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Original Citation

Zhang, Xiangchao, Jiang, Xiang and Scott, Paul J. (2009) The Orthogonal Distance Fitting Of Parametric Surfaces For Precision Coordinate Metrology. In: University of Huddersfield Research Festival, 23rd March - 2nd April 2009, University of Huddersfield. (Unpublished)

The Orthogonal Distance Fitting of Parametric Surfaces for Precision Coordinate Metrology

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1. Introduction

To evaluate the form quality of a surface, it is required to compare the measured data with the design template and determine the relative deviation between them. However, the two surfaces are not necessarily well aligned, then we need to transform the measurement data properly to best-match them.

Traditionally, only the vertical deviation is considered in the error metric,

$$E = \sum_{i=1}^{N} [z_i' - f(x_i', y_i')]^2$$

Here $[x_i', y_i', z_i']^T = \mathbf{p}_i' = R\mathbf{p}_i + \mathbf{t}, i = 1,2,\cdots,N$ is the transformed point of an arbitrary measurement point $\mathbf{p}_i = [x_i, y_i, z_i]$ and $z = f(x, y)$ is the nominal function of the design template.

The definition of error distance above does not coincide with measurement guidelines and the estimated fitting parameters will be biased [1]. Consequently we solve this problem by applying the orthogonal distances in the error metric,

$$\min_{\mathbf{R},\mathbf{t}} \sum_{i=1}^{N} \|R\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i\|^2 = \min_{\mathbf{R},\mathbf{t}} \sum_{i=1}^{N} \|\mathbf{p}_i' - \mathbf{q}_i\|^2$$

In the equation, $\mathbf{q}_i = [X_i, Y_i, Z_i]$ is the orthogonal projection of $\mathbf{p}_i$ onto the template.

2. Algorithm of Orthogonal Distance Fitting

Parametric forms are the most general ways to specify a surface, and hence extensively used in engineering design. They represent a surface with,

$$\mathbf{S}(u,v) = [x(u,v),\ y(u,v),\ z(u,v)]^T$$

To solve the optimal rotation matrix R and translation vector **t**, we calculate the projection points and transformation $\mathbf{m} = \{\theta_x, \theta_y, \theta_z, t_x, t_y, t_z\}$ alternately in a nested way,

$$\min_{m} \sum_{i=1}^{N} \min_{u_i, v_i} \|\mathbf{p}_i - \mathbf{q}_i\|^2$$

The closest projection point of each measurement point can be obtained with the Newton-Raphson algorithm [2].

We define $\mathbf{g} \in \Re^{3N}$ with $g_i = \begin{cases} x_i' - X_i & i = 3k-2 \\ y_i' - Y_i & i = 3k-1, k = 1,2,\cdots,N \\ z_i' - Z_i & i = 3k \end{cases}$

So that the objective function becomes,

$$\min_{\mathbf{m}} \sum_{i=1}^{3N} g_i^2 = \min_{\mathbf{m}} \mathbf{g}^T \mathbf{g}$$

The Jacobian matrix $J = \partial \mathbf{g} / \partial \mathbf{m} \in \Re^{3N \times 6}$ can be calculated by,

$$\begin{bmatrix} J_{3i-2} \\ J_{3i-1} \\ J_{3i} \end{bmatrix} = \frac{\partial \mathbf{p}_i'}{\partial \mathbf{m}} - \frac{\partial \mathbf{p}_i}{\partial \mathbf{u}_i} \frac{\partial \mathbf{u}_i}{\partial \mathbf{m}}$$

Here $\mathbf{u}_i = [u_i, v_i]^T$ are the foot-point parameters of $\mathbf{q}_i$, and $\partial \mathbf{u}_i / \partial \mathbf{m}$ will be derived from [3],

$$\frac{\partial}{\partial \mathbf{u}_i} (\mathbf{p}_i - \mathbf{q}_i)^T (\mathbf{p}_i - \mathbf{q}_i) = -(\partial \mathbf{q}_i / \partial \mathbf{u}_i)^T = 0$$

Omit the subscript *i* and rewrite $\partial \mathbf{q} / \partial \mathbf{u}$ as $\mathbf{q}_{\mathbf{u}}$, we obtain,

$$\mathbf{u}_{\mathbf{m}} = \left\{ \mathbf{q}_{\mathbf{u}}^T \mathbf{q}_{\mathbf{u}} - \begin{bmatrix} \mathbf{q}_{uu}^T (\mathbf{p} - \mathbf{q}) & \mathbf{q}_{uv}^T (\mathbf{p} - \mathbf{q}) \\ \mathbf{q}_{uv}^T (\mathbf{p} - \mathbf{q}) & \mathbf{q}_{vv}^T (\mathbf{p} - \mathbf{q}) \end{bmatrix} \right\}^{-1} \mathbf{q}_{\mathbf{u}}^T \mathbf{p}_{\mathbf{m}}$$

And the motion parameters can be updated with the Levenberg-Marquardt algorithm [2],

$$(J^T J + \lambda I) \delta \mathbf{m} = -J^T \mathbf{g}$$

λ is a user-set damping factor.

3. Verification

We tested the performance of this algorithm with various experiments and found its bias of fitted motion parameters and intrinsic characteristics is at least one order better than the traditional algebraic fitting method.

References

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