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An expert mill cutter selection system

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Abstract

This paper discusses the selection of tools in milling operations. To carry out this research, it has been developed an expert system hinged on numerical methods. The knowledge base is given by limitations in process variables, which let us to define the allowable cutting parameter space. The mentioned process variables are, instabilities due to tool-work-piece interaction, knowing as chatter vibration, and the power available in the spindle motor. Then, a tool cost model is contrived. It is used to choose the suitable cutting tool, among a known set of candidate available cutters, and to obtain the appropriate cutting parameters, which are the expert system outputs. An example is presented to illustrate the method.

1. Introduction

Machining, in particular milling operations, is a broad term used to define the process of removing material from a work-piece. Furthermore, the milling operation process planning is required, nowadays, to increase its productivity, reducing cost and improving the final product [1].

This paper brings forward the concept of selecting an appropriate mill cutter, among a known set of candidate cutters, and obtaining the adequate cutting parameters for milling operations through an expert system.

There are several versatile approaches for tool and/or cutting parameter selection based on expert systems on manufacturing environments. Wong and Hamouda [2] developed an on-line fuzzy expert system. The system inputs, the tool type, the work-piece material hardness and the depth of cut, and control the cutting parameters at the machine, as output. Cemal Cakir et al. [3] explained an expert system based on experience rules for die and mold operations. In that paper, the geometry and material of the work-piece, tool material and condition and operation type are considered as inputs. Then, the system provides recommendations about tool type, tool specifications, work-holding method, type of milling operation, direction of feed and offset values. Vidal et al. [4] focused on the problem of choosing the manufacturing route in metal removal process. They select the cutting parameters by optimising the cost of the operation taking into account various factors, such as, material, geometry, roughness, machine and tool. Carpenter and Maropoulus [5] designed a system, which provides reliable tool selection and cutting data for a range of milling operations. The method employs rule based decision logic and multiple regression techniques for a wide range of materials.

Here, the developed expert system consists of the relative compliance between the tool and the work-piece, and it is predicted with analytical methods. Moreover, time and frequency domain milling process simulations have been developed, which are, then, used in the expert system definition.

Then, the knowledge base is explained. Basically, it defines the allowable cutting parameters, which are known as cutting parameter space, for a given tool-work-piece configuration. It is based on the chatter vibrations avoidance, which limits the productivity of the process, and on a spindle power limitation criterion.

On the other hand, a novel tool cost function is designed. It depends on spindle power consumption, material removing rate (MRR) and on a stability criterion against possible perturbation in the spindle speed variable.

The MRR is a parameter which measures the process effectiveness. It is required to be as large as possible. But, if the MRR increases beyond certain limits, chatter vibrations are appreciated and the process becomes unstable [6]. Other variable which limits the process effectiveness is the power available in the spindle motor [7]. The third parameter taking part into the cost function is considered to ensure a well-posed behaviour of the system if a perturbation in the spindle speed happened.

In conclusion, the proposed cost function is a measure of how the milling process is being carried out at certain operation conditions. The larger the cost function, correspond to the worst operation condition. Thus, the cutter and cutting conditions which minimise the designed cost function are selected.

Then, the expert system takes tool characteristics, related tool-work-piece material parameters and milling operation as inputs and outputs the selected tool among the candidates and robust programmed cutting parameters.
2. System description

A model, which represents the dynamic compliance between the tool and work-piece in milling processes, has been developed. In this case, it is predicted with analytical methods. The model assumes the cutter to have two orthogonal degrees of freedom and the work-piece to be rigid.

2.1. Dynamic model

The dynamic model of the milling cutter is assumed to be a system with one mode of vibration in each direction, x and y, while the feed direction is along the x-axis. The milling system under consideration is shown in figure 1. The milling cutter has n teeth, which are equally spaced. The dynamics of the system is given by the differential equations [8],

\[ m_x \ddot{x} + c_x \dot{x} + k_x \cdot x = \sum_{j=0}^{n} f_x (t) = f_x (t) \]  
\[ m_y \ddot{y} + c_y \dot{y} + k_y \cdot y = \sum_{j=0}^{n} f_y (t) = f_y (t) \]  

where \( m_x, c_x, k_x \) and \( k_y \) are the mass, damping and stiffness of the tool, \( f_x \) and \( f_y \) are the components of the cutting force that is applied by the j'th tooth, which are obtained by projecting \( f \) into the two orthogonal axes.

2.2. Cutting force model

A simple model of the cutting forces will be discussed here which expresses the tangential cutting force to be proportional with the instantaneous chip thickness. Despite this simplicity, this model captures the essence of the process. Hence,

\[ f_x = k_t \cdot b \cdot h \]  
\[ f_y = k_r \cdot f_t \]  

where \( k_t \) is a proportional constant. This cutting force model has been used by several authors [6].

The most critical variable in (3) is the chip thickness because it changes not only with the geometry of cutting tool and cutting parameters, but also with the uneven surface left by the previous passes of the cutting tool. This process is known as regenerative mechanism [6].

The chip thickness is measured in the radial direction, with the coordinate transformation,

\[ v_r = -x \cdot \sin \phi_j - y \cdot \cos \phi_j \]  

where \( \phi_j \) is the instantaneous angular immersion of tooth \( j \) measured clockwise from the normal Y axis (fig.1).

The resulting instantaneous chip thickness consists of static part \( s_j \cdot \sin \phi_j \), attributed to rigid body motion of the cutter, and a dynamic component caused by the dynamic displacements or vibrations of the tool at the present, \( v_r \), and previous tooth periods, \( v_r' \). Then, the total chip load can be expressed by,

\[ b(\phi_j) = [s_j \cdot \sin \phi_j + (v_r' - v_r)] \cdot g(\phi_j) \]  

in the tool rotate angle domain, or

\[ h_1(t) = s_j \cdot \sin \phi_j + x(t) \cdot \sin \phi_j (t) + y(t) \cdot \cos \phi_j (t) \]

\[ + [x(t-T) \cdot \sin \phi_j (t) + y(t-T) \cdot \cos \phi_j (t)] \]  

in the time domain, where \( g(\phi_j) \) is a unit step function which determines whether the tooth is in or out of cut, \( s_j \) is the feed rate per tooth, \( T \) is the tooth period and, if the spindle rotates at \( N_s \cdot \left( \text{rad} \cdot s^{-1} \right) \), the immersion angle varies as \( \phi_j (t) = N_s \cdot t \), and \( \phi_j (t) = 0 \) if the \( j \)-tooth is not engaged with the part[6].

2.3. Time domain simulation

Since the system is excited by cutting forces that cannot be expressed by simple analytic functions, the equations cannot be integrated in a closed form. Hence, the 4th order Runge-Kutta method is employed to solve the differential equations (1) and (2)[8]. A simulation system, which reads the input data of cutting conditions, machine tool characteristics, and other related parameters, and outputs the forces and vibration displacements of chatter in milling has been developed.

2.4. Stability lobes

Projecting \( f_x \) and \( f_y \) determined by equations (3) and (4) into x and y axis, taking into account that the static component of the chip thickness is dropped from the expression (6), and summing for all teeth engaged and rearranging the above expressions (3) and (4) in matrix form, will yield to [2]:

\[ \begin{bmatrix} f_x \\ f_y \end{bmatrix} = \frac{1}{bh} \begin{bmatrix} a_{xx} & a_{xy} \\ a_{yx} & a_{yy} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \]  

where \( a_{xx}, a_{xy}, a_{yx}, a_{yy} \) can be easily obtained, and they are angular position dependent.
Considering that the angular position of the parameters changes with time and angular velocity, equation (8) can be expressed in time domain in a matrix form as:

\[
\begin{bmatrix}
    f_x \\
    f_y
\end{bmatrix} = \frac{1}{2} b k \begin{bmatrix} A(t) \end{bmatrix} \Delta \omega(t)
\]  

(9)

where \( \{ \Delta \omega(t) \} = \{ x(t) - x(t - T), y(t) - y(t - T) \} \).

The time directional dynamic milling force coefficients collected in \( A(t) \) are periodic function of the tooth passing period, \( T \). Furthermore, \( A(t) \) can be expanded into a Fourier series. For the most simplistic approximation, the average component of the Fourier series expansion can be considered. The dynamic milling expression for milling force will be reduced to:

\[
f(t) = \frac{1}{2} b \cdot k \begin{bmatrix} A \end{bmatrix} \{ \Delta \omega(t) \}
\]  

(10)

where \( A = \begin{bmatrix} a_{xx} & a_{xy} \\
                        a_{yx} & a_{yy} \end{bmatrix} \) is the time-invariant but immersion-dependent directional cutting coefficient matrix [6].

Thus, being \( \phi(i \omega) \) the transfer function matrix at the cutter contact zone, denoted by

\[
\begin{bmatrix} \phi_x(i \omega) \\
                \phi_y(i \omega) \end{bmatrix}
\]

\[
\begin{bmatrix} \phi_v(i \omega) \\
                \phi_v(i \omega) \end{bmatrix}
\]

Furthermore, describing the vibrations at the chatter frequency, \( \omega \), in the frequency domain using harmonic functions,

\[
\{ r(i \omega) \} = \left[ \begin{bmatrix} \phi_x(i \omega) \end{bmatrix} \{ f \} e^{i \omega t}, \begin{bmatrix} \phi_y(i \omega) \end{bmatrix} \{ f \} e^{i \omega t} \right],
\]

\[
\{ \Delta \omega(i \omega) \} = \left[ \begin{bmatrix} r_x(i \omega) - r_x(i \omega) \end{bmatrix}, \begin{bmatrix} r_y(i \omega) - r_y(i \omega) \end{bmatrix} \right],
\]

then, the equation (10) can be written as:

\[
\{ f \} e^{i \omega t} = \frac{1}{2} b k \begin{bmatrix} A \end{bmatrix} \{ \phi(i \omega) \} \{ f \} e^{i \omega t}
\]  

(11)

Obtaining the characteristic equation and its eigenvalue, \( \Lambda \):

\[
\Lambda = -\frac{N}{4 \pi} b k \left( 1 - e^{-i \omega t} \right)
\]  

(12)

For the case that the cross transfer functions of the systems are neglected, the characteristic equation will be reduced to a quadratic equation, and the eigenvalues \( \Lambda \) can be obtained [6].

The critical axial depth of cut is calculated by substituting the obtained eigenvalue into equation (12):

\[
b_{ax} = \frac{2 \pi \Lambda x}{Nk} \left( 1 + \kappa^2 \right)
\]  

(13)

where \( \kappa = \Lambda_f / \Lambda_R \) is the division between the imaginary and real parts of the eigenvalue \( \Lambda \).

Corresponding to the spindle speed \( N_s = 60f / N \cdot T \) (14) and the chatter frequency can be found [6] as:

\[
\omega T = e + 2k \pi, \quad \text{where } e = \pi - 2 \phi, \quad \text{and } \phi = \tan^{-1} \kappa .
\]

\( T \) is the spindle period, and \( k \) is the integer number of full vibration waves (i.e. lobes), imprinted on the cut arc. The lobes are calculated, selecting a chatter frequency from transfer functions around a dominant mode, solving the eigenvalue equation(12), calculating the critical depth of cut from (13), calculating the spindle speed from (14) for each stability lobes, and repeating the procedure by scanning the chatter frequencies around all dominant modes of the structure [6].

![Figure 2](image.png)

Figure 2: The milling system representation, stability chars, force time response and force frequency response

Figure 2 shows the lobes char, and the analytical time and frequency domain response for a tool 2 system, which characteristics can be seen in section 5. The chatter stability lobes make up a spindle speed (frequency) dependent dividing line between stable (down part line) and unstable (up part line) depth of cut for a certain width of cut. Stable state corresponded figures present a delimit time response, and the tooth passing frequency and its harmonics, frequency response. Unstable state corresponded figures present a not delimit time response, and the chatter frequency is appreciated.

3. Expert system

The main objective of the expert system is to obtain a mill cutter, among the available ones, which have an operating point or adequate cutting parameters, with acceptable productivity (MRR), robustness stability against spindle speed perturbations and less power consumption than the spindle motor availability.

For this purpose, it is got the allowable cutting space parameter, spindle speed, feed rate and axial depth of cut for a constant radial depth of cut, taking into account the regenerative chatter instability and the power available in the spindle motor. Then, a novel cost function is schemed. It is inversely proportional to MRR and a parameter determine as stability against spindle speed perturbation, and proportional to power consumption. Each term of the cost function have a proportionally
factor to have terms of the same magnitude. Also, there is a weight factor which measures the importance of each term. The weight factors are intended to be programmed by the machine operator.

3.1. Milling process determination and preliminary rules

In order to evaluate the system performance, it is needed to select a suitable tool and performance indices. Milling processes, basically, consists of two phases roughening and finishing the surface. The main difference between these operations is to decide the most appropriate performance index for a given tool. The quality and geometric profile of the cutting surface is of paramount importance in milling finishing operation, whereas roughing -milling consists on removing a large amount of material from a blank.

This paper deals with roughing milling operation. The rate at which the material is removed is called material removing rate (MRR). This parameter measures the productivity of machining processes. In milling operations, MRR is defined as the multiplication between axial and radial depth of cut, and feed per tooth. MRR upper limit, is given by, chatter vibrations and power deliver by the spindle motor. At certain combinations of cutting parameters, such as spindle speed, axial depth of cut and feed per tooth, either chatter vibrations are appreciate, or the power available by the spindle motor is insufficient. Then, these parameters bound the roughing milling productivity.

For those reasons, at a first approximation, the input cutting parameter space is given by the cutting parameters, which are below the line at the stability lobes char, and the power consumption is less than the power available by the spindle motor.

But, due to the approximations in constructing stability chars, the lobes are constructed, not by replacing pure imaginary roots into the characteristic equation, but adding a positive real number to them. Furthermore, to have a robust system, it has been taken into account a confine in a programmed maximum depth of cut.

Then, the following algorithmic methodologies are used, which are called preliminary rules:

• **Rule1:** Stability margin setting to ensure that the system plays in a stable region, despite the system model uncertainties.

• **Rule 1.1:** For calculating secure stability lobes char, a small stability margin is selected, i.e, it is supposed that the chatter vibrations happen at \( \delta + i \cdot \omega_0 \) instead of at \( i \cdot \omega_0 \). The reason is that the stability border line is calculated from a linear approximation. Then, \( i \cdot \omega_0 \) is replaced by \( \delta + i \cdot \omega_0, \delta > 0 \), when the stability border line is calculated. This rule is applied to the equation (13).

• **Rule 1.2:** For improving the robustness of the system, it has been taken into account a margin at the final expression for chatter free axial depth of cut, equation (18), i.e, \( b_{lim} = \alpha \cdot b_{lim}, 0 < \alpha < 1 \). This rule lets a better control capacity in the spindle speed. On the other hand, a better MRR selection is lost.

• **Rule 2:** For searching the allowable input space parameter, the set of spindle speed, \( N_s \), axial depth of cut, \( b \) and feed rate, \( s_f \).

• **Rule 2.1:** Calculate the boundary points, spindle speed and axial depth of cut pairs, which compose the line between stable and unstable zones, satisfying Rule 1. This rule is obtained by plotting the stability lobes char, which gives the line between stable and unstable zones.

• **Rule 2.2:** Calculate the admissible input space, \( Q = (N_s, b, s_f) \). The boundaries spindle speed and axial depth of cut, gives the maximum spindle speed and axial depth of cut pairs without chatter vibrations (rule 2.1). The time domain simulations output the system dynamical force shape. As it will be seen in the next section, the spindle power is force dependent, which is spindle speed, axial depth of cut and feed rate dependent. Then, for a given spindle motor power available, the admissible input cutting parameter space is obtained.

3.2. Tool selection

In this section, an approach for tool selection is suggested. For this purpose, a tool cost model function is designed. The designed tool cost model is used to select the appropriate tool between the candidates though the optimisation Rules, explained below.

Then, the study requires a given set of candidates milling cutters. Each one is characterised by the following properties:

\[ R_i = \left( \omega_{i1}, \omega_{i2}, \xi_i, \xi_{ii}, k_{ii}, n_s, D_i, \beta_i \right) \]

where, \( \left( \omega_{i1}, \omega_{i2} \right) \in W \) is the tool natural frequency, \( \left( \xi_i, \xi_{ii} \right) \in \xi \) is the tool damping ratio, \( (k_{ii}, k_{ii}) \in K \) is the tool static stiffness, \( n_s \) is the tool number of teeth, \( D_i \) is the tool diameter and \( \beta_i \) is the tool helix angle. \( R_i \in T_i \), \( i = 1, 2, ..., N \), where \( N \) is the number of tools and \( T \) is the set of tools available to the designer. \( W \) is the set of tools’ natural frequencies, composed by the pairs \( (\omega_{i1}, \omega_{i2}) \) for each tool, \( \xi \) is the set of tools’ damping ratio, composed by the pairs \( (\xi_i, \xi_{ii}) \) for each tool and tools’ static stiffness is composed by \( (k_{ii}, k_{ii}) \) for each tool.

3.2.1. Tool cost model definition

To carry out the selection of a suitable tool, a novel tool cost function has been conceived. The tool cost
model for a single milling process can be calculated using the equation (20).

\[
C\left(P, MRR, \Delta N_s; R, c_1, c_2, c_3\right) = c_1 \cdot NF_1 \cdot P_1 \\
+ c_2 \cdot \frac{\Delta N_s}{MRR} + c_3 \cdot \frac{\Delta N_s}{\Delta N_s} \quad (20)
\]

with \(c_1 = 1, R \in T\), where \(P_i = V \cdot \sum f_i(\phi)\), \(MRR = a \cdot b \cdot s\), \(\Delta N_s\) takes its definition given below, and \(q = (N_s, b, s) \in Q\). Standardizing factors, \(NF_i\), are defined as follow, \(NF_1 = \frac{P_{\text{avail}}}{MRR}\), where \(P_{\text{avail}}\) is the power available in the spindle motor, \(NF_2 = \Delta N_{\text{max}}\), where \(\Delta N_{\text{max}}\) is the maximum MRR with the chatter vibration and spindle power restrictions calculated among all the candidate cutters and \(NF_3 = \Delta N_{\text{max}}\), where \(\Delta N_{\text{max}}\) is the maximum measured value of this variable among the candidate cutters.

The tool cost function is designed to be MRR, power consumption, and a range against possible perturbations in tool rotational motion, dependent and inversely proportional to MRR and a range against possible perturbations and directly to power consumption.

These parameters have the following definitions:

Material or Metal Removing Rate (MRR)

\(MRR = a \cdot b \cdot s\), where, \(a\) is the radial depth of cut, \(b\) is the axial depth of cut and \(s\) is the linear feed rate. The MRR is a parameter, which compares, the efficiency of the milling process. A larger MRR improves the process productivity.

Cutting power draw from the spindle motor \(P_1\)

The cutting power, \(P_1\), drawn from the spindle motor is found from,

\[
P_1 = V \cdot \sum f_i(\phi) \quad (21)
\]

where \(V = \pi D \cdot N_s\) is the cutting speed and \(N_s\) is the spindle speed. The tangential cutting force is given by:

\[
f_i(\phi) = K_i \cdot b \cdot h(\phi) \quad (22)
\]

where \(b\) is the axial depth of cut, \(K_i\) is the cutting force coefficient, which are material dependent and is evaluated from experiments, and \(h(\phi)\) is the chip thickness variation, which is feed rate \(s\) (mm/rev-tooth) dependent.

Spindle speed security change \(\Delta N_s\)

An additional term, spindle speed security change, is added to the cost function model to be sure that chatter vibrations are avoided. The spindle speed security change, \(\Delta N_s\), measure the nearest spindle speed at which chatter vibrations happen to the supposed spindle speed it will be operated. This fact allows to have an error margin due to possible perturbations in this variable.

To calculate analytically, \(\Delta N_s\), the following algorithmic methodologies are carried out. They are divided in two cases:

Case I: \(k = 0\), this case corresponds to pairs, spindle speed, axial depth of cut, situated below the first lobe of the stability chart. Then, there is no lobe in the right part of the point as it can be shown in figure 3. Suppose that \((N_a, b_i)\) is the point which \(\Delta N_s\) has to be calculated:

a) If \(b_{\text{min, crI}} > b_i\)

\[
\Delta N_s = \text{abs}\left(N_{s, \text{min, crI}} - N_a\right).
\]

b) If \(b_{\text{min, crI}} < b_i\)

\[
\Delta N_s = \text{abs}\left(N_{s, \text{crI}}(b_i) - N_a\right).
\]

\(b_{\text{min, crI}}\) is the minimum value of the axial depth of cut corresponding to the border line, \(N_{s, \text{min, crI}}\) is its corresponding spindle speed, \(N_s(\text{crI})\) is the left-projection of the point \((N_a, b_i)\) into the nearest lobe.

Case II: \(k \neq 0\), in this case, the point ,which \(\Delta N_s\) has to be calculated, is situated between two lobes in the stable region. Suppose that \((N_{aI}, b_i)\) is the mentioned point, then \(\exists k\) such that \(N_{s, \text{min, crI}}(k) < N_a < N_{s, \text{min, crI}}(k + 1)\), where \(k\) is the lobe number, \(k = 0, 1, S - 1\), and \(S\) is the number of printed lobes , and \(N_{s, \text{min, crI}}(k)\) is the spindle speed corresponding to the axial depth of cut minimum value on the border line, \(b_{\text{min, crI}}(k)\), for the k-lobe. Then:

a) If \(b_{\text{min, crI}}(k) > b < b_{\text{min, crI}}(k + 1)\)

\[
\Delta N_s = \min\left(\text{abs}\left(N_{s, \text{crI}}(k) - N_a\right), \text{abs}\left(N_{s, \text{crI}}(k + 1) - N_a\right)\right).
\]

b) If \(b_{\text{min, crI}}(k) < b > b_{\text{min, crI}}(k + 1)\)

\[
\Delta N_s = \min\left(\text{abs}\left(N_{s, \text{crI}}(k) - N_a\right), \text{abs}\left(N_{s, \text{crI}}(k + 1) - N_a\right)\right).
\]

where \(N_s(\text{crI})(k)\) is the left-projection of the point \((N_{aI}, b_i)\) into the k-lobe, and \(N_s(\text{crI})(k + 1)\) is the right-projection into the k+1-lobe. The case under consideration is graphically represented in figure 4.

Note that, standardization factors, \(NF_i\), are also added to the cost function to have terms with the same magnitude. Moreover, they make to have a relative term between all the candidates cutters involved. On the other hand, these terms ensure that the cost function will be comparable among the different cutters.

The \(c_i, i = 1, ..., 3\), values are the weights of the cost function terms. They measure the importance of the cost function terms. The below optimisation Rule 3 give a pattern to program the \(c_i\) .
3.2.2. Optimisation Rules

The above defined tool cost function is used to select the appropriate tool and cutting parameters, through the following optimisation rules.

**Rule 3**: Weight factors selection

The weight factors are intended to be programmed by the machine operator. An extended explanation of their meaning and their adequate selection is given in this section. To select suitable values of \( c_1, i = 1,..,3 \), their meaning has to be perceived. The \( c_1 \), measures the importance of the spindle power consumption. The larger \( c_1 \) parameter is the more important to the spindle power consumption in the cost model function. The \( c_2 \) measures the machine productivity, if the \( c_2 \) is near to one high productivity is required, and if it is near to zero the productivity has no importance. The same reasoning is applied to the \( c_3 \), which measures the stability against possible perturbations in the spindle speed variable.

It has to take into account that the expert system, ensures that the spindle power consumption is always going to be smaller than the power available in the spindle motor, through Rule 1. Also, that the cutting parameter space has no problems due to chatter vibrations through Rule 2.

Then, a possible criterion leading to a process with acceptable productivity, which is the main objective of the milling processes, \( c_2 \), about 0.75, and the other two constants will add 0.25, suitable values are \( c_1 = 0.1 \) and \( c_2 = 0.15 \).

**Rule 4**: Tool selection criterion

A simple tool selection criterion for cutter selection has been developed. For a given values of \( c_1,c_2,c_3 \), and a given tool characteristics, the cost function value is obtained for all the admissible input cutting parameter space. The minimum value of the cost function is saved. The procedure is repeated for all the available cutters. Comparing the minimum value of the cost function for all available or candidate cutters, the corresponding cutter to the minimum value of the minimum value of the cost function is the selected tool.

The selection criterion is, mathematically, expressed as:

- **Compute**, \( C(P(q),MRR(q),\Delta N,(q),(R,c_1,c_2)) \); (23) for each \( R \in T \), \( i \in N \), and \( N \) is the set of candidate tools and \( \forall q \) is \( \{ N_q,b,s \} \in Q \) where \( j \in N_q \) is a discrete sub-space of the cutting parameters space where the cost function (20) is calculated.
- **For obtaining the selected tool, ST, compute**
  \[
  ST = \arg \min_{q \in Q} C(P(q),MRR(q),\Delta N,(q),(R,c_1,c_2))
  \]
  with \( ST \in T \), obtaining the appropriate tool according to the criterion.

Following the rules, the expert system provides an appropriate cutter among the candidates.

**Rule 5**: Cutting parameter selection

- **Rule 5.1**: General case

  To select the cutting parameters, there are two possibilities. First of all, directly, calculate the cutting parameters, which correspond to the selected tool, which gives the minimum value of the cost function. It can be expressed mathematically as,

  - **Compute**, the following equation (24)
    \[
    q = \arg \min_{q \in Q} C(P(q),MRR(q),\Delta N,(q),(R,c_1,c_2))
    \]
    obtaining an input cutting parameter for the selected tool.

  The cutting parameter space is obtained by checking all possible values of spindle speed and axial depth of cut which are below the stability line in the stability chart according to Rule 1. These values join to the allowable feed rates, which do not make consume more spindle power than the available, Rule 2, form the cutting parameter space. For the selected tool, the trio of cutting parameters which minimize the cost function are, then, selected.

- **Rule 5.2**: Refinement case

  In order to have a more accurate possibility, it has been taken into consideration that the cutting parameters can be searched with a more fine integration step around the point where the cost function gives its minimum value. Now, the cutting parameter space is given by a 3-tuple \( Q = \{ N_q,b,s \} \), around \( q^* \), for \( k = 1,..,p \), where \( p \) is the number of points to be considered, according to Rules 1 and 2. The procedure for obtaining the required cutting parameter is the same as used in Rule 5.1 through equation (24) for the above defined new cutting parameters.

  Mathematically expressed :

  - **Compute**, \( q^* = \arg \min_{q \in Q} C(P(q),MRR(q),\Delta N,(q),(ST,c_1,c_2)) \)

  Obtaining the refined cutting parameter.
Rule 6: Process malfunctions: tuning $c_1, c_2, c_3$ values

Nevertheless, in programming the selected tool and cutting parameters, malfunctions of the process may lead to a poor behaviour of the process. The most important are tool wear and burr formation. These phenomena, which are common in the manufacturing processes, make that the analytical and experimental testes are not always in concordance. If it is happened, the follow algorithmic methodology could be applied:

While $A_{\text{ampl}} > A_{\text{ampl,ctg}}$

$c_1 \leftarrow 0.99 \cdot c_1$

$c_2 \leftarrow 0.01 \cdot c_1 + c_3$

end

where $A_{\text{ampl}}$ is the chatter frequency vibration amplitude, and $A_{\text{ampl,ctg}}$ is the highest amplitude among the tooth passing frequency and its harmonics. So, a more stable state is obtained.

Finally, figure 5 shows a scheme of the expert system. The developed expert system takes the $\alpha$ and $\delta$ constants, the tools’ modal parameters such as its natural frequency, damping ratio, tool static stiffness, number of teeth, the radius of the tool, the helix angle, and the cutting constants for the work material and cutter (tools’ characteristics), the spindle power available and the cost function weight factors, as inputs and outputs the appropriate tool among the candidates and robust programmed cutting parameters.

4. Example

For the validation of this method, the above study has been applied for two practical straight cutters and a full-immersion up-milling operation. The example considers the tools to have the following characteristics, according with the section 3.2 notation, $R_1 = (603,666,3,9,3,5,5,5,9,5,7,1,5,3,3,0,0)$, and $R_2 = (900,0,9,1,1,6,5,1,3,9,1,3,8,0,8,7,9,0,9,7,1,2,1,2,7,0)$. The natural frequency is measured in hertz, the tool damping is in %, the tool stiffness is in $1\,\text{KN} \cdot \text{mm}^{-1}$ and the diameter of the tool is in mm. The work-piece is a rigid aluminium block whose specific cutting energy is chosen to be $k_{\text{el,al}} = 600\,\text{kN} \cdot \text{mm}^{-2}$ and the proportionally factor is taken to be $k_1 = 0.3$, for the tool one, and $k_2 = 0.07$ for the other one. Other expert system parameters are, the stability margin factor, $\delta = 0.05$ and the stability margin factor for the axial depth of cut, $\alpha = 0.95$.

The analytical test for mill cutter selection was conducted using spindle speeds with increments of $1000 \, \text{rpm}$, axial cutting depth started with its minimum value in the stability border line divided by ten, and it is increased in steps of this same size, for a given spindle speed. The operation constrain on the maximum feed per tooth is $0.55 \, \text{mm}$ and the step integration is taken to be $0.05$. The spindle power availability is $745.3\,\text{W}$.

The resultant tool is that leading to the minimum tool cost function value. In figure 6, it is shown the values of tool cost function as $c_1$ parameter varies, the $c_1$ has been taken as a constant $c_1 = 0.075$ and the $c_2$ follow the rule $c_2 = 1 - c_1 - c_3$. This study has been done to illustrate the influence of the $c_1$ parameters in the tool cost function. It is observed the tool $R_1$ has a better behaviour respect to the tool $R_2$ for all possible value of $c_1$ and $c_2$, with $c_1 = 0.075$. Analysis with other values of $c_1, c_2$ and $c_3$, have been carried out and the results are similar, and the tool $R_1$ has a better behaviour. Then, a more general analysis shows in figure 7, in which the minimum value of the tool cost function for all possible combinations of $c_1, c_2, c_3$, with the restriction $c_1 + c_2 + c_3 = 1$ is displayed. The analysis has revealed that the first tool has a better
behaviour than the second one for all combinations of the \( c_i \) parameters. Thus the output of the expert system is the first tool.

For the cutting parameters selection, two steps have been done. First, the cutting parameter corresponding to the minimum of the tool cost function for the selected tool for values of \( c_1 = 0.2, c_2 = 0.725, c_3 = 0.075 \) is obtained. These values are \( q^* = (5800, 0.4924, 0.2722) \).

It can be a well-done first approximation. For a more accurate solution, the tool cost function is evaluated around the above mentioned cutting parameters. Then, another integration is taken into account around \( q^* \).

The test for cutting parameters selection is conducted using spindle speeds between 5700 and 5900 rpm, and a step of 20, axial depth of cut between 0.48 and 0.52 and a step of 0.01, and the feed per tooth between 0.25 and 0.35 and a step of 0.025. The resulted programmed cutting parameters are \( q^{**} = (5780, 0.494, 0.28) \).

Figure 8 shows the situation in the stability lobes of the programmed point \( q^{**} \), the tool displacement and the power consumption. It is observed that the point is robustly stable and the power consumption is less than the power availability in the spindle motor, while acceptable MRR.

This method can be applied to any number of selected tools generating in a automatic task the best one to be used in the system. Moreover, the method can be used to schedule the relative compliance between the available tools and the used work-pieces materials. Finally, the expert system can be used to optimise the manufacturing process, in the sense of planning the adequate sequence of work-pieces to be manufactured for each tool in order to minimise the changes of tools.

5. Conclusion

An efficient approach for mill cutter selection has been developed through an expert system. The expert system is instructed with the characteristics of the candidates tools, as well as with the stability margin and constrains of operations, such as, power availability and robust. Furthermore, a tool cost model function, built from the expert systems preliminary rules, is proposed to evaluate the possible performance of each candidate tool in milling process. This performance index is then used to select an appropriate tool and cutting parameters for the operation which lead to the maximum productivity, while respecting stability and power consumptions margins though optimisation rules. A simulation example which shows the behaviour of the system is presented.

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