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Inverse problems of measurement
with application on specification of surface profile

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Introduction:
A contraction of the specification of free-form surface profile is pointed out. The inverse problem of measurement (IPM) is defined based on the representational measurement theory. For the use of IPM, a desired property of specification limits is derived and a correction for the contraction is proposed.

Specification and measurement of surface profile
The upper and lower specification limits (LSL and ULSL) of a free-form surface profile defined in ISO 1101 are two curves enclosing circles of certain diameter ρ, the centers of which are situated on the nominal surface profile (see figure 2a). For an actual surface profile, if all the points on the tolerance zone, i.e. LSL ≤ y ≤ ULSL, 1 is within the space.

The classical method of measuring surface profile is contact measurement by moving a tactile stylus along the surface to be measured to obtain the locus of the center point of the stylus tip.

Figure 1. Working principle of measuring surface profile with a tactile stylus With S as the stylus element, the locus c is the division of 1, and l can be estimated by the equation c = E₁D₁E₂(l). The combination of D₁ followed by E₂ is a closing filter C₁ = D₁E₂ E₁.

The contraction of the specification of free-form surface profile

Due to the extensive property of closing filters, the estimated profile is always above the actual profile (see Figure 1). Hence when an actual surface profile coincides with the LSL limits within space, the measurement result (without errors) would, however, be out of spec., which contradicts with the real situation.

Inverse problems of measurement

Inverse problem is a general framework of problems that infer information from observations (Saslow, 2009). In many cases, the measurements are not directly observable, they only can be inferred from the observed data of some related proxy quantities.

Definition: Inferring the values of the measurements from the observed data is the inverse problem of measurement (IPM). In forward mapping is the characteristic function of the measurement process, i.e. X → D. The inverse or pseudo-inverse of D, denoted as g: D → X, can be used to find the inverse solution; i.e. X is expected to satisfy the following equation:

\[ h = g \] is the pseudo-inverse of the measurement function. For any IPM, X and D are always determined by an ensemble ERS.

The essential reasons of the contraction

- To estimate the surface profile according to the observed locus in an inverse problem, D, is the forward mapping and its pseudo-inverse is E₁, in the sense that D₁E₁Dₙ = D₁. Essential reasons of the contraction:
  - the forward mapping D is not one-to-one;
  - the inverse solution X is a maximal point of the possible input, i.e. l ≥ E₁D₁(l) = F;

- The spec. limits should reflect the required measurement resolution, e.g. 3.000 ± 0.10mm. So the spec. limits given in ISO1101 should be amended.

- We expect that if the true value of a measured object is within spec., its measured value is also within spec. Hence the following desired property of spec. limits should be satisfied:
  Let a be limit, u < a, then g(a) ≤ u.

A proposed solution

- Correcting the curve of LSL from \( l_s \) to \( l_s' \) (see Figure 2b), \( l_s' = C_s(l_s) \) (see Figure 2b), \( C_s \) is the closing filter with the structuring element S. The diameter of the stylus S is assumed to be smaller than the dimension \( l_s' \). It can be proved with the invariant property of a closing filter that the desired property is satisfied after the correction.

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Figure 2. A correction of the tolerance zone of surface profile