DIVERSITY METRIC ASSESSMENT FOR MULTI-OBJECTIVE OPTIMIZATION

H. E. Radhi, and S. Barrans

University of Huddersfield, Queensgate, Huddersfield HD1 3DH, UK

ABSTRACT

In multi-objective optimization evolutionary algorithms (MOEA) diversity is an important issue and many researchers have investigated this notion. Due to the importance of multi-objective optimization in industry and engineering, it is essential to find a diverse set of Pareto optimal solutions which covers as much space as possible in the feasible region of the solution space. Theresearch (2012) presented in this paper aims to examine the performance and compare two metrics for diversity assessment of the non-dominated solution (i.e. Pareto set) in the solution space. These two metrics have been programmed in (MATLAB) software and implemented in number of simple bi-objective optimization cases to compare the analytical results with the visual distribution of points. The experimental results show that the circumscription diversity (CM) metric outperforms the Pair Wise (PW) metric

Keywords: Multi-objective optimization, Diversity metrics.

MULTI-OBJECTIVE OPTIMIZATION

In contrast with single optimal solution cases, in multi-objective problems it is not possible to have single solution that optimizes all objectives. Instead there usually exists a set of non- dominated solutions or Pareto optimal solutions. The mathematical multi-objective optimization statement can be defined as follows Deb (2001).

Minimize / Maximize

$$f(x) = [f_1(x), f_2(x), \dots\dots.., f_N(x)]^{T} \tag{1}$$

Subject to

$$g(x) \leq 0 h(x) = 0$$

$$x_{i,L} \leq x_i \leq x_{i,H} \qquad i = 1,2, \dots\dots, n$$

Where f(x) is the objective function, *N* the total number of objective functions, while *g* and *h* are vectors of inequality and equality constraint respectively and *x* the set of design variables Fig. 1 shows the solutions of a multi-objective optimization problem. The dotted line represents the Pareto optimal solutions which are not dominated by any other solutions, since no other solutions in the set are equal or better for both objective functions.

Note that solution 1 has a small value of f_1 but a large value of f_2. Solution 5 has large value of f_1 but small value of f_2; one cannot decide that solution 1 is better than solution 5, or vice-versa, if the goal is to minimize both objective functions. It is evident solution 6 is not a good solution; since it is dominated by solution 5.Deb (2001) used the term 'domination' and 'non-domination' to describe the Pareto front. The solution *x1* dominates a solution *x2* if and only if:

- The solution *x1* is not worse than *x2* in any of the objectives.
- The solution *x1* is strictly better than solution *x2* in one object at least.

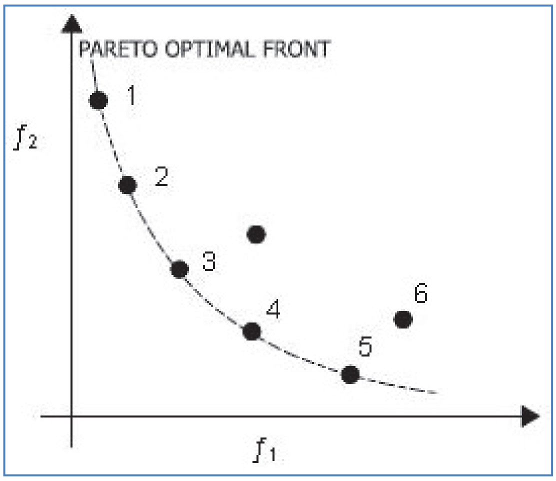


Figure 1: Main concept of Pareto dominance in a two objective problem

UNIFORMITY PERFORMANCE METRICS

The concept of a performance metric is normally used to form an understanding of which algorithm is the better and in which aspect. One very important aspect is the uniformity of the distribution of points within the Pareto set. To obtain the maximum information about the whole Pareto surface at minimum computational cost, the sample points must be uniformly distributed. Where points on the Pareto surface are generated in the objective space using multi-objective evolutionary algorithms, they will often be clustered. Accounting for clustering in the uniformity metric is therefore important.

1. **Pair Wise Metric (PW).**

In this study the Pair Wise criterion defined by Gunzburer & Burkdart (2004),has been used. For a set of N points $\{z_i\}_{i=1}^{N}$the minimum distance between the point z_i and any other points is

$$\gamma_i = \min_{j \neq i} \left| z_i - z_j \right| \tag{2}$$

The Pair Wise (PW) metric can be rewritten:

$$PW = \frac{1}{\overline{\gamma}} \left[\frac{1}{n} \sum_{i=1}^{n} (\gamma_i - \overline{\gamma})^2 \right]^{\frac{1}{2}} \tag{3}$$

Where

$$\overline{\gamma} = \frac{1}{N} \sum_{i=1}^{N} \gamma_i \tag{4}$$

For a perfectly uniform distribution of points $\gamma_1=\gamma_2=\ldots\ldots=\gamma_n=\overline{\gamma}$ so the PW=0.Small values of PW means that the result is close to uniformly distributed.

1. **Circumscription Metric (CM)**

To address the limitations of pair wise distance based diversity metrics to be illustrated here, Tahernezhadiani, K., (2012) proposed a diversity indicator based on the number of unique circles that can be defined circumscribing at least 2 points in the Pareto set and the radii of those circles. This metric used a monotonic logarithmic function:

$$circumscription\ metric = \frac{\log((1+100)*(1+C+\sqrt{R}))}{\log(1+100)} \tag{5}$$

Where C is the number of generated circles and R is the sum of the circle radii

EXPERIMENTAL RESULTS

Five different cases and visualization of point distribution are used to compare the performance of circumscription diversity metric (CM) and pair wise metric (PW), these cases are the following:

- *Case 1* uniform distribution of points Figure 2.
- *Case 2* modifying the uniformly distributed points by moving the red points from (x=4.77, y=4.77) to (x=4.5, y=4.5) Figure 3.
- *Case 3* grouping the points in 6 clusters Figure 4.
- *Case 4* uniform distribution in two dimensions Figure 5.
- *Case 5* grouping the points in 4 clusters Figure 6.

In each figure, the diversity value according to the two metrics is indicated. The comparison between case-1 and case-2 indicates that the two metrics give the same indication that the case-1 is better than case-2 with highest CM and lowest value of PW metric. However, the comparison between case-2 and case-3 shows that case-2 is the best regarding CM metric and is the worst regarding the PW metric, by comparing this results with graphical distribution of the points it is clear to notice the CM more realistic than PW and this latter metric fails to assess the uniformity distribution especially when there is clustering in points.
Another comparison is done between case-4 and case-5 which indicates that the case-4 is more uniform than case-5 regarding the CM metric, whilst the PW gives different meaning that the case-5 is ideal case with value of PW=0. The graphical distribution shows a different scenario is that the case-4 is more uniformly distributed than case-5. In conclusion the CM metric has better ability than PW metric to indicate the distribution of points especially then there is a clustering in the points.

APPLICATION TO CYLINDER OPTIMIZATION

In the following section the comparison amongst these metrics is performed in bi-objective optimization problem this problem represents optimization of cylinder; there are two geometrical variables that can change: the radius and the height of the cylinder. The objectives of optimization are to minimize the surface of the cylinder and the volume; the parameters of the optimization problem are presented in Table 1.

Table 1: Optimization parameters of cylinder problem

Input Variables	limit	Objective functions	Initial population	No. of Generations	Optimization algorithms
Radius	0.5-1.5	To minimize the surface and volume	50 Sobol	10	FMOGA-II, NSGA-II, MOGA-II
Height	0.5-5				

Three different algorithms have used during this simulation : Multiobjective Genetic Algorithm (MOGA-II) Silvia (2003), , Fast Multiobjective Genetic Algorithm (FMOGA-II), Non-dominated Sorting Genetic Algorithm (NSGA-II) Deb at al.(2000), the results of optimization which indicate the values of CM and PW metric are presented in Table 2.
As mentioned before the highest value of CM and lowest value of PW metric means better uniform distribution of points in design space, as can be seen in table 2 the two metrics indicate that the FMOGA-II has the more uniform distribution than the others algorithms whilst the diversity metrics regarding NSGA-II and MOGA-II gives different scenario that is the NSGA-II is more uniform than MOGA-II regarding CM metric whereas the MOGA-II is more uniform than NSGA-II, for the second time the PW metric shows the incorrect value of diversity measure.
The Pareto front distribution of cylinder problem obtained by FMOGA-II, NSGA-II and MOGA-II are presented in Figure (7, 8 and 9).

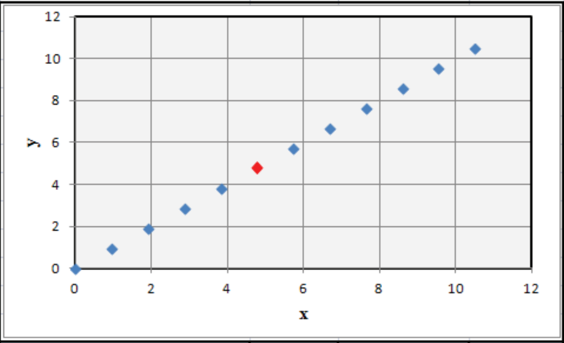
Table 2: Diversity metrics value for cylinder problem

Optimization Algorithm	CM diversity metric	PW diversity metric
FMOGA-II	1.7043	0.6991
NSGA-II	1.6863	3.2318
MOGA-II	1.6633	1.525

CONCLUSIONS

The analysis of a number of synthetic problems, in both one dimension and two dimensions has shown that the circumscription metric gives a more reliable indication of the uniformity of point distribution than the more commonly used pair wise metric. This is particularly the case where points are clustered in space.
The two diversity metrics were applied to the results of an optimization problem analysed using three different MOEA's. Again, the circumscription metric was shown to outperform the pair-wise metric.

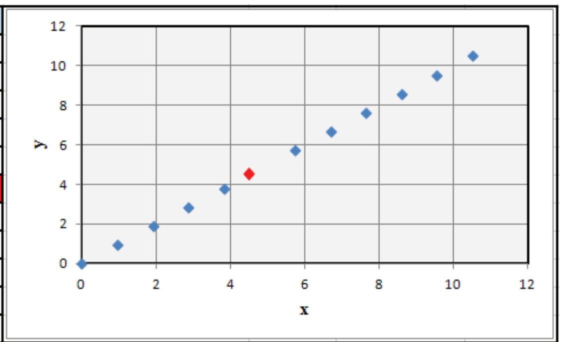
x	y
0	0
0.95455	0.95455
1.90909	1.90909
2.86364	2.86364
3.81818	3.81818
4.77273	4.77273
5.72727	5.72727
6.68182	6.68182
7.63636	7.63636
8.59091	8.59091
9.54545	9.54545
10.5	10.5



CD metric	1.4892	COV metric	0

Figure 2: Regular distributions of points in 1D

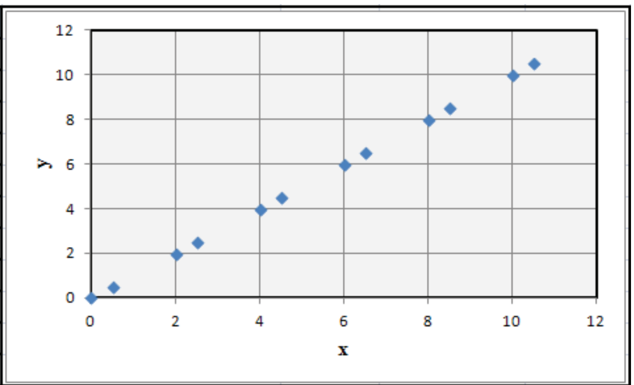
x	y
0	0
0.95455	0.95455
1.90909	1.90909
2.86364	2.86364
3.81818	3.81818
4.5	4.5
5.72727	5.72727
6.68182	6.68182
7.63636	7.63636
8.59091	8.59091
9.54545	9.54545
10.5	10.5



CD metric	1.4887	COV metric	0.111808

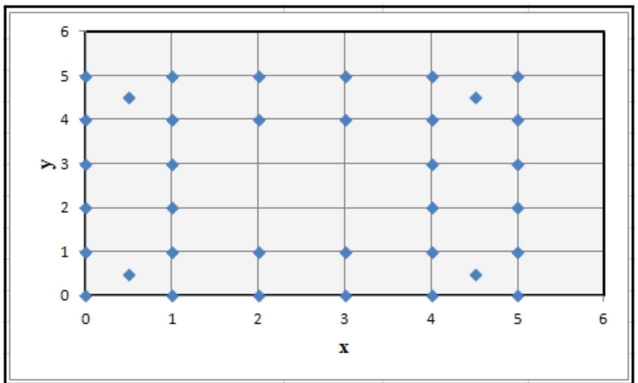
Figure 3: Modified distribution of points in 1D

x	y
0	0
0.5	0.5
2	2
2.5	2.5
4	4
4.5	4.5
6	6
6.5	6.5
8	8
8.5	8.5
10	10
10.5	10.5



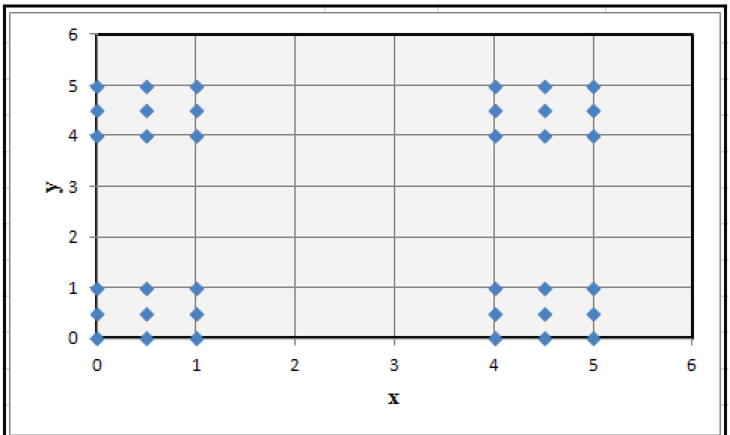
CD metric	1.4883	COV metric	3.14E-16

Figure 4: Cluster distribution of points in 1D



CD metric	1.5641	COV metric	0.173829

Figure 5: Uniform distribution in two dimensions.



CD metric	1.5235	COV metric	0

Figure 6: Cluster distribution of points in 2D

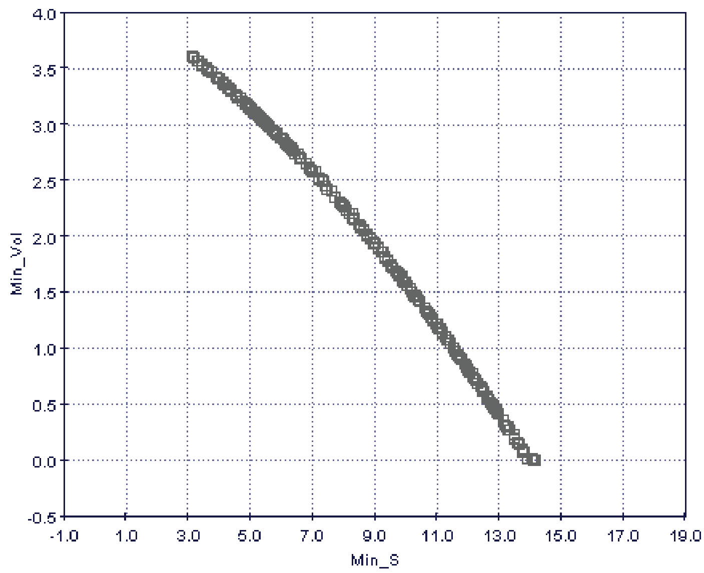


Figure 7: Pareto front distribution of cylinder problem obtained by FMOGA-II algorithm

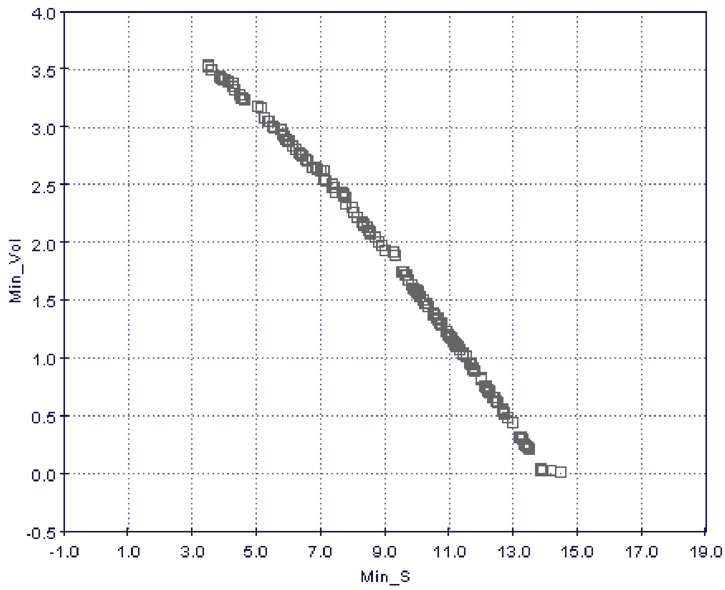


Figure 8: Pareto front distribution of cylinder problem obtained by NSGA-II algorithm

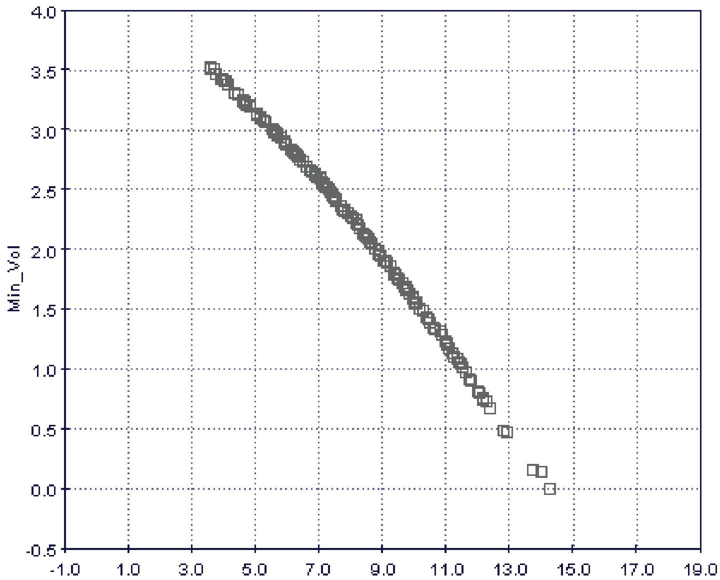


Figure 9: Pareto front distribution of cylinder problem obtained by MOGA-II algorithm

REFERENCES

Deb, K. (Ed.). (2001). *Multi-objective Optimization using Evolutionary Algorithms*. John Wiley and Sons.

Deb, K., Agarwal, S., Partap, A., &Meyarivan, T. (2000).*A Fast Elitist Non Dominated Sorting Genetic Algorithm for Multi-Objective Optimization NSGA-II:* Indian Institute of Technology.

Gunzburer, M., &Burkdart, J. (2004). Uniform measures for point samples in hypercube. fromhttp://people.sc.fsu.edu/~burkardt/pdf/ptmeas.pdf

Tahernezhadiani, K., Hamzeh, A., Hashemi, S., (2012) *Toward Enhancing Solution Space Diversity in Multi-objective Optimization: A hyrevolume-based Approach*. International Journal of Artificial Intelligence & Applications, Vol. 3, No. 1

Silvia, P. (2003). *MOGA-II An improved Multi-Objective Genetic Algorithm*: ESTECO Technical Report.