CELLULAR AUTOMATA MODELLING OF MICRO ABRASIVE MACHINING

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ABSTRACT
This paper presents the use of Cellular Automata (CA) for modelling micro abrasive machining. A one-dimensional CA was initially used to model the workpiece, as having different heights, to represent the roughness; and the forces from the abrasive grits were modelled to randomly erode the workpiece, as such smoothening out and polishing the surface. The modelling was then extended further to two dimensions. Results of the simulations have shown that the mean heights of the peaks, depicting the surface roughness of the workpiece and the mean standard deviation sharply decrease as time evolved. Good surface finish was always observed just after 20 pass steps. The work carried out has demonstrated that it is possible to model and simulate abrasive machining processes by implementing simple rules of cellular automata.

Keywords Cellular Automata, Micromachining

1 INTRODUCTION
Abrasive machining is a manufacturing process where material is removed from the workpiece using a multitude of small abrasive particles, where each of these particles acts as a miniature cutting tool. The most important property of an abrasive is its hardness. It must be significantly harder than the workpiece material, to cut effectively. Chips are formed by the mechanism of compression and shear. Examples of abrasive machining are namely; grinding, polishing, honing, lapping etc. There is now a critical requirement for these processes to achieve micro/nanometre surface finishes for many applications in medicine, aerospace, electronics and energy industrial sectors. Models for the simulation of micro abrasive machining are numerous, but the Cellular Automata (CA) method is considered in this article because it exhibits high efficiency when handling arbitrarily complex systems.

The concept of CA was initiated by Von Neumann (1966), as a means of modelling the nature of self-reproduction in biological systems. A cellular automaton is a decentralized computing model, providing an excellent platform for performing complex computation with only local interactions. It consists of a regular array of identically programmed units, called cells which interact with their neighbours subject to a finite set of prescribed rules for local transitions. In practice, cellular automata may be viewed as parallel-processing computers of simple construction Yu et al (2005) and Ganguly et al (2005). CA has been applied to many fields ranging from life sciences to science and engineering. Wolfram (2002) introduced the method to the broad scientific public with the publication of his book, “A New Kind of Science”. One of the most popular applications of CA is in the creation of the ‘game of life’. The program simulates an evolution of society of living organisms Gardner (1970). In materials science, Liu et al (1996) used CA for the modelling of the motion of grain boundaries in evolving microstructures. CA has also been used in machining applications, Karafyllidis and Thanailakis (1995) used CA for simulating the photo resist etching process, Gurney et al (1999) applied CA to roughening in the etching process; Zhu and Liu (2000) developed an anisotropic crystalline etching simulation program for micro electromechanical systems (MEMS) structures, based on CA model; and a two-dimensional CA model was used by Orbanic and Junkar (2004) for the simulation of the abrasive water jet (AWJ) cutting process.

2 THE CELLULAR AUTOMATA (CA) METHOD
A cellular automaton consists of a finite-dimensional lattice of sites whose values are restricted to finite set of integers, $Z_k = \{0,1,2,\ldots,k-1\}$. The value of each site at any time step is determined as a function of the values of the neighbouring sites at previous time step. This can be stated as follows Jen (1990);
\[ x_{i}^{t+1} = f(x_{i-r}^{t}, \ldots, x_{i-1}^{t}, x_{i}^{t}, x_{i+1}^{t}, \ldots, x_{i+r}^{t}) \]

\[ f : Z_{k}^{2r+1} \rightarrow Z_{k} \quad (1) \]

Where \( x_{i}^{t} \) is the value of site \( i \) at time \( t \)

- \( f \) is the rule defining the automaton
- \( r \) is a non-negative integer specifying the radius of the rule

**CA Neighbourhoods**

Each cell in CA can only interact with other cells, which are in its neighbourhood, and this consists of the surrounding (adjacent) cells. The simplest neighbourhood is an elementary system, consisting of a 1-dimensional row of cells, each of which can have the value 0 or 1. The CA can have different neighbourhoods, some of which are, namely: the von Neumann, Moore, first neighbours and one way, as shown in Figure 1. The state of all cells is updated simultaneously in discrete time steps with regard to the state in their neighbouring cells in the previous time step. The algorithm for calculating the new states is known as the local rule of CA (Orbanic and Junkar 2003).

The formula for calculating the rule size space, \( S_{n} \), in n-Dimension (Adapted from Speller et al (2007))

\[ S_{n} = k^{(2r+1)n} \quad (2) \]

where \( k \) is possibilities for each state and \( r \) is the radius of the neighbourhood.

For 1-Dimension CA, if \( r=1 \) and \( k=2 \), the rule space is 256, which are the so-called elementary CA.

For 2-Dimension CA, if \( r=1 \) and \( k=2 \), the rule space is \( 1.341 \times 10^{154} \)!

This type of CA is known as deterministic. For modelling, the rules can be obtained by trial and error or by other methods namely; genetic algorithms (Hajela and Kim (1999, 2000)), genetic programming (Koza et al 1999) and shape grammar (Speller et al 2007). This shows the fundamental problem in the application of deterministic CA, which is the difficulty in finding rules from the large rule space, that exhibit the desired behaviour that one is interested in. Also, it often yields configurations which lack the apparent randomness of real systems. To provide the flexibility in modelling physical systems, probabilistic CA can be used, where rules are probabilities ranging from 0 to 1 and the deterministic CA are the limits in which the probabilities are all 0 or 1 (Gurney et al 1999). For a real machining process case, the values can be decided based on the material removal rates. Orbanic and Junkar (2004), in modelling the Abrasive Water Jet (AWJ), introduced an intensity parameter \( A_{awj} \), which is related to kinetic energy, and depends on water pressure, water mass flow rate, abrasive mass flow rate, type and size of abrasive grains and geometry of the cutting head, a removal resistance parameter \( M_{mat} \), and the cutting velocity. These three input parameters are entered into the CA model to perform the material removal step. Equation 3 shows an example of a rule for material removal:

\[ M_{i,j}^{t+1} = M_{i,j}^{t} - R_{v} \times (A_{i-1,j}^{t} + A_{i+1,j}^{t}) - R_{h} \times (A_{i,j-1}^{t} + A_{i,j+1}^{t}) \quad (3) \]

where \( M_{i,j}^{t+1} \) is the material resistance at time step \( t+1 \),

- \( M_{i,j}^{t} \) is the material resistance at time step \( t \)
- \( R_{v} \) and \( R_{h} \) are the coefficients for vertical and horizontal material removal respectively, and
- \( A_{i-1,j}^{t}, A_{i+1,j}^{t}, A_{i,j-1}^{t}, A_{i,j+1}^{t} \) and \( A_{i,j}^{t} \) are the AWJ intensities in the neighbourhood cells.

### 3 THE MODELLING OF THE POLISHING METHOD

The CA was used to model the workpiece, as having different heights, to represent the roughness; and the forces from the abrasive were modelled to randomly erode the workpiece, as such smoothening out and polishing the surface. A set of random real numbers was generated to model the initial roughness of the surface. Figure 2 shows the surface asperities of a nominal surface in 3D.

**Steps for the CA**
The n-dimensional space is partitioned into discrete subset of finite n-dimensional volumes - cells
- A state is assigned to each cell, using a set of random real numbers
- At every time step, a local neighbourhood is defined for each cell
- A state change rule is defined, which computes the new state of a cell as a function of all cells in the local neighbourhood of that cell
- The simulation proceeds by discrete time steps (Adapted from Janssens (2009))

The Rules
For the 1-D case, using the ‘first neighbours’ neighbourhood, a set of 3 numbers, say (p, q and r) were generated to represent the cells. The cells p and r are ‘neighbours’ of cell q. The value of cell q in the set, was updated according to some simple rules as follows, viz:

\[
\begin{align*}
\text{If } q > p \text{ and } q > r, \text{ then } q &= q - M \text{ (where } M = 1) \\
\text{If } q < p \text{ and } q > r, \text{ and if } q > p \text{ and } q < r \text{ then } q &= M, \text{ (where } M = 0.5) \\
\text{If } q = p \text{ and } q > r, \text{ and if } q > p \text{ and } q = r \text{ then } q &= q - M, \text{ (where } M = 0.25)
\end{align*}
\]

and M is a function of parameters relating to the model

For other possibilities, the value of q is unchanged.
For the 2-D case, using the von Neumann neighbourhood, the rule generations follow the same pattern. A set of 5 numbers say (p, q, r, s, and t) was used for the modelling of the cells. The cells p, q, s and t are the ‘neighbours’ of cell q.
The influential parameters, like hardness of workpiece material, abrasive grit size, bonnet pressure, spindle speed, et cetera, can be mapped into the rules of the CA to provide characteristic material removal rate. The above M parameter can be shown to be a function of the desired influential parameters;

\[ M = f(\text{hardness, abrasive grit size, bonnet pressure, spindle speed .....}) \]

So, M can take values ranging from 0 to 1, like 0.7, 0.6 etc., which would affect the shapes of the roughness graphs, like in Figure 3. The modelling was implemented by using Mathematica 7.

4 RESULTS
1-D Case:
The final surface finish is depicted in Figure 4.
The mean of the peaks' heights for 50 iterations is shown in Figure 5.
The standard deviation of the mean of the peaks' heights for 50 iterations is shown in Figure 6.

2-D Case:
The mean of the peaks' heights for 50 iterations is shown in Figure 7.
The standard deviation of the mean of the peaks' heights for 50 iterations is shown in Figure 8.
The time evolution of the abrasive process is shown in Figure 9.

The results of the simulations have shown that the mean heights of the peaks, depicting the surface roughness of the workpiece and the mean standard deviation sharply decrease as time evolved. Good surface finish was always observed just after 20 pass steps. These present some natures of the practical aspects of polishing. By adjusting the cellular automata rules, it can be possible to model polishing process accurately. The results also show similarities to other research output in improvements of surface finish (Chen and Rowe 1999 and Chen 2009). They studied the surface finish in grinding spark-out, and found that the surface roughness follows a non-linear relationship with the grinding depth (Figure 10). The rate at which the residual stock is removed is slowed down due to plastic pile-up, which in turn slows down the improvement in surface roughness.

5 CONCLUSION
The work carried out has demonstrated that it is possible to model and simulate abrasive machining processes by implementing simple rules of cellular automata. The challenge is to be able to develop a robust polishing model, which will incorporate material properties.

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Figure 1: CA Typical Neighbourhoods (Ollinger 2008)

Figure 2: Surface Asperities of a Nominal Smooth Surface (Stokes 2008)

Figure 3: Variations Dependent on Influential Parameters

Figure 4: The Final Surface Finish in 1-D

Figure 5: The Mean of the Peaks’ Heights for 50 Iterations for 1-D
Figure 6: The Standard Deviation of the Mean of the Peaks' Heights for 50 Iterations for 1-D

Figure 7: Snapshots of the Time Evolution of the Abrasive Process in 2-D

Figure 8: The Mean of the Peaks' Heights for 50 Iterations for 2-D

Figure 9: The Standard Deviation of the Mean of the Peaks' Heights for 50 Iterations for 2-D

Figure 10: Roughness Improvement during Spark-out (Chen 1999)