The Development of Multi-axial Creep Damage Constitutive Equations for 0.5Cr0.5Mo0.25V Ferritic Steel at 590°C*

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Within the framework of a phenomenological approach a set of multi-axial creep damage constitutive equations for 0.5Cr0.5Mo0.25V ferritic steel at 590°C is developed in which a new formulation is employed. The deficiency of the previous formulation and the need for improvement became apparent after a critical review of the development of creep damage constitutive equations for 316 stainless steel(1). The need for improvement was further underpinned by a call for modification of the constitutive equations(36). Recently, a specific formulation was proposed and validated(2)-(4). This paper reports the latest developments of the multi-axial creep constitutive equations for 0.5Cr0.5Mo0.25V ferritic steel at 590°C including: 1) the fundamental requirement; 2) formulation; 3) validation; and 4) conclusion. It systematically shows the suitability of this new set of constitutive equations and the incapability of the previous ones. Furthermore, it contributes knowledge to the methodology.

Key Words: Creep Damage Constitutive Equations, Formulation, Multi-axial States of Stress, Validation

1. Introduction

Creep damage phenomena in material and structural members are one of the serious problems in our modern life and have been widely studied(5)-(49). Creep continuum damage mechanics, originally proposed by Kachanov in 1958, has been developed with the objective of dealing with it in engineering design. It is evident that significant progresses have been made in various aspects including theory (such as the phenomenological approach, unified irreversible thermodynamic formulation(7),(32), anisotropic damage theory(28)-(30)), applications(11)-(26), experimental observation and verification(35)-(38), experimental methodology(39) and even optimization. The field of creep continuum damage mechanics, as well as damage mechanics as a whole, is developing rapidly with a large number of recent publications. However, it is vitally important to distil new information and to assess and refine both the generic methodology and the specific set of constitutive equations developed to ensure their general applicability. This is the main concern of this paper.

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The phenomenological approach was originated by Kachanov (1958) and it can be broadly classified into weak coupling and strong coupling between damage and deformation. For the case of weak coupling the effect of material damage on the elastic properties is disregarded and a coupling is established by introducing damage variables into the constitutive equation of the continuum solid using the effective state variables concept (see Kachanov(12),(13); Rabotnov(14),(15); Leckie(43),(44); Hayhurst(18),(19); Leckie and Hayhurst(17),(45); etc.). In the case of the fully (strongly) coupled approach, damage evolution affects both the elastic properties of the material and the inelastic response(28)-(30).

Within the weak approach, a set of creep damage constitutive equations for uni-axial tension is developed first, with consideration of the different creep mechanisms and the coupling between the creep damage and the creep strain. This can then be generalized to multi-axial states of stress(16)-(19),(43)-(45). The success of this approach relies crucially on the development of a set of appropriate constitutive equations capable of depicting the observed multi-axial behaviour. This generalization from a uni-axial state to a multi-axial one is directed by the theory developed or adopted. The set of constitutive equations developed should be validated properly to ensure their general applicability.

0.5Cr0.5Mo0.25V ferritic steel is an important material in industries such as power generation, for example. A class of KRH (Kachanov-Rabotnov-Hayhurst) type constitutive equations for a range of temperatures was developed for this material by Perrin and Hayhurst in 1996[23]. That knowledge was immediately used to produce a specific set of constitutive equations at 590°C which, together with constitutive equations for the weld material and the Type IV heat effected zone, was used by one of the authors to predict damage evolution and failure in welded vessels, as reported in Ref.(22). However, further development was initially motivated by the realization that a deficiency existed in the KRH approach. This realization came after a critical review[1], supported by a recently reported discrepancy between predicted and experimentally observed damage, leading to a call by other researchers[36] for modification of constitutive equations. The conceptual development was partially reinforced by the general plasticity theory[50]. This paper reports the latest progress in the development of creep constitutive equations for multi-axial states of stress for 0.5Cr0.5Mo0.25V ferritic steel at 590°C including: 1) the two fundamental requirement; 2) formulation; 3) validation; and 4) conclusions. This paper systematically shows the inadequacy of the previous formulation and the suitability of a new set of constitutive equations for general engineering analysis. Furthermore, it contributes knowledge to the methodology.

2. Requirement[1]

The two fundamental requirements for the development of a set of creep damage constitutive equations are creep strain rate consistency and damage evolution consistency. This means that the creep strain rate and damage evolution predicted by a set of constitutive equations under various states of stress should be consistent with experimental observations. For simple uni-axial tests this is reduced to producing a fit to the creep strain curves whilst ensuring the correct damage evolution. For the multi-axial case, the expression of this consistency requirement is quite complicated even without mentioning the order of damage variable/variables, as the second order stress and strain tensors are involved. However, this can be achieved by comparison of a set of creep curves and damage evolution curves upon which the contours/surfaces of constant creep strain and constant damage have been constructed. In fact, the isochronous rupture loci (these are surfaces in stress space for which rupture times are constant) are one of these surfaces. The development of multi-axial creep damage constitutive equations could be thought of mathematically as a two-objective optimization problem and mechanically as damage evolution and creep deformation.

Integration of a specific set of creep damage constitutive equations from virgin material to failure will produce:

$$\omega=\omega_f, \qquad t=t_f^{multi}, \qquad \varepsilon=\varepsilon_f^{multi} \tag{1}$$

where

ω, ω_f: damage, critical value of damage;

$\varepsilon, \varepsilon_f^{multi}$: creep strain tensor, creep strain tensor at failure for multi-axial stress state;

t, t_f^{multi}: time, rupture time for multi-axial stress state.

The general applicability of a set of multi-axial constitutive equations will be ensured through validation. Both lifetime and strain at failure will be considered and a wide range of stress states and loading conditions will be included.

3. Formulation

3.1 Uni-axial form

The uni-axial form of the creep damage constitutive equations for 0.5Cr0.5Mo0.25V ferritic steel at 590°C are written in rate form as[22],[23]:

$$\dot{\varepsilon}=A\sinh\left(\frac{B\sigma(1-H)}{(1-\phi)(1-\omega)}\right) \tag{2}$$

$$\dot{H}=\frac{h}{\sigma}\left(1-\frac{H}{H^*}\right)\dot{\varepsilon} \tag{3}$$

$$\dot{\phi}=\frac{K_c}{3}(1-\phi)^4 \tag{4}$$

$$\dot{\omega}=CN\dot{\varepsilon} \tag{5}$$

where $N=1$ for $\sigma>0$ and $N=0$ for $\sigma<0$, $\dot{\varepsilon}$ is uni-axial creep strain rate and σ is uni-axial stress. A, B, C, h, H^* and K_c are material parameters. H, ϕ and ω are the three state variables. The first state variable, H, represents the strain hardening that occurs during primary creep; initially H is zero and as strain is accumulated, increases to the value of H^*. The second state variable, ϕ, describes the drop in creep resistance in particle hardened alloys such as ferritic steel due to evolution of the carbide precipitates; its initial value is zero. The third state variable, ω, represents intergranular cavitation damage and varies from zero, for material in the virgin state, to $\omega_f=1/3$, when all the grain boundaries normal to the applied stress have completely cavitated[23], at which time the material is considered to have failed.

3.2 KRH formulation

The multi-axial KRH formulation is written as[22],[23]:

$$\dot{\varepsilon}_{ij}=\frac{3S_{ij}}{2\sigma_e}A\sinh\left(\frac{B\sigma_e(1-H)}{(1-\phi)(1-\omega)}\right) \tag{6}$$

$$\dot{H}=\frac{h}{\sigma_e}\left(1-\frac{H}{H^*}\right)\dot{\varepsilon}_e \quad (7)$$

$$\dot{\phi}=\frac{K_c}{3}(1-\phi)^4 \quad (8)$$

$$\dot{\omega}=C\dot{\varepsilon}_e^*\langle\sigma_1/\sigma_e\rangle^\nu \quad (9)$$

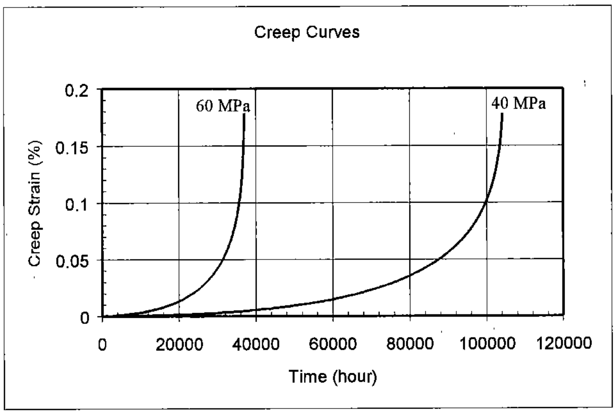
where $\langle\ \rangle$ is the Heaviside step function; v is the stress state index defining the multi-axial stress rupture criterion; $\dot{\varepsilon}_{ij}$ is the rate of the creep strain components; $\dot{\varepsilon}_e=\sqrt{2/3\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}$ is the rate of the effective creep strain; σ_1 is the maximum principal stress; S_{ij} is the deviatoric stress; $\sigma_e=\sqrt{3/2S_{ij}S_{ij}}$ is the effective stress.

The adopted function of states of stress $\langle\sigma_1/\sigma_e\rangle^\nu$ was originally proposed by Cane according to Perrin and Hayhurst[23]. Perrin and Hayhurst obtained the value of 2.8 for ν through calibration of isochronous rupture loci in the plane stress condition and a couple of multi-axial tests. A lengthy discussion on its determination can be found in the same paper.

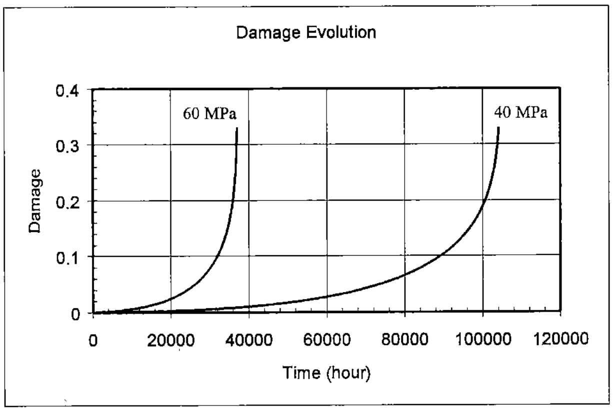
For completeness the material constants for this material at 590°C are given below:

$A=2.1618\times10^{-9}$ MPa h^{-1} $\quad B=0.20524$ MPa^{-1}

$C=1.8537$ $\quad h=2.4326\times10^5$ MPa $\quad H^*=0.5929$



(a) Creep curves under uni-axial tension



(b) Damage evolution under uni-axial tension

Fig. 1 Creep and damage evolution under uni-axial tension

$K_c=9.2273\times10^{-5}$ MPa^{-3} h^{-1} $\quad \nu=2.8$

The creep curves and damage evolutions are shown in Fig. 1. Damage evolution may be interpreted as the ratio of the lost cross section to the original cross section.

3.3 New formulation

The new form of the constitutive equations is given as[2],[3]:

$$\dot{\varepsilon}_{ij}=\frac{3S_{ij}}{2\sigma_e}A\sinh\left(\frac{B\sigma_e(1-H)}{(1-\phi)(1-\omega_d)}\right) \quad (10)$$

$$\dot{H}=\frac{h}{\sigma_e}\left(1-\frac{H}{H^*}\right)\dot{\varepsilon}_e \quad (11)$$

$$\dot{\phi}=\frac{K_c}{3}(1-\phi)^4 \quad (12)$$

$$\dot{\omega}=C\dot{\varepsilon}_e\cdot f_2 \quad (13)$$

$$\dot{\omega}_d=\dot{\omega}\cdot f_1 \quad (14)$$

where f_1 and f_2 are functions of the stress state. The newly introduced fourth state variable, ω_d, represents the effect of creep damage on deformation. Function f_2 is introduced to depict the effect of the state of stress on damage evolution, in a similar fashion to the previous formulation. The additional function f_1 is introduced to better phenomenologically depict the coupling between damage and tertiary deformation and creep rupture. The physical implication is that creep deformation and creep rupture are two different processes and two separate internal variables are needed.

3.4 Specific form of f_1 and f_2

The specific form for function f_1 was a Huddleston's formulation for strength theory, given as[34]:

$$f_1=\left(\frac{2\sigma_e}{3S_1}\right)^a\exp\left\{b\left[\frac{3\sigma_m}{S_s}-1\right]\right\} \quad (15)$$

where $S_s=\sqrt{\sigma_1^2+\sigma_2^2+\sigma_3^2}$, $\sigma_m=\frac{1}{3}(\sigma_1+\sigma_2+\sigma_3)$, $S_1=\sigma_1-\sigma_m$ and σ_1, σ_2, σ_3 are the principal stresses. a and b are two material parameters reflecting stress sensitivity.

Spindler's relation for strain at failure[35],[42],[48],[49] is adopted for function f_2:

$$f_2=\left(\exp\left\{p\left[1-\frac{\sigma_1}{\sigma_e}\right]+q\left[\frac{1}{2}-\frac{3\sigma_m}{2\sigma_e}\right]\right\}\right)^{-1} \quad (16)$$

where p and q are two material parameters. This function equals one for uni-axial conditions and coincides with the well known Rice-Tracey equation if $p=0$ and $q=1$.

Substituting Eqs. (15)-(16) into Eqs. (13)-(14) gives:

$$\dot{\varepsilon}_{ij}=\frac{3S_{ij}}{2\sigma_e}A\sinh\left(\frac{B\sigma_e(1-H)}{(1-\phi)(1-\omega_d)}\right) \quad (17)$$

$$\dot{H}=\frac{h}{\sigma_e}\left(1-\frac{H}{H^*}\right)\dot{\varepsilon}_e \quad (18)$$

$$\dot{\phi}=\frac{K_c}{3}(1-\phi)^4 \quad (19)$$

$$\dot{\omega}=C\dot{\varepsilon}_e\left(\exp\left\{p\left[1-\frac{\sigma_1}{\sigma_e}\right]+q\left[\frac{1}{2}-\frac{3\sigma_m}{2\sigma_e}\right]\right\}\right)^{-1} \quad (20)$$

$$\dot{\omega}_d=\dot{\omega}\left(\frac{2\sigma_e}{3S_1}\right)^a \exp\left\{b\left[\frac{3\sigma_m}{S_s}-1\right]\right\} \quad (21)$$

The constants a and b should be determined from experimentally obtained isochronous rupture loci.

3.5 Discussion

The KRH approach for generalization originated with the development of multi-axial constitutive equations for copper and aluminium in 1972[18]. In this approach the creep damage rate is related to the creep damage equivalent stress equation identified from the isochronous rupture loci produced under plane stress conditions.

The main concepts and/or steps used in KRH approach may be summarised as:

1) the effective creep strain is controlled by Von Mises stress;

2) the description of tertiary creep deformation is achieved by coupling with creep damage;

3) creep damage is assumed to be quasi-isotropic;

4) the effective stress is based on an equivalent strain hypothesis but it was not consistently used as recommended by Lemaitre and Chaboche[6]. No theoretical justification was given for this.

5) the material is deemed to have failed when the damage variable reaches its critical value. This critical value was originally assumed to be 1, and then modified to be a constant less than 1. It typically ranges from 0.5 to 0.9 according to Lemaitre[6];

6) the creep rupture strength theory or creep damage equivalent stress is introduced to describe the damage evolution to achieve lifetime consistency; and

7) the material constant ν is calibrated against the lifetime within bi-axial states of stress.

The fundamental deficiency in this KRH approach is that the creep strain consistency requirement is not satisfied. The new formulation introduces two functions to depict the effect of states of stress on the damage evolution and the creep strain, respectively. The possible physical justification for this is that the creep deformation process and the creep rupture process differ and two internal variables may be needed.

4. Validation[4]

4.1 General

Adequate validation should include both proportional and non-proportional loading conditions. The former is a must and the latter is highly desirable but may not be carried out due to high cost and difficulty of the experimental work. However as discussed by Xu[4], even for the proportional loading condition, the validation undertaken previously was not adequate in terms of either the items addressed (only lifetime was matched, not strain at failure) or the range of stress states. Further more, there was a degree of ambiguity between calibration and validation as the notched bar test results were reported for calibration rather than validation[23].

An improved practical validation method with consideration of the availability of experimental data was proposed by Xu[4]. This method includes:

1) Check the isochronous rupture loci under plane stress and plane strain (proportional loading) conditions;

2) Check the strain at failure under plane stress and plane strain (proportional loading) conditions;

3) Check typical creep curves under plane stress and plane strain (proportional loading) conditions;

4) Check damage development, creep strain development, strain at failure and lifetime for multi-axial stress states with complex (or non-proportional) loading conditions. The notched bar test may be used to carry out this check.

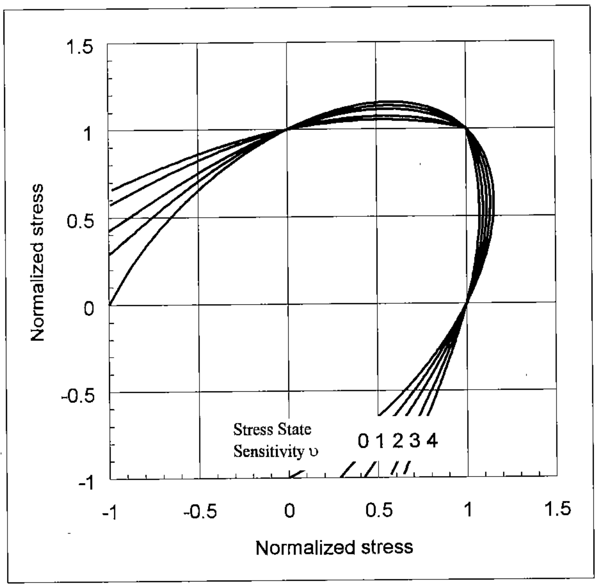
In this approach, the plane stress and plane strain stress states (proportional loading) are selected to present a wide range of stress states (under proportional loading). It is suggested that all the material constants should be determined in the first two steps. Step 3 should be used to further check the coupling of damage and creep strain, which, as far as these authors are aware, has not been addressed by other researchers. Inaccurate predictions of creep strain will result in incorrect predictions of the stress redistribution and hence destroy the general applicability of the material model. Step 4 validates the constitutive equations under multi-axial non-proportional loading conditions.

4.2 Results

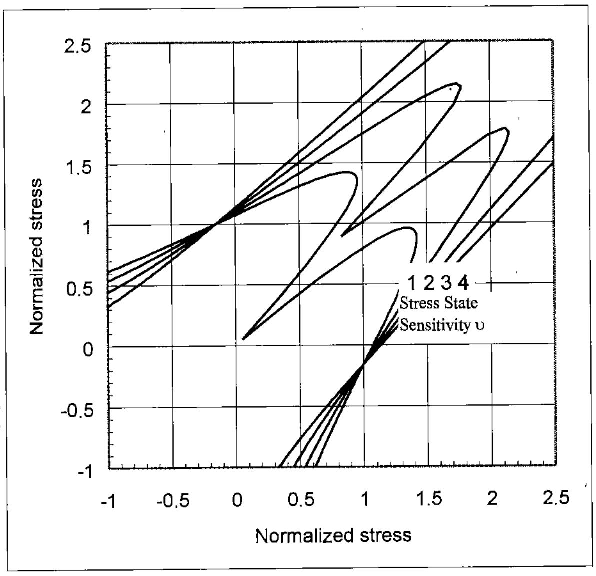
The following section will illustrate the validation process with typical results. Details of this have been presented previously by Xu[4]. The experimental data has been provided by Spindler[42].

The isochronous rupture loci and ratios of strain at failure for the previous formulation are presented in Figs. 2 and 3, respectively. The stress states sensitivity ranges from 1 to 4 with an interval of 1 and both plane stress and plane strain conditions are considered.

The concept of isochronous rupture loci has already been introduced in section 2. The numerical results (plane stress and plane strain conditions) were obtained by a specially developed computer program. This program performs the following functions: 1) finding the rupture lifetime for a given normalizing



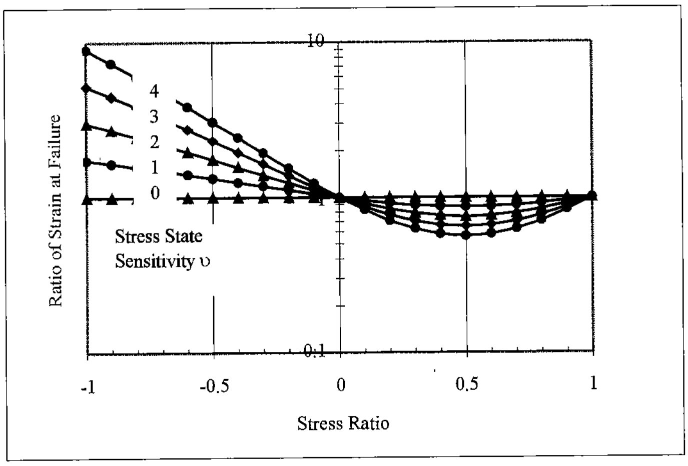
(a) Isochronous rupture loci under plane stress conditions



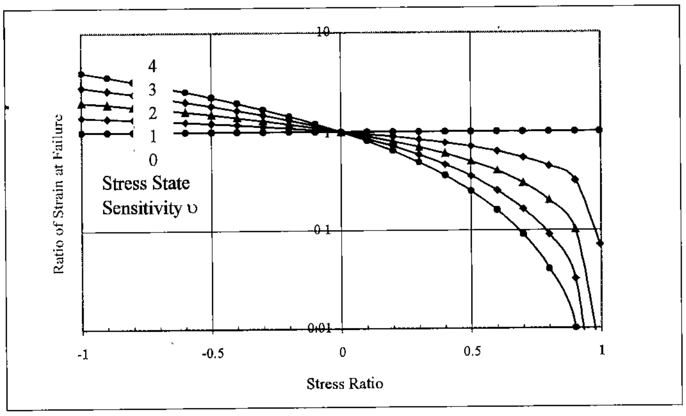
(b) Isochronous rupture loci under plane strain conditions

Fig. 2 Isochronous rupture loci for previous formulation. The normalized stress is used for each axis. The normalizing stress σ_0 is 60 MPa. The stress state sensitivity ranges from 1 to 4 with intervals of 1.

stress through integration (uni-axial case) ; 2) looping over the specified range of stress states (either plane stress or plane strain conditions) ; 3) finding the required stress level by iteration; 4) recording the stresses and strain components obtained. The graphic isochronous rupture loci were produced via Excel based upon the numerical results. The normalized stresses σ_1/σ_0 and σ_2/σ_0 are used as the two axes. The ratio of strain at failure is defined as the effective creep strain for the multi-axial stress state over the



(a) Ratios of strain at failure under plane stress conditions



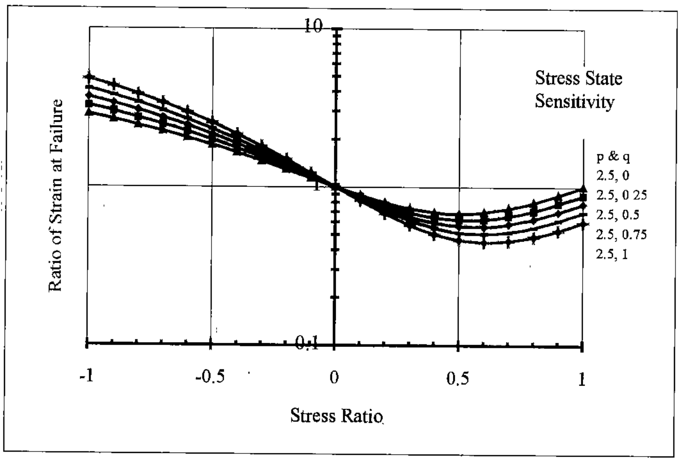
(b) Ratios of strain at failure under plane strain conditions

Fig. 3 Ratios of strain at failure for previous formulation. The stress state sensitivity ranges from 1 to 4 with intervals of 1. The normalizing stress σ_0 is 60 MPa.

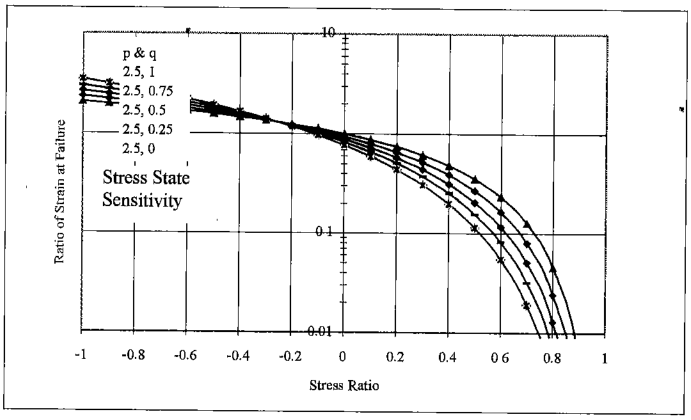
creep strain for uni-axial tension having the same rupture lifetime, while the stress ratio used for plane stress and plane strain conditions is defined as σ_1/σ_2.

The selected parametric characteristics of the ratios of effective strain at failure for the new formulation are illustrated in Fig. 4. The stress state sensitivity q ranges from 0 to 1 with an interval of 0.25 with $p=2.5$. Both the plane stress and plane strain conditions are considered. The characteristics for a wider range of p have been presented separately by Xu[(2)].

The two selected isochronous rupture loci are presented in Figs. 5 and 6. The normalized stresses σ_1/σ_0 and σ_2/σ_0 are used as the two axes. The parametric characteristics of ratios of strain at failure are shown in Fig. 4 for $p=2.5$. Figure 5 illustrates it's the rupture loci characteristics when constants a and b are zero. Figure 6 presents a better coupling of

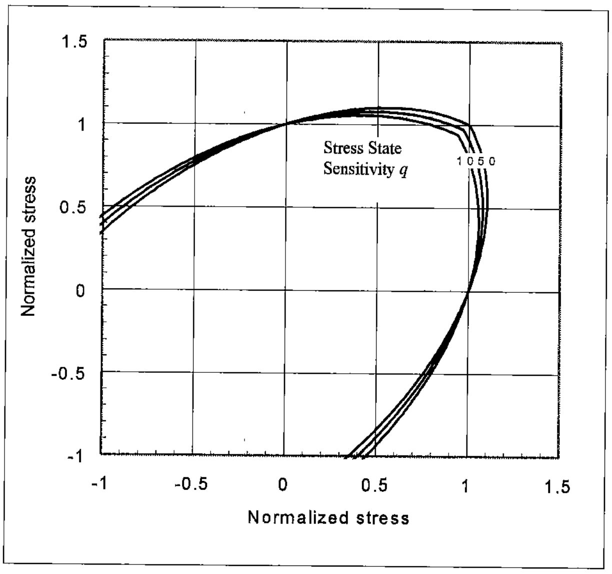


(a) Ratios of strain at failure under plane stress conditions

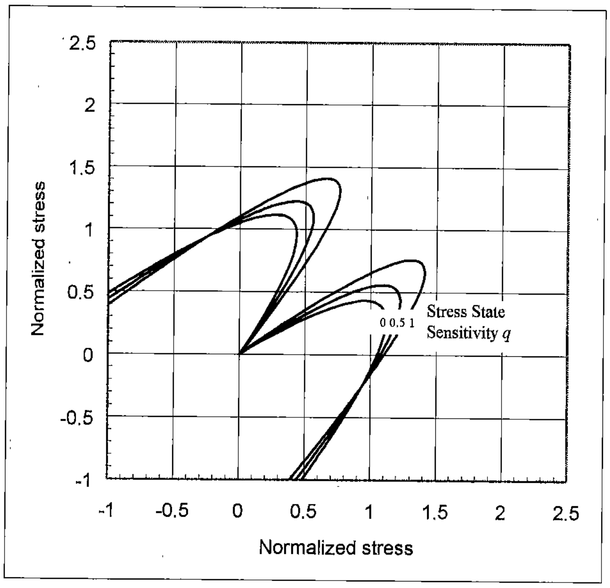


(b) Ratios of strain at failure under plane strain conditions

Fig. 4 Ratios of strain at failure for new formulation. The parameter q ranges from 0 to 1 with interval of 0.25 and $p=2.5$. The normalizing stress σ_0 is 60 MPa.



(a) Isochronous rupture loci under plane stress conditions



(b) Isochronous rupture loci under plane strain conditions

Fig. 5 Isochronous rupture loci for new formulation. The normalized stress is used for each axis. The normalizing stress σ_0 is 60 MPa. The stress state sensitivity q is 0, 0.5 and 1 while $p=2.5$, $a=0$ and $b=0$.

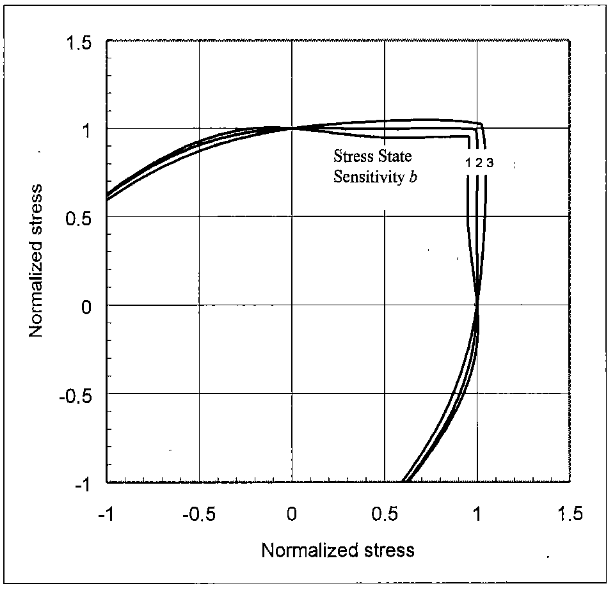
creep deformation, creep damage and lifetime. This coupling is required to be consistent with experimental observations.

4.3 Discussion

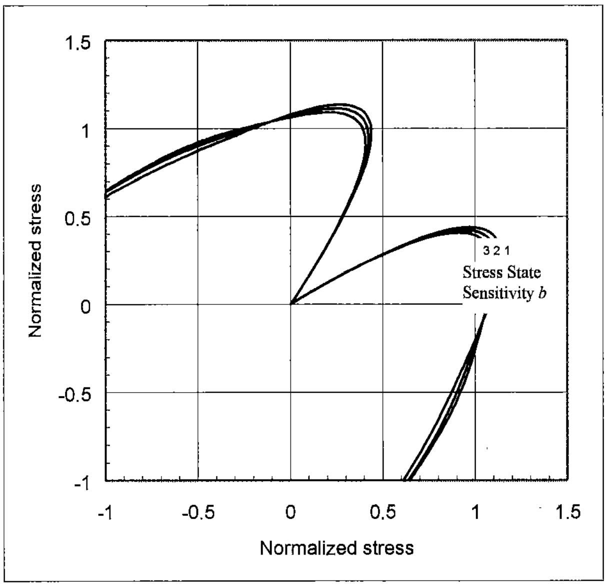
The following observations and comparisons can be drawn:

1) The isochronous loci for the plane stress condition shown in Fig. 2(a) indicate that the rupture lifetime for equal bi-axial tension is the same as that for uni-axial tension, which is not consistent with experimental observations. The creep rupture strength depicted tends to be higher than experimental observation when the stress state is approaching equal bi-axial tension. However, this is still a minor concern compared with the following observations.

2) A significant creep strength increase under plane strain conditions is shown in Fig. 2(b) when the tri-axiality is of the order of 1.5 to 2.8. This increase is not realistic according to well-known creep strength theory. Thus, this formulation is unable to find a value for the stress sensitivity ν that can match the isochronous rupture loci under plane stress and plane strain conditions simultaneously. This deficiency was not revealed in the previous constitutive equation development and would not be revealed by the validation practice reported by Perrin and Hayhurst[23]. The difficulty of determining the



(a) Isochronous rupture loci under plane stress conditions



(b) Isochronous rupture loci under plane strain conditions

Fig. 6 Isochronous rupture loci for new formulation. The normalized stress is used for each axis. The normalizing stress σ_0 is 60 MPa. The parameter b is 1, 2 and 3 while $a=2$ and $p=2.5$, $q=1$.

value for stress sensitivity needs to be re-examined.

3) Further more, the ratios of strain at failure for the previous formulation, shown in Fig. 3, are conjugate with the shape of the isochronous rupture loci shown in Fig. 2 through the common stress sensitivity parameter ν. Thus, there is no freedom provided to produce a strain at failure consistent with experimental observation. For example, the predicted ratio of strain at failure for equal bi-axial tension is equal to, rather than being smaller than, that for uni-axial tension. This further demonstrates the need for modification of previous formulation though it has been addressed previously by Xu[(1),(2)].

4) A wide range of ratios of strain at failure can be predicted by the new formulation through the stress state parameters p and q, although only one set of curves are presented here for brevity. It is important to realize that these ratios of strain at failure are independent of the shape of isochronous rupture loci. A set of values; $p=2.3$ and $q=0.8$ was found to be very close to the experimental data for plane stress conditions collected by Spindler[(42)]. Furthermore the explicit function of the ratio of strain at failure is an empirical one derived from experimental data so there is no question of inconsistency.

5) For a given ratio of strain at failure a wider rage of isochronous rupture loci can be produced, as shown in Figs. 5 and 6. There is a significant creep strength increase under plane strain conditions in Fig. 5, which indicates that the stress sensitivity parameters p and q alone, cannot predict the rupture lifetime correctly. Using the stress sensitivity parameters a and b, the isochronous rupture loci under plane stress conditions can be made to closely match experimental observations, as shown in Fig. 6. However, there are insufficient experimental results to validate the formulation under plane strain conditions. The coupling function has been designed with 2 stress state sensitivities a and b, to depict the effects of maximum principal stress and mean stress separately.

4.4 Overall conclusion

The following overall conclusions can be drawn:

1. The KRH formulation is not appropriately constructed being deficient on three counts: a) the predicted lifetime near the bi-axial tension region is longer than that observed experimentally[(27),(42),(46),(47)]; b) it is impossible to obtain a value for stress state sensitivity ν for both plane stress and plane strain conditions as an unrealistic increase of creep strength is predicted under plane strain conditions; c) the predicted ratio of strain at failure is conjugate with the isochronous rupture loci which is not consistent with experimental observations[(41)];

2. The new formulation attempts to address the deficiencies of the previous formulation. It introduces two separate functions to depict the complex influence of stress states on creep strain and damage evolution. This results in a modified damage evolution equation and a different coupling between damage and deformation.

3. The validation conducted (Steps 1 and 2 under plane stress and plane strain conditions) shows that this new set of constitutive equations is capable of

producing a wide range of ratios of strain at failure and rupture lifetimes that should be consistent with experimental observation. The fundamental reasons for this are: 1) The ratio of strain at failure is automatically consistent with experimental observations as the specific function has been derived from them; 2) Creep strength theory has been used in the derivation of the implicit equations for the isochronous rupture loci.

4. The validation of the creep curves (more accurately the strain tensors) and the damage evolution under steps 3 and 4 has been addressed but needs to be pursued further.

5. Conclusions

The development of a new set of multi-axial creep damage constitutive equations for 0.5Cr0.5Mo 0.25V ferritic steel at 590°C has been presented covering a critical review of fundamental requirement, the formulation, the specific equations, and validation of the ratio of strain at failure and rupture lifetime under proportional loading conditions. It also contains comparison with the previous formulation whenever applicable. It shows that this new set of constitutive equations is better constructed and is able to depict creep rupture and strain at failure consistent with experimental observations. It is planned to further investigate the creep strain prior to failure and to validate the formulation under non-proportional loading conditions, which will be reported in due course.

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