectives for climate and development policy. *Journal of International Development,* 22**,** 218-232.

JENELIUS, E. 2009. Network structure and travel patterns: explaining the geographical disparities of road network vulnerability. *Journal of Transport Geography,* 17**,** 234-244.

JENELIUS, E. 2010. *Large scale road network vulnerability analysis.* PhD, KTH Royal Institute of Technology.

JENELIUS, E., PETERSEN, T. & MATTSSON, L. G. 2006. Importance and exposure in road network vulnerability analysis. *Transportation Research Part A: Policy and Practice,* 40**,** 537-560.

KNOOP, V. L., SNELDER, M., VAN ZUYLEN, H. J. & HOOGENDOORN, S. P. 2012. Link-level vulnerability indicators for real-world networks. *Transportation Research Part A: Policy and Practice,* 46**,** 843-854.

KOETSE, M. J. & RIETVELD, P. 2009. The impact of climate change and weather on transport: An overview of empirical findings. *Transportation Research Part D: Transport and Environment,* 14**,** 205-221.

MALTINTI, F., MELIS, D. & ANNUNZIATA, F. Methodology for Vulnerability Assessment of a Road Network. In: OSWALD CHONG, W. K. & HERMRECK, C., eds. International Conference on Sustainable Design and Construction (ICSDC) 2011, 2011 Kansas City, Missouri. American Society of Civil Engineers, 686-693.

MURRAY-TUITE, P. M. & MAHMASSANI, H. 2004. Methodology for Determining Vulnerable Links in a Transportation Network. *Transportation Research Record: Journal of the Transportation Research Board,* 1882**,** 88-96.

ORTÚZAR, J. & WILLUMSEN, L. G. 2011. *Modelling transport,* United Kingdom, John Wiley & Sons, Ltd.

PISANO, P. & GOODWIN, L. 2004. Research Needs for Weather-Responsive Traffic Management. *Transportation Research Record: Journal of the Transportation Research Board,* 1867**,** 127-131.

RASHED, T. & WEEKS, J. 2003. Assessing vulnerability to earthquake hazards through spatial multicriteria analysis of urban areas. *International Journal of Geographical Information Science,* 17**,** 547-576.

RODRIGUE, J. P., COMTOIS, C. & SLACK, B. 2009. *The Geography of Transport Systems,* USA, Routledge.

ROSS, T. J. 2005. *Fuzzy Logic with Engineering Application,* England, Jone wiley & Sons, LTD.

SCOTT, D. M., NOVAK, D. C., AULTMAN-HALL, L. & GUO, F. 2006. Network robustness Index: A new method for identifying critical links and evaluating the performance of transportation networks. *Journal of Transport Geography,* 14**,** 215-227.

SHEPARD, R. B. 2005. *Quantifying environmental impact assessments using fuzzy logic,* USA, Springer.

SRINIVASAN, K. 2002. Transportation network vulnerability: assessment: a quantitative framework. *Southeastern Transportation Center – Issues in Transportation Security.*

STATISTICS NETHERLANDS 2012. Statistical yearbook 2012. Statistics Netherlands.

TAMPÈRE, M. J., STADA, J. & IMMERS, L. H. Methodology for Identifying Vulnerable Sections in a National Road Network. Proceedings of 86th Annual Meeting of the Transportation Research Board, 2007 Washington, DC.

TAYLOR, M. A. P. & SUSILAWATI, S. 2012. Remoteness and accessibility in the vulnerability analysis of regional road networks. *Transportation Research Part A: Policy and Practice,* 46**,** 761-771.

TAYLOR, M. P. & D'ESTE, G. M. 2007. Transport Network Vulnerability: a Method for Diagnosis of Critical Locations in Transport Infrastructure Systems. *In:* MURRAY, A. T. & GRUBESIC, T. H. (eds.) *Critical Infrastructure.* Springer Berlin Heidelberg.

TORLAK, G., SEVKLI, M., SANAL, M. & ZAIM, S. 2011. Analyzing business competition by using fuzzy TOPSIS method: An example of Turkish domestic airline industry. *Expert Systems with Applications,* 38**,** 3396-3406.

WARDROP. 1952. Some theoretical aspects of road traffic research. Proceedings of the Institution of Civil Engineers, Part II, 1, pp. 325-378.

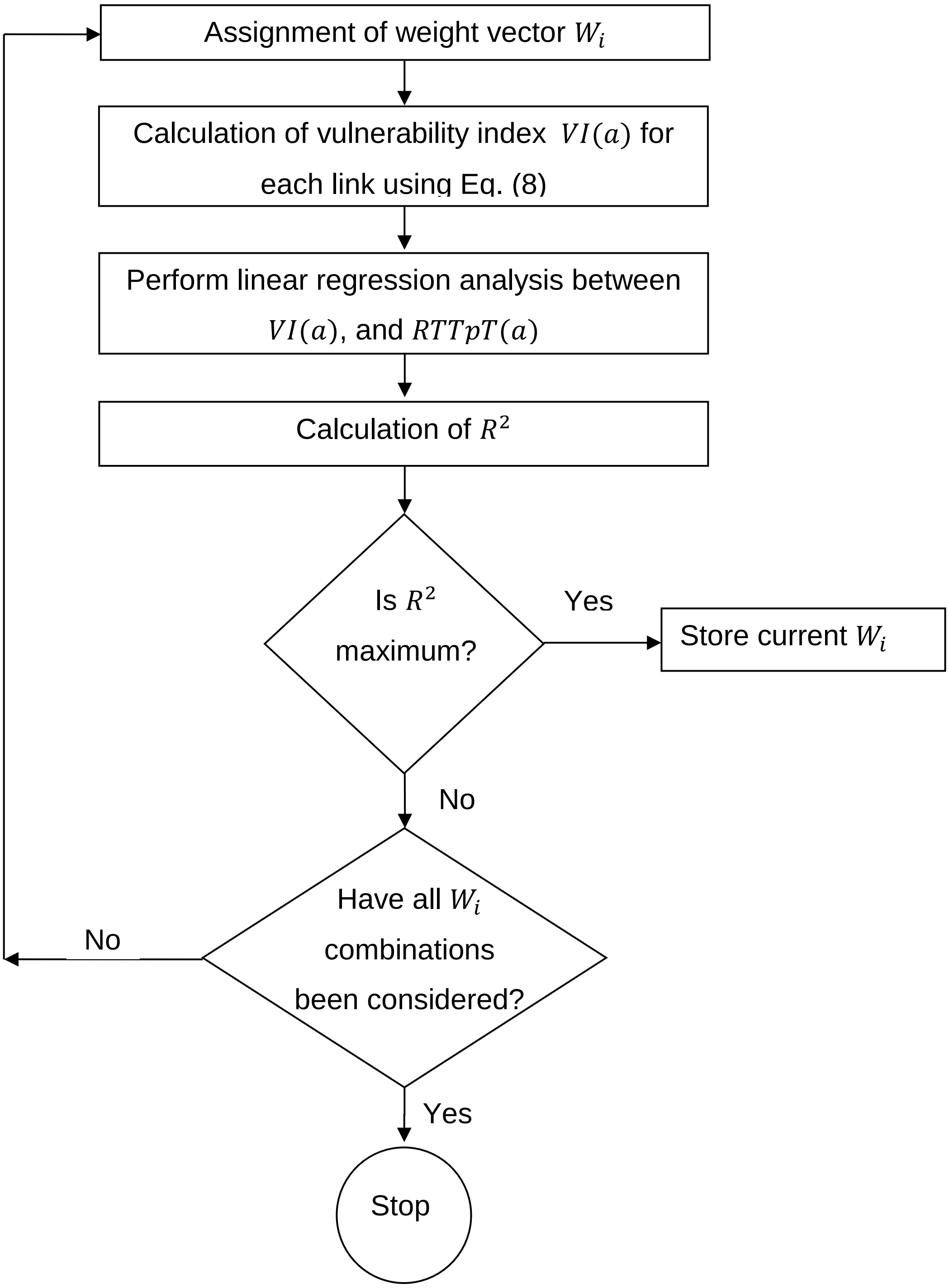


Figure 1 A flow chart for the optimum weight combination for the four attributes.

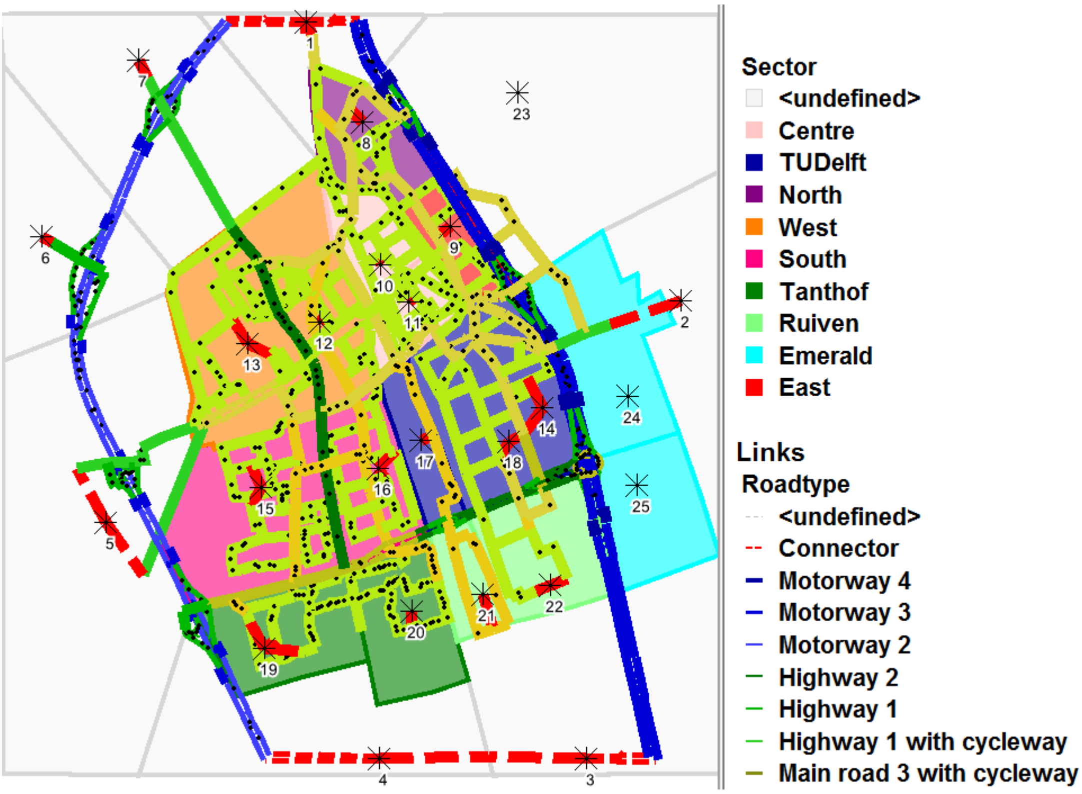
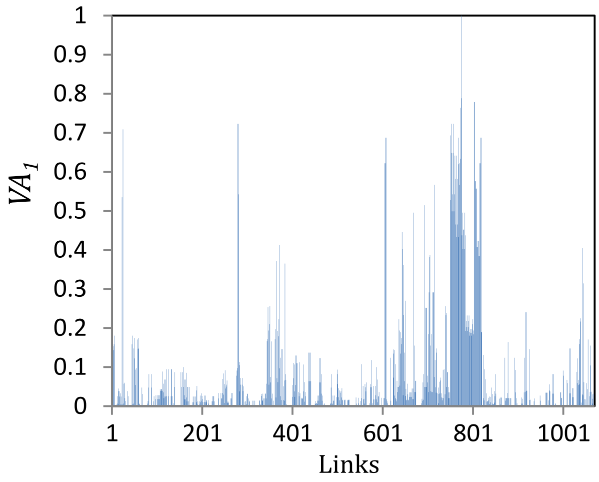
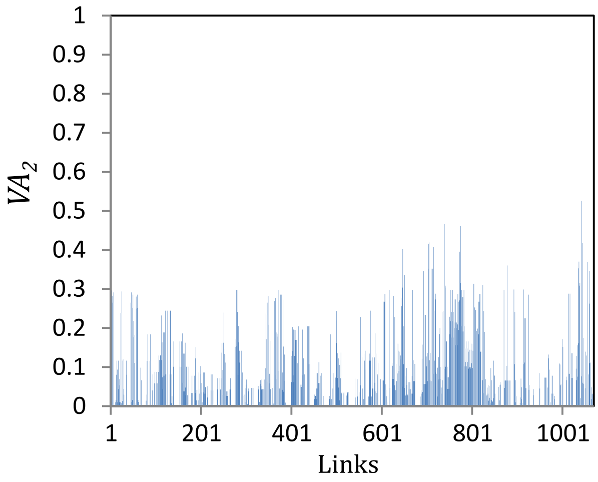


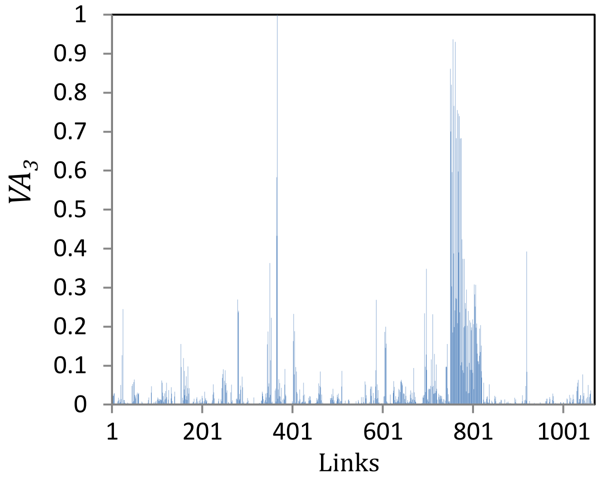
Figure 2 Delft road transport network.



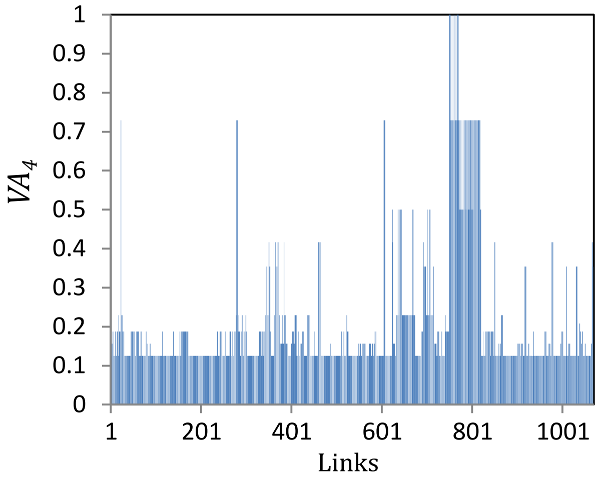
(a) VA_1



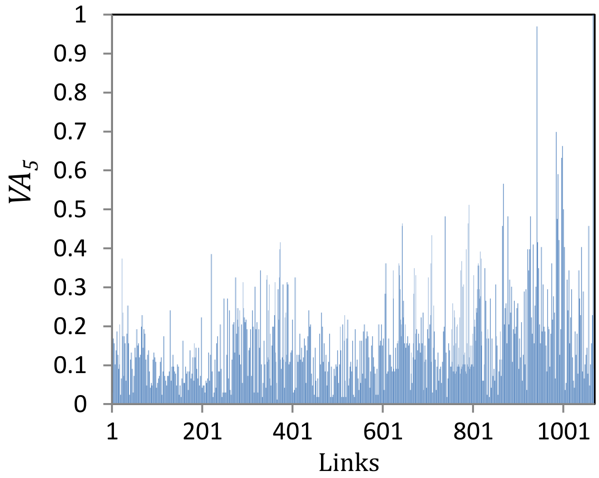
(b) VA_2



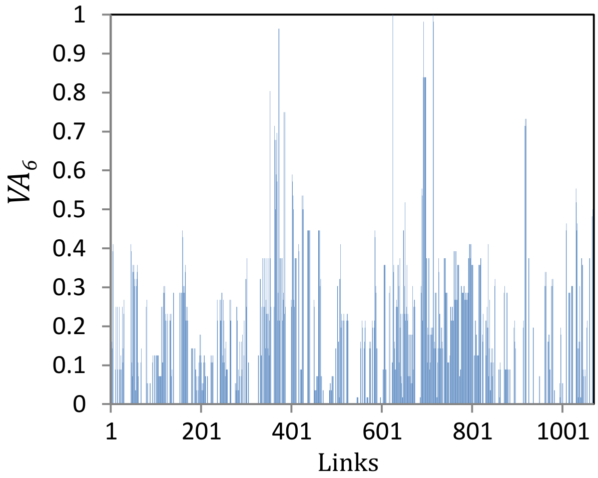
(c) VA_3



(d) VA_4

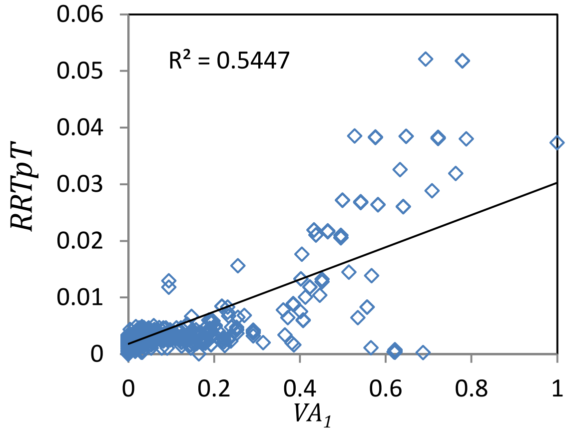


(e) VA_5

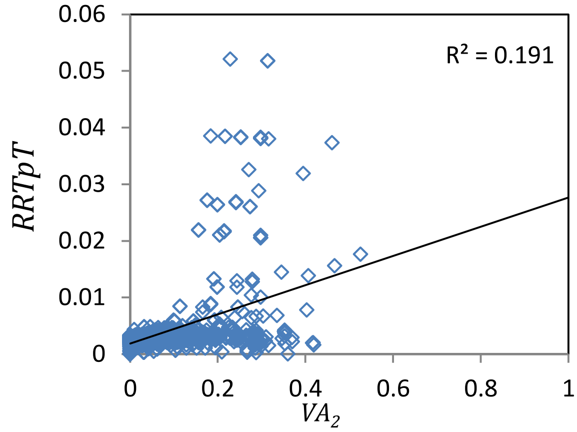


(f) VA_6

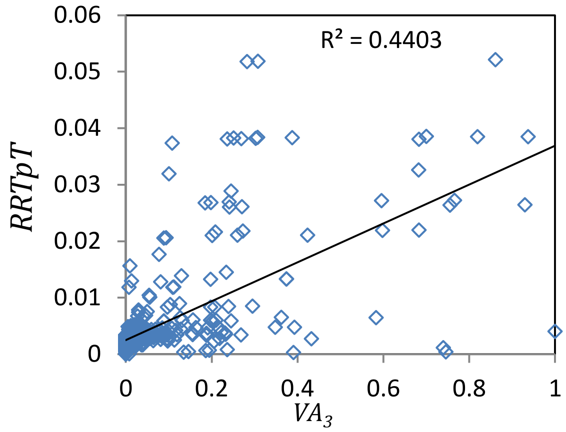
Figure 3 Variation of VAs per link.



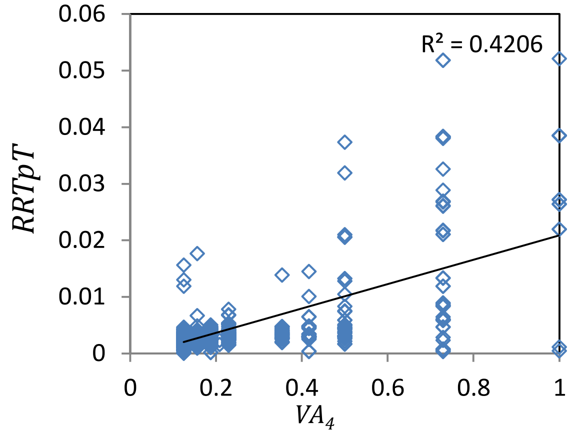
(a) VA_1



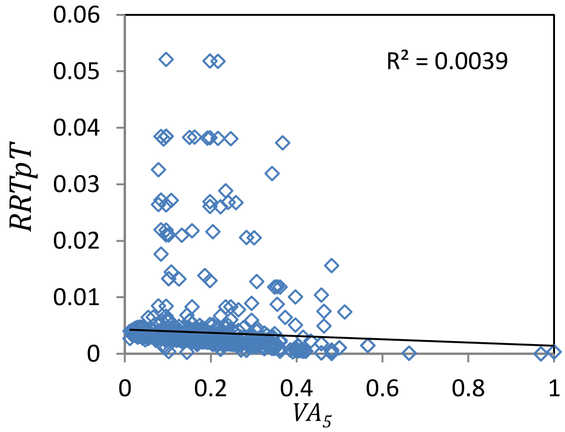
(b) VA_2



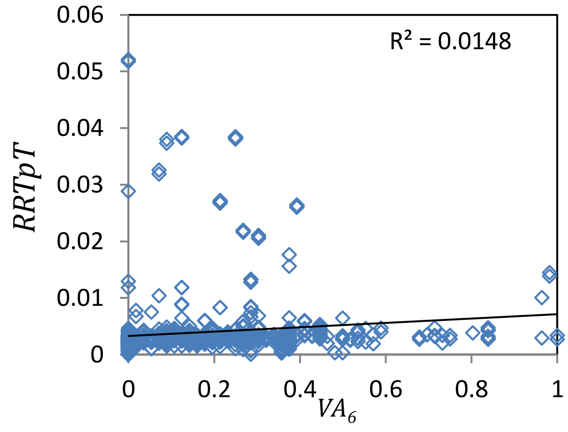
(c) VA_3



(d) VA_4



(e) VA_5



(f) VA_6

Figure 4 Correlations between VAs and $RTTpT$ for each link closure.

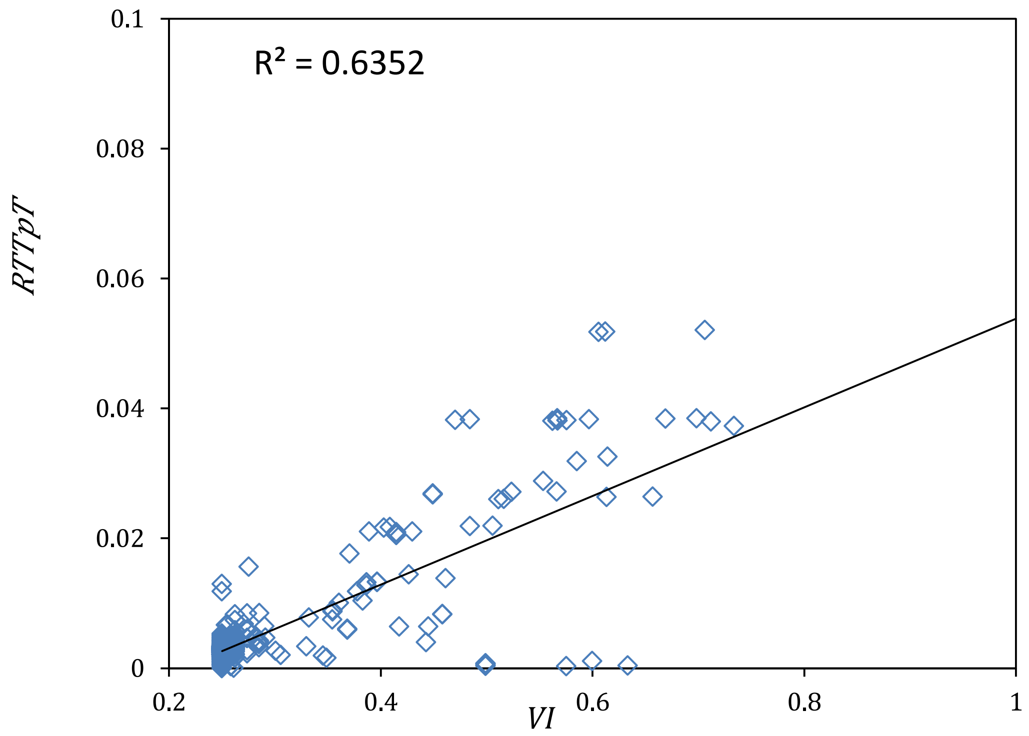


Figure 5 Vulnerability Index and $RTTpT$ for all links.

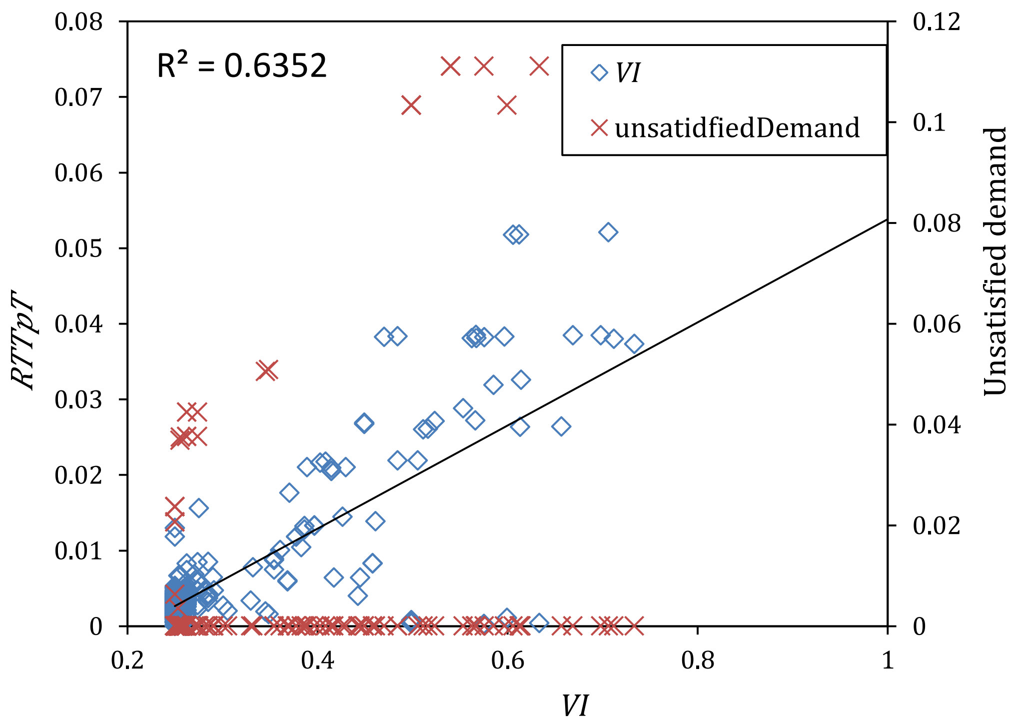


Figure 6 $RTTpT$, unsatisfied demand and vulnerability index for the network links.

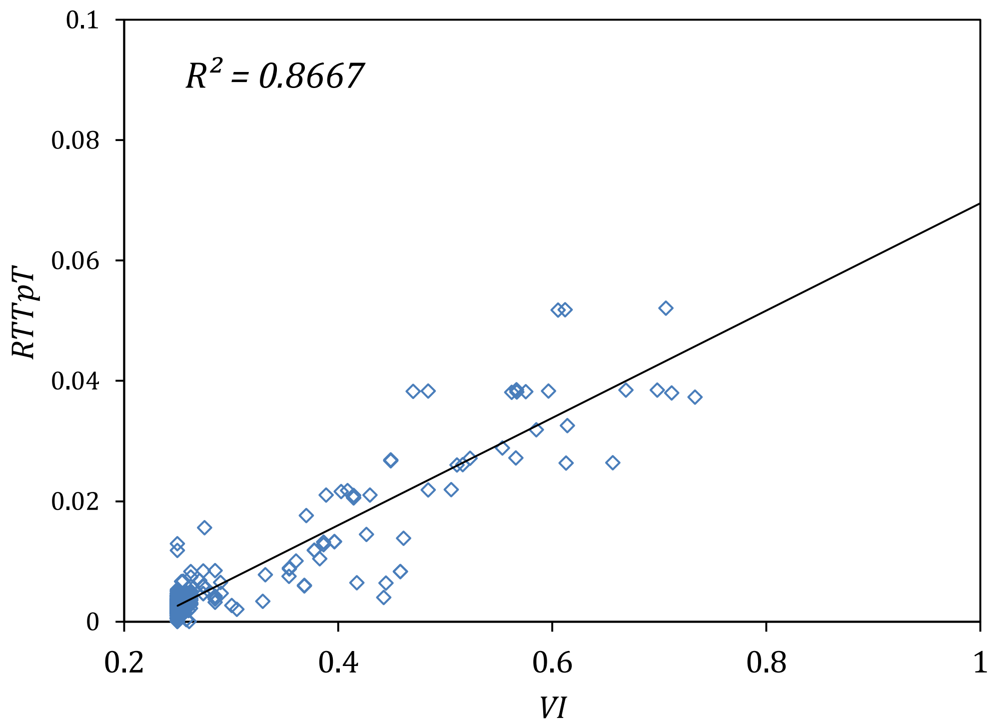


Figure 7 Correlation between vulnerability Index and $RTTpT$ excluding cut links.

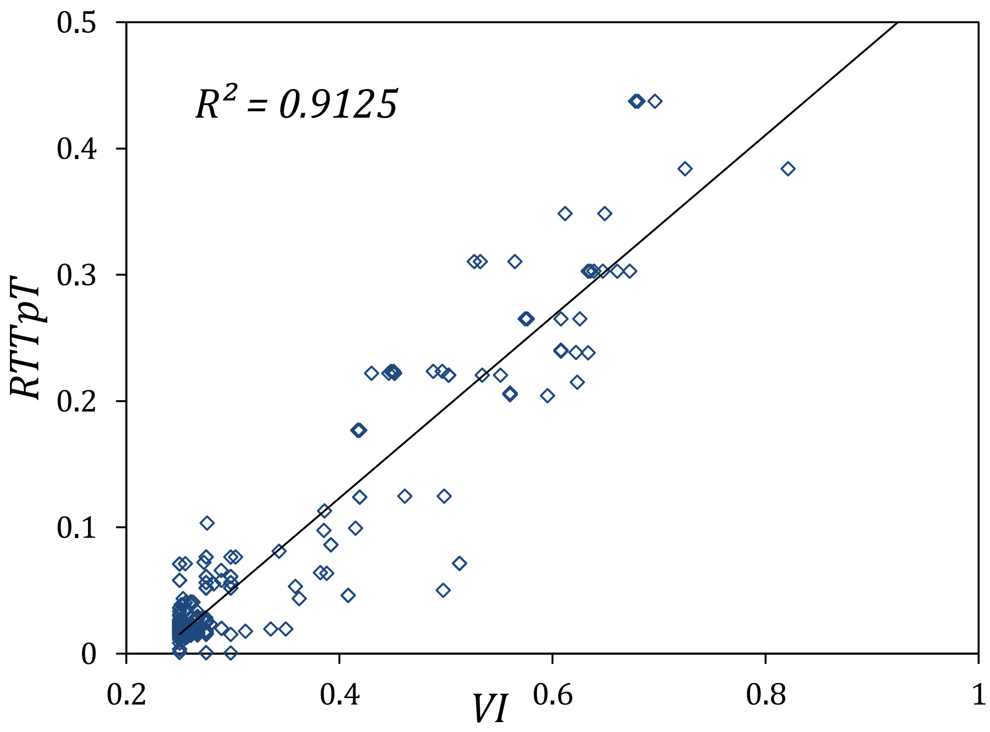


Figure 8 Correlation between VI and modified relative travel time.

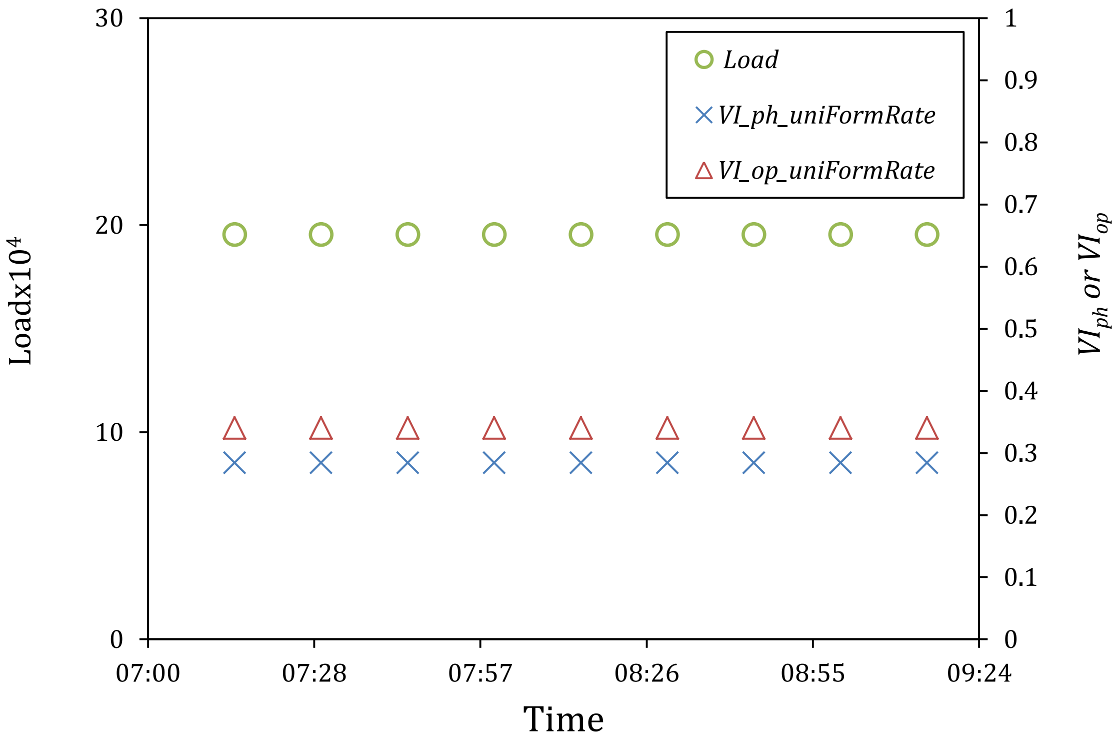


Figure 9 VI_{ph} and VI_{op} under uniform distributed departure rates.

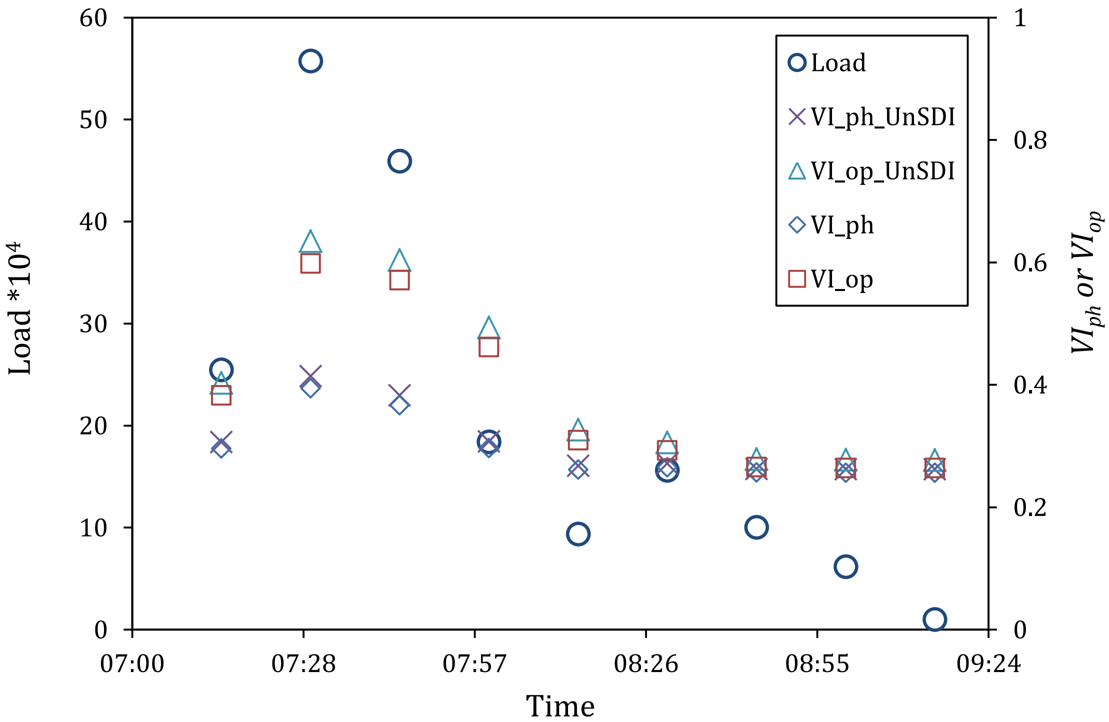


Figure 10 VI_{ph} and VI_{op} under different departure rates with and without UnSDI.

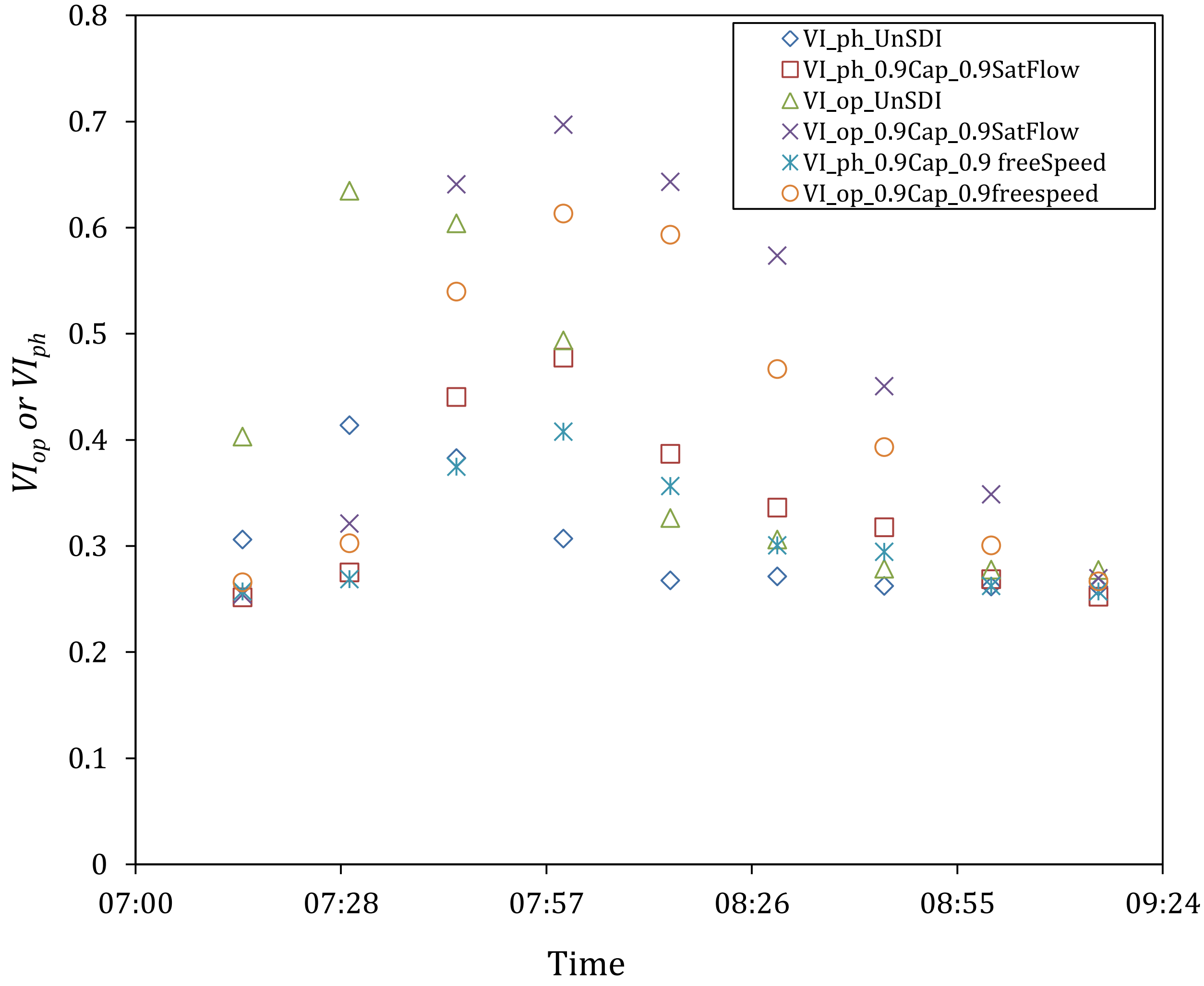


Figure 11 VI_{ph} and VI_{op} under different departure rates and network capacity