Enhancement detection of characteristic signal using stochastic resonance by adding a harmonic excitation

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Abstract. For a bistable nonlinear system, deterministic and stochastic excitations play equivalent roles in promotion of chaos according to qualitative results of Melnikov theory. When a bistable system maintains the state of stochastic resonance (SR), the output of system is chaotic, and the most effective spectral shape is obtained when the output power is distributed closet to the frequency of the Melnikov scale's peak. In classical SR, improvement of the signal-to-noise ratio (SNR) is achieved by increasing the noise intensity, but this approach may be unwieldy. Instead of it in this paper, the more effective SNR enhancement is achieved by adding a harmonic excitation with frequency based on the system's Melnikov scale factor to the system while the noise is left unchanged. The effectiveness of this method is confirmed and replicated by numerical simulations. Combined with the strategy of scale transform, the method cab be used to detect weak periodic signal with arbitrary frequency buried in the heavy noise. At last, the method for enhancement detection of machinery fault characteristic signal is discussed via a case data.

Keywords. Stochastic resonance, Chaotic dynamics, Weak signal detection, Machinery fault detection

1. Introduction

It is well known that the evaluation of the dynamic behavior of the machinery solely depends on the quality of measured signals. All the influence factors, including transmission path, transmission medium, ambient environment, etc., degrade the signal to noise ratio (SNR) of measurement. As an extreme case, the useful information is wholly buried in the noise and hardly recovered. Therefore, early signatures of possible potential faults in an operating machine may be lost entirely. In some occasions, such as the transverse crack of flexible rotors, rotor-stator rub-impact of rotors and the failure of the high-speed gearboxes, etc., it is extraordinarily important to catch the symptoms as early as possible to prevent the catastrophic failures of critical machinery. However, the early symptoms of potential faults are not always detectable. They are ambiguous and may be overlooked. In these cases, when the catastrophic failures occur, they are generally considered unforeseen and unpredictable [1].

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Thus, the importance of detecting the weak signals is quite obvious. These weak signals sometimes are symptoms of machine faults. One of the key points in machinery diagnostics is how to catch the symptoms of machine failure as early as possible. At present, one strategy to attain this object is based on measuring means and hardware. Another strategy is started with signal processing technique and software, namely, to apply more progressive processing methods. The chaotic resonator was recommended for weak signal detection [2,3]. Furthermore, several approaches have been applied for weak signal detection, such as wavelet analysis, holospectral analysis, high order statistics, Hilbert-Huang transform and so on [4].

In this paper, a method based on extended stochastic resonance theory to detect weak signal will be presented, which can detect a weak signal in the presence of heavy noise. The principle and property of classical SR have been illustrated in [5-7]. In classical SR, the SNR can be improved by increasing the noise. But the approach by increasing the noise is counterintuitive and unwieldy. According to Melnikov theory, for a wide class of systems, deterministic and stochastic excitations play qualitatively equivalent roles in inducing chaotic motions with escapes over a potential barrier. Such motions therefore possess common qualitative features that suggest the extension of SR approaches beyond classical SR, so that the SNR can alternatively be improved by keeping the noise unchanged and adding a deterministic excitation which is close to the detected signal and selected in accordance with Melnikov theory, rather than by increasing the noise. We can denote this process to be extended stochastic resonance.

2. Principle of SR explained by chaotic dynamic approach

As stated in [8, 9], for a bistable system with noise and a periodic signal, the improvement of the signal-to-noise ratio (SNR) achieved by increasing the noise intensity is known as stochastic resonance (SR) [10,11] (i.e., classical SR in these papers). Here, The signal to noise ratio (SNR) is expressed in dB as $SNR=10\log10(S/N)$, where S and N are, respectively, the ordinate of the output power spectrum and the ordinate of the broadband output power spectrum at the signal frequency ω_0. Consider the motion in a bistable double-well potential of a lightly damped particle subjected to stochastic excitation and a harmonic excitation with low frequency ω_0. The signal is assumed to have small enough amplitude that, by itself (i.e., in the absence of the stochastic excitation), it is unable to move the particle from one well to another. We denote the characteristic rate, that is, the escape rate from a well under the combined effects of the periodic excitation and the noise, by $\alpha = 2\pi n_{tot} / T_{tot}$, where n_{tot} is the total number of exits from a well during time T_{tot}. We consider the behavior of the system as we increase the noise while the signal amplitude and frequency are unchanged. For zero noise, $\alpha = 0$, as noted earlier. For very small noise we have $\alpha < \omega_0$. As the noise increases, the ordinate of the spectral density of the output noise at the frequency ω_0, denoted by $\Phi_n(\omega_0)$, and the characteristic rate α increase. Experimental and analytical studies show that, until $\alpha \approx \omega_0$, a cooperative effect (i.e., a synchronizationlike phenomenon) occurs wherein the signal output power $\Phi_s(\omega_0)$ increases as the noise intensity increases. Remarkably, the increase of $\Phi_s(\omega_0)$ with noise is faster than that of $\Phi_n(\omega_0)$. This results in an enhancement of the SNR. The synchronizationlike phenomenon plays a key role in the mechanism just described.

We consider second-order dynamical systems described by the equation [8]

$$\ddot{x}(t) = -\beta\dot{x}(t) - V'(x) + G(t) \tag{1}$$

where $V(x)$ is a potential function. The unperturbed counterpart of equation (1) is the Hamiltonian system expressed by $\ddot{x} = -V'(x)$. We assume that $V(x)$ is a double-well potential (Duffing-Holmes)

$$V(x) = -\frac{1}{2}x^2 + \frac{1}{4}x^4 \tag{2}$$

$\ddot{x} = -V'(x)$ with the potential (2) has the homoclinic orbits [12].

Firstly, it is assumed that the excitation is only periodic, that is, in equation (1) $G(t) \equiv A_0 \sin(\omega_0 t)$. According to the Smale-Birkhoff theorem, the necessary condition for the occurrence of chaos is that the Melnikov function induced by the perturbation have simple zeros. For Duffing system this condition is the Melnikov inequality

$$-(4/3)\beta + A_0 S_M(\omega_0) > 0 \tag{3}$$

where

$$S_M(\omega) = \sqrt{2}\pi\omega \operatorname{sech}(\pi\omega/2) \tag{4}$$

is a system property known as the Melnikov scale factor [13]. For the Duffing oscillator $S_M(\omega)$ is shown in figure 1.

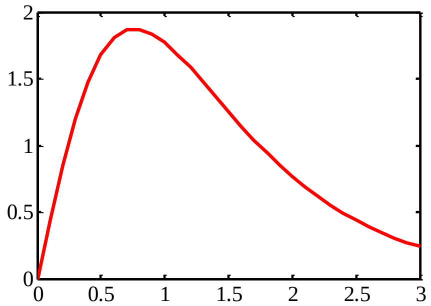

Figure 1. Melnikov scale factor $S_M(\omega)$ for double-well potential.

Secondly, assume that the excitation consists of the quasiperiodic sum

$$G(t) \equiv A_0 \sin(\omega_0 t + \phi_0) + A_a \sin(\omega_a t) + \sum_{k=1}^{K} a_k \sin(\omega_k t + \varphi_k) \tag{5}$$

For this case a generalization of the Smale-Birkhoff theorem [13] yields the Melnikov inequality as the necessary condition for chaos

$$-\frac{4\beta}{3} + A_0 S_M(\omega_0) + A_a S_M(\omega_a) + \sum_{k=1}^{K} a_k S_M(\omega_k) > 0 \tag{6}$$

Finally, assume that the system's excitation is

$$G(t) \equiv A_0 \sin(\omega_0 t + \phi_0) + A_a \sin(\omega_a t) + \sqrt{2D\beta} R(t) \tag{7}$$

where $R(t)$ is a Gaussian process with unit variance and spectral density $g(\omega)$. Over any finite time interval, however large, each realization of the process $R(t)$ may be approximated as closely as desired by a sum [8]

$$R_N(t) = \sum_{k=1}^{K} b_k \sin(\omega_k t + \varphi_k) \tag{8}$$

so that the Melnikov inequality, that is, the necessary condition for chaos, can be written as in equation (6) where $a_k = \sqrt{2D\beta} b_k$. In formula (8), $b_k = \sqrt{g(\omega_k)\Delta\omega}$, φ_k are randomly chosen phases of uniform distribution on the interval $[0, 2\pi]$ and $\omega_k = k\Delta\omega$, $\Delta\omega = \omega_{max}/K$, ω_{max} is the frequency beyond which the spectrum vanishes (the cutoff frequency).

For the damped, forced system, the existence in a plane of section of a transverse point of intersection between the stable and unstable manifolds implies the existence of an infinity of intersection points. Eventually, they may form a chaotic motion under a particular excitation. The strength of the chaotic transport, and therefore the characteristic rate α, increases as the left-hand side of in equation (6) becomes larger [13]. This is true regardless of whether the excitation is deterministic or stochastic. Moreover, again regardless of whether the excitation is deterministic or stochastic, a qualitative feature of the chaotic motions featuring escapes is that their spectral densities have a broadband portion with significant energy content at and near the system's characteristic rate α. Thus, we expect that we can build a bridge between chaos and stochastic resonance. That is to say, we can explain SR phenomena by chaotic dynamics approach.

Assume that the excitation is a sum of a harmonic signal and an added harmonic, that is, in equation (1) $G(t) \equiv A_0 \sin(\omega_0 t) + A_a \sin(\omega_a t)$. The system is therefore deterministic with, in general, quasiperiodic excitation. The necessary condition for chaos is given by in equation (6) in which $a_1 = a_2 = \cdots = a_K = 0$. We choose A_0 so that, for $A_a = 0$, the motion is confined to one well. In accordance with Melnikov theory this will be the case if the Melnikov inequality given by in equation (3) is not satisfied. We now add the excitation $A_a \sin(\omega_a t)$. For a certain region R_a of the parameter space $[A_a, \omega_a]$, the system can experience chaotic motion with jumps over the potential barrier. The Melnikov scale factor $S_M(\omega)$ provides the information needed to select frequencies ω_a such that the added excitation is effective in inducing chaotic behavior. In general, ω_a should be equal or close to the frequency for which $S_M(\omega)$ is largest (Figure 1).

Given the existence in the spectrum of a broadband portion qualitatively similar to that present in the case of classical SR, it is reasonable to expect that the synchronizationlike phenomenon that occurs in the classical SR case would similarly occur for the deterministically excited chaotic system. This was verified by numerical simulation for a large number of cases. As a typical example, we choose the same case as [8] for $\beta = 0.316$, $A_0 = 0.095$, $\omega_0 = 0.0632$ (for these values in equation (3) is not satisfied) and $\omega_a = 1.1$. Spectral densities of motions with these parameters and A_a=0.263, 0.287, and 0.332, are shown in figures 2, 3 and 4, respectively. For figure 3, $\alpha = 0.0671$ is close to the signal frequency $\omega_0 = 0.0632$. The energy in the broadband portion of the spectrum is reduced, while the energy at the signal's frequency is enhanced, with respect to their respective counterparts in figure 2 and figure 3, for which $\alpha = 0.0518$ and $\alpha = 0.1611$, respectively. The synchronizationlike phenomenon noted for classical SR is thus clearly evident in figure 3. We also noticed that the motions of figures 2, 3, and 4 are indeed chaotic by observing figures 2(a), 3(a) and 4(a). That is to say, the additive harmonic signal plays the same role as the noise in enhancement of SNR.

3. Simulation analysis for Enhance detection of weak signal by adding a harmonic excitation

3.1 weak signal with low frequency

Notice that the larger the left-hand side of in equation (6), the stronger is the chaotic transport, and therefore the larger is the rate α. Let $A_a = 0$, $a_k = \sqrt{2D\beta}\sqrt{g(\omega_k)\Delta\omega}$ in equation (6). It is therefore clear from in equation (6) that for any given power of the stochastic excitation $2D\beta$, the left-hand side of in equation (6) becomes larger and the rate α increases. We thus obtain the interesting qualitative result that, for a given Melnikov scale factor $S_M(\omega)$ and a given power of the stochastic excitation, the rate α increases as the spectral power of the excitation is distributed nearer to the frequency of $S_M(\omega)$'s peak, ω_{pk} (the greatest effectiveness being achieved by a single component with frequency equal or close to ω_{pk}).

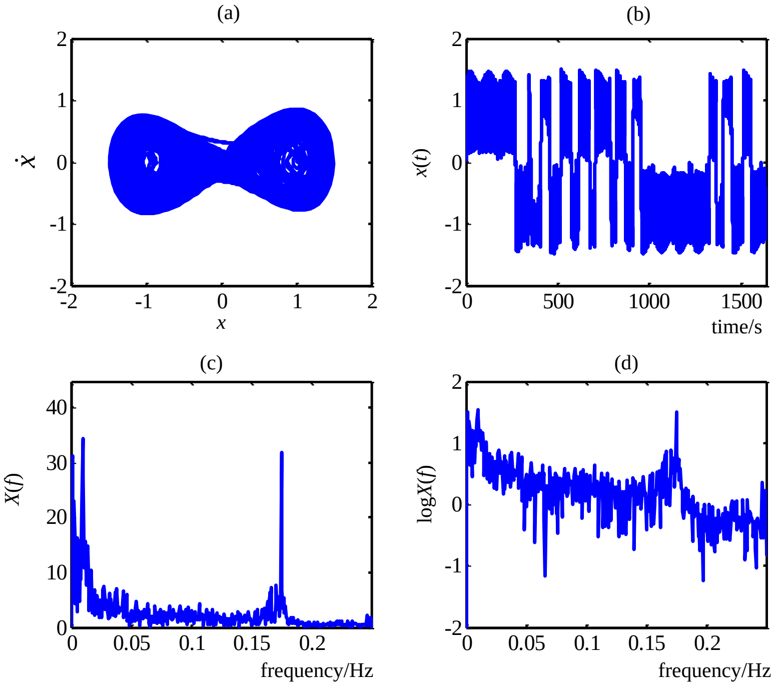

Figure 2. Amplitude spectra of system with D=0. (a) Phase plane orbit of system output. (b) Waveform of system output. (c) Amplitude spectrum of system output. (d) Logarithmic spectrum of system output. A_0 and ω_0 kept constant. The system is subjected to an additional harmonic excitation with $\omega_a = 1.1$ and $A_a = 0.263$.

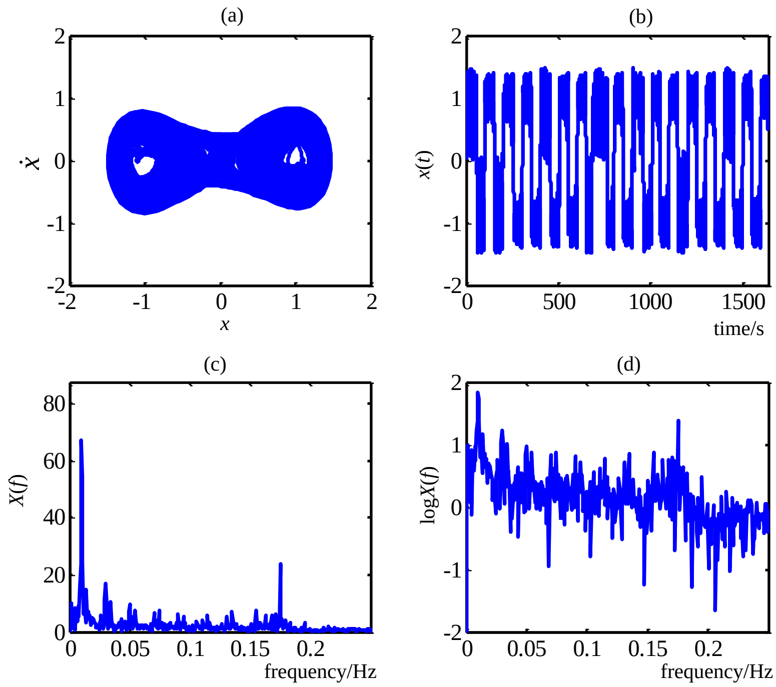

Figure 3. All settings are the same as in figure 2 except amplitude $A_a = 0.287$.

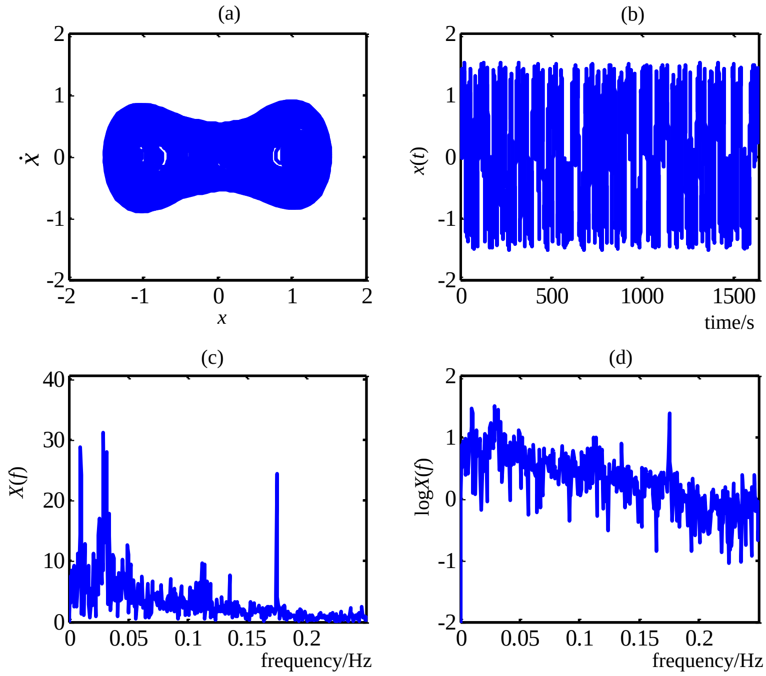

Figure 4. All settings are the same as in figure 2 except amplitude $A_a = 0.332$.

We now illustrate the usefulness of this result for a system with classical SR (i.e., one for which in equation (7) $A_a = 0$, $D > 0$). We assume $R(t)$ has the Lorentzian spectral distribution $g(\omega) = \gamma(1+\omega^2\tau^2)^{-1}$ cut off at the frequency ω_{max}; τ is the correlation time and γ is a normalization constant such that the variance of R(t) is unity. As can be seen in figure 1, the Melnikov scale factor $S_M(\omega)$ would in practice suppress contributions of components with frequencies $\omega > \omega_{max}$.

Consider the case $\tau = \tau_1 = 0.2$. Examples of averaged output spectra $P(\omega)$ for $A_0 = 0.3$, ω_0=0.069, ω_{max}=3.0, β=0.25 are shown in figures 5 (a) ~ (c) for D=0.1, 0.6, and 2.0, respectively (some parameters are the same as in [8]). The averaging was performed over 225 noise realizations approximated by in equation (6) with 100<K<500. Note that $A_0 < 4\beta / 3S_M(\omega_0)$, so that no chaotic behavior can be induced by the periodic signal alone. However, it was verified that, for the noise realizations used to obtain the results of figures 5 (a) ~ (c), the Melnikov inequality given by in equation (6) was satisfied, and that the respective motions were chaotic. Energy transfer to the signal frequency was found to be highest when the rate α for the chaotic motion was close to the signal frequency (α=0.0077, α=0.0667, α=0.1772 for figure 5(a), figure 5(b) and figure 5(c), respectively).

As illustrated earlier, assume that $A_a = 0$, and that for a set of values A_0, ω_0, β and D the system has low SNR. We could improve the SNR by increasing D. However, it is more effective to increase the SNR by keeping D unchanged and adding an excitation $A_a \sin(\omega_a t)$ such that ω_a is equal or close to the frequency of $S_M(\omega)$'s peak and A_a is so chosen as to bring about a characteristic rate comparable to the signal frequency. In figure 5(d), all parameters and the normalized spectrum $g(\omega)$ are the same as for figure 5(a), except that the system is subjected to an added excitation with amplitude $A_a = 0.23$ and frequency $\omega_a = 1.1$. This approach to increasing SNR is seen to be quite effective by contrasting figure 5(d) with figure 5(b) (in figure 5(d), $\alpha = 0.0706$ close to ω_0).

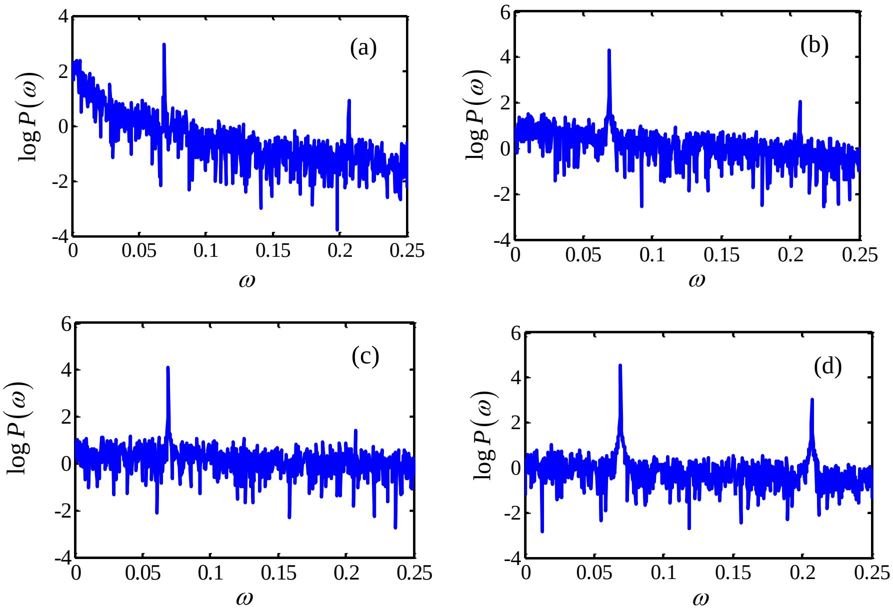

Figure 5. Averaged power spectra of output for stochastically excited system: (a) ~ (c) Increasing noise intensity *D* and $A_a = 0$. (d) The same noise intensity *D* as in (a), and $A_a = 0.23$. Noise correlation time $\tau = 0.2$.

3.2 weak signal with arbitrary frequency

From figure 1 and above analysis, we can get that $S_M(\omega)$ achieves the maximum when $\omega \approx 0.76$. Once the frequency ω_a of added harmonic excitation is equal or close to the frequency of $S_M(\omega)$'s peak, the SNR improvement is obvious. That is to say, the frequency of the detected characteristic signal only satisfies $\omega \le \omega_{\max} \approx 3.0$ and is very low. Now the problem is how to detect weak signal with arbitrary frequency by the method discussed above?

Combining equations (1) and (2), we obtain

$$\ddot{x} = -\beta\dot{x} + x - x^3 + G(t) \tag{9}$$

Where $G(t)$ is expressed by formula (7), In general ,let added harmonic $\omega_a = 1.0$, detected signal $\omega_0 < \omega_a$.

Now, assume that $t = \omega_1\tau$, (9) becomes

$$\frac{1}{\omega_1^2}\frac{\mathrm{d}x^2}{\mathrm{d}\tau^2} = -\frac{\beta\mathrm{d}x}{\mathrm{d}\tau} + x - x^3 + G(\tau) \tag{10}$$

Let $x_1 = x$, $x_2 = \dfrac{1}{\omega_1}\dfrac{\mathrm{d}x}{\mathrm{d}\tau}$, rewrite (10) to be state equation

$$\begin{aligned} \frac{\mathrm{d}x_1}{\mathrm{d}\tau} &= \omega_1 x_2 \\ \frac{\mathrm{d}x_2}{\mathrm{d}\tau} &= \omega_1\left(-\beta x_2 + x_1 - x_1^3 + G(\tau)\right) \end{aligned} \tag{11}$$

Thus, equation (11) can be applied to enhancement detection of weak characteristic signal with arbitrary frequency. It is important to emphasize that we can use normalized scale transform of SR described in [1] instead of equation (11).

Now, as a typical example, assume that we want to detect a characteristic signal with amplitude A_0 = 0.3 and frequency $\omega_0 = 0.069\times 2\pi \times 1000$ (i.e., 69Hz). In this case, $\omega_1 = 2\pi \times 1000$, $\omega_a = 1.0\times 2\pi \times 1000$. Substitute these parameters into equation (11) and the solution is shown in figure 6 It can be seen that the SNR improvement is obvious from figure 6(b).

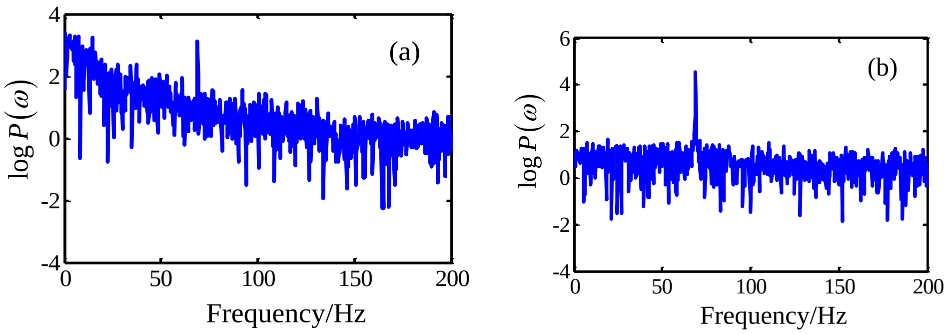

Figure 6. Averaged power spectra of output for stochastically excited system:(a) Increasing noise intensity *D*=0.1 and $A_a = 0$ (α =70.9466 lower than ω_0 =433.5398(i.e. 69HZ). (b) The same noise intensity *D* as in (a), and $A_a = 0.23$, here α =429.5146(i.e.68.3594Hz). Noise correlation time $\tau = 0.2$.

4. A case – weak characteristic signal detection for small fault of bearing

Experimental data of bearing faults come from Centre for Diagnostic Engineering in University of Huddersfield leaded by Professor Andrew Ball. The test rig consists of a three-phase electrical induction motor and a dynamic brake. The motor drives the brake by means of three shafts, which are joined by pairs of matched flexible couplings. The bearing used in the experiments is a N406 roller bearing. The tested bearing was fitted in the bearing housing on the driven side. The specifications of the tested bearing are: Bore diameter 30mm, Outer diameter 90mm, Bearing width 23mm, Pitch circle diameter (PD) 59mm, Ball diameter (BD) 14mm, Number of balls 9. In the experiments, the load applied to the test rig was 42.0 Nm and the rotational speed was 1,456 rpm (24.3 Hz). The three vibration signals were collected by accelerometers which were fixed on the cage near the tested bearings. Two different sizes of line defect were seeded on the outer race of bearings: a medium defect (approximately 0.3×16mm^2) and a small defect (approximately 0.11×6mm^2). Based on the geometric sizes and the rotational speed, the characteristic defect frequency (CDF) of the outer race was calculated to be 83.4 Hz. In the vibration data acquisition, the sampling rate was 64938Hz and the length of data was 810439. For the convenience of analysis, the vertical radial acceleration signal was used to validate the equation (11) for early detection of incipient fault.

A segment of FFT spectrum of envelope signal analyzed from acceleration signal of small defect in outer race of bearings is shown in figure 7. The characteristic defect frequency of the outer race (about 83Hz) can not be found in figure 7. Let envelope signal with small defect and added harmonic signal drive the normalized scale transform of SR model in [1] and equation (11), respectively. The results are shown in figure 8 and figure 9 respectively. In these two figures, the characteristic defect frequency component of the outer race is relatively obvious (about 84Hz).

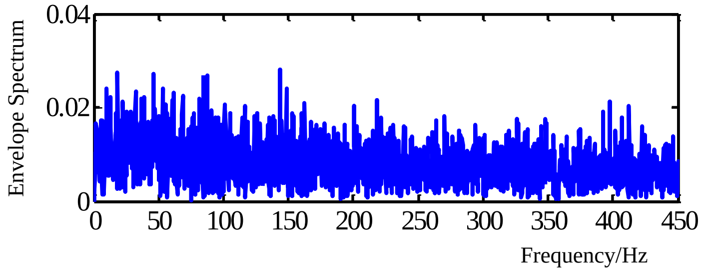

Figure 7. A segment of FFT spectrum of envelope signal with small defect.

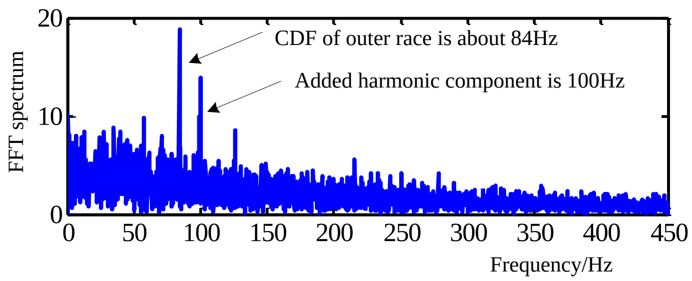

Figure 8. A segment of FFT spectrum of output of scale transform of SR model in [1] was driven by envelope signal with small defect and added harmonic signal with $\omega_a = 2\pi \times 100$.

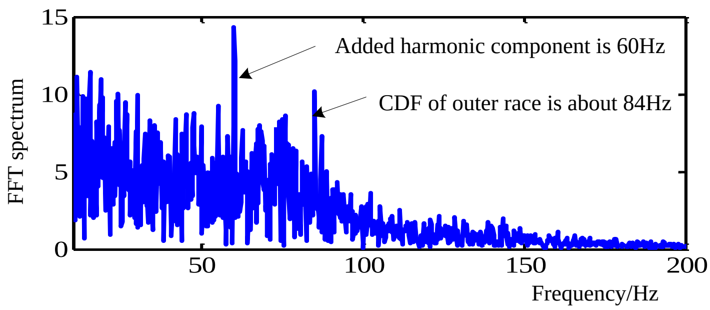

Figure 9. A segment of FFT spectrum of output of equation (11) excited by envelope signal with small defect and added harmonic signal with A_a =0.07 and $\omega_a = 2\pi \times 60$.

5. Conclusion

The enhancement of the SNR achieved by increasing the noise intensity is viewed as a particular case of a type of chaotic behavior that includes, as another particular case, the enhancement of the SNR by adding a harmonic excitation while leaving the noise unchanged. Based on this result, One qualitative result is the existence, independent of the deterministic, stochastic, or mixed character of the excitation, of a broadband portion of the output spectrum, which allows the occurrence of a synchronizationlike phenomenon that is the key to the enhancement of the SNR. These qualitative results suggested the investigation of the alternative mechanism for enhancing SNR, wherein the noise intensity is left unchanged and a harmonic excitation is added instead. This mechanism is more effective — allows a better SNR to be obtained — than the mechanism that relies on increasing the noise intensity.

In this paper, we applied these results further to enhancement detection for weak signal with arbitrary frequency buried in heavy noise. Some numerical simulation and case analysis show that the alternative mechanism for enhancing SNR is very effective and operable.

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