Method for estimating tyre cornering stiffness from basic tyre information

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Abstract: This paper proposes a simple mathematical tyre model that estimates tyre cornering stiffness. The model is derived by considering the tyre to be a combination of two independent systems. The sidewalls are assumed to be negligibly stiff in the lateral direction, and hence their influence on the lateral dynamics of the tyre will be ignored. The belt and tread area of the tyre will be considered to be an homogeneous uniform band, and its stiffness will be estimated with reference to measured tyre data. The resulting model is estimated to yield cornering stiffness values within about 30 per cent of the actual measured values.

Keywords: tyres, vehicle dynamics, tyre modelling, vehicle handling, cornering stiffness, tyre properties

1 INTRODUCTION

Modern computers now have the power to solve very complex vehicle dynamics models, and software such as ADAMS has become widespread in the industry [**1**, **2**]. The accuracy with which vehicle handling performance may be modelled is dependent upon the accuracy of the parameters used in the model. One set of parameters that is often difficult to establish is the characteristics of the tyres, specifically their ability to generate forces. Complex handling models may require complex tyre models, and a number of such models have been developed. One such model that has rapidly become well regarded and used in the industry is called the 'magic formula', which has been developed by Pacejka over the last 20 years [**3–7**].

One problem with such models is the generation of the many coefficients required in them, which is only possible with extensive laboratory testing and hence can be an expensive process. However, many handling studies can be successfully carried out by limiting the scope of the study to the 'linear region' of the vehicle handling domain. When considering a vehicle, there are many non-linear parameters, and in the study of vehicle handling the most important of these are probably the tyres, but, if the lateral acceleration of the vehicle is restricted to the region below about 3 m/s^2, then it is often sufficiently accurate to consider the tyres to have a linear relationship between slip angle and lateral force [**8**]. Unfortunately, the only way accurately to establish the actual cornering stiffness of a tyre is to test the tyre, once again resulting in high costs.

This paper proposes a method for estimating tyre cornering stiffness on the basis of nothing more than the major dimensions of the tyre. It must be stated at this point that the results are not expected to be highly accurate, since tyres are too complicated to be accurately predicted using simple measures, but the resulting values should provide a starting point for the vehicle handling modelling process in the absence of better data. The method also provides a simple way to compare the likely change in cornering stiffness that results from a change in tyre dimensions (all other factors being constant).

2 MODEL DERIVATION

In a radial tyre the sidewalls are designed to be highly flexible in the lateral direction. The belt is wrapped around the plies of the carcass and, by virtue of the width–depth ratio, is designed to be stiff in lateral bending while remaining flexible in vertical bending. It is this resistance to lateral bending that provides the bulk of the structural support when moments are applied through the contact patch. Hence, the lateral stiffness of the belt is the critical factor in determining the cornering stiffness of the tyre. It should be noted that in tyre construction the 'belt' has a specific meaning relating to the internal circumferential bands in the structure of the carcass. For this paper, however, the belt refers to all parts of the tyre within the tread region (i.e. only excluding the sidewalls).

If the tyre is running at a small slip angle, α, and the belt of the tyre is imagined to be 'cut' at top dead centre and unwrapped, the belt would look as shown in Fig. 1 (exaggerated). It will be noted that the belt has assumed a characteristic 'S' shape. The belt is aligned with the direction in which the tyre is pointing at top dead centre, the contact patch is aligned with the direction the tyre is travelling, and the angle between the two, α, is the slip angle of the tyre. In reality, the contact patch will be laterally displaced from the centre-line of the belt, and the entire belt will be laterally displaced from the centre-line of the wheel since the flexible sidewalls of a radial tyre allow sideways movement of the entire belt in response to side loads. However, neither of these net lateral displacements has any significant influence on the following analysis, and hence they are not shown in Fig. 1.

For small slip angles the belt of the tyre may be considered to be symmetrically identical, and hence only one-half of the belt need be analysed, as illustrated in Fig. 2. In Fig. 2 the contact patch has been omitted, and the forces are applied to the trailing (or leading) edge of the contact patch. Inclusion of the contact patch would cause a further small lateral displacement at the point of force application. This additional displacement will generally be less than 3 mm for typical tyres in their linear operating range. Including this aspect would result in significantly more complication to the mathematical model, and so is not considered necessary in this simple model.

The shape of the belt results from two influences, the self-aligning moment, M, and the shear force, W, which combine to generate contact patch twist during cornering. The easiest way to visualize the shear force is to consider the top dead centre of the tyre. With no shear force, the contact patch moment would cause the two 'ends' of the belt at top dead centre to be laterally displaced from each other.

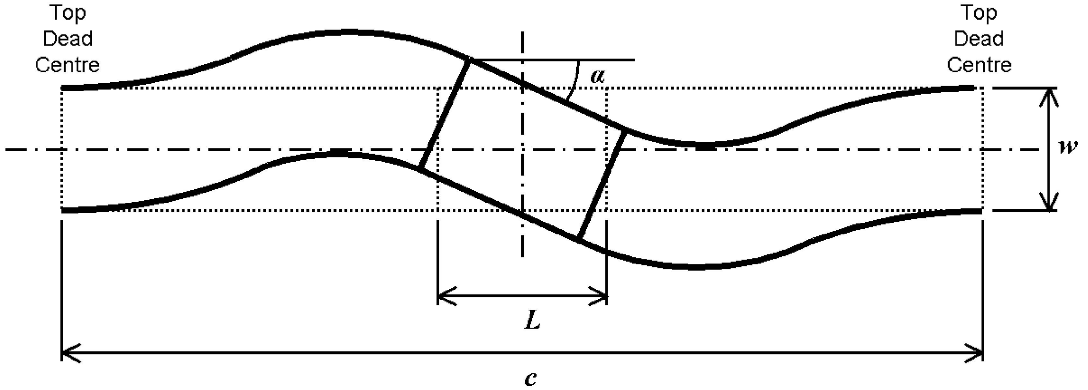

Fig. 1 Deflected shape of a tyre belt with slip angle α

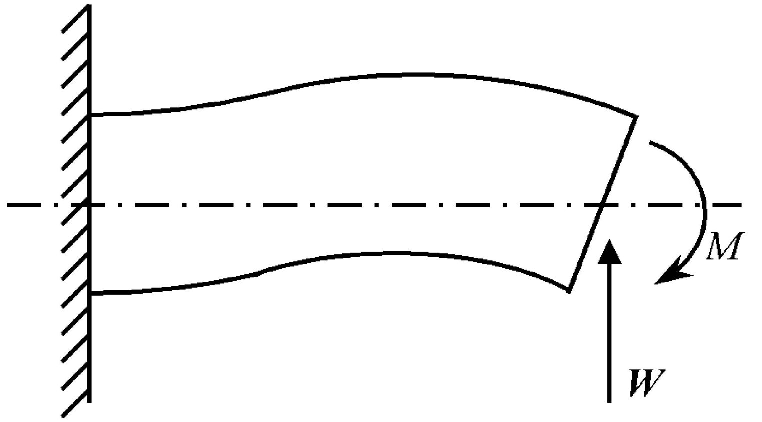

Fig. 2 Simple beam model of a tyre belt with a slip angle

This displacement discontinuity is prevented by the action of the shear force. The shear force may be considered to be two equal but opposite lateral forces acting at the contact patch. Each of these forces acts on one-half of the belt and is of a magnitude to return the contact patch to the centre-line. Determination of the magnitude of W is given in Appendix 2.

In the development of this beam model, shear force deflection has been neglected. It is conventional to neglect shear deflection when the length–width ratio exceeds about 5:1 [**9**], since with such a ratio the error introduced is less than 4 per cent. The belt of a typical tyre has a circumference–width ratio of between 12:1 and 8:1, so the equivalent 'beam' shown in Fig. 2 would have a length–width ratio of between 6:1 and 4:1 (or a little less if the contact patch is allowed for), which implies that the validity of the assumption to neglect shear deflection is marginal. The intention of this model is, however, to represent the belt of the tyre with a very simple model, and then apply statistical methods to ensure conformance with real data. It is not the intention of this model accurately to predict exact behaviour on the basis of the absolute physical properties of the tyre. It is therefore suggested that the assumption of pure bending and no shear deflection is sufficiently valid for typical passenger car tyres.

The slip angle resulting from these two forces is analogous to the end slope of a simple cantilever beam. For simplicity it will be assumed that the belt is linear, elastic, and isotropic. Therefore the slip angle is given by

$$\alpha = \frac{M[(c-L)/2]}{4EI} \tag{1}$$

The complete derivation of equation (1) is given in Appendix 2. Moment M is defined as

$$M = Fx \tag{2}$$

Tyre cornering stiffness is defined as F/α, and hence

$$\text{Tyre cornering stiffness} = \frac{8EI}{x(c-L)} \times 2 \tag{3}$$

(Note that at this point the cornering stiffness is doubled to take account of the fact that there are actually two 'beams'.)

Parameters I and c are given by

$$I = \frac{bw^3}{12} \tag{4}$$

$$c = 2\pi(r + wa) \tag{5}$$

Substituting equations (4) and (5) into equation (3) yields

$$\text{Tyre cornering stiffness} = \frac{4Ebw^3}{3x[2\pi(r+wa) - L]} \tag{6}$$

Equation (6) defines the cornering stiffness. It contains the pneumatic trail, x, and the contact patch length, L, which are not necessarily known. Fortunately, the contact patch length can be estimated with reasonable accuracy if the sidewall vertical deflection is known [**10**]. Tyres always tend to run in quite a small band of sidewall deflections, probably as a result of the need to avoid excessive heat build-up (suggesting smaller sidewall deflection) while maintaining the largest possible contact patch (suggesting larger sidewall deflection). Experience suggests that properly inflated road car tyres tend to run with a sidewall deflection of about 15 per cent of the undeflected sidewall height, while race car tyres run nearer 10 per cent (primarily owing to their extreme widths). Simple geometry leads to the following expression for the contact patch length

$$L = 2(r + wa)\sin\left[\operatorname{acos}\left(1 - \frac{swa}{r+wa}\right)\right] \tag{7}$$

The derivation of equation (7) is given in Appendix 2. Once the contact patch length is found, the pneumatic trail is also easy to establish. For small slip angles the contact patch meets the ground at the undeflected position, then follows a straight line at an inclination equal to the slip angle away from the undeflected position. This is shown in Fig. 3.

At any point along the length of the contact patch the lateral force produced is approximately proportional to the lateral displacement from the undeflected position, and hence the triangle thus described represents a lateral force distribution. Longitudinal forces are not represented on the diagram. The resultant sum of the lateral forces can be considered to act at the centroid of the triangle. Hence, the pneumatic trail is given by

$$x = \frac{L}{6} \tag{8}$$

Substituting equations (7) and (8) into equation (6) leads to

$$\text{Tyre cornering stiffness} = \frac{2Ebw^3}{(r+wa)^2 \sin\{\operatorname{acos}[1-(swa)/(r+wa)]\} \times (\pi - \sin\{\operatorname{acos}[1-(swa)/(r+wa)]\})} \tag{9}$$

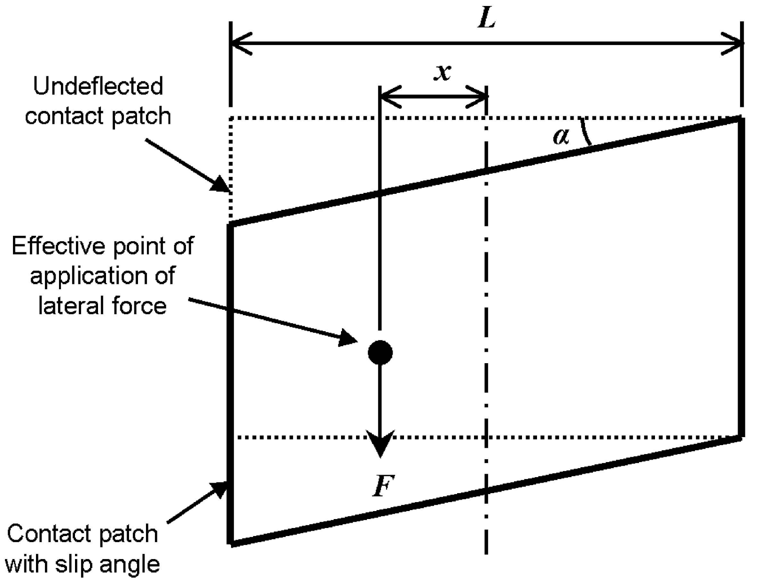

Fig. 3 Detail of the contact patch shape

Equation (9) gives an estimate of the tyre cornering stiffness using parameters that can easily be found.

1. Parameters *r*, *w*, and *a* can be read off the sidewall of the tyre.
2. Parameter *s* can always be assumed to be 0.15 for road tyres and 0.1 for racing tyres, unless a better figure is known.
3. Parameter *b* is the material thickness of the belt. It is suggested that a value of 0.015 m be used for road tyres, and a value of 0.01 m for race tyres (to allow for lighter construction methods). The value of *b* is not critical since the belt is, in reality, a complex non-homogeneous structure, and hence the stiffness of any particular belt cannot be known with any accuracy. The value of *b* should be considered in conjunction with the value of *E*, with the factor *Eb* representing the typical stiffness of belts in general. The value of *E*, for any given value of *b*, may be established by comparison with real tyre test data, as demonstrated in section 3 of this paper.
4. Parameter *E* is the belt compression modulus. The suggested value is $27 \times 10^6\ \mathrm{N/m^2}$, and section 3 describes how this was arrived at.

3 ESTIMATION OF *E*

The belt primarily consists of rubber, and hence the compression modulus should be similar to that of rubber. Unfortunately, the compression modulus of rubber is highly variable and dependent on many factors, including shape factor, strain rate, and rubber compound. Although the strain rate can be assumed to be very low, and the hardness of the rubber for typical tyres could be assumed to be between 50 and 65 Shore A [**11**], the shape factor is an unknown quantity. The purpose of the plies in the belt of the tyre is to provide structural support for the rubber. They are bonded to the rubber and restrict the strain of the rubber in particular directions. In other words, the plies are there specifically to increase the effective shape factor of the rubber, but the resulting shape factor is almost impossible to estimate, and thus it is not possible to establish a value for *E* on the basis purely of the physical properties of the belt.

To address this problem, the value of *E* may be derived from experimentally measured tyre data by establishing the value that best conforms to a range of tested tyres. Figure 4 shows a comparison of theoretical cornering stiffness, as predicted by equation (9), against measured cornering stiffness for a range of road tyres. The range of tyres covered is from 145/80R13 to 235/75R15. It should be noted that equation (9) yields results in the SI units of N/rad, but the units used in Fig. 4 are N/deg. This is simply because the units N/deg are more typically used by vehicle and tyre engineers. A simple regression technique gave the best fit when a value of $27 \times 10^6\ \mathrm{N/m^2}$ was chosen for *E*. This value falls within the typical range for rubber and suggests quite a high shape factor (perhaps somewhere between 1 and 3). The solid diagonal line is the line of best fit for the data and is, by definition, a perfect correlation between the predicted and the measured data. The dotted lines show the 95 per cent confidence limits

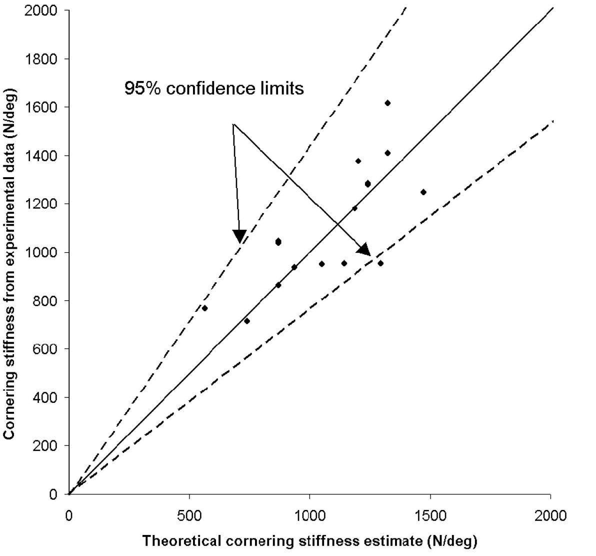

Fig. 4 Measured tyre cornering stiffness versus predicted tyre cornering stiffness (measured data supplied with the kind permission of MSC Software Limited)

(i.e. two standard deviations) for the data presented. These confidence limits show that for the range of data studied there is a 95 per cent likelihood that a predicted value will be within ±30 per cent of the actual measured value.

The values for *b* and *E* have been established with reference to only a very limited set of measured tyre data. The relationship between the thickness of the belt, the hardness of the rubber compound, and the structure and number of the belt plies could be investigated in more detail. The number of belt plies is usually quoted on the tyre, the hardness of the rubber compound is relatively easy to measure, and the belt thickness could easily be established. Analysis of these parameters could lead to a more reliable estimate for the value of *E*, and may reduce the scatter of the data presented in Fig. 4.

4 CONCLUSIONS

A simple model has been derived to estimate the tyre cornering stiffness on the basis only of tyre information that is readily available. Comparison with measured data indicates that, to two standard deviations (95 per cent confidence), a result predicted using this model is likely to be within 30 per cent of the actual measured result, although it must be noted that the measured dataset is probably too small for reliable statistical analysis.

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APPENDIX 1

Notation

Symbols in final model

a	tyre aspect ratio (tyre section height/tyre section width)
b	belt thickness (m)
E	compression modulus of the belt (N/m^2)
r	wheel radius (m)
s	sidewall vertical deflection when loaded (unitized per cent)
w	belt width (m)

Symbols used in derivation

c	tyre outside circumference
F	lateral (cornering) force
I	second moment of area of the belt radial cross-section
L	length of the contact patch

M moment on the tyre contact patch (self-aligning moment)
W shear force in the belt
x pneumatic trail

α slip angle

APPENDIX 2

Derivation of the shear load, *W*, and the slip angle, *α*

From the 'Myosotis formulae', the end deflections caused by M and W are given by

$$\mathrm{Def}_M = \frac{M[(c-L)/2]^2}{2EI} \tag{10}$$

$$\mathrm{Def}_W = \frac{W[(c-L)/2]^3}{3EI} \tag{11}$$

To avoid a discontinuity in the belt, the end deflection caused by M must equal the deflection caused by W

$$\frac{M[(c-L)/2]^2}{2EI} = \frac{W[(c-L)/2]^3}{3EI} \tag{12}$$

which simplifies to

$$W = \frac{3M}{c-L} \tag{13}$$

The Myosotis formulae also provide expressions for the end slope of the beam, and, by the principle of superposition, the total end slope of the beam (and hence the slip angle of the tyre) is given by

$$\alpha = \frac{M[(c-L)/2]}{EI} - \frac{W[(c-L)/2]^2}{2EI} \tag{14}$$

Substituting equation (13) into equation (14) leads to

$$\alpha = \frac{M[(c-L)/2]}{4EI} \tag{15}$$

Derivation of the length of the contact patch

The tyre is assumed to deform as shown in Fig. 5. This assumption neglects the fact that the tyre belt is longitudinally stiff, and so, while Fig. 5 appears to show a reduction in the length of the belt when loaded, this is actually compensated for by a certain amount of circumferential bulging away from the contact patch area.

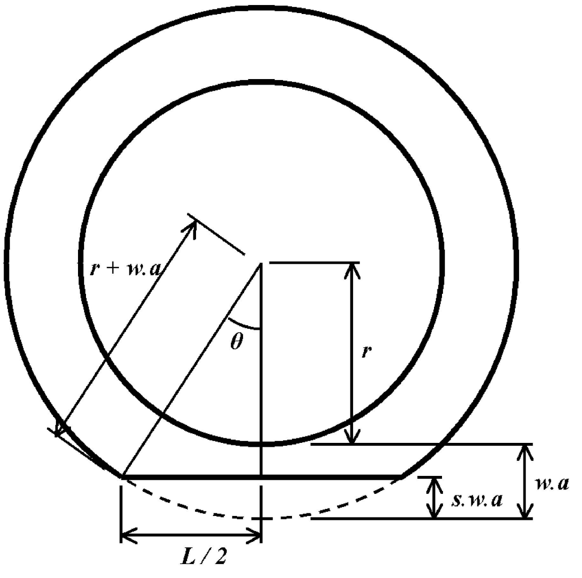

Fig. 5 Tyre under vertical load, showing the deformation at the contact patch

From simple trigonometry

$$\theta = \mathrm{acos}\left(\frac{r + wa - swa}{r + wa}\right) \tag{16}$$

and

$$\theta = \mathrm{asin}\left(\frac{L/2}{r + wa}\right) \tag{17}$$

Combining equations (16) and (17) yields

$$\mathrm{asin}\left(\frac{L/2}{r + wa}\right) = \mathrm{acos}\left(\frac{r + wa - swa}{r + wa}\right) \tag{18}$$

which leads to

$$L = 2(r + wa)\sin\left[\mathrm{acos}\left(1 - \frac{swa}{r + wa}\right)\right] \tag{19}$$