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Low frequency room excitation using Distributed Mode Loudspeakers

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ABSTRACT

Conventional pistonic loudspeakers excite the modes of an enclosed sound field in such a way as to introduce modal artefacts which may be problematic for listeners to high-quality reproduced sound [1]. Their amelioration may involve the use of highly space-consumptive passive absorptive devices or active control techniques [eg 2,3,4]. Other approaches have concentrated on the design of the driver used to excite the room. Distributed sources ranging from the dipole [5] to more complex configurations [6] may be expected to interact with the room eigenvectors in a complicated manner which may be optimised in terms of the spatial and frequency-domain variance of the soundfield. Recent interest in distributed sources has centred on the Distributed Mode Loudspeaker (DML), and this paper reports an investigation into the interaction of DMLs with modal soundfields. It is shown that large DMLs may be expected to modify the low-frequency soundfield. Producing useful low-frequency control remains difficult but may be achieved in some circumstances.

I - INTRODUCTION

Low frequency modal behaviour has been shown to be a major problem in critical listening rooms [1]. The spatial and frequency response irregularities associated with room modes may distract a careful listener from the correct perception of sound being monitored, especially in the case of small rooms where the sparse concentration of first modes is at audible frequencies. For this reason, the spatial relationship between source and receiver in a room are critical to ensure a correct perception of sound. This has been a subject of research for many acoustic designers when trying to design rooms for critical listening. One way of trying to reduce the effect of room modes is to use space-consumptive passive absorptive devices, which by changing the impedance at the boundaries of the space effectively reduce the Q factor of the predominant modes. This means that modal residues overlap over a wider range of frequencies. Other approaches have used active control techniques which employ other sources in the space to modify the radiation from the primary sources. These techniques have achieved some success in controlling room modes [2,3,4].

This paper results from work growing out of the authors' interests in the control of low frequency modes and the behaviour of DML sources. Although much published work exists concerning the behaviour of DML sources - for example see [8,9,10] for an introduction - little material is available describing the interaction of panel- and room- modal behaviour. Because of their peculiar way of radiating sound, the interaction between these sources and room modes may lead to a way to improve the excitation of frequencies at the lower end of the audio spectrum. This paper sets out a generalised technique for looking at this interaction using analytical and numerical solutions.

A brief review is given of the wave solution for a sound field in a three dimensional enclosure, excited by point [11] and then by distributed plane sources [12]. In this paper the authors take a generic approach based on classical vibrating plate theory [7]. A computer model based on an analytical evaluation of the response of a lightly damped rectangular enclosure to a simply supported plate provides the basis for investigating plate aspect ratios, materials and positioning.

A numerical simplification is introduced which can be used to generate equivalent results, but which does not require the analytical evaluation of the integral of the product of source and room eigenvectors. This numerical method is then used to investigate the behaviour of sources with more complex boundary conditions, and in particular plates with clamped and free boundaries. Finally, some scale model practical measurements are presented to evaluate the success of the concept.

II - THEORY

1. The point source and rectangular piston

As proposed by Morse [11], the steady state sound field solution for a point source in a room may be described as the sum of the contribution of each mode at a certain frequency, depending on source and receiver locations. This is often referred to as a *Modal Decomposition Model.*

$$p(\bar{r},\omega)=\sum_{n}^{(3)}\frac{\psi_{src}\psi_{rcv}}{\omega^2-\omega_n^2-2i\delta_n\omega_n} \quad \textbf{(1)}$$

where,

$$\psi_{src}=\cos\left(\frac{n_x\pi}{l_x}x_0\right)\cos\left(\frac{n_y\pi}{l_y}y_0\right)\cos\left(\frac{n_z\pi}{l_z}z_0\right) \quad \textbf{(1.1)}$$

$$\psi_{rcv}=\cos\left(\frac{n_x\pi}{l_x}x\right)\cos\left(\frac{n_y\pi}{l_y}y\right)\cos\left(\frac{n_z\pi}{l_z}z\right) \quad \textbf{(1.2)}$$

correspond to source and receiver shape functions, respectively.
δ_n is the damping associated with the room boundaries. ω and ω_n are forcing and resonant frequencies respectively. l_x, l_y and l_z are the dimensions of the room.
Index 3 above the summation sign implies a triple summation or integral.

The above equation describes the sound field at a location in a room, when using a point source. Bullmore[12] has also defined the radiation of a rectangular source in an enclosure, vibrating with pistonic movement. The authors have derived a similar solution, which can be evaluated using

$$\psi_{PP}=\frac{1}{k_yk_z}\cos(k_xx_0)[\sin(k_yy_2)-\sin(k_yy_1)].. \quad \textbf{(2)}$$
$$[\sin(k_zz_2)-\sin(k_zz_1)]$$

as the source shape function in (1). The cases where either or both of the k_y and k_z are zero require a different eigenvector solution, given by

$$\psi_{PP}=\frac{1}{k_z}\cos(k_xx_0)[y_2-y_1][\sin(k_zz_2)-\sin(k_zz_1)] \quad \textbf{(2.1)}$$

for $k_y=0$ and $k_z\neq 0$

substituting *y* for *z* and *z* for *y* in the case of $k_z=0$ and $k_y\neq 0$.

For the case where $k_y=0$ and $k_z=0$, then

$$\psi_{PP}=\cos(k_xx_0)[y_2-y_1][z_2-z_1] \quad \textbf{(2.2)}$$

The radiation behaviour of such a source in an enclosed space is expected to be similar to that of a point source if the receiver location is sufficiently distant from the source position. However, if the pressure is calculated very close to the source there will be some differences associated with convergence problems related to the dimensions of the source.

Figure 1 shows the response calculated 1cm away from each source.

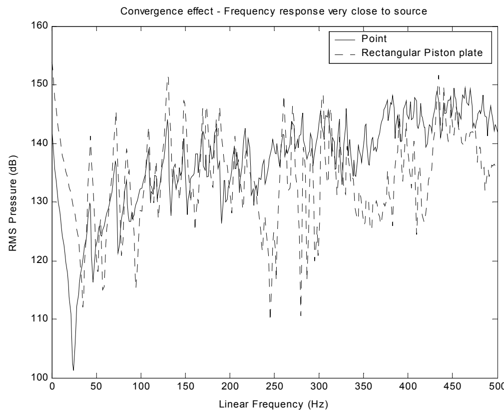

Figure 1

As the source-receiver spacing reduces, the radiation from the source appears to predominate. For a point source the result approaches a line describing spherical radiation into free space, because of the infinitely small size of the source. However, for a pistonically moving plate, the response shows larger maxima and minima due to the distributed nature of the source. As a large radiating surface, each elemental plate area will be responsible for an elemental radiation function. The differing distances from the various radiating areas to the receiver position give rise to effects which are analagous to comb-filtering. As the size of the plate is reduced, the effect is only noticeable at increasingly small distances.

2. Distributed mode sources

As published by Warburton[13] and Avis, Copley[7] the displacement on the surface of a rectangular vibrating plate can be described as:

$$w(x,z)=i\omega F\sum_{n}^{(2)}\frac{\psi_n(x,z)\psi_n(x_0,z_0)}{\Lambda_n[\omega_n^2(1+i\eta)-\omega^2]} \quad \textbf{(3)}$$

It is not surprising that the above equation is similar to equation 1, for modal response in a room. Both situations can be described as modal systems, in terms of three dimensions in the room and two dimensions on a plate. The number of summations is indicated in each case.

Different types of boundary impedances give different solutions and will therefore imply different receiver shape functions. The following are the solutions for simply supported, clamped and free edges on a plate undertaking vibrational movement. *a* and *b* represent the dimensions of the plate in the *x* and *z* dimension.

$$\psi_{SS}=\sin\left(\frac{n_x\pi}{a}x\right)\sin\left(\frac{n_z\pi}{b}z\right) \quad \textbf{(3.1)}$$

where the resonant frequency of the plate is given by

$$\omega_n=\sqrt{\frac{D}{M}\left[\left(\frac{n_x\pi}{a}\right)^2\left(\frac{n_z\pi}{b}\right)^2\right]} \quad \textbf{(3.1.1)}$$

For clamped and free edges the receiver eigenvectors assume a more complex form. Expressions are presented for one spatial dimension only. The shape function is obtained by multiplying the given expression and the one resulting from substituting *x* with *z*. The *n* indexes correspond to the number of nodal lines in the plate for each mode. For example a second mode on a clamped edge plate has three nodal lines – the two edges and the one through the centre line.

For edges clamped at x=0 and x=a:

$$\psi_{Cl}(x) = \cos\left(\gamma\left(\frac{x}{a}-\frac{1}{2}\right)\right) + k\cosh\left(\gamma\left(\frac{x}{a}-\frac{1}{2}\right)\right) \quad \textbf{(3.2.1)}$$

for $n_x = 2,4,6...$

where $k = \frac{\sin\left(\gamma/2\right)}{\sinh\left(\gamma/2\right)}$ and $\tan\left(\gamma/2\right) + \tanh\left(\gamma/2\right) = 0$

$$\psi_{Cl}(x) = \sin\left(\gamma\left(\frac{x}{a}-\frac{1}{2}\right)\right) + k\sinh\left(\gamma\left(\frac{x}{a}-\frac{1}{2}\right)\right) \quad \textbf{(3.2.2)}$$

for $n_x = 3,5,7...$

where $k = -\frac{\sin\left(\gamma/2\right)}{\sinh\left(\gamma/2\right)}$ and $\tan\left(\gamma/2\right) - \tanh\left(\gamma/2\right) = 0$

For edges free at x=0 and x=a:

$$\psi_F = 1 \text{ for } n_x = 0 \quad \textbf{(3.3.1)}$$

$$\psi_F = 1 - \frac{2x}{a} \text{ for } n_x = 1 \quad \textbf{(3.3.2)}$$

$$\psi_F(x) = \cos\left(\gamma\left(\frac{x}{a}-\frac{1}{2}\right)\right) + k\cosh\left(\gamma\left(\frac{x}{a}-\frac{1}{2}\right)\right) \quad \textbf{(3.3.3)}$$

for $n_x = 2,4,6...$

where $k = -\frac{\sin\left(\gamma/2\right)}{\sinh\left(\gamma/2\right)}$ and $\tan\left(\gamma/2\right) + \tanh\left(\gamma/2\right) = 0$

$$\psi_F(x) = \sin\left(\gamma\left(\frac{x}{a}-\frac{1}{2}\right)\right) + k\sinh\left(\gamma\left(\frac{x}{a}-\frac{1}{2}\right)\right) \quad \textbf{(3.3.4)}$$

for $n_x = 3,5,7...$

where $k = \frac{\sin\left(\gamma/2\right)}{\sinh\left(\gamma/2\right)}$ and $\tan\left(\gamma/2\right) - \tanh\left(\gamma/2\right) = 0$

Approximate resonant frequencies for clamped and free edge plates are obtained from

$$\omega_n = \frac{2\lambda h\pi^2}{a^2}\left[\frac{E}{48\rho\left(1-\nu^2\right)}\right]^{1/2} \quad \textbf{(4.1)}$$

$$\lambda = G_x^4 + G_z^4\frac{a^4}{b^4} + 2\frac{a^2}{b^2}\left[\nu H_x H_z + (1-\nu)J_x J_z\right] \quad \textbf{(4.2)}$$

Expressions for G, H and J can be found at the appendix.

In order to simulate the modal distribution of a room when a distributed source is used, the triple integral of the product of source and room eigenvectors must be determined. This task escalates in complexity for the cases of free and clamped edge plates.

3. Radiation from distributed sources in rooms

To be able to evaluate the behaviour of distributed sources in enclosed spaces with low damping, equation 1 needs to be redefined to include a different source shape function.

In the case of a simply supported plate, defined in equation 3.1, the following integral, representing the radiation of a simply supported source in a 3 dimensional room, needs to be solved.

$$\int_{z_1}^{z_2}\int_{y_1}^{y_2}\int_{0}^{l_x} \delta(x-x_0)\sin(k_{py}y_p)\sin(k_{py}y_{p0})\sin(k_{pz}z_p)\sin(k_{pz}z_{p0})... \quad \textbf{(5)}$$
$$\cos(k_{rx}x)\cos(k_{ry}y)\cos(k_{rz}z)dxdydz$$

y_1, y_2 and z_1, z_2 are the limits of plate in the room.

y_p, y_{p0}, z_p and z_{p0} may be defined in terms of y, y_1, z and z_1, and define displacement and exciter positions on plate respectively.

k_p and k_r represent the wave numbers for each dimension of plate and room respectively.

The result is an expression which allows the evaluation of the SPL at any given point in a room, using a simply supported plate. Equation 6 describes the modified source shape function which may be used in equation (1):

$$\psi_{src} = \psi_{SS} = \sum_n^{(2)} \frac{1}{\omega^2 - \omega_n^2 - 2\delta'_n\omega_n}\sin(k_{py}y_{p0})\sin(k_{pz}z_{p0})\cos(k_{rx}x_0)... \quad \textbf{(6)}$$
$$\left(\frac{1}{\left(k_{py}^2 - k_{ry}^2\right)\left(k_{pz}^2 - k_{rz}^2\right)}\right)\left[\cos(k_{py}y_1)A_y + \sin(k_{py}y_1)B_y\right]...$$
$$\left[\cos(k_{pz}z_1)A_z + \sin(k_{pz}z_1)B_z\right]$$

where,

$$A_y = k_{py}\left[\cos(k_{py}y_2)\cos(k_{ry}y_2) - \cos(k_{py}y_1)\cos(k_{ry}y_1)\right] + ...$$
$$k_{ry}\left[\sin(k_{py}y_2)\sin(k_{ry}y_2) - \sin(k_{py}y_1)\sin(k_{ry}y_1)\right]$$

$$B_y = k_{py}\left[\sin(k_{py}y_2)\cos(k_{ry}y_2) - \sin(k_{py}y_1)\cos(k_{ry}y_1)\right] + ...$$
$$k_{ry}\left[\cos(k_{py}y_1)\sin(k_{ry}y_1) - \cos(k_{py}y_2)\sin(k_{ry}y_2)\right]$$

The presence of hyperbolic functions in the modeshapes of clamped and free plates means that the analogous solution in these cases is rather laborious.

Another method of simulating the response of a distributed-type source in a room is to use a superposition technique, where the response of the room to a large plate may effectively be calculated from the contribution of a number of smaller pistonically radiating plates. The phase and amplitude of velocity of each of these pistonic plates is calculated using the group of equations (3) for simply supported, clamped and free edges. The results for analytical and superposition models for a simply supported plate are similar. The superpostion technique can therefore be used to evaluate the response of clamped and free edge plates as sources in a room.

The resulting expression for pressure in the room is:

$$p_{\sup}\left(\bar{r},\omega\right)=\sum_{h}^{\substack{numberof\\elements}} i\omega\rho_0 c^2 Q_h \sum_{n}^{(3)} \frac{\psi_{PP}(h)\psi_{rcv}}{\omega^2 - \omega_n^2 - 2i\delta_n\omega_n} \quad \textbf{(7)}$$

where Q_h, represents the complex amplitude of velocity for each element, and takes the form of 3.1, 3.2 or 3.3 depending on the type of plate in use.

III - ANALYSIS

1. Applications to low frequency room excitation

Using the superposition expressions for radiation of distributed sources in a room, a simulation may be carried out to determine the transfer function for a given source/receiver combination. The first case to consider is that of a room which is long in one dimension. This is effectively a rectangular duct (along the x dimension), where length axial modes (the largest dimension) are at much lower frequencies than other modes. If the source is then placed normal to this dimension and next to one of the walls (at a pressure antinode), it is expected that strong axial modes will be excited in the plane of duct length. Figure 2 shows simulation results for each plate type along with excitation by a pistonic plate of similar size. In this case each plate is almost as large as the parallel wall just behind it. The centre point of the source is defined at the centre of the room wall where it is placed. The parameters of each plate are the same. These were chosen arbitrarily so that the simply supported plate would produce a first plate resonance at 20Hz. This choice dictates plate physical parameters that may not represent real physical materials.

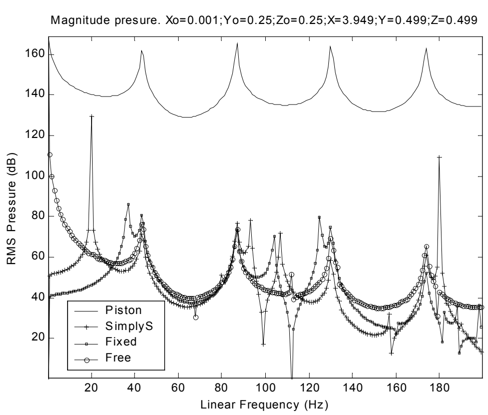

Figure 2

It is clear that the pistonic plate excites the strong axial modes which occur at 44Hz, 88Hz and higher harmonics. All distributed mode sources superimpose their own resonant frequencies to produce a somewhat different overall response. At first analysis the interaction between plate source and axial room modes seems to be common to all types of plate. That is, even though each plate imposes its own specific resonant frequencies, the main axial room modes are still noticeable for all cases. There is no apparent benefit in terms of controlling any of the room modes. For regions such as that between 88Hz and 130Hz, the average frequency variance is smaller due to the addition of more resonances. This suggests that the overall perception of room modes may be altered by the presence of extra plate resonances in a favourable manner. This suggestion requires verification by rigorous subjective testing.

Although the excitation of modes normal to the plate are of interest, it is those in the plane of the plate which are most likely to be affected by the modal behaviour of the source. If the simulation is changed such that the plate is placed along the longest room dimension, the pressure variance associated with strong axial modes appears along the length of the plate rather than perpendicular to it. Figure 3 show this.

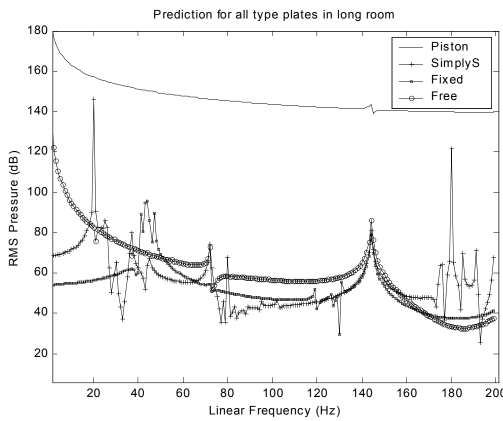

Figure 3

Here, strong axial room modes form a series 72Hz, 144Hz, etc. The plate parameters are calculated so that the first resonance for a simply supported case is at 20Hz. Again, this is achieved with the use of arbitrary parameter values, which have been retained for the other boundary cases.

It is interesting to note that the pistonic plate does not excite the anticipated axial modes along the room. Other simulations by the authors show that if the size of the pistonic plate is reduced to a certain extent, the 144Hz mode is more noticeable, although the 72Hz mode is still not excited at all. Only when the centre point of the plate is moved off centre does the first resonant mode become noticeable. This effect is expected, as the pistonic plate is driving positive and negative phase points of the first room mode. This also gives further insight into how distributed sources may affect the driving of low frequency modes in rooms.

The distributed mode plates excite modes at anticipated frequencies, although at reduced levels. It can be said that some degree of control, especially for the first mode, is achieved. The clamped edge plate fails to excite the first mode, in common with the pistonic plate. The model distribution for this type of plate may be directly involved in this effect. As was the case in figure 2, the simply supported resonances are more numerous and are superimposed on the overall room frequency response.

Further predictions using these models show that if a plate could be artificially manipulated such that it vibrated only in even combinations of modes (such as (2,2), (4,2) or (10,2)) then depending on the source location with regard to room mode shapes, some of the room modes would not be excited at all. These combinations of modes in plates are peculiar as for each positive phase modal 'cell' there exists a negative phase counterpart. As these regions are equal in number, cancellation occurs.

This scenario is only achievable in simulation, not least because practical DMLs have large damping and low Q modes. Residues from adjacent modes combine to make the surface displacement on the plate more complex than the binary phase model anticipated in the 'cell-cancellation' hypothesis [7]. The location of plate exciter(s) may also introduce significant assymetries which complicate the plate radiation further.
It has been suggested that the mechanisms involved in driving room modes may be associated with phase-matching of the

radiating platemode with room eigenvectors. The following simulation (Figure 4) shows this effect for the first axial mode of a room whose length greatly exceeds its width or height. The driving plate chosen is of the simply supported type and covers the entire length of the room.

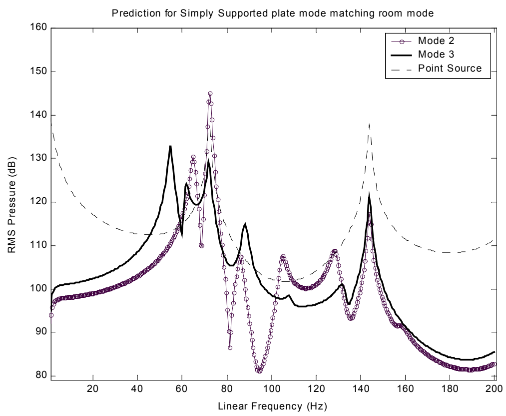

Figure 4

Once again, expected modes are at 72Hz and 144Hz. The first case tested is that where the 2nd mode on the plate matches the frequency of the 1st room mode. In general terms, the plate is divided into two modal cells with anti-phase surface displacement. Compared to the point source result, it can be seen that the 1st room mode is still driven strongly, and the 2nd room mode is slightly attenuated. This behaviour is expected given that at this mode the plate is effectively a dipole source[5]. If however the plate is defined so that its 3rd mode matches the 1st room mode, then the effect is a clear reduction of peak amplitude on that mode.

This result is indicative of the interaction of room and DML, in terms of a visualisation of the behaviour of the spatial integration of their modeshape products. This may be instructive in the practical control of room modes using DML sources, enabling an insight into the significance of plate physical parameters and also positioning within the enclosed sound field.

IV - PRACTICAL MEASUREMENTS

In order to identify the differences between an enclosure excited by a point source and a vibrating plate, an experiment was carried out using a scale model of a real listening room. For the purpose of the experiment, it can be said that the room is effectively a lightly damped rectangular enclosure. The sources used were a 6 cm diameter pistonic driver in a sealed box, and an 80 by 60cm DML made of a composite panel board material mounted on a metal frame, with a single exciter attached to the panel.

A dual FFT analyzer was used, and the noise signal driving the speaker was compared to the signal received using a microphone at a fixed location in the scale room. The results are presented in terms of *p/V* transfer function magnitudes. The measurement point was chosen to be one of the corners of the room where all modes have anti-nodal points.

As a reference the modal behaviour of the room was measured by placing the pistonic source at the corner opposite the microphone. A prediction model for a point source was used which compared to this measurement favourably. This measurement was used in the determination of simulation parameters such as room absorption. The model used was 1:5 scale, which means that the results shown represent frequencies up to 100Hz, concentrating on the very low end of the auditory range and where modal density in small rooms is sparse and problematic. Measurements were then made using the plate at various selected locations in the scale room.

Figure 5 shows the resulting modal response when the plate is placed flat in the corner opposite the microphone, with the longest plate dimension along the length of the room.

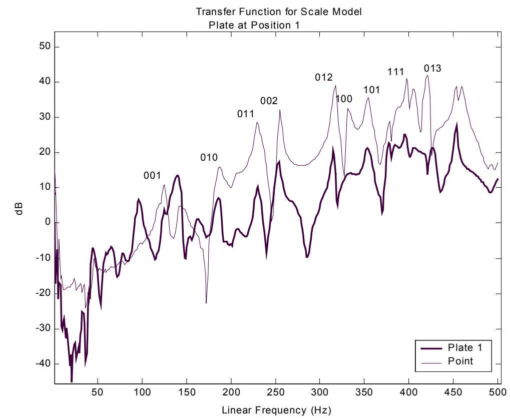

Figure 5

The amplitude of peaks associated with room modes is smaller when using the plate. At modes such as [011], the peak amplitude is reduced by up to 10dB due to the interaction between DML and room shape functions. The region between 300Hz and 500Hz also shows an attenuation in level using the DML, but the frequency domain variance of the results is similar to that recorded with the point source suggesting that this effect is simply due to a difference in sensitivity between the two sources and is not significant in terms of the excitation of the room modal structure.

A further test was carried out, now placing the plate in the middle of the floor, with its longest dimension running perpendicular to the length of the room. The plate used is almost as long as the width of the room, so that when placed in this manner it spans the width of the model. Figure 6 shows the resultant sound field.

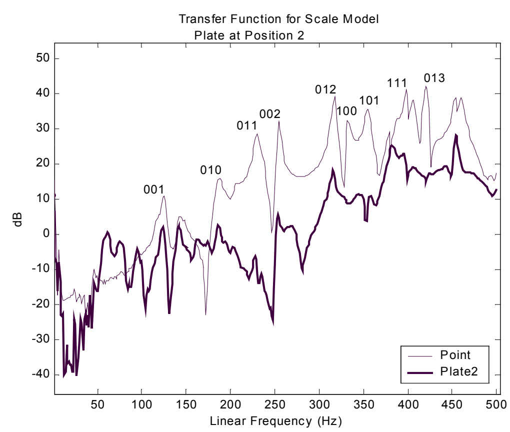

Figure 6

Some modes are clearly not excited by the DML. Careful observation reveals that in general the plate does not drive room modes associated with the *y dimension*, which describes the width of the room. Other plate positions were measured and the general trend is that although many of the main room modes are still present, their peak amplitude is reduced with some modes attenuated very significantly at the reciever location. The overall response, although still showing peaks and dips, shows less frequency domain variance. This is a significant result, since it arises from the interaction of source and room modeshapes.

The DML plate models used in this and earlier [7] work are based on classical plate theory for an isotropic panel with certain well-defined boundary conditions. The practical measurements reported above employ a DML which cannot be strictly regarded as isotropic; neither are its boundary conditions well-defined. Figure 7 shows a comparison between the prediction model and a scale model measurement.

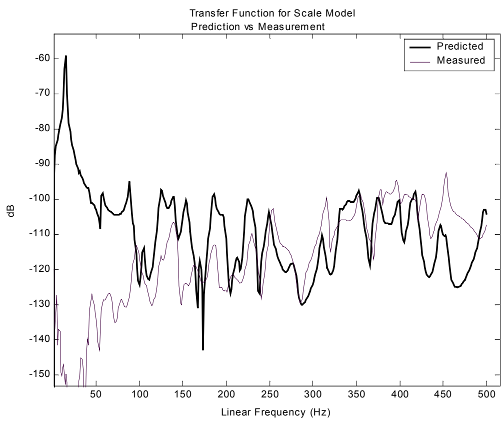

Figure 7

The two lines show agreement at some of the room modes. Some differences, especially at the lowest frequencies are associated with limitations in the accuracy of plate and exciter physical parameters. However, the modelling technique has provided a useful insight into the physical processes which govern DML radiation in an enclosed rectangular volume, and the trends noted in the numerical models have been clearly demonstrated in practical measurement.

V - CONCLUSION

A derivation for the radiation of a rectangular piston in an enclosure has been given. An analytical solution for a rectangular plate with simply supported boundary impedances has also been presented. A prediction model based on the superposition principle has been used to calculate numerical solutions for the interaction of plates with more complex boundary conditions in rooms. It has been shown that DML sources excite room modes in a different way to point sources. The distributed nature of the source accounts for these differences as indicated in the case of a pistonically moving plate when placed along the dimension of the modes to be controlled.

The prediction model of the interaction of plate and room modes is indicative of the mechanisms involved, and provides useful insight into the effect of changing plate parameters and room dimensions. When designed and placed in specific positions of the room, the excitation of detrimental room modes can be controlled and reduced. These models have limited utility in predicting the exact behaviour of a given plate in a given room, since the identification of physical parameters and boundary impedances necessarily limits the model accuracy.

Prediction results indicate another possible benefit in using distributed sources for low frequency excitation. This is associated with the introduction of closely spaced plate resonant peaks near to the problematic room modes. The higher concentration of resonances may alter the perception of a listener depending on Q factor and frequency. This, however, is a concept that will require some further subjective testing.

Scale model measurements have indicated that if the plate is defined to span one of the dimensions of the room, the modes associated with this dimension may be beneficially reduced. With accurate values for plate physical parameters, the prediction model can be used to define realizable plates with complex modal behaviour, which when positioned optimally in a given room will contribute to a more controlled and subjectively more accurate low frequency excitation.

If the modal frequencies are successfully controlled there is evidence to support [2] that also the low frequency spatial variance associated with these will be reduced.

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Appendix

FREQUENCY COEFFICIENTS IN EQUATION 4.2

Boundary condition	$\mathbf{n_x}$	$\mathbf{G_x}$	$\mathbf{H_x}$	$\mathbf{J_x}$
Fixed	2	1.506	1.248	1.248
	3,4,5…	$n_x - \frac{1}{2}$	$\left(n_x - \frac{1}{2}\right)^2 \left[1 - \frac{2}{\left(n_x - \frac{1}{2}\right)\pi}\right]$	$\left(n_x - \frac{1}{2}\right)^2 \left[1 - \frac{2}{\left(n_x - \frac{1}{2}\right)\pi}\right]$
Free	0	0	0	0
	1	0	0	$12/\pi^2$
	2	1.506	1.248	5.017
	3,4,5…	$n_x - \frac{1}{2}$	$\left(n_x - \frac{1}{2}\right)^2 \left[1 - \frac{2}{\left(n_x - \frac{1}{2}\right)\pi}\right]$	$\left(n_x - \frac{1}{2}\right)^2 \left[1 + \frac{6}{\left(n_x - \frac{1}{2}\right)\pi}\right]$

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