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**Projector calibration method based on optical coaxial camera**

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**ABSTRACT**

This paper presents a novel method to accurately calibrate a DLP projector by using an optical coaxial camera to capture the needed images. A plate beam splitter is used to make imaging axis of the CCD camera and projecting axis of the DLP projector coaxial, so the DLP projector can be treated as a true inverse camera. A plate having discrete markers on the surface will be designed and manufactured to calibrate the DLP projector. By projecting vertical and horizontal sinusoidal fringe patterns on the plate surface from the projector, the absolute phase of each marker's center can be obtained. The corresponding projector pixel coordinate of each marker is determined from the obtained absolute phase. The internal and external parameters of the DLP projector are calibrated by the corresponding point pair between the projector coordinate and the world coordinate of discrete markers. Experimental results show that the proposed method accurately obtains the parameters of the DLP projector. One advantage of the method is the calibrated internal and external parameters have high accuracy because of uncalibrating the camera. The other is the optical coaxes geometry gives a true inverse camera, so the calibrated parameters are more accurate than that of crossed-optical-axes, especially the principal points and the radial distortion coefficients of the projector lens.

**Keywords:** DLP calibration, fringe projection, optical coaxial system, absolute phase calculation.

**1. INTRODUCTION**

With the mature of Digital Micromirror Device (DMD) techniques, Digital Light Processing (DLP) projectors based on DMD have been widely utilized in 3D shape measuring systems to project digital structure-light patterns due to their advantages of high contrast, high intensity, flexible programming, and cost efficiency [1]. 3D calibration of these systems is an important step to obtain the accurate shape data [2, 3]. In actual applications, most off-the-shelf DLP projectors are made for lecture presentation and have significant lens distortion due to nonlinear factors which cannot be compensated by the classical pinhole camera model [4]. Also, because of the manufacturing and assembling deviations of the projecting lens, nonlinear projection distortion of lens is inevitable, especially for measuring large-size objects. Therefore, it is vital to calibrate the geometric parameters of DLP projectors to improve the precision of 3D shape measuring systems.

A projector has the same physical principles and mathematical models as a camera, which can be regarded as an inverse camera [5]. In fact, the projector cannot capture images like a camera. Therefore, many projector calibration methods have been studied and they can be divided into three categories. The first category is to use a calibrated camera to determine the geometric parameters of projectors. Song et al proposed to first calibrate the CCD camera and then the DLP projector [5]. Falcao et al rely on a pre-calibrated camera to establish the relationship between the camera and a projector [6]. Ma et al have studied lens distortion of the projector [7]. However, projector calibration still depends on camera calibration. Because these methods used a calibrated camera to determine the parameters of projectors, the accuracy of the projector calibration is influenced by that of the camera calibration. The error of the calibrated camera will accumulate to the projector. Some experiments demonstrated that the error of projector calibration is one order of magnitude larger than that of camera calibration [5].

The second category of projector calibration is to find one homography transformation between a calibration plane and the projector image plane [8-10]. Chen et al presented a two-stage easy-to-deploy strategy to robustly calibrate both

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intrinsic and extrinsic parameters of a projector [9]. Based on the incremental strategy, the calibration process first establishes a set of initial parameters, and then it iteratively upgrades these parameters incrementally using the projected and captured images of dynamically-generated calibration patterns. Anwar et al and Drar`eni et al adopted different approaches which need neither a calibrated camera nor a printed pattern [10, 11]. Instead, they moved the projector to several locations so that the projected calibration pattern on a fix plane changes the shape. The homography between projector and camera was found to calculate the parameters of the projector. However, moving the projector might be inconvenient, or impossible in the mounted systems. Therefore, these methods are complicated in actual applications because of an iterative procedure. Moreover, the calibrated results are not high because homography is a linear operator and cannot model non-linear distortions of the projector lenses.

The third category of projector calibration is to create DMD images. Zhang et al [12] and Li et al [13] presented novel projector calibration methods by using DMD images. The DLP projector was treated as an inverse camera to create DMD images, which need establishing the correspondence between camera pixels and projector pixels. Instead of directly computing the corresponding point of the projector from the images captured by the camera, they created new synthetic images from the projector’s view. However, the used system structure is unreasonable because of the crossed-optical-axes geometry of the projector and camera. Therefore, the projector is not a true inverse camera.

The existing methods either have complicated procedures by iteratively adjusting a projected pattern until it overlaps a print pattern, or low accurate obtained parameters because of dependence on the calibrated camera. This paper presents a novel method to accurately calibrate a DLP projector by using an optical coaxial camera to capture the needed images. A plate beam splitter is used to make imaging axis of the CCD camera and projecting axis of the DLP projector coaxial, so the DLP projector can be treated as a true inverse camera. The proposed method can accurately calibrate a DLP projector by establishing corresponding point pair between projector pixel coordinate and world coordinate of discrete markers on a plate surface. The corresponding projector pixel coordinate of each marker is determined by measuring its absolute phase from the projected vertical and horizontal sinusoidal fringe patterns on the plate surface. Like camera calibration by using a checkerboard [4], if there are several corresponding point sets at different orientations between the world (object) coordinate system and the projector pixel coordinate, the internal and external parameters of the DLP projector can be calibrated.

2. PRINCIPLE

2.1 The coaxial structure of camera and projector

Because a projector can be regarded as the inverse of a camera, they have the same physical principles and mathematical models. The projector cannot capture images like the camera. However, the projector can indirectly capture images by establishing a relationship between the projector and the camera, so that the projector is calibrated like camera calibration [13].

When calibrating the camera via a checkerboard, the calibration plate needs to be arbitrarily placed at several symmetrical orientations along the imaging axis. Otherwise, the obtained results of camera parameters by optimizing algorithm will not be accurate. For the same reason, a calibration plate for calibrating projectors needs to be placed at several symmetrical positions along the projecting axis of the projector. If the calibration plate is placed symmetrically along the imaging optical axis of the camera for the crossed-optical-axes geometry of the projector and camera, the plate is unsymmetrical along the projecting axis and the calibrated parameters of the projector are not accurate. The calibration plate can be placed along the imaging axis of the camera and along the projecting axis of the projector, as demonstrated in Figs.1 (a) and (b) respectively. In both situations, the calibration plate cannot be symmetrically placed along the two axes because of the crossed-optical-axes geometry of the projector and camera.

The coaxial structure of the projector and camera can solve this problem by using a plate beam splitter, as shown in Fig. 2. The plate beam splitter makes imaging axis of the CCD camera and projecting axis of the DLP projector coaxial, so the DLP projector can be treated as a true inverse camera. A black plate is used to avoid the effects of environment and objects in front of the camera on the captured calibration plate image, as shown in Fig. 2. After building the pixel relationship between the projector and the camera, the projector can “capture” images like a camera. Therefore, the calibrated parameters of coaxial structure of the projector and camera are more accurate than that of crossed-optical-axes, especially for principal points and radial distortion of the projector lens.
2.2 Realization of the coaxial structure

A beam splitter with both the transmittance and the reflectance of 50% is used to realize the coaxial structure, as shown in Fig. 2. The DLP projector projects fringe patterns onto a calibration plate. The diffuse reflected light by the plate is incident to the beam splitter from another direction. The reflected light on the calibration plate is captured by the CCD camera. The following two steps are needed to ensure the coaxial structure of the projector and camera.

Step 1: The projector and the camera’s optical axes are in the same plane parallel to the surface of an optical tabletop.
a) Height and angle adjustment of the projector. In principle, the optical projecting axis of the projector is at the center position of the projected images. Therefore, a white cross at an image center can be generated by software and then projected onto a white plate from the projector. When the white plate is moved back and forth on an accurate translating stage, the projected cross position on the plate surface does not move up and down, which proves that optical projecting axis of the projector is parallel to the optical tabletop. The projected cross position on the white plate surface is marked as a reference for the following camera adjustment.

b) Height adjustment of the camera. To adjust the imaging axis of the camera having the same height to the optical tabletop surface, the marked white plate in Step 1 is moved back and forth in front of the camera along the translating stage. If the captured marker by the camera is always being center position in the living images, the imaging axis of the camera is parallel to the optical tabletop surface and has the same height as the projecting axis.

Step 2: Angle adjustment of the plate beam splitter. A two-dimensional Manual Tilt Platforms is used to ensure the beam splitter perpendicular to the optical tabletop surface, so that the diffuse reflecting light from the calibration plate can be captured by the CCD camera. At the same time, a black plate having a diffuse reflective surface is used to avoid the effects of environment and objects in front of the camera so that the captured images of the calibration plate are clear.

2.3 Mathematical model of the projector

The mathematical model of the projector is demonstrated in Fig.3.

![Figure 3. Mathematical model of the projector](image)

O-XYZ and o-xyz are the 3D world coordinate system and the projector coordinate system, respectively. o-mn is the 2D projector pixel coordinate system. \( P_w = (X, Y, Z, 1) \) are the 3D homogeneous coordinates of a point in the world coordinate system. \( P_u = (m, n, 1) \) are the corresponding 2D homogeneous coordinates of the point in the projector pixel coordinate system. Like a camera model [17], the projector model is represented as

\[
s[m \ n \ 1]^T = A[R \ T][X \ Y \ Z \ 1]^T
\]

where, \( R \) is a matrix representing the three rotating angles, \( T = [T_x \ T_y \ T_z] \) is a vector representing the three translating distance, \( s \) is an arbitrary scaling factor and \([ \ ]^T\) denotes the transposition. Three rotating angles and three translating...
distance are called as external parameters of the projector. A is a matrix of partial intrinsic parameters of the projector, which can be represented as

\[
A = \begin{bmatrix}
    f_m & 1 & m_n \\
    0 & f_n & n_n \\
    0 & 0 & 1
\end{bmatrix}
\]

where \([m_n, n_n]\) are the pixel coordinates of the principal point, \([f_m, f_n]\) are the focal lengths along m and n-axis direction. To calibrate the projector model, we use a planar target assumed to lie on the plane \(Z = 0\). Thus, the 3x4 projection matrix \([R, T]\) reduces to a 3x3 homography. In practice, due to the distortion of the projector lens, point \(P_w\) at the world coordinate system does not directly correspond to point \(P_u\) at the projector pixel coordinate system, but at distorted position of point \(P_d\). To compensate for the lens distortion, the projection model is augmented with two radial distortion terms \(k_1\) and \(k_2\), two tangential distortion terms \(p_1\) and \(p_2\). The coordinates \(P_d\) can then be corrected using the following relation

\[
P_d = P_u + \delta(P_d, P_u)
\]

where,

\[
\delta(P_d, P_u) = \begin{bmatrix}
    m_n(k_1r_u^2 + k_2r_u^4) + 2p_1m_nr_u + p_2(r_u^2 + 2m_u^2) \\
    n_n(k_1r_u^2 + k_2r_u^4) + 2p_2m_nr_u + p_1(r_u^2 + 2n_u^2)
\end{bmatrix}
\]

\[
r_u^2 = m_u^2 + n_u^2
\]

\([k_1, k_2, p_1, p_2]\) are called distortion coefficients of the projector lens. The four coefficients along with principal point \([m_n, n_n]\) and focal lengths \([f_m, f_n]\) are internal parameters of the projector lens. Projector calibration means to obtain the eight internal parameters \([m_n, n_n, f_m, f_n, k_1, k_2, p_1, p_2]\) and six external parameters \([R, T]\).

### 2.4 Relationship establishment of a projector and a camera

According to the proposed model of Eq. (1), it is possible to calibrate a projector by a plate having discrete markers on the surface provided the corresponding projector pixel coordinate of each marker can be accurately determined in the projector pixel coordinate system. A white plate with a scattered surface was designed and manufactured for projector calibration [3]. There are 9x12 discrete black hollow ring markers on the surface, as illustrated in Fig. 4. The separation of neighboring markers along vertical and horizontal direction has the same value of 15 mm with an accuracy of 1 µm.

In order to build up the correspondence of discrete markers between the world coordinate system and the projector pixel coordinate system at each plate orientation, vertical and horizontal sinusoidal fringe patterns are generated in a computer and projected onto the plate surface through the DLP projector, respectively. At each fringe direction, twelve sinusoidal fringe patterns (including three fringe pattern sets with the optimum numbers and each set having four phase-shifted fringes) are used to calculate the absolute phase of the center position of each marker and one texture map illuminated under white illumination to determine the center position of each marker [15]. The main procedure includes the following several steps to establish the pixel relationship between the projector and the camera, as shown in Fig. 5.

A set of horizontal and vertical sinusoidal fringe patterns are projected onto the calibration plate surface at each plate orientation. At each fringe direction, the fringe numbers are 56, 63, 64. The coaxial camera captures the fringe pattern images and the texture map illuminated under white light.
Sub-pixel coordinates of the center position of each marker on the calibration plate are extracted by using an automatic extraction algorithm from the captured texture map image.

1) The horizontal and vertical wrapped phase maps are calculated from the captured horizontal and vertical sinusoidal fringe patterns respectively by using the four-step phase-shifting algorithm. Assuming four captured vertical fringe patterns are \( I_1, I_2, I_3, I_4 \) and they have \( \pi/2 \) shift in between, a wrapped phase map is calculated by using the four-step phase-shifting algorithm

\[
\theta = \tan^{-1} \left( \frac{(I_4 - I_2)}{(I_1 - I_3)} \right)
\]

After applying Eq. (4) to the three fringe pattern sets, three wrapped phase maps are obtained. Because of the inverse trigonometric function in Eq. (4), the obtained modulo \( 2\pi \) phase information needs to be unwrapped. In order to obtain the absolute phase information at the center of each marker, the optimum multiple-fringe numbers selection method is used to calculate the absolute fringe order pixel by pixel [16]. With three fringe sets having fringe numbers of 56, 63 and 64, the method is referred to as the optimum three-fringe numbers selection method, which is used in the proposed method. This method resolves fringe order ambiguity as the beat obtained between 64 and 63 is a single fringe over the full field of view and the reliability of the obtained fringe order is maximized as fringe order calculation is performed through a geometric series of beat fringes with \( N, N-1, N-\sqrt{N} \). For horizontal fringe patterns, the corresponding wrapped and absolute phase maps are calculated.

2) The absolute phase of each marker along vertical and horizontal direction is directly extracted from the two obtained absolute phase maps, denoted as \( \phi_m, \phi_n \). The corresponding point \((m, n)\) in the projector pixel coordinate system is calculated by the following equations

\[
m = M \phi_m / (2\pi F)
\]

\[
n = N \phi_n / (2\pi F)
\]

where, \( M \) and \( N \) are the vertical and horizontal resolution of the DLP projector. \( F \) is the projected fringe numbers for phase calculation.
2.5 Calculation of projector parameters

The calibration plate are placed at several orientations in front of the DLP projector. At each plate orientation, the previous-mentioned steps are repeated to get the corresponding point of each markers in the projector pixel coordinate system. Similar to camera calibration using a checkerboard [17], the DLP projector can be calibrated to determine the internal and external parameters by using the obtained projector pixel coordinate on the white plate and their corresponding world coordinate [18]. To verify the accuracy of the proposed projector calibration method, the obtained projector pixel coordinates can be re-projected onto the calibration plate. If the projected point coincides with the corresponding marker center on the calibration plate, the pixel relationship between the projector and the CCD camera is accurately established [19].
3. EXPERIMENTS AND RESULTS

3.1 Experimental system

The system comprises a portable DLP (Digital Light Processing) video projector, a 3-CCD color camera with IEEE 1394 port, a beam splitter and a personal computer (PC), as illustrated in Fig. 6. The projector is from BenQ (Model CP270) with one-chip digital micro-mirror device (DMD) and a resolution of up to 1024 x 768 pixels (XGA). The 3-CCD camera from Hitachi (Model HVF22F) has a resolution of 1360 pixel_1024 pixel. The camera has a standard zoom lens from Pentax with focal length from 12mm to 36 mm and an adjustable aperture. The beam splitter has the transmittance and the reflectance of 50%. The personal computer provides system control. The PC graphics card is setup to drive two monitors, one for the DLP and the other for the control software and viewing the captured data. A white cross in center of the generated image is projected onto the middle of the field from the projector to locate the object in the calibrated field of view. The software can also display the intensity profile in the three color channels along the middle row direction in order to give a proper fringe modulation depth and to avoid intensity saturation.

![Figure 6. The experimental system. (a) a DLP projector, (b) a color 3-CCD camera, (c) a beam splitter, and (d) a personal computer](image)

3.2 Experiments and results

The DLP projector can be calibrated to determine the internal and external parameters by using the obtained corresponding points in projector pixel coordinate system at different orientations. The calibrated internal parameters of the projector are listed in Table 1. The calibration plate orientations (external parameters of the projector) are demonstrated in the three-dimensional coordinates in Fig.8.

Table 1 Internal parameters of the calibrated projector

<table>
<thead>
<tr>
<th>parameters</th>
<th>m_0 (pixel)</th>
<th>n_0 (pixel)</th>
<th>F_m (pixel)</th>
<th>F_n (pixel)</th>
<th>K_1</th>
<th>K_2</th>
<th>p_1</th>
<th>p_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>coaxial</td>
<td>505.72</td>
<td>219.38</td>
<td>2175.85</td>
<td>2160.49</td>
<td>-0.11138</td>
<td>0.78327</td>
<td>-0.00902</td>
<td>0.00350</td>
</tr>
<tr>
<td>Non-coaxial</td>
<td>483.21</td>
<td>223.87</td>
<td>2209.62</td>
<td>2205.82</td>
<td>-0.11413</td>
<td>0.90586</td>
<td>-0.01069</td>
<td>0.00350</td>
</tr>
</tbody>
</table>
For the coaxial structure, the points are much closer to the ideal principal points 512 and 384. In order to evaluate the calibrated parameters, the re-projection error distributions of the projector are demonstrated in Figure 9. For the coaxial structure of the projector and camera, the average re-projection errors are 0.07646 and 0.09627, respectively. While for the crossed-optical-axes geometry of the projector and camera, the average re-projection errors increase to 0.08779 and 0.09849, respectively.

(a) \( \text{err} = [0.07646 \ 0.09627] \)  
(b) \( \text{err} = [0.08779 \ 0.09849] \)

Figure 9. Re-projection error. (a) Re-projection error of coaxial structure, and (b) Re-projection error of crossed-optical-axes structure

The experimental results testify that the proposed projector calibration method accurately determines the internal and external parameters of DLP projectors. Since the white plate can be placed randomly in the projecting field volume, the proposed projector calibration method is simple and flexible.

4. CONCLUSION

In conclusion, a novel projector calibration method has been proposed by using an optical coaxial camera to capture the calibration plate images. A plate beam splitter has been used to make imaging axis of the CCD camera and projecting axis of the DLP projector coaxial, so the DLP projector can be treated as a true inverse camera. Vertical and horizontal absolute phase has been used to establish the point pair correspondence of the markers between the world coordinate system and the projector pixel coordinate system. The experimental results demonstrate the accuracy and flexibility of
the proposed projector calibration method. Because the proposed method needs neither a calibrated camera nor a complicated procedure, it can give high accurate geometric parameters of the projector by using an optical coaxial camera in a simple and flexible way.

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REFERENCE