FINITE ELEMENT ANALYSIS OF METAL WEAPONS IN A TIME-DOMAIN TRANSIENT MAGNETIC FIELD

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ABSTRACT

A method of secondary magnetic field analysis of metal objects in time-domain transient magnetic field is established by modification of finite element analysis. Secondary current distribution of complex metal geometries is analyzed at different time steps in a transient magnetic field environment. A unique sensor system is currently being designed using this analysis to identify the location of potentially dangerous metal objects.

Keywords Transient magnetic field, Metal detection and classification, Finite element analysis

1 INTRODUCTION

Threats from people carrying metal weapons have increased in recent years [1]. Although, there is controlled screening for threat objects, available in secured establishments, the high rates of false alarms from screening equipment seriously affects the efficiency of the security system. The problem with currently available screening equipments is that it detects all metal objects without deciphering between different types of threat or non-threat metal objects leading to false alarms [2]. Ultimately, the increased need for manual intervention required to resolve both valid and false alarms leads to the unnecessary restriction of individuals within an establishment. Therefore, it is a necessity to build a robust metal detection classification system for concealed threat metal objects with a high detection probability and a low false alarm rate.

This paper discusses the application of Finite Element Analysis (FEA) to analyze secondary magnetic field behaviour of metal objects in time-varying magnetic fields. In this method, transient pulse current is applied to a transmitter coil to illuminate the metal target. The primary field induces surface current on the metal target, generating a secondary magnetic field, which is then sensed by a receiver coil. Surface current on the metal target flows inwards as shown in figure 1. and the secondary field of the target secondary current decays.

2 FINITE ELEMENT MODEL

The finite element method is based on division volume of space in which the equation is satisfied into small volumes (the finite elements). Within each finite element a simple polynomial is used to approximate the solution [3]. In the analysis of electromagnetic fields, it is essential to define the potential function of at least one domain of the model [4]; otherwise an infinite number of solutions could be generated by adding an arbitrary constant to the solution. The domain of the model is divided into line elements, in order to simplify the solution. A shape function in terms of polynomial, is defined and validated at each node of the model, where each particular node is only defined in the elements that use the node and it is zero outside the elements [5]. The following discrete methods are used to approximate the field values using the characteristic nodal values and associated shape functions of the model, these are; Variation methods, least squares and weighted residual methods. Out of these three methods, weighted residuals methods have wide application and are used in the software to develop a numerical solution [6]. The weighted residual method is used over to defined whole domain in finite element mode. The Galerkin weighted residual method is the best choice for the types of equation arising in electromagnetism [7]. In this case, the basis functions approximating potential are also used for the weights. Discretisation of a particular domain into line elements with associated nodes gives a set of independent weighting functions, from which a set of equations is developed by requiring that equation to be satisfied independently for each weight function. For non-linear materials, Newton-Raphson method can be used to solve non-linear equations [8].

In electromagnetic field calculations special finite elements are not required to solve the equivalent of shell and plate geometries that are common in mechanical design [9]. However, electromagnetic fields are required to be computed to much higher accuracy than is required in other disciplines, the geometry is frequently complicated with wide range of dimensions and the actual results required is often derived from the field solution by integration or differentiation. The basic limitation of finite elements solutions is that the accuracy of solution is related to size of elements. Thus, the numbers of elements in the background region were reduced and a fine mesh was established on surface of weapon to calculate distribution of eddy current accurately.

A 3D model of a weapon (Gun) and a transmitter coil was modelled in Opera 3D modeller. The physical characteristics of the model are shown in figure 2. The 3D model was resolved within boundary condition symmetry for linear permeability materials to establish that the model was consistent with the associated physical model.

2.1 BOUNDARY CONDITION

Boundary conditions are used in two ways; firstly to provide a way of reducing the size of finite element representation of symmetrical models; secondly to approximate magnetic field at large distances from the model. Tangential magnetic boundary condition was applied to the model and the field symmetry of the model is represented in equation (2.1). In order to simplify calculations and save computing power only quarter of the model was analyzed.

Field Symmetry of Tangential Magnetic, $\mathbf{H}.\mathbf{n} = 0$ (2.1)

Where **H** is magnetic field and **n** is the normal unit vector to the surface being considered. A non-zero value for the electric scalar potential V on an external surface is used to drive current into the model. The total and reduced vector potential formulation used in transient electromagnetic solution does not require the vector or electric scalar potentials to be specified unless needed to represent the field symmetry of the model. The magnitudes of the potentials are automatically aligned with model symmetry and are shown in figure 3.

An external mesh surface was created on the faces of a conductor to calculate surface eddy currents by the applied voltages being separated by a small gap in the mesh. Tangential magnetic boundary conditions are required on all boundary surfaces of the gap. Electromagnetic fields are frequently not contained within a finite volume [10]. By applying derivative boundary conditions on the model, it is restricted to finite domain solution. In order to restrict the field generated by inside regions of a gun and conductor reflecting at the surface, the interior region of the gun was modelled as reduced potential which minimizes the errors on the model. Relatively small fields from the inside region of gun are only reflected in the false open boundary.

Low frequency electromagnetic fields are described by the quasi-static limit of Maxwell's equation [12] which excludes displacement current. The magnetic field produced by the transmitter coil can be calculated by integration of Biot-Savart's equation [12]. The vector potential describing the magnetic field excluding the field from the transmitter coil is known as reduced vector potential (A_R) and it is defined by

$$B = \mu_0 H_s + \nabla \times A_R \qquad (2.2)$$

Total vector potential of the outer regions of target is represented in equation (2.3) [12]

$$\nabla \times \frac{1}{\mu} \nabla \times A = -\sigma \frac{\partial A}{\partial t} - \sigma \nabla V \qquad (2.3)$$

The electric scalar potential (V) emerges because of the non-uniqueness of the potential, which arises during integration of Maxwell's equation.

Reduced vector potential is represented in equation (2.4)

$$\nabla \times \frac{1}{\mu_0} \nabla \times A_R = 0 \tag{2.4}$$

In the transient analysis, it is necessary to log magnetic field parameters of the model at each analysis, this allows tracking of key characteristics as a function of time, which is achieved by the time stepping method.

2.2 TIME STEPPING

Transient analysis of electromagnetic fields is time dependent and analyzes eddy currents of the target where a current driven source is changed in a predetermined way in the model. Transient analysis equations are solved using a time stepping algorithm. Applying the Galerkin procedure [4] to equation (2.3) produces a matrix equation of the form;

$$R.A + S\frac{\partial A}{\partial t} + b = 0 \tag{2.5}$$

where A is now a vector of unknown potentials and B is vector of the driving terms. Discreting A and B as first order functions in time:

$$A(t) = (1-\tau)a_n + \tau a_{n+1} \tag{2.6}$$

$$B(t) = (1-\tau)b_n + \tau b_{n+1} \tag{2.7}$$

where
$$\tau = \frac{t - t_n}{t_{n+1} - t_n} \tag{2.8}$$

a_n and b_n are values of A and B at time t_n. Using τ as the weight in a Galerkin weighted residual solution of equation (2.5) leads to a recurrence relationship between a_{n+1} and a_n:

$$(R(1-\theta) - \frac{S}{\Delta t})a_n + (R\theta + \frac{S}{\Delta t})a_{n+1} + b_n(1-\theta) + b_{n+1}\theta = 0 \tag{2.9}$$

where the time step $\Delta t = t_{n+1} - t_n$. The value of θ in this model is 1.

A transient ramp drive function was defined to drive an external current driven circuit using a transmitter coil. Transient time-table is defined to allow step analysis control of a transient magnetic field. The control is achieved through the use of a command script, which is executed several times through the course of a time step. Also, key state values are set as system variables and are used for conditional execution of commands, allowing switches in behaviour.

An external circuit in transmitter explained in section 2.3 executes time steps for transient analysis of the model.

2.3 EXTERNAL CIRCUITS AND CURRENT IN TRANSMITTER

In the developed 3D model, an external current drive circuit is used to excite the transmitter coil for a transient solution. The current density defined for each region is supplied by external current source. Transmitter current is supplied as a function of time and is independent of property of the coil. The external circuit consists of a loop, which consists of a time varying drive function for source current, resistors which define resistance of target and coil. Transmitter coil is meshed to allow source current in the coil and the external impedance of the coil is assumed to be zero. The resistance of the transmitter model is defined by the conductivity of the region. Time steps for the external circuit are saved to a script file, which is analyzed with the model during analysis.

It is necessary to calculate current in transmitter model, which is achieved by solving an extra equation for each grouped region of the transmitter coil. The equation limits the total current, I, flowing in terms of $\frac{\partial A}{\partial t}$ and a potential gradient ∇V, which comes from the integration of equation (2.3) and is usually zero [11].

$$-\int_{\Omega_J} \sigma(\frac{\partial A}{\partial t} + \nabla V)\partial\Omega = I \qquad (2.10)$$

The effect of the potential gradient is a spatially uniform current density over the conductor, *J* given by

$$J = -\sigma\nabla V \qquad (2.11)$$

and becomes an extra unknown in the modified equation (2.2). The following two equations are solved together.

$$-\nabla.\frac{1}{\mu}\nabla A_Z - \nabla \times H_c = J - \sigma\frac{\partial A_Z}{\partial t} \qquad (2.12)$$

$$\int_{\Omega_J} (-\sigma\frac{\partial A}{\partial t} + J)\partial\Omega = \int_{\Omega_J} J_s\partial\Omega \qquad (2.13)$$

The equation (2.13) is repeated for every group of regions within the transmitter and symmetry non-zero. J_s is the current density. The computation of total current in a conductor with equation (2.13) depends on the total current in the transmitter coil, and equation (2.13) depends on the spatial integral of the time differential of the vector potential over the entire transmitter coil area. A small time step is established to calculate current density over transmitter coil area during the time when transmitter is switched on. The basic method of analysis is based on the principle of Electro Magnetic Induction (EMI) technology and is described in section 3

3 METHODS OF ANALYSIS

The detection and classification of metal objects described in this paper uses time varying transient magnetic field to create a primary magnetic field in a transmitter coil. According to Faraday's law [12], time-varying electric current through a transmitter coil creates a primary magnetic field. As there is no energy after current ceases to flow in the transmitter coil, the primary field collapses in time. This collapsing primary magnetic field induces Electro Motive Force (EMF) in nearby metal weapons, such as a gun. Induced EMF generates eddy current to flow in the weapon, which decay with time and induce a secondary magnetic field around the object. The rate of decay is proportional to the object material, size and shape creating individual signatures for different metal objects. Hence, the detection of secondary magnetic field decay rate signatures is of interest to this research

A model of a gun with an external loop of current driven circuit was established in the Opera 3D software to include transmitter coil and a target. Transient Direct Current (DC) pulse of 5 Amps was supplied to transmitter coil for 1 milliseconds. The time rate of decay of eddy current of metal parts of the weapon was measured and is represented in figure 4.

4 RESULTS AND DISCUSSION

The time decay rate of eddy currents is characteristic property of metal type, size and shape, leading to a specific signature. An object signature of threat metal object is currently being designed using decay time constant of metal weapons. Furthermore, an Automatic Target Recognition (ATR) algorithm [13] is currently being investigated for more accurate classification of threat objects. In this analysis, eddy current distribution of both interior and exterior parts of the weapon has been analyzed. The figure 5 shows eddy current distribution in the interior region of a gun. The total current density was calculated for the target as shown in figure 1 and the total current density of the model is shown in figure 6. Electrical characteristics of model was analyzed after the transmitter was switched off and figure 7 shows field parameters of the model after 1 milliseconds of analysis. Utilising the above analysis on an array of sensors coils is currently being designed to capture eddy current from metallic parts of weapons in transient magnetic fields. Distribution of 3D sensor array would assist in utilising all the behaviours of metals in transient magnetic field.

The circuit representation of transient EMI model [14] is shown in figure 8. This transient EMI model is used as external circuit to calculate eddy current, magnetic field behaviour in transmitter, receiver and metal object. In this model three loops are used to represent transmitter, receiver and metal object in transient EMI environment. The subscript T, R, O represents transmitter, receiver and object respectively.

5 CONCLUSIONS

Transient time-domain analysis of eddy current distribution of complex weapon geometry was analysed using the finite element method. The rate of decay of a secondary field is a unique property of the metal; determined by the conductivity, magnetic permeability, shape and size of the target, hence a weapon database could be created with the use of time constant as the object signature. A sensor array is currently being designed, using the above analysis in this paper, for metal weapon detection system.

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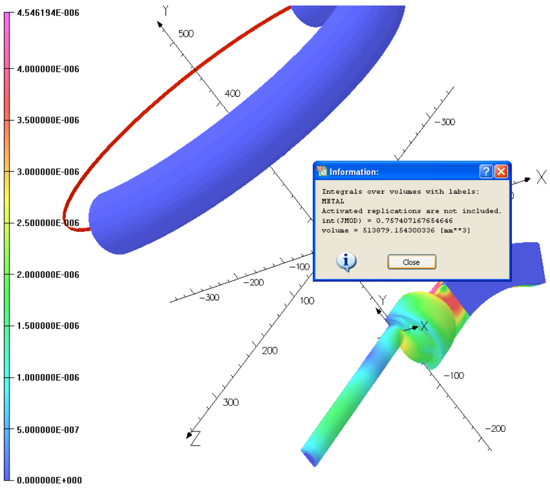

Figure 1: Total current densities of metal region of weapon

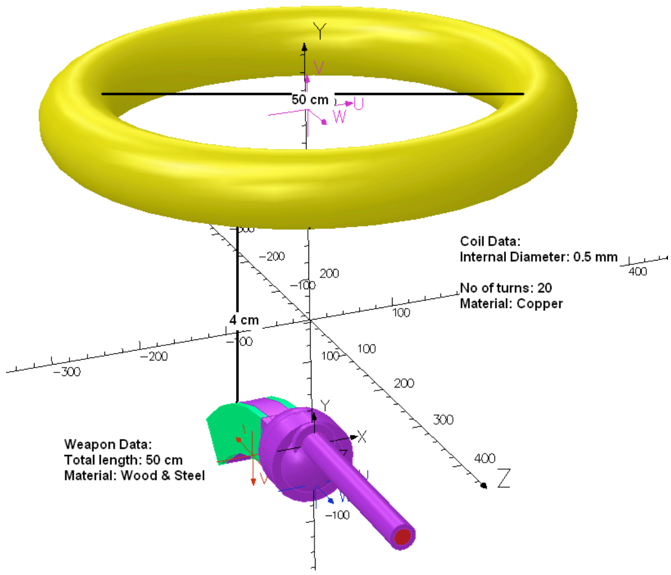

Figure 2: CWD Model Physical characteristics

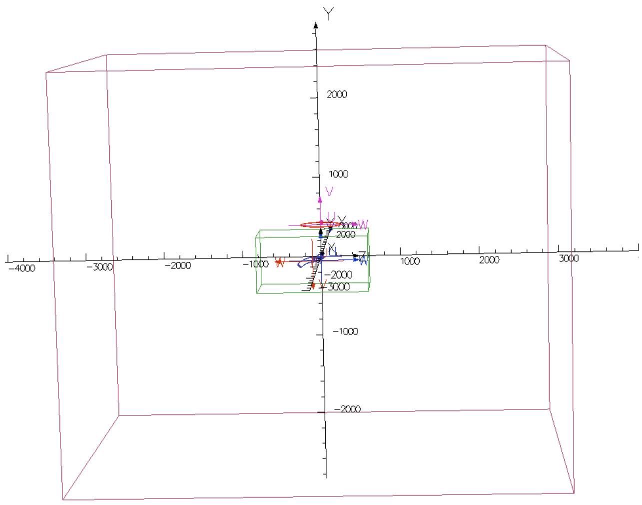

Figure 3: CWD Model with background air regions

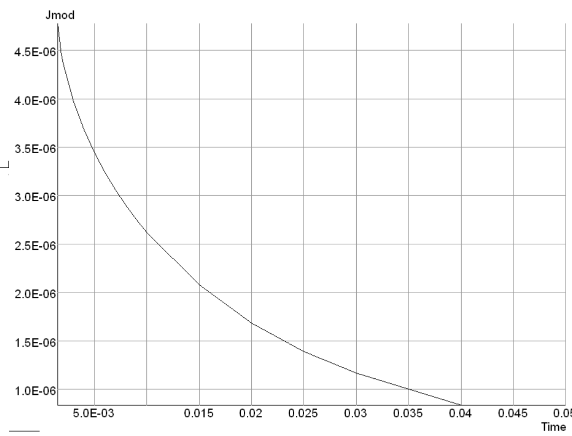

Figure 4: Decay current vs time graph for weapon

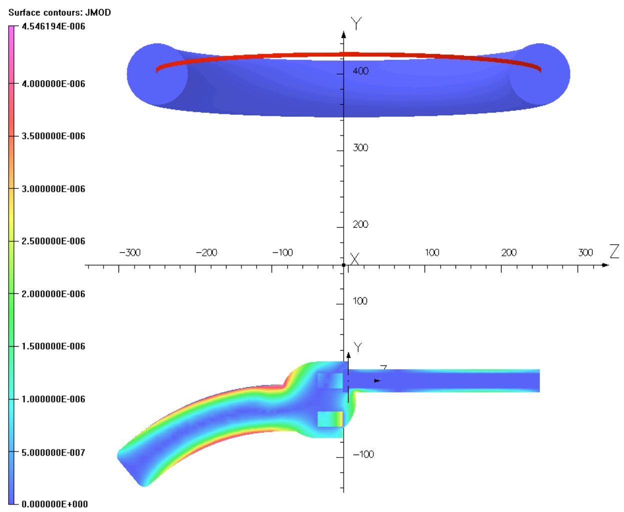

Figure 5: Eddy current distribution of interior of weapon

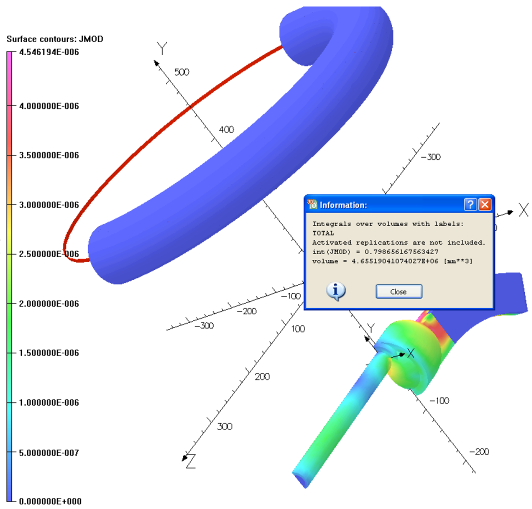

Figure 6: Total secondary current distribution of the model

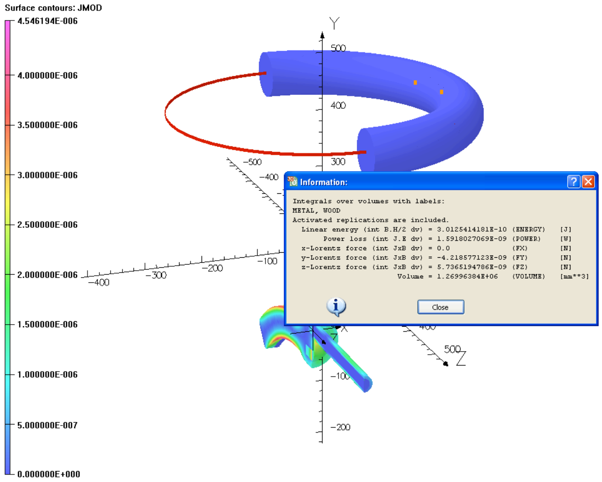

Figure 7: Electrical characteristics of weapon after transmitter was switched off

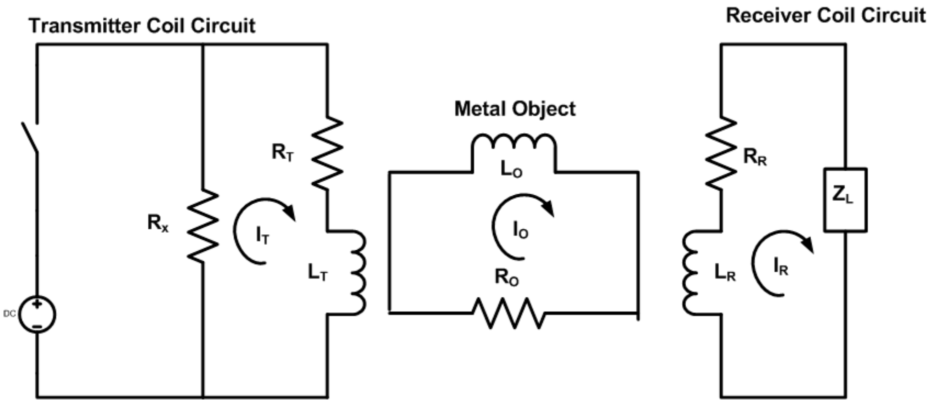

Figure 8: Circuit Model of CWD system in time-domain transient electromagnetic field