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Inverse problems of measurement with application on specification of surface profile

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Introduction:
A contradiction of the specification of free-form surface is pointed out. The inverse problem of measurement (IPM) is defined based on the representational measurement theory. By using the concept of IPM, a desired property of specification limit is derived and a correction for solving the contradiction is proposed.

Specification and measurement of surface profile
The upper and lower specification limits (LSL and LSL) of a free-form surface profile defined in ISO 1101 are two curves enclosing circles of certain diameter r, the centers of which are situated on the nominal surface profile (see figure 2a). For an actual surface profile l, if all the points are within the tolerance zone, i.e., LSL < l < LSL, l is within the spec.

The canonical method of measuring surface profile is contact measurement by moving a tactile stylus along the surface to be measured to obtain the locus of the centre point of the stylus tip.

Figure 1. working principle of measuring surface profile with a tactile stylus

A Contradiction of the Specification of Free-form Surface
The contradiction
Due to the extensive property of closing filter, the estimated profile is always above the actual profile (see figure 1). Hence when an actual surface profile coincides with the LSL (flat within spec.), the measurement result (without error) would, however, be out of spec., which contradicts with the real situation.

Figure 2. A correction of the tolerance zone of surface profile

Invers problems of measurement

Inverse problem is a general framework of problems that infer information from observations (Kalantari, 2009).

In many cases, the measurements are not directly observable, they can only be inferred from the observed data of some related proxy quantities.

Definition: Inversion of the measurements from the observed data is the inverse problem of measurement (IPM). Inversor mapping is the characteristic function of the measurement process, i.e., \[ X \sim D \rightarrow X \sim \text{D} \]

The inverse or pseudo-inverse of \( h \), denoted as \( D \sim X \sim D \), can be used to find the inverse solution \( h \). It is expected to satisfy the following equation:

\[ h \sim h \]

For any IPM, \( X \sim D \) and \( D \sim X \) are always determined by an ensemble ERS.

Representational model of measurement

Representational measurement theory allows measurement to be defined as the assignment of numbers to attributes of objects in such a way as to describe them (Fechner, 1902). Here a measurement can be considered as a mapping from the measured objects to the measured values.

For the measurement of a attribute, one or more empirical relations would be defined between the measured objects. E.g., pressure \( \sim P \) is a very general empirical relation.

The set of the measured objects with the empirical relations, \( R \sim R \rightarrow R \), can be taken as a mathematical object \( \text{Meas} = (M, R_1, R_2, \ldots, R_n) \), called an empirical relational system (ERS). E.g., the ERS of length (or mass, force) is \( (M, \sim \leq \leq \sim) \), where \( \sim \leq \leq \) is a preorder, \( \sim \) is an association operation.

Measurability: A measurement is possible only if there exists a structure-preserving mapping from the ERS to a specified numerical relational system (NRS). E.g., the NRS representing the length is \( (R_1, \sim \leq \leq \sim) \). The numbers in the NRS are the values of the measured quantity to be measured.

The principle of correcting the contradiction

To estimate the surface profile according to the observed data to an inverse problem, \( D \sim X \) is the forward mapping and its pseudo-inverse is \( D \sim X \), in the sense that \( D \sim X \sim D \sim X \).

Essential reasons of the contradiction:
- The forward mapping \( D \) is not one-one.
- The inverse solution \( h \) is a maximal point of the possible input, i.e., \( l \sim E, D(l) = \ell \).

The spec. limits should also reflect the required measurement resolution, e.g., \( 3.00 \pm 0.1 \) mm. So the spec. limits given in ISO 1101 should be amended.

We expect that if the true value of a measured object is within spec., its measured value is also within spec. Hence the following desired property of spec. limits should be satisfied:
Let \( h \) be spec. limits, \( \exists h \) such that:

The essential reasons of the spec. limit

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