THEORETICAL CONCEPTS AND MATLAB MODELLING OF VLC BASED MIMO SYSTEMS
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ABSTRACT
This paper presents the theoretical concepts and techniques involved in developing MATLAB models to simulate the behaviour of a short-range 4x4 multiple-in, multiple-out (MIMO) visible light communication (VLC) system. Within the scope of this paper, the material presented is contained in four sections: the first section provides an introduction to light emitting diode (LED) based VLC systems; the second describes the theoretical concepts and MATLAB modelling of these systems, specifically transmission, channel, receiver and channel estimation modelling; the third presents and describes a unidirectional 4x4 MIMO VLC system model; and finally, the fourth presents conclusions and further work.

Keywords light emitting diode (LED), multiple-in–multiple-out (MIMO), solid-state lighting (SSL), visible light communication (VLC)

1 INTRODUCTION
In recent years there has been an increase in the use of solid-state lighting (SSL) for domestic and industrial lighting applications. SSL is gaining momentum as replacement lighting for standard incandescent and fluorescent lighting technologies. This is due to the fact that SSL uses LED technology which has significantly higher efficiency, and longer life expectancy than traditional technologies (Table 1).

<table>
<thead>
<tr>
<th>Type</th>
<th>Light Output (% of total power consumption)</th>
<th>Lifespan (h)</th>
<th>Cost (60W bulb equivalent comparison)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incandescent</td>
<td>5%</td>
<td>~1000</td>
<td>~$1.00</td>
</tr>
<tr>
<td>Compact Fluorescent Lamps (CFL)</td>
<td>10%</td>
<td>~10,000</td>
<td>~$5.00</td>
</tr>
<tr>
<td>Solid-state LED (VLC)</td>
<td>50%</td>
<td>~50,000</td>
<td>~$35.00</td>
</tr>
</tbody>
</table>

Table 1 Lighting type comparison [1, 2]
In addition to these qualities, LEDs possess wide bandwidths enabling them to be switched at very high frequencies. This property enables the LED to be used as a free-space optical data transmitter. Unifying the lighting and high speed switching properties of the LED, gives rise to a dual purpose lighting and VLC system.

The LEDs considered in this paper emit visible light at wavelengths extending from 375 nm to 780 nm. Free-space optical systems operating over this range are referred to as VLC systems [3]. The bandwidth of these systems is only limited by the optical transducers i.e. the transmitting LED and the receiving photodiode (PD), whereas equivalent radio frequency (RF) based systems, such as Bluetooth and Wi-Fi, are largely constrained by spectrum allocation. VLC offers potentially higher data rates, over an unrestricted region of the electromagnetic spectrum, when compared to RF based systems (Table 2). VLC is ideally suited to indoor short-range applications, such as computer networking, control and audio/video streaming systems.

<table>
<thead>
<tr>
<th>System</th>
<th>Channel Medium</th>
<th>Data Rate (bits per second)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bluetooth</td>
<td>RF wireless</td>
<td>10x10^6</td>
</tr>
<tr>
<td>WiFi (IEEE 802.11a/b/g/n)</td>
<td>RF wireless</td>
<td>100x10^6</td>
</tr>
<tr>
<td>VLC</td>
<td>Optical wireless</td>
<td>&gt;100x10^6</td>
</tr>
</tbody>
</table>

Table 2 Comparison of data rates of short-range RF systems versus a VLC system
In order to successfully model a MIMO VLC system, a number of theoretical concepts need to be understood: free-space light propagation; transmitter definition; channel definition; intensity modulation (IM) schemes; error correction algorithms; receiver definition; MIMO channel estimation; characteristics of optical transducers: LED and photodiode (PD), and analogue circuits. The following sections present the theoretical concepts needed to develop models for a short-range 4x4 MIMO VLC system.
2 THEORY AND MODELLING OF INDOOR OPTICAL WIRELESS LINKS

Transmitter Definition: In order to design and develop systems using LEDs as an optical data transmitter, it is necessary to model the radiant intensity of the LEDs light emission at any point within 3D space. Previous research has shown that the light emission of an LED is Lambertian in nature [4].

Lambertian radiant intensity is defined by,

\[ R_o(\phi) = \left[ \frac{m + 1}{2\pi} \right] \cos^m(\phi) \]  

(1)

Where \( m \) is the order of the Lambertian emission, and is related to the semi-angle (half power) \( \phi_{1/2} \) of the LED emission,

\[ m = -\frac{\ln 2}{\ln \left( \cos \left( \frac{\phi_{1/2}}{2} \right) \right)} \]  

(2)

Assuming the semi-angle \( \phi_{1/2} \) is 60° (or \( \pi/180 \times 60 \) = 1.047197551 radians), applying this to equation (2) yields \( m = 1 \), substituting this into equation (1) yields,

\[ R_o(\phi) = \left[ \frac{1}{2\pi} \right] \cos(\phi) \]  

(3)

Using equation (3), it is possible to determine the radiant intensity of the LED at any point within a 3D space using a Cartesian coordinate (i, j, k notation) system and trigonometry.

Figure 1 LED and PD located within a Cartesian coordinate system

In Figure 1, an LED transmitter and PD receiver are located within a 3D space defined by a length and width of 8m, and a height of 2m. The LED is positioned at \( i_{LED} = 4m, j_{LED} = 4m, k_{LED} = 2m \) and the PD at \( i_{PD} = 4.5m, j_{PD} = 4.5m, k_{PD} = 1m \), relative to the datum. Normalising all the coordinates to the LEDs position, results in \( i_{LED} = 0m, j_{LED} = 0m, k_{LED} = 0m \) and the PD at \( i_{PD} = 0.5m, j_{PD} = 0.5m, k_{PD} = 1m \). The distance and angle of the PD relative to the LED is calculated using equations [4] and [5].

Note that the LED and PD are treated as infinitesimally small source and sink points respectively.

\[ LEDPD = \sqrt{a_h^2 + b_h^2 + d_v^2} \]

(4)

Where \( LEDPD \) is the relative distance between the PD and the LED, and \( a_h \) and \( b_h \) are the normalized horizontal \( i \) and \( j \) coordinates, and \( d_v \) is the normalized vertical \( k \) coordinate.

\[ \cos(\phi) = \frac{d_v}{LEDPD} \]  

(5)
The power received at the PD, $P_{PD}$ (Watt), is computed by multiplying the total power emission of the LED, $P_{LED}$, by equation (3),

$$P_{PD} = P_{LED} \times R_0(\theta) [W]$$

(6)

In addition, it is possible to compute the time delay, $\tau_{LEDPD}$ (seconds), of the transmitted light from the LED to the PD ($c = \text{speed of light} \sim 3 \times 10^8 \text{ m/s}$),

$$\tau_{LEDPD} = \frac{LEDPD}{c} [s]$$

(7)

A MATLAB simulation, based on equations (3, 4, 5) for a single LED, is presented in Figure 2. The radiant intensity at any given set of i, j and k coordinates is represented using colour coding. The colour bar shows areas of highest intensity in dark red, and areas of low intensity in dark blue. Note 1: all surfaces bounding the 3D space are considered non-reflective in this simulation. Note 2: the power, $P_{LED}$, emitted by the LED is unity.

![Figure 2 4D Plot of a single LEDs Lambertian intensity within a 3D Space](image)

Analysis of Figure 2 shows that the highest radiant intensity is located directly under the LED, when the angle between the LED and PD is zero. This results in the highest intensity of $1/\pi$ ($\sim 0.32$) as described by equation (3).

The single LED transmitter model was expanded to a 2x2 LED transmitter array. The LED array was positioned in the same 3D space (8mx8m2m) as the single LED example, with the LED coordinates as shown in Figure 3.

![Figure 3 2 x 2 LED Array (plan view)](image)

The resultant simulation of the 2x2 LED array is shown in Figure 4.
Analysis of Figure 4 shows that the highest radiant intensity is once again located directly under the LEDs, when the angle between the LED and PD is zero. However, in the case of multiple LEDs, the highest intensity achievable is four times that of the single LED simulation. This is because the individual LED intensities are summed resulting in $4 \times 1/\pi \approx 1.28$. This is illustrated by the colour bar scale to the right of Figure 4.

**Channel Definition:** The output signal $Y(t)$ as a function of the input signal $X(t)$ of a single channel VLC system can be modelled using the transfer function shown in equation (7). The symbol $\otimes$ denotes convolution, where $X(t)$ is convolved with $Rh(t)$. $X(t)$ is the intensity modulating signal, $R$ is the responsivity of receiving PD, $h(t)$ is the optical impulse response of the LED, and $N(t)$ the additive noise introduced by the system [5].

$$Y(t) = X(t) \otimes Rh(t) + N(t)$$ (7)

Responsivity is the measure of a PDs sensitivity to light, and has the units Amperes per Watt (AW).

$$R_\lambda = \frac{I_p}{P} \text{ [A/W]}$$ (8)

Where, $I_p$ is the photocurrent flowing through the PD, and $P$ is the light power incident on the PD.

Suitable candidates for the IM signal, $X(t)$, are pulse position modulation (PPM) or on-off keying schemes, such as decode PPM (di-PPM), offset PPM, non-return-to-zero on–off keying (NRZ-OOK), and duo binary PPM. In addition, the IM signal can be coded using suitable error correction algorithms, such as maximum likelihood sequence estimation (MLSE) or Reed-Solomon (RS), which can significantly improve bit error rate (BER) performance.

In the line-of-sight (LOS) channel, the DC gain is estimated by only considering the LOS propagation. In other words the LEDs have a direct LOS to the receivers PDs, and no reflections from walls, ceiling, floor or other surfaces are present at the receiver PDs. In this condition the channel transfer function is,

$$H(0)_{LOS} = \begin{cases} \frac{A_{rx}}{LEDPD^2} R_0(\theta) \cos(\psi) & \text{for } 0 \leq \psi \leq \psi_c \\ 0 & \text{for } 0 \leq \psi \leq \psi_c \end{cases}$$ (9)

Where $A_{rx}$ is defined as the PD area, $LEDPD$ is the distance between the LED transmitter and the PD receiver, $R_0(\theta)$ is the LED radiant intensity, $\psi$ is the angle of incidence, and $\psi_c$ is the field of view (FOV) of the PD [4, 6].

**Receiver Definition:** Combining the transmitter and channel definitions yields the receiver output,
\[ Y(t) = R \cdot P_{LED} \cdot H(0)_{LOS} \cdot [X(t) \otimes h(t)] + N(t) \]  

\[ (10) \]

**Channel estimation**: Where multiple transmitters and receivers are employed, MIMO techniques can be exploited to increase the data rate/throughput of the system. MIMO permits simultaneous parallel transmission of multiple data streams, which are transmitted separately by multiple transmitters and then similarly received by multiple receivers. In this configuration, multiple paths from the transmitters to the receivers exist, forming a channel matrix or H matrix. Coefficients of the H matrix can be established by training and calibration sequences, which must be transmitted periodically between the data transmissions to enable data recovery through a channel estimation algorithm. The transmitter to receiver mapping of 4x4 MIMO is described in Figure 5 and equation (11) [7].

\[
\begin{bmatrix}
R_1 \\
R_2 \\
R_3 \\
R_4
\end{bmatrix} =
\begin{bmatrix}
h_{11} & h_{12} & h_{13} & h_{14} \\
h_{21} & h_{22} & h_{23} & h_{24} \\
h_{31} & h_{32} & h_{33} & h_{34} \\
h_{41} & h_{42} & h_{43} & h_{44}
\end{bmatrix}
\begin{bmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4
\end{bmatrix} +
\begin{bmatrix}
N_1 \\
N_2 \\
N_3 \\
N_4
\end{bmatrix}
\]

\[ (11) \]

Where \( R \) is a receiver, \( T \) is a transmitter, and \( N \) is a noise source. The \( h \) coefficients are generated by transmitting calibration and training sequences.

\[
R = H \cdot [T + N] \\
[T + N]' = H^{-1} \cdot R
\]

\[ (12) \]

\[ (13) \]

Equation (12) shows the simplified expression of equation (10). Applying the inverse matrix, as shown in equation (13), demonstrates how channel estimation is used to recover the transmitted data stream, inclusive of additive noise from the system.

**3 4x4 MIMO VLC UNIDIRECTIONAL SYSTEM MODEL**

![Figure 6 Block diagram of a 4x4 MIMO VLC unidirectional system model](image-url)
Based on the theoretical concepts described in section 2, a system model for the 4x4 MIMO VLC system was developed, this is shown in Figure 6. The system consists of a 2x2 LED transmitter array and a 2x2 PD receiver array. The system is described as follows: A serial pseudo random bit sequence (PRBS) is presented to the input of a shift register which converts the serial data stream to a parallel data stream. This parallel data stream is then passed to four transconducance amplifiers (TCAs) which translate the shift register output logic voltages to currents which then drive four independent LEDs. These LEDs have an impulse response h(t), which is convolved with the parallel data streams. The resultant light emission of the LEDs is transmitted through the free-space optical channel, and is received by four independent PDs (PDn) located some distance away from the transmitting LEDs. The four LEDs light emissions follow multiple paths to the PDs, leading to a 4x4 (16 combinations) receive matrix (H-matrix) which is received by the 2x2 PD receiver array. Following the PDs, noise is added to the received signal currents prior to their conversion to voltages and pre-amplification by the transimpedance amplifier (TIA) stages. Matching filters (MF) stages then provide pulse shaping, which is then followed by post-amplification (PA) stages which provide additional gain, before being passed to the inverse matrix (H-1) function which estimates the four independent data streams. Finally, filtering is performed by the LPF stages prior to conversion of the PRBS from parallel to serial format by the output shift register [8].

4 CONCLUSIONS AND FURTHER WORK

This paper presented SSLs advantages over traditional incandescent and CFL counterparts in terms of efficiency and life expectancy. It also presented the use of SSL as a high bandwidth optical data transmitter in a VLC system. SSL clearly demonstrated its added value and unique capabilities over traditional technologies.

In addition, VLC system throughput was compared against existing RF based technologies, such as Bluetooth and WiFi, and VLC was shown to have favourable data rates, exceeding 100x10^6 bps.

This paper also explained the key theoretical concepts needed in order to construct MATLAB based models. 3D spatial simulations of Lambertian radiant intensity were presented for a single LED, and a 2x2 LED array. Intensity modulation schemes were mentioned: PPM and OOK. LOS channel and receiver equations, and channel estimations techniques for a 4x4 MIMO were also presented.

Finally, these concepts were used to develop a block diagram of a 4x4 MIMO VLC unidirectional system model. This block diagram was presented along with an overview of the operation of the blocks and the propagation of the signals through the system.

Further work is needed to develop the model to include the convolution of the PPM/OOK IM signals with the LEDs impulse response, and the channel equation. Training and calibration sequences to enable channel estimation for the 4x4 MIMO will also be tested. Diffused light propagation will be experimented with, and MLE or RS error correction algorithms will be implemented.

REFERENCES