n^k-pulse position modulation for optical wireless communication

R.A. Cryan and M.J.N. Sibley

An original analysis is presented for n^k-PPM in which the information is conveyed by the position of k pulses, each within their own frame of n slots, giving n^k PPM symbols. Comparisons are made with multiple PPM (MPPM) and it is demonstrated that n^k-PPM offers improved orthogonality, a simplified circuit implementation and comparable receiver sensitivity.

Introduction: Conventional n-ary pulse position modulation (PPM) has been proposed for optical fibre [1], optical wireless [2] and optical satellite [3] channels, owing to its enhanced sensitivity performance. In this format, L bits (where L is termed the coding level) of pulse code modulation (PCM) are encoded into PPM by positioning a single pulse in one of 2^L time slots. Since a single pulse is conveying L bits of information there is an immediate reduction in average power when compared to PCM. However, this comes at the cost of increased bandwidth consumption. For example, with a coding level of $L = 10$, 1024-ary PPM conveys 10 bits of information by a single pulse but the slot duration is 10/1024 narrower than the PCM bit period leading to a prohibitive bandwidth expansion of $1024/10 = 102.4$. Multiple PPM [4–8] has been proposed as a technique for offering improved receiver sensitivity over PCM under bandwidth-constrained environments such as the optical wireless channel. In multiple PPM, k-pulses are positioned within a time frame containing S slots leading to $\binom{S}{k}$ symbols per frame. Unfortunately, $\binom{S}{k}$ is rarely a power of two, which leads to complex encoding and decoding circuitry. In [6], a new format, termed as compound-K single-pulse MPPM, was proposed for the optical satellite channel. Here it is adapted for the optical wireless channel and denoted n^k-PPM. In this modulation format, the information is conveyed by the position of k pulses, each within their own frame of n slots, giving n^k-PPM symbols. By ensuring that n is a power of 2 the practical implementation is significantly simplified when compared to MPPM.

In this Letter an original analysis is presented for the optical wireless channel, which compares the performance of n^k-PPM with both MPPM and conventional PPM in terms of bandwidth expansion, orthogonality and average power for a given error probability.

Bandwidth expansion: To ensure the same data throughput, L bits of PCM must be mapped onto 2^L PPM, MPPM and n^k-PPM symbols within the same time frame. Since the bandwidth is approximately the reciprocal of the slot duration, the bandwidth of PCM is $B_{PCM} \simeq 1/T_b$ and the normalised bandwidth of PPM is

$$\frac{B_{PPM}}{B_{PCM}} \simeq \frac{2^L}{L} \tag{1}$$

For MPPM, $\binom{S}{k}$ is rarely a power of 2, and so the PCM symbols are encoded onto a subset of MPPM symbols such that S is selected to ensure that $L = \left\lfloor \log_2 \binom{S}{k} \right\rfloor$, giving a normalised bandwidth expansion of

$$\frac{B_{MPPM}}{B_{PCM}} \simeq \frac{S}{L} \tag{2}$$

For n^k-PPM the normalised bandwidth is

$$\frac{B_{n^k-PPM}}{B_{PCM}} \simeq \frac{k2^{(L/k)}}{L} \tag{3}$$

These bandwidth expansions are shown in Fig. 1 against coding level. As can be seen, the bandwidth of conventional PPM increases rapidly with increased coding level and indeed, at $L = 10$, the required bandwidth is 102 times that required for PCM, which is of course impractical. In contrast, with $k = 2$ and $L = 10$, MPPM and n^2-PPM only require 4.6 and 6.2%, respectively, of the bandwidth of conventional n-PPM, representing significant reductions.

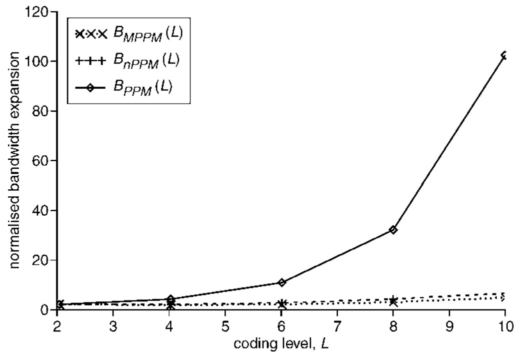

Fig. 1 *Bandwidth expansion against coding level, L*

Orthogonality: Unlike conventional PPM, where only one pulse per frame is transmitted, multiple PPM and n^k-PPM are not orthogonal. By mapping the $\binom{S}{k}$ MPPM symbols it can be demonstrated that the number of non-orthogonal symbols is given by

$$\binom{S}{k} - \binom{S-k}{k} - 1 \tag{4}$$

Likewise, by mapping the n^k-PPM symbols it can be shown that

$$n^k - (n-1)^k - 1 \tag{5}$$

are not orthogonal. For the same coding level, L, and number of pulses, k,

$$n^k - (n-1)^k - 1 < \binom{S}{k} - \binom{S-k}{k} - 1 \tag{6}$$

and so n^k-PPM offers increased orthogonality.

For example, with $L = 6$ and $k = 2$, 8^2-PPM would have 14 symbols that were not orthogonal. An equivalent MPPM system would require an alphabet of $2^6 = 64$ symbols that would be achieved using a $\binom{12}{2} = 66$ symbol MPPM scheme in which 20 symbols are not orthogonal.

Maximum likelihood detection: The union bound on symbol-error probability for multiple PPM with maximum likelihood detection is [7, 8]

$$P_e \leq \sum_{i=1}^{k} \frac{M_i}{2} \operatorname{erfc}\left(\sqrt{\frac{A^2 i}{N_0}}\right) \tag{7}$$

where M_i is the number of codewords with Hamming distance 2_i from a particular codeword, A is the channel gain and $N_0/2$ is the two-sided noise power spectral density. For MPPM

$$M_i = \binom{k}{i}\binom{S-k}{i} \tag{8}$$

and for n^2-PPM $M_1 = 2(n-1)$ and $M_2 = n^2 - 2(n-1)$. In order to make comparisons, the systems should operate at the same error probability and this facilitates a normalisation to the performance of a conventional pulse code modulation (PCM) system. For PCM

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{2P_{PCM}}{N_0 B_{PCM}}}\right) \tag{9}$$

where P_{PCM} is the average optical power required by PCM, and B_{PCM} is the bit-rate. Making use of (9) and (7) then, for an $\binom{S}{2}$ MPPM system the normalised average optical power is

$$\frac{P_{MPPM}}{P_{OOK}} \simeq \sqrt{\frac{8}{S\left\lfloor \log_2\left(\frac{S(S-1)}{2}\right)\right\rfloor}}$$

Taking the same approach for n^2-PPM

$$\frac{P_{n^2-PPM}}{P_{OOK}} \simeq \sqrt{\frac{2}{n \log_2 n}} \tag{10}$$

and for conventional PPM

$$\frac{P_{PPM}}{P_{OOK}} \simeq \sqrt{\frac{2}{2^L L}} \tag{11}$$

Fig. 2 shows the sensitivity, normalised to PCM, against the number of bits, L. As expected, conventional PPM offers the best sensitivity since a single pulse is being used to convey the L bits of information. However, as Fig. 1 demonstrates, this is at the cost of significant bandwidth expansion. For example, with $L = 10$, conventional PPM offers an improvement of 18.5 dB over PCM but the required bandwidth expansion is 102, which makes the modulation format impractical for high data rate systems. In contrast, with $L = 10$, MPPM and n^2-PPM only require bandwidth expansions of 4.6 and 6.4, respectively, and so would be physically implementable at high data rates. Fig. 2 demonstrates that n^2-PPM offers marginally improved sensitivity over MPPM and, with $L = 10$, they offer improvements of 9.5 and 8.8 dB, respectively, over PCM. n^2-PPM has the advantage that n is selected to be a power of two and so circuit implementation is significantly simplified when compared to MPPM for which $\binom{S}{2}$ is rarely a power of two.

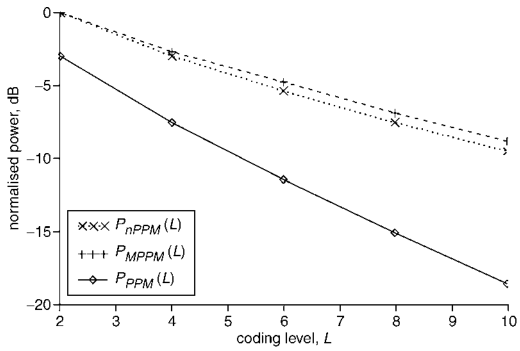

Fig. 2 *Normalised power against coding level, L*

Conclusions: n^k-PPM has been proposed and analysed for the optical wireless channel. It has been demonstrated that this new modulation format offers improved sensitivity over PCM and has similar sensitivity performance and bandwidth requirements as MPPM. Furthermore, n^k-PPM offers improved orthogonality over MPPM and it is easier to implement since n is a power of 2 and so it lends itself to simple digital circuit design. For a 32^2-PPM scheme, the sensitivity improvement over PCM is 9.5 dB but the bandwidth requirement is only 6.2% of that required for conventional PPM.

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