Numerical investigation of turbulent flow heat transfer and pressure drop of AL$_2$O$_3$/water nanofluid in helically coiled tubes

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Abstract

Passive convective heat transfer enhancement can be achieved by improving the thermo-physical properties of the working fluid, changing flow geometry or both. This work presents a numerical study to investigate the combined effect of using helical coils and nanofluids on the heat transfer characteristics and pressure losses in turbulent flow regime. The developed computational fluid dynamics models were validated against published experimental data and empirical correlations. Results have shown that combining the effects of alumina (Al$_2$O$_3$) nanoparticles and tube coiling could enhance the heat transfer coefficient by up to 60% compared with that of pure water in straight tube at the same Reynolds number. Also, results showed that the pressure drop in helical coils using Al$_2$O$_3$ nanofluid for volume fraction of 3% was six times that of water in straight tubes (80% of the pressure drop increase is due to nanoparticles addition), while the effect of Reynolds number on the pressure drop penalty factor was found to be insignificant.

Keywords: nanofluids; helical coils; turbulent flow; heat transfer; CFD; fluent

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1 INTRODUCTION

Passive heat transfer enhancement techniques can improve compactness and thermal efficiency of heat exchangers. They are preferred due to their simplicity, longer operating life and lower cost and power requirements. Although there are various methods for achieving passive heat transfer enhancement, they all depend on changing flow geometry or improving the thermo-physical properties of the base working fluid. Helical coils have been shown to enhance single-phase heat transfer [1, 2], boiling heat transfer [3, 4] and condensation heat transfer [5, 6]. Nanofluids formed by mixing nanoparticles of metals or metal oxides such as copper, alumina, copper oxide with base fluid such as water, oil, ethylene glycol are investigated as a passive heat transfer enhancement technique. Nanoparticles improve the energy transport properties of the base fluid by increasing the effective thermal conductivity which enhances the heat transfer rate of the nanofluid. The applications using these nanofluids include engine cooling to reduce the engine weight and fuel consumption [7], increasing the critical heat flux in boilers [8] and developing compact heat exchangers for medical applications [9].

Recently, many researchers investigated numerically and experimentally the effect of nanofluids in enhancing the heat transfer in the turbulent flow regime in straight tubes. Li and Xuan [10] measured the heat transfer coefficient of Cu dispersed in water with 0.3–2% volume fraction in a straight tube 10 mm diameter and 0.8 m long. They developed a new correlation for Nusselt number in laminar and turbulent flow regimes as a function of volume concentration, Reynolds, Prandtl and Particle Peclet numbers. Nguyen et al. [11] investigated numerically the utilization of two nanofluids γ-Al$_2$O$_3$/water and γ-Al$_2$O$_3$/ethylene glycol (volume fractions 0–7.5%) for microprocessor cooling. The reduction in microprocessor temperature using the nanofluid was insignificant at lower levels of heat supplied. Rostamani et al. [12] numerically investigated turbulent flow of water with copper oxide (CuO), alumina (Al$_2$O$_3$) and titanium oxide (TiO$_2$) nanoparticles (volume concentrations 0–6%). Results showed that increasing the nanoparticles’ volume concentration gave increased heat transfer coefficient and shear
pressure drop characteristics of the multi-walled carbon nanotubes (MWCNT)/oil nanofluids with weight concentration of 0.1, 0.2 and 0.4% in vertical helically coiled tubes. Their results indicated a heat transfer enhancement ratio of up to 10 times that of pure oil flow in straight tubes [27] while pressure drop of 3.5 times compared with base fluid in straight tube [29] were achieved.

Turbulent base fluid flow in helical coils was investigated by many researchers. Seban and Mclaughlin tested two coils using water with 7.37 mm internal diameter and coil to diameter ratio of 17 and 104 using direct electrical heating with constant heat flux. They correlated their experimental results with the thermo-physical properties calculated at the film temperature (the average between bulk fluid temperature and wall temperature) [30]. Mori and Nakayama tested two coils with tube diameter to coil ratio of 18.7 and 40. They correlated their results with the thermo-physical properties calculated using the bulk average temperature [31].

Numerical and experimental research of heat transfer and pressure drop using nanofluids in helical coils in turbulent flow regime is very limited [16]. Kannadasan et al. [32] compared the water base fluid heat transfer in helical coil turbulent flow with CuO/water nanofluid at 0.1 and 0.2 volume fractions. At 0.2% volume fraction, 45 and 49% enhancement in Nusselt number were found for horizontal and vertical positions with an increase in pressure drop of 21 and 25%, respectively, compared with pure water flow in coils. Wallace [33] measured the heat transfer rate using nanofluids in a helically coiled cooler. However, the author did not report any measurements of heat transfer coefficients or wall temperatures. This work presents a CFD modelling study to investigate the heat transfer enhancement and pressure drop in turbulent flow with nanofluids through helical coil tubes. To model the heat transfer characteristics of nanofluids, researchers used different approaches including the homogeneous approach [12], the two-phase mixture approach [34], the Eulerian–Eulerian approach [35] and the Lagrangian trajectory model [36]. In the current investigation, the homogeneous approach is used as it requires less computational time and provides accurate prediction [12] and [35].

2 FLOW GOVERNING EQUATIONS AND THERMOPHYSICAL PROPERTIES

Al₂O₃ nanofluid was treated as incompressible, steady state, homogeneous and Newtonian fluid with negligible effect of viscous heating. The Navier–Stokes flow governing equations in the Cartesian co-ordinates are as follows:

Continuity:

\[ \frac{\partial}{\partial x_i} (\rho u_i) = 0. \]
Momentum:
\[
\frac{\partial}{\partial x_j}(\rho u_i u_j) = -\frac{\partial P}{\partial x_j} + \rho g_i + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] + \frac{\partial}{\partial x_j} (-\rho u_i u_j').
\]

Energy:
\[
\frac{\partial}{\partial x_j}(\rho u_i T) = \frac{\partial}{\partial x_j} \left[ (\Gamma + \Gamma_i) \frac{\partial T}{\partial x_j} \right] = \mu \quad \text{and} \quad \Gamma_i = \frac{\mu_i}{Pr_i}.
\]

\(\Gamma\) and \(\Gamma_i\) are the molecular thermal diffusivity and turbulent thermal diffusivity, respectively. The Boussinesq hypothesis is used to relate the Reynolds stresses (last term in momentum equation) to the mean velocity as:
\[
(-\rho u_i u_j') = \mu_i \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right).
\]

The turbulent viscosity term is computed using the \(k\)-\(\varepsilon\) turbulent model with two additional equations namely turbulent kinetic energy (TKE, \(k\)) and turbulent dissipation rate (\(\varepsilon\)) so that:
\[
\mu_i = \rho C_{\mu} \frac{k^2}{\varepsilon}.
\]

The modelled equation of the TKE, \(k\), is:
\[
\frac{\partial}{\partial x_j}(\rho k u_i) = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_k}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + G_k + p\varepsilon.
\]

where \(p\varepsilon\) is the turbulence destruction rate (TDR) of TKE and \(G_k\) is the rate of generation of the TKE given by:
\[
G_k = -\rho u_i u_j' \frac{\partial u_i}{\partial x_j}.
\]

The dissipation rate of the TDR, \(\varepsilon\), is given by the following equation:
\[
\frac{\partial}{\partial x_j}(\rho \varepsilon u_i) = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_k}{\sigma_k} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + C_{1e} \frac{\varepsilon}{k} G_k + C_{2e} \rho \frac{\varepsilon^2}{k}.
\]

The boundary values for the turbulent quantities near the wall were determined using the two layers enhanced wall treatment. The values of the empirical constants in the turbulence transport equations are as follows:
\[
C_{\mu} = 0.09, \quad C_{1e} = 1.44, \quad C_{2e} = 1.92, \quad \sigma_k = 1, \quad \sigma_\varepsilon = 1.3 \quad \text{and} \quad Pr_i = 0.85.
\]

\(Pr_i\) is the turbulent Prandtl number at the wall. These default values have been determined from experiments with air and water for fundamental turbulent shear flows. They have been found to work fairly well for a wide range of wall-bounded and free shear flows [37]. The effective thermo-physical properties of the nanofluid are [38]:

\[
\rho_{nf} = (1 - \varphi)\rho_{bf} + \rho_p \varphi.
\]

Specific heat:
\[
C_{nf} = ((\rho C)_p \varphi + (\rho C)_{bf}(1 - \varphi))/\rho_{nf}.
\]

Thermal conductivity:
\[
\lambda_{nf} = (1 + 4.5503\varphi)\lambda_{bf}.
\]

Dynamic viscosity:
\[
\mu_{nf} = \exp(4.91\varphi/(0.2092 - \varphi))\mu_{bf}.
\]

where \(\eta_f\), \(bf\) and \(P\) denote the nanofluid, base fluid and particle, respectively. The base fluid thermo-physical properties have been fitted as polynomial functions in temperature (Kelvin) using Engineering Equation Solver (EES) data as shown in equations (13–15).

\[
\rho_{bf} = 2813.77E(-01) + 6351.93E(-03)T
- 1761.03E(-05)T^2 + 1460.96E(-08)T^3,
\]

\[
k_{bf} = -1056.42E(-03) + 1011.33E(-05)T
- 1772.74E(-08)T^2 + 7994.88E(-12)T^3,
\]

\[
\mu_{bf} = 9684.22E(-05) - 821.53E(-0)T
+ 2345.21E(-09)T^2 - 2244.12E(-12)T^3.
\]

These properties were formulated as user-defined functions (UDF) subroutines and incorporated into Fluent 6.3 solver. Fluent is a computational fluid dynamics software based on finite volume method for solving the continuity, momentum and energy partial differential equations of fluid flow.

3 MODELLING DESCRIPTIONS AND VALIDATION

3.1 Straight tube

The CFD analysis for the base fluid flow in straight tube was investigated to provide a reference case using the experimental setup of Williams et al. [38]. Half of the 9.4-mm internal diameter and 2819-mm long tube has been modelled using the symmetry along the tube axis to reduce the computational time. Two adiabatic sections with 1 and 0.5 m long, respectively, were positioned before and after the heated section to ensure fully developed flow. The boundary conditions at the inlet and outlet of the tube was specified as velocity inlet and pressure outlet, respectively. The heated section was meshed with 40 and 1600 nodes in the radial and axial directions, respectively. The positioner before and after the heated section to ensure fully developed flow. The boundary conditions at the inlet and outlet of the tube was specified as velocity inlet and pressure outlet, respectively.
section. The coupled algorithm was used with Courant number set to one for solving the pressure–velocity coupling [39]. Courant number is the ratio of the time step to the cell residence time which expresses the ratio of the distance travelled by a disturbance in one time step to the length of a computational distance step. In the CFD calculations, the Courant number must be less than or equal to unity so as to ensure the convergence of the discretized equations.

The average heat transfer coefficient was calculated using the average heated wall temperature and average fluid temperature at the inlet and outlet of the heated section. Figure 1a shows the CFD predicted heat transfer coefficient and those reported by Williams et al. [38] at various Reynolds numbers with ±9% agreement with experimental data and those predicted by Petukhov (Equation (16)) and Gnielinski correlations (Equation (17)) who correlated the Nusselt number as a function of Reynolds and Prandtl numbers in the following form [40].

Petukhov correlation:

\[ Nu = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7(f/8)^{0.5}(Pr(2/3) - 1)^{1.64}}, \]  

where \( f = (1.82 \log_{10}(Re) - 1.64)^{-2} \)

Figure 1b presents the predicted heat transfer coefficient of Al2O3 nanofluid in a straight tube compared with the experimental results of Williams et al. [38] at volume concentration ratios of 0.9, 1.8 and 3.6% at Reynolds numbers ranging from 8000 to 60 000 with ±12% agreement. The Pak and Cho correlation (Equation 18, [15]) was in a good agreement with the CFD prediction. On the other hand, the Vajjha et al. (Equation 19, [41]) correlation tends to under predict the experimental measurement and the Maiga correlation (Equation 20, [42]) was found to over predict the experimental results.

Pak and Cho correlation [15]:

\[ Nu_{nf} = 0.021Re_{nf}^{0.8}Pr_{nf}^{0.5}. \]  

Vajjha correlation [41]:

\[ Nu_{nf} = 0.065(Re_{nf}^{0.65} - 60.22)(1 + 0.0169\psi^{0.15})Pr_{nf}^{0.542}. \]  

Maiga correlation [42]:

\[ Nu_{nf} = 0.085Re_{nf}^{0.71}Pr_{nf}^{0.35}. \]  

3.2 Helically coiled tube

For simulating the flow of a base fluid inside helically coiled tube, coil with tube length and diameter similar to those used in the straight tube analysis was used. The coil pitch was selected as 15 mm and number of turns of 5 leading to a coil diameter of 179.5 mm. The discritization schemes utilized were second order for energy, first order for momentum and SIMPLEC algorithm with skewness factor of one for coupling the velocity and pressure parameters. The mesh contains 1 026 000 cells with the number of nodes in the axial direction are 500, 1500 and 250 for the inlet straight, helically coiled, outlet straight tubes, respectively. Tri-quad elements has been utilized to mesh the inlet face and hex/wedge cooper elements used to mesh the coil volume with six layers close to the wall and 50 nodes in the radial direction as shown in Figure 2. The mesh quality has been checked by controlling the turbulent wall function \( y^+ \) value to be <5 as depicted in Figure 3. The required simulation time for each run was 8 h using 2.4 GHz core Quad processor with 2 GB RAM memory computer.

Figure 4 compares the CFD predicted heat transfer coefficients with the empirical correlations (Equations 21–22) of Seban and Mclaughlin [30] and Mori and Nakayam [31] at a heat flux of 30 kW/m². The percentage mean absolute deviation between the CFD prediction and those of the Seban
and Mclaughlin [30] correlation was found to be less than ±3.2%.

\[
\alpha_{\text{Seban-Mclaughlin}} = 0.023Re^{0.85}Pr^{0.4}\left(\frac{d_i}{d_{\text{coil}}}\right)^{0.1}\frac{\lambda}{d_i},
\]

\[6000 \leq Re \leq 65 600\quad 2.9 < Pr < 5.7.\]

\[
\alpha_{\text{Mori-Nakayama}} = \frac{1}{41}Re^{5/6}Pr^{0.4}\left(\frac{d_i}{d_{\text{coil}}}\right)^{1/12}\left(1 + 0.061/\left(Re(d_i/d_{\text{coil}})^{2.5}\right)^{1/6}\right)\frac{\lambda}{d_i}.
\]

\[10 000 \leq Re \leq 200 000\quad Pr > 1.\]

**4 HEAT TRANSFER AND PRESSURE DROP OF NANOFLUIDS IN HELICAL COILS**

This section investigates the effect of using nanofluids on the heat transfer and pressure drop for helical coils. Similar methodology for modelling the nanofluid flow in straight tubes (Section 3.1) was used. Figure 5 compares the velocity contours of the nanofluid (concentration 2%, Figure 5a) with that of the base fluid (Figure 5b) at Reynolds number of 20 000 and different positions (inlet, 1 turn, 2.5 turns and 5 turns) in the coil. This figure shows that, for the same Reynolds number, higher velocities were produced due to the larger kinematic viscosity of the nanofluid. The flow enters the coil as hydrodynamically fully developed turbulent flow as shown by the coil inlets in Figure 5a and b. Inside the helical coil, the fluid elements with high velocities are pushed to the outer side of the coil due to the centripetal force thus generating secondary flow in the coil that results in better mixing of bulk fluid and decreases the wall temperature.

Figure 6 shows the variation of the heat transfer enhancement ratio (nanofluid in the helical coil divided by that of base fluid in the straight tube) with the volume concentrations of the...
nanofluids at various Reynolds numbers. This figure shows that the heat transfer enhancement ratio increases with the nanofluid volume fractions due to enhanced nanofluid thermal transport properties and helical coil enhanced flow mixing. Also, increasing the Reynolds number further increases the heat transfer enhancement ratio. An enhancement ratio of up to 1.6 (60% enhancement) was achieved with 3% nanofluid volume concentration at Reynolds number of 50,000. At the same Reynolds number, the base fluid in the helical coil (\( \phi = 0 \)), produced an enhancement ratio of 1.12. Thus, the addition of nanoparticles contributed with 80% of the 60% enhancement while the helical coiling contributed 20%.

Figure 7 shows the CFD results for the pressure drop ratio (nanofluid in helical coil to base fluid in straight tube) at various Reynolds numbers and volume concentrations. Results show that increasing the volume fraction increases the pressure drop ratio where a volume fraction of 3% produced six times the pressure drop of the base fluid in straight tube. This could be explained by the significant increase in the viscosity of the nanofluid for the same Reynolds number.
Using Darcy equation, the pressure ratio of nanofluids in helical coils compared with base fluid in straight tube can be expressed as:

$$\frac{\Delta P_{Hc}}{\Delta P_{St}} = \left( \frac{f_{Hc} + \frac{f_{Hc}L_{t}}{L_{tHc}}}{f_{St}L_{t}} \right) \left( \frac{\mu_{nf}}{\mu_{bf}} \right)^{2} \left( \frac{d_{w}}{d_{col}} \right)^{2} \left( \frac{\rho_{nf}}{\rho_{bf}} \right)^{-1},$$  \hfill (23)

where $L_{t}$, $L_{tHc}$ are the total straight tube lengths including the adiabatic parts and the coil length with 4319 and 2819 mm, respectively. The friction factor of nanofluid in helical coil $f_{Hc}$ was calculated using the White correlation [43] for turbulent flow. The friction factor of the nanofluid in the straight tube $f_{St}$ was taken as equal to that of the base fluid in a straight tube at the same Reynolds number, as recommended by Li and Xuan [10]. Thus:

$$\frac{f_{Hc}}{f_{St}} = \frac{f_{Hc}}{f_{St}} = \frac{4(0.08\text{Re}_{nf}^{-0.25} + 0.012(d_{w}/d_{col})^{0.5})}{0.316\text{Re}_{nf}^{-0.25}}$$ \hfill (24)

where $f_{Hc}$ and $f_{St}$ are the friction factors of the nanofluids in helical coils and straight tubes based on White [43] and Blasius correlations [44] ($f_{\text{Blasius}} = 0.316\text{Re}_{nf}^{-0.25}$) using the nanofluid thermo-physical properties. Figure 8 compares the pressure drop ratio calculated using equations (23) and (24) with those predicted by the CFD with ±5% relative deviation.

Figure 9 presents the heat transfer enhancement ratio (heat transfer coefficient of nanofluid in the helical coil divided by the base fluid in the straight tube) versus Reynolds number for the present study compared with those of Kumar et al. [2], Fakoor-Pakdaman et al. [29], Kannadasan et al. [32] and Eslayed et al. [45] for both nanofluids and base fluids. It can be seen that nanofluid are more effective in enhancing the heat transfer in the laminar flow regime compared with the turbulent flow. In the laminar flow regime with thick boundary layers, the addition of nanoparticles increases the thermal conductivity and reduces the specific heat of the fluid resulting in a better dispersion of heat inside the fluid leading to steeper temperature gradient close to wall. As the flow becomes turbulent, the thermal

5 CONCLUSIONS

The combined effect of using helical coils and nanofluids heat transfer enhancement and pressure losses in turbulent flow was numerically investigated. The developed CFD models were validated against published experimental data and empirical correlations. CFD results showed that using 3% volume fraction of Al2O3/water nanofluids in helical coils increased the heat transfer coefficient by up to 60% of that for pure water in straight tubes at the same Reynolds number. The contribution of tube coiling to this enhancement was shown to be only 10% on average.

Pressure drop values from CFD prediction and developed correlation were in close agreement, with less than ±5% deviation. The pressure drop in helical coils using Al2O3 with volume fraction of 3% was six times that of water in straight tubes. With such modest improvement in heat transfer and high pressure losses, the adoption of such heat transfer enhancement techniques can only be justified in applications where improvements in heat transfer is very critical.

The long-term stability of nanofluids was reviewed by Ghadimi et al. [46]. They concluded that the stability of nanoparticles in the fluid depends mainly on the techniques used to prepare the nanofluid. Methods like ultrasonic, pH-control and using dispersants (Arabic Gum) have produced stable solutions.

REFERENCES


