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The Design, Implementation and Evaluation of a Desktop Virtual
Reality for Teaching Numeracy Concepts via Virtual
Manipulatives

Lamya Fouad Muhammad Daghestani

A thesis submitted to the University of Huddersfield in partial fulfilment of
the requirements for the degree of Doctor of Philosophy

School of Computing and Engineering

The University of Huddersfield in collaboration with

King Abdulaziz University

March 2013

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ABSTRACT

Virtual reality offers new possibilities and new challenges for teaching and learning. For students in elementary mathematics, it has been suggested that virtual reality offers new ways of representing numeracy concepts in the form of virtual reality manipulatives. The main goal of this thesis is to investigate the effectiveness of using desktop virtual reality as a cognitive tool to enhance the conceptual understanding of numeracy concepts by elementary school children, specifically addition and subtraction. This research investigated the technical and educational aspects of virtual reality manipulatives for children beginning to learn numeracy by implementing a prototype mathematical virtual learning environment (MAVLE) application and exploring its educational effectiveness.

This research provides three main contributions. First, the proposed design framework for the virtual reality model for cognitive learning. This framework provides an initial structure that can be further refined or revised to generate a robust design model for virtual reality learning environments. Second, the prototyping and implementation of a practical virtual reality manipulatives application ‘MAVLE’ for facilitating the teaching and learning processes of numeracy concepts (integer addition and subtraction) was proposed. Third, the evaluation of conceptual understanding of students’ achievements and the relationships among the navigational behaviours for the desktop virtual reality were examined, and their impacts on students’ learning experiences were noted.

The successful development of the virtual reality manipulatives provides further confirmation for the high potential of virtual reality technology for instructional use. In short, the outcomes of this work express the feasibility and appropriateness of how virtual reality manipulatives are used in classrooms to support students’ conceptual understanding of numeracy concepts. Virtual reality manipulatives may be the most appropriate mathematics tools for the next generation. In conclusion, this research proposes a feasible virtual reality model for cognitive learning that can be used to guide the design of other virtual reality learning environments.

DEDICATION

*The dissertation is dedicated to my mother 'Nadia' who inspired
my passion for learning, inquiry, and discovery...*

To my: sisters and brothers, lovely children and sincere friends

LIST OF PUBLICATION

- Daghestani, L., Al-Nuaim, H. and Al-Mashat, A., (2005). Case Study: Investigating the Use of VLE in Teaching Mathematical Concepts to Third Grade Students. Proc. 2nd Saudi Sci. Conf. Fac. Sci., KAU, 15-17 March 2004. Saudi Arabia, Jeddah, 61-69.
- Daghestani, L., Ward, R.D., Xu, Z. and Al-Nuaim, H., (2008). The Design, Development and Evaluation of Virtual Reality Learning Environment for Numeracy Concepts Using 3D Virtual Manipulatives. Fifth International Conference on Computer Graphics, Imaging and Visualisation, 2008. CGIV '08. IEEE, p.93-100.
- Daghestani, L., Ward, R.D., Xu, Z. And Al-Nuaim, H., (2010). Virtual reality potential role in numeracy concepts using virtual manipulatives. Advances in Computer Science and Engineering: Proceedings of the Sixth IASTED International Conference (ACSE 2010). ACTA Press, Calgary, Canada, p.249-255.
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CHAPTER 1 Introduction

1.1 Background

When learning math, educators stress the importance of children conceptually understanding primary mathematical concepts in order to comprehend a higher-level of mathematical thinking in future grade levels (Canobi, 2002; Brown, 2007). It was discovered that children's advancements in math at the elementary level are below the required standard because they are incapable of deciphering abstract concepts on their own (Holmes and Adams, 2006). Research studies have highlighted a lack of development in the conceptual understanding of addition and subtraction in children's early primary school years (Robinson and Dubé, 2009; Ginsburg, 2009). However, the reason for the poor understanding of addition and subtraction concepts might be that learning them is multifaceted, requiring knowledge about a different but related concepts (Fuson and Briars, 1990; Canobi, 2009).

Math educators realised that the sense of touch kinesthetically sparked children's interest and enthusiasm in class (Clements, 1999). Therefore, hands-on manipulatives (physical manipulatives) were often used in elementary mathematics classrooms to explore mathematical concepts and procedures. Manipulatives, defined as physical objects, are used as teaching tools to engage students in the hands-on learning of mathematics (Resnick and Ford, 1981). Physical Manipulatives can come in a variety of forms such as Dienes block/base-10 blocks, Cuisenaire rods and Unifix cubes, and they are treated as external-action representation systems during classroom activities.

It has not been common practice to link physical manipulative representations cognitively with their symbolic representations, which has resulted in a limited amount of serial translations of the actions (Goldin and Kaput, 1996). Ball (1992) expressed that “students do not automatically make connections between actions with physical manipulatives and manipulations using the symbolic notation system”. This notion caused math educators to examine new ways to teach math concepts using computer technology, which led to the replacement of physical manipulatives with computer manipulatives (Clements, 1999). The necessity of developing children’s conceptual understanding of math as an abstract concept led math educators to utilize computer technology.

With the advancement of computer technology, the basic idea of math physical manipulatives was extended to computer-based manipulatives or ‘virtual manipulatives’ (Moyer et al. (2002). Virtual manipulatives are an embodiment of physical manipulatives in computer-generated programs and come in the form of two-dimensional (2D) or three-dimensional (3D) graphical representation (Moyer et al., 2002). Virtual manipulatives are similar to the physical type in that they may be manually slid, flipped, turned and rotated using the computer mouse as though it was a 3D object (Moyer et al., 2001). However, virtual manipulatives have additional benefits not obtainable with physical manipulatives, such as dynamic visualisation, immediate feedback, simplicity of access anywhere and multiple linked representations (Duffin, 2010).

Furthermore, virtual manipulatives saved the teachers and students a lot of time and created a real interest in the teaching process. This technology helped children control their actions and interactions, which enabled them to internalise the concepts they were learning (Sarama and Clements, 2009). Other researchers found that virtual manipulatives could be individually customised for each child. The children became more attentive and interested in class because of the ease and flexibility they found using the virtual manipulatives.

Because some researchers believed that the virtual manipulatives were superior, math educators began to focus their research on how they could benefit from virtual manipulatives. Nevertheless, some researchers discovered that physical manipulatives were better than virtual manipulatives because the information obtained using the virtual manipulatives may be misunderstood due to the differences in format between each, whereas they were understood when interacting with the physical manipulatives. These mixed results suggest that there may be advanced virtual manipulatives that are able to more closely resemble physical manipulatives. Ultimately, the researchers suggested using a combination of physical and virtual manipulatives because they may be more effective together than either is alone (Reimer and Moyer, 2005; Moyer et. al., 2005; Suh and Moyer, 2007).

Even with all the technology that has been introduced in the natural and social sciences in the last two decades, virtual reality technology was used. Virtual reality can be described as a multi-sensory, highly interactive computer environment that makes the user believe they are actually experiencing a situation, although in reality, they are looking at artificial environment (Gigan, 1993). The capabilities of virtual reality

technology can help educators build a new visual language that bridges the gap between the natural world and the abstract world (Yair, 2001). Being concerned with improving virtual manipulatives, this research aimed to develop a new approach to virtual manipulatives.

1.2 Statement of the Problem

Research on numeracy using virtual reality technology is slight. It is not well understood whether the use of virtual reality manipulatives enhances children's conceptual understanding at the elementary level, specifically in Saudi Arabian schools. The current study focused on whether virtual reality technology could be used as a cognitive tool for enhancing children's conceptual understanding of abstract mathematical concepts. One of the main issues addressed in this research is the design necessary for virtual reality manipulatives to serve as cognitive tools.

1.3 Purpose of the Study

The purpose of this study was to verify the effect of applying virtual reality manipulatives as cognitive tools in teaching the numeracy concepts of addition and subtraction to elementary students in Saudi Arabia, the Jeddah area. The study aimed to determine whether students were able to make connections between concrete and abstract numeracy concepts using virtual reality manipulatives.

1.4 Rationale of the Study

Based on what cognitive developmental theorists believe (Piaget, 1952), it was realised that the successful and reasonable process of learning for young children should start with concrete concepts and then proceed to abstract symbols. Given that children at early primary grade levels obtain knowledge through interactive experimental learning methods, they will, consequently, be able to apply that knowledge afterwards in higher grades (Piaget, 1972). In accordance with Piaget views, Manches et al. (2009) stated that if Manipulatives were knowledgeably used in math learning, students' achievements would increase and this could foster more positive attitudes towards math learning.

A review of previous studies about the use of physical manipulatives (Fey, 1979; Suydam, 1984) and other studies concerning the use of virtual manipulatives (Herrere, 2003; Lamberty and Kolodner, 2004; Suh et al., 2005; Lyon, 2006; Brown, 2007; Suh and Moyer, 2007; Yuan , 2009; Manches et al., 2010; Duffin, 2010) showed that children were able to harness and use meanings of new mathematical concepts. However, other studies expressed little difference between using physical manipulatives and virtual manipulatives (Steen et al., 2006; Suh and Moyer, 2007; Moyer-Packenham, 2012).

Concerning the learning outcomes some studies noted that virtual reality technology had its own impact on the educational process because of its unique capabilities (Gigante, 1993; Kameas et al., 2000; Roussou, 2004) and that using virtual reality technology as a teaching tool has bridged the gap between the real world and the abstract world (Yair, 2001). Based on the fact that virtual reality provides the sense of being included in the learning process, in that, the user is no longer looking at the data

on a screen but rather is immersed as an active participant (Jackson and Fagan, 2000). As far as it is crucial to improve the level of the children's conceptual understanding, virtual reality technology appears to fulfil this need.

1.5 Thesis Scope

The scope of this thesis draws attention to three primary areas related to this research. These interdisciplinary areas are interrelated with each other (Figure 1.1). The conceptual diagram in Figure 1.1 illustrates the priorities of the research, and the review in Chapter 2 relates to this illustration.

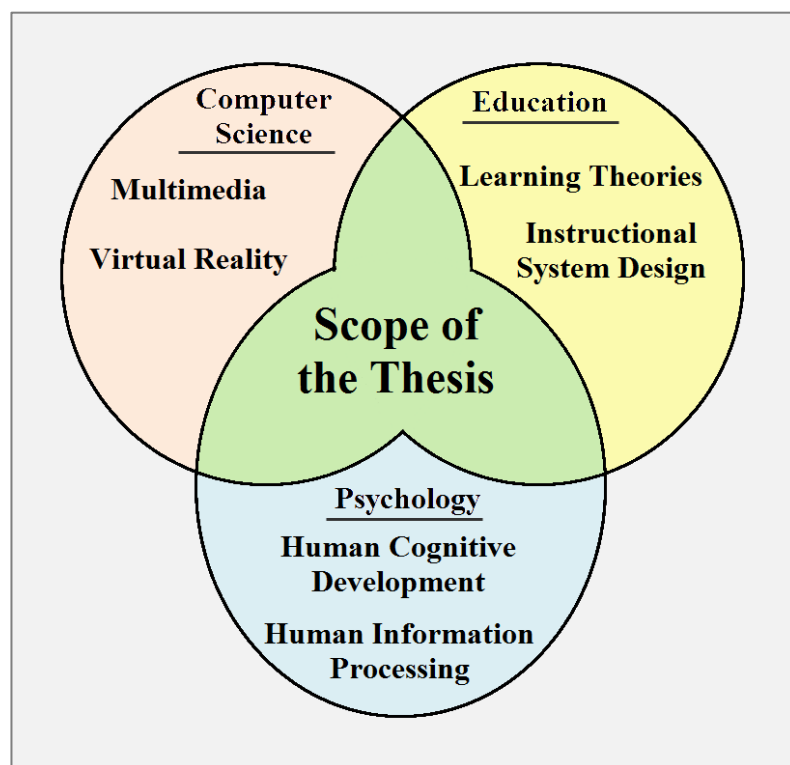


Figure 1.1 The conceptual diagram of thesis scope

1.6 Organization of the Thesis

This thesis is organised into six chapters. Chapter 2 reviews and summarises the relevant literature regarding math manipulatives (physical and virtual), the aspects of virtual reality technology and its application as a cognitive learning tool and the cognitive approaches used for multimedia instructional design to lay a foundation for examination of virtual reality manipulatives. Chapter 3 presents the MALVE theoretical and technical framework development. Chapter 4 provides details about the research methodology. Chapter 5 presents the data analyses results and discussion. Chapter 6 provides a summary of the research and concludes with the contributions, implications and possible future research related to this thesis.

CHAPTER 2 Literature Review

The focus of this chapter is to present a summary of the literature relevant to aspects of this research. There are three areas of linked work that are relevant to the research presented in this thesis: math manipulatives (physical and virtual), virtual reality learning environments and cognitive approaches used for multimedia instructional design.

Regarding math manipulatives, most of the literature has reviewed their usefulness in the development of children's conceptual understanding as it relates to an overview of historical theories. Research that has investigated the effectiveness of using math manipulatives in the classroom as instructional tools are presented, as well as teachers' perceptions regarding the use of math manipulatives. Then, a definition of numeracy concepts (addition and subtraction) and an analysis of the common difficulties and errors in learning them are highlighted, as are their influence on student achievement.

Virtual reality technologies are discussed in terms of their capabilities to engage learners in the exploration, construction and manipulation of the virtual learning environment. To examine cognitive approaches used for multimedia instructional design, an overview of cognitive theory architectures that are relevant to multimedia instructional design is provided first. Second, the cognitive processes used when designing an interactive multimedia and virtual reality learning environment are considered.

2.1 Math Manipulatives

Math manipulatives are materials often used in the classroom, especially in the early grades. Children who use manipulatives may better comprehend both mathematical ideas and their implementation in real-life situations (Marzola, 1987). Manipulatives are tangible objects that represent or embody abstract mathematical concepts and may be seen and handled by the students (Moyer, 2001). Ultimately, math manipulatives are essential tools for students, and they must be selected to appropriately signify the particular objectives of the mathematics lesson (Boggan et al., 2010).

The work of Piaget (1952) and Bruner (1960), amongst others, has helped provide a theoretical foundation for children's cognitive development and identified a possible role of physical objects in exploring and articulating ideas when children lack the ability to do so more abstractly. Dienes' (1960) work is of great significance in that it describes not only the types of activities, but also the types of materials that can be used to support certain mathematical concepts.

2.1.1 Relevance of manipulatives in math learning

Constructing a suitable meaning for learning mathematical concepts and processes has been attempted by a few cognitive psychologists, including Piaget (1952), Bruner (1960) and Dienes (1960). In the following sections, we review the theories related to impactful mathematics learning among young children.

2.1.1.1 Piaget's developmental theory

Jean Piaget was a child psychologist. Piaget's (1952) theory of cognitive development describes a child's advancement through certain stages of mental development. Piaget's theory also gives an approximate age range for each stage, as shown in chronological order in Figure 2.1.

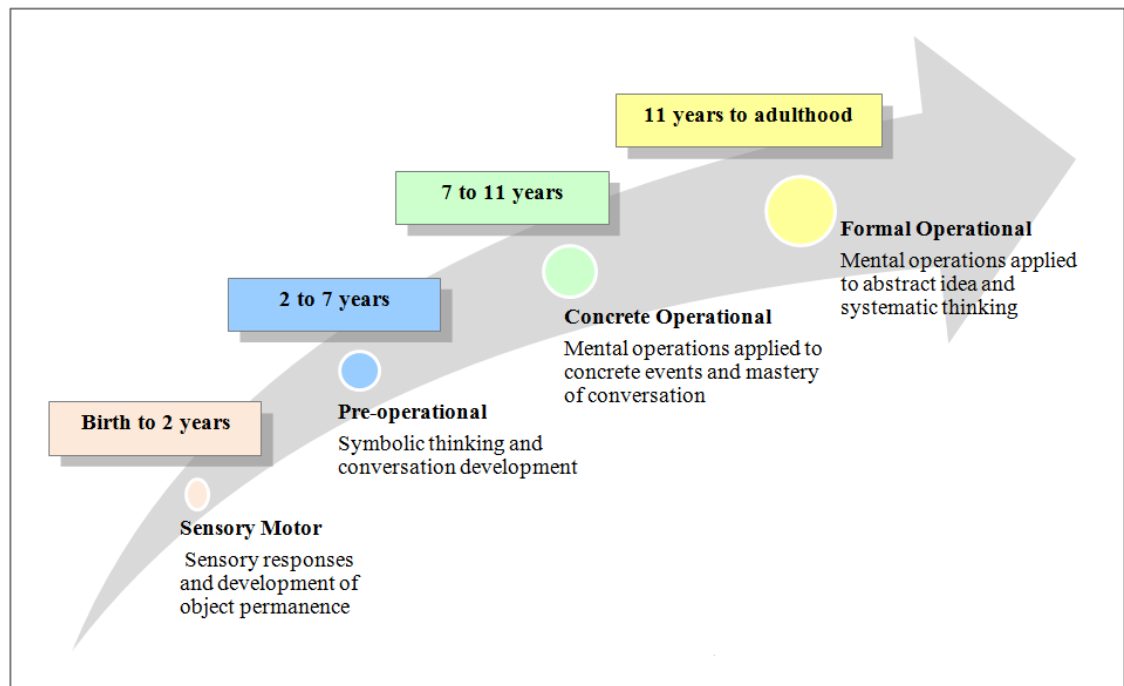


Figure 2.1 Piaget's stages of cognitive development

Piaget recognised four main stages in a child's life. The **Sensory Motor** stage is when infants and babies are more concerned with learning about the physical world, objects and their own physical development. The **Preoperational stage** is when a child learns and develops verbal skills, including reading and writing. The **Concrete Operational stage** is when a child begins to understand abstract concepts, such as numbers and relationships. Finally, the **Formal Operational stage** is when a child begins to reason logically and systematically.

The cognitive development of a child and their ability to understand concepts typically move from concrete to abstract understanding, according to Piaget's theory of mental development at the concrete operational stage (ages 7-11, elementary school level). Therefore, a child in this age range needs more practice with concrete materials first to allow for the grasping of subsequent abstract mathematical concepts. Hence, children begin to understand symbols and abstract concepts only after experiencing the ideas on a concrete level (Piaget, 1952).

Piaget's theory proposed that children in the concrete operational stage do not have a significant mental maturity; thus, he suggested that in order for children to be able to recognise words or symbols used in abstract mathematical concepts, they should practise with hands-on concrete materials. Therefore, constructing an interactive learning environment is very important for increasing children's learning abilities and facilitating their initial constructions of connections between different elements in the learning environment.

2.1.1.2 Bruner's cognitive development theory

Jerome Bruner was the founder of cognitive psychology and one of the key figures in the cognitive revolution of the 1960s. Bruner (1960) was a strong believer of constructive learning. Constructive learning requires hands-on activities in which the child can experience and test their ideas. Bruner considered the child to be an active participant in learning and believed they should be urged to participate in the learning process. According to Bruner's (1966) theory of cognitive development, three modes of representation link to signify a child's demonstration and development of conceptual understanding: (a) **Enactive Representation**: acting on concrete objects; (b) **Iconic**

Representation: forming images of the concrete constructions; and (c) **Symbolic Representation:** adopting symbolic notations. These three modes of representation form the path in which information or knowledge is stored and encoded in the mind (Figure 2.2).

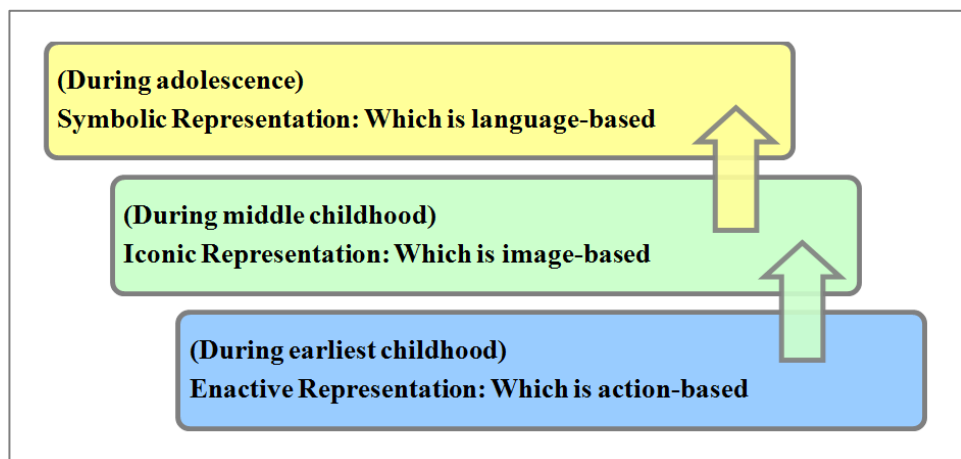


Figure 2.2 Bruner's stages of cognitive development

Bruner believed that it was particularly crucial for a teacher to introduce various tangible embodiments of the same concept before gradually moving children to more symbolic ones. He also emphasised that different forms of representation of a single concept may be more appropriate for children at various ages or stages of learning than others. Bruner viewed a child's development as being reactive to the learning environment, and the most important aspect was discerning the appropriate means of producing the material to help a child progress through the stages of learning.

Bruner's theory was established on the idea that children built new concepts based on previous knowledge. Therefore, they used their existing knowledge to create assumptions and assist them in solving problems, as well as exploring relationships. This idea supports the notion that fully establishing a child's understanding of a concept

is needed before proceeding to the next one. In regards to how children learn, it becomes essential for a teacher to recognise that it will be hard for a child to decipher new concepts without possessing the knowledge about how to relate the new information.

Based on the previous descriptions of Piaget's and Bruner's theories, learning mathematical concepts in the classroom may demand additional learning materials to enhance children's conceptual understanding. Multi-sensory materials, such as math manipulatives, assist a child's use of visual, tactile and auditory interactions (Rains et al., 2008). These materials can help elementary teachers bridge the gap between the use of physical materials and the understanding of abstract concepts (Bullock, 2003). Similarly, Bruner proposed that teachers construct and organise children's activities with well-designed concrete materials to allow children to learn the required concepts.

2.1.1.3 Zoltan Dienes

Unlike Piaget and Bruner, Zoltán Dienes (1973) was concerned entirely with the learning of mathematics, because he believed it differed from other sciences regarding the nature of structural relationships among concepts. Dienes' stated four principles theory of learning mathematics emphasised the necessity of students' direct interactions with their environment as they learn mathematics. He believed that various tangible materials (i.e., physical manipulatives) were essential for students to obtain abstract ideas and crucial for learning mathematics.

Dienes drew upon Piaget's that learning was an active process and proposed four principles of learning instruction (dynamic principle, perceptual variability principle, mathematical variability principle and constructively principle). He asserted that children should learn with materials that varied perceptually but were all consistent in their structural correspondence with the concept being learnt.

Arguably, Dienes' work that relates to the structured materials he developed for supporting children's concepts of place value is one of the greatest legacies, notably, the base-10 blocks of his multi-base arithmetic blocks known as Dienes' blocks (Manches et al., 2010). Dienes argued that learning base-10 numeration system consisted of understanding the place value relationships and applying the resulting concepts to real-world situations.

2.1.2 Physical Manipulatives

Manipulatives tools "embody the core relationships and structures of mathematics, and they stimulate intuition and inquiry" (Resnick and Ford, 1981). Manipulatives represent the objects used to teach math. They are modelled to be accessible and easy for students to manage as concrete materials to understand abstract ideas. They make concept awareness more realistic to enhance learning.

The use of manipulatives allows children to grasp mathematical operations and helps connect mental images and abstract ideas to their learning experiences. Although extensive research has been conducted that supports the use of manipulatives across grade levels, teachers in the classroom scarcely use them to teach math concepts (Fey, 1979; Suydam, 1984).

According to one study, teachers only used manipulatives for a restricted number of children and only a few times throughout the school year (Scott, 1983). Furthermore, another study found that children who used a variety of manipulatives had clearer mental images and could represent abstract thoughts better than those who did not use hands-on materials (Hsiao, 2001).

2.1.3 Virtual Manipulatives

Virtual manipulatives are simply a version of physical manipulatives on the computer screen rather than a student's desk. Virtual manipulatives have been defined as an "interactive, web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer et al., 2002). In other words, virtual manipulatives are defined as "computer-based simulations of physical manipulatives that are accessed via the Internet or computer software" (Bouck and Flanagan, 2009).

Virtual manipulatives are usually in the form of Java or Flash Applets, and the mouse or keyboard clicks can be used to select and flip, rotate and turn the manipulatives. These manipulatives are useful when teaching mathematical skills such as place value, carrying values and borrowing values (Kamii et al., 2001). Virtual manipulatives are effective in facilitating students' understanding of mathematical concepts, and positive results seem to be caused by the visual nature of these manipulatives and the students' abilities to use them interactively (Reimer and Moyer, 2005; Moyer-Packenham et al., 2008).

The standard argument for using mathematical virtual manipulatives for young children is that virtual manipulatives provide a tangible representation that children can use to bootstrap the acquisition of abstract concepts in math (Paek et al., 2011). The following terms represent the strengths of virtual manipulatives (Duffin, 2010):

- Constraints – Constraints can be imposed that help focus student attention on mathematical rules. For example, in base blocks, ten blocks can only be allowed in the tens column.
- Seeding – Virtual manipulatives can be seeded or configured for use in specific activities, increasing focus and saving start up time.
- Dynamic visualization – Visual representations of mathematical concepts can be interacted with, allowing the learner to explore relationships.
- Multiple, linked representations – Representations can be linked to help draw attention to the relationships between the representations and deepen understanding.
- Hints and immediate feedback – Software can make learning more efficient by giving hints when students request it and providing feedback when it recognizes mistakes.
- Simulations – Learners can run simulations that otherwise would be prohibitive in a classroom setting. For example, a spinner can be spun 1000's of time in only a few seconds.
- Instructional sequences – Sequences of instructional activities can be built into online materials that scaffold and focus student learning.
- Cost – Many websites provide virtual manipulatives for free on the internet.
- Saving – The state of virtual manipulatives can be easily printed or saved so that it can be reviewed, modified, and discussed later.
- Maintenance – In contrast to physical manipulatives, virtual manipulatives don't get lost or broken.
- Access – Virtual manipulatives can be accessed anywhere there is internet access including in student homes.

There are varieties of virtual manipulatives found on the web that can be used in the classroom. The NLVM has the most well-known developed virtual manipulatives available on the Internet (Moyer et al., 2002; Yuan, 2009; Bouck and Flanagan, 2010). The NLVM is a National Science Foundation (NSF)-supported project that ran from 1999 to 2010 to develop a library of a unique interactive set of web-based virtual manipulatives. These are primarily provided in the form of interactive 2D/3D Java applets. NLVM contains the most useful mathematical topics, and a teacher or student can pick a topic and grade level from the matrix and then choose from a list of math topics (Figure 2.3).

Index	Pre-K – 2	3 – 5	6 – 8	9 – 12
Number & Operations				
Algebra				
Geometry				
Measurement				
Data Analysis & Probability				

Credits | Contact | © 1999-2010 Utah State University. All Rights Reserved.
English | Español | Français | 中文

Figure 2.3 National Library of Virtual Manipulatives (<http://nlvm.usu.edu/>)

2.1.4 Research Studies on the Use of Physical and Virtual Manipulatives

A great deal of information was found concerning the use of manipulatives in teaching mathematics to elementary-aged students. In fact, many researchers studied the impact of using math manipulatives (physical or virtual) in mathematics classroom teaching and learning. A study conducted by Moreno and Mayer (1999) compared two different types of virtual manipulatives aimed at learning mathematical addition of whole numbers. One manipulative represented the problems using only a symbolic form, and the second manipulative presented the problems using symbolic, visual and verbal forms. The results showed significantly higher achievement in students who used the manipulative with multiple representations (i.e., symbolic, visual and verbal).

In a research study dealing specifically with rational numbers, Reimer and Moyer (2005) studied 19 third-grade students during a two-week classroom session using several interactive virtual fraction manipulatives. Data were collected from pre- and post-test levels of student conceptual knowledge and procedural computation, as well as student interviews and attitude surveys. Results from the post-test analysis indicated that students showed significant advancements in conceptual knowledge. Furthermore, the results showed a significant positive relationship between conceptual and procedural knowledge of the post-test scores. The interviews and attitude surveys showed that advancements in conceptual knowledge at post-test may be attributed to the active manipulation and immediate feedback provided by the virtual fraction applets.

Suh et al. (2005), in their study “Developing Fraction Sense Using Virtual Manipulative Concept Tutorials”, used two virtual manipulative applets from the National Council of Teachers of Mathematics (NCTM) electronic standards and from the NLVM to reinforce fraction concepts in three fifth-grade classes with students of different ability levels (low, average and high achievement based on standardised testing results from the school). The advantageous characteristic of these virtual fraction manipulatives was that they allowed students to experiment and test hypotheses in a safe environment. This study was conducted in three 1-hour class sessions to investigate the learning characteristics afforded by virtual manipulatives technology tools.

The results of the observations and analyses from Suh et al.’s study (2005) showed that the students identified as low achievers seemed to benefit the most of the three groups of students from working with the virtual manipulatives tutorials. This study concluded by urging teachers to use virtual manipulative technologies. In addition, they suggested that teachers, researchers, and educational technology developers should ensure that effective computer programs and applets continue to progress for mathematics teaching (Suh et al., 2005).

In a quasi-experimental pre-test and post-test design study Bolyard and Moyer-Packenham (2006) investigated the impact of virtual manipulatives on student achievement in learning the concepts of integer addition and subtraction. The participants included 99 sixth-grade students in six mathematics classes. This study used three different treatment groups of virtual manipulatives: virtual integer chips, virtual integer chips with context and virtual number line.

Overall, the findings indicated that students in each of the three treatment groups made significant pre- to post-test gains in understanding both integer addition and subtraction concepts. Moreover, the analysis of the differences among the three treatment groups at post-test indicated that students' performances on integer addition and subtraction items were similar. The general conclusion is that the virtual manipulative environments supported students' learning of these concepts regardless of group assignment (Bolyard and Moyer-Packenham, 2006).

Steen et al. (2006), in their study, investigated the existing differences in the academic achievement of first-grade students in a geometry unit who used virtual manipulatives and those students who used the traditional text-recommended practice activities. Thirty-one students were randomly assigned to either a virtual manipulatives group or control group. The virtual manipulatives used in this study were from the NLVM, the Arcytech, the NCTM Illuminations and Math Cats. A pre-test and post-test were conducted in both groups. The tests and assessment activities used for both groups were in compliance with the 'Grade One' and 'Grade Two' levels from the text's publisher.

Results showed that the virtual manipulatives group had significant improvements from pre-test to post-test at both the Grade One and Grade Two test level. The control group only showed a significant improvement at the Grade Two test level. These results indicated that applying virtual manipulatives as an instructional tool was extremely effective for the virtual manipulatives group and, perhaps, more effective than using the traditional text activities (Steen et al., 2006).

A mixed-methods study was conducted by Suh and Moyer-Packenham (2007) that compared mathematics achievement in two third-grade classrooms using physical and virtual manipulatives. This study examined the representational connections between visual and verbal/symbolic codes and their effect on understanding fraction concepts. The study used a within-subject crossover repeated-measures design and contained an examination of quantitative data (pre- and post-test). Qualitative data (field notes, students' written works, student interviews and classroom videotapes) were also collected to help the researchers to further interpret the results of the quantitative findings.

The results from this study revealed statistically significant differences in student achievement in favour of the virtual manipulative treatment. For a further interpretation of these results, Suh and Moyer applied the framework of the dual-coding theory to individual test items. An analysis of students' representations showed evidence of pictorial and numeric connections among their work, indicating that the multi-representational presentation of the fraction-addition process stimulated interrelated systems of coding information. Although, the physical manipulatives group performed better on the dual-coded items than the single-coded items, the virtual manipulatives treatment group performed significantly better overall on all test items than the physical manipulatives group. This study concluded by suggesting that the use of dual-coded representations in virtual manipulative environments that associate visual images with symbolic notation systems have the potential to be effective in teaching mathematical processes (Suh and Moyer-Packenham, 2007).

In another study, Suh and Moyer (2007) applied a classroom project that included two groups of third-grade students in a week-long unit focusing on algebraic relationships using physical and virtual manipulative. The target of the unit was to engage students with different algebraic models and motivate students to use informal strategies to represent their relational thinking. The virtual manipulatives had unique features that promoted student thinking such as (a) explicit linking of visual and symbolic modes; (b) guided systematic support in algorithmic processes; and (c) immediate feedback and a self-checking system. However, the physical manipulatives had unique features such as (a) tactility; (b) opportunities for invented strategies; and (c) mental mathematics.

Suh and Moyer (2007) recorded field notes, interviewed students and videotaped class sessions in order to identify unique features of the learning environments. Result from the pre and post-test measures showed that students in the physical and virtual manipulative environments gained significantly in achievement and revealed elasticity in interpreting and representing their understanding in multiple representations. These results showed that although the different manipulative models had different features, both the physical and virtual manipulatives were effective in assisting students' learning and stimulating relational thinking and algebraic reasoning (Suh and Moyer, 2007).

Brown (2007) designed his study to investigate the impact of using virtual manipulatives and physical manipulatives on 48 sixth-grade students' learning skills and concepts in equivalent fractions. These students will be divided into two treatment groups: one group will receive mathematics instruction with virtual manipulatives and the other will receive mathematics instruction with physical manipulatives.

A pre-test and post-test was conducted in both groups and a students' attitudes survey was distributed at the end of the study. Brown's major interest was whether or not students who used virtual manipulatives would out-perform students who used physical manipulatives on the post-test. A minority interest was students' attitudes about using Manipulatives in the mathematics classroom.

Post-test results showed that physical manipulative use had a greater impact on students' achievements than the virtual manipulative use had. The possible reasons for the scoring differences in the post-tests were as follows: (a) the students in the physical manipulatives group began with higher pre-test scores and, therefore, had a better understanding of equivalent fractions at the onset of the study and (b) the instructions for the use of the physical manipulatives were more efficient than those given for the use of the virtual manipulatives.

This study also measured students' attitudes about using physical and virtual manipulatives in the mathematics classroom. Students reflected positive attitudes towards using both manipulatives, but they tended more towards the virtual manipulatives than the physical manipulatives. Based on these findings, Brown concluded that students who received equivalent fraction instruction with physical manipulatives surpassed students who received equivalent fraction instruction with virtual manipulatives. He also concluded that the use of manipulatives, both virtual and physical, boosted the learning environment in the elementary mathematics classroom (Brown, 2007).

Moyer-Packenham et al. (2008) examined teacher use of virtual manipulatives across Grades K-8 after participating in a professional development institute where manipulatives and technology were the main resources used in all of the activities. The collected data for the study depended on the researchers' analyses of 95 lesson summaries where teachers explained their uses of virtual manipulatives within their classroom mathematical instruction. It was familiar for teachers to use the virtual manipulatives alone or as a follow-up to physical manipulatives use. One essential finding of this study was that teachers used the virtual manipulatives during their regular mathematics instruction.

Another remarkable finding from this study was the elucidation of the most common virtual manipulatives used across the grade levels: geoboards, pattern blocks, tangrams and base-10 blocks. The results also provided the manner in which these manipulatives were used by teachers as cognitive technological tools. Further findings suggested that teachers' choices about which virtual manipulatives to use, what content to teach while using them and whether to use virtual manipulatives along with physical manipulatives were primarily affected by their acquaintance with similar physical manipulatives and beliefs about the mathematical, cognitive and pedagogical fidelity of virtual manipulative use. The results ultimately expressed that virtual manipulatives were central to mathematics learning and were frequently used in conjunction with physical manipulatives. Finally, Moyer-Packenham et al. (2008) recommended that further examinations were required that used in-depth interviews with teachers and observations of classroom implementation to reveal additional insights into these results.

Daher (2009) explored the use of virtual manipulatives applets by pre-service teachers to solve mathematical problems and how they understood this work. This understanding indicated whether and in what manner they would use the applets as teaching tools. Moreover, this study further explored the functions, effectiveness and benefits of the applets while tackling mathematical problems. To analyze the participants' solutions, difficulties and needs for the applets, content analysis was performed to help verify the occurrence of specific words or concepts within the text and the relationships among these words and concepts.

The factors that affected the participants' understanding of the need for applet use were their capabilities in implementing the activity correctly, the applets' related actions and operations and the type of problems that hampered implementation. This means that pre-service teachers should initially be introduced to applets that have no operations or compatibility problems so they may, consequently, become interested in their work with the applets and see them as worthy teaching tools without encountering difficulties that could decrease their importance. Although many of the participants believed that mathematical problems could be solved without using the applets, they still stressed the role of the applets as boosting, simplifying and explaining mathematical problems' statements and solutions. Simultaneously, they indicated that applets were tools that learners enjoyed using and that motivated them to solve mathematical problems.

Daher (2009) study concluded by reporting that pre-service teachers were likely to use applets in their future teaching practices when they want to improve and develop their students' learning or generate interest in mathematical problem solving. The researcher suggested that designers of educational applets may benefit from this research by taking its findings into consideration when making efficient design decisions.

To propose a multi-representative construction model, Hwang et al. (2009) developed an innovative virtual manipulatives and whiteboard (VMW) system that combined virtual manipulatives and multimedia whiteboards. The VMW system allowed users to manipulate virtual objects in a 3D space that were viewed from any perspective to find clues and solve geometry problems. The purpose of the VMW was to promote a multi-representative construction model based on a pedagogical theory that states, "Children would construct their geometry concepts from multiple representations like mapping the concrete items to abstract ideas through physical or mental manipulation" (Hwang et al., 2009).

The intended system was evaluated with one pilot study to investigate its perceived ease of use and effectiveness. The results showed that the proposed system was recognised as useful and enabled students to understand the processes of geometry problem solving, such as using various solving strategies, as well as revealing geometrical misconceptions. Furthermore, students' solving strategies were analyzed using their manipulations in the 3D space, and the solutions were recorded in the whiteboards.

Results indicated that the VMW system could afford more elastic thinking than paper and pencil practices, or even manipulation of actual physical objects, to let students approach their utmost potential in understanding and solving geometry problems. Moreover, it was discovered that most students agreed that the VMW system helped them use various representations for solving geometry problems and simplified and widened their thinking to incorporate different viewpoints in the 3D arena. Additionally, the students felt that the VMW system could help them show their solutions more completely (Hwang et al., 2009).

Yuan (2009) investigated how elementary school teachers in Taiwan applied web-based virtual manipulatives to mathematics teaching and the issues that emerged from these applications. The chosen virtual manipulatives applets were taken from the NLVM. Four elementary school teachers in Taiwan were chosen to participate in this study. Each teacher applied their own case study at a certain grade level, and their selections of applets were based on their individual interests. In other words, these teachers decided when and how to use a specific applet for their students. The teachers incorporated the virtual manipulatives into their classroom teaching to help students visualise mathematical relationships and to actively involve them in their learning. The study included four case studies: Case 1 used base blocks in Grade 2; Case 2 used base blocks decimals in Grade 4; Case 3 used the Diffy game in Grade 5; and Case 4 used isometric geoboards in Grade 5.

The results were obtained through self-reported observations from the four teachers. The results indicated that virtual manipulatives are able to be used by students with different learning capabilities, such as lower achievers and higher achievers. It seems that teachers in the lower grades were more willing to apply virtual manipulatives applets than teachers at the higher grade levels. Regardless, insufficiencies in modelling the tools and teacher training for their use may minimise the effectiveness of the applications. In his recommendations, Yuan proposed that future research should include more teachers to understand their instructional points of view about the application of virtual manipulatives in their teaching methods. Additionally, to obtain crucial and adequate results, it is essential to have the researcher attend the classroom and observe the students' work in class.

Fishwick and Park (2009) presented a method and application for teaching the distributive law of algebra and basic algebraic computations within a multi-user environment called Second Life. The main goal of this work was to use the technology of this environment and investigate how it could be applied to provide substitute methods of representation. The first illustration of the distributive-law concept was through the use of 2D projections. However, with Second Life, the algebraic variables and their operators emerge to the user in an immersive setting, and the objects are positioned by dragging and placing them using the computer mouse. Additionally, with Second Life, multiple users can perform operations at the same time; thus, this process can be used within a teacher-student setting, with the teacher representing one avatar and the students representing their own avatars.

Fishwick and Park (2009) concluded that there was a significant learning curve when working with 3D Second Life compared with the 2D version because algebraic variables and their operators in Second Life are given to the user in an immersive setting. Furthermore, they predicted that this type of immersion system could be used to deepen the experience and subsequent understanding because users could move around in the environment and be pulled towards the virtual manipulatives. However, 3D interfaces still require development to stimulate a 'comfortable feeling' to users within them.

In a study by Yuan et al. (2010), they developed virtual manipulative polyominoes kits for junior high school students in Taipei County, Taiwan, to investigate polyominoes use. Sixty eighth-grade students (27 boys and 33 girls) from two different classes participated. To compare the problem solving performance differences between using physical manipulatives and virtual manipulatives, non-equivalent quasi-experimental group pre- and post-tests were conducted. Students in the experimental group used virtual manipulatives to explore polyominoes, and those in the control group used physical manipulatives. Students' 'responses from attitudes' surveys for the virtual manipulative group were also analysed to understand their perceptions about using the virtual manipulatives.

The results of the comparisons between the effectiveness of virtual manipulatives and that of physical manipulatives indicated that using the virtual manipulatives was as effective as using the physical manipulatives for boosting the learning of polyominoes. This study also showed that virtual and physical representations enhanced students' problem solving performances. This meant that

merely substituting the physical manipulatives with virtual manipulatives did not influence the amount of learning. It was actually the instructional design that affected performance. Hence, the research stated that regardless of the instructional tools used, teachers should focus on instructional design to allow for adequate use of the manipulatives (Yuan et al., 2010).

Furthermore, based on the students' responses of attitudes surveys, the students expressed that the virtual manipulatives were easier to operate during problem solving for polyominoes, and virtual manipulatives could dedicate their awareness to group discussion. It was also noticed that there were different problem-solving behaviours between the groups, despite the fact that students in both groups were given the same instructions to solve for the number of polyominoes. It was effectively recommended that if the physical environment could be modified to create more space for manipulating physical manipulatives, such as on tables so they could easily fix and rotate them, the students would similarly attain the ideas and attitudes of those who used virtual manipulatives (Yuan et al., 2010).

A meta-analysis study comparing the use of virtual manipulatives with other instructional treatments was conducted by Moyer-Packenham and Westenskow (2011) to combine quantitative results from research involving virtual manipulatives and inspect the effects of virtual manipulatives as an instructional tool in studies of differing durations. Comparisons were made using Cohen's *d* effect size scores, which reported treatment effect magnitude independent of sample size.

The outcomes from 29 research reports yielded 79 effect size scores that were grouped and averaged to determine the total effects of virtual manipulatives use alone and in conjunction with physical manipulatives or other instructional treatments. The results from the meta-analysis revealed that virtual manipulatives had a moderate average effect on student achievement compared with other methods of instruction. The results also proposed that the length of treatment for virtual manipulatives affected the average effect size scores, and larger effect size scores resulted when lessons were of longer durations.

Moyer-Packenham and Westenskow (2011) study concluded that virtual manipulatives were influential instructional tools for teaching mathematics because they have distinctive characteristics that positively influence students' achievements compared with other instructional methods. This study also suggested that further research was required to decipher whether the use of virtual manipulatives as instructional tools was more influential for some students than others because little is known about how learner characteristics, virtual manipulatives applet features or instructional methods affect student learning.

Akkan and Çakir (2012) investigated pre-service teachers' points of view about using virtual and physical manipulatives in mathematics teaching. The virtual manipulatives in this study were taken from the NLVM. The sample for this research consisted of 187 pre-service teachers (92 of which were in their first year and 95 were in their third year) in the Department of Classroom Teaching at Kafkas University. In this context, questionnaires and interviews were conducted.

The results of the questionnaire analyses regarding the reasons for choosing virtual manipulatives were as follows: They have more positive effects on the motivation of students, have more influence on developing problem solving skills, provide immediate feedback, are pleasurable and fun, are both time and economically efficient and provide opportunities for individual successive trials at different times by students at different levels of achievements.

However, some of the pre-service teachers preferred the physical manipulatives and explained their reasons as follows: They allow for simultaneous visual and tactile discovery, easy obtain ability and group work. This study concluded that most of the pre-service teachers stated that both physical and virtual manipulatives were essential for teaching mathematical concepts, discovering mathematical relationships and boosting mathematical thinking. They also indicated that the use of manipulatives would further the development of students' academic achievements. Therefore, teachers and pre-service teachers need to be motivated, encouraged and trained on the activities designs to effectively use these types of manipulatives (Akkan and Çakir, 2012).

Moyer-Packenham and Suh (2012) explored the effect of virtual manipulatives on various achievement groups through a teaching exploration that included 58 fifth-grade students in four classes at the same school. There was one low group ($n = 13$), two average groups ($n = 12$ and $n = 12$) and one high group ($n = 21$). During a two-week unit focusing on two rational number concepts, three groups (low, high and one average) used virtual manipulatives from the NLVM website and the NCTM electronic resources, and the other average group used physical manipulatives.

Data sources included pre- and post-test scores for students' mathematical content knowledge and videotapes of classroom sessions. The results from the pre- and post-tests indicated significant improvements. The lower-achieving students showed significant gains as an individual group, whereas only numerical gains for students in the average and high achieving groups were recognised. Qualitative data gathered from videotapes of classroom sessions suggested that the different achievement groups experienced the virtual manipulatives in different ways. The high-achieving group noticed patterns easily and moved to the use of symbols, whereas the average- and lower-achieving groups were more dependent on pictorial representations as they systematically worked stepwise through the processes and procedures using mathematical symbols.

Eventually, virtual manipulative applets that include multiple capacities may be considered beneficial for the higher-achieving students because of the inclusion of multiple examples, whereas the same applets may be considered as hindrances for lower-achieving students because of the limited guiding feedback. Moyer-Packenham and Suh (2012) concluded by stating that it was essential to consider which applets are more influential and beneficial to students of different achievement levels and how interrelated affordances may affect various students during mathematics instruction. The different effects on students of different achievement levels are important aspects to consider when designing mathematics instruction that uses technology. These different impacts may have been caused by the visual and pictorial representations that shaped the students' perceptions of the concepts.

2.1.5 Numeracy Concepts and Instruction

Numeracy, as defined by the National Numeracy Strategy, is a proficiency that requires an understanding of the numerical system (Doig et al., 2003; Brown et al., 2003). Numeracy is a core part of early childhood development (Doig et al., 2003). Developing a solid understanding of a positional base-10 numeration system during the pre-kindergarten to second-grade years are essential for every child (NCTM, 2000). Therefore, the NCTM defines the principles and standards of numbers and operations as follows:

- Understanding the place-value structure of the base-10 numeration system and representing and comparing whole numbers and decimals;
- Understanding the various concepts regarding addition and subtraction of whole numbers, as well as the relationship between the two operations;
- Computing and using strategies to develop fluency with basic number combinations that focuses on addition and subtraction.

The multi-digit base-10 positional numeration system is represented in terms of 1's, 10's, 100's, etc.; this means that each numeral has a different value depending on its place (i.e., place value) (Ball, 1988). According to Fuson (1990), multi-digit understanding is difficult because it requires children to understand not only how numbers can be partitioned according to the values of the Base-10 numeration system, but also how these values interrelate. Thus, to understand written numbers in multi-digit numbers, children must build name-value and positional base-10 numeration conceptual structures for the words and the numbers and relate these conceptual structures to each other, as well as to the words and the numbers (Fuson and Briars, 1990).

The difficulty in understanding numeracy concepts is well researched. Thomas et al. (2002) investigated children's understanding of the number system and found that children do not develop sufficient understanding of numeracy as a positional base-10 numeration system. Moreover, numeracy procedures involving the manipulation of digits, such as the regrouping concept (i.e., carrying over and borrowing) in addition and subtraction, are influenced by a sufficient understanding of the base-10 numeration system of numbers (Fuson, 1990).

According to Fuson (1992), children are often taught multi-digit addition and subtraction as sequential procedures of single-digit numbers, and digits are written in certain locations. More specifically, these procedures deal with multi-digit numbers as single-digit numbers situated next to each other rather than using a multi-digit place-value meaning for the digits in different positional base-10 numeration. Thus, they seem to be using a concatenated single-digit conceptual structure for multi-digit numbers.

The computational algorithms standard for addition and subtraction are typically performed according to the concept of place value. When addition and subtraction are approached, previous knowledge of place value must be considered by children. Addition and subtraction for multi-digit numbers must be done digit by digit beginning with the number on the right side (i.e., from lower place value to higher place value or from ones to tens and so on).

Many children who implement addition and subtraction computational algorithms accurately apply this procedurally and do not understand the essential aspects of the procedure and are not able to provide the values from the regrouping concept (i.e. carry and borrow) that they have already written (Canobi et. al., 1998; Canobi et. al., 2002). Actually, students must have some knowledge of the place-value concept to be able to regroup numbers through addition and subtraction. Therefore, through the learning of addition and subtraction, children build up knowledge about the place-value concept (i.e., ones, tens, hundreds, etc.) (Fuson, 1990; Fuson and Briar, 1990).

Piaget (1952) and Bruner (1966) recommended the use of physical materials for learning based on their observations of the way children interacted with their environment and the way they handled concepts not part of their previous knowledge. Piaget emphasised that children's conceptual development is based on their active interactions with objects. In other words, he stated that children who learned mathematical relationships using physical objects could build more accurate and inclusive mental representations than those who did not have these experiences.

When introducing place value, Dienes' blocks manipulatives were often mentioned. It is believed that Dienes invented and developed base-10 blocks, which are referred to as Dienes' blocks, to teach place value (Sriraman, 2007). Base-10 blocks consist of individual units and pieces: long pieces contain 10 units, flats pieces contain 10 long pieces and blocks contain 10 flat pieces. They are used to show place-value for numbers and to increase understanding of addition and subtraction algorithms (see Figure 2.4).

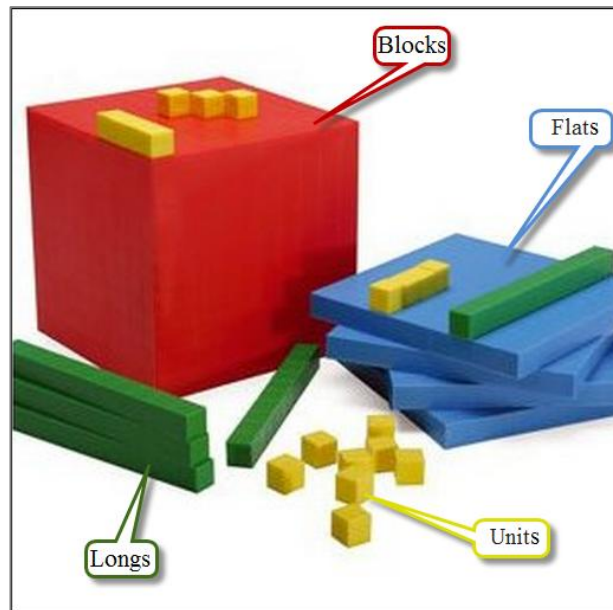


Figure 2.4 Dienes base-10 blocks manipulatives

The use of base-10 blocks enables children to create well-established, intuitive conceptual understanding for more formal arithmetic operations such as addition and subtraction (Goldstone and Son, 2005). Burris (2010) stated that using base-10 blocks increased perception of the positional base-10 numeration system, as traditionally viewed, and directly correlated to the mathematical algorithms for addition and subtraction. Once children understand place value, they can deal with complex algorithms without the need to memorise rules that may blur their understanding (Richardson, 1999). These topics were recognised as being difficult to comprehend at this early stage of knowledge development. Two research studies, conducted by Robinson and Dube (2009) and Ginsburg (2009), highlighted the lack of development of conceptual understanding of addition and subtraction concepts during children's early elementary school years. Similarly, math educators stressed the need for children's conceptual understanding of primary mathematical concepts. According to Brown (2007), this understanding improved mathematical thinking in later grades.

2.2 Virtual Reality

Virtual Reality is defined by Moshell et al. (2002) as “a real-time graphical simulation with which the user interacts via some form of analogy control, within a spatial frame of reference and with user control of the viewpoint's motion and view direction.” Another definition of virtual reality is as a 3D graphical simulation model where a user can control the viewpoints and motions and interact intuitively in real time, causing the virtual experience to feel more real (Wilson, 1999; Moshell et al., 2002). Furthermore, virtual reality can be described as a multi-sensory, highly interactive computer environment that makes the user believe they are actually experiencing a situation, even though they are, in reality, participating in an artificial environment (Gigante, 1993; Kameas et al., 2000; Roussou, 2004).

Virtual reality systems have three main characteristics: multi-sensory, interactive and inherent engagement of its users in an artificial environment (Burdea and Coiffet, 2004). Moreover, virtual reality is classified according to the level of immersion it provides ranging from semi-immersive (or desktop) to fully immersive (Scalese et al., 2008). Most virtual reality systems attempt to support users by providing the ability to interact with the system as they would with real objects in the real world. One of the central foundations for why virtual reality has been used for training and educational purposes is the aspect of high interactivity and the ability to present a virtual reality environment that is similar to the real world (Lee and Wong, 2008).

2.2.1 Virtual Reality Essential Attributes

Virtual reality can be seen as a continuation of the spectrum by which the real world is perceived (i.e., 2D, 3D and virtual reality). The 2D graphics are representations of any picture providing only the width and height but no depth, whereas 3D graphics are representations of an object providing the dimensions of width, height and depth (volume), but the viewer cannot see what is behind the image, and therefore, it is as if the viewer is looking at a 2D image of a 3D object (Giambrouno, 2002). In 3D images, only the sides of certain objects can be seen no matter how the head is moved around that image (Giambrouno, 2002).

In virtual reality environments, 3D objects can be rotated with respect to the user, or they can remain stationary, allowing the user to move around them, select a certain portion to view or zoom in and out for more or less detail; it is as if the user is looking through the viewfinder of a rapidly moving video camera (Foley et al., 1997). Animation can illustrate the movements of 2D or 3D objects, but virtual reality provides users with a stronger sense of 'being there' (Trindade et al., 2002).

In psychology, it is understood that a 2D image is represented in the human brain as a recognised set of 3D shapes arranged in a 3D space; this scenario is referred to as the mental map of our understanding of the world conveyed by the image. However, in 3D computer graphics, the equivalent of the mental map is the graphical scene, and the snapshots of the map are the equivalent of 3D modelling objects (Larnder, 2002).

Therefore, one has to make the distinction between 3D graphical environments and virtual reality environments. The distinguishing features of a virtual reality environment from other 3D modelling systems are its concentration on real-time graphics, interactive capabilities and immersion, rather than the simple dimensions of the graphics (Wilson et al., 1996). There are three main features that distinguish virtual reality systems from other graphical multimedia applications (Wilson et al., 1996; Stry, 2001; Trindade et al., 2002):

- Navigation: This is the most prevalent user action in a virtual reality environment. It presents challenges such as supporting spatial awareness and providing efficient and comfortable movement between distant locations so that users can focus on more important tasks.
- Interaction: Interactions between the user and the environment are in real-time with 3D objects, and these interactions generate the subjective feeling of being present.
- Immersion: immersion means the feeling of presence, where presence is interpreted as the sense of being in the environment that is depicted by virtual reality technology and the ability to act within that environment.

These features are distinctive from other visual educational technologies such as film, television and multimedia. A photo or movie may show students the internal geometry of objects, but only virtual reality allows them to enter inside and observe it from any perspective (Stry, 2001; Trindade et al., 2002).

Hedberg and Alexander (1996) argued that the defining attribute of virtual reality environments was the range of interactive multimedia environments capable of displaying various degrees of the previously mentioned distinguishing features through what has been termed virtual worlds, or varying degrees of virtuality (Figure 2.5).

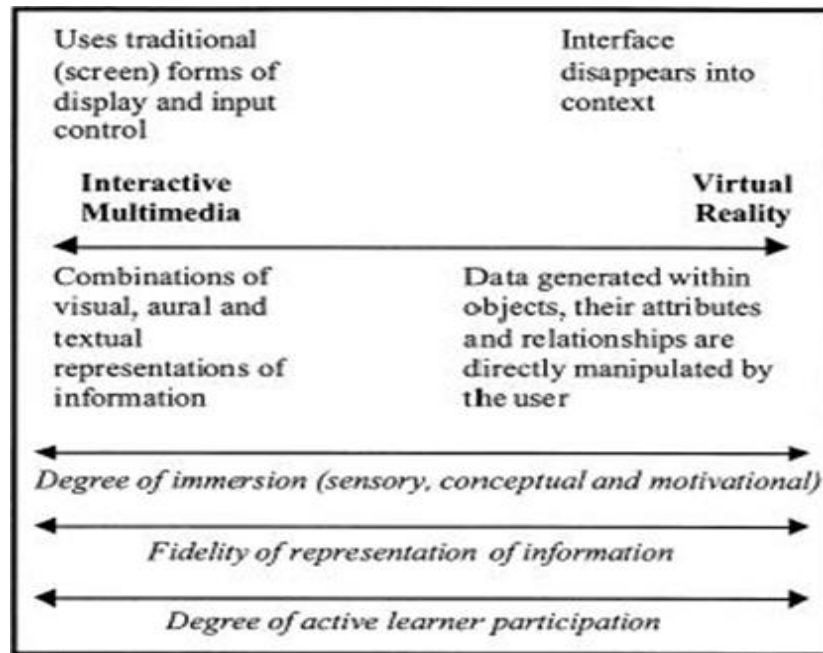


Figure 2.5 Comparisons between interactive virtual reality and multimedia (adapted from Hedberg and Alexander, 1996).

2.2.2 Types of Virtual Reality Systems

Virtual reality systems vary according to the type of technological equipment used, such as displayed hardware and interaction devices. Virtual reality systems (Figure 2.6) are generally classified according to the level of immersion they provide, ranging from semi-immersive (or desktop) virtual reality to fully immersive virtual reality to augmented reality (AR) (Mantovani et al. 2003; Christou, 2010). Desktop and fully immersive virtual reality systems have been widely used by industrial enterprises to train their personnel and in educational systems as learning tools (Ritke-Jones, 2010).

The simplest ways to display a virtual reality world in a semi-immersive system is through the use of large-screen projection (see Figure 2.6[1]) with or without stereo, a table projection system or with a conventional monitor (see Figure 2.6 [2]) where the interaction is performed using a regular mouse and keyboard (Christou, 2010).

A virtual world is another form of desktop virtual reality that has the ability to offer new competence for users to enhance and support learning. Collaborative virtual world systems provide interactions among two or more avatars controlled by humans (Christou, 2010). Many open-source software packages are available to enable the creation of a virtual world. Some of the most well-known examples of virtual worlds include the following (Varcholik et al., 2009):

- Second Life (<http://secondlife.com/>),
- Active World (<http://www.activeworlds.com/>),
- Open Wonderland (<http://openwonderland.org/>),
- Open Simulator (http://opensimulator.org/wiki/Main_Page)

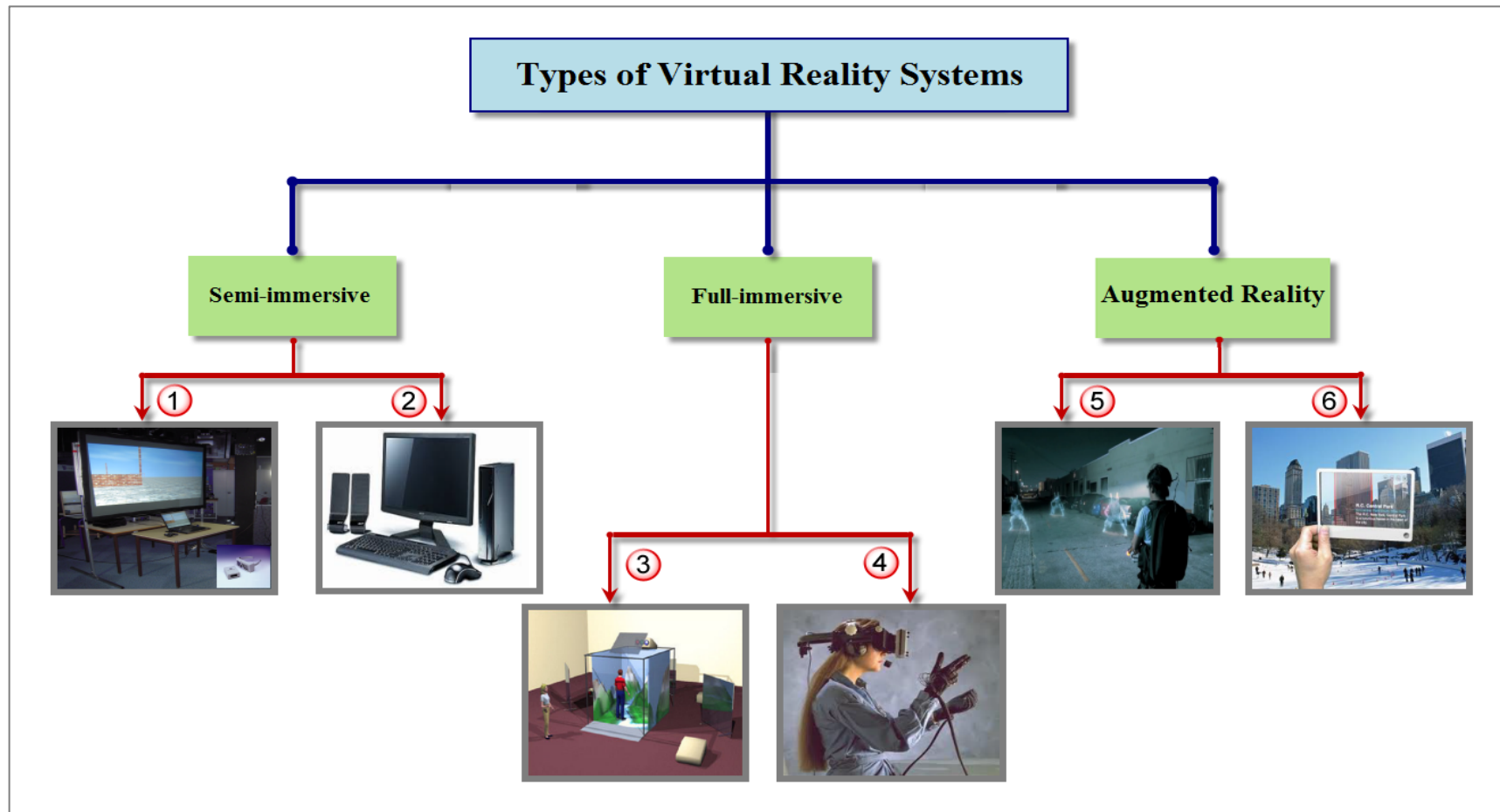


Figure 2.6 Types of Virtual Reality Systems: (1) projection screen, laptop and i-glasses; (2) conventional monitor, keyboard and mouse; (3) CAVE; (4) HMD and Data Gloves; (5) AR Game; (6) AR Smart glasses.

Fully immersive virtual reality systems use full-scale representations via the CAVE system (Figure 2.6 [3]) or a head-mounted display (HMD) (Figure 2.6 [4]), and the interaction for both may be controlled by using a tracked handheld input device such as data gloves. Fully immersive virtual reality systems motivate users through visual, auditory and other sensory stimuli to increase their virtual reality experience to seem as though they are situated in the real world (Garcia-Ruiz et al., 2010). The financial investment is quite expensive because of the cost of the hardware needed for a powerful graphical workstation, including a head-mounted display, data gloves, etc. (Sun et al., 2011).

Augmented reality technology was defined by Ucelli et al. (2005) as a “blend of manipulation and visualisation through the overlay of synthetic environments over real ones” (Figure 2.6 [5]). Furthermore, AR systems are a combination of virtual reality and real-world attributes (Figure 2.6 [6]) by integrating computer graphic objects into a real-world scene (Lee and Wong, 2008). They take virtual reality a step further by allowing the user to interact with real and virtual objects simultaneously.

Interacting with virtual reality environments to manipulate objects in the virtual world is not completely natural because of the use of sensors, effectors and input devices. Interaction can be defined as the ability of the user to take action within the virtual reality environment (Jackson and Fagan, 2000). This interaction task is performed according to inputs generated by the user in the virtual reality environment, as is the case for both immersive and desktop virtual reality environments.

An interaction technique outlines the mapping path between the user and the virtual reality environment and determines how the environment will react when the user interacts using the input devices (Dix et al., 2004). Bowman et al. (2001) stated that there are three types of interaction tasks that can be implemented within virtual reality environments using a given input device. These three types of interaction tasks are: navigation, selection/manipulation and system control. Navigation is choosing a particular orientation, as well as a particular location (Dix et al., 2004). The navigation interaction tasks refer to the tasks of efficiently moving the viewpoint within the 3D space using environmental cues and artificial aids (Bowman, 2002; Haik et al., 2002).

The navigation tasks can generally be classified into the following three categories (Bowman et al., 2002): exploration, search tasks and manoeuvring that enable users to place the viewpoint at a more advantageous location to perform a particular task. The selection task refers to the specification of one or more objects to which a command will be applied. It might also denote the beginning of a manipulation task. A manipulation task refers to the modification of various object attributes including position and orientation or other properties (Bowman, 2002). System control refers to a task in which a command is applied to change either the state of the system or the mode of interaction (Bowman et al., 2001).

In virtual reality learning environments, the user can have direct experiences with objects and is able to possibly gain a full spectrum of information, exploration and feedback regarding those objects. Therefore, users need to navigate through the virtual world to manipulate the virtual objects. The navigational behaviours in desktop virtual reality systems can be performed using a conventional keyboard and mouse.

Designers have been struggling to find comparably easy-to-use mechanisms for full six-degrees-of-freedom navigation (x , y and z position and yaw, pitch and roll (see Figure 2.7)) (Wu et al., 2011).

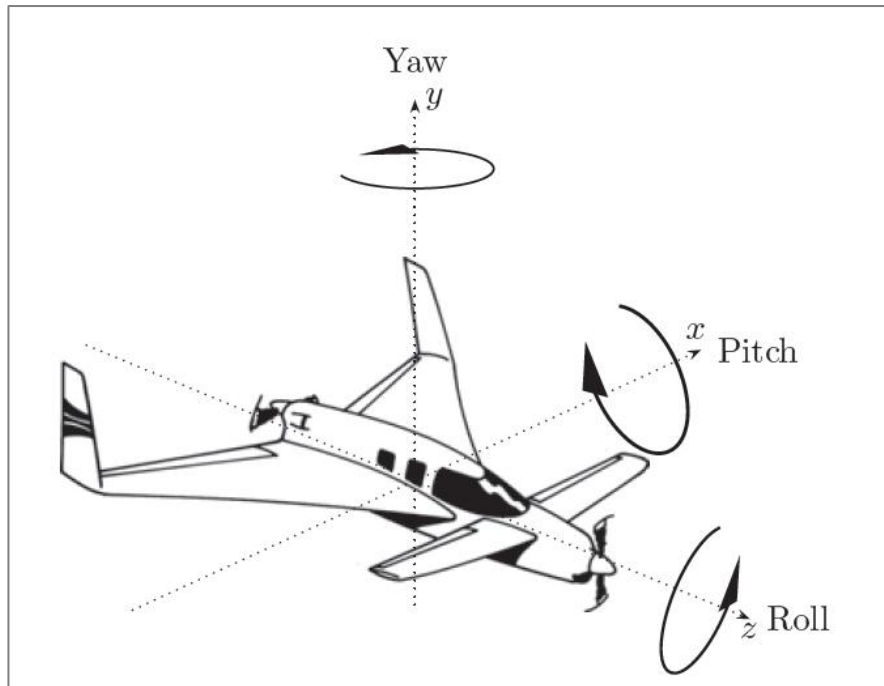


Figure 2.7 The aircraft Yaw, Pitch and Roll around the 3-axis (x , y , z) in 3D graphics

Although immersive virtual reality technology is less expensive than it was a decade ago, such immersive virtual reality devices are weighty such as wearing the head mounted display (see Figure 2.6[4]) and can cause cyber-sickness (Aoki et al., 2008), which restricts their use in schools and colleges (Lee et al., 2009). Moreover, the use of virtual worlds (e.g., Second Life) seems useful for learning, but these systems are online, and in order to use them successfully, there must be a good network infrastructure, which is not available in many schools.

The major advantage of desktop virtual reality systems over immersive virtual reality systems is the cost, which makes it an attractive solution for many applications (Gaoliang et al., 2009). Moreover, designing desktop virtual reality systems is more economical and requires only less expensive devices, such as a mouse and conventional display monitor (Gaolianga et al., 2010). Therefore, a desktop virtual reality system is a suitable educational tool in the classroom and is useful for many educational applications (Sun et al., 2010); however, this type of virtual reality technology does not provide the feeling of full immersion in the environment to the user (Limniou et al., 2008).

In general, desktop virtual reality systems offer an affordable solution in many virtual reality applications due to its low cost and portability (Gaoliang et al., 2009). The cost, availability and flexibility of desktop virtual reality systems make them easily adoptable and adaptable by teachers and students without major capital expenses and effort (Sun et al., 2010). Future desktop virtual reality technology will have a growing demand in both education, training and different government and economic sectors (Garcia-Ruiz et al., 2010).

2.2.3 Virtual Reality as Visualisation of Knowledge

Visualisation is defined as “mechanisms by which humans perceive, interpret, use and communicate visual information” (McCormick et al., 1987). Visualisation of cognition is identified by Sánchez et al. (2000) as an “externalisation of mental representations embodied in artificial environments”. Sánchez et al. (2000) also defined the visualisation of knowledge as exploring information in order to add understanding and insight to it.

Three different modes of knowledge acquisition that are constructed and generated in the virtual domain were introduced by Peschl and Riegler (2001): empirical, constructive and synthetic. In a virtual reality learning environment, the user is no longer looking at data on a screen but rather is immersed as active participants within the data. It seems that the capabilities of virtual reality learning environments facilitate learning through a process of self-paced exploration and discovery. Messinis et al. (2010) showed that there is a positive relationship between interaction and a sense of presence.

The strong interaction within virtual reality learning environments provides users with an increased sense of presence. This sense of presence is caused by navigating through a virtual world in a way similar to that of a moving camera; this concept assumes that there is a real person viewing and interacting with the virtual world.

It seems that the virtual reality learning environments function as cognitive tools that are capable of making intangible things tangible; they can also be designed to make the abstract more concrete and able to be seen (Chen et al., 2004). However, interactions in virtual reality learning environments allow students to construct knowledge from direct experiences, not from the description of the experience, and this satisfies the constructivist theory by Piaget (1972). Similarly, Doolittle (1999) defined constructivism as a theory of knowledge acquisition, not a theory of pedagogy.

Visualisation in virtual reality is used to simulate a concept not able to be comprehended in a 3D model (Mills and Araujo, 1999). In fact, Trindade et al. (2002) attributed the difficulties in visualising a 3D model to the lack of one's ability to mentally rotate a 3D model, short depth perception or an inadequate sense of perspective. Consequently, with the help of virtual reality learning environment capabilities, educators are building a new visual language that bridges the gap between the physical world and the abstract world (Yair et al., 2001). This emphasises the notion that virtual reality learning environments are ideal for allowing students to explore ideas and construct knowledge based on their experiences without relying on symbol systems. The distinguishing characteristics of virtual reality learning environments (Dalgarno and Lee, 2010) are classified as either representational fidelity or learner interaction:

- Representational fidelity:
 - Realistic display of environment,
 - Smooth display of view changes
 - Object motion, Consistency of object behaviour, User representation, Spatial audio, Kinesthetic and tactile force feedback.
- Learner interaction:
 - Embodied actions including view control,
 - Navigation and object manipulation,
 - Embodied verbal and non-verbal communication,
 - Control of environment attributes and behaviour,
 - Construction of objects and scripting of object behaviours

2.2.4 Studies of Virtual Reality in Education

For educational purposes, in general, virtual reality has been widely proposed as a significant technological breakthrough that possesses an immense potential to facilitate learning. In recent decades, the constructivism learning theory has become increasingly popular as a virtual reality learning theory (Winn, 1993); Winn concluded that the constructivist theory provided a conceptual framework for virtual reality in education and was considered as a valid and reliable theory of learning in virtual reality. The constructivist idea that users can build their own worlds has been shown to be useful in teaching children abstract ideas and concepts (Piaget, 1972). The key to the compatibility of virtual reality with constructivism lies in the notion of immersion (Rose, 1995).

Virtual reality technology allows for the creation of virtual reality learning environments where students can learn by interacting with virtual objects similar to how they would with real objects. The most important reasons cited for developing virtual reality in education are as follows (Pantelidis, 1995):

- Explore things and places that without alterations of scale in size, time and distances, could not otherwise be effectively examined.
- Teaching using the real thing is impossible, dangerous and inconvenient.
- Interacting with a model is as motivating as or more motivating than interacting with a real thing.
- Experience of creating a simulated environment or model is important to the learning objective.
- Visualisation, manipulation and rearrangement of information are needed, so as to become more easily understood.

The most established, well-known and successful research work involving virtual reality for general education can be traced to the Human Interface Technology (HIT) Lab at the University of Washington, Seattle. Since 1990, their focus has been to allow students to build their own virtual environments by applying the constructivist approach. The Virtual Reality Roving Vehicles (VRRV) project, started by Rose (1995) at HIT Lab, was developed to evaluate the experience of immersive virtual reality as a tool for students and teachers. Osberg et al. (1997) performed the first pilot study for the VRRV project through the development of wetlands ecology life cycles for water, carbon, energy and nitrogen.

The Science-Space project at George Mason University, Fairfax, Virginia, was based on fully immersive virtual reality and the non-collaborative learning mode (Salzman et al., 1999). This project represented Newton's laws and the laws of conservation, electrostatics and molecular representations and quantum-molecular bonding. All these virtual environments focused on using immersive virtual reality to convey abstract scientific concepts and to aid complex conceptual learning.

The Electronic Visualization Laboratory (EVL) at the University of Illinois at Chicago is a graduate research laboratory specialising in virtual reality, visualisation and advanced networking. Roussou et al.'s (1995-1999) project, the Narrative-Based Immersive Constructionist/Collaborative Environments project, was conducted at EVL and applied virtual reality for the creation of a family of educational environments for young users with the aim to build an experiential learning environment that would engage children in authentic activities.

Their approach was based on constructivism, where real and synthetic users, motivated by an underlying narrative, build persisting virtual worlds through collaboration.

The potential of virtual reality to benefit education is widely recognised, and a number of studies have conclusively demonstrated the ability to teach content using virtual reality technologies. In geometry, Hwang et al. (2009) stated that using 3D virtual reality and its manipulation enhances and supports understanding the concepts of geometry. Fishwick (2009) created a multi-user, meta-gaming Second Life virtual environment with the aim of exploring its use for performing basic algebra operations. Figueira-Sampaio et al. (2009) developed a virtual balance environment that allowed the concepts and procedures of mathematical algebra equations in elementary school to be represented in a virtual reality environment.

The Desktop Virtual Reality Earth Motion System was designed and developed to be applied in the classroom (Chen et al., 2007). The system was instigated to aid elementary school students in understanding the motions of the earth using virtual reality. Statistical results reported that students were able to comprehend concepts when their learning was supported by virtual reality. An interactive immersive virtual reality learning environment developed by Roussou (2009) specifically for fractions in mathematics showed that children who fully interacted with the virtual reality environment were able to problem solve, but there was no evidence supporting the expected conceptual change.

A study conducted by Song and Lee (2002) concluded that virtual reality to visualise geometric objects is a good visual aid tool in middle school mathematics classes, as well as for any class that requires a detailed description of physical reality beyond what is possible using a verbal approach. Cyber-Math is an extendable, avatar-based shared virtual environment used to teach and explore non-trivial mathematics relating to a variety of mathematical subjects (Taxén and Naeve, 2002).

Virtual reality applications play an important role in supporting a great variety of fields. Virtual reality applications in learning represent a promising area with a high potential of enhancing and modifying the learning experience. Moreover, virtual reality learning environments are an experiential learning tool that helps students to learn in a natural, interactive, engaging educational context. The challenge for researchers of virtual reality applications in learning is to demonstrate that they can produce learning outcomes that are different, if not better, than outcomes achieved by other means (Jackson and Fagan, 2000).

Although virtual reality can serve as valuable supplemental teaching and learning resources to augment and reinforce traditional learning methods (Dean et al., 2000), it cannot entirely replace conventional classroom teaching techniques. Virtual reality learning environments can serve as valuable additions to traditional learning systems by supporting the learning of different concepts and skills. However, virtual reality still presents a challenge to learners and developers.

All these virtual reality applications, whether immersive or non-immersive, exploit the visual strength of virtual reality, which is known to be important in gaining conceptual understanding. More relevant in the present context is to mention that one of the main values of virtual reality is its ability to give substance to abstract concepts.

2.3 Cognitive Approach for Multimedia Instructional Design

The term cognitive can be defined as mental activities that require both memory storage and information processing such as attention, perception, action and problem solving (Craik and Lockhart, 1972). Cognitive, in this research, means to act with human cognition or represents models of human cognition. Understanding cognition understands the cognitive process. In psychology, the cognitive process is the process by which information is encoded, stored and retrieved in the mind (Winn and Snyder, 1996).

Cognitive learning is defined as the acquisition of knowledge through cognitive processes (Clark and Harrelson, 2002). It is a creation of mental representations of physical objects in our memory. The foundation of the cognitive process is using knowledge to direct and adapt actions towards world goals. The way we learn is constrained by our memory system (Clark and Mayer, 2008). The human information-processing approach focuses on how the human memory system acquires, transforms, compacts, elaborates, encodes, retrieves and uses information (Moore et al., 1996).

The memory system is divided into three main storage structures (Baddeley et al., 2009; Craik and Lockhart, 1972; Clark and Harrelson, 2002): sensory memory, working memory and long-term memory, which are defined as follows:

- Sensory memory is the short-term part of memory. It is the capability to retrieve impressions of sensory information after the origin stimuli have ended. It acts as a sort of buffer for stimuli received through the five senses of sight, hearing, smell, taste and touch.
- Working memory functions as a type of “scratch-pad” for temporary retrieval of the information which is being processed at any time. It can be considered as the ability to remember and process information simultaneously.
- Long-term memory is, clearly enough, proposed for storage of information over a long period of time. It can keep a seemingly infinite amount of information more or less indefinitely.

Working memory keeps a small amount of information (typically around seven items or less) in the mind in an active, readily available state for a short period (typically from 10-15 seconds or sometimes up to a minute) (Baddeley et. al., 2009). The information in working memory transferred to long-term memory by a process of consolidation involving rehearsal and meaningful association. Contrasting working memory, long-term memory encodes information for storage semantically, or based on meaning and association.

To determine the conditions that maximise learning, it is important to examine human cognition. Once we have established the processes of human cognition, including why those processes have their particular characteristics, we are in a position to design learning environments in accordance with the human cognitive architecture.

Cognitive scientists seek to understand human cognitive processes such as perceiving, thinking, remembering, understanding language and learning (Sorden, 2005). Cognitive scientists, such as Allan Paivio, Alan Baddeley, John Sweller and Richard Mayer, provide numerous distinguished assumptions for cognitive processes, which set up a framework for using experiential theories of cognition and learning that improve multimedia instruction and help humans in learning more effectively.

Paivio's dual-coding theory and Baddeley's model of working memory suggest that humans process information through dual channels: one auditory and the other visual. This belief, combined with Sweller's theory of cognitive load, provides a convincing argument for how humans learn. Accordingly, Mayer's cognitive theory of multimedia learning presented how multimedia instruction can be designed to maximise learning. Mayer's cognitive theory of multimedia learning provides empirical guidelines that may help designers create multimedia instruction more effectively.

In the following sections, we provide more details about cognitive learning theories in regards to human cognitive architecture and explain how these theories could help multimedia designers gain insight for the intention of designing an effective multimedia cognitive learning tool.

2.3.1 Paivio's Dual Coding Theory

Paivio's dual-coding theory is possibly one of the most fundamental theories of multimedia learning. This theory was initially developed in the 1971. The basic idea is that visual and verbal information is processed differently along distinct channels in working memory.

Paivio discusses the idea that the cognitive process occurs within two separate information coding systems: a visual coding system for processing visual knowledge and a verbal coding system for processing verbal knowledge. These visual and verbal coding systems act as two channels in which information travels down either individually or simultaneously. As information goes through these channels, many connections are developed during the process of cognition.

Figure 2.8 is a visual representation of the dual-coding theory. The dual-coding theory identifies three types of connections: (a) representational connections, which are made between verbal or non-verbal information received by the learner; (b) referential connections, which indicate the activation of the verbal system by the non-verbal system or vice-versa; and (c) associative connections, which are made within the verbal and non-verbal channels. A given task may require any or all three kinds of processing. These connections are activated depending on the learner's previous knowledge or experiences. These types of connections are advocating for by supporters of multimedia instruction based on the belief that if information is coded verbally and visually simultaneously, the information will likely be remembered because one can activate the other.

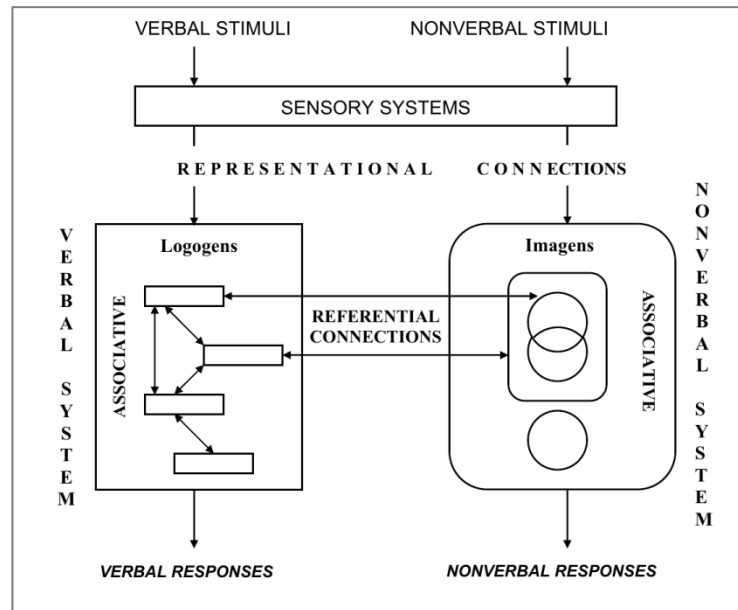


Figure 2.8 Verbal and nonverbal symbolic systems of Dual Coding Theory. Adopted from *Mental Representations: A Dual-Coding Approach*, Paivio, 1986.

Suh and Moyer-Packenham (2007) examined the application of Paivio's theory in multi-representational virtual mathematics environments. This study compared mathematics achievement in two third-grade classrooms using two different representations (virtual and physical manipulatives) in the lesson concerning rational numbers and algebraic concepts. The participants in this study were 36 third-grade students; all students participated in both treatment groups. The results showed that the virtual manipulatives treatment group performed significantly better on all test items than the physical manipulatives group.

Paivio's work has inferences in many areas regarding human factors, interface design and the development of educational multimedia. The dual-coding theory is complemented with the theory by Baddeley in which working memory is divided into a visuospatial sketchpad and a phonological loop.

2.3.2 Baddeley's Working Memory Model

Alan Baddeley and Graham Hitch proposed a model of working memory (1974) in an attempt to provide a more accurate representation than previously provided. The original model of Baddeley (Figure 2.9) was composed of three main components: the central executive is a flexible system that is responsible for the control of cognitive processes; the phonological loop deals with sound or phonological information; and the visuospatial sketchpad are assumed to hold information about what we see.

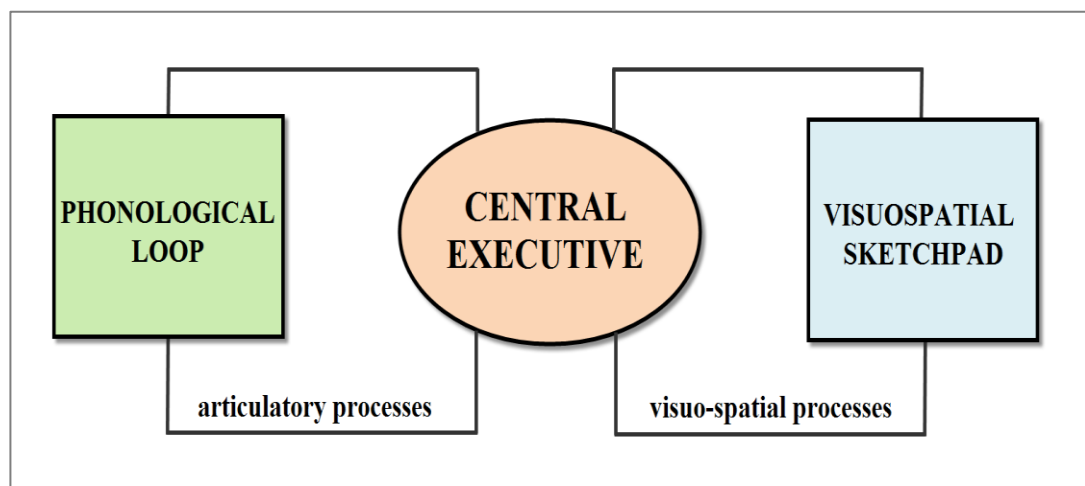


Figure 2.9 The working memory model, adapted from Baddeley and Hitch (1974)

The original model supposed a limited capacity controller, the central executive, supported by two tentative storage systems, the phonological loop and the visuospatial sketchpad. The phonological loop enables the processing of either spoken or written verbal information, whereas the visuospatial sketchpad is responsible for the processing of visual and spatial information (Baddeley et al., 2011).

The central executive system is in direct contact with both the phonological loop and the visuospatial sketchpad. This three-component system has been revised by Baddeley (2000) to include a fourth component, the episodic buffer. The episodic buffer is a temporary multimodal storage component that works as a limited capacity store that can incorporate information from the visuopatial sketchpad and from the phonological loop, creating a multimodal code (Zheng et al., 2011).

2.3.3 Sweller's Cognitive Load Theory

Sweller's (1999) cognitive load theory states that intensive information given simultaneously causes cognitive overload, inadequate information acquisition and processing by learners. Cognitive load represents the information load placed on working memory through instruction. The cognitive load theory was designed to introduce guidelines to assist in the presentation of information in a manner that motivates learners to become engaged in activities that improve their intellectual performance (Reed, 2006).

Sweller argued that the cognitive load is minimised by the use of dual-mode (visual-auditory) instructional techniques, and the limited capacity of working memory is boosted if information is processed using both the visual and auditory channels, based on Baddeley's model of working memory (Gyselinck et. al., 2008; Sorden, 2005).

2.3.4 Mayer's Multimedia Theory

Mayer (1997) was convinced that one of the most important avenues of cognitive psychology understood how technology, such as multimedia, can be used to foster student learning. Mayer assumed that multimedia could improve learning by presenting different types of information to the user at the same time. Mayer (2002) presented the cognitive theory of multimedia learning (Figure 2.10) as three assumptions about how people's minds work with pictures and words in multimedia learning: the dual-channel assumption, the limited capacity assumption and the active processing assumption (Bradford, 2011; Austin, 2009; Mayer, 2005, 2002; Robinson, 2004).

First, the dual-channel assumption is a central feature of Paivio's 'Dual-Coding' theory and Baddeley's theory of 'Working Memory'. This assumption denotes that the human possesses separate information processing channels for verbal and visual material: a visual-pictorial channel and an auditory-verbal channel.

Second, the limited capacity assumption is a central assumption of Baddeley's theory of 'Working Memory' and Sweller's 'Cognitive Load' theory. This assumption implies that there is only a limited amount of cognitive processing capacity available in the verbal and visual channels.

Third, the active processing assumption is drawn from Wittrock's 'Generative-Learning' theory and Mayer's active learning processes (selecting–organizing–integrating). This assumption asserts that meaningful learning demands significant

active learning processes within the verbal and visual channels in the working memory at the same time.

Figure 2.10 presents the cognitive theory of multimedia learning. The boxes in Figure 2.10 represent memory stores, including sensory memory, working memory and long-term memory. A multimedia lesson consists of pictures and words in printed or spoken form (indicated on the left side of the figure). The pictures and printed words enter the sensory memory through the eyes and the spoken words enter through the ears (indicated in the ‘Sensory Memory’ box).

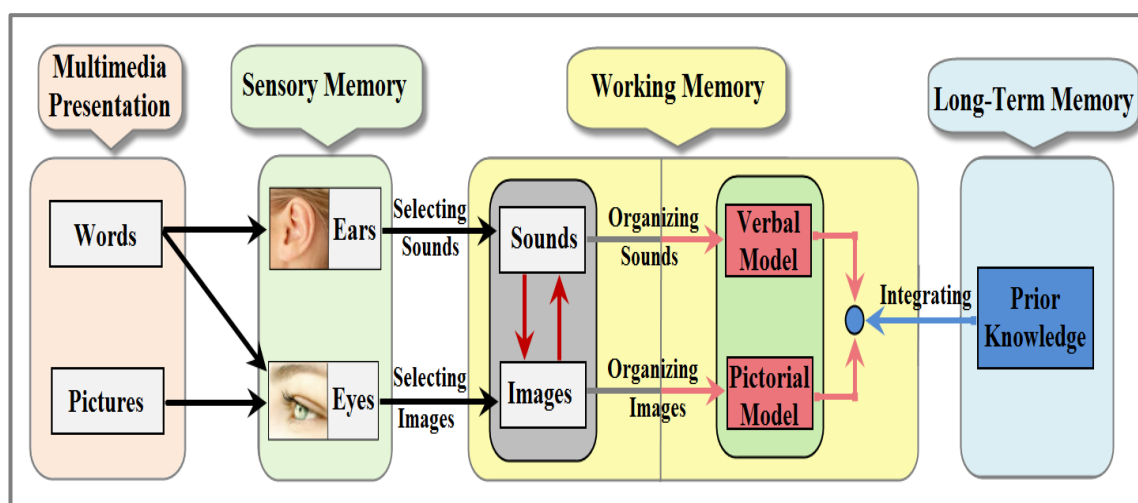


Figure 2.10 Cognitive theory of multimedia learning, adapted from (Mayer, 2002)

Sensory memory allows pictures and printed text to be captured as exact visual images for a short time period in the visual sensory memory (at the top) and for spoken words and other sounds to be captured as exact auditory images for a short time period in the auditory sensory memory (at the bottom). The arrow from ‘pictures’ to the eye corresponds to a picture being recorded in the eyes; the arrow from ‘words’ to the ear corresponds to spoken text being recorded in the ears; the arrow from ‘words’ to the eye corresponds to printed text being recorded in the eyes. If the learner pays attention,

some of the materials will be selected for further processing in the working memory (represented by the arrow labelled 'Selecting Images' and 'Selecting Sounds').

The left side of the box labelled 'Working Memory' in Figure 2.10 represents the raw material that comes into the working memory—the learner processes a few pieces of information at one time in each channel. The arrow from 'Sounds' to 'Images' represents the mental conversion of a sound (such as the spoken-word horse) into a visual image (such as an image of a horse); thus, when the word horse is spoken, a mental image of a horse may also be formed. The arrow from 'Images' to 'Sounds' represents the mental conversion of a visual image (such as a mental picture of a horse) into a sound image (such as the sound of the word horse); thus, the word horse may be heard in the mind when looking at a picture of a horse.

In contrast, the right side of the "Working Memory" box in Figure 2.10 represents the knowledge constructed in the working memory—the learner can mentally organise selected images into pictorial models (represented by the arrow labelled 'Organising Images') and selected words into verbal models (represented by the arrow labelled 'Organising Words').

Finally, the box on the right labelled 'Long-Term Memory' (Figure 2.10) corresponds to the learner's storehouse of knowledge. For learners to actively think about material stored in the long-term memory, it must be brought into the working memory (as indicated by the arrow from 'Long -Term Memory' to 'Working Memory'). Meaningful learning occurs when the learner appropriately engages all of these

processes. Thus, Mayer argued that there were three important cognitive processes, which are indicated by the arrows in Figure 2.10:

- **Selecting:** The first step is to pay attention to relevant words and images in the presented material.
- **Organizing:** The second step is mentally to organize the selected materials in coherent verbal and pictorial representation.
- **Integrating:** The final step is to integrate the incoming verbal and pictorial representations with each other along with prior knowledge.

The cognitive theory of multimedia learning was extensively studied by Mayer and his associates to investigate how best to design multimedia presentations. Eight principles were concluded by Mayer and his associates that yielded an understanding of how to use multimedia to help students grasp a scientific explanation (Mayer, 2002): multimedia, contiguity, coherence, modality, redundancy, interactivity, signalling and personalisation. The definitions of the eight principles are presented as follows (Robinson, 2004; Mayer, 2011):

- **Multimedia principle:** Deeper learning occurs from the use of words and pictures than from words alone. According to the cognitive theory of multimedia learning, further understanding occurred when students mentally linked pictorial and verbal models of an explanation.
- **Contiguity principle:** Deeper learning results from presenting words and pictures simultaneously rather than successively. The cognitive theory of multimedia learning stated that a simultaneous presentation would increase the number of opportunities to match the pictures and words that need to be simultaneously processed to enable the construction of connections between them.

- Coherence principle: Deeper learning occurs when extraneous words, sounds or pictures are excluded rather than included. The cognitive theory of multimedia learning assumes that adding interesting but unrelated material to a multimedia presentation overloaded one or both of the processing channels and obstructed the integration of pictorial models, verbal models and previous knowledge.
- Modality principle: Deeper learning occurs when words are presented as narration rather than as on-screen text. The cognitive theory of multimedia learning indicated that the use of on-screen text and animation could overload the visual channel, whereas the use of narration could free up the visual resources to focus on the animation.
- Redundancy principle: Deeper learning occurs when words are presented as narration rather than as narration and on-screen text. The cognitive theory of multimedia learning indicated that students learned more from animation and narration than from the combination of animation, narration and on-screen text.
- Interactivity principle: Deeper learning occurs when learners are allowed to control the presentation rate of information. The cognitive theory of multimedia learning stated that adding an interactive user control could improve learning because it could allow students to activate their cognitive processes at their own rates and reduce the chances of cognitive overload.
- Signalling principle: Deeper learning occurs when key steps in the narration are signalled rather than nonsignalled. According to the cognitive theory of multimedia learning, if signalling directed the learner's attention to key events and the relationships among them, this action could enhance integration.
- Personalisation principle: Deeper learning occurs when words are presented in a conversational style rather than a formal style.

Mayer supported his suggested principles by referring to results from the abundance of research started in 1997 and entering their third decade of continuous data collection that ultimately reached a consistently recognised state (Moreno et al., 2011; McLaren et al., 2011; Mayer, 2011; Johnson and Mayer, 2010; Campbell and Mayer, 2009; Clark and Mayer, 2008; Harskamp et al., 2007; Mayer et al., 2006; Atkinson et al., 2005; Mayer et al., 2005; Mayer, 2003; Plass et al., 2003; Mayer and Moreno, 2003, 2002; Mayer, 1997, 1998; Baker and Mayer, 1999; Moreno and Mayer, 1999; Moreno et al., 2001; Mayer, 2001; Mayer et al., 2001; Quilici and Mayer, 2002). The outcomes from the experiments have partly verified many of the eight fundamental principles and have resulted in new various, but not static, principles that demonstrate the dynamic nature of the cognitive theory of multimedia learning (Sorden, 2012).

The adaptation and expansion of the theory appeared frequently in the literature. The cognitive theory of learning with media (Figure 2.11), proposed by Moreno (2006), expanded the cognitive theory of multimedia learning (Mayer, 2002) to include media such as virtual reality, agent-based and case-based learning environments, which may present the learner with instructional materials other than words and pictures .

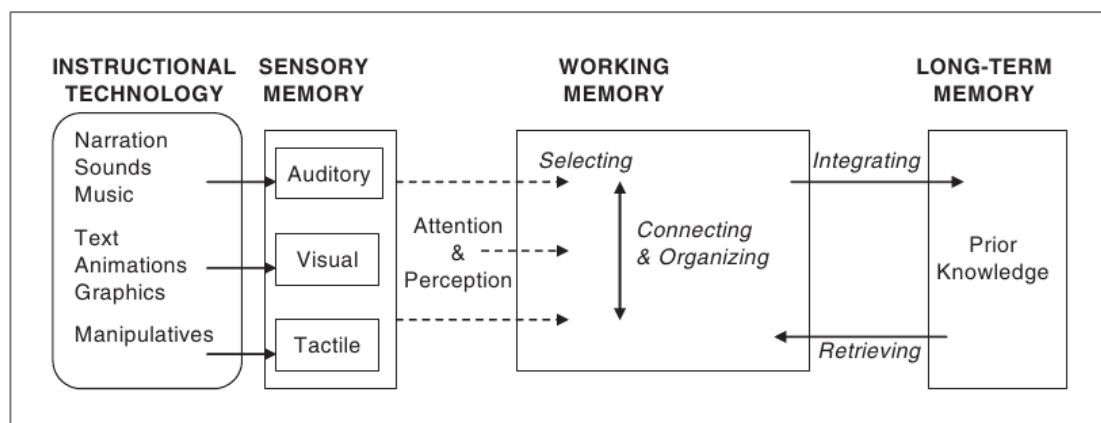


Figure 2.11 A framework of a Cognitive Theory of Learning with Media, Moreno (2006)

Moreno's (2006) cognitive theory of learning with media is based on the following learning assumptions: "(a) Learning starts when information is processed in separate channels for different sensory modalities; (b) only a few pieces of information can be consciously processed at one time in the working memory; (c) long-term memory consists of a vast number of organised schemas; (d) knowledge may be represented in long-term memory in verbal and nonverbal codes; (e) after being sufficiently practised, schemas can operate under automatic processing; and (f) conscious effort needs to go into selecting, organising and integrating the new information with existing knowledge (i.e., active processing)".

In conclusion, Mayer's research explained how cognitive science could inform instruction and how research regarding instruction could reinforce the theories of cognitive science. Despite the fact that some principles, such as the interactivity principle, were based on the notion that learner interaction has its own impact on the learning process, Mayer did not change his model to include the tactile modality.

CHAPTER 3 Design and Implementation of Mathematical Virtual Learning Environment (MAVLE)

This chapter presents a description of the processes for the design, implementation and evaluation of a virtual reality manipulatives (VRM) learning environment entitled the mathematical virtual learning environment (MAVLE). The developed MAVLE system was based on a proposed design framework called the virtual reality model for cognitive learning. This system explicitly intensifies the use of base-10 blocks manipulatives to support the learning and teaching of the numeracy concepts addition and subtraction.

3.1 Virtual Reality Model for Cognitive Learning

The focus of this research was how best to use virtual reality, specifically navigational behaviours (exploration, manoeuvring and manipulation), for the design of instructional cognitive learning content for math. The researcher's reliance on a cognitive view of the science of learning was made explicit throughout with reference to Mayer's (2002) cognitive theory of multimedia learning and Moreno's cognitive theory of learning with media that was an expansion of Mayer's theory. In order to design multimedia instructional learning content to promote deep understanding in learners, Mayer (2002) used the three basic assumptions of the cognitive theory of multimedia regarding how people learn from words and pictures: the dual channel assumption by Paivio (1990), the limited capacity assumption by Sweller (1999) and the active processing assumption by Mayer (2001). Mayer's cognitive theory of multimedia

learning (Figure 3.1) fits into a constructivist paradigm for learning in which instruction must become personally relevant to the learner.

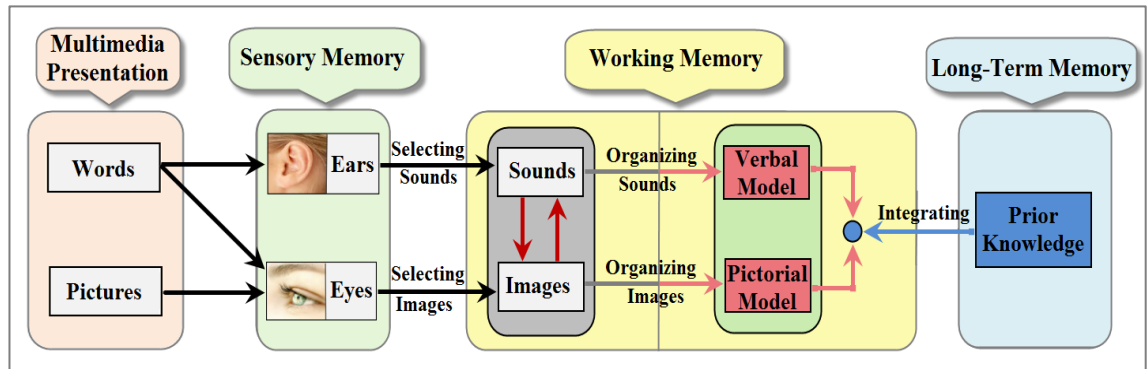


Figure 3.1 Multimedia Cognitive Learning Theory, Mayer (2002)

In order to design an effective virtual reality learning environment as a cognitive tool, a cognitive model is needed to integrate the technological attributes of virtual reality's learning systems into cognitive processing. It is the view of this researcher that Mayer's model (Figure 3.1) is not an exhaustive representation of all learning activities. Mayer's model stresses only two main sensory inputs: the visual and auditory modalities. It lacks the inclusion of user interactions, such as selection and manipulation, which have an effect on knowledge construction based on the principle of the constructivist theory, which places an importance on learning by doing.

Therefore, the cognitive theory of learning with media proposed by Moreno (2006) expanded the cognitive theory of multimedia learning (Mayer 2002) to include other media, such as virtual reality, by adding tactility. Therefore, to build upon the strength of Mayer's model, avoid the limitations previously mentioned and include aspects of Moreno's model, a design framework entitled interactive multimedia model for cognitive learning was proposed, as shown in Figure 3.2.

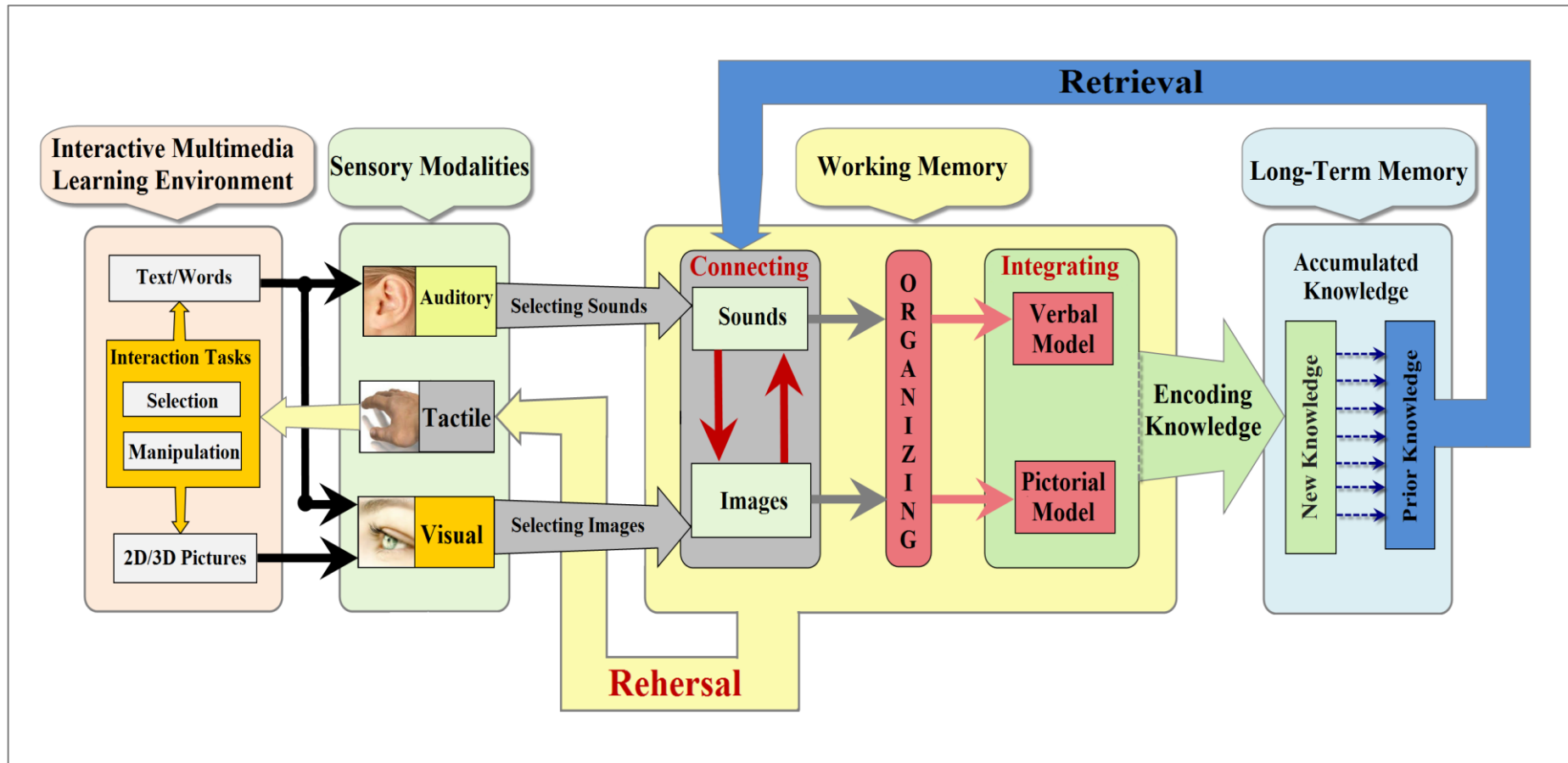


Figure 3.2 The proposed design framework for the interactive multimedia model for cognitive learning

The proposed design framework was an attempt to demonstrate the effect of motor tactile modality on what learners hear and see. In fact, the difference between these models (Figures 3.1 and 3.2) lies in the type of interaction tasks (selection and manipulation) allowed using the interactive multimedia: what the user is able to produce with this model differs from what the user is able to produce in the Mayer's multimedia model, where the learner's role is passive.

Information received by the sensory modalities from the learning environment. The sensory modalities will act as a filtering system, and the output will transform into input in the working memory. The working memory will integrate and organise the processed information, and the output will be integrated with the recalled previous knowledge from long-term memory. Consequently, the output of this integration will create new knowledge that will be stored in the long-term memory, and this new knowledge will act as previous knowledge in the next cognitive information processing cycle.

The learner constructs new knowledge by integrating previous knowledge with their current experience within the interactive multimedia learning environment. This new knowledge, according to Johnson et al. (2001) is a mapping of a novel system onto an already familiar one; thus, in the long-term memory, new knowledge is incorporated with previous knowledge to form accumulated knowledge (Figure 3.2).

While examining the relationship between multimedia and virtual reality, Hedberg and Alexander (1996) proposed that the defining attributes of virtual reality could produce better interaction than in multimedia. For this reason, the MAVLE

system uses virtual reality technology in terms of navigational behaviours to advance the learning effect. In addition, Osberg (1997) compared virtual reality with multimedia and stated that virtual reality progresses to no less than one level above multimedia in terms of perceptual richness and locus of control. Moreover, multimedia is a representation of 2D and 3D images, whereas virtual reality is a simulation of 3D objects intended to manipulate the senses into believing the environment is real (Daghestani et. al., 2008).

Regarding the relationship between multimedia and virtual reality, it could be suggested that the proposed design framework for the interactive multimedia model for cognitive learning (Figure 3.2) has been expanded to include a third channel (the perceptual channel) that encompasses the unique immersion characteristics of virtual reality. Therefore, for the purpose of this research, we proposed design framework for the virtual reality model for cognitive learning (Figure 3.3).

The key properties that underline the proposed design framework combine three sensory modality channels: **Visual** (seeing images with the eyes), **Auditory** (hearing sounds with the ears) and **Tactile** (handling tasks with the hands). The selected perceptions from these three channels are then processed and transformed into the working memory. These transformations are performed using previous knowledge contained in the long-term memory (Nunez, 2004). The output of this processing in the working memory represents temporary mental model structures (auditory and pictorial). The accumulated knowledge in long-term memory determines our perceptions. At the same time, our interpretation of sensory perceptions requires the retrieval of knowledge from the long-term memory (Wickens, 1992).

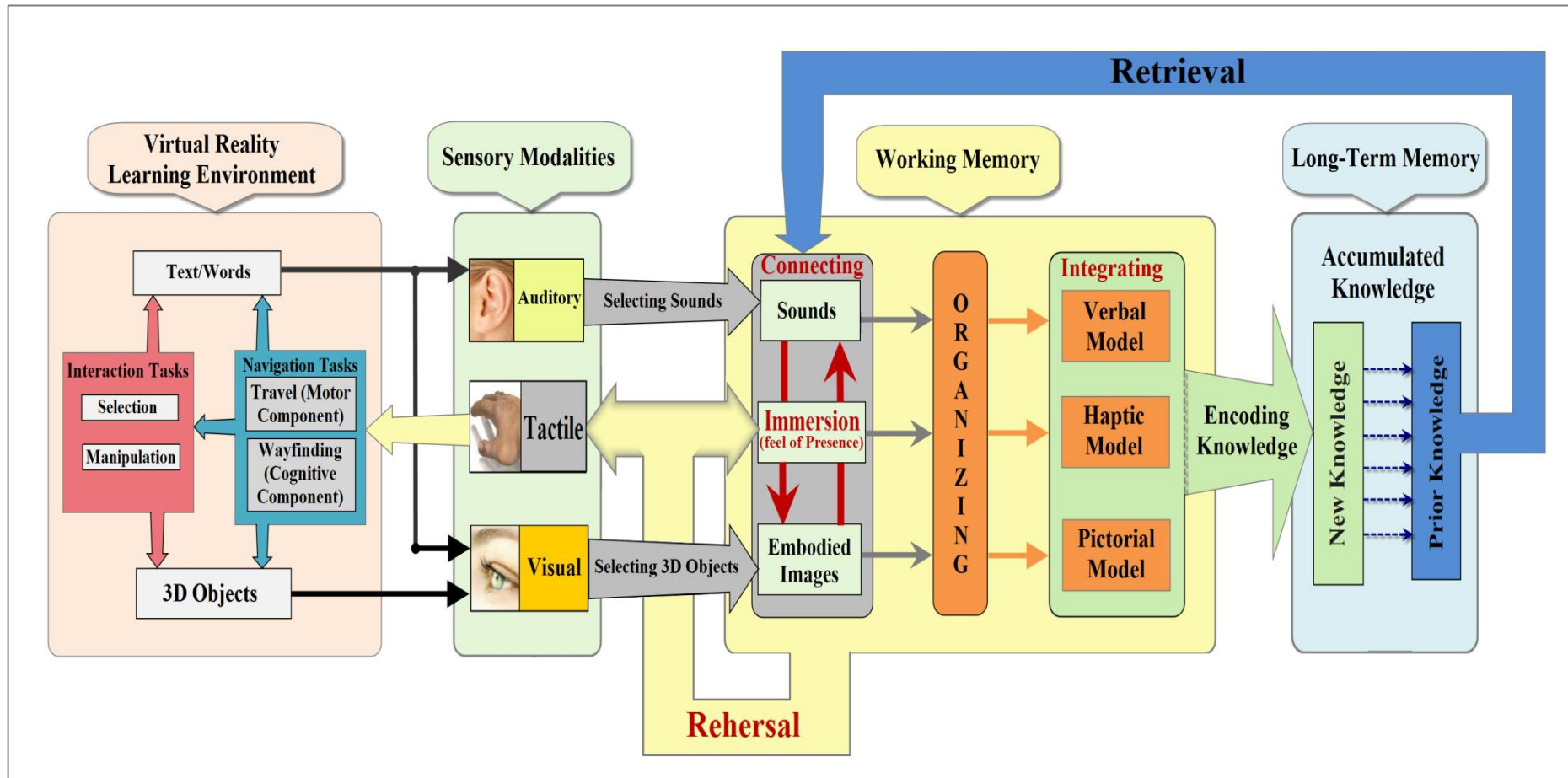


Figure 3.3 The proposed design framework for the virtual reality model for cognitive learning

The kind of interactions and perceptions achievable within virtual reality environments are different from 2D or 3D interactive multimedia in one specific way: the sense of immersion that is generated by navigation behaviours. The proposed design framework for the virtual reality model for cognitive learning (Figure 3.3) allows students to examine realistic 3D images of objects from various angles and distances. Indeed, this assumption of immersion, which emphasises the importance of not overloading the working memory during the learning process, is closely associated with the cognitive load theory by Sweller (1999). The interactive capabilities of virtual reality technologies permit students to manipulate various mental models that deepen conceptual understanding.

Table 3.1 shows a complete comparison between Mayer's cognitive theory of multimedia learning model and the proposed design framework for the virtual reality model for cognitive learning. As we can see from Table 3.1, the main difference between models involves interaction tasks and navigation tasks. The sensory memory acts as a processing system and the output will be as an input to the working memory. The working memory will organize the processed information in specific orders that formulate mental models. Finally the output from working memory will act as an input to the long-term memory. The output of the integration is an accumulated knowledge that represents the total knowledge gained due to cognitive virtual reality information processing. New knowledge stored in long-term memory will act as a prior knowledge in the next cognitive information processing cycle.

This research, therefore, proposes an extension to the interactive multimedia cognitive model, where the learning environment is a virtual reality learning environment. In this proposed design framework for the virtual reality model for cognitive learning (Figure 3.3), because of the sense of immersion produced by the navigation of virtual reality-based environments, a third channel for tactile interactions was added to the verbal and pictorial channels represented in Mayer's model of cognitive theory of multimedia. The navigation tasks of the virtual reality learning environments have two components: motor (travel) and cognitive (wayfinding).

Table 3.1 Cognitive model of Multimedia learning vs. virtual reality model for cognitive learning

Technology	Input Channels	Cognitive Information Processing							
		Sensory Memory			Working Memory			Long-Term Memory	
		Sensor	Processing	Output	Input	Processing	Output	Input & Integration	Output
Multimedia	Auditory	Ears	Selecting Sounds	Sounds	Sounds	Organizing	Verbal Model	Verbal Model + Prior Knowledge + Pictorial Model	Accumulated knowledge
	Visual	Eyes	Selecting Images	Images	Images		Pictorial Model		
Virtual Reality	Auditory	Ears	Selecting Sounds	Sounds	Sounds	Organizing	Verbal Model	Verbal Model + Prior Knowledge + Haptic Model + Pictorial Model	Accumulated knowledge + New knowledge
	Tactile	Hands	Selecting Perceptions	Immersion	Feel Of Presences		Haptic Model		
	Visual	Eyes	3D Objects	Embodied Images	Embodied Images		Pictorial Model		

3.2 MAVLE Methodology Framework

The proposed design and implementation methodology framework (virtual reality model for cognitive learning) provided the researcher with the necessary elements for the design phase of the MAVLE system. The virtual reality model for cognitive learning mainly focused on the inner workings of human input senses, working memory, long-term memory and executed actions. For designers, the model can help guide thinking about interaction tasks (selection and manipulation) and navigation tasks (travel and wayfinding). The four phases of the methodology framework for the MAVLE illustrated Figure 3.4.

The four phases of the methodology framework for the MAVLE system are as follows:

- Phase 1: This phase included an evaluation of 2D manipulatives (2DM). The evaluation of two teachers was conducted to explore how 2DM could best be used in a classroom to support student understanding of numeracy concepts. This would allow for a deeper insight into the behaviour of the 2DM and their interaction features. Furthermore, this phase helped identify the specific 2DM aspects that could be used to design MAVLE system.
- Phase 2: This phase primarily defined the requirements of the virtual reality model for cognitive learning. This model was used to guide the design and implementation of the MAVLE.
- Phase 3: This phase consisted of MAVLE design and implementation based on the requirements that resulted from previous phases.
- Phase 4: This phase involved the iterative evaluation-design process of the MAVLE graphical user interface. Feedback and suggestions were obtained from teachers and students for further development of the MAVLE system.

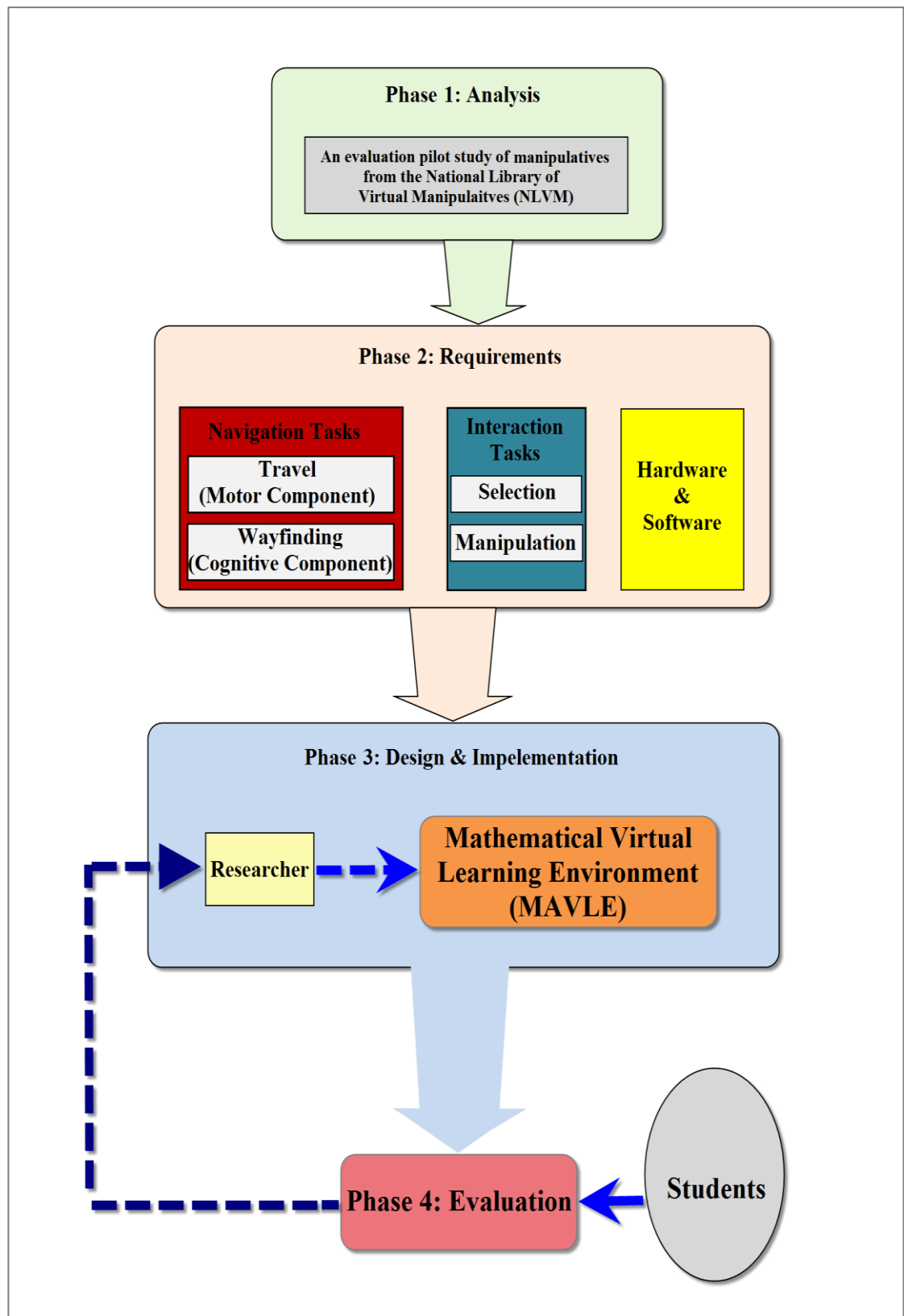


Figure 3.4 The MAVLE methodology framework

3.3 Phase 1: Evaluation of NLVM

The aim of this evaluation was to understand how primary school teachers apply base-10 blocks, 2D-based manipulatives from NLVM to mathematics teaching and address the issues that may arise from these applications. This evaluation was conducted with math teachers from a private primary school for girls in Jeddah, Saudi Arabia.

The evaluation sessions with the second-grade teachers lasted for approximately 1 hour and consisted of a series of problems with integer addition. The teachers were asked to begin their interaction session by first completing the exploration phase of the NLVM. Their interaction sessions were observed, and after they completed the tasks, they completed an evaluation form concerning the ease of use, motivation and support for problem solving, as recommended by Moyer et al. (2002) (see Appendix A). Each teacher was observed by the researcher while exploring the NLVM.

The researcher took notes about the interaction tasks used while the teachers created new exercises and any verbal comments of frustration or excitement they expressed while using the application. These notes were only to be used if the data on the evaluation forms were contradictory to their actual experience during the observation. A discussion was conducted with the teachers to determine whether they considered the application useful for their students, beneficial as support for the curriculum and limited by the English interface.

The teacher's responses reflected their satisfaction levels with its ease of use and learning potential, but they were not satisfied that the interface did not provide the user with feedback on whether or not their attempt was correct. The designers of the NLVM depended mainly on the instructor's feedback to the students. Their verbal responses were supported by their answers on the evaluation form and what the researcher observed.

It must be noted that the researcher also identified another limitation of the NLVM, which was when the user physically moves the base-10 blocks on the screen from one place to another, they can cover other base-10 blocks, hence, obstructing the view of other objects. This is because the NLVM application does not support collision detection between objects.

3.4 Phase 2: Requirements

System requirements are a set of functionalities and constraints that the end-user expects from the system. The requirements of the MAVLE system were taken from the features of the NLVM. The following list provides an overview of MAVLE's requirements for numeracy concepts (addition and subtraction) using a base-10 block:

- Addition: Addition is accomplished by using the dragging procedure of base-10 blocks in each place-value digit.
- Carry concept: is to make a regrouping in each place-value digits when tenth of the base-10 blocks are countered. This will create a group of 10; carry and drag it into the next higher place value. This action demonstrates the understanding of the place-value concept.

- Subtraction: To do subtraction, start by pulling out the correct type and number of blocks to take away the represent minuend.
- Borrow concept: To make a fair trade down, you cannot take 8 from 3, so to perform the subtraction using base-10 blocks, you will need to borrow a rod from the leftmost column (i.e., place-value concept) to make 10 additional units.

3.4.1 Choice of Device

Desktop virtual reality systems can often run on standard PC hardware. For the purpose of this research, we used an Intel PC platform with the Microsoft Windows XP operating system. The interactions were achieved using a standard mouse and keyboard.

3.4.2 Interaction Tasks

Jackson and Fagan (2000) stated that the interaction is the ability of the user to take action within the virtual reality environment. Indeed, interaction with the virtual reality environment represents how humans exchange information with the environment. Interaction with objects includes two basic tasks: the selection of the objects and how they can be manipulated (hold, move, release, throw, etc.). The MAVLE theoretical models of interaction have been adapted from Bowman (2002) and can be divided into two groups: interactions that relate to human action and the virtual reality interaction design model (Table 3.2).

Table 3.2 The interaction design model adapted from Bowman (2002)

Generic guidelines by Bowman		Suggested implementation for MAVLE
Existing HCI Guidelines	Practice user-centered design and follow well-known general principles from HCI research.	Use of iterative design with target users for the MAVLE prototypes
Choice of Device	Use HMD or Spatially Immersive Displays (SIDs) when immersion within a space is a performance requirement. Use workbench displays when viewing a single object or set of objects from a third-person point of view.	Because MAVLE is a desktop application, the immersive display devices, such HMD, are not needed but rather the conventional computer screen is required
	In SIDs, design the system to minimize the amount of indirect rotation needed.	
	Use an input device with the appropriate number of degrees of freedom for the task.	These input devices are not needed but rather the regular mouse and Keyboard are required instead
	Use physical props to constrain and disambiguate complex spatial tasks.	
	Use absolute device for positioning tasks and relative device for tasks to control the rate of movement.	
Interacting in 3D Space	Take advantage of the user's proprioceptive sense for precise and natural 3D interaction.	n/a
	Use well-known 2D interaction metaphors if the interaction task is inherently 1D- or 2D.	Menu and Buttons are used
	Allow two-handed interaction for more precise input relative to a frame of reference.	n/a
	Provide redundant interaction techniques for single task.	Use of constrained navigational using mouse and keyboard.

Table 3.3 The interaction design model adapted from Bowman (2002)

Generic guidelines by Bowman		Suggested implementation for the MAVLE
Travel/Navigation	Make travel tasks simple by using target-based techniques.	Constrained Navigational
	Use physical head motion for viewing orientation if possible.	“Camera in hand” metaphor
	Avoid the use of teleportation; instead, provide smooth transitional motion between locations.	Applied
	If steering techniques are used, train users in strategies to acquire survey knowledge. Use target-based or route-planning techniques if spatial orientation is required but training is not possible.	Applied
	Consider integrated travel and manipulation techniques if the main goal of viewpoint motion is to maneuver for object manipulation.	Use of 5 DOF accomplished using a combination of mouse and keyboard
	Use non-head-coupled techniques for efficiency in relative motion tasks.	n/a
	Provide way-finding and prediction aids to help the user decide where to move, and integrate those aids with the travel technique.	Use of a reset-position button in order to reset the view to a predefined camera position and orientation.
Selection	Use the natural virtual hand technique if all selections are within arm’s reach.	Selection done by conventional mouse
	Use ray-casting techniques if speed of remote selection is requirement.	n/a
	Ensure that the chosen selection technique integrates well with the manipulation technique to be used.	Applied
	Consider multimodal input for combined selection and command tasks.	n/a
	If possible, design the environment to maximize the perceived size of objects.	Applied
Manipulation	Reduce the number of degree of freedom to be manipulated if the application allows it.	A regular mouse and keyboard
	Provide general or application-specific constraints or manipulation aids.	Applied
	Allow direct manipulation with the virtual hand instead of using a tool.	n/a
	Avoid repeated, frequent scaling of the user or environment.	Applied
	Use indirect depth manipulation for increased efficiency and accuracy.	n/a
System Control	Reduce the necessary number of commands in the application	Applied
	When using virtual menus, avoid submenus and make selection at most a 2-D operations.	Applied
	Indirect menu selection may be more efficient over prolonged periods of use.	Applied
	Voice and gesture-based commands should include a method of reminding the user of the proper utterance or gesture.	Not applicable
	Integrate system control with other interaction tasks.	Applied

The MAVLE interaction design model was based on concepts created by Bowman (2002) as follows:


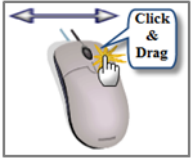

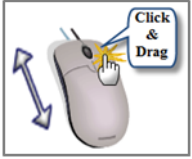

- Selection: This refers to the specification of one or to select any.
 - Picking: To pick a base-10 block, 'Click Mouse Left Button'.
 - Move: To move a base-10 block, 'Click and Drag Mouse Left Button'.
- Manipulation: The user can use the mouse and different buttons as tools to manipulate objects within the MAVLE system. Pressing the middle mouse button inside any place value compartment will drop as much as base-10 blocks as the exercises request. The buttons on the top panel, including 'Hand', 'Glue Bottle', 'Hammer' and 'Broom', will help the user to manipulate base-10 blocks so they can move, glue or breaks them from one compartment to another. However, errors of manipulation are considered as warnings and indicated with a 'ping' sound, and errors in the results will be indicated through dialogue boxes. In manipulation, the user or system moves a data object, modifying the content of the world that the user sees. An important point here is that, as with data manipulation, it is important for the system to maintain the illusion of the virtual world or reality.
- System Control:
 - Language: Everything is in the Arabic language, including the digit menus, buttons, dialog box text and the numbers.
 - Exercise modes: Are two types
 - ❖ Classroom mode: In this mode, the student will create an exercise from the class board or textbook without restrictions on grade level or levels of difficulties.
 - ❖ Regular mode: The exercises will work as defined in the next sections (grade, difficulty, etc.).
 - Sound modes: The action of dropping base-10 blocks in predefined places in each compartment is indicated with a specific sound.

- Collision detection: Base-10 block versus base-10 block and base-10 block versus compartment separators (illegal movements) are denoted by a specific sound.
- Gluing the final base-10 block to form a set of 10 is indicated by a specific sound during the carry operation.
- Hammering to break a set of base-10 blocks are indicated by a specific sound during the borrow operation.
- Each compartment can hold only 10 base-10 blocks. The user cannot add additional base-10 blocks; if this is attempted, a warning will be issued (a ping sound).
- The movement strategy of the base-10 blocks signifies that the user must finish the rows from right-to-left in strict order, i.e., they have to fill the results compartment with the right answer before moving to the next row using the following conditions:
 - ❖ Move the base-10 block from the first to the second compartment in the Addition state or from the second to the first compartment in the Subtraction state.
 - ❖ Glue 10 base-10 blocks in the Addition state, and then move them to the first row of the upper level.
 - ❖ Break the base-10 block in the Subtractions state; if the first row is empty, the 10 new base-10 blocks will be moved automatically to the next lower level (i.e., borrow).

3.4.3 Navigational behaviours

Navigation refers to the behaviours of moving the viewpoint within the 3D space and includes both a cognitive component (wayfinding) and a motor component (travel—also called viewpoint motion control) (Table 3.4).

Table 3.4 MAVLE navigational tasks based on the Virtual Reality Cognitive Model

Navigation Tasks	Description	Implementation
Cognitive Component (Wayfinding)	A reset position button used in order to reset the viewpoint seen to the first scene settings	
Motor Component (Travel)	Walk: Click the right mouse button and drag left/right in order to move sideways	
	Rotate: Press Left/Right or Up/Down keyboard arrows in order to either turn viewpoint left/right or tilt viewpoint up/down.	
	Fly: Click the right mouse button and drag forward/backward in order to move up/down	
	Zoom in/out: Roll mouse wheel forward/backward in order to move in/out	

3.5 Phase 3: Design and Implementation

During the development of the MAVLE prototypes, the same teachers were involved as evaluators during the iterative design process. Initially, the MAVLE interface was to be divided into compartments; each compartment would represent a place value (1's, 10's, 100's and 1000's) (Figure 3.5).

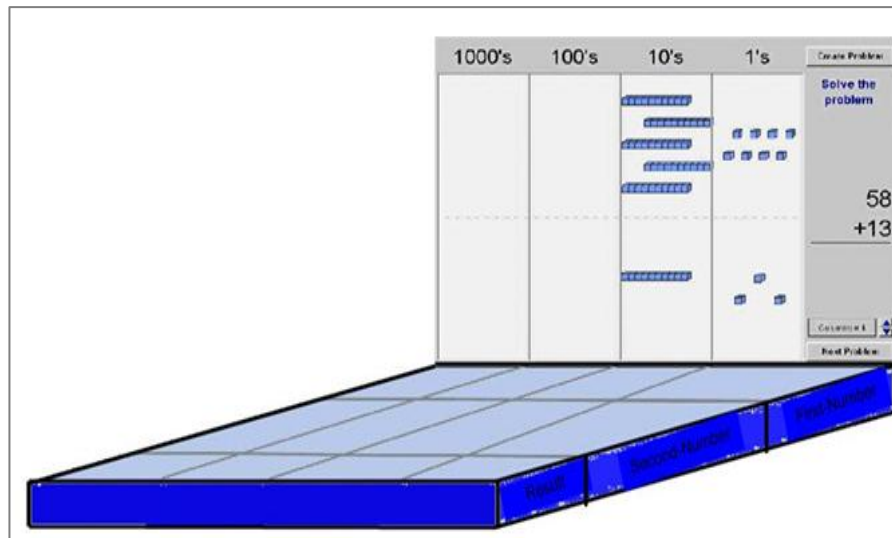


Figure 3.5 MAVLE initial interface prototype sketch

3.5.1 Metaphor

Metaphors are often used to provide the user with a mental model to assist their use of computers (Coschurba et al., 2001). According to Dix et al. (2004), the use of natural metaphors can aid the usability of virtual reality technology. Metaphors also create a bridge between real and virtual environments (Sánchez et al., 2000). The design processes required both an understanding of visual metaphors and how learning tasks can be accommodated within the metaphor. Metaphors used in an interface should resemble something familiar to the users to help the learners get started and then to allow them to explore new concepts. From a virtual reality perspective, the metaphors serve to map the concepts of the virtual world into graphical representations.

For the purpose of this study, the MAVLE metaphor was the embodiment of the physical-colour base-10 block manipulatives, which were useful in developing mental images of numbers, place value and operations. These base-10 blocks represented the standard concrete base-10 block manipulatives.

The elements of the manipulatives were implemented as follows: a unit was implemented as a blue box; rods, which include 10 units vertically tiled, were implemented as green boxes; a flat, which includes 100 units, were implemented as 10 yellow rods horizontally tiled; and the cube, which includes 1,000 units, was implemented as 10 red flats tiled to form a cube (see Figure 3.6). Therefore, the initial interface of the MAVLE was modified to include an open space with the base-10 blocks.

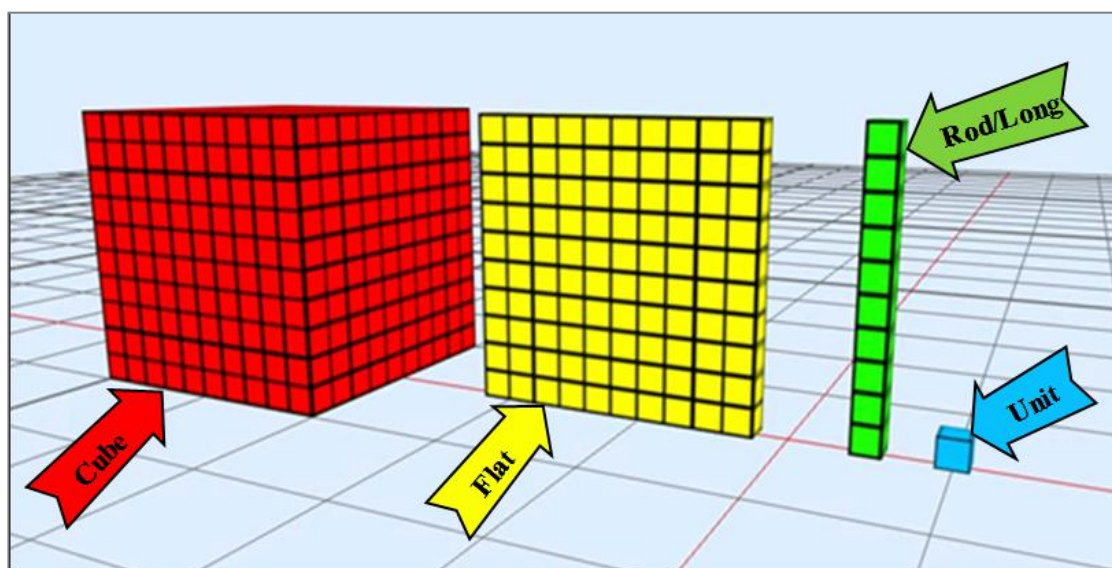


Figure 3.6 MAVLE base-10 blocks

All the blocks had three basic functions: move, rotate and collide. There were also other tools or icons that could be used to change the action by clicking the mouse or changing the mode of the program (Figure 3.7):

- The hammer icon breaks any of the large pieces into the next size down. For instance, by selecting the hammer and clicking on a flat, the flat will break into 10 rods or a rod will break into 10 units, etc.

- The glue icon does the opposite of what the hammer does. For example, if you align 10 units in a straight line to form a rod, clicking on the glue button will arrange the blocks and glue them together to form the corresponding shape.
- The broom icon clears all base-10 blocks and other blocks from the working area at once.

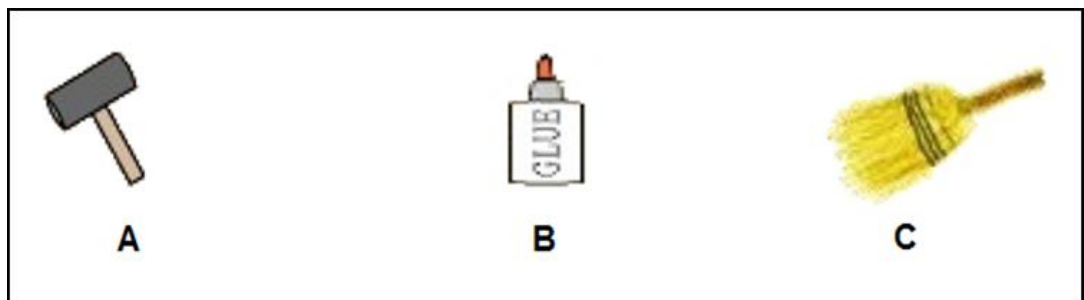


Figure 3.7 A Metaphor used in MAVLE software A) Hammer, B) Glue, and C) Broom

3.5.2 Selection of the 3D Modelling program for MAVLE Implementation

Many programming languages are available for creating 3D graphical applications, each with drawbacks and advantages. There are different technologies that make the variety of virtual world application areas possible. Some of these applications are proprietary, and some are open source. Applications in this domain are often developed using the main programming languages of the virtual reality modelling language (VRML) and the Java 3D Application Programming Interface API extension of the Java language. Currently, most virtual reality tools are individual plug-ins for a general web browser. Most of the tools are built on OpenGL or Direct3D, such as the VRML browser and the Java 3D programming environment (Vani et al., 2010; Selman, 2002).

The main differences between Java 3D and VRML, as suggested by Ko and Cheng (2009), are summarised as follows:

- Program approach: VRML adopts a content-centric approach, whereas Java 3D uses a program-centric approach for building 3D worlds.
- Flexibility: Java 3D is more flexible in terms of programming style and the available functions. Essentially, the larger number of functions available under Java 3D makes it a better tool for developing more specialised and customised behaviour and applications. Java 3D provides more extensive support for behaviours, interpolators, clipping and collision detection.
- Application complexity: VRML may be more suitable for simple graphics applications where the development time is at a premium. When the content or 3D world to be created is more complicated, Java 3D will be more suitable.
- File format: As a text-based modelling language for dynamic interpretation based directly on the source code, VRML has a file format that is more standardised. This is not the case for Java 3D, which has the capability to support compiled codes using low-level API for faster 3D graphics rendering.
- Compatibility: Java 3D is able to support VRML objects through the VRML97 loader. However, it is not possible for VRML to run Java 3D programs.
- Dynamic variation of scene graph: Because Java 3D nodes in the scene graph are instances of the corresponding classes, the scene graph that describes the virtual 3D world created with Java 3D can be dynamically changed. This is not possible for VRML.

Selection of the appropriate technologies was very critical for successful implementation of the MAVLE. Choosing the right tool that would satisfy the user's desire for interactivity and realism was essential. The MAVLE system was first developed as small programs called units. Each unit was developed and tested for its functionality; this has been referred to as unit testing (Vliet, 2000). Unit testing mainly verifies whether the modules or units meet their specifications.

One of the main features needed for implementing the requirements for interaction in the MAVLE was collision detection for moving objects (Figure 3.8). "Collision detection (CD) is a fundamental component to simulate realistic and natural object behaviors in virtual reality-based system. The collision detector is responsible for finding and handling collision between geometric models." (Galen et al., 2009). Collision detection has been a fundamental issue in many areas, such as physics-based modelling, computer-simulated environments, computer animation and robotics (Watt, 2000).

The ability to detect collisions (Figure 3.9) in the virtual world is an important building block for the walk-around navigation behaviour; it can also serve as the basis for spatial-change detection (Barrilleaux, 2000). The issues surrounding collision detection have been widely studied in the literature. The object-oriented scene graph included in Java 3D begets the potential to increase the efficiency of the collision detection process (Watt, 2000).

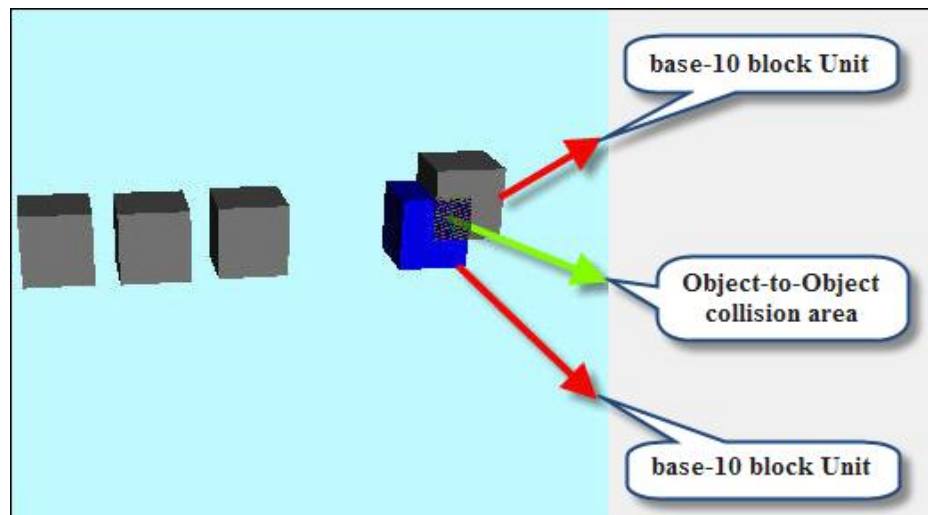


Figure 3.8 Objects penetrates each other

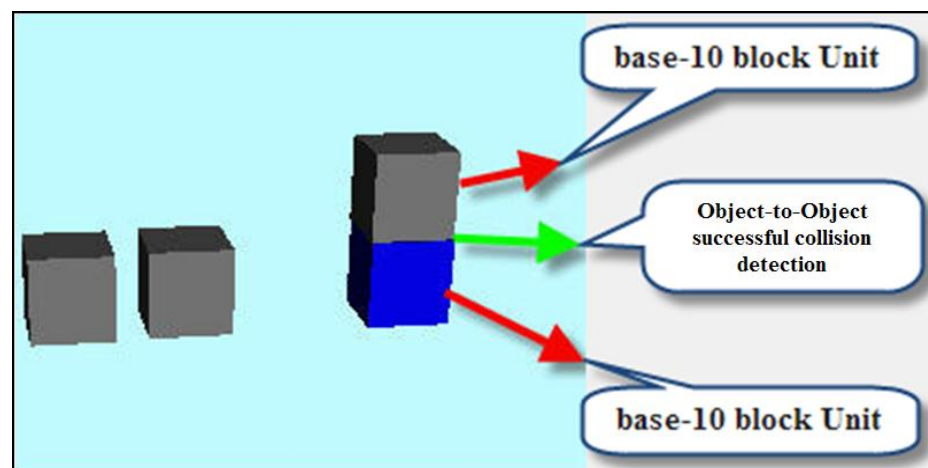


Figure 3.9 Successful collision detection

The basic collision detection requirement for the MAVLE was to detect the presence of an object in relation to a target object and act accordingly. The MAVLE prototype unit was developed using two different virtual reality languages (Java 3D and VRML). Both prototypes were compared to analyse their suitability for implementing the following types of collision detection:

- Viewer-to-object collision detection.
- Object-to-object collision detection.

Based on the design principles, the MAVLE prototype implemented with Java and Java 3D using Sun Microsystem's NetBeans 5.5 integrated development environment (IDE) was used; it provided complete support for the entire Java platform (Java Platform Standard Edition, Java Platform Micro Edition and Java Platform Enterprise Edition). NetBeans 5.5 IDE is a modular, standards-based program written in the Java programming language.

The Java 3D 'ViewPlatform' object represented the user's location and orientation; it had a built-in transformation object that controlled this movement. Navigation through virtual worlds was programmed using the 'Flying Platform' object that controls the interactions between input devices, mouse and keyboard and the ViewPlatform object.

The MAVLE was also implemented using ParallelGraphics VrmIPad running in the Cortona VRML client. ParallelGraphics is a VRML-authoring software tool based on ISO standards for VRMLs. Early evaluation of the MAVLE prototype helped this research detect advantages and disadvantages at an early stage in the development of this software program. This section examines the preliminary outcomes from the comparative study of the MAVLE prototype implemented using both Java 3D and VRML. Although Java 3D and VRML both seemed to target the same application area, i.e., virtual worlds, fundamental differences between them existed with regard to implementation of the scene graph. Unfortunately, it was proven to be difficult to implement all of the prototype's behaviour capabilities, and the VRML performance was less than expected.

The development of the MAVLE prototype brought to light some VRML issues and difficulties that VRML programmers are likely to face. Foremost of these was a problem involving collision detection. The object-to-object collision detection in MAVLE was expected to be one of the most important features and had to be implemented successfully and efficiently. Whereas Java 3D and VRML were frequently used for 3D graphics development, Java 3D was, in general, a more specialised tool for creating customised 3D graphical applications. Therefore, Java 3D was used for rendering the MAVLE prototype at the next stage of this research project; the selection of Java 3D for developing the MAVLE was based on the following conclusions:

- The Java 3D API is available free of charge.
- Java 3D easily integrates with Java, using Swing and AWT (Abstract Window Toolkit) components.
- Java 3D is portable across various platforms.
- Java 3D's scene graph acts as a querying structure that supports collision detection.
- Its application can be easily made available on the Internet with some modifications.

3.5.3 MAVLE system design

Cognitive learning tools are tools that can support a learner's ability to perform a task (Lee and Wong, 2008). With this aim in mind, this research intended to design the MAVLE to be a cognitive tool. Mayer's design principles suggested that learning was enhanced when related words and pictures were positioned close together on the screen. The manipulatives in this study were based on VRM base-10 blocks. The VRM provided in the MAVLE only use words for labelling buttons.

3.5.3.1 MAVLE interface description

The MAVLE shown in Figure 3.10 was obtained as a result of the design process described in the previous section. MAVLE application consisted of a three panels: top, right and working. As we can see in the figure, the top panel contains many icons, and each is used to change an action resulting from a mouse click. Table 3.5 describes all the functions available on the top panel.

The right panel consists of the exercise display panel and contains two boxes. The first box displays the first number, and the second box display the second number. The second is result display panel; it is used to reflect the number of virtual manipulatives inside each compartment.

The working panel contains four compartments that are used to represent the place values (1's, 10's, 100's and 1000's). One of the best uses for this program is to present the concepts to the class, making it a very tidy, quick and simple way to teach the material. A second use is obviously for the students to practice problems in the computer labs. A student simply needs to click on the 'New' icon to start a new problem.

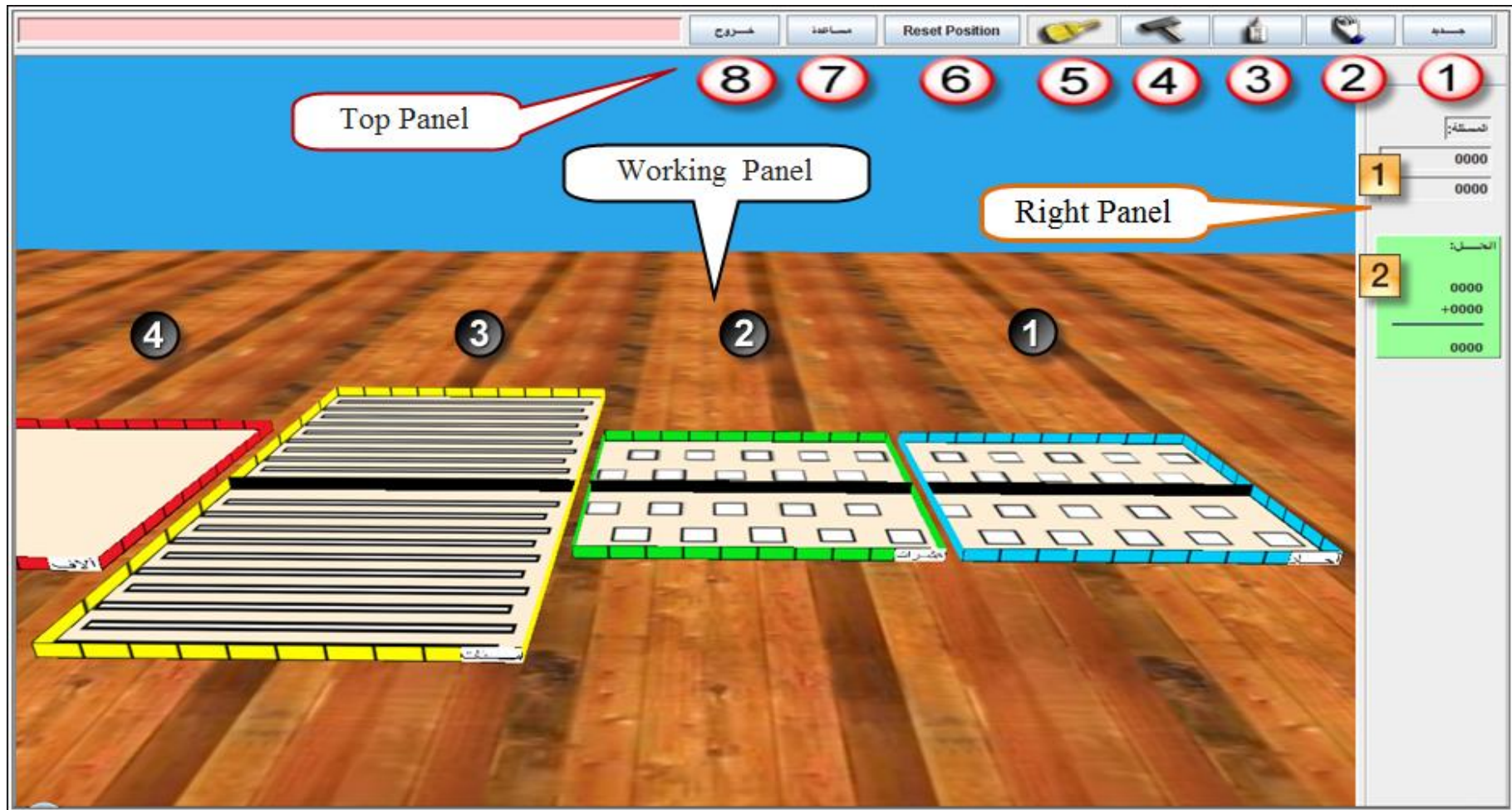
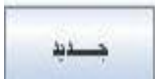









Figure 3.10 MAVLE main screen interface

Table 3.5 MAVLE top panel icons description

Icon Symbol		Descriptions
1		Clicking on this icon will Start new exercise in order to (create own problem / generate problem / start problem).
2		The Hand icon used to put MAVLE in normal mode which is the most common one. This is the mode where you can make perform different interaction tasks ('drop' new VM inside any compartments in order to represent the required number of VM according to the exercise that have been created; selects, and drag 'pick' in order to move any VM.
3		The glue icon used to align 10 units of VM in a straight line or 10 rods of VM to be glued together to form the corresponding size up the block. Click on the Hand icon to revert to normal mode.
4		The hammer does the opposite of what the glue does. This hammer is used to break any of the large pieces into the next size down. There is a distinct sound emphasising the use of a hammer. Select the hammer and click on a 100-block to break it into 10 10-blocks or click on a 10-block to break it into 10 1-blocks. Click on the Hand icon to revert to normal mode.
5		You click on the broom to clear, all at once, all the VM blocks from the working panel in order to start new exercise.
6		Clicking on this icon will reset the view to a predefined camera position and orientation.
7		Clicking on Help give general help about the MAVLE application and how to use it.
8		Clicking on the Exit icon will exit from MAVLE.

A new exercise dialog box will appear (Figure 3.11). The dialog box described as follows:

- Student name: will ask the student to enter his/her name.
- Student age: will ask the student to enter his/her age.
- Grade level: the grade level will be either 2nd or 3rd grade.
- Exercise mode: either 'Classroom Mode' or 'Regular Mode'.
- Classroom Mode: in this mode the student will create the exercise from the class board or textbook, without restriction on the grade level or level of difficulties.
- Regular Mode: the exercises will work according what will be selected in the next box (grade level, level of difficulty)
- Level of difficulties: the exercise level of difficulties with each grade level will be either 'Easy' or 'Difficult'.
- Type of operation: the type of operation will be either 'Addition' or 'Subtraction'.
- Ok button: when this button pressed the new exercise will start.
- Cancel button: when this button pressed the new exercise dialog box cancelled.

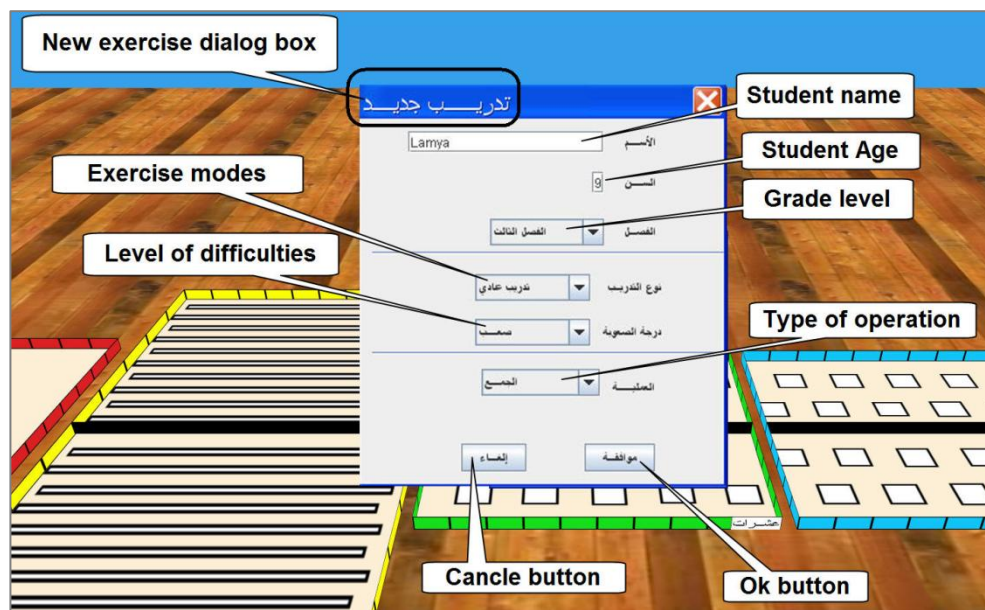


Figure 3.11 MAVLE new exercise dialog box

The MAVLE travel technique tasks are achieved using a standard mouse and keyboard manoeuvres. These travel tasks are accomplished with the following actions (Figure 3.12):

- Click right mouse button + drag up/down (1) in order to move along the Y axis.
- Press keyboard up/down (2) arrows in order to turn up/down around Y axis (Yaw).
- Press keyboard left/right (3) arrows in order to turn left/right around X axis (Pitch).
- Click right mouse button + drag sideways left/right (4) in order to move along X axis.
- Roll mouse wheel forward/backward (5) in order to move along the Z axis.

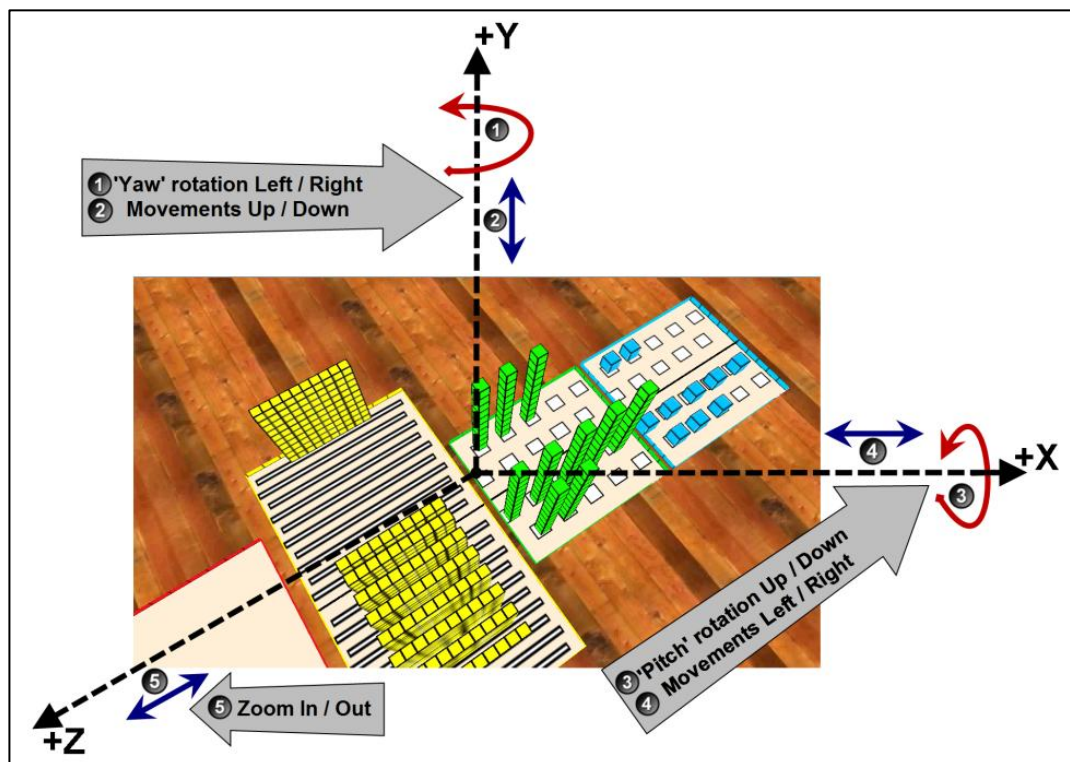


Figure 3.12 MAVLE travel techniques accomplished using mouse and keyboard

3.6 Phase 4: Evaluation of MAVLE System Interface

An informal pilot study was conducted with a small group of teachers and students at a school similar to the one used in the real study. Conducting a pilot study allowed the researcher to ask teachers for suggestive feedback on the MAVLE application. A usability evaluation of the MAVLE prototype was conducted. The goals of the usability evaluation were to assess usability problems with the MAVLE system interface. Our aim was to evaluate the ease of use and ease of learning the interface.

3.6.1 Description of the Sampling

According to the education system in Saudi Arabia, gender segregation was required at all levels of public and private education, which means female access to male schools was prohibited, and vice versa. The Saudi Arabian educational system is unique among all Middle Eastern countries because of its structure and strategies for the reproduction of cultural gender divisions through gender-segregated schools and colleges (El-Sanabary, 1994).

However, there is a divergence between computer education in public and private schools in terms of content and the stage at which schools start to teach computer studies. Computing is only taught to high school students in public schools, whereas private schools start teaching computer skills at the elementary level (Abu-Hassana and Woodcock, 2006).

Due to educational gender segregation and differences in computer experience, the target samples for this study were from the second grade of a private girl's elementary school in western-central Jeddah, Saudi Arabia. The second-grade students were aged between 7 and 8 years old. Regarding ethical principles, which are vital aspects of the research process (Greig et al., 2007), the researcher made clear statements regarding the ethical approval and rights of the parents and students in terms of disclosure and confidentiality (see Appendices B and C).

3.6.2 Participants

The target group for the evaluation was students from a private girls' elementary school in western-central Jeddah, Saudi Arabia. The chosen school was similar to that planned for the primary study. The school had two second-grade classes with approximately 30 students in total. More specifically, the participants were composed of only two groups of students.

3.6.3 Material

The MAVLE application is a stand-alone application. It provides the students with a real-time, 3D-interactive environment where they can manipulate and test objects using the mouse and keyboard found on any PC.

3.6.4 Procedure

Students were introduced to navigation in the MAVLE and applied practices that consisted of a sample environment where each user was asked to perform navigational behaviours to guarantee full understanding. The practice stage ensured that the user would have some knowledge of how to use the mouse in navigating in a desktop virtual reality environment. The tasks were designed to examine the students' abilities in solving the exercise involving the addition of two- and three-digit numbers. The addition of three-digit numbers was generally taught at the end of the second term for students in the second grade.

The researcher led the participants through a series of three-digit addition exercises. The exercises were created when the students pressed the 'New' button. A new exercise dialog box appeared (Figure 3.13), allowing the students to enter their names and ages. The students then chose options from the combo boxes: grade level (second or third), exercise mode (classroom or regular), level of difficulties (easy or difficult) and type of operation (addition or subtraction). After configuring the exercise, the students pressed the OK button to start.

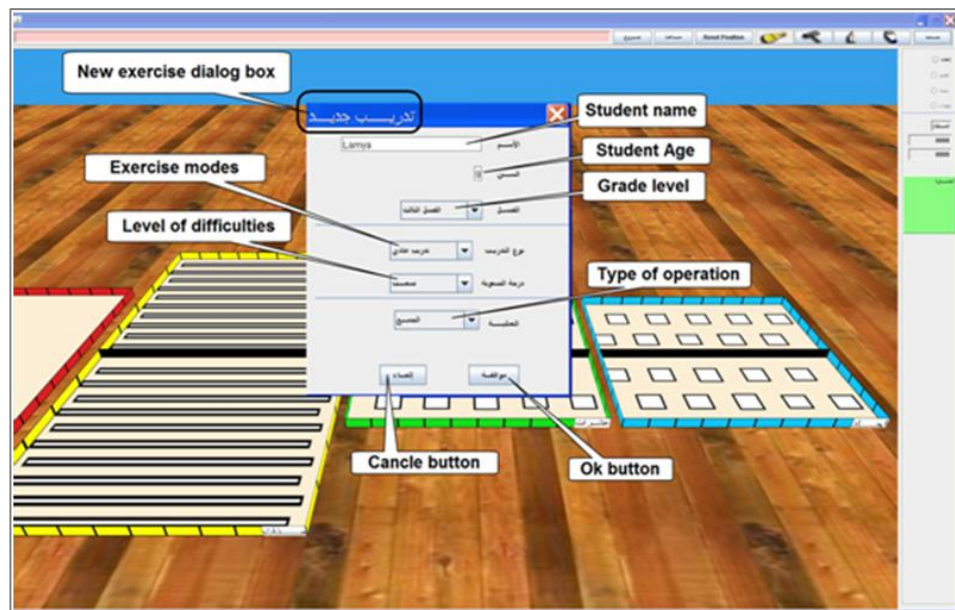


Figure 3.13 MAVLE new exercise dialog box

The addition exercise created was: $163 + 977$. The first number was 163, and the second number was 977. In Figure 3.14, the right panel area is labelled (1), (2) represents the 1's place-value digit, which is three in the first number compartment and seven in the second number compartment, (3) represents the 10's place-value digit, which is six in the first number compartment and seven in the second number compartment, (4) represents the 100's place-value digit, which is one in the first number compartment and nine in the second number compartment, and finally, (5) represent the 1000's place-value digit, which is zero (i.e., empty). In this example, the student started to solve the exercise by placing the cursor inside the compartment and pressing the roll button on the mouse; to enables them to drop the base-10 blocks into the 1's, 10's and 100's compartments. Feedback for each dropped base-10 blocks appeared in the result display panel on the right side of the screen Figure 3.14.

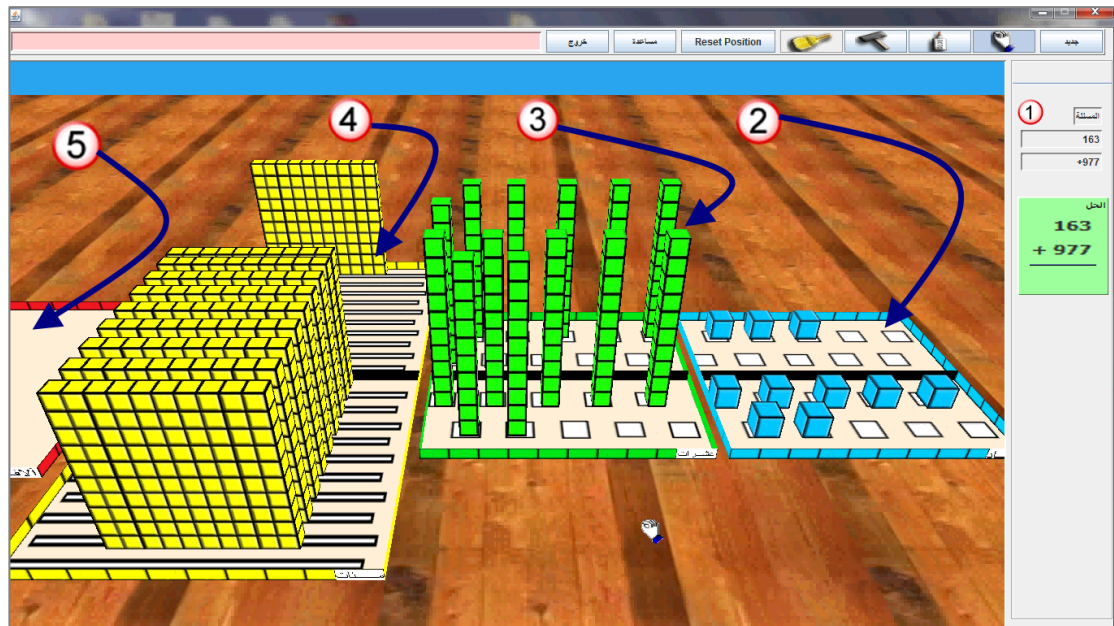


Figure 3.14 New addition exercises $163 + 977$

Next, the student began to interact with the base-10 blocks, starting from the 1's place-value digit, which satisfied the numeracy place-value concept. The student then moved the base-10 blocks from the first number compartment to the second number compartment. Each movement of any base-10 blocks was reflected in the result display panel. According to addition operation rules, when the second number compartment contains 10 units of base-10 blocks, the student should perform a regrouping (i.e., carry concept).

This regrouping concept is achieved by pressing the glue button on the top panel ([1] from Figure 3.15). In doing so, each base-10 block from the 1's compartment is laid out as if waiting in front of the 10's place-value compartment at the base of the blocks ([2] from Figure 3.15). After the last block is selected, they will automatically be glued together to form the new rod base-10 block and moved directly to the first number in the 10's place-value digit ([3] from Figure 3.15). This process is replicated for all digits,

and all actions and movements are reflected in the result display panel (see Figures 3.16 and 3.17).

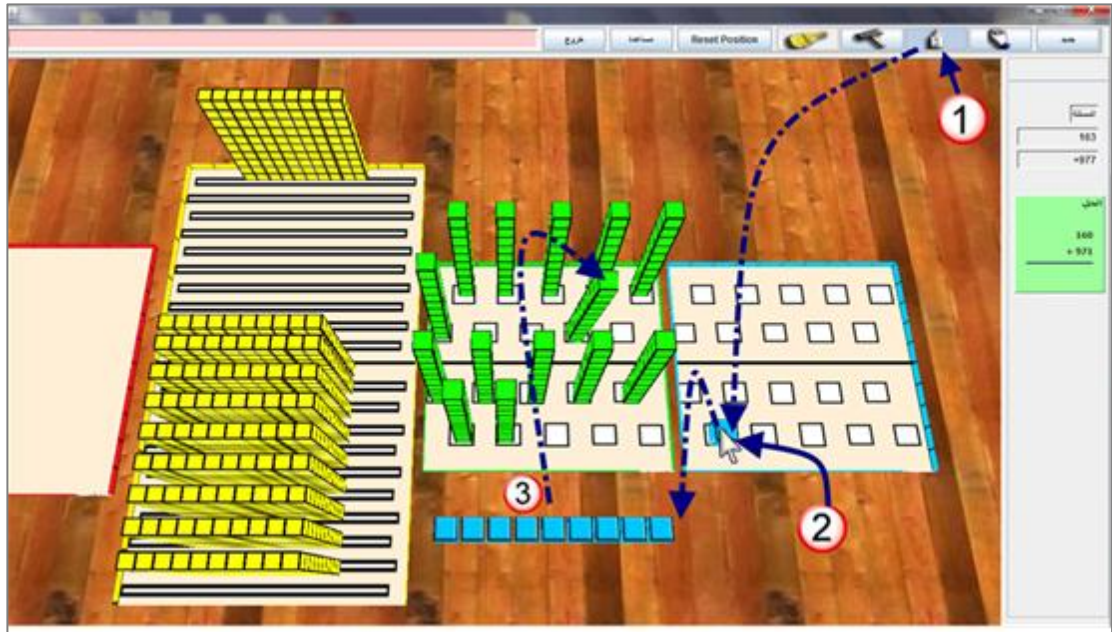


Figure 3.15 Screen shot shows the regrouping 10's concept (i.e. carry), top viewpoint for the working area, this was a result of VIRTUAL REALITY travel

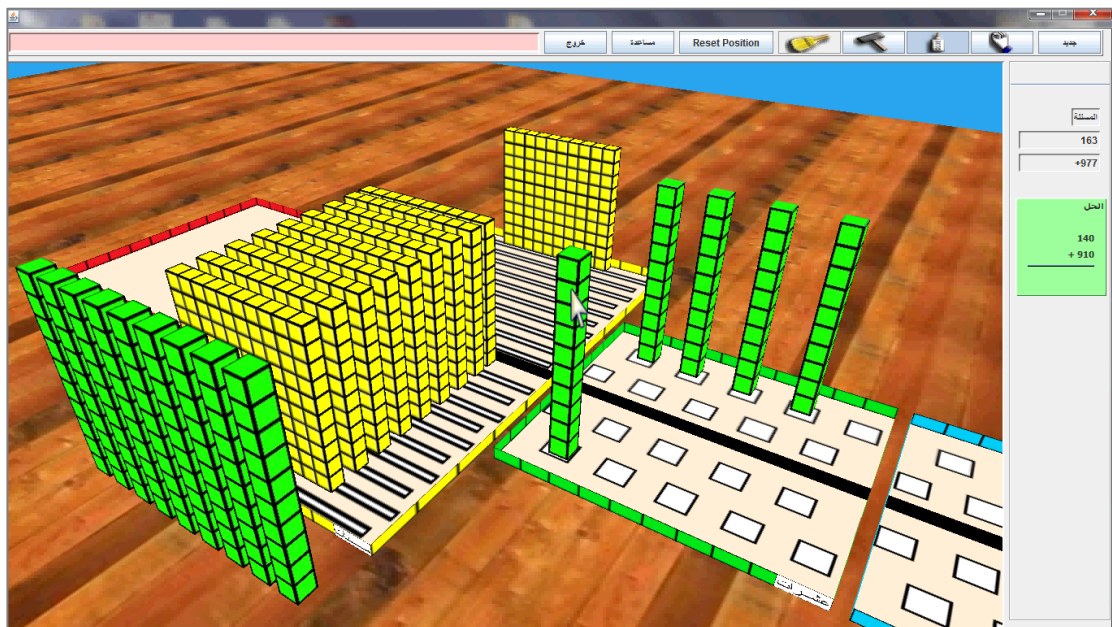


Figure 3.16 Screen shot shows the regrouping 100's concept (i.e. carry), different viewpoint for the working area, this was a result of VIRTUAL REALITY travel

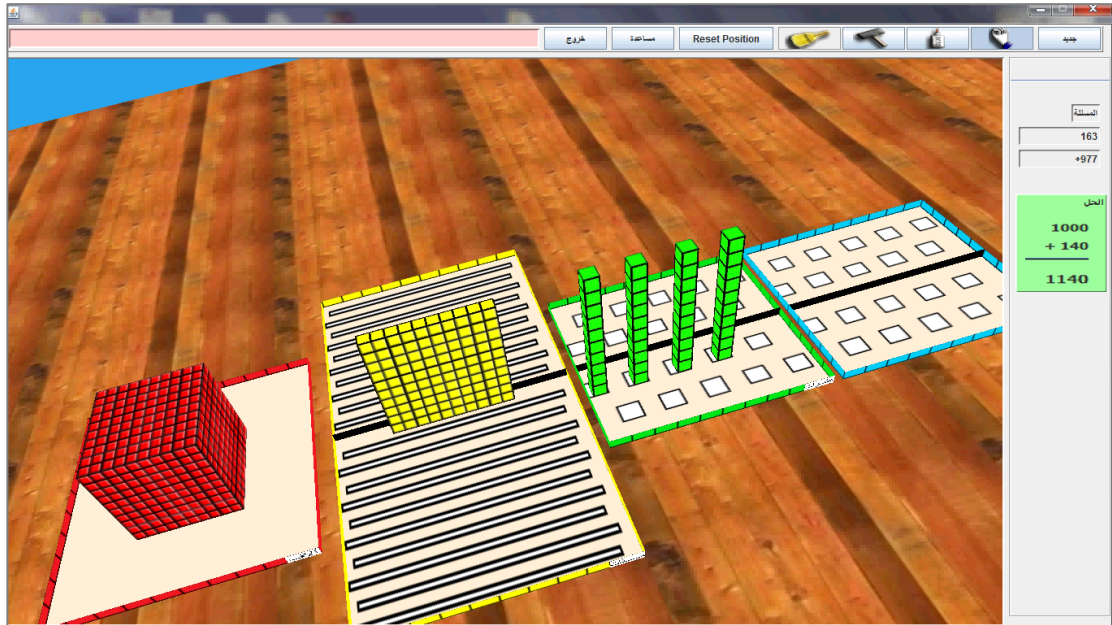


Figure 3.17 Screen shot shows the final result for the problem: $163 + 977 = 1140$, top viewpoint for the working area, this was a result of travel tasks

3.6.5 Results and Recommendations

The pilot study indicated that the navigational behaviours of the MAVLE were easy to use by students. Verbal comments made by several students during their use of the MAVLE specified that they felt immersed in the environment, although no special immersive virtual reality hardware was used. The types of comments heard from the students demonstrated they were thinking about their actions and were expressing them to their peers. The following comments are provided as a small sample of what was expressed: “Look, I moved my 100 block to the 10’s and then all my 10’s over so that I would have ten 10’s”; “Watch, this is how I moved my 10 blocks to the 1’s and now back again. See, I still have the same amount”; and “Watch me make 1000; watch it break into ten 100’s.”

These types of comments were not heard before when students were working with the traditional classroom teaching methods. Their math teacher was excited to see self-discovery of the addition concept in her students using the knowledge they had gained in class. These comments were based on an immediate feedback of their action placement and choice of base-10 blocks. Finally, their comments, such “This is fun” and “Yeah, computer time”, and shouts and cheers showed sheer excitement when working with the MAVLE.

In conclusion, the initial evaluation indicated that the following changes were required for the software design:

- The mouse shape could be made into a hand shape. It could be modified to transform into the glue, broom or hammer icon depending on which button is pressed. This would help young students differentiate between mouse and mode actions.
- The 10's compartment needs to be enlarged to reflect its place-value position when compared with the 1's compartment. In addition, the 1000's compartment will be enlarged to represent its place-value position.
- In the right panel, the finish button on the activity panel will be removed because it produces the same action as the broom button on the top panel. This removal will help avoid confusion in the students.
- Due to the extra effort needed for students to enter their data and preferences in the new exercise dialog box, it was modified to exclude unnecessary items and add needed items. The three items on the new exercise dialogue box are as follows (Figure 3.18):
 - Operation type: The operation type will be either ‘Addition’ or ‘Subtraction’.
 - Training type: 2-digit or 3-digit exercises.

- Difficulty type: In addition exercises, regrouping tens, regrouping hundreds and regrouping thousands, while in subtraction, regrouping tens and regrouping hundreds.
- The MAVLE prototype for the subtraction operation mode will be implemented.

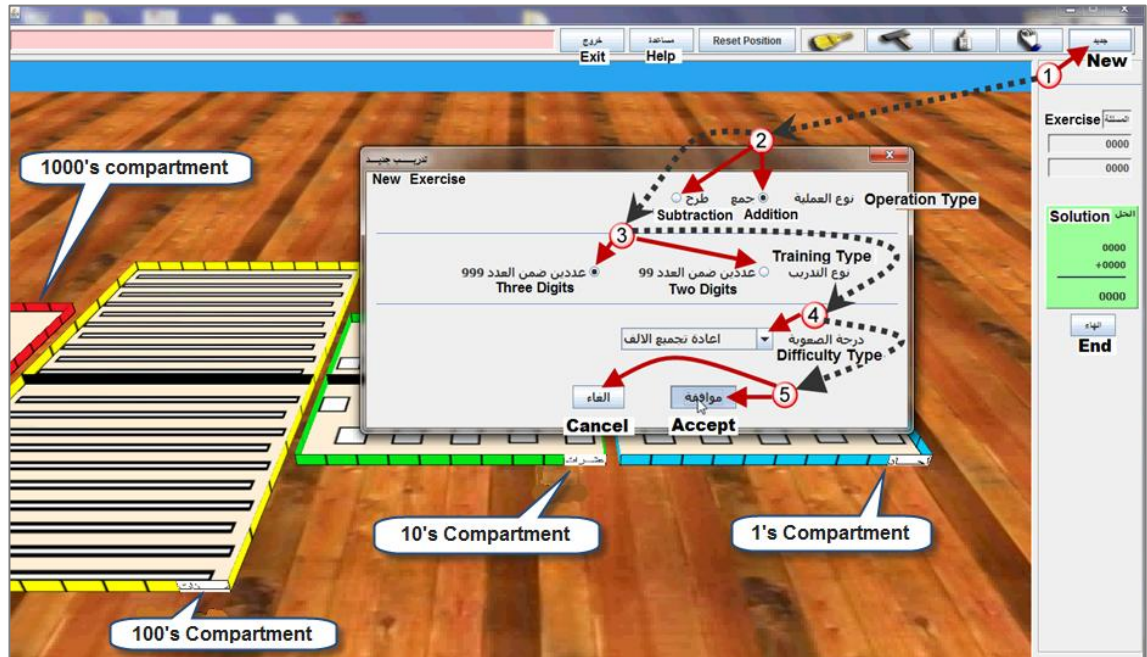


Figure 3.18 MAVLE new exercise dialogue box

CHAPTER 4 Research Methodology

The previous chapters summarised the necessity for this research within the relevant context of the existing scholarly literature. The purpose of this research was to examine the impact of using VRM as a cognitive learning tool to aid second-grade students' conceptual understanding of addition and subtraction numeracy concepts. This chapter outlines the research methodology used during the implementation and analysis of this study. The research methodology describes the virtual manipulatives applications and an overview of the design, participants, data collection process and data analysis procedures for the studies performed.

4.1 Instrumentation and Materials

The virtual manipulatives applications investigated in this study were as follows:

- The MAVLE application: is a stand-alone application developed for the purpose of this study. The MAVLE application was used by the VRM group.
- The NLVM application is a free-trial version 2.0 downloaded from the (<http://nlvm.usu.edu/en/nav/vlibrary.html>). The students worked specifically with the 'Base Blocks Addition' and 'Base Blocks Subtraction' applets in the Grades Pre-K–2 number and operation section. The NLVM application was used by the 3DM group.

4.1.1 National Library of Virtual Manipulatives (NLVM)

The NLVM website provided the 3DM used in this study. During the numeracy unit, students worked specifically with the ‘Base Blocks Addition’ and ‘Base Blocks Subtraction’ applets in the grades Pre-K–2 number and operation section. The NLVM base-10 blocks virtual manipulatives used in this research were presented in a text and 3D-graphics format and did not include sound or animation. The NLVM had a ‘back’ button that was present on each page. The back button took the user back to the main menu.

4.1.1.1 Description of Addition Operation Using NLVM

When ‘Base Blocks Addition’ first loads, the student is requested to complete an addition problem using base-10 blocks. The student clicks the ‘New Problem’ button to obtain a new exercise. The addition-carry operation is performed by means of the amalgamating procedure, in which the user amalgamates the base-10 blocks to make a carry into the next higher place value. To accomplish this, the student clicks and holds down the mouse key while dragging a rectangle to lasso the base-10 blocks together (Figure 4.1).

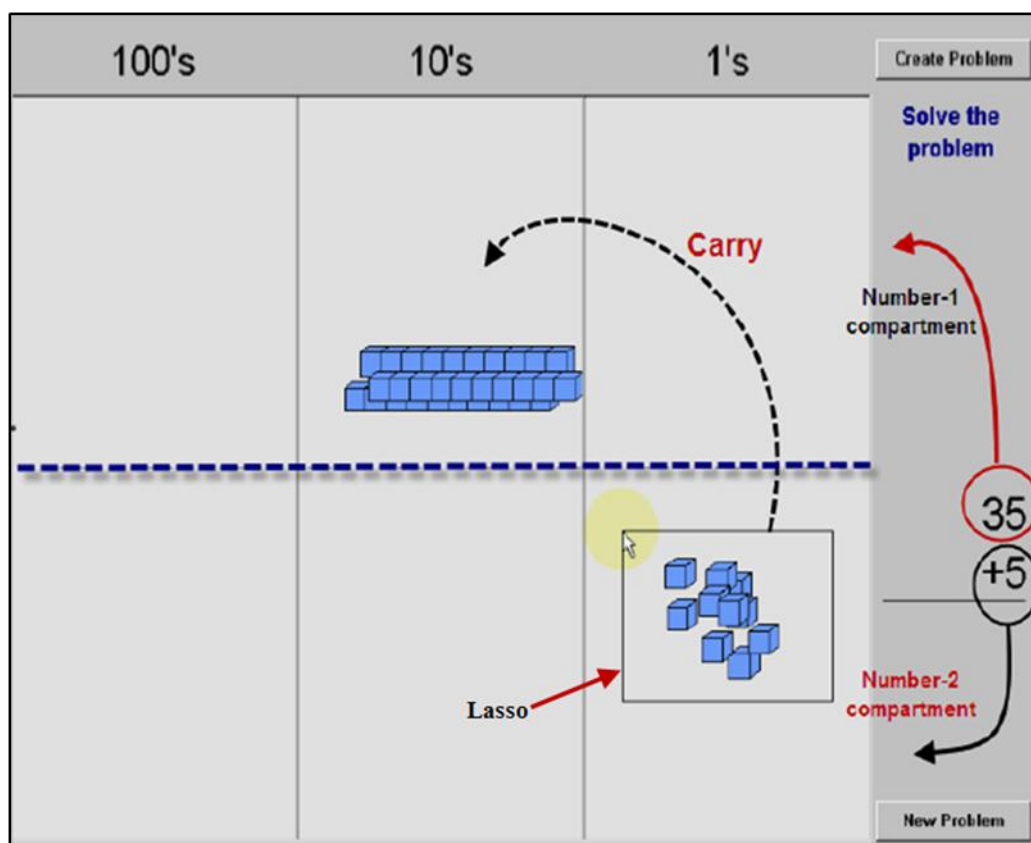


Figure 4.1 NLVM 'Base Blocks Addition' illustrating the carry operation

4.1.1.2 Description of Subtraction Operation Using NLVM

When the 'Base Blocks Subtraction' first loads, the student is requested to complete a subtraction problem using base-10 blocks. The student clicks the 'New Problem' button to obtain a new exercise. Positive numbered base-10 blocks are displayed using blue blocks (first number compartment). Negative numbered base-10 blocks are displayed using red blocks (second number compartment). If a student touches a blue block with a red block of the same size (click-hold-drag with the mouse), the blocks will disappear. The subtraction-borrow operation is used when the student moves a base-10 block from a higher place value to a lower place value. The base-10 blocks are then broken apart to show that the student made a borrow (Figure 4.2).

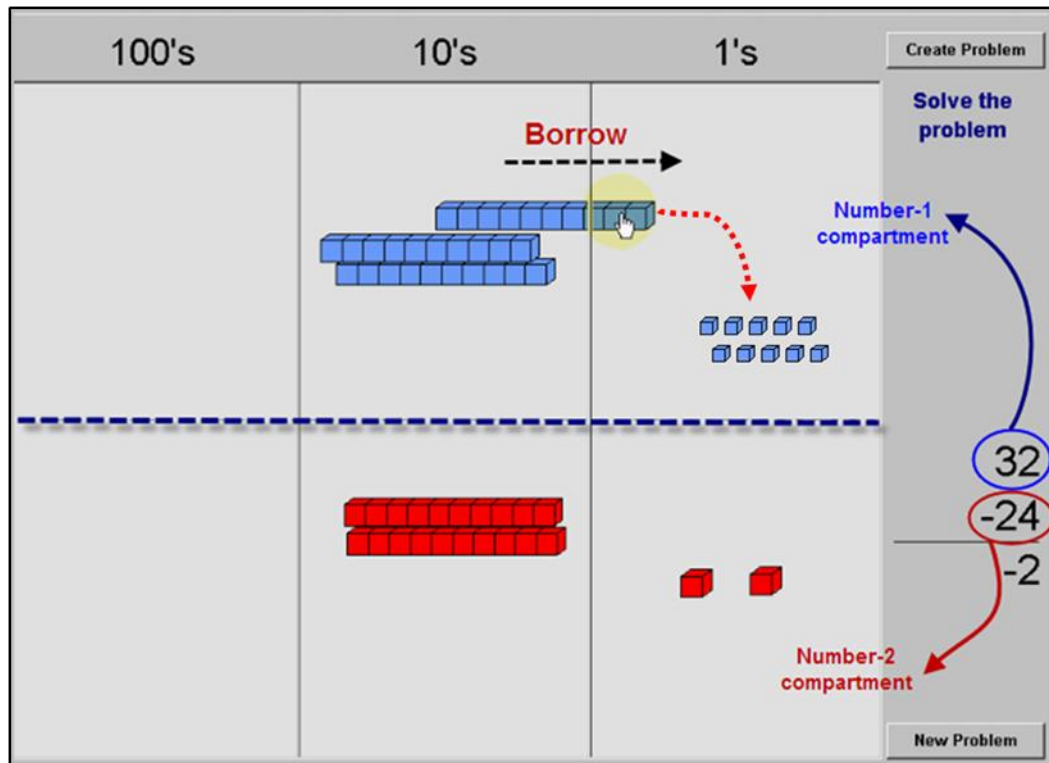


Figure 4.2 NLVM 'Base Blocks Subtraction' illustrating the borrow operation

4.1.2 Mathematical Virtual Learning Environment (MAVLE)

4.1.2.1 Description of Addition Operation Using MAVLE

When MAVLE first loads, the student clicks the 'New Problem' button (Figure 4.3 [1]) to obtain a new exercise. The student then clicks the hand icon (2) to pick up the base-10 blocks from the first number compartment and places them in the second number compartment to begin the addition process. The addition-carry operation is accomplished by using the glue icon (3) from the menu bar. This icon enables the student to start amalgamating the base-10 blocks (4) to form a rod that will be automatically moved to the right place-value compartment (5).

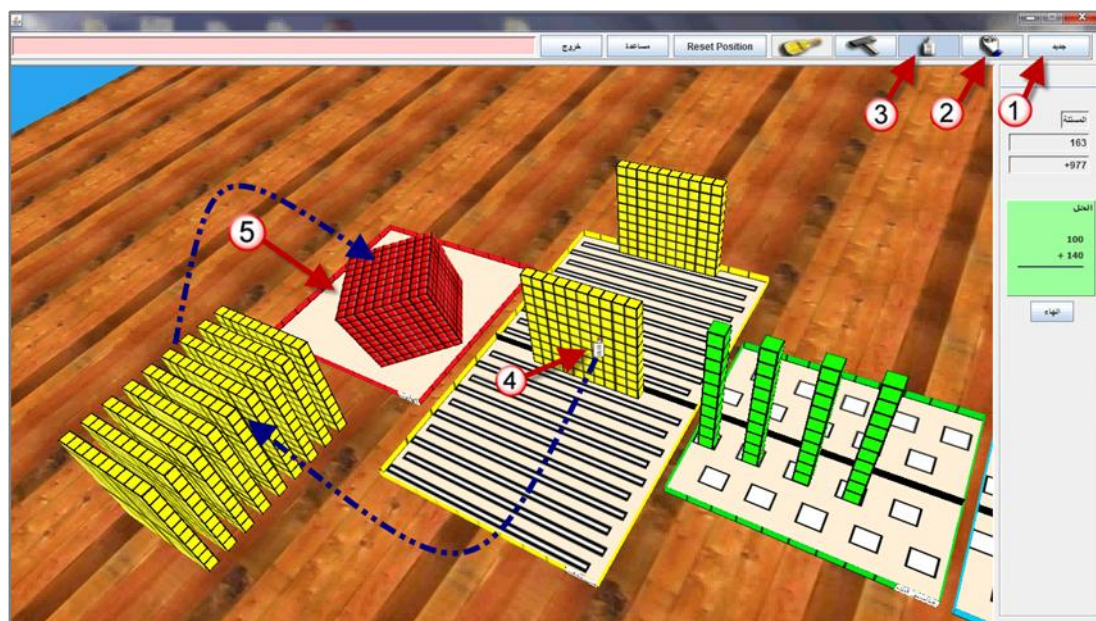


Figure 4.3 MAVLE ‘Base-10 Blocks Addition’ illustrating the carry operation

4.1.2.2 Description of Subtraction Operation Using MAVLE

The student commences the subtraction exercise by clicking a ‘New Problem’ button (Figure 4.4 [1]). The student then clicks the hand icon (2) to pick up a base-10 block from the second number compartment and drop it on a base-10 block in the first number compartment, which will cause both blocks to disappear. The subtraction-borrow operation is performed by the hammer icon (3). The student uses the hammer icon to break apart a group of base-10 blocks into the next smallest unit grouping (4). The student clicks on the hand icon to revert to normal mode in order to continue the exercise.



Figure 4.4 MAVLE 'Base-10 Blocks Subtraction' illustrating the borrow operation

4.2 Study 1: The Impact of Virtual Reality Manipulatives on Students' Performance in Numeracy Concepts

In this study, our primary goal was to engage second-grade students in exploring the VRM and 3DM, which support visualisation of abstract numeracy concepts (i.e., addition and subtraction), and observe their interactions. The study specifically attempted to investigate the following research hypotheses:

- The VRM group is predicted to have a significant positive performance outcome (i.e., regarding the number of solving problems for addition and subtraction) than those in the 3DM group.
- The number of errors in the place-value concept is predicted to be significantly different between the VRM and 3DM groups.
- The number of errors in the regrouping concept is predicted to be significantly different between the VRM and 3DM groups.
- The number of errors in the concept of regrouping positively correlates with the number of errors in the concept of place value.
- In the VRM group, a greater number of solving problems correlates with a high level of virtual reality navigation behaviour.

4.2.1 Design

This study used a between-subjects experimental design. Students participated in the study during their regularly scheduled mathematics class sessions. Participating students were randomly assigned to one of two treatment groups (VRM and 3DM). According to Gall et al. (2003), an experimental study is the most influential research method for verifying cause and effect relationships among two or more variables. As such, the procedure for this study involved the assignment of students into groups for the delivery of instructional interventions, illustrating the sequence of study procedures and data collection.

4.2.2 Participants

The participants in this study were second-grade students from four primary schools. These students were not novices regarding computer use because computers are used in general applications for different subjects. In total, 104 students were included in this study from the following schools: 36 from Alandalus, 24 from DarAlhuda, 18 from Alferdaous and 26 from Alebdaa. After comparing the students' math scores, referred to as a mathematical achievement in the schools, the highest level of mathematical achievement was found in the Alandalus school (high = 34% and medium = 35.2%). In contrast, the lowest mathematical level of achievement was found on the Alferdaous school (high = 20% and medium = 14.8%).

Figure 4.5 illustrates that there is a difference between schools in the students' mathematical levels of achievements. This led us to ask the question, 'Do the school influence student achievement levels in mathematics?' To answer this question, a Pearson chi-square test was performed to determine whether there was a significant relationship between the school and level of achievement in mathematics. The results indicated that there was no statistical significant association between schools and mathematical levels of achievement ($\chi^2 [3] = 1.733$, $p = 0.630$). Therefore, we considered all students, regardless of their school, as if they were from one school.

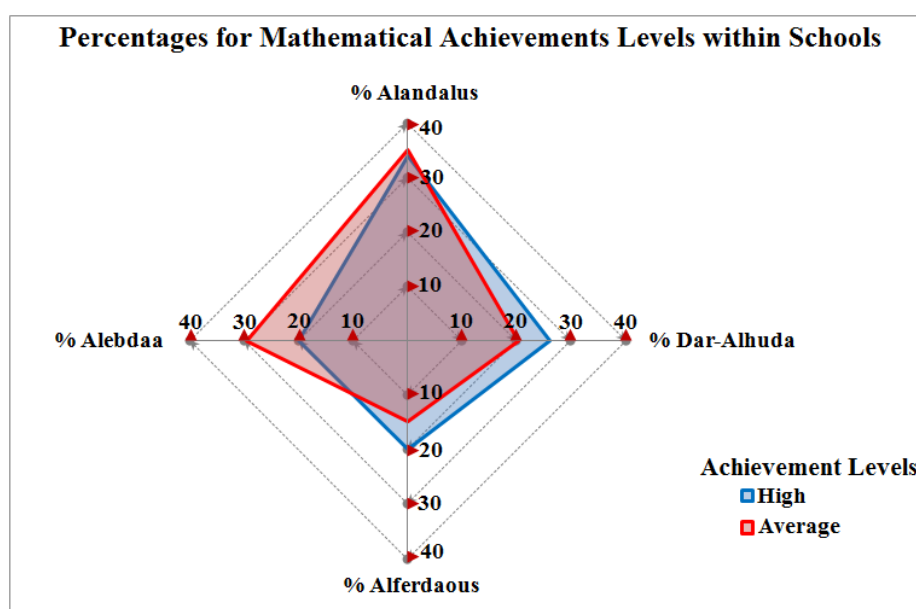


Figure 4.5 Percentages of student's mathematical level of achievements per schools

Accordingly, the participants in this study were selected from each school randomly. The students in class (A) were assigned to the VRM group, and the students from class (B) were assigned to the 3DM group. Thus, the total number of students in both groups equalled 52 (Figure 4.6).

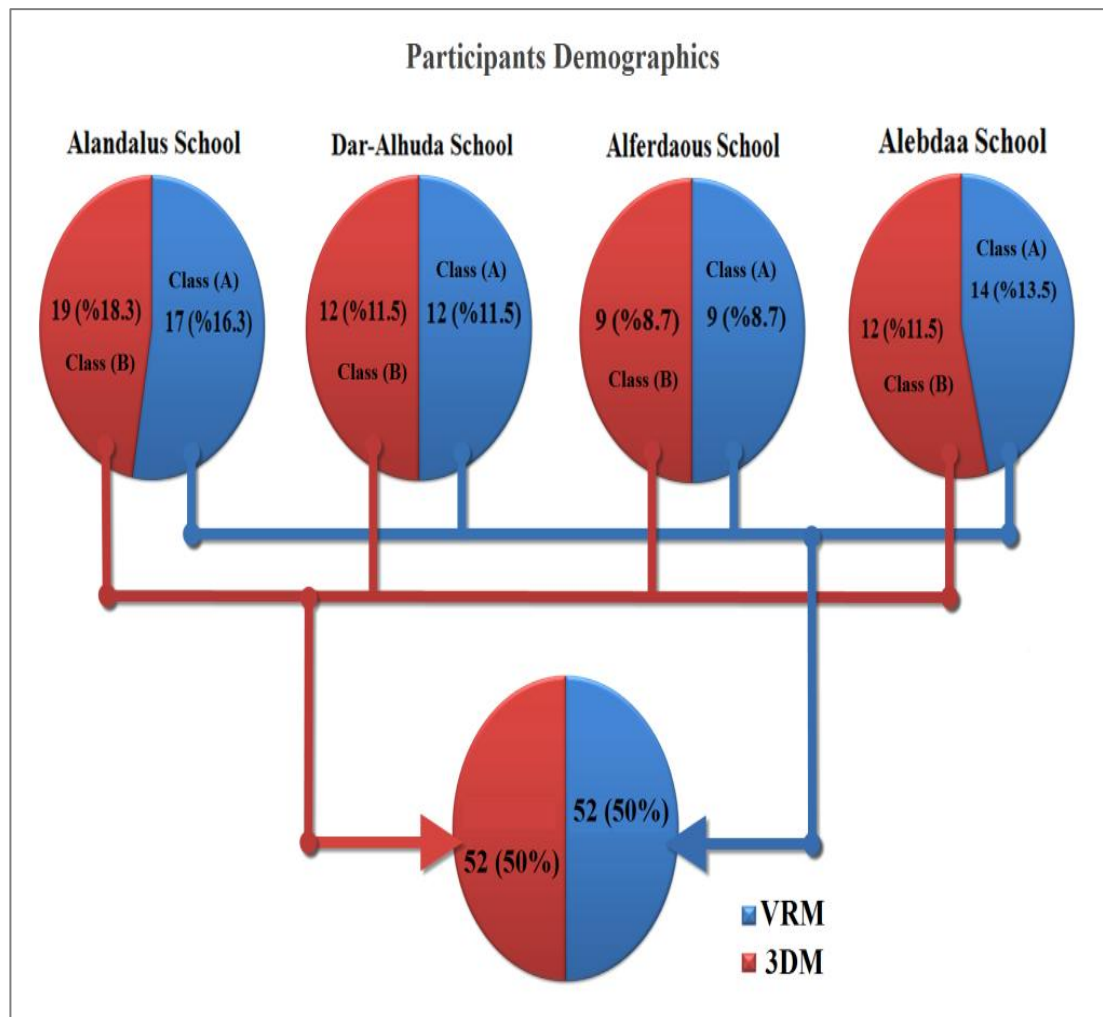


Figure 4.6 Participant distributions across experimental groups

4.2.3 Procedure

This study took place in computer labs at the four individual schools during regular school hours and over the course of five days. The students attended regular 45-minute computer lab sessions scheduled every day. In all the schools, a formal coordination existed between the researcher and the math teachers to ensure that students were taught addition and subtraction before the week of the experiment. Thus, the length of time between the classroom teaching lesson of addition and subtraction and the date of the experiment was constant in all schools.

On the first day of the study, the researcher downloaded the NLVM and MAVLE experimental applications to all computers in each school. On the second day, the researcher presented a one-day NLVM and MAVLE workshop to introduce teachers to the purpose of the study and to guide them in how to teach and implement the numeracy concepts of addition and subtraction. On the third experimental day, the teachers and the researcher introduced the students to the application interface and tools.

Students began working with the exercises based on their previous knowledge of addition and subtraction of two- and three-digit integer numbers (see Appendix D and E). Each new task was introduced only when the researcher was satisfied that the students had successfully understood the previous task. This process continued until the students became familiar with the NLVM and MAVLE applications. On Days 4 and 5, the students conducted addition and subtraction exercises freely; however, these lessons were related to the lessons of the numeracy curriculum unit.

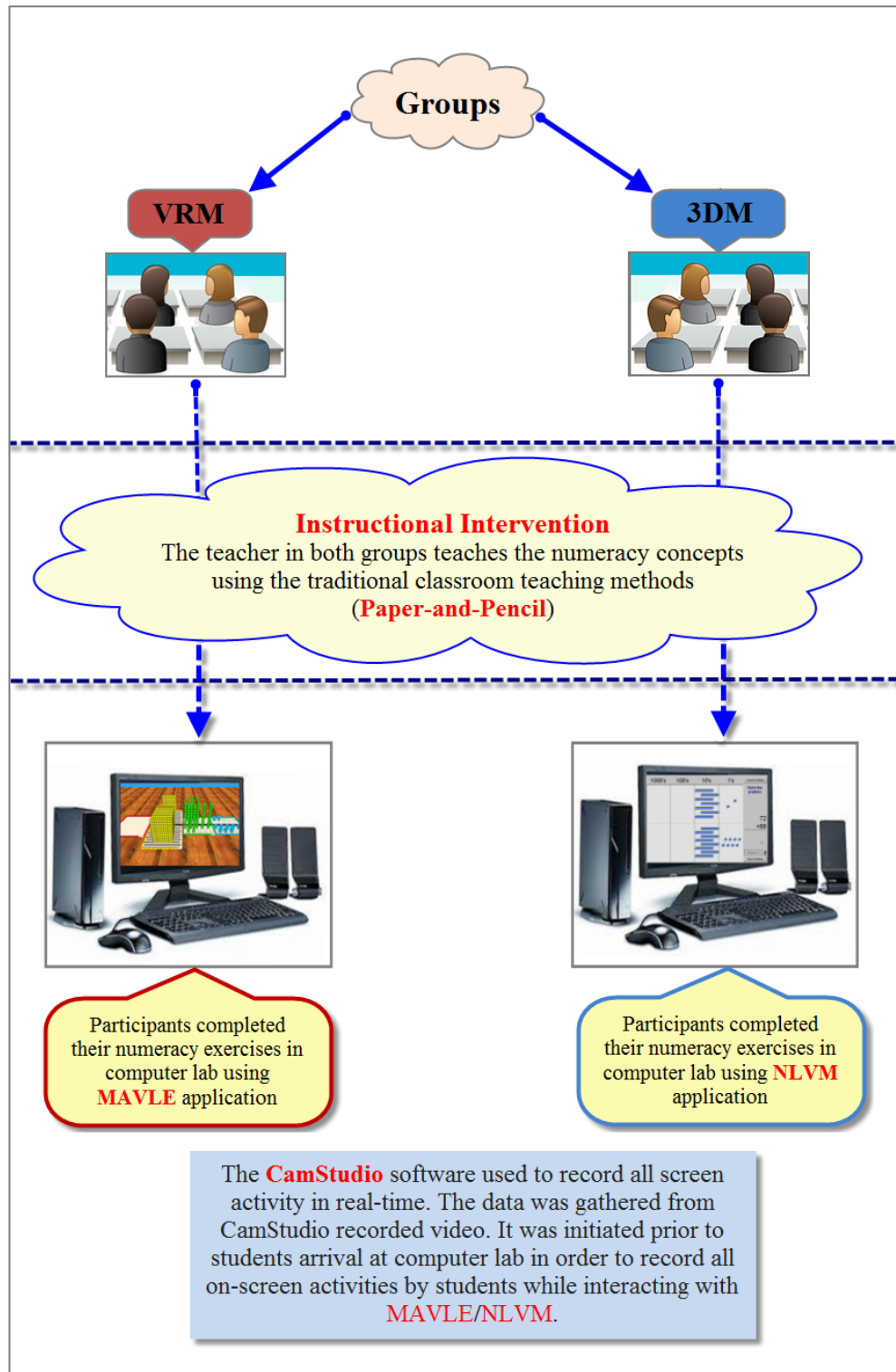


Figure 4.7 The sequences of study-1 procedures and data collection

4.2.4 Data Collection

The computer lab setting was prepared by the researcher by placing a numbered sticker on each computer screen. This number was linked to the name of each student listed on the class register in order to track the student's place in the computer lab. Data was collected using the screen video capturing program, CamStudio (released by RenderSoft). The flexible nature of screen video-based data collection allows a researcher to rewind and review materials repeatedly. The advantages of using screen video-based data lie in its permanence as a record, its uncomplicated restoration and its ability to make findings readily available to other researchers.

The CamStudio software was used to record students on-screen activity and the interaction of keyboard/mouse input during the experimental computer lab sessions. The CamStudio software was initiated before the arrival of the students. The researcher requested that all students freely interact with the math application and start their individual exercises. After the session ended, the researcher saved the recorded screen video on a hard disk. The researcher carefully watched and transcribed the data from the recorded video screen files.

4.2.4.1 Preparing to analyze video data

Before data analyses could begin, understanding the type of elements that were most important to note for both groups was required. This research sought to find evidence related to the predictions made in the hypotheses for this study. Therefore, we were particularly interested in counting the number of times children started new numeracy problems (addition or subtraction) and solved them successfully.

In addition, we were also interested in determining the number of errors for the regrouping and place-value concepts when children failed to solve the numeracy problems correctly. Furthermore, for the VRM group, extra data were collected from the screen-capturing videos by observing the frequency of navigational behaviours such as wayfinding, flying, walking and zooming, as shown in Table 3.4. The purpose of collecting these data was because we expected there might be a pattern between the number of solved problems and the virtual reality navigational behaviours that could help us make a certain prediction about how virtual reality navigational behaviours help in the learning process.

4.2.5 Data Analysis Plan

Variables used in the study are summarised in Table 4.1. The dependent variables were collected by counting the number of problem-solving achievements and the number of errors in the regrouping and place-value concepts. Specifically, for the VRM group, the total number of navigational behaviours (walk, fly, zoom in/out and wayfinding) in the MAVLE was collected, and these navigational behaviours were performed by the students while solving problems.

Table 4.1 Summary of Dependent and Independent Variables in Study-1

Type	Variables	Description
Independent	3DM	Refers to the 3D manipulatives representation of base-10 block manipulatives in NLVM.
	VRM	Refers to the VRM representation of base-10 block manipulatives in the MAVLE.
Dependent	Navigational behaviours	Refers to the total number of navigational behaviours patterns (fly, walk, zoom in/out and wayfinding) performed by each participant in the MAVLE.
	Solved problems	Refers to the number of successful completions of addition and subtraction problems.
	Regrouping errors	Refers to the number of errors in the regrouping concept (i.e., carry and borrow) while solving addition and subtraction problems.
	Place-value errors	Refers to the number of errors in the place-value concept (i.e., 1's, 10's, 100's and 1000's) while solving addition and subtraction problems.

Multiple data analysis techniques were used for the collected data. For the analysis of the data, the SPSS 17.0 (Statistical Package for Social Sciences) and Excel 2007 were used. At the beginning of the study, an independent sample t-test at the level of significance 0.05 was conducted to look for significant differences between the students who were interacting with either the MAVLE or the NLVM. Then, correlations and linear regression analysis methods were used to explore the relationships among the variables.

4.3 Study 2: The Effectiveness of Virtual Reality Manipulatives on Student Achievements in Numeracy Concepts

In this study, our primary goal was to investigate the effectiveness of VRM on student achievement in numeracy concepts. The research hypothesis addressed in this study was as follows:

- All groups are predicted to have a significant positive achievable outcome from pre-tests to post-tests for both addition and subtraction.
- The VRM group is predicted to have a significant positive achievable outcome in post-tests for both addition and subtraction than the 3DM or traditional classroom teaching (TCT) groups.

4.3.1 Design

This study used a quasi-experimental design pre-test, post-test and control group (Campbell and Stanley, 1966). As such, the procedure for this study involved assignment of students to groups, implementation of pre-tests, delivery of instructional interventions and implementation of post-tests, Figure 4.8 illustrates the sequence of study events and procedures.

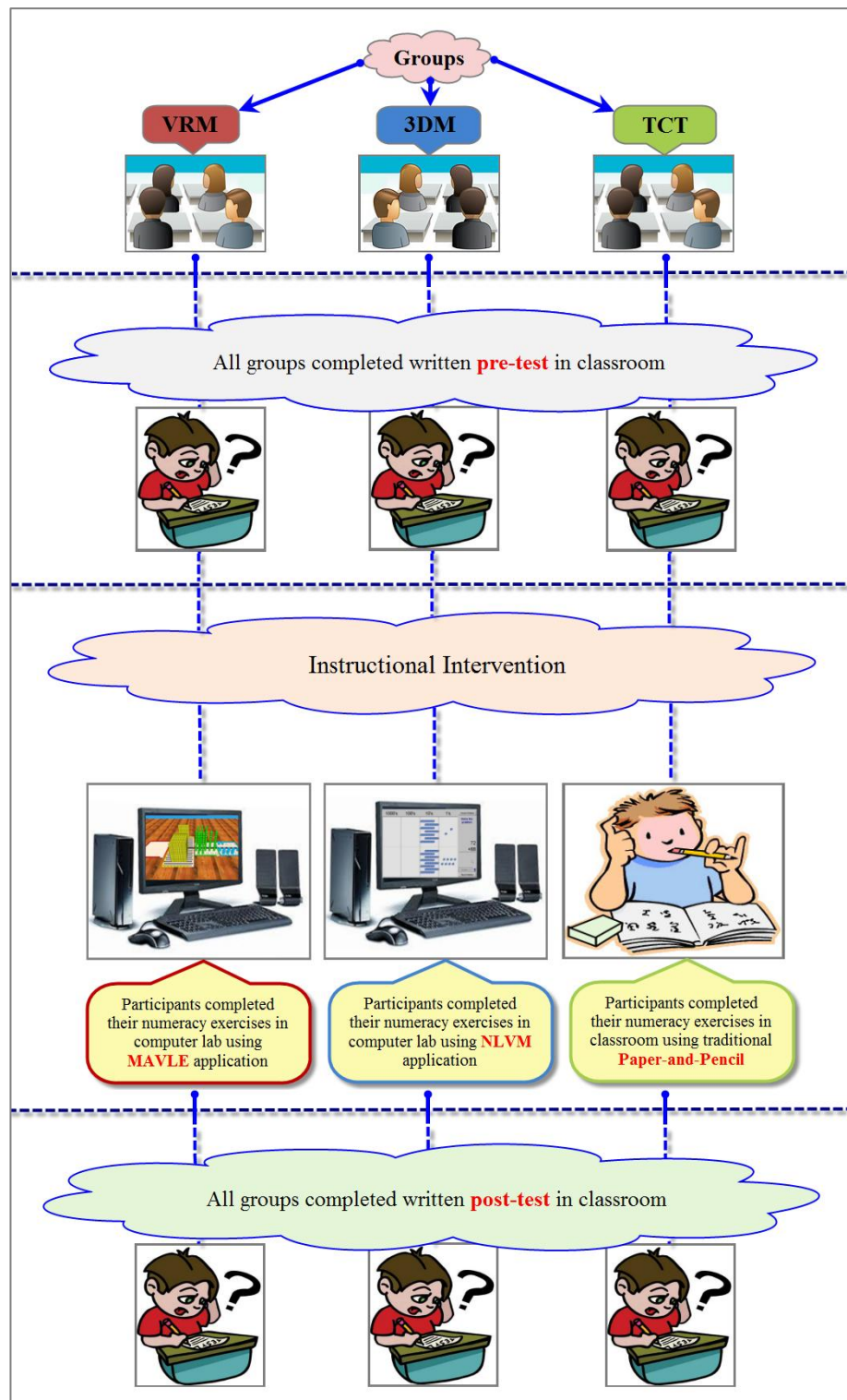


Figure 4.8 The sequences of study-2 procedures and data collection.

4.3.2 Participants

The participants were 59 students from three different classrooms. All three classes were taught by the same teacher. Class (A) had 17 students, Class (B) had 20 students and class (C) had 22 students.

4.3.3 Procedure

The most common tools for measuring student achievement levels are pre- and post-tests. The researcher designed in-class paper-and-pencil tests of numeracy lessons for both addition and subtraction. The tests were based on second-grade level objectives for learning addition and subtraction. Students completed the test before the addition/subtraction unit (i.e., the pre-test) and on the last day of the addition/subtraction unit (i.e., the post-test).

4.3.3.1 Pre/Post Tests Implementation

Pre- and post-tests were developed to determine student learning associated with target concepts. The researcher administered the pre-tests before the interventions and the post-tests immediately after the interventions for all three groups. The pre- and post-tests were given during their assigned class period without exceeding the standard class time of 45 minutes. Pre- and post-tests for both addition and subtraction were distributed to all students. The teacher asked the students to place their assigned student identification number in the space provided, read the written instructions to them and encouraged them to do their best work. When the students had finished the tests, the teacher collected them and placed them in a folder provided by the researcher.

4.3.3.2 Homogeneity of Pre-test

Before the data could be analyzed to answer the research questions, it was important to examine the equivalence of the instruments by analyzing the data produced by the three instruments. Two separate statistical tests were performed to determine the equivalence of the pre-test: Levene's test of equality of variances and a one-way analysis of variance (ANOVA).

The results of the Levene's test evaluated one of the assumptions of the one-way ANOVA, which was whether the population variances for the three groups were equal or not. Based on the Levene's test of equality of variances, it could be assumed that the homogeneity of variances was not violated ($p = 0.142$, $p > 0.05$) in the study. This indicated that the homogeneity assumption was valid for all group comparisons. The first test, a Levene's test, showed that the p-value was greater than 0.05; thus, the variance in pre-test scores was not significantly different in the three groups. This indicated that the homogeneity assumption was valid for all group comparisons.

The second test, a one-way ANOVA, was conducted to determine if the three learning modes were homogeneous in terms of existing knowledge of addition and subtraction, which was measured by the pre-test. Statistical tests were conducted at a p-value significance level of 0.05. The result showed that there was no statistically significant difference in the pre-test scores between the VRM group ($M = 24.94$, $SD = 10.262$), 3DM group ($M = 22.85$, $SD = 6.815$) and TCT group ($M = 21.45$, $SD = 6.688$; $F [2, 56] = 0.932$, $p = 0.400$). It was, thus, inferred that there were no significant differences regarding previous knowledge on the subject matter for all learning modes.

Classes were chosen randomly for each group: Class A = the VRM group, Class B = the 3DM group and Class C = the TCT group.

4.3.3.3 Description of addition/subtraction instructional setting sequence

During this study, lessons were conducted in the second-grade classrooms and a computer lab. There were 25 computers in the computer lab, as well as a teacher computer station with a display screen. Every student had their own computer and worked independently in the lab.

The addition pre-test was administered in one day for all three classes. The subtraction pre-test was administered the following day. The lessons for the TCT group started with an introduction to the numeracy topic for the day; this was followed by several mathematical tasks where students used paper and a pencil. Students completed worksheets and teacher-made task sheets that provided practice with the physical manipulatives. At the end of each computer lab and classroom session, the teacher used the last 10 minutes of the class to hold a discussion with the students to elicit thinking and connect ideas that students explored during the sessions.

After these two days, the teacher started a new lesson concept each day followed by a practice exercise. The teacher taught the lesson in each class at their scheduled times. Each assigned class was taught with the use of one treatment application (i.e., VRM, 3DM or TCT).

The teacher explained new concepts on Day 1 and Day 3. On Days 2 and 4, the teacher gave the students' numeracy problems to solve to confirm their understanding. The days (2 and 4) were either spent in the computer lab for Classes A and B or in the classroom for Class C.

The computer lab sessions started with an introduction to the virtual manipulative (VRM or 3DM); this was followed by several mathematical tasks for the students to complete independently. Each day, students received teacher-made task sheets with instructions for using the virtual manipulatives and space to record their work. The teacher modelled how to use the virtual manipulative applets before students worked independently.

Classroom Instructional Settings

The conversational framework by Laurillard (2002) can be used to explain how an active conversation between teachers and students may support student mathematical learning in math classrooms. From this perspective, we based our design of the classroom setting on Laurillard's conceptual level of actions (Figure 4.9). The teacher helped the students to build their mathematical numeracy knowledge of the concepts through the processes of iterative negotiation.

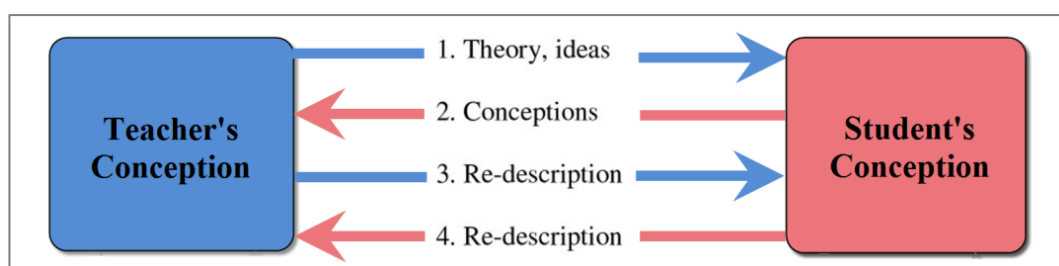


Figure 4.9 Laurillard's 'Conceptual' level of actions

The class started with the teacher greeting the students and asking them to open their books to that day's lesson about an addition or subtraction. Similarly, in all classes, the teacher projected the same page on the screen or board in front of the class. The classroom setting was the same for all groups (Figure 4.10). The researcher was present with the teacher in the class at all times to observe without interference.

After introducing the new concept on the screen or board, the teacher solved the examples in each class according to the class-assigned treatment; Class A used the MAVLE application, Class B used the NLVM application and Class C used the paper and pencil method.

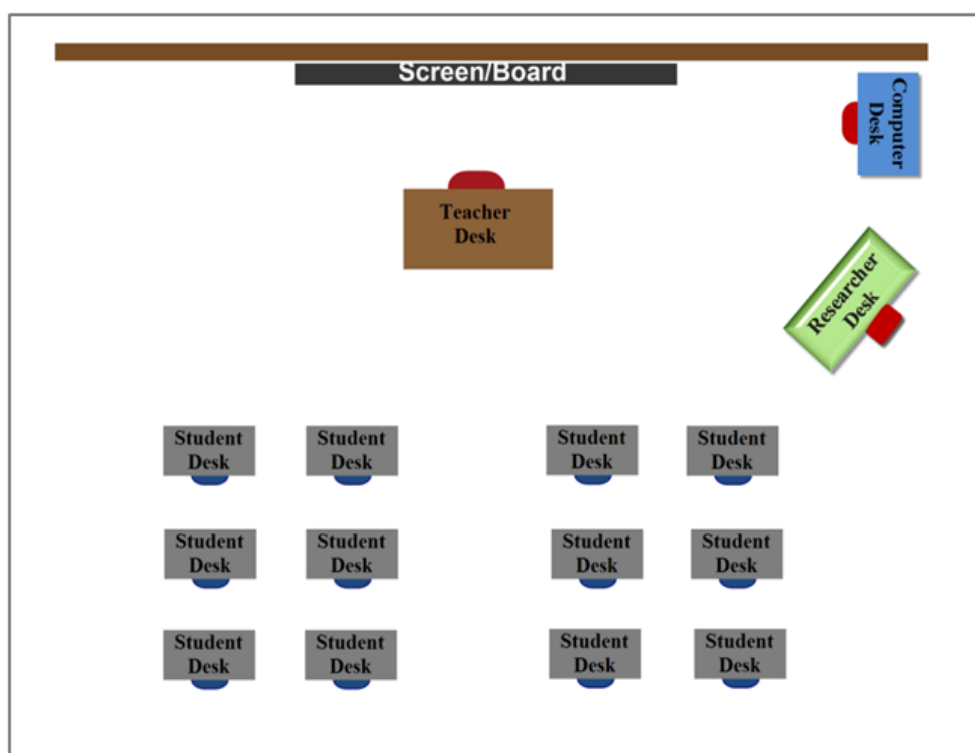


Figure 4.10 The classroom setting

Computer-Lab Instructional Settings

Laurillard's (2002) conversational framework viewed the learning process as a conversation between the teacher and student. From this perspective, we based the design of the computer lab setting on Laurillard's experiential level of actions (Figure 4.11). At this level, the teacher sets out practices for the students to improve their understanding of the concepts.

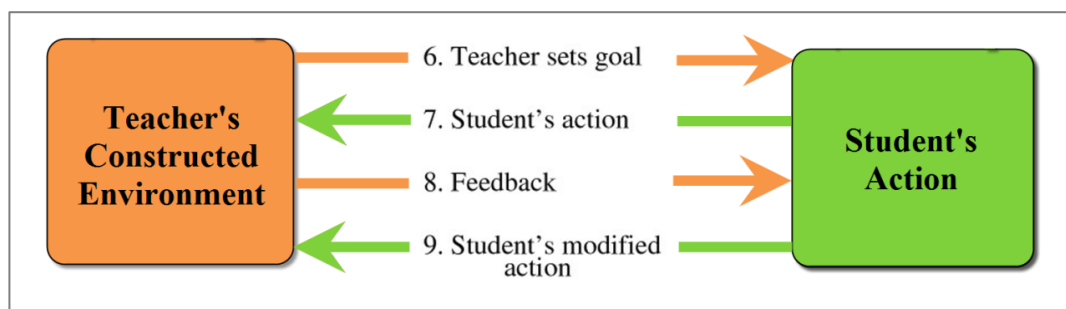


Figure 4.11 Laurillard's 'Experiential' level of actions

Class A went to the computer lab with their teacher and the researcher. Each student sat at a desktop computer station and started applying the concepts learned during class at their own pace, with the opportunity to manipulate the exercises freely. The lab setting (Figure 4.12) was the same for the VRM and 3DM groups.

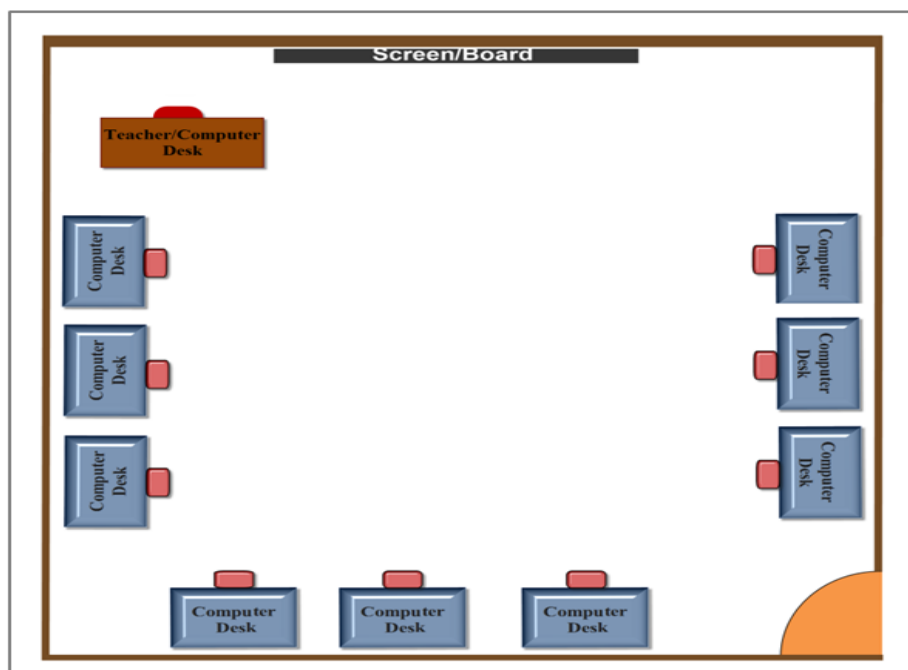


Figure 4.12 The computer lab setting

After the teacher explained and solved one example on the computer lab screen, the teacher asked the students to start solving more exercises by themselves to elicit their understanding of the concepts using the assigned treatment (VRM or 3DM) while doing so. The teacher moved around the class to observe the students' performances. The same sequence took place with Class B in the computer lab at their scheduled class time.

4.3.4 Data Collection

Data sources included pre- and post-tests used to examine student addition and subtraction content knowledge. The pre- and post-tests had eight questions for both addition and subtraction (Appendices D and E). Table 4.2 provides a list of all the numeracy concepts measured by the pre- and post-tests. Three subject matter specialists were requested to review the test questions and provide an assessment of how well the test questions represented the numeracy concepts of addition and subtraction.

These three subject matter specialists were math teachers in Jeddah—the location of the study. Comments from the subject matter specialists were taken into consideration before the tests were used.

Both pre and post-tests were similar in content. The total grade was out of 28 for each test. Grading systems for each question on the pre- and post-tests were as follows for both addition and subtraction:

- One mark was given for each correct answer in addition or subtraction for any digit (1's, 10's and 100's) and zero for each incorrect answer.
- One mark was given for each correct answer in regrouping concepts (i.e., carry and borrow) and zero for each incorrect answer.

Table 4.2 Addition and subtraction operation concepts for the second grade level

Addition Concepts			Subtraction Concepts		
Concept	Description	Example	Concept	Description	Example
1-	Add a two-digit and a one-digit number - without regrouping	$\begin{array}{r} 23 \\ + 5 \\ \hline 28 \end{array}$	1-	Subtract a one-digit number from a one-digit number	$\begin{array}{r} 5 \\ - 3 \\ \hline 2 \end{array}$
2-	Add a two-digit and a one-digit number - with ten's regrouping (i.e. carry)	$\begin{array}{r} \boxed{1} \\ 36 \\ + 5 \\ \hline 41 \end{array}$	2-	Subtract a one-digit number from a two-digit number - without regrouping	$\begin{array}{r} 36 \\ - 5 \\ \hline 31 \end{array}$
3-	Add two two-digit numbers - without regrouping	$\begin{array}{r} 34 \\ + 15 \\ \hline 49 \end{array}$	3-	Subtract a one-digit number from a two-digit number - with ten's regrouping (i.e. borrow)	$\begin{array}{r} \boxed{2} \boxed{14} \\ 34 \\ - 7 \\ \hline 27 \end{array}$
4-	Add two two-digit numbers - with ten's regrouping (i.e. carry)	$\begin{array}{r} \boxed{1} \\ 47 \\ + 25 \\ \hline 72 \end{array}$	4-	Subtract two two-digit numbers - without regrouping	$\begin{array}{r} 47 \\ - 25 \\ \hline 22 \end{array}$
5-	Add two two-digit numbers - with ten's and hundred's regrouping (i.e. carry)	$\begin{array}{r} \boxed{1} \boxed{1} \\ 87 \\ + 34 \\ \hline 121 \end{array}$	5-	Subtract two two-digit numbers - with ten's regrouping (i.e. borrow)	$\begin{array}{r} \boxed{7} \boxed{13} \\ 83 \\ - 34 \\ \hline 49 \end{array}$
6-	Add two three-digit numbers - without regrouping	$\begin{array}{r} 523 \\ + 415 \\ \hline 938 \end{array}$	6-	Subtract three-digit numbers - without regrouping	$\begin{array}{r} 625 \\ - 412 \\ \hline 213 \end{array}$
7-	Add two three-digit numbers - with ten's and hundred's regrouping (i.e. carry)	$\begin{array}{r} \boxed{1} \boxed{1} \\ 123 \\ + 298 \\ \hline 421 \end{array}$	7-	Subtract three-digit numbers - with ten's regrouping (i.e. borrow)	$\begin{array}{r} \boxed{6} \boxed{13} \\ 673 \\ - 258 \\ \hline 415 \end{array}$
8-	Add two three-digit numbers - with ten's, hundred's and thousand's regrouping (i.e. carry)	$\begin{array}{r} \boxed{1} \boxed{1} \boxed{1} \\ 723 \\ + 398 \\ \hline 1121 \end{array}$	8-	Subtract three-digit numbers - with ten's and hundred's regrouping (i.e. borrow)	$\begin{array}{r} \boxed{11} \\ \boxed{8} \boxed{1} \boxed{13} \\ 923 \\ - 295 \\ \hline 628 \end{array}$

4.3.5 Data Analysis Plan

This section describes the various statistical tests used to analyze and test data. Each research hypothesis were investigated using the collected data for both addition and subtraction numeracy concepts. SPSS Version 17 and Excel 2007 were used to run the statistical tests, which included a paired sample t-test data analysis procedure to determine whether any of the groups (VRM, 3DM and TCT) demonstrated significant improvement from the pre-test to the post-test. An ANOVA was performed on the addition post-tests and the subtraction post-tests to look at the differences in test scores among groups.

The effect size was used to tell if the effect to be tested was weak or strong (Cohen, 1988). The effect size of an ANOVA-type model test is known as partial eta squared (i.e., η^2). When the η^2 is: 0.1 it assumes that the effect size is small, 0.25 it assumes that the effect size is moderate and 0.4 it assumes that the effect size is strong. The paired sample t-test data analysis procedure was performed to determine whether any of the groups demonstrated significant improvements from the pre-test to the post-test.

Furthermore, to determine whether significant differences existed among the groups on post-test performance, an analysis of covariance (ANCOVA) was performed with the groups serving as the principal independent variable and the post-test score as the dependent variable; the pre-test score was the covariate. When subjects are randomly assigned to treatment groups and the experimental design includes pre- and post-tests (Schochet, 2008), the ANCOVA are the ideal method for adjusting for possible extraneous variables.

CHAPTER 5 Data Analysis Results

In this chapter, data analysis results are reported separately for the two studies. For each study, the quantitative analysis results, for which data were collected, are reported.

5.1 Study 1: The Impact of Virtual Reality Manipulatives on Student Performance in Numeracy Concepts

This study examined the delivery of numeracy activities for addition and subtraction, using VRM and 3DM in the computer lab. A comparison was made for the accomplished activities of addition and subtraction and the encountered errors in the place-value and regrouping concepts. The goals of the study were to determine the impact of virtual reality on student performance in numeracy concepts (addition and subtraction) and which manipulatives format, VRM or 3DM, had the greatest effect on student achievement performance and children's behaviours. In this study, data were obtained using a screen-capturing software (CamStudio) that recorded all on-screen interactions performed by each student who used their keyboard and mouse. Data were analysed using descriptive summaries and tests to determine the significant differences among groups, correlations and regression models.

The study specifically attempted to investigate the following research hypotheses:

- The VRM group is predicted to have a significant positive performance outcome (i.e., regarding the number of solving problems for addition and subtraction) than those in the 3DM group.
- The number of errors in place-value concept predicted to be significantly different between VRM group and 3DM group.
- The number of errors in the regrouping concept is predicted to be significantly different between the VRM and 3DM groups.
- The number of errors in the concept of regrouping positively correlates with the number of errors in the concept of place value.
- In the VRM group, a greater number of solving problems correlates with a high level of virtual reality navigation behaviour.

5.1.1 Results

Our aim of this study was to verify that the VRM were more useful in helping students comprehend numeracy concepts. The first hypothesis was that *‘The VRM group is predicted to have a significant positive performance outcome (i.e., regarding the number of solving problems for addition and subtraction) than those in the 3DM group’*.

Descriptive statistics (Figure 5.1) showed that the students in the VRM group had a higher mean score of 10.83 (SD = 6.392) than the students in the 3DM group with a mean score of 6.81 (SD = 3.543).

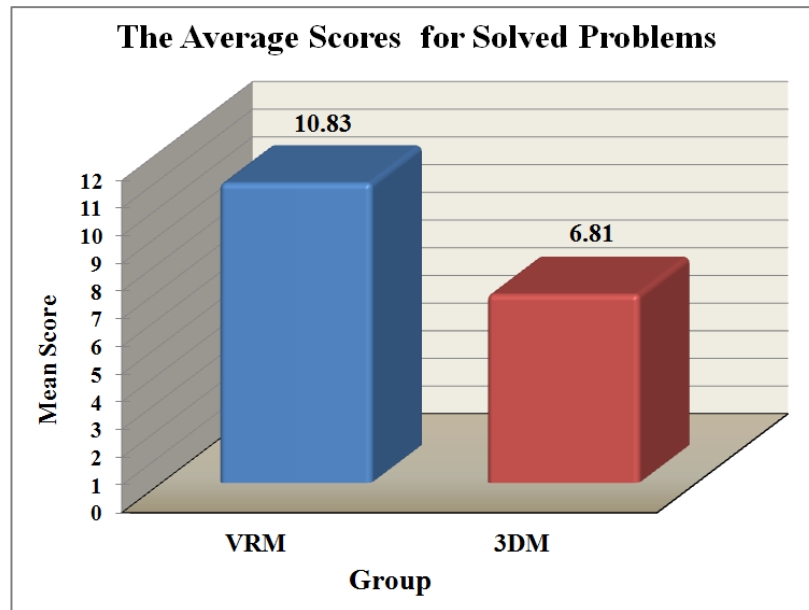


Figure 5.1 Mean number of problems solved by groups

An independent sample t-test was performed to test the previous hypothesis. The difference between these two groups (see Table 5.1) was found to be statistically significant ($t = 3.966$, $df = 102$, $p = 0.000$).

Table 5.1 Mean number of numeracy problems solved by group (N = 52).

Group	Mean	Std. Deviation	t	Sig. (2-tailed)
VRM	10.83	6.392	3.966	.000
3DM	6.81	3.543		

We looked for the percentage of concept errors in regrouping (i.e., carry and borrow) and place value (i.e., base-10 numeration systems: 1's, 10's, 100's and 1000's) during the performance of the addition and subtraction operations, we noticed that the percentage number of conceptual errors in the 3DM group for regrouping were 81.44% and 60.67% for the place-value concept.

Whereas, in the VRM group, the percentage number of conceptual errors for the regrouping concept was 18.56% and 39.33% for the place-value concept. These data indicated that the amount of regrouping and place-value concept errors in the 3DM was greater than those in the VRM group. This may reveal that the 3DM group most likely has difficulties with the regrouping and place-value concepts more than VRM group (Figure 5.2).

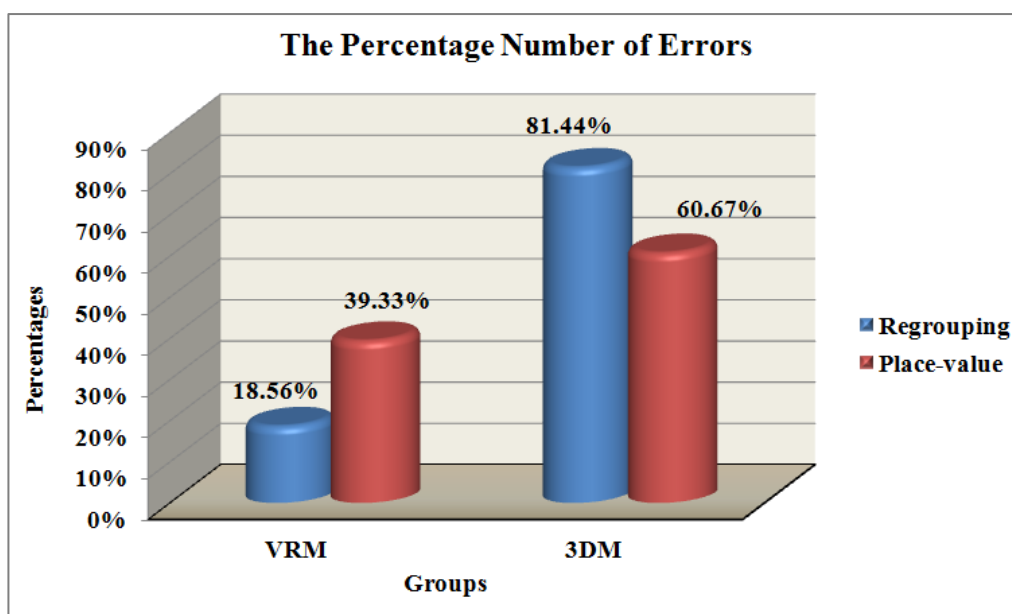


Figure 5.2 The percentage number of conceptual errors in the regrouping and place-value concepts per groups.

This tremendous variation in the number of errors for the regrouping concept between groups (3DM = 81.44% versus VRM = 18.56%) could be caused by students' misconceptions of the base-10 numeration system (i.e., place value), as suggested by Price (1998). Understanding the place-value concepts is a necessary prerequisite in computations (Price, 1998). In other words, Nataraj and Thomas (2009) stated that children need to understand base-10 number system (i.e., place value) structures to develop their conceptual understanding of numbers and operations.

The second hypothesis stated, '*The number of errors in the place-value concept is predicted to be significantly different between the VRM and 3DM groups.*'

The mean score for the place-value errors in the VRM group was 1.35 (SD = 1.644), and the mean score was 2.08 (SD = 2.641) in the 3DM group. An independent sample t-test was performed to test the previous hypothesis. The difference between these two groups was not found to be statistically significant ($t = -1.694$, $df = 102$, $p = 0.093$).

The third hypothesis stated, '*The number of errors in the regrouping concept is predicted to be significantly different between the VRM and 3DM groups.*'

The mean score for regrouping errors in the VRM group was 1.29 (SD = 1.601), and the mean score was 5.65 (SD = 4.191) in the 3DM group. An independent sample t-test was performed to test the previous hypothesis. The difference between these two groups was found to be statistically significant ($t = -7.017$, $df = 102$, $p = 0.000$).

The fourth hypotheses stated, '*The number of errors in the concept of regrouping correlates positively with the error number in the concept of place value.*'

Further analysis was undertaken to investigate if there was a correlation between regrouping and place-value concept errors for both groups. In order to detect this correlation, the Pearson's r correlation coefficient was calculated for both groups. In the VRM group, there was no statistically significant correlation ($r = 0.259$, $n = 52$, $p = 0.063$) between regrouping and place-value concept errors. Whereas in the 3DM group,

the correlation between regrouping and place-value concept errors were statistically significantly and positively correlated ($r = 0.536$, $n = 52$, $p < 0.0005$).

Interestingly, the results showed that the number of concept errors in regrouping (i.e., carry and borrow) performed by the second-grade students in the 3DM group were significantly correlated with the number of errors in the place-value concept (i.e., base-10 numeration systems: 1's, 10's, 100's and 1000's). In contrast, in the VRM group, the absence of a significant correlation between the number of errors in the regrouping and place-value concepts could be related to navigational behaviours that allow students to visualise and explore the base-10 numeration system place values by travelling (walk, fly and zoom in/out). If they lose their way while travelling, they can click on the wayfinding navigational aid 'rest-position' button included on screen with the MAVLE system while they continue to solve numeracy problems for addition and subtraction simultaneously.

The final hypothesis stated, '*In the VRM group, a greater number of solved problems correlate with high levels of virtual reality navigation behaviour*'.

A scatter diagram showing the spreading of the variables (walk, zoom, fly and wayfinding) in navigational behaviours is presented in Figure 5.3. The scattered plotted points predicted the strength and direction of the relationships among walk, zoom, fly and wayfinding. By examining the scatter plot, we noticed the positive linear pattern and saw how close the points of walk, zoom, fly and wayfinding were to each other.

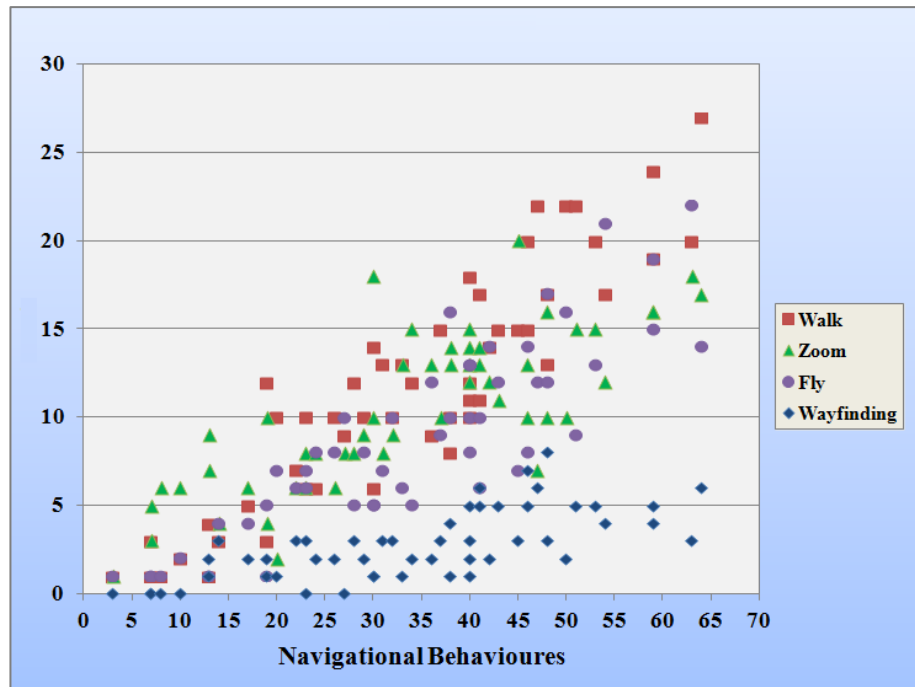


Figure 5.3 Scatter diagram showing the spreading of variables (walk, zoom, fly and wayfinding) in navigational behaviours.

To determine the strength of the relationships among walk, zoom, fly and wayfinding variables, Pearson's correlation were used. A significant positive moderate correlation at the 0.01 level was found among the navigational behavior variables. Table 5.2 summarises the analysis.

Table 5.2 Pearson's correlation among navigational behaviours variables (walk, zoom, fly and wayfinding)

Variables		Walk	Zoom	Fly	Wayfindin g
Walk	Pearson Correlation	1	0.598**	0.680**	0.614**
	Sig. (2-tailed)		.000	.000	.000
Zoom	Pearson Correlation		1	0.569**	0.431**
	Sig. (2-tailed)			.000	.001
Fly	Pearson Correlation			1	0.550**
	Sig. (2-tailed)				.000
Wayfindin g	Pearson Correlation				1
	Sig. (2-tailed)				

Clearly, it seems, from the above table, that there was a moderate positive correlation among the navigational behaviours variables. Therefore, we decided to combine the variables under one name, which was navigational behaviours.

A scatter diagram was drawn for the two variables of performance while solving problems and performance in navigational behaviours (Figure 5.4), in addition to calculating Pearson's r correlation coefficient. This scatter-plot diagram showed a clear pattern of a greater number of solving problems being achieved by those participants who used high levels of navigational behaviours.

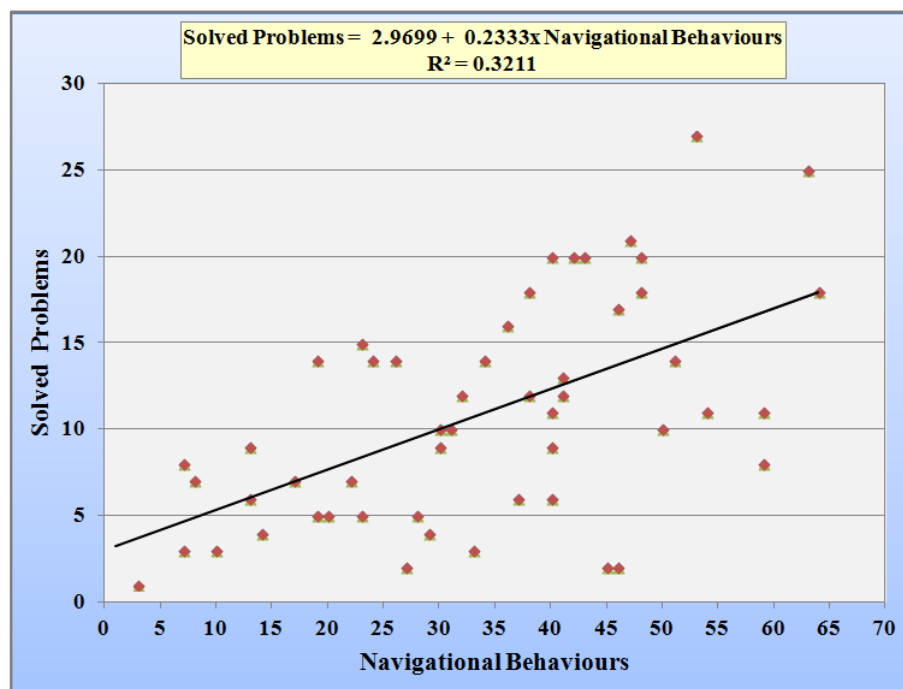


Figure 5.4 The relationship between solving problems and navigational behaviours.

This scatter plot was used to visually identify relationships between navigational behaviours and solved problems. Each plotted dot in this scatter diagram represented one student's navigational behaviours versus solved problems. As shown in Figure 5.4, it seemed that the data somewhat followed a linear path. Thus, the question was whether

there was a correlation between navigational behaviours and solved problems? Presumably positive because as the navigational behaviours increased, the number of solving problems increased, Pearson's r correlation coefficient analysis was calculated between the navigational behaviours and solved problems to determine whether there was a positive association between them.

The results revealed that there was a positive moderate linear correlation between navigational behaviours and solved problems, which was statistically significant ($r = 0.442$, $n = 52$, $p = .001$). Furthermore, not only did we want to determine whether there was a positive correlation between navigational behaviours and solved problems, but we also wanted to use the navigational behaviours to help predict the number of solving problems. Thus, regression analysis was used to determine how many problems could be expected to be solved if students increased their navigational behaviours.

The linear regression analysis was a model-based technique that was an extension of Pearson's correlation. Therefore, the question became, 'If the student increases his navigational behaviours, how many problems could expect to be solved?' By performing linear regression analysis, we tried to predict this answer; thus, the dependent predicted variable was solved problems and the independent predictor variable was navigational behaviours.

In the model summary, the R-square revealed the ‘goodness of fit’ of the model ($R^2 = 0.321$), which determined that the navigational behavior could explain approximately 32% of the variability in solved problems. The regression model ($R^2 = .321$, $F [1, 50] = 23.648$, $p = 0.000$) indicated that the variability of solving problems seemed to increase with increased navigational behaviours.

The regression equation may help us predict future results in order to decipher the number of solved problems determined by navigational behaviours.

5.1.2 Discussion

The power of virtual reality in visualisation is the removal of the need for the user to construct a mental 3D image of objects. The 3D models are very useful in familiarising students with the features of the different shapes and objects and can be particularly useful in teaching younger students. This 3D technology has brought new possibilities and challenges and explores virtual reality affordances from a new perspective, depending on which subject matter it is being used for.

The idea of place value and the structure of the number system gain added importance because they not only strengthened the understanding of the operations on numbers, fractions and decimals, but they are also the basis of algebra, which, in turn, forms the foundation for all higher mathematics (Nataraj and Thomas, 2009).

When the students fly to examine the base-10 blocks from a bird's eye view, they are experiencing the advantage of forming configuration knowledge directly (i.e., to get an overview of the spatial information).

This study examined how virtual reality navigation tasks affect student performance on the MAVLE system. The results revealed that there was a positive linear relationship between student performance and virtual reality navigation tasks, and this may be because virtual reality navigational attributes provide richer perceptual experiences to students. This may be linked to the third channel 'Immersion' in the virtual reality model for cognitive learning (see section 3.1).

5.2 Study 2: The Effectiveness of Virtual Reality Manipulatives on Student Achievements in Numeracy Concepts

This study examined the delivery of addition and subtraction content using the VRM or 3DM for activities on the computer. A comparison was made of the pre-test and post-test scores among the VRM, 3DM and TCT groups. The goal of this study was to determine which manipulatives, VRM or 3DM, had the greatest effect on students' achievements. The research hypotheses addressed in this study were as follows:

- All groups are predicted to have a significant positive achievable outcome from pre-tests to post-tests in both addition and subtraction.
- The VRM group is predicted to have a more significant positive achievable outcome of the post-tests for both addition and subtraction than the 3DM and TCT groups.

5.2.1 Results

Our primary goal was to investigate the effectiveness of VRM on student achievement in numeracy concepts. The hypothesis was that *‘All groups are predicted to have a significant positive achievement outcome from pre-tests to post-tests in both addition and subtraction.’*

Figure 5.5 demonstrates the percentages of mean gain scores for the addition operation (post-test to pre-test) for all groups. The VRM group had an average increase of 44.43% in the post-test, the 3DM group had an average increase of 30.46% in the post-test and the control group had an average increase of 26% in the post-test. The results reveal that the TCT group had the lowest average increase in scores in the post-test, and the VRM group recorded the highest average score increase in the post-test.

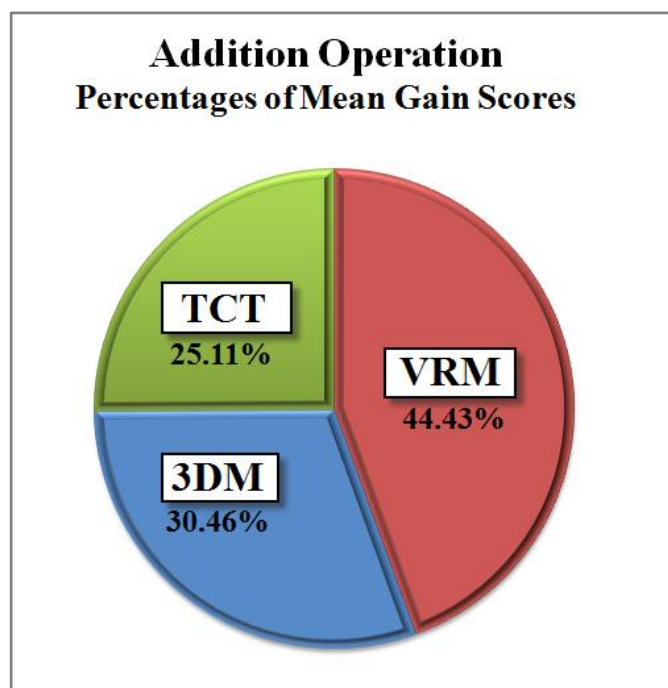


Figure 5.5 Percentages of mean gain scores for addition operation

To analyze the overall change for each group from pre-test to post-test (Figure 5.6), the researcher used a paired sample t-test with a confidence level of 0.05. The VRM group had a mean change of 10.94 with an SD of 7.013. The t-test showed a significant difference between the pre- and post-test scores ($t = 6.432$, $p = 0.000$). The 3DM group had a mean change of 7.50 with an SD of 6.629. The t-test analysis yielded a significant difference between the pre- and post-test scores ($t = 5.060$, $p = 0.000$). The TCT group had a mean change of 6.12 with an SD of 3.594. The t-test revealed a significant difference between pre- and post-test scores ($t = 8.067$, $p = 0.000$).

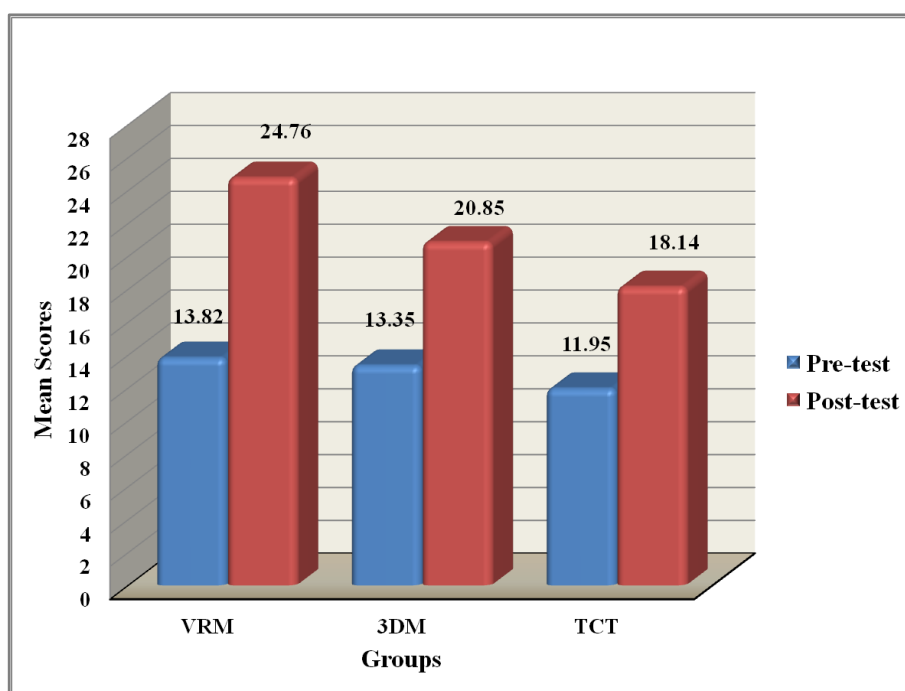


Figure 5.6 Differences between mean scores of the pre- and post-tests for the addition operation

Figure 5.7 demonstrates the percentages of mean gain scores for the subtraction operation (post-test to pre-test) for all groups. The VRM group had an average increase in scores of 38.49% in the post-test. The 3DM group had an average increase in scores of 33.96% in the post-test, and the TCT group had an average increase in scores of 27.10% in the post-test. The results reveal that the TCT group had the lowest average

increase in scores in the post-test, and the VRM group recorded the highest average increase in score in the post-test.

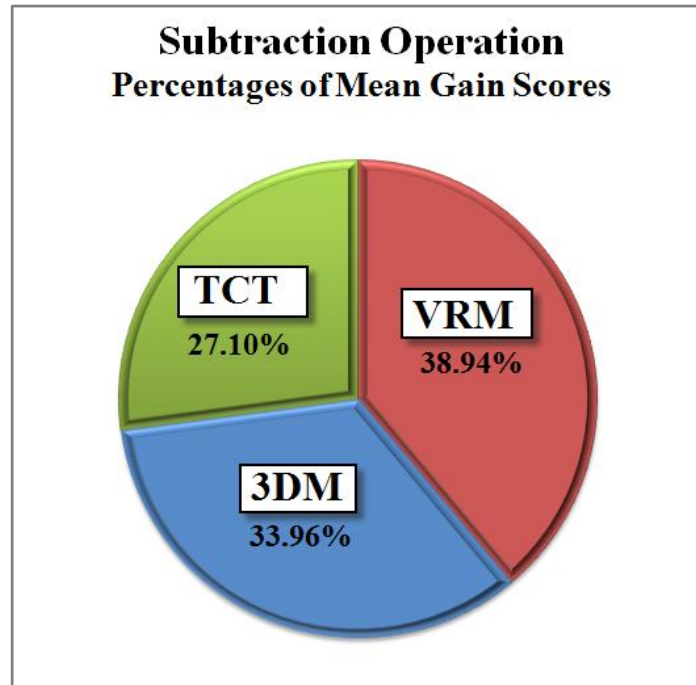


Figure 5.7 Percentages of mean gain scores for the subtraction operation

To analyze the overall change for each group from pre-test to post-test (Figure 5.8), the researcher used a paired sample t-test with a confidence level of 0.05. The VRM group had a mean change of 13.59 with an SD of 2.830. The t-test showed a significant difference between pre and post-test scores ($t = 19.799$, $p = 0.000$). The 3DM group had a mean change of 11.85 with an SD of 3.703. The t-test analysis yielded a significant difference between pre and post-test scores ($t = 14.311$, $p = 0.000$). The TCT group had a mean change of 9.45 with an SD of 4.480. The t-test revealed a significant difference between pre and post-test scores ($t = 9.899$, $p = 0.000$).

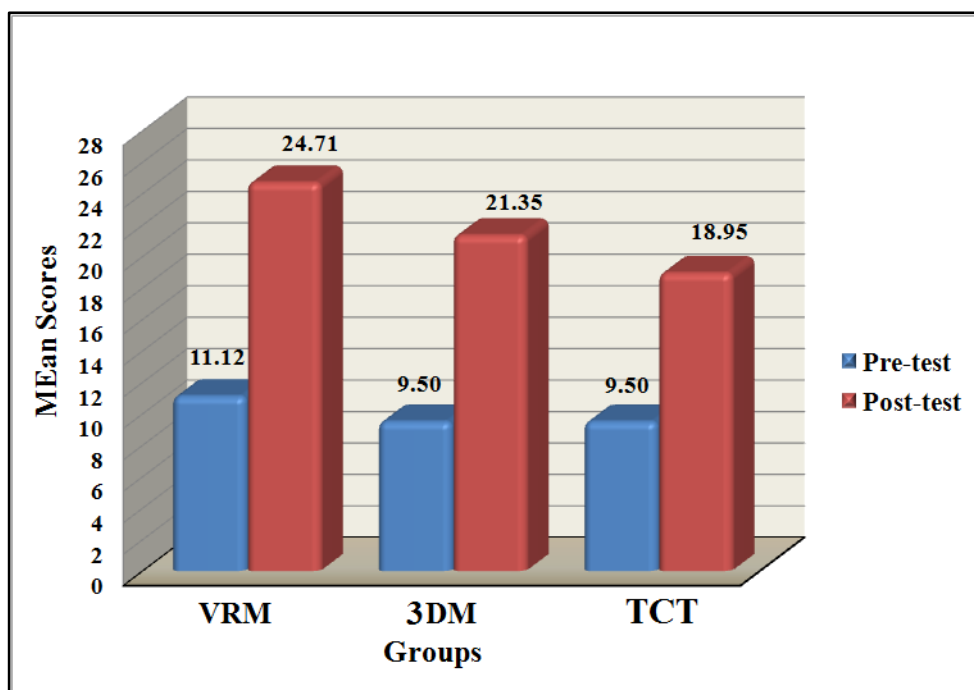


Figure 5.8 Differences between mean scores of the pre- and post-tests for the subtraction operation

The next hypothesis stated, *‘The VRM group was predicted to have a more significant positive achievable outcome of the post-tests for both addition and subtraction than the 3DM and TCT groups.’*

The scores of the pre- and post-tests for addition were from 28 scores. The VRM group pre-test mean score was 13.82 with an SD of 7.135, and the post-test mean score was 24.76 with an SD of 1.985. The 3DM group pre-test mean score was 13.35 with an SD of 6.635, and the post-test was 20.85 with an SD of 3.801. Finally, the TCT group mean score was 11.95 from the pre-test with an SD of 5.113 and 18.14 from the post-test with an SD of 4.764.

A one-way ANCOVA and a post hoc analysis (least significant difference [LSD] method) were performed to investigate the differences in student achievement outcomes among second-grade students using VRM, 3DM and TCT (the control group) applications to enhance their understanding of the concept of numeracy addition. The independent variable was the treatment group (VRM, 3DM and TCT), and the dependent variable consisted of post-test scores on the numeracy concept for the addition operation. The pre-test scores for the numeracy addition concept were used as the covariate in this analysis.

Preliminary checks were conducted to ensure that there was no violation of the assumption of homogeneity of variance; the result of the Levene's test ($F = 1.237$, $p = 0.298$, $p > .05$), indicated that no significant difference was found between treatment groups (VRM, 3DM and TCT). In other words, the basic assumption of homogeneity of variance was not violated.

A one-way ANCOVA was used to measure and analyze the collected data. After adjusting the pre-test scores, there was a significant difference between the three groups on the post-test scores ($F [3, 59] = 142.792$, $P < 0.01$). The effect size, calculated using partial eta squared, was 0.175, which in Cohen's (1988) terms would be considered a small effect size. Furthermore, treatment group ($F = 1170.723$, $P < 0.01$) was the significant factor in the post-test scores of the addition operation for second-grade students. The effect size, calculated using partial eta squared, was 0.840, which in Cohen's (1988) terms would be considered a large effect size. The means, SD, adjusted means and standard error of the dependent variable post-test scores by group are shown in Table 5.3.

Table 5.3 Means (M), SD, adjusted M and standard error (SE) of the addition post-test scores by group.

Groups	M	SD	Adjusted M ^a	SE
VRM	24.76	1.985	24.546	.850
3DM	20.85	3.801	20.752	.782
TCT	18.14	4.764	18.394	.749

^a Evaluated at covariate appeared in the model: Pretest = 12.97

The follow-up post-hoc pairwise comparisons were conducted when the result of the one-way ANCOVA was found to be statistically significant. In this study, a post hoc analysis (LSD method) was performed for further comparison. It showed that the VRM group performed significantly better than the 3DM group concerning post-test mean scores (mean difference = 3.794, $p = 0.002$) and the TCT group (mean difference = 6.152, $p = 0.000$). However, the performance of the 3DM group was better than that of the TCT group (mean difference = 2.358, $p = 0.034$).

In summary, the post-hoc analysis revealed that the VRM group performed better than the 3DM and TCT groups, and the performance of the 3DM group was significantly better than that of the TCT group.

The scores of pre- and post-tests for subtraction were from 28 scores. The VRM group pre-test mean score was 11.12 with an SD of 3.998, and the post-test mean was 24.71 with an SD of 3.177. The 3DM group pre-test mean score was 9.50 with an SD of 1.987, and the post-test mean was 21.35 with an SD of 3.990. Finally, the TCT group mean score was 9.50 from the pre-test with an SD of 3.700 and 18.95 from the post-test with an SD of 4.337.

A one-way ANCOVA and a post hoc analysis (LSD method) were performed to investigate the differences in the numeracy subtraction concept among second-grade students using the VRM, 3DM and TCT applications. The independent variable was the treatment group (VRM, 3DM and TCT), and the dependent variable consisted of post-test scores for the numeracy subtraction concept. The pre-test scores for the numeracy subtraction concept were used as the covariate in this analysis.

Preliminary checks were conducted to ensure that there was no violation of the assumption of homogeneity of variance, and the result of the Levene's test ($F = 2.756$, $p = 0.075$, $p > .05$), indicated that no significant difference was found among the treatment groups (VRM, 3DM and TCT). In other words, the basic assumption of homogeneity of variance was not violated.

A one-way ANCOVA was used to measure and analyze the collected data. After adjusting the pre-test scores, there was a significant difference between the three groups on the post-tests scores ($F [3, 59] = 14.578$, $P < 0.01$). The effect size, calculated using partial eta squared, was 0.210, which in Cohen's (1988) terms would be considered a small effect size. Furthermore, treatment group ($F = 41.388$, $P < 0.01$) was the significant factor in the post-test scores for the subtraction operation for second-grade students. The effect size, calculated using partial eta squared, was 0.693, which in Cohen's (1988) terms would be considered a large effect size. The means, SD, adjusted means and standard error of the dependent variable post-test scores by group are shown in Table 5.4.

Table 5.4 Means (M), SD, adjusted M and standard error (SE) of the subtraction post-test scores by group.

Groups	M	SD	Adjusted M ^a	SE
VRM	24.71	3.177	24.085	.868
3DM	21.35	3.990	21.601	.788
TCT	18.95	4.337	19.206	.753

^a Evaluated at covariate appeared in the model: Pretest = 9.97

The follow-up post-hoc pairwise comparisons were conducted when the result of the one-way ANCOVA was found to be statistically significant. In this study, a post hoc analysis (LSD method) was performed for further comparison. It showed that the VRM group performed significantly better than the 3DM group concerning the post-test mean scores (mean difference = 2.483, $p = 0.040$) and better than the control group (mean difference = 4.879, $p = 0.000$). However, the performance of the 3DM group was better than that of the control group (mean difference = 2.395, $p = 0.032$).

In summary, post-hoc analysis revealed that the VRM group performed better than the 3DM and control groups, and the performance of the 3DM group was significantly better than that of the control group.

5.2.2 Discussion

An analysis of the relationships between the use of the VRM, 3DM and TCT numeracy learning achievement applications presented in Table 5.3 shows the result of applying the ANOVA to examine whether there was a significant difference between the pre-test and post-test scores of the three groups. Before the experiment, the pre-test scores of the three groups showed no significant difference, which further showed no significant difference in the students' previous knowledge. After eight weeks of the

experiment, the post-test scores reached a statistically significant difference among the three groups: students in the VRM experimental group attained greater improvement on numeracy learning achievements. Consequently, the result implies that the MAVLE could enhance students' numeracy learning.

Based on the results of this study, we conclude that working with 3DM may increase the cognitive load on children, forcing them to mentally reconstruct the 3D shape of base-10 blocks. In contrast, using the VRM as the cognitive tool may improve the teaching and learning process and enhance the students' learning experiences.

It has been claimed that the use of a 2D and 3D manipulatives not only increases student achievement, but also allows them to improve their conceptual understanding and problem solving skills (Lamberty and Janet, 2002, 2004; Herrere, 2003; Reimer and Moyer, 2005; Suh et al., 2005; Lyon, 2006; Steen et al., 2006; Brown, 2007; Suh and Moyer, 2007; Yuan, 2009; Manches et al., 2010; Duffin, 2010).

This study also suggests that the use of a virtual reality manipulatives could promote a more positive attitude towards mathematics in students. In short, the outcomes of this work prove the feasibility and appropriateness of considering the virtual reality manipulatives as a cognitive tool and discussing how they can be used in classrooms to support students' conceptual understanding of numeracy concepts. Virtual reality manipulatives may be the most appropriate mathematics tool for the next generation. In conclusion, this research proposes a feasible virtual reality cognitive learning model that can be used to guide the design of other virtual reality learning environments.

Addition and subtraction errors could be caused by a lack of understanding of the basic place-value concept of regrouping, as seen in Problems 1 and 2 in Figure 5.9. Conversely, student errors could also be attributed to students misunderstanding the algorithm itself, as noted in Figure 5.9:

- Student 1: The student does not have an understanding of regrouping because she is treating each column as a separate problem.
- Student 2: The student is writing the 10's and carrying the 1's by writing 1 instead of 3 from 13 in the 1's column.
- Student 3: This student is able to regroup from 10's to 1's but does not change the 10's digit.

1.	$\begin{array}{r} 86 \\ + 8 \\ \hline 814 \end{array}$
2.	$\begin{array}{r} 27 \\ + 36 \\ \hline 81 \end{array}$
3.	$\begin{array}{r} 97 \\ + 8 \\ \hline 95 \end{array}$

Figure 5.9 Samples of students's errors in addition operation

CHAPTER 6 Discussion and Conclusions

Manipulatives are tools used for teaching basic math concepts in early elementary school grades. The practice of using manipulatives at this level was guided by the cognitive development theories of Piaget and Bruner. Afterwards, Dienes used their theories to develop his base-10 block manipulatives, which are used to teach the numeration base-10 number system. Despite the validity of base-10 blocks, Goldin and Kaput (1996) stated that there was a lack of linking between the cognitive representation of physical manipulatives with symbolic representations, resulting in limited serial translation of action.

To overcome this problem and to meet the recommendations of math educators who support using computer technology, virtual manipulatives emerged as a solution to fulfil these requirements. In order to examine the effectiveness of the virtual manipulatives on student conceptual understanding, many studies were conducted that compared virtual manipulatives with physical manipulatives. Some studies concluded that virtual manipulatives were more effective, whereas others concluded that physical manipulatives were more effective. Still, other studies recommended using both physical and virtual manipulatives (Reimer and Moyer, 2005; Moyer et. al., 2005; Suh and Moyer, 2007).

However, some studies evaluated the effectiveness of using virtual manipulatives from the teachers' perspectives, which stated that the effectiveness of virtual manipulatives depends on their design and how realistic the representation is

compared with physical manipulatives. Whenever a design is mentioned, a reference must be made to Mayer's principles of multimedia instructional design, which he derived and tested based on his model of cognitive theory of multimedia learning. This theory was based on three assumptions (previously referred to Chapter 2): the dual-channel assumption (visual and auditory), the limited-capacity assumption of the working memory and the active processes assumption (selecting, organising and integrating).

Mayer, after many studies, concluded that the best way to design multimedia instruction was through visual animation and voice narration, which uses the advantage of both verbal and visual dual channels without overloading one over the other in the WM. As such, it was recognised that Mayer restricted the cognitive learning process to two modalities, auditory and visual, without any reference to the tactile modality.

In reference to the human information processing model, we distinguished the cognitive processor component from the motor processor component. The cognitive processor component is activated through sensory stimuli (eyes and ears), whereas the motor processor component is activated through hand movements in response to the cognitive processor component. The motor processor component performed by hand movements is recognised as an input tactile modality in the computer through devices such as a keyboard, mouse, joystick, touch screens and wands. This motor processor is a vital component in the learning cognitive process, according to the human information processing theory.

The exclusion of the tactile modality in Mayer's model and the ambivalence to the importance of this modality puts the student in a passive position while learning, and, thus, they have no interactive role in the learning process. Furthermore, the learner is restricted only attaining knowledge from the presented multimedia learning environment. Considering these issues, this research proposed a model, derived from Mayer's model of cognitive theory of multimedia learning, where the tactile modality was added to the other two modalities because of its essential role in the learning cognitive process, as suggested by the human information processing theory. The proposed design framework is named as the 'virtual reality model for cognitive learning'.

6.1 Discussion

This chapter summarises the contributions made by this research to the field of virtual reality technology and its applications, specifically in the area of elementary mathematics. This research draws attention to the application of virtual reality systems based on a cognitive learning model.

This research explored the potential role of desktop virtual reality technology as a cognitive tool in the design and development of the MAVLE system, which embodies VRM for young children to use when they practice exercises for numeracy concepts (addition and subtraction). The researcher built and tested the MAVLE prototype, in which two software languages (Java 3D and VRML) were tested to investigate which would be most compatible with the MAVLE system. Although the Java 3D and VRML languages are generally used for the development of 3D graphics, Java 3D is a more exclusive tool for creating customised 3D graphical applications.

The results emerging from the study are presented in the previous chapter and signify the research directed by the following hypotheses:

- The VRM group is predicted to have a significant positive performance outcome (i.e., regarding the number of solving problems for addition and subtraction) than those in the 3DM group.
- The number of errors in the place-value concept is predicted to be significantly different between the VRM and 3DM groups.
- The number of errors in the regrouping concept is predicted to be significantly different between the VRM and 3DM groups.
- The number of errors in the concept of regrouping positively correlates with the number of errors in the concept of place value.
- In the VRM group, a greater number of solving problems correlates with a high level of virtual reality navigation behavior.
- All groups are predicted to have a significant positive achievable outcome from pre-tests to post-tests in both addition and subtraction.
- The VRM group is predicted to have a more significant positive achievable outcome of the post-tests for both addition and subtraction than the 3DM and TCT groups.

This research aimed to systematically explore the feasibility of completing the process of designing, implementing and evaluating a virtual reality learning environment for use as a cognitive tool from the initial conception to evaluation and classroom application. The steps in this research were as follows:

An interactive multimedia learning model was developed based on the learning cognitive model presented by Mayer (2002) and modified to include the interaction tasks of selection and manipulation.

The proposed design framework was named the virtual reality model for cognitive learning (Figure 3.3) based on the interactive multimedia model for cognitive learning (Figure 2.3). The latter model improved upon to include virtual reality navigational tasks (travel and wayfinding), providing a third channel (Immersion) to address the immersive nature of virtual reality systems within the working memory. Based on this model, the researcher designed, developed and tested an application to conceptually and virtually represent the base-10 block manipulatives for addition and subtraction used in a traditional classroom setting.

Study 1 was conducted to compare the performance of two groups of second-grade students using either the VRM or the 3DM. Each student was observed while solving problems generated by the application, and their interactions were recorded by a screen-capture application. Study 2 was conducted to compare the performance of three groups of second-grade students using the VRM, the 3DM or TCT. The latter group was designated as the control group. Each student in the three groups was given a pre-test, the appropriate treatment and then a post-test. The classroom and computer lab settings for Study 2 were based on the Laurillard's (2002) conversational framework.

The results of Study 1 show statistically significant differences in the number of numeracy problems solved between the groups of second grade students. The VRM group solved more problems than the 3DM group. In addition, the number of conceptual errors among the 3DM group was higher, whereas there was no statistically significant difference in the place-value error between the two groups. Yet, further analysis showed that the large number of errors in the 3DM has a high correlation with the carry and borrow concepts, which could be attributed to the students' lack of understanding of the

place-value concept. Analysing the linear regression of the navigation of the VRM revealed that they had significant moderate correlations with the number of solved exercises.

The results of Study 2 show that among the three groups, there was a statistically significant difference between the pre-test and the post-test scores. This difference was expected and could easily be attributed to knowledge gain because the addition and the subtraction lessons were explained to all groups using the VRM, the 3DM and TCT. The results of Study 2 also show that among the three groups, there was statistically significant difference between the VRM and 3DM and between the 3DM and TCT. Students using the VRM performed better than students in the 3DM and the control group, whereas students using the 3DM performed better than students in the control group.

In attempting to answer the research hypotheses positively, it can be concluded that the navigation feature of the virtual reality had a positive effect on the students' conceptual understanding of numeracy concepts. It must be noted, however, that this virtual reality navigation feature is an additional feature to the regular point-and-click navigation of the 3D graphics, which is considered the interaction task of the interactive multimedia learning model. Whereas the navigation features of virtual reality are the navigation tasks of the proposed design framework for virtual reality model for cognitive learning (Figure 3.3) that contributed in forming the third channel 'Immersion' that provides the user the feeling of presence associated with any virtual reality learning system.

The immersive dimension of the virtual reality model of cognitive learning could help students reduce cognitive overload on the working memory by immediately seeing abstract concepts that they used to have to make an effort to try to mentally visualise.

The significance of this research offers ground for further investigations into the value of virtual reality systems in the already well-researched area of comparing the positive effects of incorporating 3D applications within the learning process. In addition, virtual reality has not been established as a learning tool to simulate situations where the objects are hard to reach or are dangerous, such as in the fields of astronomy, chemistry, biology and flight training. This research has shown that virtual reality is becoming a practical cognitive tool for visualising abstract concepts in young learners.

6.2 Contribution

The contribution to knowledge is our attempt to disseminate the virtual reality learning process from design to classroom application. The research reported in this thesis resulted in novel contributions, which are as follows:

- The cognitive theory of multimedia learning was extended to an interactive multimedia model for cognitive learning and a virtual reality model for cognitive learning.
- The prototyping and implementation of a practical desktop virtual reality manipulatives application (MAVLE) for facilitating the teaching and learning processes of numeracy concepts (integer addition and subtraction) was proposed.

- The evaluation of conceptual understanding of students' achievements and the relationships among the navigational behaviours of desktop virtual reality were examined, and their impacts on students' learning experiences were noted.

6.3 Limitations

It is emphatically stressed that the results from this research took place within the constraints of a real school setting of four private schools in western-central Jeddah, Saudi Arabia. These constraints were caused by the following factors:

- The Saudi educational system is strictly segregated, and hence, because the researcher is female, she was only permitted to conduct this research within the constraints of female educational institutions.
- Computer labs were only available in private primary schools because computer lessons are compulsory in their curriculum.

6.4 Future Work

Further development of the MAVLE would incorporate a voice recording to highlight the invalid operations. The MAVLE could be further developed with interactive voice recognition to ensure that the student articulates the actual concept of the exercise. For instance, the number 465 would be voiced by the student as 4 100's, 6 10s and 5 1's. This re-iteration of the number confirms the actual understanding of the place-value concept. Perhaps an introduction of an automatic assessor (intelligent tutoring system) is needed. The automatic assessor would evaluate the exercises while the student was completing them by counting the amount of times certain operations and navigational tools were used to derive the correct answer. The automatic assessor

could count the amount of correct answers achieved and then recommend other exercises according to the learning needs of the student.

The MAVLE could also be further developed to work with an assortment of hardware, such as a touch screen or joystick, empowering users to use the MAVLE who have difficulties using a mouse. For a multiple-user experience, the MAVLE could be integrated with other systems, such as the Nintendo Wii, and this would enable multiple users to work with each other to solve more complex exercises that would be displayed on split screens.

This research provides a direction for further studies because it is evident that virtual reality systems motivate students; however, more research is required to determine how to design desktop virtual reality systems for the greatest positive impact on student achievement. Finally, the researcher informally observed the joy of the students interacting with the MAVLE, and this reaction should be further studied because math has historically been associated with anxiety for many young students.

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Appendix A: Evaluating Virtual Manipulatives

A list of possible evaluation questions to determine the effectiveness of using virtual manipulatives in their classrooms.

Category	Manipulative Characteristic	Yes	No	N/A
Ease of Use Presentation	Is the manipulative easy to use?			
	Are the directions easy to find?			
	Are the directions clear?			
	Are the manipulatives interesting?			
	Are there distractions on the page (e.g. ads, unrelated images or sounds)			
Motivation	Is there something that provides motivation for the user?			
	Is the activity engaging?			
Support for Problem Solving	Does the manipulative help the user construct knowledge of a concept (rather than drill and practice)?			
	Does it allow for users to experiment?			
	Is there any reward for using the manipulative in a meaningful way rather than using it to just guess at the answer?			
	Is feedback provided? If so, is the feedback meaningful?			

Appendix B: Letter of Informed Consent-Parent

Letter of Informed Consent-Parent

This study is being conducted to investigate the use of computerized instructional strategies that can be used in mathematics classrooms, specifically the use of virtual manipulatives. Your child will be asked to participate in three days of instruction learning about addition and subtraction concepts. This research will add to the literature and findings on how to effectively use virtual manipulatives to help students build conceptual understanding of mathematics. There are no foreseeable risks.

All data collected in this study will be confidential; all person-identifiable data will be coded so that your child cannot be identified. Your participation is voluntary and you may withdraw from the study at any time and for any reason.

This research is being conducted by Lamya Daghestani (She may be reached at 0556136132), Doctoral Candidate at University of Huddersfield in collaboration with King Abdulaziz University under the direction of Dr. Robert D. Ward at University of Huddersfield and Dr. Hana Al-Nuaim at King Abdulaziz University. You may contact the King Abdulaziz University Office of Sponsored Programs at (02-695 2937) if you have any questions or comments regarding your rights as a participant in the research.

Consent

I have read this form and give permission for my child to participate in this study.

Parent's Signature: _____

Child's name: _____

Appendix C: Letter of Student Assent

Letter of Student Assent

Hi. This is Mrs. Lamy from King Abdulaziz University. I am doing a study to find better ways to help students learn math using tools called virtual manipulatives. For three days, you will exercises addition and subtraction within computer lab. This will not be part of your grade but you will help me learn how children learn math. I will record every lab session you work on computer so that I can learn more about the way students learn but your name will not show up in any of my work. If you do not want to be in this study, you can let me know at any time for any reason.

Your Schools headmistress and your math teacher have given me permission to do this study in your class. I look forward to working with you on this study and thank you for your help.

Student Assent:

I have read this form and I would like to be in the study.

Student's name: _____

Student's Signature: _____

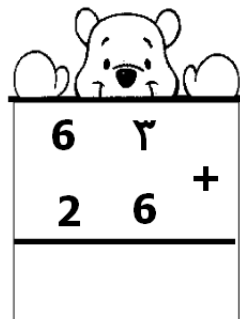
Appendix D: Addition Pr and Post-test for the Second Grade

English version

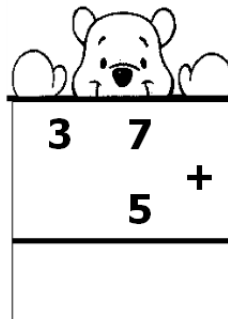
Name: _____

ID: _____

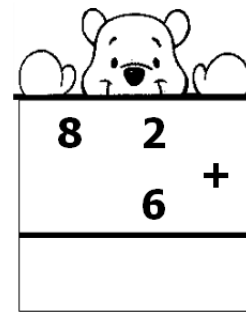
Find the solution for the following addition problems



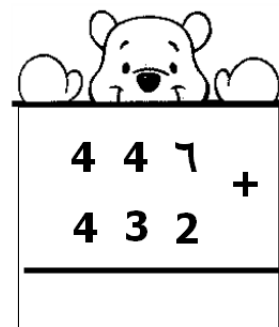
$$\begin{array}{r} 6 \quad 7 \\ 2 \quad 6 \quad + \\ \hline \end{array}$$



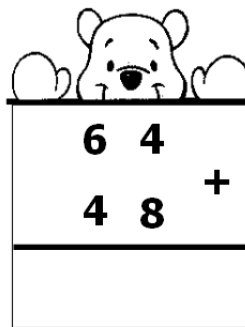
$$\begin{array}{r} 3 \quad 7 \\ \quad 5 \quad + \\ \hline \end{array}$$



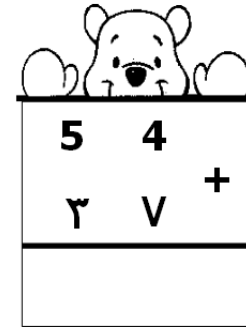
$$\begin{array}{r} 8 \quad 2 \\ \quad 6 \quad + \\ \hline \end{array}$$



$$\begin{array}{r} 4 \quad 4 \quad 7 \\ 4 \quad 3 \quad 2 \quad + \\ \hline \end{array}$$

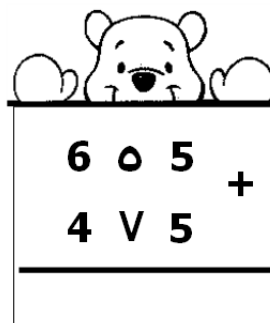


$$\begin{array}{r} 6 \quad 4 \\ 4 \quad 8 \quad + \\ \hline \end{array}$$

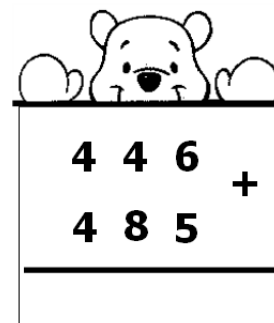


$$\begin{array}{r} 5 \quad 4 \\ 7 \quad 7 \quad + \\ \hline \end{array}$$





$$\begin{array}{r} 6 \quad 0 \quad 5 \\ 4 \quad 7 \quad 5 \quad + \\ \hline \end{array}$$

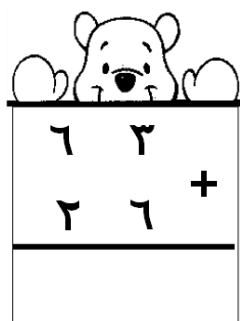


$$\begin{array}{r} 4 \quad 4 \quad 6 \\ 4 \quad 8 \quad 5 \quad + \\ \hline \end{array}$$

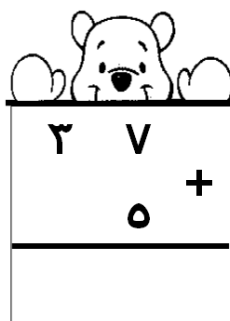
الرقم:

الاسم:

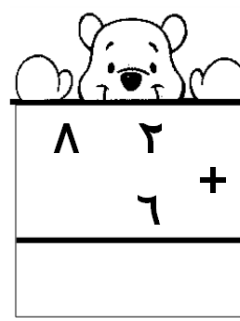
- أوجد ناتج عمليات الجمع التالية :



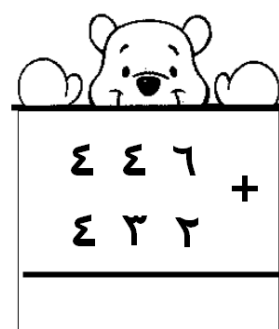
$$\begin{array}{r} 1 \quad 2 \\ 2 \quad 1 \quad + \\ \hline \end{array}$$



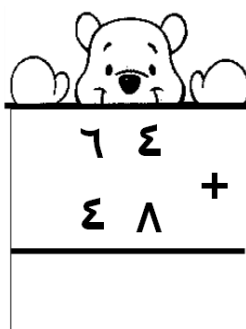
$$\begin{array}{r} 2 \quad 7 \\ \quad \quad 5 \quad + \\ \hline \end{array}$$



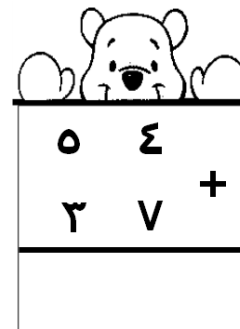
$$\begin{array}{r} 8 \quad 2 \\ \quad \quad 6 \quad + \\ \hline \end{array}$$



$$\begin{array}{r} 3 \quad 3 \quad 1 \\ 3 \quad 2 \quad 2 \quad + \\ \hline \end{array}$$

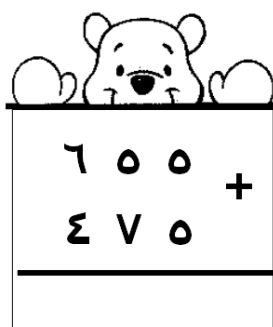


$$\begin{array}{r} 1 \quad 3 \\ 3 \quad 8 \quad + \\ \hline \end{array}$$

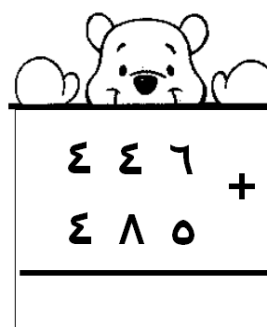


$$\begin{array}{r} 5 \quad 3 \\ 2 \quad 7 \quad + \\ \hline \end{array}$$





$$\begin{array}{r} 1 \quad 0 \quad 0 \\ 3 \quad 7 \quad 0 \quad + \\ \hline \end{array}$$



$$\begin{array}{r} 3 \quad 3 \quad 1 \\ 3 \quad 8 \quad 0 \quad + \\ \hline \end{array}$$


Appendix E: Subtraction Pr and Post-test for the Second Grade

English version


Name:

ID


Find the solution for the following subtraction problems:




$$\begin{array}{r} 33 \\ - 9 \\ \hline \end{array}$$




$$\begin{array}{r} 58 \\ - 4 \\ \hline \end{array}$$




$$\begin{array}{r} 7 \\ - 3 \\ \hline \end{array}$$



$$\begin{array}{r} 743 \\ - 431 \\ \hline \end{array}$$




$$\begin{array}{r} 77 \\ - 58 \\ \hline \end{array}$$




$$\begin{array}{r} 67 \\ - 36 \\ \hline \end{array}$$





$$\begin{array}{r} 864 \\ - 675 \\ \hline \end{array}$$

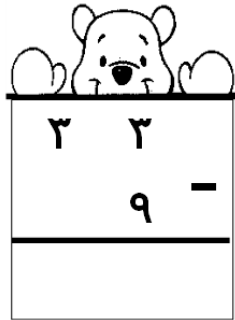


$$\begin{array}{r} 673 \\ - 145 \\ \hline \end{array}$$

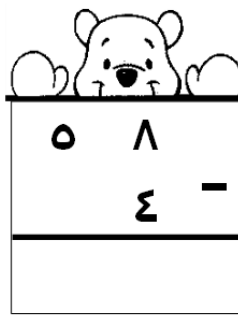
الرقم:

الاسم:

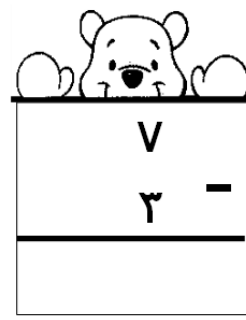
- أوجد ناتج عمليات الطرح التالية :



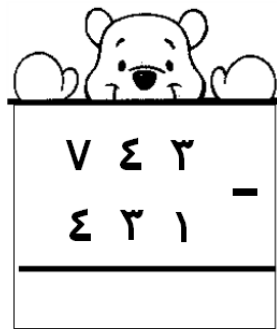
$$\begin{array}{r} 2 \quad 2 \\ - \quad 9 \\ \hline \end{array}$$



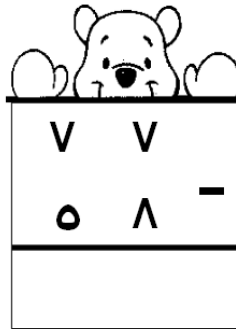
$$\begin{array}{r} 5 \quad 8 \\ - \quad 4 \\ \hline \end{array}$$



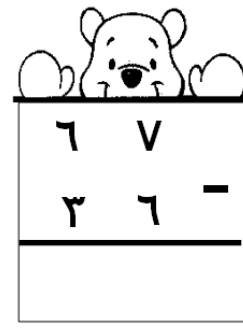
$$\begin{array}{r} 7 \\ - \quad 2 \\ \hline \end{array}$$



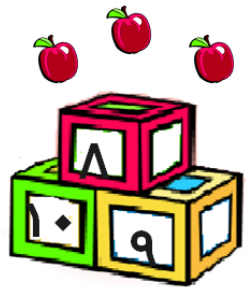
$$\begin{array}{r} 7 \quad 4 \quad 2 \\ - \quad 4 \quad 2 \quad 1 \\ \hline \end{array}$$

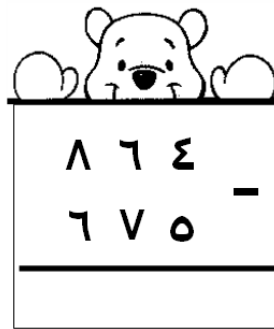


$$\begin{array}{r} 7 \quad 7 \\ - \quad 5 \quad 8 \\ \hline \end{array}$$

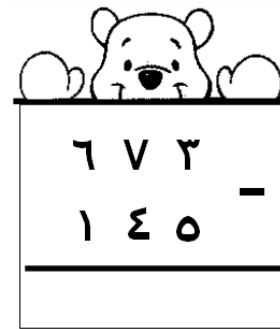


$$\begin{array}{r} 6 \quad 7 \\ - \quad 2 \quad 6 \\ \hline \end{array}$$





$$\begin{array}{r} 8 \quad 6 \quad 4 \\ - \quad 6 \quad 7 \quad 5 \\ \hline \end{array}$$



$$\begin{array}{r} 6 \quad 7 \quad 2 \\ - \quad 1 \quad 4 \quad 5 \\ \hline \end{array}$$