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Fault Diagnosis of Rolling Bearings using Multifractal Detrended Fluctuation Analysis and Mahalanobis Distance Criterion

Jinshan Lin, Qian Chen*
State Key Laboratory of Mechanics and Control of Mechanical Structures
Nanjing University of Aeronautics and Astronautics
Nanjing, China
E-mail: q.chen@nuaa.edu.cn

Xiange Tian, Fengshou Gu
Centre for Diagnostic Engineering
University of Huddersfield
Huddersfield, UK
f.gu@hud.ac.uk

***Abstract*—Vibrations of a defective rolling bearing often exhibit nonstationary and nonlinear characteristics which are submerged in strong noise and interference components. Thus, diagnostic feature extraction is always a challenge and has aroused wide concerns for a long time. In this paper, the multifractal detrended fluctuation analysis (MF-DFA) is applied to uncover the multifractality buried in nonstationary time series for exploring rolling bearing fault data. Subsequently, a new approach for fault diagnosis is proposed based on MF-DFA and Mahalanobis distance criterion. The multifractality of bearing data is estimated with the generalized Hurst exponent and the multifractal spectrum. Five characteristic parameters which are sensitive to changes of bearing fault conditions are extracted from the spectrum for diagnosis of fault sizes. For benchmarking this new method, the empirical mode decomposition (EMD) method is also employed to analyze the same dataset. The results show that MF-DFA outperforms EMD in revealing the nature of rolling bearing fault data.**

Keywords-multifractal; detrended fluctuation analysis; Mahalanobis distance; rolling bearing; fault diagnosis

I. INTRODUCTION

Rolling bearings are key parts of mechanical systems and always play crucial roles in mechanical power and motion transmission, which means that render condition monitoring and fault diagnosis of rolling bearings are critically important [1]. Rolling bearings usually work under atrocious conditions and are very complex dynamical systems. In addition, vibration data of defective bearings are ordinarily nonstationary and nonlinear, and contain quite weak fault features. Consequently, the underlying nonlinear dynamical mechanism, which inherently rules complex dynamical systems and is the most fundamental features of complex systems, is normally buried deep in nonstationary vibration data and rather difficult to be uncovered. Therefore, feature extraction of bearing fault data is a fairly intractable issue.

Many methods have been applied to bearing vibration data for detection feature extraction, including statistical parameters [2], envelope analysis [3], spectral kurtosis [4] and different pattern classification methods [5]. Additionally, it is worth specially mentioning that some time-frequency methods, such as wavelet transform (WT) [6] and empirical mode decomposition (EMD) [7], have been used widely in condition monitoring and fault diagnosis of rolling bearings. Nonetheless, when used to analyze complex bearing vibration data, all the previous methods often produce unsatisfactory results because of their own drawbacks [8-10]. Thus, further researches on these methods are in progress.

In 1994, C.K. Peng et al. presented a new method called detrended fluctuation analysis (DFA) for detecting the long-range correlations of DNA sequences [11]. By DFA, the monofractality of nonstationary time series can be unveiled. Recently, E. de Moura et al. have made good attempts to apply DFA to vibration data compression [12, 13]. In [12], DFA was used as a transform tool to compress the gearbox vibration data containing 2048 points into the fluctuation curves of 37 values. The extracted fluctuation curves fully expressed the various working conditions contained in the original gearbox data. Afterwards, in [13], the bearing vibration data containing 4096 points were transformed into the fluctuation curves of 37 values and the extracted fluctuation curves can represent the various conditions of severity of defects in the original bearing data. Consequently, it is proven that DFA may be an excellent tool for data compression nearly without information loss. However, DFA is only a monofractal analysis method and barely able to expose the underlying nonlinear dynamical mechanism hiding in multifractal time series with much fewer parameters. In 2002, multifractal detrended fluctuation analysis (MF-DFA) [14], an extension of the monofractal DFA, was proposed by J.W. Kantelhardt et al. for examining the multifractality of nonstationary time series. Recently, MF-DFA has been applied to condition prediction and recognition of complex chemical systems [15, 16].

In this paper, MF-DFA was used to analyze the nonstationary and nonlinear bearing vibration data and then a new approach to fault diagnosis of rolling bearings was proposed based on MF-DFA and Mahalanobis distance criterion. Primarily, the multifractality of bearing vibration data was quantified with the generalized Hurst exponent and the multifractal spectrum based on MF-DFA. Afterwards, the multifractal spectrum was exploited to characterize different types of bearing fault data. Subsequently, five characteristic parameters were extracted from multifractal spectrum of bearing vibration data and employed to form fault feature sets. In contrast, the EMD was employed in analyzing the same bearing vibration data. Next, Mahalanobis distance was used to classify bearing test data and also measure the abilities of MF-DFA as well as EMD to extract fault features from bearing fault data. The results show that the proposed approach to fault diagnosis of rolling bearings in this paper delivers satisfactory performances. Furthermore, despite only using half the number of characteristic parameter used in EMD, MF-DFA performs far better than EMD in feature extraction of bearing vibration data. Hence, it is deduced that MF-DFA might be a more concise and effective method for feature extraction of bearing vibration data compared with EMD.

II. MULTIFRACTAL DETRENDED FLUCTUATION ANALYSIS *(MF-DFA)*

A. Description of MF-DFA

MF-DFA [14] which is an extension of the monofractal DFA [11] can reveal the multifractality of nonstationary time series effectively. The procedure of executing MF-DFA for a time series x_k with the length N is divided into following five steps [14]:

(1) Define the "profile"

$$Y(i) \equiv \sum_{k=1}^{i} [x_k - \langle x \rangle], \ i = 1,...,N \ . \quad (1)$$

Since the detrending in the third step can eliminate the mean $\langle x \rangle$, the subtraction in this step is not compulsory.

(2) Divide profile $Y(i)$ into $N_s \equiv \mathrm{int}(N/s)$ non-overlapping segments with equal length s. Since the length N of a real series is scarcely an integral multiple of the time scale s, a short part data at the end of the profile may remain unused. To make full use of the profile, the same procedure is performed to divide the profile into segments by starting at the other end of the profile. Therefore, $2N_s$ segments are derived in all.

(3) Use the least-square algorithm to fit the local trend for each of $2N_s$ segments. Then calculate the variance

$$F^2(v,s) \equiv \frac{1}{s}\sum_{i=1}^{s}\left\{Y[(v-1)s+i] - y_v(i)\right\}^2 \quad (2)$$

for the v^{th} segment when $v = 1,...,N_s$ and

$$F^2(v,s) \equiv \frac{1}{s}\sum_{i=1}^{s}\left\{Y[N-(v-N_s)s+i] - y_v(i)\right\}^2 \quad (3)$$

for segments when $v = N_s + 1,...,2N_s$. Here, $y_v(i)$ is the fitting polynomial in the v^{th} segment. Different order polynomials can be used in the fitting procedure (linear, quadratic, cubic, or higher order polynomials are conventionally called DFA1, DFA2, DFA3,…, respectively) [14]. Since the detrending of the time series is done by subtracting the polynomial fits from the profile, different orders of DFA may derive different detrended results in the series. The term of (MF-) DFA*m* (m^{th} order (MF-) DFA) means to eliminate trends of order *m* from the profile. Thus, by attempting different orders of DFA and comparing their results, the type of the polynomial trend in the time series can be determined [14].

(4) The q^{th} order fluctuation function $F_q(s)$ can be obtained by the average of all $2N_s$ segments:

$$F_q(s) \equiv \left\{\frac{1}{2N_s}\sum_{v=1}^{2N_s}[F^2(s,v)]^{q/2}\right\}^{1/q}, \quad (4)$$

where, the index q can be assigned any real value except zero. If $q = 2$, MF-DFA is equivalent to the standard DFA. For different time scales s, steps (2)-(4) will be repeated. Thus, the fluctuation $F_q(s)$ can be obtained as a function of the variables of the time scale s and the index q. In addition, it is emphasized that $F_q(s)$ may relate to the DFA order m and is defined only for $s \geq m+2$ in which m is the order of the polynomial fits.

(5) Analyze the logarithmic relations of $F_q(s)$ and s for each q to determine the scaling behavior of the fluctuation functions $F_q(s)$. If the series x_k are long-range power-law correlations, $F_q(s)$ increases for large values of s as a power-law

$$F_q(s) \sim s^{H(q)} \quad (5)$$

Generally, the exponent $H(q)$ may be dependent on q. For a stationary time series, $H(2)$ is equivalent to the famous Hurst exponent H [14]. Thereby, the function $H(q)$ will be called as the generalized Hurst exponent.

Since the exponent $1/q$ in (4) diverges for $q = 0$, the value $H(0)$ can not be directly determined by (5). To overcome this issue, a logarithmic averaging procedure is used as a substitute for the averaging procedure in (4),

$$F_0(s) \equiv \exp\left\{\frac{1}{4N_s}\sum_{v=1}^{2N_s}\ln[F^2(s,v)]\right\} \sim s^{H(0)} \tag{6}$$

Monofractal time series has the identical scaling behaviors for all segments and thus the corresponding $H(q)$ is independent of q. In contrast, multifractal time series may have very different scaling behavior for different segments and then $H(q)$ greatly depends on q.

For the positive q, the average $F_q(s)$ will be largely dominated by the segments v with large variance (e.g., large deviation from the corresponding fit). In this case, the corresponding $H(q)$ mainly describes the scaling behavior of the segments with large fluctuations which ordinarily hold a small scaling exponent for multifractal time series. However, for the negative q, the average $F_q(s)$ will be largely dominated by the segments v with small variance. In this case, the corresponding $H(q)$ mostly describes the scaling behavior of the segments with small fluctuations which commonly hold a large scaling exponent for multifractal time series [14].

B. Relation between MF-DFA and standard multifractal theory

As the simplest multifractal analysis method, multifractal based on the standard partition function is only suitable for processing normalized stationary time series [17, 18]. In such applications the relations between the generalized Hurst exponent $H(q)$ from MF-DFA and the scaling exponent $\tau(q)$ from the standard partition function can be described as [14]:

$$\tau(q) = qH(q) - 1 \tag{7}$$

Then, through a Legendre transform [19], the corresponding singularity exponent α and multifractal spectrum $f(\alpha)$, which construct a set of parameters for describing multifractal series, can be derived as:

$$\alpha = \tau'(q) = H(q) + qH'(q) \tag{8}$$

$$f(\alpha) = q\alpha - \tau(q) = q[\alpha - H(q)] + 1 \tag{9}$$

C. Physical explanation

In this paper, the five characteristic parameters extracted from multifractal spectrum obtained by MF-DFA have clear physical meanings. The parameter $\alpha_{\max}$ corresponds to the singularity exponent for $q \to -\infty$ and represents the singularity exponent of the smallest fluctuations of time series. On the contrary, the parameter $\alpha_{\min}$ indicates the singularity exponent for $q \to +\infty$ and thus means the singularity exponent of the largest fluctuations of time series. Next, the parameter α_{ext} denotes the singularity exponent for $q = 0$ and then the fractal dimension of the corresponding fractal subsets accurately agrees with the well-known Hausdorff dimension. Also, the singularity spectrum width $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$ hints the strength of multifractality of time series. The larger the value $\Delta\alpha$ the stronger the multifractality of time series is. Additionally, the difference $\Delta f = f(\alpha_{\max}) - f(\alpha_{\min})$ implies the shape characteristics of multifractal spectrum: if $\Delta f > 0$, the multifractal spectrum is a unimodal convex curve with a right hook-like shape; if $\Delta f < 0$, the multifractal spectrum is a unimodal convex curve with a left hook-like shape; and the case $\Delta f = 0$ suggests that the investigated time series is monofractal.

III. MAHALANOBIS DISTANCE CRITERION

Mathematically, Mahalanobis distance is a statistical method for measuring the similarities of two sets of data. Compared with Euclidean distance, Mahalanobis distance considers the correlations between data and is scale-invariant. Mahalanobis distance between a sample $x = (x_1, x_2, ..., x_N)^T$ and the i^{th} group of data with the mean $u_i = [u_{i1}, u_{i2}, ..., u_{iN}]^T$ and the covariance matrix C_i is defined as

$$MD_i = \sqrt{(x-u_i)^T C_i^{-1}(x-u_i)} \ \ and\ i = 1,...,M \tag{10}$$

$$MD_j \le MD_i, i = 1,..., j-1, j+1,...,M \tag{11}$$

then the sample x can be classified as the j^{th} set. Hence, by checking Mahalanobis distances between unknown samples and known sets, it is possible to classify these unknown samples into corresponding classes.

IV. FAULT DIAGNOSIS OF ROLLING BEARINGS

Based on MF-DFA and Mahalanobis distance criterion, a novel method can be proposed for fault diagnosis of rolling bearings. For more clarification, the method can be summarized as follows:

(1) Utilize MF-DFA to analyze bearing vibration data, derive multifractal spectrum and select the characteristic parameters sensitive to changes of bearing fault conditions.

(2) Extract the characteristic parameters from multifractal spectrum of the reference data to form a

baseline feature set and then perform the same processes for testing datasets to form corresponding testing feature sets.

(3) Calculate Mahalanobis distance to identify bearing fault types.

V. APPLICATIONS TO SEVERITY IDENTIFICATIONS OF ROLLING BEARING FAULTS

To confirm the performance of the method, MF-DFA will be utilized to analyze bearing datasets from Bearing Data Center of Case Western Reserve University. The investigated bearing vibration data was sampled at the frequency of 12000Hz and contains of four different sizes of defects on the ball a roller bearing: normal, 0.1778mm, 0.3556mm and 0.5334mm which are referred thereafter as ‘normal’, ‘fault size 1’, ‘fault size 2’ and ‘fault size 3’ respectively. For performing data analysis, each type of data is equally divided into two parts: one for reference data and the other for test data; furthermore, each type of reference data as well as test data is separated into ten equal segments with a length of 6000 points. Fig. 1-4 show a typical sets of the vibration data for four types of conditions respectively. It can be seen that it is difficult to differentiate between them accurately.

Following the steps suggested in section IV. MF-DFA was used to examine the multifractality of bearing vibration data, where the index q in MF-DFA was set to a range from -5 to 5 with a unity increment. Fig. 5 reveals the generalized Hurst exponent for the ten groups of data, where a data group includes four different fault sizes. As revealed in Fig. 5, the generalize Hurst exponent of four types of bearing vibration data greatly depends on the index q, which indicates an occurrence of the multifractality.

Fig. 6 displays the multifractal spectrum for the ten groups of data. It can be seen that the multifractal spectrums of bearing ball fault data with different fault sizes exhibit very different shapes and distributions and extreme positions. Therefore, bearing fault data with different fault sizes can be clearly separated based on these differences. A careful study shows that three characteristic points: the left endpoint, the extreme point and the right endpoint of each multifractal spectrum can give a nearly full description of the changes in shapes, distributions and extreme positions of multifractal spectrum. Thus five feature parameters: $\alpha_{\max}$, $f(\alpha_{\max})$, α_{ext}, $\alpha_{\min}$ and $f(\alpha_{\min})$ were extracted to represent the three characteristic points and used to form fault features, where $\alpha_{\max}$, $\alpha_{\min}$, $f(\alpha_{\max})$ and $f(\alpha_{\min})$ mean the maximum singularity exponent, the minimum singularity exponent, the multifractal spectrum for $\alpha_{\max}$ and the multifractal spectrum for $\alpha_{\min}$, respectively, and α_{ext} indicates the singularity exponent corresponding to the extremum of multifractal spectrum.

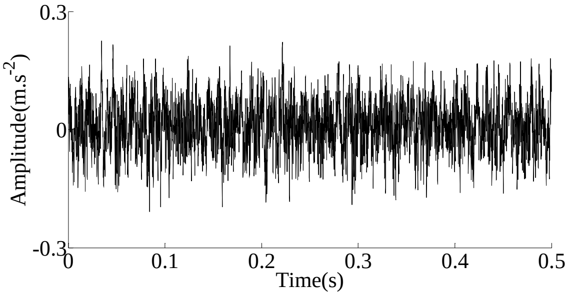

Figure 1. The normal bearing vibration data

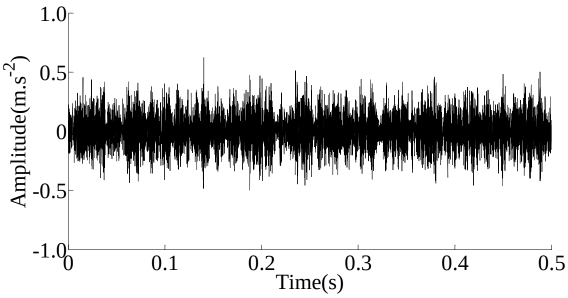

Figure 2. The bearing vibration data with ball fault size 0.1778mm

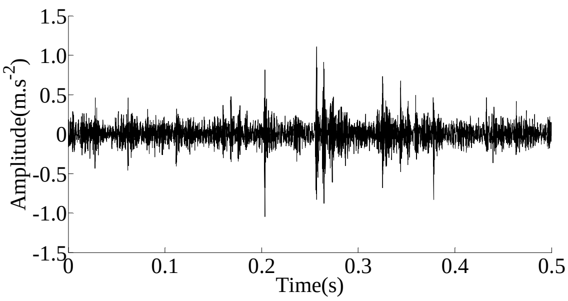

Figure 3. The bearing vibration data with ball fault size 0.3556mm

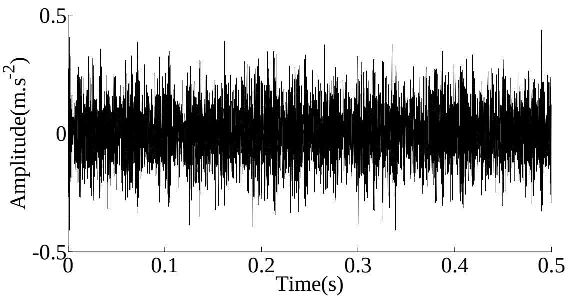

Figure 4. The bearing vibration data with ball fault size 0.5334mm

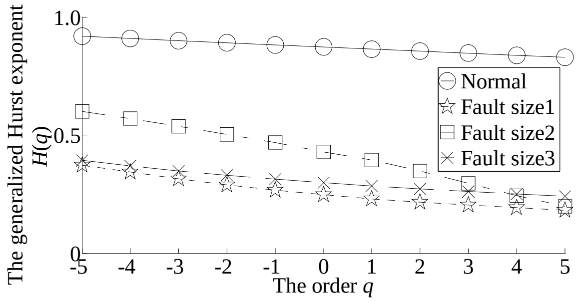

Figure 5 Generalized Hurst exponent curves of bearing vibration data for four defect sizes

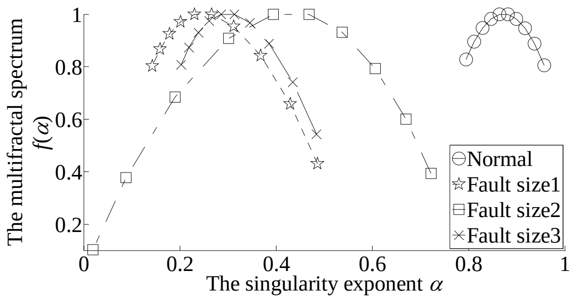

Figure 6 Multifractal spectra of bearing vibration data for four different fault sizes

In the meantime, EMD was also used to explore the same bearing fault data to extract fault features from bearing vibration data. According to the self-adaptive decomposition results of EMD, each type of bearing fault data was uniformly decomposed into ten components from high frequency to low frequency. Thereby, a series x_k can be expressed as

$$x_k = \sum_{j=1}^{10} c_j + r \tag{12}$$

where c_j indicates the j^{th} component of the series x_k and r denotes the global trend of the series x_k. Then, a ten-dimension vector $(e_1, \ldots, e_{10})$ was constructed to represent the fault features of each type of bearing fault data and the i^{th} element of the vector was defined as

$$e_i = c_i^2 \Big/ \sum_{j=1}^{10} c_j^2 \,, i = 1, \ldots, 10 \,. \tag{13}$$

Subsequently, Mahalanobis distance criterion was exploited to classify bearing test data and compare the abilities between MF-DFA and EMD. Table 1 shows comparisons between MF-DFA and EMD for extracting fault features from faulty ball bearing data based on Mahalanobis distance criterion. Clearly, the results from the MF-DFA produce much higher classification rate and give satisfactory performances in fault diagnosis of rolling bearings.

In addition, MF-DFA method uses only 5 feature parameters, which is half the number of characteristic parameters from EMD. This shows MF-DFA is more effective and efficient in feature extraction of bearing fault data.

TABLE I. COMPARISONS BETWEEN MF-DFA AND EMD FOR EXTRACTING FAULT FEATURES FROM BEARING VIBRATION DATA BASED ON MAHALANOBIS DISTANCE CRITERION

Compared terms	Methods for Fault Feature Extraction	
	MF-DFA	***EMD***
Number of characteristic parameter	5	10
Number of test samples	40	40
Number of correctly identified samples	37	21
The correct rate (%)	92.50	52.50

VI. CONCLUSIONS

This paper uses MF-DFA to explore rolling bearing vibration data and has proposed a novel method for fault diagnosis of rolling bearings. It has shown that multifractal spectra for different bearing faults have clear differences which can be characterized by 5 feature parameters. Mahalanobis distance criterion is then used to quantify the difference of the features for fault classification. Results obtained in separating four different conditions of the bearing reach a correct rate as high as 92%. In contrast, EMD based method produces a correct rate of 52%. Therefore, it is concluded that MF-DFA might be essentially a more concise and effective method for feature extraction in bearing vibration data.

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