Inverse problems of measurement with application on specification of surface profile

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Introduction:
A contradiction of the specification of free-form surface is pointed out. The inverse problem of measurement (IPM) is defined based on the representational measurement theory. By using the concept of IPM, a desired property of specification limit is derived and a contradiction for solving the contradiction is proposed.

Specimen and measurement of surface profile
The upper and lower specification limits (LSL and LSL) of a free-form surface profile defined in ISO 1101 are two curves enclosing circles of certain diameter r, the centres of which are situated on the nominal surface profile (see figure 3a). For an actual surface profile, if all the points on it are within the tolerance zone, i.e. LSL ≤ l ≤ LSL, it is within the spec.

The traditional method of measuring surface profile is contact measurement by moving a tactile stylus along the surface to be measured to obtain the locus of the centre point of the stylus tip.

Figure 1. Working principle of measuring surface profile with a tactile stylus
With S as the stylus, the locus s is the division of l, and l can be estimated by the division of (s - 1). The combination of D follows by D2 is a closing filter, C2 = D2 E2.

In the case of measuring, the accuracy of l is determined by the precision of the measuring device, the quality of l, and the quality of D. The quality of l is determined by the accuracy of the division of S, and the quality of D is determined by the precision of D2.

Figure 2. Accommodation of the tolerance zone of surface profile
A correction of the specification of free-form surface

The contradiction
Due to the extensive property of closing filters, the estimated profile is always above the actual profile (see figure 1). Hence, when an actual surface profile coincides with the LSL (fig. 3a), the measurement result (without error) would, however, be out of spec., which contradicts with the real situation.

Figure 3. The inverse problem of contact surface measurement

Representational model of measurement
Representational measurement theory allows measurement to be defined as the assignment of numbers to attributes of objects in such a way as to describe them (Fiehler, 1992). Hence, measurement can be considered as mapping from the measured objects to the measured values.

For the measurement of a feature, one or more empirical relations would be defined between the measured objects. E.g. source l ≤ LSL, 1 is within the spec.

The empirical relation between the measured objects and the empirical relation is called an empirical relational system (ERS). E.g. ERS: ERS = (M, R1, R2, ... , Rn), where M is a source, i.e. l ≤ LSL, 1 is within the spec.

Measurement: a measurement is possible only if there exists a structure-preserving mapping from the ERS to a specified numerical relational system (NRS). E.g. the NRS representing the length is (R1, S1). The numbers in the NRS are the values of the measured quantity to be measured.

Figure 4. A correction of the specification of free-form surface

Inverse problems of measurement
Inverse problem is a general framework of problems that infer information from observations (Sabbas, 2009).

In many cases, the measurement is not directly observable, they can only be inferred from the observed data of some related proxy quantities.

Definition: inferring the values of the measurements from the observed data is the inverse problem of measurement (IPM). An inverse mapping is the characteristic function of the measurement process, i.e. D = X.

X : Measured data
D : Estimated values of measurement
h : Inverse of pseudo-inverse of h, denoted as h = D X, can be used to find the inverse solution, i.e. X = D h.

For any IPM, X and D are always determined by an ensemble ERS.

Principle of correcting the contradiction

To estimate the surface profile according to the observed data in an inverse problem, D2 is the forward mapping and its pseudo-inverse is D2, i.e. D2 E2 D2 = D2.

Essential reasons of the contradiction:

1. The forward mapping D2 is not one-one function.
2. The inverse solution X is a point of the possible input, i.e. l ≤ LSL, 1 is within the spec.

We expect that if the true value of a measured object is within spec., its measured value is also within spec. Hence the following desired property of spec. limits should be satisfied:

Let a be a spec. limit, and then.

A proposed correction

- Correcting the curve of LSL from l to l = C2(l) (see figure 2b), where C2 is the closing filter with the structuring element S.
- The diameter of the stylus S is assumed to be smaller than the dimension r. It can be proved that the unknown property of a closing filter that the desired property is satisfied after the correction.

Acknowledgement: The authors gratefully acknowledge the UK’s Engineering and Physical Sciences Research Council (EPSRC) funding of the EPSRC Centre for Innovative Manufacturing in Advanced Metrology (Grant Ref. EP/J036243/1).

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