Representing and Reasoning over Topological Relations in OWL

Sotiris Batsakis
University of Huddersfield
S.Batsakis@hud.ac.uk

Grigoris Antoniou
University of Huddersfield
G.Antoniou@hud.ac.uk

Ilias Tachmazidis
University of Huddersfield
Ilias.Tachmazidis@hud.ac.uk

ABSTRACT

Representing topological information for the Semantic Web often involves qualitative defined natural language terms such as "Into" or "Overlapping". This can be the case when exact coordinates of spatial regions are not available, they are incomplete or unreliable. Topological spatial relations are the most important aspect of spatial representation and reasoning, thus embedding such relations into an ontology along with their semantics as expressed using reasoning rules is an important issue. In this work we propose a representation of RCC-5 topological relations using OWL object properties and axioms, combined with reasoning rules expressed using SWRL embedded into the ontology.

Three alternative representations are proposed and compared: the first is based on a straightforward implementation of the path consistency method for spatial reasoning, the second is an optimized version of the path consistency based representation implemented using an alternative representation of the topological equality relation and the third is based on the decomposition of RCC-5 relations to simpler ones. To the best of our knowledge this is the first work dealing with topological RCC-5 relations representation for the Semantic Web. In addition, our work improves the performance of topological RCC-8 relations reasoning when the best method, in terms of reasoning time, for RCC-5 relations is applied over RCC-8 relations.

Categories and Subject Descriptors

H.2.8 [**Database Applications**]: Spatial databases and GIS; I.2.4 [**Knowledge Representation Formalisms and Methods**]: Representations (procedural and rule-based); F.4.1 [**Mathematical Logic**]: Logic and constraint programming

General Terms

Semantic Web

Keywords

Spatial reasoning, SWRL, OWL

1. INTRODUCTION

Ontologies form the basis of knowledge representation required for implementing the vision of the Semantic Web[1]; Semantic Web technologies are used for automating tasks handled manually by users, tasks such as reasoning over Linked Data and answering complex questions. Understanding the meaning of Linked Data in RDF[1] format requires formal definitions of concepts and their properties, using the Semantic Web Ontology definition language OWL[2]. OWL [11] provides the means for defining concepts, their properties and their relations and allowing for reasoning over the definitions and the assertions of specific individuals using reasoners such as Pellet[15]. Furthermore, reasoning rules can be embedded into the ontology using the SWRL[7] rule language.

Representing spatial aspects of objects is very important for several applications such as Geographical Information Systems (GIS). Spatial information in turn can be defined using quantitative (e.g. using coordinates) and qualitative terms (i.e., using natural language expressions such as "Into"). Qualitative spatial terms have specific semantics which can be embedded into the ontology using reasoning rules. In previous work [3] such a representation embedding semantics by means of SWRL rules, is proposed for spatial and temporal information in OWL. Specifically directional and topological information (based on the RCC-8 set of topological relations) were covered. No optimizations were proposed for the representation of topological relations.

Current work deals with the case of RCC-5 topological relations which was not covered in [3] and to the best of our knowledge such a representation for the Semantic Web has not been proposed previously. In addition, different representations and optimizations for representing topological information based on RCC-5 relations are proposed and evaluated. Representations are compared in terms of compactness (required number of axioms, rules and asserted facts), and reasoning speed over the set of asserted facts.

The basic representation is based on the path consistency method [14, 3] applied for the topological RCC-5 relations. An alternative representation is based on the decomposition

[1] http://www.w3.org/standards/techs/rdf
[2] http://www.w3.org/TR/owl-features/

of RCC-5 relations. Following the decomposition based approach each RCC-5 relation is transformed to a combination of three other relations [8]. Reasoning over these relations requires fewer rules than reasoning over RCC-5 relations directly, thus the representation based on decomposition of RCC-5 relations is more compact that the representation based on reasoning directly over the RCC-5 relations. On the other hand implied facts are more, since each RCC-5 topological relation between two regions is transformed to three new relations holding between the two regions. This makes reasoning more complicated as demonstrated by the experimental evaluation in this work. Specifically, reasoning over the composition based representation is slower than reasoning over the initial representation (not using decomposition).

The third representation is based on the first one (reasoning over RCC-5 relations directly) but the following optimization is applied: equality relation is replaced by the sameAs OWL keyword, an assertion that is consistent with the intended semantics of the equality relation. Evaluation demonstrates that this optimization is efficient in terms of both compactness of representation and reasoning speed.

The compactness of representation and the increased reasoning performance obtained using the third approach over RCC-5 relations allows for applying the proposed representation for RCC-8 relations as well. Experimental evaluation demonstrates the fact that replacing the equality relation with the OWL sameAs keyword improves performance in terms of compactness of representation and reasoning speed in the case of RCC-8 relations as well.

Current work is organized as follows: related work in the field of spatial knowledge representation is discussed in Section 2. The proposed representations are presented at Section 3 and the corresponding reasoning mechanisms at Section 4. The extension to RCC-8 relations is presented at Section 5 followed by evaluation in Section 6 and conclusions and issues for future work in Section 7.

2. BACKGROUND AND RELATED WORK

Reasoning over Linked Data requires formal definitions of concepts and their relations. Definitions of ontologies for the Semantic Web is achieved using the Web Ontology Language OWL[3]. OWL is more expressive than the RDFS[4] that can be also used for defining concepts and their properties and relations. The current W3C standard is the OWL 2[5] language, offering increased expressiveness while retaining decidability of basic reasoning tasks. OWL 2 is based on the $\mathcal{SROIQ}(\mathcal{D})$ [9] description logic, thus decidability of basic reasoning tasks on OWL 2, such as consistency, is derived from the decidability of the underlying description logic. Reasoning tasks are applied both on the concept and property definitions into the ontology (TBox) and the assertions of individual objects and their relations (ABox). Reasoners include among others Pellet[6], Fact++[7], RacerPro[8], KAON2[9] and Hermit[10].

Reasoning rules can be embedded into the ontology using SWRL[11]. To guarantee decidability, the rules are restricted to *DL-safe rules* [12] that apply only on named individuals in the ontology ABox. *Horn Clauses* (i.e., a disjunction of classes with at most one positive literal), can be expressed using SWRL, since Horn clauses can be written as implications (i.e., $\neg A \vee \neg B ... \vee C$ can be written as $A \wedge B \wedge ... \Rightarrow C$). The efficiency of reasoning over Horn clauses using forward chaining algorithms is the reason for choosing this form of rules. The antecedent (body) of the rule is a conjunction of clauses. Notice that, neither disjunction nor negation of clauses is supported in the body of rules. Also, the consequence (head) of a rule is one positive clause. Neither negation nor disjunction of clauses can appear as a consequence (head) of a rule. These restrictions improve reasoning performance but complicate qualitative spatial reasoning, since disjunctions of clauses typically appear in the head of spatial reasoning rules.

Qualitative spatial reasoning (i.e., inferring implied relations and detecting inconsistencies in a set of asserted relations) typically corresponds to Constraint Satisfaction Problems which are NP problems, but tractable sets (i.e., solvable by polynomial algorithms) are known to exist [14]. Topological relations based on Region Connection Calculus (RCC)[6] are one of the most important aspects of spatial information. This is illustrated by the fact that topological relations are directly represented and supported in the GeoSPARQL[12] query language. Formal spatial representations have been studied extensively within the the Semantic Web community. Relations between spatial entities in ontologies can be topological, directional or distance relations. Furthermore, spatial relations are distinguished into qualitative (i.e., relations described using lexical terms such as "Overlaps") and quantitative (i.e., specifying the coordinates of all points that define a closed region).

A representation of topological relations using OWL class axioms has been proposed in [10], but an alternative representation using object properties offered increased performance [16]. Embedding spatial reasoning into the ontology by means of SWRL rules applied on spatial object properties forms the basis of the representation proposed at [3].

Based on the representation proposed at [3] the dedicated Pellet-Spatial reasoner [16] has been extended for directional relations in the CHOROS system [5]. CHOROS achieved improved performance over the SWRL based representation of [3] but the spatial reasoner is not embedded into the ontology, thus requiring specific software which must be properly adjusted whenever modifications into the ontology occur. The SWRL based representation on the other hand

[3]http://www.w3.org/TR/owl-ref/
[4]http://www.w3.org/2001/sw/wiki/RDFS
[5]http://www.w3.org/TR/owl2-overview/

[6]http://clarkparsia.com/pellet/
[7]http://owl.man.ac.uk/factplusplus/
[8]http://www.racer-systems.com/
[9]http://kaon2.semanticweb.org/
[10]http://hermit-reasoner.com/
[11]http://www.w3.org/Submission/SWRL/
[12]http://www.opengeospatial.org/standards/geosparql

offers greater flexibility since it can be used and modified freely using only standard Semantic Web tools such as the Protégé editor and the Pellet reasoner. Furthermore, a rule based approach directly benefits from optimizations that are applied to reasoning tools.

In this work a representation of RCC-5 relations is proposed based on SWRL and OWL axioms implementing path consistency. An alternative representation based on the decomposition of RCC-5 relations and the corresponding reasoning rules [8] is also implemented using the appropriate modifications required for compliance with Semantic Web standards such as SWRL. Decomposition based representations for the Semantic Web have been proposed for directional spatial relations [2] and temporal interval relations [4] (where interval relations were transformed into relations between the endpoints of intervals), but to the best of our knowledge this is the first such representation for the Semantic Web over topological relations.

The optimization based on replacement of a spatial equality relation by the OWL SameAs keyword [3, 5] is applied for the SWRL and OWL axioms based implementation of path consistency for topological relations. In this work this optimization is applied on both RCC-5 and RCC-8 relations combined with the path consistency reasoning method, yielding improved reasoning times and more compact representation in both cases. To the best of our knowledge this is the first work dealing with representation of RCC-5 relations for the Semantic Web and the first that proposes and evaluates different approaches for achieving such representation.

3. SPATIAL REPRESENTATION

Topological RCC-5 relations in this work are represented as object properties between OWL objects representing regions. For example if *Region*1 is *Into Region*2 user asserts the binary relation *Region*1 *PP* (proper part) *Region*2, or equivalently *PP(Region1, Region2)*. This approach is similar to the approach used in [3]. The first representation proposed in this work implements reasoning rules applied on RCC-5 relations. The second approach is based on decomposing RCC-5 relations and the third on eliminating the equality relation and using the OWL SameAs keyword instead. All three approaches are presented in the following.

3.1 RCC-5 representation

Region Connection Calculus[6] is one of the main ways of representing topological relations. There are several variants of the calculus corresponding to different levels of detail of the represented relations, variant such as RCC-5, RCC-8, RCC-15 and RCC-23.

RCC-5 relations is a set of 5 topological relations namely DR (discrete), PO (partially overlapped), EQ (equals), PP (proper part) and PC (contains). Figure 1 illustrates these relations between two regions X and Y. Relations DR, PO and EQ are symmetric, i.e., $DR(x, y) \rightarrow DR(y, x)$, $EQ(x, y) \rightarrow EQ(y, x)$ and $PO(x, y) \rightarrow PO(y, x)$. Relation PP is the inverse of PC, i.e., $PP(x, y) \rightarrow PC(y, x)$ and $PC(x, y) \rightarrow PP(y, x)$. All these 5 basic RCC-5 relations are pairwise disjoint. Also EQ, PP and PC are transitive. Transitivity rules are implemented using additional SWRL rules implementing path consistency.

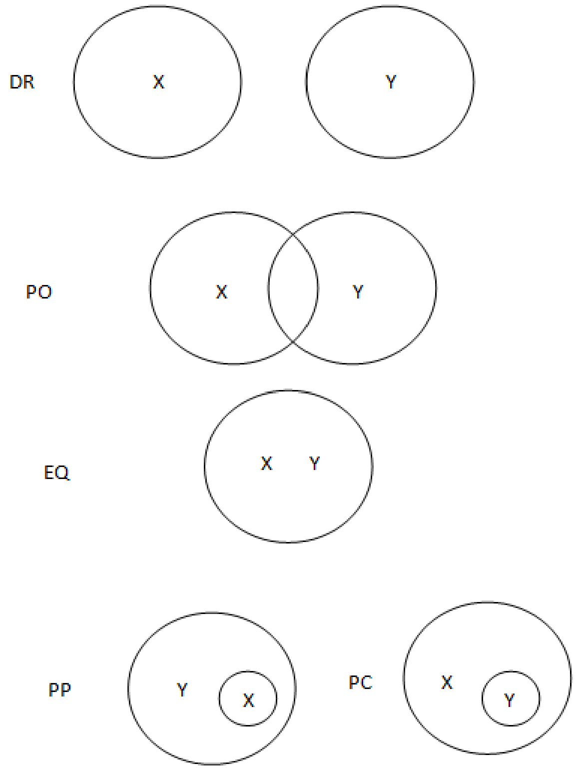

Figure 1: Topological RCC-5 Relations

In addition to the five relations of Figure 1 additional relations are required for representing disjunctions of these five basic relations. These additional relations are required for implementing reasoning rules, specifically the reasoning rules implementing path consistency [14] that are presented in Section 4.1.

The additional relations representing disjunctions of basic relations are:

$$DR\_PO \equiv DR \vee PO$$
$$DR\_PO\_PC \equiv DR \vee PO \vee PC$$
$$DR\_PO\_PP \equiv DR \vee PO \vee PP$$
$$PO\_EQ\_PP\_PC \equiv PO \vee EQ \vee PP \vee PC$$
$$PO\_PP \equiv PO \vee PP$$
$$PO\_PC \equiv PO \vee PC$$

The above representation and the corresponding reasoning mechanism can be expressed and implemented using only OWL 2 axioms and SWRL rules, thus requiring only standard tools such as Protégé[13] and the Pellet reasoner [15]. No additional software is required for spatial reasoning.

[13]http://protege.stanford.edu/

Twelve object relations are required in total, 5 basic RCC-5 relations, 6 additional relations representing disjunctions and the *null* (or $\perp$) relation representing inconsistency detection between two regions (i.e., inferred or asserted facts between two regions are incompatible). For example two regions cannot be both discrete (DR) and equal (EQ). This is expressed as:

$$DR(x, y) \land EQ(x, y) \rightarrow null(x, y)$$

Object property axioms combined with SWRL reasoning rules yield a representation fully compliant with existing Semantic Web standards and tools. Total number of axioms (OWL and SWRL) are 119 for directly representing RCC-5 formalism. These rules are presented in Section 4.1.

3.2 Decomposition of RCC-5 relations

Reducing the complexity of the previous representation can be achieved by translating RCC-5 relations to a new set of relations that need fewer rules to reason with. This approach is similar to the decomposition of temporal Allen interval relations to end-point relations[17] that has been applied for Semantic Web representation of time at [4] and the decomposition of directional relations proposed at [2] that yield a more compact representation than the representation presented at [3].

In this work the representation is based on the decomposition proposed at [8]. In [8] three relations are used for the decomposition. C representing the fact that two regions have common points, P representing the fact that a region is part of the another region and P_i that is the inverse of P (i.e., a region contains another region). In addition, C is symmetric and P and P_i are transitive. The proposed decomposition in [8] is the following:

$$\begin{aligned}
DR(x, y) &\equiv \neg P(x, y) \land \neg P_i(x, y) \land \neg C(x, y)\\
PP(x, y) &\equiv P(x, y) \land \neg P_i(x, y) \land C(x, y)\\
PC(x, y) &\equiv \neg P(x, y) \land P_i(x, y) \land C(x, y)\\
PO(x, y) &\equiv \neg P(x, y) \land \neg P_i(x, y) \land C(x, y)\\
EQ(x, y) &\equiv P(x, y) \land P_i(x, y) \land C(x, y)
\end{aligned}$$

Reasoning over the resulting relations C, P and P_i requires fewer rules than reasoning directly with the RCC-5 relations. But between two regions three different relations hold simultaneously, C, P and P_i (or their negations) instead of one basic relation using RCC-5. Since negation is used on the decomposition presented above, and negation is not directly supported in SWRL additional relations are introduced representing negations. Specifically $notC$, $notP$ and $notP_i$ are the negations of C, P and P_i respectively. Each relation is disjoint with it's negation thus: *disjoint(C,notC), disjoint(P,notP) and disjoint(*P_i*,not*P_i*).*

SWRL does not directly support rules with conjunctions of multiple atoms as heads, thus the above transformation rules are implemented using multiple rules with identical bodies and single atoms as heads, each one being one atom of the conjunction. Thus, the decomposition is implemented using SWRL as follows:

$$\begin{aligned}
&DR(x, y) \rightarrow notP(x, y)\\
&DR(x, y) \rightarrow notP_i(x, y)\\
&DR(x, y) \rightarrow notC(x, y)\\
&PP(x, y) \rightarrow P(x, y)\\
&PP(x, y) \rightarrow notP_i(x, y)\\
&PP(x, y) \rightarrow C(x, y)\\
&PC(x, y) \rightarrow notP(x, y)\\
&PC(x, y) \rightarrow P_i(x, y)\\
&PC(x, y) \rightarrow C(x, y)\\
&PO(x, y) \rightarrow notP(x, y)\\
&PO(x, y) \rightarrow notP_i(x, y)\\
&PO(x, y) \rightarrow C(x, y)\\
&EQ(x, y) \rightarrow P(x, y)\\
&EQ(x, y) \rightarrow P_i(x, y)\\
&EQ(x, y) \rightarrow C(x, y)
\end{aligned}$$

The representation based on decomposition consists of the new relations C, P and P_i, their negations $notC$, $notP$ and $notP_i$ the axioms and the transformation rules presented above, and the reasoning rules of Section 4.2. Combining all these, 46 OWL axioms and SWRL rules are required for representation and reasoning, thus the representation is more compact than the representation supporting direct reasoning over RCC-5 relation of section 3.1.

3.3 Equality relation elimination

Another representation is proposed based on the following observation: RCC-5 equality relation (EQ) can be replaced with the OWL $SameAs$ keyword. This is based to the fact that if two regions x, y are Equal, or equivalently $EQ(x, y)$, then x must has the same topological relations with all other regions that y has and vice versa. This is exactly the semantics of the OWL $SameAs$ keyword. Thus, by replacing EQ relation with the $SameAs$ keyword all OWL compliant reasoners will treat objects representing two equal regions as identical and there is no need to add SWRL rules for reasoning over the EQ relation. Specifically $SameAs$ keyword enforces the symmetry, reflexivity and transitivity that the EQ relation also represents. Thus all reasoning rules involving the equality relation can be replaced with the following rule:

$$EQ(x, y) \rightarrow SameAs(x, y)$$

In addition to the five relations of Figure 1 additional relations are still required for representing disjunctions of these relations in order to implement the reasoning mechanism of Section 4.3.

The additional relations representing disjunctions of basic relations are:

$$DR\_PO \equiv DR \vee PO$$
$$DR\_PO\_PC \equiv DR \vee PO \vee PC$$
$$DR\_PO\_PP \equiv DR \vee PO \vee PP$$
$$PO\_PP\_PC \equiv PO \vee PP \vee PC$$
$$PO\_PP \equiv PO \vee PP$$
$$PO\_PC \equiv PO \vee PC$$

The above representation and the corresponding reasoning mechanism can be expressed and implemented using only OWL 2 axioms and SWRL rules, as in the case of the two previous representations.

Twelve object relations are required in total, 5 basic RCC-5 relations, 6 additional relations representing disjunctions and the *null* (or $\perp$) relation representing inconsistency detection between two regions. This is the same number as in the case of the representation of Section 3.1 but since all SWRL reasoning rules involving the equality relation can be removed the total number of axioms (OWL and SWRL) are 90 instead of 119 that are required for directly representing RCC-5 formalism. Furthermore reasoners can treat equal regions as identical objects, i.e., as one object having all relations that the two identical regions have, thus reducing the total number of individuals that must be handled by the reasoner.

The representation based on equality elimination is more compact than the representation of Section 3.1, but still less compact than the representation of section 3.2. On the other hand it does not introduce additional relations since it handles RCC-5 relations directly, thus yielding better reasoning performance as illustrated in Section 6.

4. SPATIAL REASONING

Reasoning is realized by introducing a set of SWRL[14] rules operating on spatial relations. Reasoners that support DL-safe rules such as Pellet[15] can be used for reasoning over topological RCC-5 relations. Reasoning rules for all three alternative representations are presented in the following.

4.1 RCC-5 reasoning

Reasoning is realized by a set of SWRL rules applied on spatial relations of Section 3.1. All reasoners that support DL-safe SWRL rules can be used for inference and consistency checking over these relations. Defining compositions of relations is a basic part of the spatial reasoning mechanism. Table 1 represents the result of the composition of two topological RCC-5 relations of Figure 1.

Composition Table can be interpreted as follows: if relation R_1 holds between $Region1$ and $Region2$ and relation R_2 holds between $Region2$ and $Region3$, then the entry of the Table 1 corresponding to line R_1 and column R_2 denotes the possible relation(s) holding between $Region1$ and $Region3$. For example if $Region1$ is *Proper Part* (PP) of $Region2$ and $Region2$ is *Proper Part* (PP) of $Region3$ then $Region1$ is *Proper Part* of $Region3$. Entries in the composition table are determined using the formal semantics of Region Connection Calculus as defined at [6].

[14]http://www.w3.org/Submission/SWRL/
[15]http://clarkparsia.com/pellet/

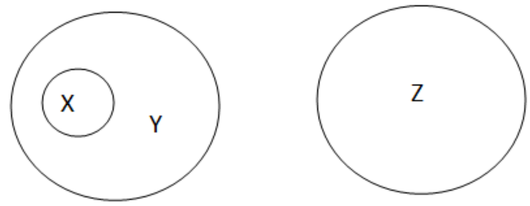

Figure 2: Composition of PP and DR Relations Example

Figure 2 illustrates the following example of composition: Region X is *Proper Part (PP)* of region Y, which in turn is discrete (DR) from region Z. The corresponding entry in Table 1 yields the relation DR as the composition of PP and DR, thus it can be inferred that X is discrete (DR) from Z as illustrated in Figure 2.

A series of compositions of relations may yield relations which are inconsistent with existing ones (e.g., the above example will yield a contradiction if X *overlaps* Z has been also asserted into the ontology). Consistency checking is achieved by ensuring path consistency by applying formula:

$$\forall x, y, k \; R_s(x, y) \leftarrow R_i(x, y) \cap (R_j(x, k) \circ R_k(k, y))$$

representing intersection of compositions of relations with existing relations (symbol $\cap$ denotes intersection, symbol $\circ$ denotes composition and R_i, R_j, R_k, R_s denote topological relations). The formula is applied until a fixed point is reached (i.e., the application of the rules above does not yield new inferences) or until the empty set is reached, implying that the ontology is inconsistent. Implementing path consistency formula requires rules for both compositions and intersections of pairs of relations.

Compositions of relations R_1, R_2 yielding a unique relation R_3 as a result are expressed in SWRL using rules of the form:

$$R_1(x, y) \wedge R_2(y, z) \rightarrow R_3(x, z)$$

The following is an example of such a composition rule:

$$PP(x, y) \wedge DR(y, z) \rightarrow DR(x, z)$$

Rules yielding a set of possible relations cannot be represented directly in SWRL since, disjunctions of atomic formulas are not permitted as a rule head. Instead, disjunctions of

Relations	DR	PO	EQ	PP	PC
DR	All	DR,PO,PP	DR	DR,PO,PP	DR
PO	DR,PO,PC	All	PO	PO,PP	DR,PO,PC
EQ	DR	PO	EQ	PP	PC
PP	DR	DR,PO,PP	PP	PP	All
PC	DR,PO,PC	PO,PC	PC	PO,EQ,PP,PC	PC

Table 1: Composition Table for RCC-5 Topological Relations.

relations are represented using new relations whose compositions must also be defined and asserted into the knowledge base. For example, the composition of relations PO and PP yields the disjunction of two possible relations (PP and PO) as a result:

$$PO(x,y) \wedge PP(y,z) \rightarrow (PO \vee PP)(x,z)$$

If the relation PO_PP represents the disjunction of relations PO and PP, then the composition of PO and PP can be represented using SWRL as follows:

$$PO(x,y) \wedge PP(y,z) \rightarrow PO\_PP(x,z)$$

A set of rules defining the result of intersecting relations holding between two regions must also be defined in order to implement path consistency. These rules are of the form:

$$R_1(x,y) \wedge R_2(x,y) \rightarrow R_3(x,y)$$

where R_3 can be the empty relation. For example, the intersection of relations DR and PC yields the empty relation ($\perp$ or *null*), and an inconsistency is detected:

$$DR(x,y) \wedge PC(x,y) \rightarrow \perp$$

Intersection of relations PO and PO_PP (representing the disjunction of *Overlaps* and *Proper Part* yields relation PO as a result:

$$PO(x,y) \wedge PO\_PP(x,y) \rightarrow PO(x,y)$$

Thus, path consistency is implemented by defining compositions and intersections of relations using SWRL rules and OWL axioms for inverse relations as presented in Section 3.

Another important issue for implementing path consistency is the identification of the additional relations, such as the above mentioned PO_PP relation, that represent disjunctions. Specifically the *minimal* set of relations required for defining compositions and intersections of all relations that can be yielded when applying path consistency on the basic relations of Figure 1 is identified. The identification of the additional relations is required for the construction of the corresponding SWRL rules.

In this work the closure method [14] of Table 2 is applied for computing the minimal relation sets containing the set of basic relations: starting with a set of relations, intersections and compositions of relations are applied iteratively until no new relations are yielded forming a set closed under composition, intersection and inverse. Since compositions and intersections are constant-time operations (i.e., a bounded number of table lookup operations at the corresponding composition table is required) the running time of closure method is linear to the total number of relations of the identified set.

Applying the closure method over the set of basic RCC-5 relations yields a set containing 12 relations. These are the 5 basic relations of Figure 1 and the relations DR_PO representing the disjunction of DR and PO, DR_PO_PC representing the disjunction of DR, PO and PC, DR_PO_PP representing the disjunction of DR, PO and PP, $PO\_EQ\_PP\_PC$ representing the disjunction of PO, EQ, PP and PC, PO_PP representing the disjunction of PO and PP, PO_PC representing the disjunction of PO and PC and *All* denoting the disjunction of all relations.

```
Input: Set S of tractable relations
Table C of compositions
WHILE S size changes
    BEGIN
        Compute C:Set of compositions of relations in S
        S=S ∪ C
        Compute I:set of intersections of relations in S
        S= S ∪ I
    END
RETURN S
```

Table 2: Closure method

A reduction to required relations and rules can be achieved by observing that the disjunction of all basic relations when composed with other relations yields the same relation, while intersections yield the other relation. Specifically, given that *All* represents the disjunction of all basic relations and, R_x is a relation in the supported set then the following holds for every R_x:

$$All(x,y) \wedge R_x(x,y) \rightarrow R_x(x,y)$$

$$All(x,y) \wedge R_x(y,z) \rightarrow All(x,z)$$

$$R_x(x,y) \wedge All(y,z) \rightarrow All(x,z)$$

Since relation All always holds between two regions, because it is the disjunction of all possible relations, all rules involving this relation, both compositions and intersections, do not add new relations into the ontology and they can be safely removed. Also, all rules yielding the relation All as a result of the composition of two supported relations R_{x1}, R_{x2}:

$$R_{x1}(x,y) \wedge R_{x2}(y,z) \rightarrow All(x,z)$$

can be removed as well. Thus, since intersections yield existing relations and the fact that the disjunction over all basic relations must hold between two points, all rules involving the disjunction of all basic relations and consequently all rules yielding this relation can be safely removed from the knowledge base. After applying this optimization the required number of axioms for implementing path consistency over the set of directional relations of Figure 1 is 119.

4.2 RCC-5 reasoning using decomposition

Reasoning over RCC-5 relations can be also implemented using the decomposition of Section 3.2. Reasoning over the resulting set of relations has been implemented in [8] using a set of 12 rules over the relations derived from the RCC-5 relations after decomposition is applied. These rules are:

$$\begin{gathered}
\neg P(x,y) \wedge P_i(y,z) \rightarrow \neg P(x,z)\\
\neg P_i(x,y) \wedge P(y,z) \rightarrow \neg P_i(x,z)\\
P_i(x,y) \wedge \neg P(y,z) \rightarrow \neg P(x,z)\\
P(x,y) \wedge \neg P_i(y,z) \rightarrow \neg P_i(x,z)\\
P_i(x,y) \wedge P_i(y,z) \rightarrow P_i(x,z)\\
P(x,y) \wedge P(y,z) \rightarrow P(x,z)\\
C(x,y) \wedge P_i(y,z) \rightarrow C(x,z)\\
\neg C(x,y) \wedge P(y,z) \rightarrow \neg C(x,z)\\
C(x,y) \wedge \neg C(y,z) \rightarrow \neg P_i(x,z)\\
P_i(x,y) \wedge \neg C(y,z) \rightarrow \neg C(x,z)\\
P(x,y) \wedge C(y,z) \rightarrow C(x,z)\\
\neg C(x,y) \wedge C(y,z) \rightarrow \neg P(x,z)
\end{gathered}$$

Adjusting the above rules by replacing negation (that is not supported by SWRL) to the equivalent relations introduced at Section 3.3 results to the following set of SWRL rules implementing reasoning for the decomposition based representation:

$$\begin{gathered}
notP(x,y) \wedge P_i(y,z) \rightarrow notP(x,z)\\
notP_i(x,y) \wedge P(y,z) \rightarrow notP_i(x,z)\\
P_i(x,y) \wedge notP(y,z) \rightarrow notP(x,z)\\
P(x,y) \wedge notP_i(y,z) \rightarrow notP_i(x,z)\\
P_i(x,y) \wedge P_i(y,z) \rightarrow P_i(x,z)\\
P(x,y) \wedge P(y,z) \rightarrow P(x,z)\\
C(x,y) \wedge P_i(y,z) \rightarrow C(x,z)\\
notC(x,y) \wedge P(y,z) \rightarrow notC(x,z)\\
C(x,y) \wedge notC(y,z) \rightarrow notP_i(x,z)\\
P_i(x,y) \wedge notC(y,z) \rightarrow notC(x,z)\\
P(x,y) \wedge C(y,z) \rightarrow C(x,z)\\
notC(x,y) \wedge C(y,z) \rightarrow notP(x,z)
\end{gathered}$$

The above SWRL rules combined with relations, axioms and transformation rules of Section 3.2 are used for implementing the representation and reasoning mechanism for RCC-5 relations using decomposition. 46 OWL axioms and rules are required for the decomposition based representation.

4.3 RCC-5 reasoning using equality elimination

The 5 basic RCC-5 relations form the basis of this representation. Reasoning is based on SWRL rules implementing the path consistency method as in Section 4.1 but the difference with the reasoning mechanism of Section 4.1 is that all rules involving the equality relation are removed and they are replaced by one equivalent axiom:

$$EQ(x,y) \rightarrow SameAs(x,y)$$

Applying the closure method of Table 2 over the set of basic RCC-5 relations while ignoring compositions with the EQ relation (since it has been removed from the composition table and replaced by the *sameAs* axiom) yields a set containing 12 relations. These are the 5 basic relations of Figure 1 and the relations DR_PO representing the disjunction of DR and PO, DR_PO_PC representing the disjunction of DR, PO and PC, DR_PO_PP representing the disjunction of DR, PO and PP, PO_PP_PC representing the disjunction of PO, PP and PC, PO_PP representing the disjunction of PO and PP, PO_PC representing the disjunction of PO and PC and All denoting the disjunction of all relations. Again the disjunction of all relations can be removed as in Section 4.1 resulting in a representation and reasoning mechanism consisting of 90 rules and axioms.

5. RCC-8 REPRESENTATION AND REASONING

Another important set of topological relations is the RCC-8 set of relation which is a refinement of RCC-5 relations set. Specifically the DR relation is refined into two distinct relations; DC (Disconnected) representing the fact that two regions don't have common points, and EC (Externally connected) representing the fact that two regions have common boundary points but not common internal points. Similarly the PP (Proper part) relation is refined into two different relations TPP and $NTPP$. TPP is representing the fact that a region is a proper part of another region and also has common points with the boundary of the enclosing region. $NTPP$ on the other hand represents the fact that the enclosed region does not have common points with the boundary of the enclosing region. $NTPP_i$ and TPP_i are the inverses of $NTPP$ and TPP respectively. RCC-8 relations are illustrated in Figure 3. The composition table for RCC-8 relations has been defined in [6] and an implementation based on path consistency using SWRL and OWL axioms was presented in [3].

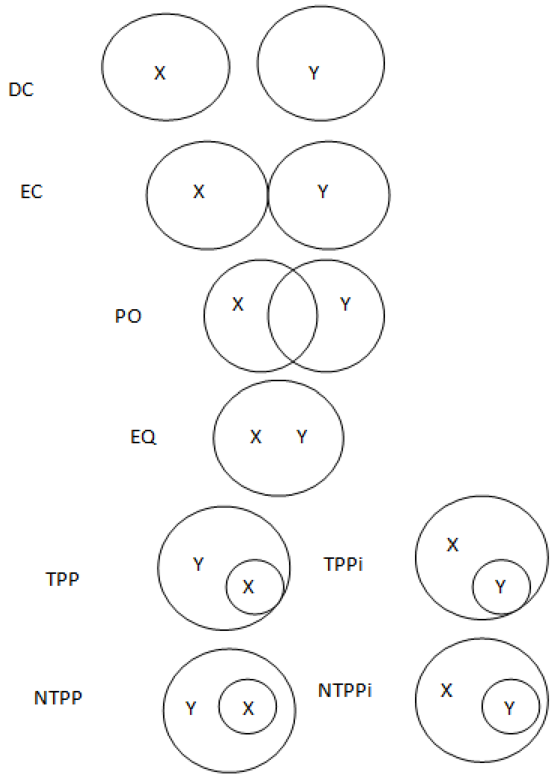

Figure 3: RCC-8 Relations

Table 3 represents the result of the composition of two topological RCC-8 relations of Figure 3 (* denotes the disjunction of all relations).

The implementation presented in [3] required additional relations representing disjunctions in addition to the basic 8 RCC-8 relations and definitions of all compositions and intersections between them as in the case of RCC-5 relation in Section 4.1. In total 49 relations and 1410 axioms and rules were used for implementing the representation and reasoning mechanism, after removing the disjunction of all relations *all* as proposed in Section 4.1.

In this work the elimination of equality relation using the $SameAs$ axiom that was used for RCC-5 relations yielding a more compact representation and improved reasoning performance is applied also for the RCC-8 relations, since equality relation is one of the basic relations of the RCC-8 formalism. Applying the closure method of Table 2 and defining all compositions and intersections of resulting relations after eliminating the equality relation, and then removing the disjunction of all basic relations, since it does not add new information, results in a representation that requires 37 object properties and 824 axioms.

6. EVALUATION

In the following the proposed representations and reasoning mechanisms are evaluated both theoretically and experimentally.

6.1 Theoretical Evaluation

The required expressiveness of the proposed representations are within the limits of OWL 2 expressiveness. Reasoning is achieved by employing DL-safe rules expressed in SWRL that apply on named individuals in the ontology ABox, thus retaining decidability. Furthermore, since the proposed representations cover basic RCC-5 and RCC-8 relations which are decided by path consistency, reasoning using the polynomial time path consistency algorithm is sound and complete [14].

Specifically, any region can be related with every other region with one basic topological relation (or exactly three after the decomposition of Section 3.2 is applied). Since basic relations are mutually exclusive, between n regions, at most $n(n-1)$ relations can be asserted (in case of decomposition the total number of relations is $3n(n-1)$) . Furthermore, path consistency has $O(n^5)$ time worst case complexity (with n being the number of regions). In the most general case where disjunctive relations are supported in addition to the basic ones, any region can be related with every other region by at most k relations, where k is the size of the set of supported relations (containing six additional relations for RCC-5 relations besides the basic ones). Therefore, for n regions, using $O(k^2)$ rules, at most $O(kn^2)$ relations can be asserted into the knowledge base.

Applying the *closure method* over the basic RCC-5 relations (Section 4.1) the total number of relations required for RCC-5 representation is 12 while the decomposition method (Section 3.2) introduces six new relations. Required axioms are 119 and 46 respectively, while equality elimination based representation requires 90 axioms. In case of RCC-8 relation required axioms are 1410 and 824 without or with equality elimination respectively.

The $O(n^5)$ upper limit for path consistency running time referred to above is obtained as follows: At most $O(n^2)$ relations can be added in the knowledge base. At each such addition step, the reasoner selects 3 variables among n regions which corresponds to $O(n^3)$ possible different choices. Clearly, this upper bound is pessimistic, since the overall number of steps may be lower than $O(n^2)$ because an inconsistency detection may terminate the reasoning process early, or the asserted relations may yield a small number of inferences. Also, forward chaining rule execution engines employ several optimizations (e.g., the Rete algorithm used in the SWRL implementation of Pellet), thus the selection of appropriate variables usually involves fewer than $O(n^3)$ trials. Nevertheless, since the end user may use any reasoner

Relations	DC	EC	PO	TPP	NTPP	TPPi	NTPPi	EQ
DC	*	DC,EC,PO,TPP,NTPP	DC,EC,PO,TPP,NTPP	DC,EC,PO,TPP,NTPP	DC,EC,PO,TPP,NTPP	DC	DC	DC
EC	DC,EC,PO,TPPi,NTPPi	DC,EC,PO,TPP,TPPi,EQ	DC,EC,PO,TPP,NTPP	EC,PO,TPP,NTPP	PO,TPP,NTPP	DC,EC	DC	EC
PO	DC,EC,PO,TPPi,NTPPi	DC,EC,PO,TPPi,NTPPi	*	PO,TPP,NTPP	PO,TPP,NTPP	DC,EC,PO,TPPi,NTPPi	DC,EC,PO,TPPi,NTPPi	PO
TPP	DC	DC,EC	DC,EC,PO,TPP,NTPP	TPP,NTPP	NTPP	DC,EC,PO,TPP,TPPi,EQ	DC,EC,PO,TPPi,NTPPi	TPP
NTPP	DC	DC	DC,EC,PO,TPP,NTPP	NTPP	NTPP	DC,EC,PO,TPP,NTPP	*	NTPP
TPPi	DC,EC,PO,TPPi,NTPPi	EC,PO,TPPi,NTPPi	PO,TPPi,NTPPi	PO,TPP,TPPi,EQ	PO,TPP,NTPP	TPPi,NTPPi	NTPPi	TPPi
NTPPi	DC,EC,PO,TPPi,NTPPi	PO,TPPi,NTPPi	PO,TPPi,NTPPi	PO,TPPi,NTPPi	PO,TPP,NTPP,TPPi,NTPPi,EQ	NTPPi	NTPPi	NTPPi
EQ	DC	EC	PO	TPP	NTPP	TPPi	NTPPi	EQ

Table 3: Composition Table for RCC-8 Topological Relations.

supporting SWRL, a worst case selection of variables can be assumed in order to obtain an upper bound for complexity. Notice that, retaining control over the order of variable selection and application of rules yields an $O(n^3)$ upper bound for path consistency [16].

6.2 Experimental Evaluation

Measuring the efficiency of the proposed representations (RCC-5, RCC-5 using decomposition and RCC-5 using equality elimination) requires a spatial ontology, thus a data-set of 200 to 1000 regions generated randomly was used for the experimental evaluation. Reasoning response times of the spatial reasoning rules are measured as the average over 5 runs. Pellet 2.2.0 running as a plug-in of Protégé 4.2 was the reasoner used in the experiments. All experiments run on a PC, with Intel Core CPU at 2.4 GHz, 4 GB RAM, and Windows 7.

In addition to that required TBox axioms and ABox assertions are presented as a measure of the compactness of each representation for randomly generated datasets of 200 to 1000 regions. The experiments are repeated for the case of RCC-8 relations for a random ontology of 200 regions. RCC-8 representations without and with equality elimination are compared in terms of required axioms and reasoning speed.

The first experiment compares the three alternative RCC-5 representations in terms of required axioms (TBox and ABox assertions) . The three representations are abbreviated as *RCC-5* (direct implementation of reasoning over RCC-5 relations of Section 3.1), *RCC-5DE* (decomposition of RCC-5 relations of Section 3.2) and *RCC-5EQ* (equality elimination based representation of Section 3.3).

Number of regions	Required axioms		
	RCC-5	RCC-5DE	RCC-5EQ
200	718	645	689
400	1318	1245	1289
600	1918	1845	1889
800	2518	2445	2489
1000	3118	3045	3089

Table 4: Required axioms as a function of the number of regions

Table 4 illustrates the fact that the decomposition yields a more compact representation because of the fewer reasoning rules compared with the other two approaches into the ontology TBox. On the other hand ABox assertions are numerically equal and moreover transforming each RCC-5 relation into three other relations decreases reasoning performance as illustrated in Table 5.

Table 5 illustrates the fact that the equality elimination

Number of regions	Reasoning Time (ms)		
	RCC-5	RCC-5DE	RCC-5EQ
200	438	426.4	300.2
400	762.6	1006.2	568.6
600	1282	1562.6	1004.6
800	1579.6	2268.6	1316.8
1000	2019.6	2610.4	1774.4

Table 5: Average reasoning time for topological RCC-5 relations as a function of the number of regions

method outperforms the other methods in terms of reasoning speed. The decomposition method has inferior performance because of the increased (by a factor of 3) number of inferred relations, since each topological relation is represented using 3 new relations between two regions. Thus decomposition offers a compact representation but in terms of reasoning speed has inferior performance when compared to the other two representations based on path consistency.

Finally RCC-8 representations without and with equality elimination (abbreviated as *RCC-8* and *RCC-8EQ* respectively) are compared in terms of required axioms and reasoning speed for a random ontology containing 200 regions. The average value over 5 such measurements is presented.

Number of axioms		Reasoning Time (ms)	
RCC-8	RCC-8EQ	RCC-8	RCC-8EQ
2009	1423	1531.2	1135.6

Table 6: Comparison of RCC-8 and RCC-8 with equality elimination

Table 6 illustrates the fact that the equality elimination method outperforms the RCC-8 representation without the equality elimination in terms of both compactness of representation and reasoning speed as in the case of RCC-5 relations. Similar results were obtained using dedicated reasoners (instead of OWL axioms and SWRL rules as in this work) for topological RCC-8 relations at [5]. On the other hand decompositions offer more compact representations but assert an increased number of spatial relations. In the case of RCC-5 relations this results in inferior reasoning speed performance.

7. CONCLUSIONS AND FUTURE WORK

In this work, three representations for handling topological spatial information in ontologies expressed using RCC-5 relations are introduced. The proposed representations and the corresponding reasoning mechanisms are implemented and evaluated in terms of both compactness and reasoning speed. The representation based on decomposition of RCC-5 relations offers a more compact representation than the direct representation based on path consistency method, but yields inferior reasoning performance. The representation based on path consistency can be improved by applying the equality elimination optimization. When applying this optimization, performance in case of both RCC-5 and RCC-8 topological relations is improved.

All representations are fully compliant with existing semantic Web standards and tools and they do not require additional software, besides standard editors and reasoners such as Pellet. This greatly increases the applicability of the proposed approaches for Linked Data, since spatial information can be easily distributed, shared and modified in case only standard tools are required for handling it.

Directions of future work include the development of real world applications based on the proposed representations for handling spatial datasets. Such applications can combine temporal with topological spatial representations for handling dynamic information (e.g., moving objects). Developing even more compact and superior in terms of reasoning speed representations and making use of parallelism for spatial reasoning are also directions of future work.

8. REFERENCES

[1] G. Antoniou and F. Van Harmelen. A semantic web primer. 2004. *MIT press.*

[2] S. Batsakis. Reasoning over 2d and 3d directional relations in owl: a rule-based approach. In *Theory, Practice, and Applications of Rules on the Web*, pages 37–51. Springer, 2013.

[3] S. Batsakis and E. G. Petrakis. Sowl: a framework for handling spatio-temporal information in owl 2.0. In *Rule-Based Reasoning, Programming, and Applications*, pages 242–249. Springer, 2011.

[4] S. Batsakis, K. Stravoskoufos, and E. G. Petrakis. Temporal reasoning for supporting temporal queries in owl 2.0. In *Knowledge-Based and Intelligent Information and Engineering Systems*, pages 558–567. Springer, 2011.

[5] G. Christodoulou, E. G. Petrakis, and S. Batsakis. Qualitative spatial reasoning using topological and directional information in owl. In *Tools with Artificial Intelligence (ICTAI), 2012 IEEE 24th International Conference on*, volume 1, pages 596–602. IEEE, 2012.

[6] A. G. Cohn, B. Bennett, J. Gooday, and N. M. Gotts. Qualitative spatial representation and reasoning with the region connection calculus. *Geoinformatica*, 1(3):275–316, 1997.

[7] I. Horrocks, P. F. Patel-Schneider, H. Boley, S. Tabet, B. Grosof, M. Dean, et al. Swrl: A semantic web rule language combining owl and ruleml. *W3C Member submission*, 21:79, 2004.

[8] J. Hué, M. Westphal, and S. Wolfl. An automatic decomposition method for qualitative spatial and temporal reasoning. In *Tools with Artificial Intelligence (ICTAI), 2012 IEEE 24th International Conference on*, volume 1, pages 588–595. IEEE, 2012.

[9] O. K. I. Horrocks and U. Sattler. The even more irresistible sroiq. *In: Proc. KR 2006, Lake District, UK,*, 2006.

[10] Y. Katz and B. Grau. Representing qualitative spatial information in owl-dl. *In Proc. of Int. Workshop: OWL Experiences and Directions, Galway, Ireland*, 2005.

[11] D. L. McGuinness, F. Van Harmelen, et al. Owl web ontology language overview. *W3C recommendation*, 10(2004-03):10, 2004.

[12] B. Motik, I. Horrocks, R. Rosati, and U. Sattler. Can owl and logic programming live together happily ever after? In *The Semantic Web-ISWC 2006*, pages 501–514. Springer, 2006.

[13] J. Renz and D. Mitra. Qualitative direction calculi with arbitrary granularity. *In PRICAI 2004: Trends in Artificial Intelligence: 8th Pacific Rim International Conference on Artificial Intelligence, Auckland, New Zealand*, 2004.

[14] J. Renz and B. Nebel. Qualitative spatial reasoning using constraint calculi. *In Handbook of Spatial Logics, Springer, Netherlands*, pages 161–215, 2007.

[15] E. Sirin, B. Parsia, B. C. Grau, A. Kalyanpur, and Y. Katz. Pellet: A practical owl-dl reasoner. *Web Semantics: science, services and agents on the World Wide Web*, 5(2):51–53, 2007.

[16] M. Stocker and E. Sirin. Pelletspatial: A hybrid rcc-8 and rdf/owl reasoning and query engine. *In: CEUR Workshop Proceedings, vol. 529-OWLED 2009,*, pages 2–31, 2009.

[17] P. van Beek and R. Cohen. Exact and approximate reasoning about temporal relations. *Computational intelligence, Vol 6(3)*, pages 132–147, 1990.