Determining a Robust, Pareto Optimal Geometry for a Welded Joint

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Abstract. Multi-criteria optimization problems are known to give rise to a set of Pareto optimal solutions where one solution cannot be regarded as being superior to another. It is often stated that the selection of a particular solution from this set should be based on additional criteria. In this paper a methodology has been proposed that allows a robust design to be selected from the Pareto optimal set. This methodology has been used to determine a robust geometry for a welded joint. It has been shown that the robust geometry is dependent on the variability of the geometric parameters.

Introduction

The concept of robust design was perhaps first expounded by Taguchi [1] although the principle of accounting for manufacturing variability by applying tolerance limits to designs has existed much longer. The desire to produce optimal designs is also long standing. For single criteria problems linear and non-linear programming and goal seeking methods have been developed. It has been recognized that many engineering design problems have multiple objectives and approaches for multi criteria optimization in design were introduced by [2] for example. Key to these approaches was the Pareto concept of a design solution being optimal if no other solution existed that was superior with respect to all criteria. More recent advances in multi-criteria optimization have tended to use population based techniques such as genetic algorithms [3].

Recently the concept of robust optimization has been investigated much more thoroughly, especially within the field of operations management with a recent review being provided by Bertsimas et al [4]. One of the simplest forms of robust optimization is to apply expected tolerances to the design variables and then seek designs which minimize variability in the performance criterion. Unfortunately, as Iancu and Trichakis [5] have identified, when these methods are applied to multi-criteria problems, they can result in solutions which are not Pareto optimal. Koksoy and Yalcinoz [6] have introduced robustness into the multi-criteria optimization problem by considering the standard deviation of the criteria as an additional set of criteria themselves to form a dual response problem. The standard deviation criterion was introduced by adding it to the mean criterion value with weights being applied to the mean and the standard deviation. This introduced the difficulty of determining the correct weights to use. The problem of using a weighted sum of multiple objectives was observed by Chen et al [7] who created additional objectives to minimize worst case outcomes (as well as maximizing desired outcomes) in their multi-criteria optimization algorithm.

In this paper a different approach to robust yet optimal designs is investigated. Rather than treating robustness as another criterion in the optimization problem, instead robustness has been used as a second stage in the design optimization process. This has allowed the identification of a subset of the Pareto optimal set which is the most robust. The technique has been applied to a classic, simply calculated robust design example and a stress concentration problem where the performance criteria have been calculated using finite element analysis.

Robust design of a tank

The stimulus for the approach to robust optimization being presented here came serendipitously from an attempt to carry out a benchmark analysis using the robust design features within the modeFRONTIER [8] software package. This software package allows multi-criteria optimization problems to be solved with a range of genetic algorithms, as discussed in [8]. The package also allows robust solutions to be determined. This is achieved by placing a normally distributed scatter of subsidiary points around each point in the design space. The variance on the objective functions due to the variance in the design variables can therefore be determined. The user is then able to determine the robust design set by running an optimization study with minimization of the variance in objective functions as the aim of the study.

The robust design benchmark problem being analyzed is presented by Ullman [9]. The design being considered is a cylindrical tank which is required to have a volume of 4 m^3. The designer is free to specify any radius, r, or length, h, for the tank within certain constraints. However, during manufacture tolerances t_r and t_h will be allowed on these dimensions giving rise over a number of products to standard deviations s_r and s_h on radius and length. The objective of the design optimization analysis is therefore to minimize the standard deviation of the volume, s_V. The standard deviation of the volume can be determined from:

$$s_v = \left[\left(\frac{\partial V}{\partial h}\right)^2 s_h^2 + \left(\frac{\partial V}{\partial r}\right)^2 s_r^2\right]^{1/2} \tag{1}$$

where: $V = \pi r^2 h$ (2)

Hence: $s_v = \pi r[r^2 s_h^2 + 4h^2 s_r^2]^{1/2}$ (3)

For an example where $s_r = 0.01$ and $s_h = 0.05$, it can be quickly determined that a long, small diameter cylinder will have a lower standard deviation than a short, fat cylinder.

In attempting to introduce this example to modeFRONTIER it was immediately apparent that simply minimizing the variance in the volume would not produce an acceptable design since the volume would not be 4m^3. A constraint was therefore introduced to only include those points with a volume of 4m^3 in the feasible set. The results obtained then seemed to be almost randomly dependent on the size of the initial population and the type of genetic algorithm used with robust design points appearing at any position on the V = 4m^3 curve in the design space. The reason for this of course was that only a very small number of designs would have a volume sufficiently close to 4m3 to be included in the feasible set. To allow for this a tolerance around the 4m^3 volume target was introduced. The results of this are shown in Fig. 1 for tolerances of 20% and 2.5% on volume. Within these plots, the data points have been grouped according to standard deviation of the volume.

The methodology employed has allowed the more robust designs to be identified. However, the designs with the lowest standard deviation on volume in both cases are not those predicted by the theory but is instead at a radius of approximately 0.75 m. This is due to two factors. Firstly, the normally distributed sample of points around each trial point in design space is limited to 100 points with a pseudo random variation in the scatter between trial points. Hence, the scatter around some trial points will contain individuals that produce poor results whilst better results are produced at other trial points. Secondly, the genetic algorithm used to refine the search will cross individuals with a similar performance (in this case the standard deviation of the volume). This has produced generations that increasingly occupy a region of the solution space towards the middle of the 0.034 to 0.04 standard deviation set. Within these later generations there will again be variation in the distribution of analysis points around each trial point. With the number of trial points generated (approximately 800), it is almost inevitable that one of these later generations will give rise to an individual with a particularly low scatter in the design space and hence a low standard deviation of volume.

Pareto robust design methodology

This benchmark problem has suggested a methodology for determining a robust design which is also close to being Pareto optimal. This is summarized in Fig. 2.

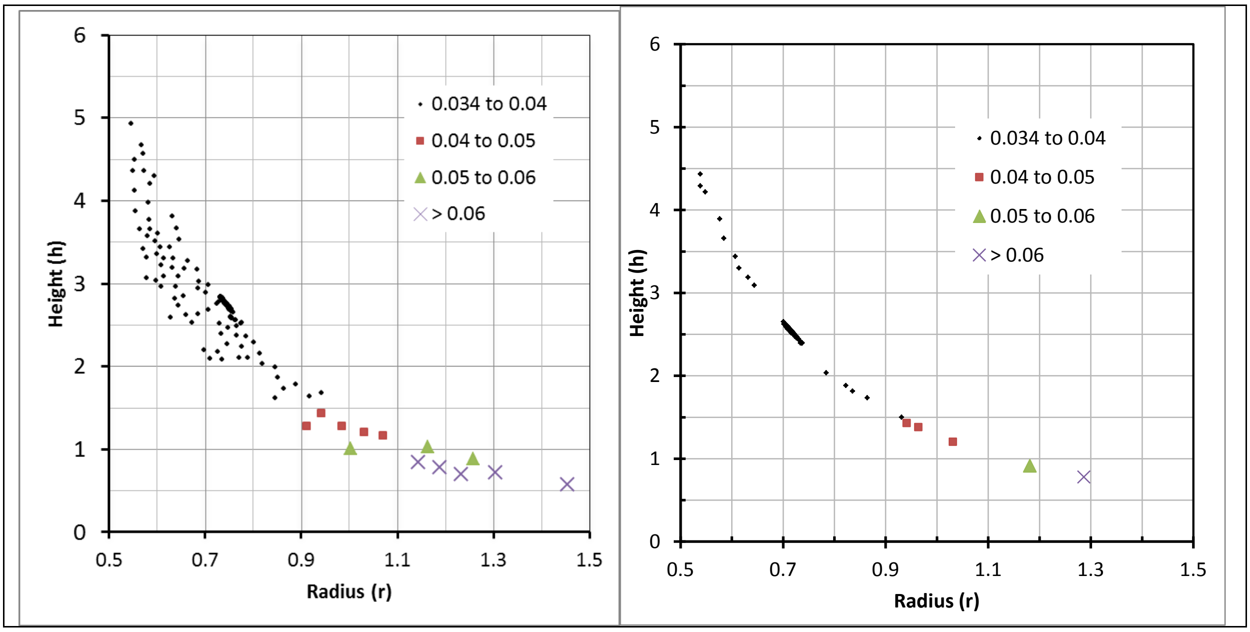

Figure 1. Tank design outcome indicating standard deviation of volume with 20% (left) and 2.5% (right) tolerance on volume

The initial Pareto optimal set can be determined in a number of ways. In the work presented here, the Fast Multi-Objective Genetic Algorithm (FMOGA) available in modeFRONTIER has been used.

All the examples presented here have been restricted to just two criteria. Hence, a curve has been used to approximate the Pareto front. A power series function with parameters determined using a least squares approach has been found to give a visually satisfactory fit. It has then been a simple task to find the functions defining the tolerance band.

It is important to remember that the determination of design points giving low performance criteria standard deviations will be a multi-criteria problem if there is more than one performance criterion. Hence, in this work the FMOGA optimization routine has again been used.

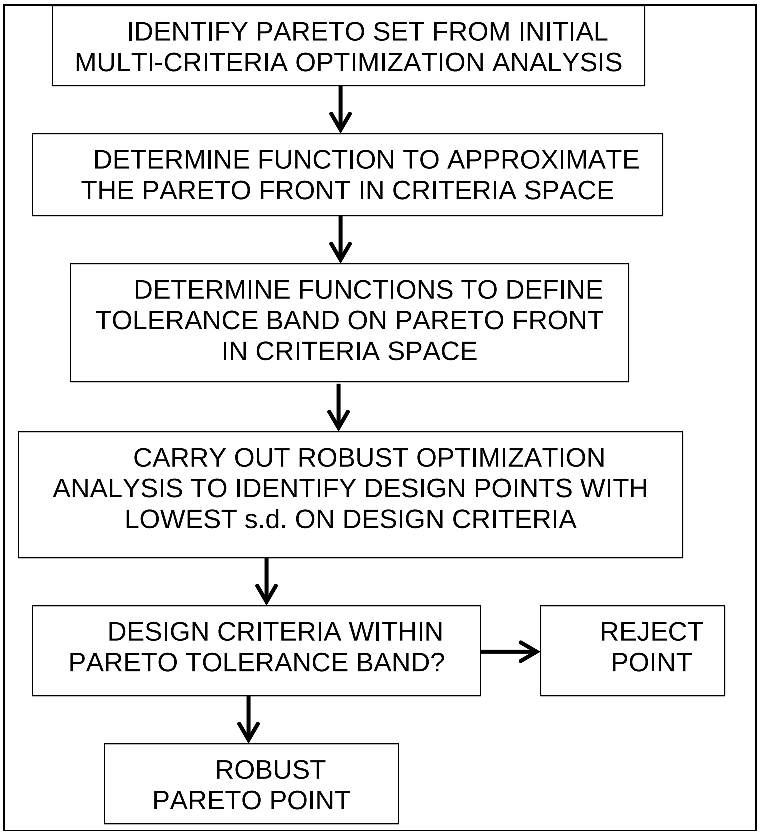

Figure 2. Pareto robust design methodology

Butt weld analysis

The example selected to demonstrate the new methodology is that of a butt welded joint subjected to both tension and bending. When such joints fail due to fatigue, the crack typically initiates at the weld toe where there can be a large stress concentration factor. In the literature this geometry is typically characterized using the parameters shown in Fig. 3 and Table 1. The ranges of parameters here were taken from the literature. The standard deviations on those parameters were derived from measurements of a test weld. This was a laser butt weld between two hot rolled steel plates (S335JR

27), the type of plate often used for boilers, tanks and associated equipment. In order to measure the weld geometry, a silicon replica was cast over the surface of the weld in 11 places top and bottom of the plate. A photograph of the replica along with a graduated scale was then imported into AUTOCAD and the weld dimensions determined by superimposing 'best fit' geometric features over the photograph. Fig 3 illustrates this process.

In order to analyze this problem an ABAQUS finite element model was created using linear plane strain elements with free form, quad dominated meshing. A parametric version of this model was then generated using the Python scripting language. In order to correctly capture the stress concentration at the weld toe a highly refined mesh was required. A convergence study was carried out on a model with the smallest weld toe radius (judged to be the most difficult case to analyze) to ensure that the mesh was sufficiently refined. This refined mesh is shown in Fig 4.

Two load cases were analyzed using the finite element model: in-plane tension and bending in the plane of the figure. For both cases a symmetry boundary constraint at the right hand side of the model was appropriate. On this edge a single point was also constrained in the vertical direction to prevent rigid body motion. To simulate tension a uniform traction acting normal to the left hand edge was applied. For the bending load case a linearly varying normal traction was applied to this edge.

Table 1. Weld geometry parameters

GEOMETRIC PARAMETER	Lower bound	Upper bound	s.d.
UPPER WELD TOE RAD, RU [mm]	0.5	5	0.1
LOWER WELD TOE RAD, RL [mm]	0.5	5	0.1
UPPER WELD TOE ANGLE, qU [°]	12	25	0.95
LOWER WELD TOE ANGLE, qL [°]	20	30	0.95
UPPER REINFORCE, FU [mm]	1	3	0.125
LOWER REINFORCE, FL [mm]	1	3	0.125

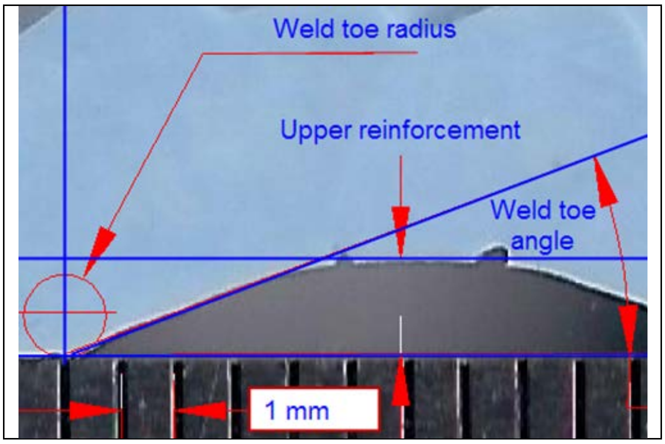

Figure 3. Weld geometry parameters (upper surface)

Robust Pareto optimization

The parametric finite element model described above was used within an optimization study to determine the two criteria to be minimised: the peak von-Mises stress arising from the two load cases. As expected, these two cases gave similar results but a Pareto front was still generated. According to the procedure given in Fig. 2, a curve was then generated to approximate this Pareto front in criteria space along with a pair of bounding curves to give a tolerance on that Pareto front.

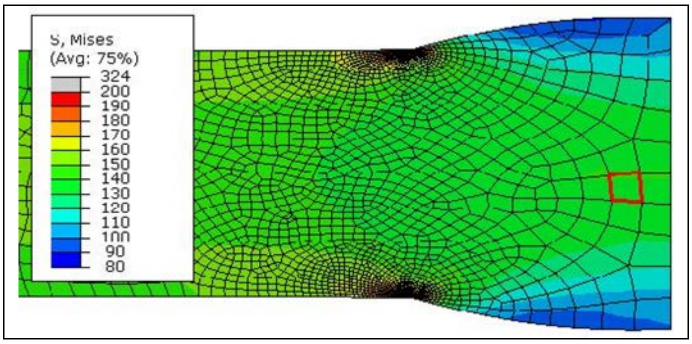

Figure 4. Finite element model showing von-Mises stress under tension (N/mm^2)

The second stage of the analysis was to carry out a robust optimization with the objective of minimizing the standard deviation of the two maximum stress criteria with the standard deviations shown in Table 1 applied to the design variables. The solutions in this stage of the analysis were constrained to lie within the tolerance band on the Pareto front defined in the first stage.

The robust optimization analysis produced three designs. These are represented on the radar plot in Fig. 5. The results here have been normalized over the parameter ranges shown in Table 1. It is interesting here that the designs are very similar. This was not the case with the initial optimization of the two stress criteria which produced a range of quite different designs.

The analysis described above was repeated but with the standard deviation on the parameters reduced by a factor of two. The results of this analysis are shown in Fig 6. It is immediately apparent the weld geometry shown here is quite different from that shown in Fig. 5. A further reduction of the standard deviation, again by a factor of two produced very similar results to those shown in Fig. 6.

It is of course to be expected that the geometry giving the most robust design will depend upon the variation allowed on parameters defining the geometry. What is remarkable in this case is that maintaining the same relative variability on those parameters but uniformly changing their magnitude will also alter which designs are more robust.

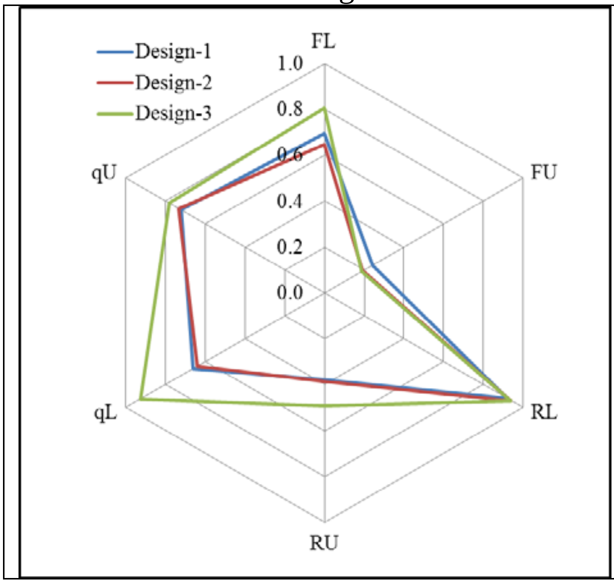

Figure 5. Robust design, large design tolerance

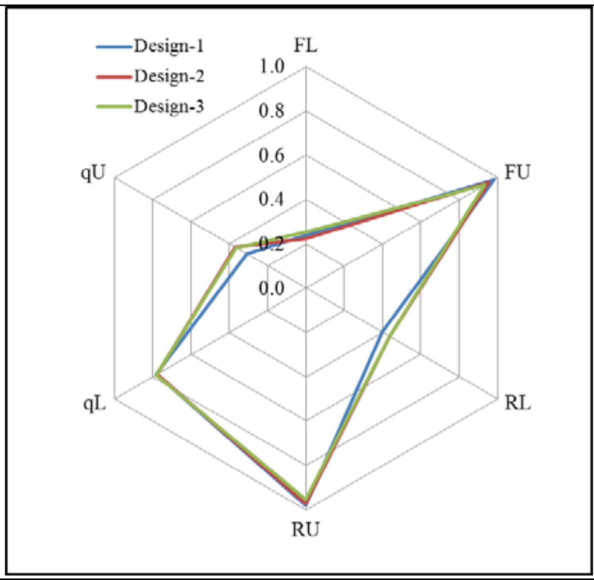

Figure 6. Robust design, small design tolerance

Conclusions

A methodology for identifying robust designs that exist within the Pareto optimal set for multicriteria problems has been proposed.

Robust, Pareto optimal geometries for a butt welded joint were identified. It was shown that these geometries were dependent on the magnitude of the variation allowed on the weld geometry parameters.

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