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Comparison of Robust Filtration Techniques in Geometrical Metrology

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Abstract— **Filtration is one of core elements of analysis tools in geometrical metrology. Filtration techniques are progressing along with the advancement of manufacturing technology. The modern filtration techniques are requested to be robust against outliers, applicable to surfaces with complex shape and reliable in whole range of measurement data. A comparison study is conducted to evaluate commonly used robust filtration techniques in the field of geometrical metrology, including the two-stage Gaussian filter, the robust Gaussian regression filter, the robust spline filter and morphological filters. They are compared in terms of four aspects: functionality, mathematical computation, capability and characterization parameters. As a result, it offers metrologists an instruction to choose the appropriate filter for various applications.**

Keywords-Geometrical metrology; robust Gaussian regression filter; robust spline filter; morphological filters

I. INTRODUCTION

Filtration is one of core elements of analysis tools in geometrical metrology. It is the means by which the information of interest is extracted from the measured data for further analysis [1]. For instance, filtration techniques are employed in surface metrology to separate the roughness component from the waviness component and the form component so that suitable characterization parameters can be derived aiming to control the manufacturing process [2]. They also serve in dimensional metrology for data smoothing. In such a manner, noises are removed by filters before fitting routines are applied to figure out the geometrical shape for dimensional evaluation. Filtration techniques are progressing along with the advancement of manufacturing technology, by which functionally and geometrically complicated surfaces are produced as requested by modern products. In response to these new features, filtration techniques are requested to be robust against outliers, applicable to surfaces with complex shape and reliable in whole range of measurement data. These motivation brings out a set of robust filtration techniques, most of which are presented in ISO 11610 [3], including the robust Gaussian regression filter, the robust spline filter and morphological filters.

Although these robust filters are detailed in ISO standards and other research literatures, differences in their usages and capabilities are not fully recognized and clearly stated yet. As a result they are confusing for metrologists and users to choose the correct type for surface assessment. The paper sets out to carry out a comparison study of robust filtration techniques, which are commonly used in geometrical metrology. The paper is structured in the following fashion. Section II presents a brief review of filtration techniques. Section III gives an introduction to four specific robust filters, i.e. the two-stage Gaussian filter, the robust Gaussian regression filter, the robust spline filter and morphological filters. A thorough comparison of these filters is conducted in Section IV in four aspects: functionality, mathematical computation, capability and characterization parameters. Finally Section V gives the conclusion.

II. FILTRATION TECHNIQUES

ISO 16610 presents a category of modern advanced filtration techniques encompassing linear filters, robust filters, morphological filters and segmentation filters. It provides a powerful and useful toolbox of filtration techniques, allowing metrologists to analyze various surface textures. Most of these filters could date back to two traditional filtration systems emerged in 1950s, i.e. the mean-line based system (M-system) and the envelope based system (E-system).

The Gaussian filter is a typical mean-line based filter, which is a convolution operation of the surface under evaluation and the Gaussian weighting function. The reference line generated from the Gaussian filter is called as the mean line due to the fact that the profile portions above and below the reference line are equal in the sum of their areas. Acting totally differently, the envelope filter is obtained by rolling a disk over the profile and the covering envelope formed by the rolling disk is viewed as the reference profile. The E-system gains its basis from the simulation of the contact phenomenon of two mating surfaces, whereby peak features of the surface play a dominant role in the interaction operation. There have been some arguments between these two systems in terms of their capability and superiority [4], however the facts proved that they are complement to each other rather than compete against each other and none of them can fulfill all the practical demands by themselves alone [5].

Motivated by the advancement of cutting-edge manufacture technology and also driven by modern product design intents and functional requirements, more sophisticated surfaces emerge. They not only are incredibly smooth, but also have the specification of surface form at levels approaching atomic magnitude, for instance, optics in ground- and space-based telescope [6]. In response to these advancements, filtration techniques as core tools for surface assessment are motivated to be enhanced in their capability and performance.

The M-system was greatly enriched by incorporating advanced mathematical theories. The Gaussian regression filter overcame the problem of end distortion and poor performance of the Gaussian filter in the presence of significant form component [7], while the robust Gaussian regression filter solved the problem of outlier distortion in addition [8, 9]. The spline filter is a pure digital filter, more suitable for form measurement [10]. Based on L_p norm, the robust spline filter is insensitive with respect to outliers [11, 12].

In the meanwhile the E-system also experienced significant improvements. By introducing mathematical morphology, morphological filters emerged as the superset of the early envelope filter but offering more tools and capabilities [13]. The basic variation of morphological filters includes the closing filter and the opening filter. They could be combined to achieve superimposed effects, referred as alternating symmetrical filters (ASF). A sequence of ASF leads to scale-space techniques [14].

III. ROBUST FILTRATION TECHNIQUES

In ISO 16610-21, robustness is defined as the insensitivity of the output data against specific phenomena (outliers, scratches and steps etc) in the input data. An example of such kind of data is the inner surface of the cylinder liner which has functional stratified properties. This kind of surfaces is composed of deep valleys superimposed by plateaux. The plateaux support force bearing and friction while the valleys serve as lubricant reservoirs and distribution circuits. The standard phase corrected Gaussian filter [15] is unable to generate a reasonable reference line. As illustrated in Figure 1, the mean line of a cylinder line profile yield by the Gaussian filter (cut-off wavelength 0.8 μm) tends to drop down toward the valleys. This subsequently distorts the evaluation of roughness component of the surface texture which could be clearly detected in the figure. Robust filters should overcome this shortcoming.

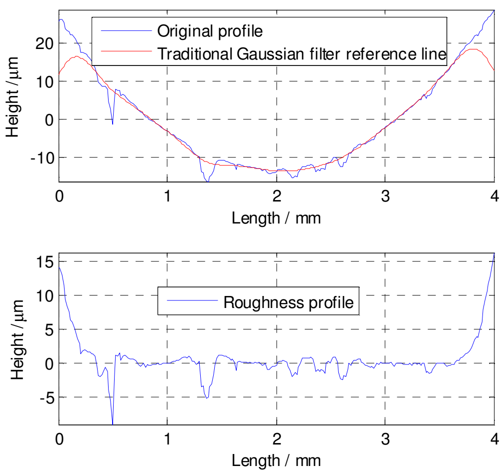

Figure 1. The reference profile and roughness profile obtained by the standard Gaussian filter.

Another noticeable issue concerning with the robust filter, although not stated by ISO standard, is the ability for form measurement. Robust filters should be able to deal with surfaces with large form components. With the enhancement of measurement capability of current instruments, there is a trend that the measured data consists of both the dimensional information (size, form, etc.) and that of the surface texture, which traditionally are measured in a separate manner. Thus the ability of separating two differing components in such a combined data set is of great importance.

A. *Two-stage Gaussian filter*

The two-stage Gaussian filter, presented by ISO 13565 [16], is an empirical approach for the analysis of functional stratified surfaces. At the first stage, a standard Gaussian filter is applied on the measured profile to gain a mean line. All the profile portions that lie under the mean line are removed and replaced by the mean line itself. The modified profile is then filtered by the same Gaussian filter again to obtain a second mean line, which is referred as the final reference line for the assessment of roughness. Although this method is effective in certain cases, it has a couple of limitations. Firstly, it was derived from empirical foundation with a significant assumption: surface contains a relative small amount of waviness, which is ambiguous and confusing. Secondly, running-in and running-out sections are generated from the Gaussian filter. These sections truncate the profile and only 20%-60% of the measurement data are used in evaluation [17].

B. *Robust Gaussian regression filter*

The traditional standard (linear) Gaussian filter can be described by the following minimization problem:

$$\int_{-\infty}^{+\infty} (z(\xi) - w(x))^2 \cdot s(\xi - x) d\xi \Rightarrow \underset{w(x)}{Min}, \qquad (1)$$

where

$z(x)$ is the input measured profile,

$s(x)$ is the Gaussian weighting function $\frac{1}{\alpha\lambda_c} e^{-\pi\left(\frac{x}{\alpha\lambda_c}\right)^2}$

with λ_c the cutoff wavelength,

$w(x)$ is the output reference profile.

The reference profile $w(x)$ can be solved as:

$$w(x) = \int_{-\infty}^{+\infty} z(\xi) s(x - \xi) d\xi = z(x) \otimes s(x), \qquad (2)$$

which in essence is a convolution operation over the interval $-\infty \le x \le +\infty$. The Gaussian weighting function has the same shape at each data point.

For the linear second order Gaussian regression filter, the minimization problem is given by

$$\int_0^l \left(z(\xi) - \beta_1(x)(\xi - x) - \beta_2(x)(\xi - x)^2 - w(x)\right)^2 \cdot s(\xi - x) d\xi \Rightarrow \underset{w(x), \beta_1(x), \beta_2(x)}{Min}, \qquad (3)$$

where a second order polynomial curve $\beta_1 x + \beta_2 x^2 + w$ is employed in order to remove the form component of the measured profile $z(x)$ with β_1 and β_2 being the polynomial coefficients. This procedure is evaluated at each sampling point over the whole length of the measured profile $[0, l]$.

For the robust (non-linear) second order Gaussian regression filter, a robust statistical estimate $\rho(x)$ is employed as the vertical weighting function aiming to eliminate the distortion caused by the outliers and abrupt features. Then the minimization problem changes to:

$$\int_0^l \rho\left(z(\xi) - \beta_1(x)(\xi - x) - \beta_2(x)(\xi - x)^2 - w(x)\right) \cdot s(\xi - x)d\xi \quad . \quad (4)$$
$$\Rightarrow \underset{w(x),\beta_1(x),\beta_2(x)}{Min}$$

$\rho(x)$ is also called as the error metric function. A commonly used error metric function is the Tukey estimator:

$$\rho(x) = \begin{cases} {c^2}/{6}\left(1 - \left({x}/{c}\right)^2\right)^3 & if\ |x| \le c \\ {c^2}/{6} & if\ |x| > c \end{cases} \quad . \quad (5)$$

where c is a constant equal to $4.4478 * Median\left(|z - w|\right)$. The computation is an iterative procedure which terminates when the deviation of c is within the given tolerance.

Figure 2 presents an example of applying the second order Gaussian regression filter on the cylinder liner profile with the cut-off wavelength 0.8 µm. The reference profile is subtracted from the raw measured profile to generate the roughness profile.

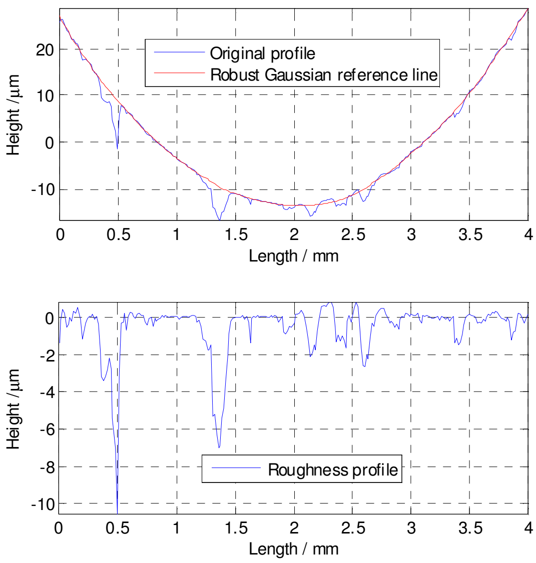

Figure 2. The reference profile and roughness profile obtained by the robust Gaussian regression filter.

C. Robust spline filter

In contrast to the Gaussian filter, the spline filter is a pure digital filter. It is specified by the filtration equation instead of the weighting function. The reference line resulted from the filter is a spline which could be described by a constrained optimization problem: find $w(x)$ minimizing the square of residual errors $\sum_{i=1}^{n}\left(z_i - w(x_i)\right)^2$ under the condition of minimizing the bending energy of spline $\int_{x_1}^{x_n}\left(\frac{d^2 w(x)}{dx^2}\right)^2 dx$ [18], i.e.

$$\sum_{i=1}^{n}\left(z_i - w(x_i)\right)^2 + \mu\int_{x_1}^{x_n}\left(\frac{d^2 w(x)}{dx^2}\right)^2 dx \rightarrow \underset{w(x),\mu}{Min} \quad (6)$$

with μ the Lagrange coefficient.

The solution of this optimization problem leads to a filtration equation:

$$(I + \alpha^4 Q)w = z \quad (7)$$

where

$$Q = \begin{pmatrix} 1 & -2 & 1 & & & & \\ -2 & 5 & -4 & 1 & & & \\ 1 & -4 & 6 & -4 & 1 & & \\ & & \ddots & \ddots & \ddots & \ddots & \\ & & 1 & -4 & 6 & -4 & 1 \\ & & & 1 & -4 & 5 & 2 \\ & & & & 1 & -2 & 1 \end{pmatrix} \quad (8)$$

$$\alpha = \frac{\Delta x^3}{16\sin\left(\frac{\pi\Delta x}{\lambda_c}\right)} \quad (9)$$

with Δx sampling interval and I is the identity matrix.

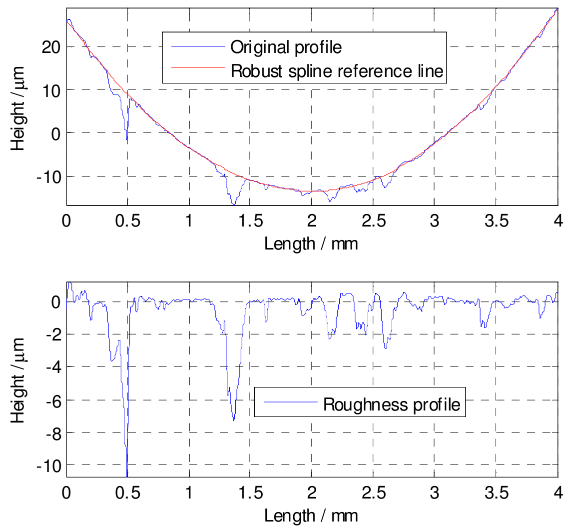

Figure 3. The reference profile and roughness profile obtained by the robust spline filter.

Similar to the robust Gaussian filter, the robust spline filter integrates the robust statistic estimate function $\rho(x)$, for example the Tukey function, thus the optimization problem of the spline filter turns to:

$$\sum_{i=1}^{n} \rho\left(z_i - w(x_i)\right)^2 + \mu \int_{x_1}^{x_n} \left(\frac{d^2 w(x)}{dx^2} \right)^2 dx \rightarrow \underset{w(x)}{Min}, \quad (8)$$

which afterwards lead to the filtration equation

$$(\Delta + \alpha^4 Q) w = \Delta z, \quad (9)$$

where Δ is the diagonal matrix of vertical weights.

Figure 3 demonstrates the reference line generated by the robust spline filter applying on the cylinder liner profile employed in the example aforementioned. The cut-off wavelength is also 0.8 µm.

D. Morphological filters

The closing filter and the opening filter are two primary types of morphological filters. As illustrated in Figure 4, the closing filter is obtained by placing an infinite number of identical disks in contact with the profile from above along all the profile and taking the lower boundary of the disks [14]. On the contrary the opening filter is archived by placing an infinite number of identical disks in contact with the profile from below along all the profile and taking the upper boundary of the disks. Alternating symmetrical filters are the combination of openings and closings with the same structuring element, which will suppress both peaks and valleys.

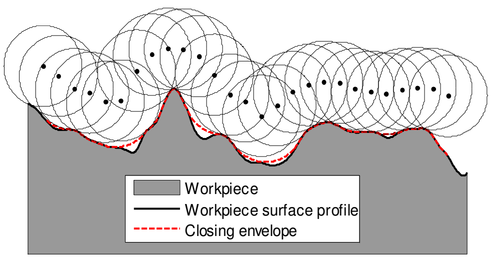

Figure 4. Closing envelope obtained by rolling a disk over the

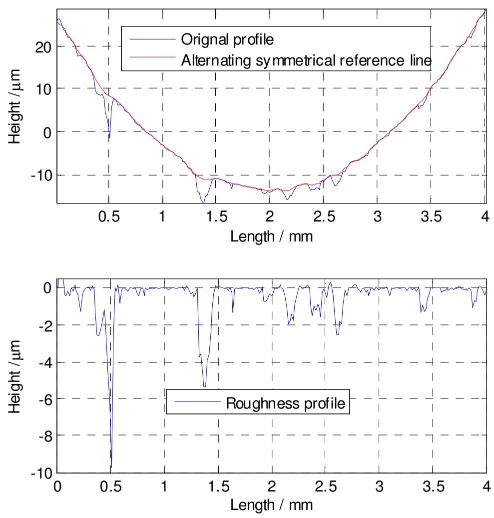

Figure 5. The reference profile and roughness profile obtained by the morphological alternating symmetrical filter.

Figure 5 presents the reference line obtained by applying the alternating symmetrical filter with disk radius 5 mm on the cylinder liner profile data. It should be mentioned that the closing operation is applied before the opening operation. The resulting reference profile basically follows the form of the closing envelope. Thus it suits for surfaces where valley features play a dominant role.

IV. COMPARISON STUDY

A. Functionality

From a functionality oriented point of view, the Gaussian filter and the spline filter are more suitable for monitoring the manufacturing condition. They are specified by the cut-off wavelength. Analyzing frequency contents of the data set is reasonable because the vibration of the machine and the tool wear will cause the corresponding frequency changes in the surface texture. The robust variation of these filters provides more powerful tools in analyzing complex surfaces. In contrast, morphological filters are believed to give better results in the functional prediction in that they are more relevant to geometrical properties of the surface itself which is critical in contact phenomenon and optical reflection. However the above statement is not absolute. Malburg [19] presented a good example whereby the spline filter and the morphological closing filter are employed to simulate the conformable interface of two surfaces of a solid block and a sealing soft gasket, respectively.

B. Mathematical computation

The standard Gaussian filter is a convolution operation of the input surface and the Gaussian weighting function. The robust Gaussian regression filter enhances it in three aspects. Firstly, it incorporates the robust statistic estimate method as the vertical weighting function, supplementing the Gaussian weighting function in the horizontal direction. Secondly, a polynomial function with certain order is introduced to approximate the form component of the surface under evaluation. It could eliminate the distortion of the standard Gaussian filter of which the polynomial curve is zero order, namely the surface is planar. Finally, the weight in both horizontal and vertical direction are normalized, which means the convolution operation at end regions could be calculated without padding extra zeros or truncating the surface.

In constant to the Gaussian filter, the spline filter is a purely digital filter specified by the filtration equation instead of the weighting function. It is a constrained optimization problem, which could be switched to an unconstrained problem by means of the Lagrange method. In a similar vein to the robust Gaussian filter, the robust statistical estimate techniques could be employed to offer the ability to deal with outliers in the data.

Morphological filters lay their basis on mathematical morphology. Morphological operation is the convolution of the sets [13], i.e. the input set and the structuring element set. The traditional algorithm, acting in a similar manner to image processing, is implemented on the basis of the set convolution [20]. Nevertheless there are more capable and efficient computational methods available for use [21].

C. Capability

Aiming to thoroughly evaluate the capability of these robust filters, they are examined in terms of following five factors: end distortion, robustness to outliers, form filtering, non-uniform sampling data filtering and roundness (closed profile) filtering. See TABLE I. In terms of end distortion in the filtration of open surfaces, the two-stage Gaussian filter suffers from data truncation at two ends of the profile in each filtering process, while the robust Gaussian regression filter and robust spline filter behave well in this aspect. Morphological filters experience the end distortion to some degree, but not as serious as the Gaussian filter. The two-stage Gaussian filter is an empirical method to handle the data with outliers. However, strictly, the two-stage Gaussian filter is not a real robust filter since it relies on the standard Gaussian filter and therefore inherits its shortcomings. The robust Gaussian filter, as the enhanced version of the standard Gaussian filter by embedding the statistical weighting function in vertical direction, is robust against outliers. Same for the robust spline filter. As to morphological filters, the primary filters (the closing filter and opening filter) are partially robust against outliers in that the closing filter only suppresses valley features and the opening filter only removes peak features. Alternating symmetrical filters, being the combination of the closing filter and opening filter, are naturally robust against both valleys and peaks.

The other three factors are concerned with the dimensional measurement. The two-stage Gaussian filter, based on the traditional Gaussian filter, is not suitable for form filtering because its reference line will be distorted by the large form component. The high order Gaussian regression filter could approximate the form component using the high order polynomial fitting in a least square manner. The spline filter originates from the form of a flexible natural cubic spline under the load of the measured profile [10]. Therefore it could handle most of surfaces in form measurement. Morphological filters are more straightforward in this aspect since it simulates rolling a ball over the surface without considering whatever the surface being rolled is. With respect to non-uniform sampling data, both the two-stage Gaussian filter and the robust Gaussian regression filter are unable to deal with this kind of data. On the contrary, the spline filter and morphological filters are applicable if the appropriate algorithms are taken [18, 22]. The standard Gaussian filter could be modified to handle the roundness data. However in contrast to the robust spline filter, there is no robust Gaussian filter for closed profile [23]. Theoretically morphological filters could apply to the roundness (closed profile) data. Recently a novel implementation of morphological filters based on the alpha shape algorithm offers the possibility in dealing with roundness data [22].

D. Parameters

It is of interest to compare the results of characterization parameters of various robust filtration techniques. Figure 6 presents the superposition of three reference profiles obtained by the robust Gaussian regression filter, the robust spline filter and the morphological alternating symmetrical filter. TABLE II lists the arithmetical mean deviation R_a and the root mean square deviation R_q of the roughness profile respectively. It is evident that these obtained values differ in their values, which could be confusing. However it is the change of these values, not their absolute values, reflects the changes in manufacturing process. Thus the best filter is the one which is most accurate in capturing the change of manufacturing condition.

There is an extra merit brought by morphological filters. The morphological closing envelope and opening envelope with the flat square structuring element can help to compute the fractal dimension, which serves as an indicator to the geometric complexity or intricacy components of a fractal or partially fractal surface [24].

TABLE I. CAPACITY COMPARISON

Filter	two- stage Gaussian filter	Robust Gaussian regression filter	Robust spline filter	Morphological filters
End distortion	Yes	No	No	Yes
Robustness to outliers	Partial	Yes	Yes	C/O: partial ASF: Yes
Form filtering	No	Yes	Yes	Yes
Non-uniform	No	No	Yes	Yes
Roundness	No	No	Yes	Yes

TABLE II. PARAMETER COMPARISON

Filter	Robust Gaussian regression filter	Robust Spline filter	Morphological ASF
R_a	0.89 μm	0.93 μm	0.68 μm
R_q	1.45 μm	1.50 μm	1.16 μm

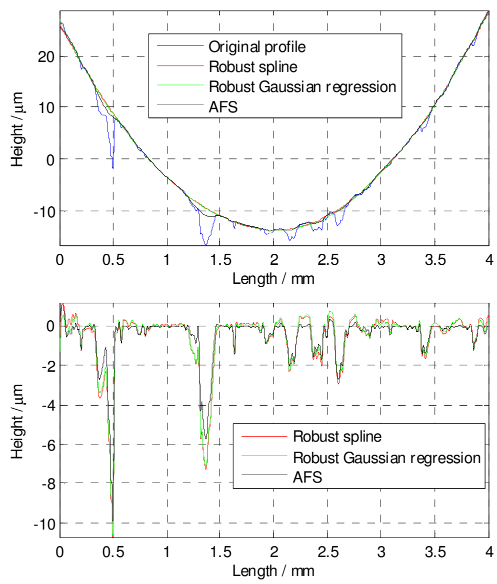

Figure 6. The reference profile and roughness profile obtained by three robust filters.

V. CONCLUSION

Filtration techniques are motivated by the demands in analyzing modern complicate surfaces produced by the cutting-edge manufacturing technology. The advanced robust filters are superior over their predecessors with more capabilities and better performances. This paper presents a comparison study on existing popular robust filters, consisting of the two-stage Gaussian filter, the robust Gaussian regression filter, the robust spline filter and morphological filters. The mechanisms of these filters are discussed in a brief manner, while more work dedicates to the comparison of these filters in terms of functionality, mathematical computation, capability and characterization parameters. As a result, it offers metrologists an instruction to choose the appropriate filter for various applications. It could be foreseen that more filtration techniques will appear by incorporating advanced mathematical tools as a result of the stimulation of the advancement of modern manufacture technologies, metrologists should more carefully compare the usages of these analysis tools and choose the correct type.

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