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SPECTRAL CHARACTERISATION OF THE n^k PULSE POSITION MODULATION FORMAT

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Indexing terms: n^k-PPM, PSD, PPM

n^k pulse position modulation (PPM) is a new modulation format that has recently been proposed for the optical wireless channel. This Letter considers, for the first time, a full spectral characterisation of n^k-PPM and presents original expressions, which are validated numerically, for predicting both the continuous and discrete spectrum.

Introduction: Conventional *n*-ary pulse position modulation (PPM) has been proposed for the optical fibre [1], optical wireless [2] and optical satellite [3] channels due to its enhanced sensitivity performance. However, this is at the cost of significant bandwidth expansion and so alternative, more bandwidth efficient schemes, have been proposed such as multiple PPM [4,5] in which *k*-pulses are positioned within a time frame containing S-slots leading to $\binom{S}{k}$ symbols per frame. Unfortunately, $\binom{S}{k}$ is rarely a power of two, which leads to complex encoding and decoding circuitry. In [6], a new format, termed as n^k-PPM, was proposed for the optical wireless channel. In n^k-PPM, the information is conveyed by the position of k pulses, each within their own frame of *n*-slots, giving n^k-PPM symbols. By ensuring that n is a power of 2 the practical implementation is significantly simplified when compared to MPPM.

This Letter evaluates, for the first time, the power spectral density (PSD) of n^k-PPM. By making use of the cyclostationary properties of the modulation format, original expressions are derived for predicting both the continuous and discrete spectrum and these are verified numerically by taking the Fast Fourier Transform of the n^k-PPM pulse stream.

Spectral Characterisation: Following the approach outlined in [7] the data pulse stream can be represented as

$$m(t) = \sum_{q=-\infty}^{\infty} a_q p(t - qT)$$

where $\{a_q\}$ is the n^k-PPM sequence and $p(t)$ is the pulse shape. To compute the discrete PSD of $m(t)$, namely, $S_m^d(f)$, the statistical correlation function, $R_m(t;\tau) = \overline{m(t)m(t+\tau)}$, must first be averaged over t and then the Fourier transform taken:

$$\begin{aligned} S_m^d(f) &= \mathrm{F}_\tau\left\{\left\langle R_m(t;\tau)\right\rangle_t\right\} \\ &= \frac{1}{T_f^2}\sum_{l=-\infty}^{\infty}\left|P\left(\frac{l}{T_f}\right)\right|^2\left|\sum_{q=1}^{kn}\mathbf{E}\left\{a_q\right\}e^{+j\left(\frac{2\pi}{kn}lq\right)}\right|^2\delta\left(f-\frac{l}{T_f}\right) \end{aligned}$$

where T_f is the n^k-PPM frame-time. The term $\sum_{q=1}^{kn}\mathbf{E}\left\{a_q\right\}e^{+j\left(\frac{2\pi}{kn}lq\right)}$ represents the characteristic function of the data distribution on the n^k-PPM frame and so makes the cyclostationary property explicit. Evaluating this and assuming a rectangular pulse of height, A, and width, t_p, allows $S_m^d(f)$ to be written as:

$$S_m^d(f) = \sum_l \left| A \frac{t_p}{T_f} \operatorname{sinc}\left(l \frac{t_p}{T_f} \right) \right|^2 \left| \frac{1}{n} \frac{\sin(\pi l)}{\sin(\pi l / nk)} \right|^2 \delta\left(f - \frac{l}{T_f} \right) \qquad (1)$$

where $\operatorname{sinc}(x) = \sin(\pi x)/(\pi x)$.

The continuous PSD can be determined by evaluating the Fourier transform of the autocorrelation function of a zero-mean n^k-PPM sequence. The autocorrelation function is given by

$$\begin{aligned} R_M(t;\tau) &= \mathbf{E}\left\{ M(t) M^*(t+\tau) \right\} \\ &= \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \int_y \int_z K_{a,\varepsilon}(n; m-n) \\ &\quad \times P(y) P^*(z) e^{-j2\pi ynT} e^{+j2\pi zmT} \\ &\quad \times e^{+j2\pi(y-z)t} e^{-j2\pi z\tau} dydz \end{aligned}$$

where $K_a(n; m-n) = \mathbf{E}\left\{ a_n a_m^* \right\} - \mathbf{E}\left\{ a_n \right\} \mathbf{E}\left\{ a_m^* \right\}$. Taking the Fourier transform of this gives

$$\begin{aligned} S_M^c(f) &= \mathrm{F}\left\{ \left\langle R_M(t;\tau) \right\rangle_t \right\} \\ &= \frac{1}{T} \left| P(f) \right|^2 \sum_{l=-\infty}^{\infty} \left[\frac{1}{N} \sum_{n=1}^{N} K_a(n; l) \right] e^{-j2\pi flT} \end{aligned}$$

Which, through the contents of the square-brackets, makes the cyclostationary nature of the sequence explicit. Evaluating this for n^k-PPM gives

$$S_M^c(f) = \frac{k}{T_f} \left| A t_p \operatorname{sinc}(f t_p) \right|^2 \left[1 - \left| \frac{1}{n} \frac{\sin\left(\frac{\pi}{k} f T_f \right)}{\sin\left(\frac{\pi}{nk} f T_f \right)} \right|^2 \right] \qquad (2)$$

Results: In order to validate the analytic results of (1) and (2), the PSD of n^k-PPM was evaluated numerically using the Fast Fourier Transform. A sampling rate of 256 samples per n^k-PPM slot duration was used and 50 FFT's were averaged in order to decrease the noise due to the randomness of the data sequence.

Fig. 1 shows the power spectral density, calculated both numerically and with the new analytic expressions of (1) and (2), for a 4^2-PPM system with the pulse width set at the slot duration, $t_p = T_f/(nk)$. Note that the frequency axis is normalised to the slot repetition frequency. As can be seen, there are no discrete lines at the frame repetition frequency nor the slot repetition frequency. The absence of the discrete spectrum can be understood by consideration of the 2nd term in (1) which represents the characteristic function of the probability density function (pdf) of the data distribution on the frame. As the pdf is uniform there will be no component at the frame repetition frequency and this is reflected in the $\sin(\pi l)$ in the numerator of the second term of (1). At the slot repetition frequency (when l/nk is an integer) the second term becomes k^2. However, for full-width pulses, the nulls of the sinc-function in the first term occur at the slot repetition frequency and so this masks the discrete spectrum. At $f = 0$, (1) predicts -12 dB which is in exact agreement with the numerical results. Fig. 1 also demonstrates that there is excellent agreement between the numerical and analytical results for predicting the continuous spectrum so validating the accuracy of expression (2).

Fig. 2 shows the PSD, calculated both numerically and analytically, for an 8^3-PPM system with the pulse width set at half the slot duration ($t_p = T_f/(2nk)$). Again, the numerical and analytical results are in excellent agreement so confirming the validity of (1) and (2) for predicting the PSD of n^k-PPM. The results demonstrate that there is a strong discrete line at the n^k-PPM slot-rate and so this can be extracted for synchronisation purposes directly from the pulse stream. Again, due to the uniform distribution of the data within the frame, no component is available at the frame repetition frequency and so alternative methods of frame extraction are required. This may be achieved by following the approach used in conventional PPM [8] through tracking naturally occurring sequences within the n^k-PPM data stream.

Conclusions: Consideration has been given to the spectral characterisation of the new n^k-PPM modulation format and original expressions, which have been validated numerically using the FFT, presented for predicting both the continuous and discrete spectrum. It is shown that, due to the uniform data distribution on the frame, there will be no discrete component at the frame repetition frequency and that when full-width rectangular pulses are deployed, there will be no discrete component at the slot-repetition frequency. However, both slot and frame synchronisation can be facilitated by careful choice of the pulse shape and by tracking naturally occurring sequences within the n^k-PPM data stream.

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Figure captions:

Fig. 1 Power spectral density of n^k-PPM with $n = 4$ and $k = 2$

Fig. 2 Power spectral density of n^k-PPM with $n = 8$ and $k = 3$

Figure 1

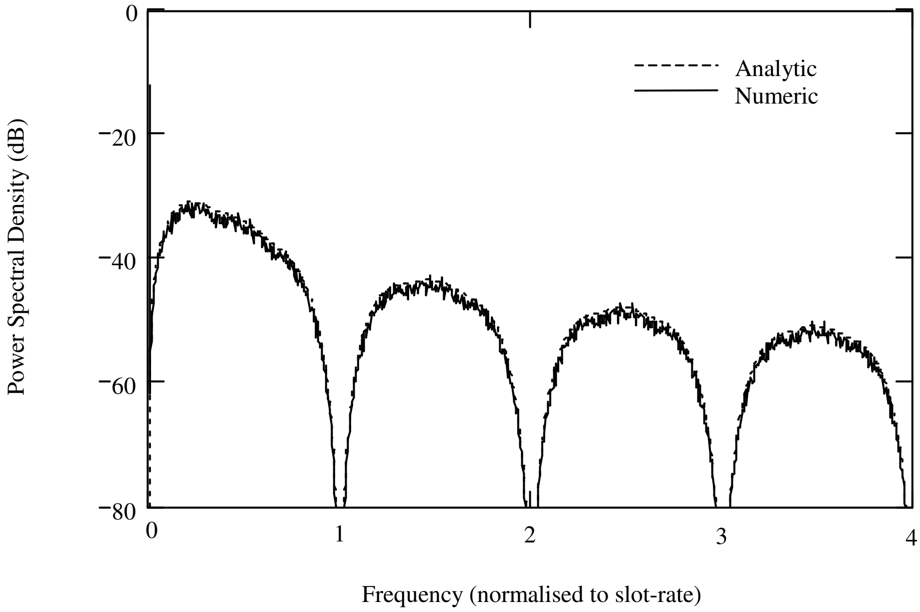
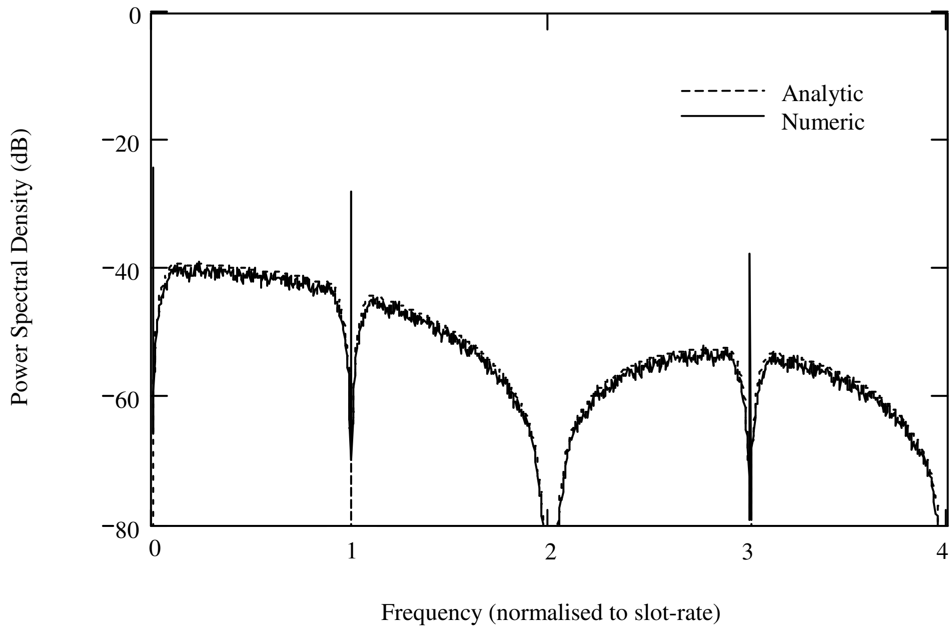

0
−20
−40
−60
−80
Power Spectral Density (dB)
Analytic
Numeric
0
1
2
3
4
Frequency (normalised to slot-rate)

Figure 2


0
−20
−40
−60
−80
Power Spectral Density (dB)
0
1
2
3
4
Frequency (normalised to slot-rate)
Analytic
Numeric