Discrete-time Model Reference Control of Milling Forces under Fractional Order Holds. Part I: Known Plant

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***Abstract*— This paper introduces a novel model-reference discrete-time control scheme applied to milling processes. The novelty of the scheme relies on the use of a fractional order hold (FROH) instead of a traditional ZOH used in the manufacturing literature to obtain a discrete-time model of the continuous system. The additional degree of freedom introduced by the FROH through its correcting gain allows the designer to improve the transient response of the closed-loop system by an adequate choice of its value. Simulation examples showing the influence of the correcting gain in the closed-loop response are presented and compared. The control scheme is applied in this paper to known piecewise-constant plants being subsequently extended to unknown ones through adaptive control.**

I. INTRODUCTION

Milling is a cutting process widely used in the manufacturing of mechanical components. It is carried out by feeding a work-piece clamped on a table against a rotating multi-tooth cutter. In order to avoid machine malfunctions such as tool wear or breakage and to achieve a certain degree of quality in the finishing of the working-piece, the peak cutting force on the working piece has to be maintained below a prescribed safety upper-bound. This fact implies that a control strategy has to be implemented on the system in order to fulfill such safety and performance requirements. Moreover, cutting parameters may be unknown or time-varying as a consequence of a complex milling geometry. Thus, the control law should be able to attain the desired objectives even in the presence of uncertainties or variations in the system parameters. In this way, the nature of the system suggests to use an adaptive controller to address the milling force control problem.

In this two-papers set work, we present the design of an adaptive control law for milling processes which improves the behavior, specially the quality of the finishing of the working piece through a more precise tool-work-piece interaction force control, in comparison with previous approaches. The work is organized as follows. In the first part, a novel discrete-time model-reference control strategy is proposed to design a force control law in the case when system parameters are known at all time. The proposed control law is then extended to the case of unknown parameters in the following paper (part II) by considering a parameter estimation algorithm running online simultaneously with the controller which leads to the design of a fully adaptive control law. Thus, in this paper, the continuous milling system described in [1] is treated as a perfectly known system but with time-varying parameters, which implies that the evolution of these parameters should be 'a priori' known by the designer.

The key point to achieve such an improved behavior of the system is the use of fractional order holds (FROH) to obtain a discrete-time model of the system. The advantage of using a FROH instead of a traditional ZOH is that FROHs incorporate an additional degree of freedom, the gain of the FROH, which can be used to modify the overall closed-loop response of the system, improving, for instance, the stability of the discrete zeros or reducing the overshoot or bad transient responses which could lead, for example, to break the cutter shank, tool breakage or tool wear, [2, 3]. Hence, the model reference control is the designed from the so obtained FROH based discrete model.

Thus, in this first paper the influence of the FROH gain in the system's behavior is studied showing that an adequate tuning of it can lead to an improved closed-loop performance. The study is carried out by means of a cost function which compares the system transient responses when different gains of the FROH are used. These results are to be extended to the unknown parameters case in the second part of this paper.

II. SYSTEM DESCRIPTION

A. Continuous model

The milling system can be modeled as the series decomposition of a Computerized Numerical Control (CNC), which includes all the circuitry involving in the table movement (amplifiers, motor drives), and the tool-work-piece interaction model itself. A feed rate command f_c (which plays the role of the control signal) is sent to the CNC unit. This feed rate represents the desired velocity for the table movement. Then, the CNC unit manages to make the table move at an actual feed velocity of f_a according to the CNC dynamics. Even though the machine tool drive servos are typically modeled as high order transfer functions, they can usually be approximated as a second order transfer function within the range of working frequencies [4]. Besides, they are tuned to be over-damped without overshoot, so that they can be modeled as the first order system [1]:

$$G_S(s) = \frac{f_a(s)}{f_c(s)} = \frac{1}{\tau_S s + 1} \tag{1}$$

where f_a and f_c are the actual and command velocity values of the table in (mm/s) respectively and τ_S is an average time constant, which depends on the type of the machine tool. In this study, it is assumed to be 0.1 ms.

In addition, the chatter vibration and resonant free cutting process can be approximated as the first order system [1, 4]:

$$G_p(s)=\frac{F_p(s)}{f_a(s)}=\frac{K_c b a(\phi_{st},\phi_{ex},N)}{N\cdot n}\frac{1}{\tau_c s+1} \qquad (2)$$

where K_c $\left(N/mm^2\right)$ is the cutting pressure constant, b (mm) is the axial depth of cut, $a(\phi_{st},\phi_{ex},N)$ is the adimensional immersion function, ranging between 0 and ~ N depending on the immersion angle and the number of teeth in cut, N is the number of teeth on the milling cutter and $n(rev/s)$ is the spindle speed. The axial deep of cut function *b* in (2) may be time-varying leading to a potential time-varying system. In particular, the cutting process is assumed to be in this work piecewise constant, admitting sudden changes in the cutting parameters at certain time instants while remaining invariant between changes. This assumption allows us to consider the cutting process to be described by the transfer function (2) with the time interval between changes.

The combined transfer function of the system, obtained from (1) and (2) is

$$G_c(s)=\frac{F_p(s)}{f_c(s)}=\frac{B_c(s)}{A_c(s)}=\frac{1}{(\tau_m s+1)}\frac{K_c ab}{Nn(\tau_c s+1)}= \\ =\frac{K_p}{(\tau_m s+1)(\tau_c s+1)} \qquad (3)$$

where the process gain is $K_p\ (N\cdot s/mm)=K_c ab/Nn$.

Figure 1 shows the sample work-piece depicting basic cutting geometry features with changes in the axial depth of cut used in the simulations. The spindle speed remains constant, $715 rpm$; the work-piece is made of Aluminum 6067 whose specific cutting pressure is assumed to be $K_c=1200\,N/mm^2$. A 4-fluted carbide mill tool, full-immersed and rouging milling operation will be taken into consideration in the present paper.

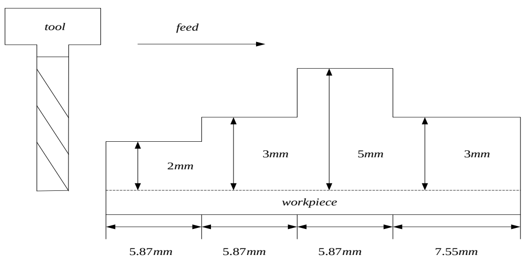

Figure 1: Work-piece profile to test control algorithms.

Also, note that the desired final geometry of the piece to be milled involves changes in the axial deep of cut which implies suddenly changes in its value, according to the sudden changes assumption presented before. On the other hand, it has been taken into account that the control law, computes new feed-rate command value at each sampling interval. Furthermore, it is worth to be mentioned that the CNC unit has its own digital position law executed at small time intervals in comparison with the sampled time of the control law, even though if high speed milling tool drives are used [1].

B. Discrete model under $\beta-FROH$

In this paper, the problem of controlling a continuous plant is addressed by using a discrete controller. The discrete controller is obtained applying a model-reference pole-placement based control design to a discrete model of the plant (3) obtained by means of a FROH with a certain correcting gain β. The additional “degree of freedom” β provided by the FROH can be used with a broad variety of objectives such as to improve the transient response behavior, to avoid the existence of oscillations in the continuous time output of the system or to improve the stability properties of the zeros of the discretized system [5, 6]. In this way, this work is especially focused on the use of these devices to improve the transient response of the closed-loop system by selecting an adequate value of the fractional order hold. Thus, in the following sections a comparative study of the behavior of the closed-loop system under different values of the correcting gain is developed. The influence of the value of β will be extended to the adaptive case in the subsequent paper. Hence, the discretization of (3) under a FROH is calculated as [7]:

$$H_\beta(z)=Z[h_\beta(s)\cdot G_c(s)] \qquad (4)$$

where $h_\beta(s)=\left(1-\beta e^{-sT}+\frac{\beta\left(1-e^{-sT}\right)}{Ts}\right)\frac{1-e^{-sT}}{s}$ is the transfer function of a $\beta-FROH$, where z is the argument of the $Z-transforn$, being formally equivalent to the one step ahead operators, q, used in the time domain representation of difference equations. This allows us to keep a simple unambiguous notation for the whole paper content. The sampling time *T* has been chosen to be the spindle speed, *n*, as it is usual for this kind of systems [1,4, 8-10]. Note that when $\beta=1$, the $FROH$ hold becomes a first order hold (FOH) and when $\beta=0$, the zero order hold (ZOH) is obtained, being both particular cases of $\beta\in[-1,1]$. Furthermore, $H_\beta(z)$ may be calculated using just ZOH devices in the following way:

$$H_\beta(z)=\frac{B_\beta(z)}{z^{\delta_\beta}\cdot A(z)}=\frac{z-\beta}{z}Z[h_o(s)G_c(s)]+\frac{\beta(z-1)}{Tz}Z\left[h_o(s)\frac{G_c(s)}{s}\right]=$$

$$=\frac{B_\beta(z)}{z^{\delta_\beta}\cdot\left(z-e^{T/\tau_m}\right)\cdot\left(z-e^{T/\tau_c}\right)} \qquad (5),$$

where $h_o(s)=\frac{1-e^{-sT}}{s}$ is the transfer function of a ZOH and $\delta_\beta=1$ if $\beta\neq0$ and $\delta_\beta=0$ if $\beta=0$, which means that a fractional order hold with $\beta\neq0$ adds a pole at the origin.

C. Desired response: reference model

A second order system $G_m(s)=\dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$ is selected to represent the system model reference. This system is characterized by a desired damping ratio, ξ and a natural frequency, ω_n. It is known that small ξ leads to a large overshoot and a large setting time. A general accepted range value for ξ to attain satisfactory performance is between 0.5 and 1, which corresponds to the so-called under-damped systems. In this way, a damping ratio of $\xi=0.75$ and a rise time, T_r, equal to four spindle periods is usually selected for practical applications [1,4,11]. Furthermore, the natural frequency is then usually suggested to be $\omega_n=2.5/T_r\ rad/s$. This continuous-time reference model is then discretized with the same FROH as the real system was in order to obtain the corresponding discrete-time reference model for the controller. Thus, a number of different discrete models obtained from a unique continuous reference model are considered depending on the value of β used to obtain the discretization.

III. MODEL-FOLLOWING CONTROL SCHEME.

The aim of the model-following control strategy is to force the closed-loop system to behave as a prescribed reference model. Thus, the following control scheme is applied:

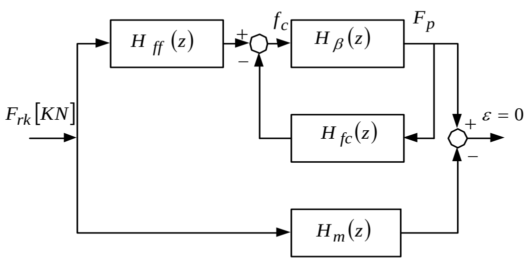

Fig.2: Model following control scheme.

where $H_{ff}(z)=\dfrac{T(z)}{R(z)}$ is the feed-forward compensator, $H_{fc}(z)=\dfrac{S(z)}{R(z)}$ is the feedback compensator, $H_\beta(z)$ is the discrete plant, $H_{m,\beta}(z)$ is the discrete-time reference model and F_{rk} is the reference force. A complete description of the design procedure for the model-following pole-placement based control scheme showed in Figure 2 can be found, for instance, in [12] being omitted here for space reasons.

In this section, the above introduced control scheme is applied to different choices of the $\beta-FROH$. Despite exhaustive simulations have been performed for a wide range of values of β, only some selected cases are presented in this section in figures 3 and 4. For each value of β, four figures are plotted. The first one depicts the model reference and the plant output signals versus sample time; the second one shows the evolution of the tracking error signal, $(e=F_p-F_{pm})$; the third figure represents the controller response and, finally, the four graphic shows the continuous-time domain system response obtained using the corresponding FROH. For all the considered cases, the figures show that the steady-state force tracks the reference force which is set to $1.2\,KN$, except for the peaks appreciated when the axial depth of cut, and then the transfer function, is suddenly altered. It is also appreciated that the discrete-time transient response follows exactly the discrete model reference at each sampling time as a consequence of the perfect knowledge of the plant

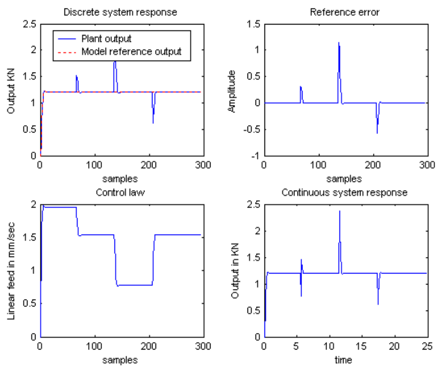

Fig.3: Responses corresponding to $\beta=-1/3$.

parameters.

The programmed feed rate is feasible and smooth, even though the axial depth of cut varies. Also, note that the finally applied control law is FROH sensitive leading to a different control signals for different discrete models of the plant. In the present case, the average value of the feed velocity along the tool path is less if the fractional order hold rather a ZOH is used. This fact is considered an advantage from the point of view of feed motor maintenance and energy consumption.

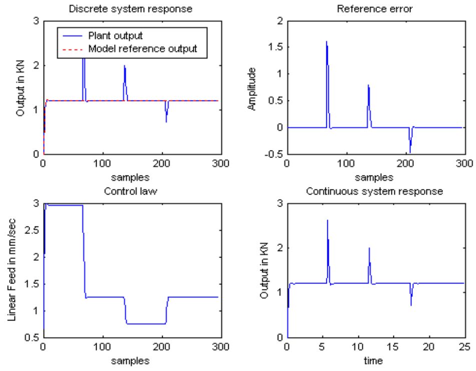

Fig.4: Responses corresponding to $\beta = 0$.

Also, when a parameter of the system changes abruptly (in this case the axial depth of cut) the model-reference control leads to large output overshoots, due to the intrinsic structure of the output. Thus, if the reference force is selected near the tool breakage limit, the large overshoot would lead to tool breakage [2, 3]. In that case, some 'a priori' information about the work-piece geometry is required to design a successful control, as in [9], where a CAD model of the work-piece is used to modify the control command when the axial depth of cut changes in order to minimize the overshoots due to abrupt changes in the transfer function

IV. TRANSIENT RESPONSES CHARACTERIZATION

The use of FROH devices allows the designer to use the β value in order to achieve an improved closed-loop transient response. In order to compare time domain transient behaviors when different β-values are used to design the control, the following cost functional is defined:

$$J_c(\beta,T)=\int_0^{T_p}\left|F_{p,\beta}(t)-F_{p,m}(t)\right|dt \cong \sum_{j=1}^{T_0 N_p}\left|F_{p,\beta}(jT_0)-y(jT_0)\right|\cdot T_0 \cong \int_0^{T_p}\left|F_{p,\beta}(\tau)-y(\tau)\right|dt \qquad (6)$$

where $F_{p,\beta}$ is the continuous time domain response, y is the continuous model reference response, T is the sampling time, T_o is the time of the computer, T_p is the tested time and N_p is the number of samples of the T_o period over T_p.

Thus, the cost function calculates an approximation of the area between time domain response and continuous model reference system response. The smaller this area is, the smaller cost function is and the corresponding control associate to the corresponding FROH will give an improved output response.

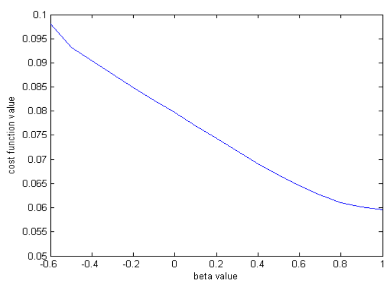

Fig.5: Cost function value versus β values associate to the control.

Figure 5 shows the cost functional (6) value as β is varied. It can be appreciated the great influence the value of β possesses in the closed-loop transient response. In this particular case, exhaustive simulation results point out that the FOH is the best hold. However, in the case of unknown plants, which is considered in the next paper, it will be shown that the most appropriate value corresponds to negative values of β which highlights the usefulness of the proposed approach in a more general a setting-up.

The cases when $\beta < -0.6$ have not been taken into consideration since they lead to non-minimum phase discrete models of the plant.

V. CONCLUSION

In this paper, a discrete-time model following control strategy for a known continuous-time milling systems has been developed. The novelty of the control schemes relies on the use of a FROH in the discretization process instead of the usual ZOH appearing in the manufacturing literature. The introduction of an additional "degree of freedom" provided by the FROH correcting gain allows the designer to improve the transient behavior of the continuous-time closed-loop system by an adequate selection of its value as simulation results have pointed out.

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REFERENCES

[1] Y. Altintas, "Manufacturing Automation", Cambridge University Press, 2000.
[2] Y. Altintas, I. Yellowley and J. Tlusty, "The Detection of Tool Breakage in Milling Operations", Journal of Engineering for Industry, November 1988, Vol. 110.
[3] Y. Altintas, "Prediction of Cutting Forces and Tool Breakage in Milling from feed drive current measurements", Journal of Engineering for Industry, pp.386-392, November 1992, Vol. 114.
[4] Y. Altintas, F. Sassani, and F. Ordubadi, "Design and Analysis of Adaptive Controllers for Milling Process. Transactions of the CSME. pp. 17-25, no. 1/2, 1990, Vol. 14.
[5] K.J. Astrom, P. Haganger and J. Sternby, "Zeros of sampled systems", Automatica, Vol. 20, nº 1, pp. 31-38, 1984.
[6] M. Ishitobi, "Stability of zeros of sampled systems with fractional order hold", IEE Proc.-Control Theory Appl., Vol. 143, nº 3, May 1996.
[7] A. Bilbao-Guillerma, M. De la Sen and S. Alonso-Quesada, "On a Root Locus-based Analysis of the Limiting Zeros of Plants of Nominal Order at most two under FROH-discretization", Proceedings of the 2005 American Control Conference.
[8] L.K. Lauderbaugh and A.G. Ulsoy, "Model Reference Adaptive Force Control in Milling", Journal of Engineering for Industry, February 1988, Vol. 111.
[9] A.Spence and Y. Altintas, "CAD Assisted Adaptive Control for Milling", Transaction of the ASME, September 1991, Vol. 113.
[10] Y.H Peng,, "On the performance enhancement of self-tuning adaptive control for time-varying machining processes", International Journal of Advanced Manufacturing Technology, pp. 395-403, 2004, 24.

[11] B.C. Kuo, (1991). Automatic Control Systems, Prentice-Hall, Englewood Cliffs, New Jersey, 1991.

[12] K.J. Astrom and B. Wittermark, “Adaptive Control”, 2^{nd} edition, Addison-Wesley, 1995.