Effect of Eccentricity on Pipe Boundary Layer Growth for Flows in Annulus

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ABSTRACT

Laminar vortex rings are produced in the laboratories using a piston/cylinder arrangement. The dynamics of these rings are being extensively studied over the last couple of decades. Research has shown that the most important flow feature which describes the generation and propagation of these rings is the boundary layer growth. The present study focuses on the effect of eccentricity on the pipe boundary layer growth for flows in annulus. Two geometries with different eccentricities are modeled and numerically solved using commercial CFD package. The results are then compared with a concentric geometry. The results indicate toward a faster flow development in eccentric annulus. As vortex rings are generated when the boundary layer is still developing, eccentricity adversely effects the production of laminar vortex rings. This faster flow development hints towards a faster transition of Navier-Stokes equation's solution from Rayleigh to Blasius.

Keywords boundary layer, eccentricity, CFD

1 INTRODUCTION

The solution to the Navier-Stokes equations for an impulsively started flow over a flat plate was derived by Rayleigh and is stated by Rosenhead (1963) and Schlichting (2000) as:

$$u = U * \mathrm{erf}\left(\frac{y}{2\sqrt{\nu t}}\right) \tag{1}$$

Where u is the flow velocity inside the boundary layer, U is the velocity of the flow outside the boundary layer, v is the kinematic viscosity of the fluid and t corresponds to time. This solution is valid only in the vicinity of the leading edge of the plate and for a very short initial time period. This means that the solution given in equation (1) is valid for developing flows only. For fully developed flows, Blasius derived the solution to this problem which is stated by Rosenhead as:

$$u = U * \mathrm{erf}\ \sqrt{\frac{Ux}{\nu}} \tag{2}$$

Hence, as the flow starts, the solution to this problem is given by equation (1) for initial times, when the flow is developing. Once the flow becomes fully developed, the solution to this problem is then given by equation (2). There is a need to analyze the development of the flow and the growth of the boundary layer in concentric and eccentric annulus. Computational fluid dynamics technique has been used in the present study to analyze the effect of eccentricity on pipe boundary layer growth for flows in annulus.

2 Numerical Modeling

Table (1) outlines the geometric features of the models being used in this study. The length of the pipes shown in figure (1) is 20cm. Three dimensional unsteady Navier-Stokes equations are numerically solved using commercial CFD package FLUENT 13. The solution has been initialed with zero velocities in order to start the flow impulsively. Second order spatiotemporal schemes and PISO method of pressure-velocity coupling has been used. The solution is advanced with time intervals of 0.002 seconds which corresponds to 1500 time steps for the piston, having a constant velocity of 8cm/sec, to reach the end of the pipe.

Nomenclature

u	Axial velocity inside the boundary layer (m/s)
U	Axial velocity outside the boundary layer (m/s)
erf	Error function (-)
x	Axial distance (m)
y	Radial distance (m)
t	Time interval (s)
v	Kinematic viscosity of the fluid (m^2/s)
d	Diameter of the inner pipe (m)
D	Diameter of the outer pipe (m)

TABLE 1: Geometrical dimensions of models

	D	**d**	**δ1**	**δ2**
	(m)	(m)	(m)	(m)
Concentric	0.025	0.015	0.005	0.005
Eccentric 1	0.025	0.015	0.0035	0.0065
Eccentric 2	0.025	0.015	0.002	0.008

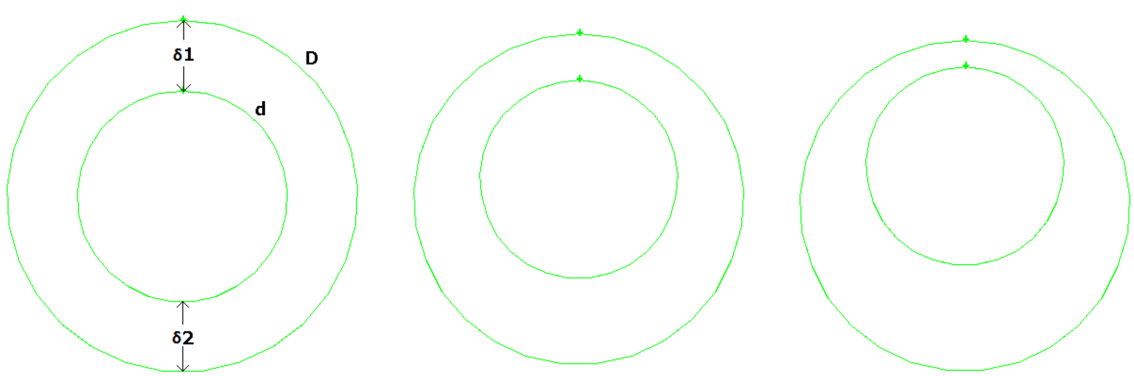

Figure 1: Concentric, Eccentric 1 and Eccentric 2 geometries

3 RESULTS

In order to analyze the effect of the eccentricity on the boundary layer growth in annulus, velocity profiles have been drawn from the results obtained from CFD. These profiles are drawn at time intervals of 0.5sec and 2sec in order to trace the development of the flow in the geometries shown in figure (1). For the case of concentric pipe geometry, as δ1 = δ2, only one profile has been drawn as shown in figure (2). For the case of eccentric geometries, velocity profiles have been drawn at top and bottom sections where the clearance between the pipe walls is minimum and maximum respectively.

Figure (2) shows the axial velocity profiles for the case of concentric annulus at time 0.5sec and 2sec. It can be seen that the flow is in developing stage at time 0.5sec. As the time progresses, the profile tends to attain the shape of a parabola, which is expected in the case of laminar flows. At time 2sec, the flow has developed and its solution is given by equation (2).

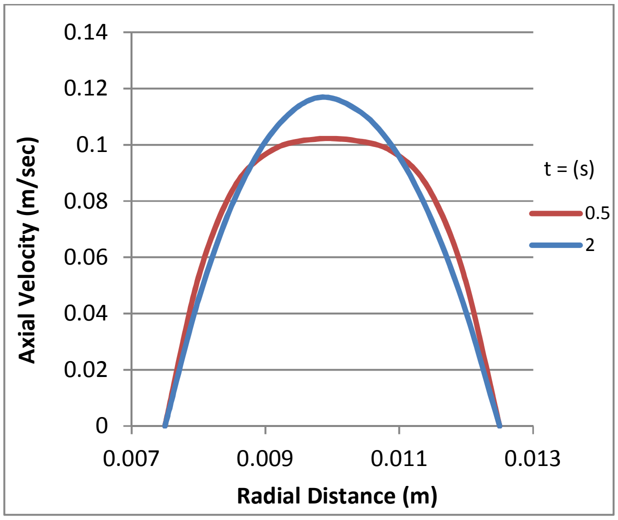

Figure 2: Axial velocity profiles in concentric annulus

Figure (3) shows the axial velocity profiles at the top section, which corresponds to δ1 in figure (1), of Eccentric 1 case. The flow has developed as early as 0.5sec from its impulsive start. Both the profiles shown are parabolic and correspond to Blasius solution for the calculation of axial velocity in the boundary layer.

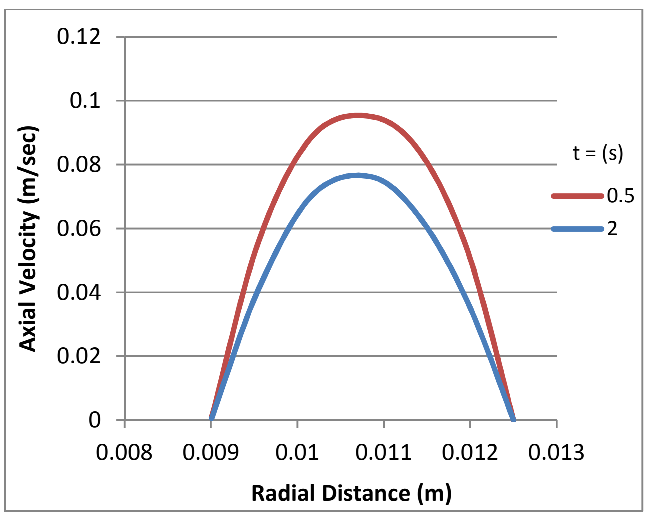

Figure 3: Axial velocity profiles in eccentric 1 annulus (top section)

Figure (4) shows the axial velocity profiles for Eccentric 1 case at the bottom section of the pipe where the clearance between the pipe walls is at its maximum. The plot shows that the flow was in developing stage at 0.5sec. At 2sec, the flow completely developed and the velocity profile took the shape of a parabola. The solution changed from equation (1) to equation (2) between these time intervals.

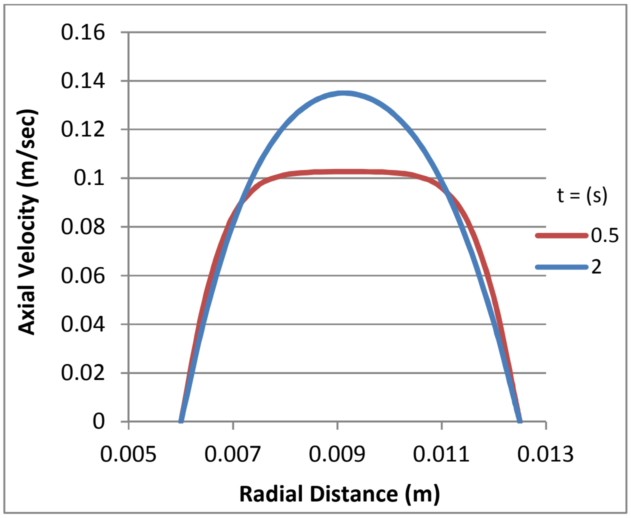

Figure 4: Axial velocity profiles in eccentric 1 annulus (bottom section)

Figure (5) shows the axial velocity profile for the case Eccentric 2 at top section which corresponds to ō1. The plot shows that at both the time intervals the velocity profile is parabolic and the flow developed as early as 0.5sec. This trend is similar to that for the case of Eccentric 1. The solution is given by equation (2) i.e. Blasius solution to the Navier-Stokes equations for the boundary layer flow.

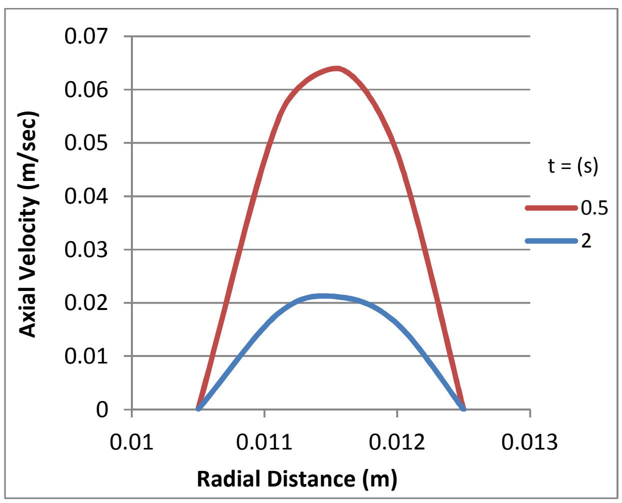

Figure 5: Axial velocity profiles in eccentric 2 annulus (top section)

Figure (6) shows the velocity profiles for the case of Eccentric 2 at bottom section. It can be easily seen that the trend is the same as that for the bottom section of Eccentric 1 case. The flow is developing in the earlier stages. The parabolic profile at 2sec shows that the flow has developed and the solution is that of Blasius. These results indicate that the flow develops at a faster rate in eccentric annulus. Furthermore, as the clearance between the pipe walls decreases, the flow develops much faster.

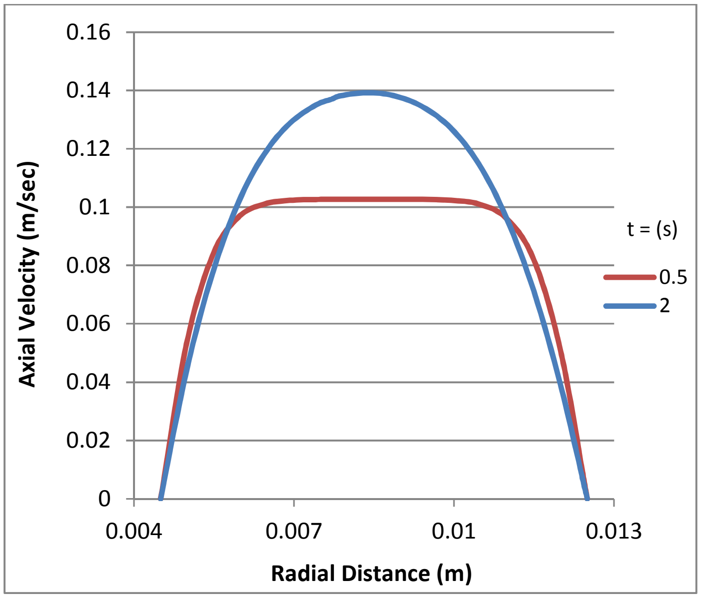

Figure 6: Axial velocity profiles in eccentric 2 annulus (bottom section)

4 CONCLUSIONS

CFD based analysis of flow in annulus of different eccentricities has been carried out. The results obtained are compared against flow in concentric annulus. The results indicate that as the eccentricity of the model increases, the flow develops at a faster rate. The solution for undeveloped laminar flow is given by Rayleigh while the solution for fully developed flow is given by Blasius. The eccentricity plays an important role toward the boundary layer growth and hence the generation of laminar vortex rings in annulus.

REFERENCES

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