DIGITAL BALANCING USING ARTIFICIAL LABYRINTH

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Abstract: Stability analysis of an improved model of balancing is presented. Simple models do not consider the problem of sensing the vertical direction. Human beings are able to balance themselves in spite of the inability to detect the exact vertical line. The human balancing organ is described briefly and a mechanical model is constructed according to the structure and functioning of this organ. Human control strategy takes care for human reflex delay. Time delay is also present in case of computer control due to sampling. Increasing time delay tends to destabilize the system and above a critical value the balancing is impossible. Stability conditions depending on sampling delay are determined and the critical value of time delay depending on parameters describing the system is calculated.

1. Introduction

Healthy human beings balance themselves successfully during standing and walking. The human balancing organ is responsible for successful balancing. It is called labyrinth and it consists of two general parts from mechanical viewpoint referred to as static and dynamic receptors. Static receptors sense gravitation and liftreactions and dynamic receptors sense angular acceleration.

The simplest model of such a man-machine system is the balancing of a stick on fingertip as an inverted pendulum. Balancing of an inverted pendulum is a basic example of stabilization of unstable equilibria of mechanical systems, so it has extended specialized literature [1,2,3,4,5,6]. The problem is interesting not only in biology, but also in the dynamics of machines, for example in robotics to construct and stabilize walking biped or quadruped robots [7,8].

Increasing reflex delay of ill or elderly people causes balancing problems and fall-overs. Time delay is also present in computer control, although it is much shorter due to the great sampling frequency of the digital processors. Another important difference between human and computer control is that the computer needs an absolute reference line as vertical direction, while human balancing is successful without knowing it.

The examined model involves an artificial labyrinth. Computer control is successful without knowing the absolute vertical direction even with sampling delay.

2. The mechanical model of labyrinth

Main parts of the balancing organ [9,10] from mechanical viewpoint are the static and dynamic receptors. Dynamic receptors are situated in the semicircular canals. The artificial semicircular canal [11] modelled by a disc with its inertia J, and the torsional damping k_e presents its viscosity. The maculas are responsible for sensing the spatial position of the head as well as gravitation and liftreactions. Maculas include static receptors standing at the bottom of the utricle and hanging from the top of the saccule. The artificial macula [12] is a second inverted pendulum attached to the upper end of the pendulum.

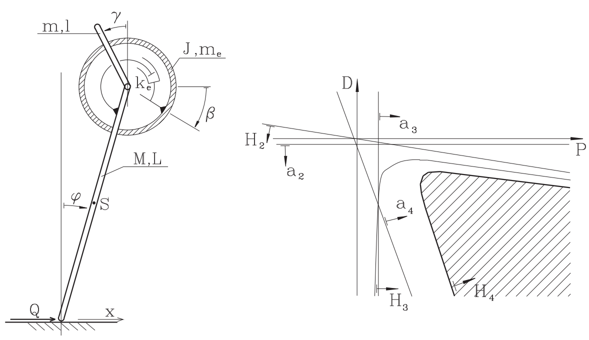

Figure 1: *The model of artificial labyrinth and its stability map*

Thus, the mechanical model of balancing is constructed in regard to the structure and functioning of the human balancing organ as it is presented in Figure 1.

The system has 4 DOF, namely x, φ, γ and β. If x is eliminated [12], the linearized system of the 3 equations of motion assume the form

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{K}\dot{\mathbf{y}} + \mathbf{S}\mathbf{y} = \mathbf{b}\,, \tag{1}$$

where

$$\mathbf{y} = \begin{pmatrix} \varphi \\ \gamma \\ \beta \end{pmatrix}, \ \mathbf{M} = \begin{pmatrix} m_{11} & m_{12} & 0 \\ m_{21} & m_{22} & 0 \\ 0 & 0 & J \end{pmatrix}, \mathbf{K} = \begin{pmatrix} k_e & 0 & -k_e \\ 0 & 0 & 0 \\ -k_e & 0 & k_e \end{pmatrix},$$

$$\mathbf{S} = \begin{pmatrix} -\left(\frac{M}{2} + m + m_e\right) gL & 0 & 0 \\ 0 & -\frac{1}{2}mgl & 0 \\ 0 & 0 & 0 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -\frac{\frac{M}{2}+m+m_e}{M+m+m_e} LQ \\ \frac{1}{2}\frac{m}{M+m+m_e} lQ \\ 0 \end{pmatrix},$$

$$m_{11} = \left(\frac{M}{3} + m + m_e\right) L^2 - \frac{(m+m_e)^2}{M+m+m_e} L^2 - \frac{\frac{M}{4} + m + m_e}{M+m+m_e} ML^2 ,$$

$$m_{12} = m_{21} = -\frac{1}{4} \frac{M}{M+m+m_e} mlL ,$$

$$m_{22} = \frac{1}{3} ml^2 - \frac{1}{4} \frac{m^2 l^2}{M+m+m_e} .$$

The control force is assumed in the form of

$$Q(t) = P(\dot{\varphi}(t_j - \tau) + \dot{\gamma}(t_j - \tau)) + D\left(\dot{\varphi}(t_j - \tau) - \dot{\beta}(t_j - \tau)\right) , \quad t \in [j\tau, (j+1)\tau) ,$$

where τ is the sampling delay.

3. The stability analysis

The stability analysis of the system without digital effects $\tau = 0$ can be carried out by the Routh-Hurwitz criterion. The trivial solution of (1) is asymptotically stable if and only if coefficients of the characteristic equation and Hurwitz-determinants are positive. The stability chart in case of the system without the delay effect is given in Figure 1. Values of parameters describing the system are based on quantities describing the human body and realistic approximate conditions $M >> m$, $M >> m_e$ and $L >> l$ are used [12].

The little pendulum and the disc cannot be constructed in the small size of the real maculas and semicircular canals, therefore the above mentioned approximations are not valid for the case of computer control. The stability analysis is accomplished for the system presented in the previous chapter. Sampling delay makes it discrete, but after the Moebius-transformation, stability conditions can be obtained using the Routh-Hurwitz criterion.

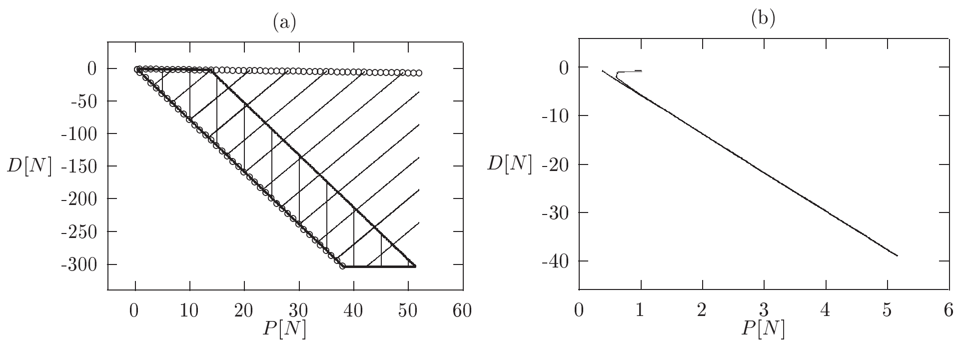

Figure 2: The stability charts (a) dotted line: $\tau = 0ms$, continuous line: $\tau = 1ms$, (b) $\tau = 4.8ms$, the critical value

The stability charts without time delay $\tau = 0$ and with small delay $\tau = 1ms$ are given in Figure 2(a). The stability domain is bordered by a straight line and a curve (stability conditions respect to the coefficient of constant in the characteristic equation and the maximum sized Hurwitz-determinant). Increasing the time delay makes the straight line move in the direction of negative D, so the stability domain becomes longer, but narrower. After the straight line reaches the point where it is tangent to the curve, the stability domain is bordered by only one curve and increasing the time delay further, it will be shorter and narrower. At a certain critical value of the sampling delay the loop disappears, the curve ends in a peak as it is shown in Figure 2(b). This is the critical time delay, balancing is impossible above it.

The critical sampling delay depends on parameters describing parts of the artificial labyrinth. It is not necessary that the little pendulum is so short as the length of maculas, quite the contrary, the critical time delay and the stability domain also increase as the length of the second pendulum increases. If the torsional damping modelling viscosity of the endolymph is greater, but small enough, then the critical value of time delay increases again. The torsional damping has an optimal value where the stability domain and the critical time delay have a maximum. Increasing the torsional damping over the optimal value, the critical value of the delay decreases and above a certain value it becomes 0. It means that the disc with the torsional damping is necessary for successful balancing. These functions are given in Figure 3(a) and (b).

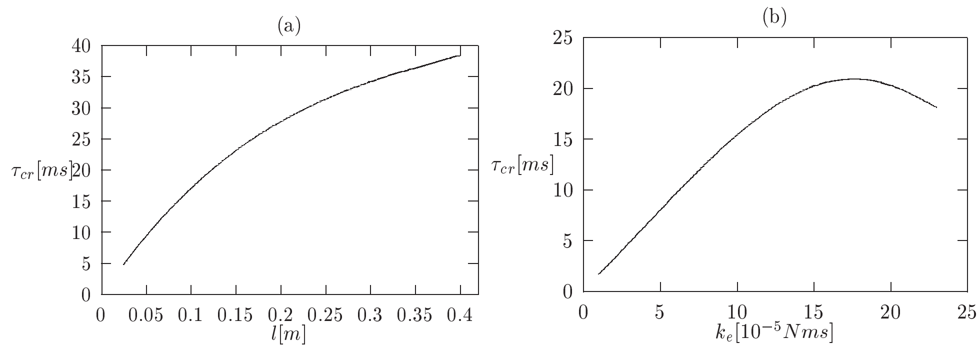

Figure 3: (a) the critical sampling delay vs. the length of the little pendulum, (b) the critical sampling delay vs. the torsional damping

4. Conclusions

An artificial balancing organ has been designed and its functioning using computer control has been examined. Human reflexes causes time delay in balancing, but this delay also appears in digital control because of taking samples. These are different types of time delay, but they cause the same problem, the decrease of stability domains, or above a critical value, the instability. Healthy balancing organ is responsible for successful human balancing and the parameters of its model also determine stability conditions in case of digital control. Angles or angular velocities relative to the absolute vertical line are not sensed. The computer controls this system using relative angular velocities, the angular velocity between the two pendula and the angular velocity between the big pendulum and the disc. This control is successful even on a surface with unknown inclination.

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