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# A hierarchical category model for geometrical product specifications (GPS)

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**Abstract**

International standards for tolerancing (ISO GPS) have undergone considerable evolutionary changes to meet the demands of the modern information age. Their expanding quantity and complexity have proposed a great obstacle to their informatisation progress. In this paper, a solution to reduce the complexity is coarse-graining the GPS knowledge into five hierarchy levels. A high-level abstraction mathematical theory − category theory is employed to model the GPS hierarchy, in which structures are modelled by categorical concepts such as categories, morphisms, pullbacks, functors and adjoint functors. As category theory is hierarchically structured itself, it can prove that the multi-level GPS framework is constructed in a rigorous manner and is expected to facilitate the future autonomous integration between design and measurement in the manufacturing system.



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*Keywords:* Geometrical Product Specifications (GPS); category theory; hierarchy; coarse-graining

## 1. Introduction

Over the last decade, International standards for tolerancing (ISO GPS) have undergone considerable evolutionary changes to meet the demands of the modern information age [1,2]. The standard system is expanding on both quantity and complexity, which have proposed a great obstacle to its informatisation progress [3,4]. There have been continuing efforts that directed toward developing knowledge models of ISO GPS [5–10], as well as incorporating GPS information into computer-aided systems [2,11]. Yet there is no comprehensive tool/model that naturally support the structural knowledge of GPS and enriched GPS concepts and semantics.

In the GPS system the interactions between design (specification) and measurement (verification) are dual. An inspector measures the surface with guidance from technical drawing/symbols. The observed (measured) data can only be considered meaningful if it can be interpreted in the range of theoretical model. When the meaningfulness of the observed data is proved the conformance process can then be taken place. The two stable mappings between the specification and verification serve the structure of adjoint functors in category theory, a high-level abstraction mathematical theory which was invented by Samuel Eilenberg and Saunders Mac lane in 1942-1945 [12]. The concept of adjoint functors is seen as central to category theory. Some category theorists consider adjoint functors as dictionaries that translate back and forth between categories[13]. If the two categories are two languages (say English and Chinese) which are equally expressive, then a good dictionary will be an explicit exchange of ideas. Employing the adjoint functors and other structures of category theory to translate specification information into verification and vice versa has the great potential to bring the ISO GPS system toward an autonomous manner.

From 1980s to the present, we have seen many successful category-theoretical applications in theoretical computer science, theoretical physics and biological. Researchers are using category theory to study complex systems [14], cognitive neural networks [15,16], biological networks [17] and model management [18]. Category theory has also been employed for the framework of knowledge representation in relational [13] and object-oriented styles [9,10]. Using object-oriented language to code the categorical model has recently been proved successful in the case of surface texture [19], one of the most complicated geometrical specification and verification systems in GPS.

In this paper, using the categorical model, structural knowledge of GPS is coarse-grained into five hierarchy levels, which is expected to reduce the complexity of the design and measure-

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ment, guarantee their stability and traceability. In this approach, the measurement process is modelled by a top-down approach (from the highest hierarchy to the lowest). The designing process of specification elements can then be conducted using a bottom-up approach. Adjoint functor is utilised in the categorical model to ensure the two mappings between specification and verification are structure-preserving and stable.

The paper is constructed as follows. Basic knowledge of category theory including adjoint functors is introduced in section 2. Five levels of hierarchy category model are structured in section 3. How the hierarchical model can be applied for the automation of specification and verification has been discussed in section 4. Section 5 summaries the paper.

## 2. A brief of category theory

Category theory (CT) itself is hierarchically structured which can be summarised into three levels as shown in Fig.1.

A category is construed as a collection of 'things' and a type of relationship between pairs of such 'things' [20]. The 'things' are called objects and the 'relationships' are called morphism in the category.

**Definition 2.1** A category $C$ consists of a collection of objects $A, B, C...$, denoted as $Ob(C)$; for every pair of objects $A, B \in Ob(C)$, a set $Hom_C(A, B)$ is called the *hom-set* from $A$ to $B$; its elements called morphisms from $A$ to $B$, and satisfy the identity law and associativity law.

The universal constructs (middle part of Fig. 1) are objects and morphsims. It is also including operations between objects within a category, such as product and coproduct.

The lower order includes the properties of universal constructs, which includes domain, codomain, epic, monic, isomorphic, initial objects, terminal object etc. An object $I$ is said to be initial if for every other object $X$ there is exactly one morphism $f : I \to X$. An terminal object $T$ is that for every other object $X$ there is exactly one morphism $f : T \to X$. More details of those properties can be found in Refs[13,21,22].

In the higher order, a set of objects constructs a category, morphsims between categories are functors, morphsims between functors are natural transformations, and if there is a functor has an inverse functor, the pair is called adjoint functor.

**Definition 2.2** Let $C$ and $\mathcal{D}$ be categories. An adjunction between $C$ and $\mathcal{D}$ consists of two functors $F : C \to \mathcal{D}$ and $F^+ : C \to \mathcal{D}$, and a natural isomorphism whose component for any objects $D \in Ob(\mathcal{D})$ and $C \in Ob(C)$ is:

$$\eta_{C,\mathcal{D}} : Hom_{\mathcal{D}}(F(C), D) \cong Hom_C(C, F^+(C))$$

This isomorphism is called the adjunction isomorphism for the $(F, F^+)$ adjunction, and for any morphism $f : F(C) \to D$ in $\mathcal{D}$, we refer to $\eta_{C,\mathcal{D}}(f) : C \to F^+(D)$ as the adjunct of $f$.

The functor $F$ is called the left adjoint and the functor $F^+$ is called the right adjoint. $C$ might be called the sending category and $\mathcal{D}$ the receiving category. This setup often denote by

$$F : C \rightleftarrows \mathcal{D} : F^+$$

Amongst concepts in CT, adjoint functor is seen as central. We often have two categories that are not on the same conceptual world, and the adjoint functors connect two different structures by structure-preserving mapping. That is why adjoint functors often come in the form of 'free' and 'forgetful'. One particular example is a forgetful functor which is defined from a category of algebraic structures (group or vector spaces) to the category of sets. The forgetful functor forges the arrows, remembering only the underlying set and regardless of their algebraic properties.

## 3. Categorical modelling schema - a hierarchy structure

Theoretically speaking, the GPS system is structured by geometrical features which defined by geometrical operations. All geometrical features can be classified into three invariance types: simple class, generated class and complex class (freeform), and each of which has different types of features.

The operations that define these features can be summarised by a pair of operations: decomposition and composition. Decomposition is an operation that decomposes a surface into different features, and composition is an operation that builds a surface up from different features. The two operations can be decomposed into an ordered set of operations, which can be refined into elements of operations that can still be gradually detailed into different levels.

Therefore the behaviour of the system can be resolved at multiple scales and the interactions at different scales inform each other. There are two ways that this information can be propagated. Top-down: the behaviour at larger scales is used to inform the interactions at more detailed scales. Bottom-up: information at smaller scales is used to inform models at larger scales.

Thereby in this section, a hierarchy structure of GPS is developed using categorical modelling schema. Five levels of the hierarchy are modelled using CT with reference to the structure of features and operations.

### 3.1. The Top level

The top level of the hierarchy is set to identify the specification features and the measurement features.

In the world of design, an artefact is presented by skin model which can be decomposed into surface features. This operation is also called 'partition', an operation that identifies bounded features such as point, straight line or plane. Specification features can then be defined by a series of decomposition and composition operations carried on the separated features, such as plane surface, a cylindrical surface or a prismatic surface.

From the introduction of CT, decomposition and composition can be view as a pair of adjoint functors. The two basic operations defined seven feature operations (defined by ISO TC 213), which are termed 'partition', 'extraction', 'filtration', 'association', 'collection', 'construction' and 'reconstruction' [23]. The set of ordered operations define the specification operator/operators for a specified feature. Note that there might be more than one operator for a specified feature. Specified features that are location, orientation or run-out are always with at least two or more operators as each of their required datum has related operator as well.

A specification feature from a skin model cannot be determined by the skin model itself. Assembly relationships between skin models and constraints between features in different skin models will be combined to determine the specified feature type. As shown in Fig.2, morphism $A_1$ and its inherited mor-

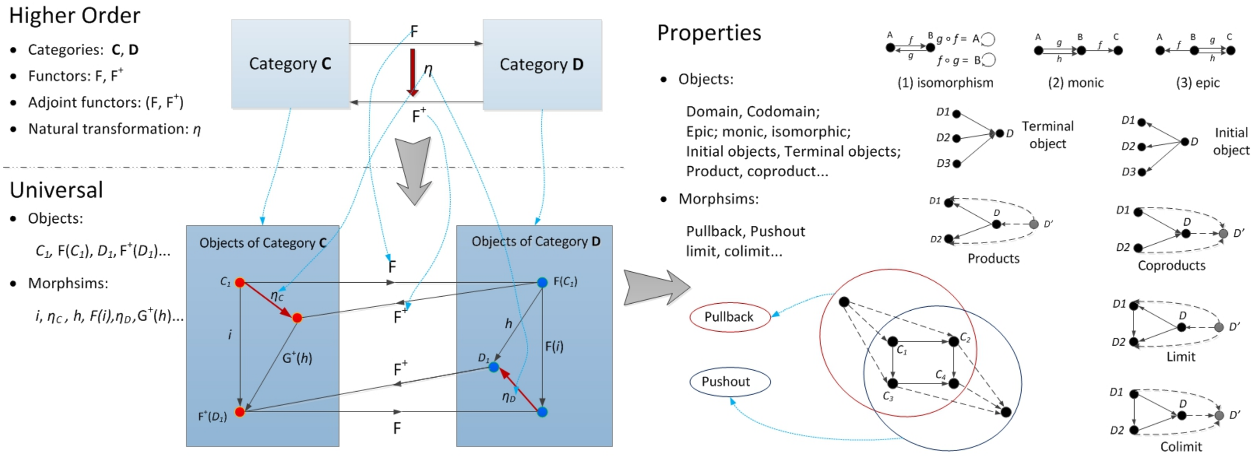



Fig. 1. Concept map of category theory

phisms $A_{11} - A_{13}$ indicate the assembly relationships between two features from different skin models.

The skin model in the design will become the real surface model in the world of measurement. And all of the operations will be mapped from specification and become associated physical operations. To ensure an accurate measurement result, the mapping from a specification operator to a verification operator should be structure-preserving, which indicates that the structure of the total order set of operations in specification should be mapped to a total order set of operation in verification. If a verification operator is known, an ordered set of specification operations should be also structure-preserving mapped.

### *3.2. The Second level*

The second level is the refinement of the specification operator and verification operator, that is, the refinement of decompositions and compositions together with related geometrical scale.

According to the geometrical scale, the features with large scale are dimensional; features that within the smaller scale are surface texture; and features with scale that between dimensional and surface texture are geometrical, which includes form, location, orientation and run-out according to their functional requirements.

For each geometrical feature, the structures of the operators are different. Fig.3 lists examples of possible operator for each feature. As they both follow the same pattern, with more decomposition/composition methods available for general and complex surface features, the details of the operations will be more flexible.

In section 1 we have discussed that the two stable mappings between the specification and verification serve the structure of adjoint functors. Let Category $\mathcal{S}$ for specification, Category $\mathcal{V}$ for verification of each geometrical characteristic. In Fig.4, the left adjoint $L : \mathcal{S} \rightarrow \mathcal{V}$ is the forward mapping from cateogry $\mathcal{S}$ to category $\mathcal{V}$; and the right adjoint $R : \mathcal{V} \rightarrow \mathcal{S}$ is the inverse from category $\mathcal{V}$ and category $\mathcal{S}$. The two functors $L$ and $R$ is a pair of adjoint functors.

Category $\mathcal{S}$ and $\mathcal{V}$ consists an ordered set of $Op_n \in Ob(\mathcal{S})$, $VOp_m \in Ob(\mathcal{V})$ are which are constructed from the seven operations listed above. Therefore, the second level is to identify the set of $Op_n$ in category $\mathcal{S}$, and set of $VOp_n$ in category $\mathcal{V}$. The morphsims between objects are indicated as $s_1 - s_n$ for category $\mathcal{S}$ and $v_1 - v_{n-1}$ for category $\mathcal{V}$, note that a single $s_1$ here may not be one morphism but a *hom-set* of morphisms $Hom_{\mathcal{S}}(Op_1, Op_2)$.

According to the definition of functors, for each object and morphism in category $\mathcal{S}$, there is a mapped object and morphism in category $\mathcal{V}$. Therefore, for $Op_1, Op_2 \in Ob(\mathcal{S})$, there are $L(Op_1), L(Op_2) \in Ob(\mathcal{V})$, and $L(Op_1) = VOp_1, L(Op_2) = VOp_2 \in Ob(\mathcal{V})$. Similarly, for morphisms $s_1$ and $s_2$ in category $\mathcal{S}$, there are $L(s_1), L(s_2)$ in *hom-set* of $\mathcal{V}$, $L(s_1) = v_1$ and $L(s_2) = v_2$. The functor $L$ here is a covariant functor which preserves the directions of morphism, i.e., every morphism $s_i : Op_1 \rightarrow Op_2$ is mapped to an morphism $F(s_i) : F(Op_1) \rightarrow F(Op_2)$.

Here, $Ob(\mathcal{S})$ in specification and $Ob(\mathcal{V})$ in verification are independent, but they are however related by the 'Duality Principle' in GPS. For example, if the object $Op_1 \in Ob(\mathcal{S})$ is the Filtration operation in the specification operator, the object $VOp_1 \in Ob(\mathcal{V})$ will be the physical filtration operation when the specification is interpreted in the verification process.

### *3.3. The Third level*

The third level identifies objects and relationships for each operation of the specification operator and verification operator, including the inheritance of categories.

The inheritances of categories in the categorical model are in accordance with the philosophy of GPS. The definitions and terms defined in GPS determine the family tree and relationships between them. To give an example, Fig.5 shows the category $\mathcal{STO}$ representing the specification operator for roundness as a set of partition objects, extraction objects and filtration objects. Categories $\mathcal{PA}$, $\mathcal{EX}$ and $\mathcal{FI}$ are inherited from the three objects in category $\mathcal{STO}$ respectively; and category $\mathcal{TB}$ is inherited from object transmission band in category $\mathcal{FI}$.

The inheritances of categories actually are adjoint functors. Let subcategory $\mathcal{SPA}$ with only one object partition is from category $\mathcal{STO}$. There are two functors between $\mathcal{SPA}$ and $\mathcal{PA}$ which are $F : \mathcal{SPA} \rightarrow \mathcal{PA}$ and $G : \mathcal{PA} \rightarrow \mathcal{SPA}$. Functor $F$ denotes category $\mathcal{SPA}$ is the family of category $\mathcal{PA}$, the object *partition* is the family of all the objects in category $\mathcal{PA}$. Functor $G$ express that category $\mathcal{PA}$ is derived from category

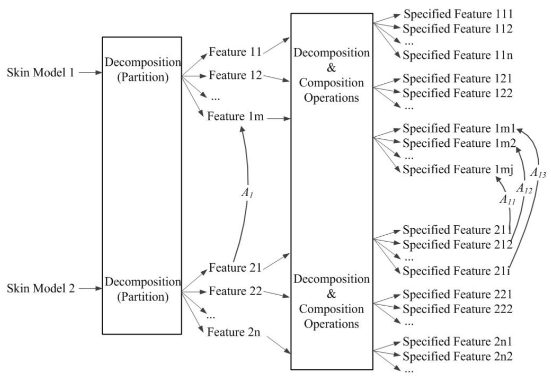



Fig. 2. Operations of skin model to establish specification features

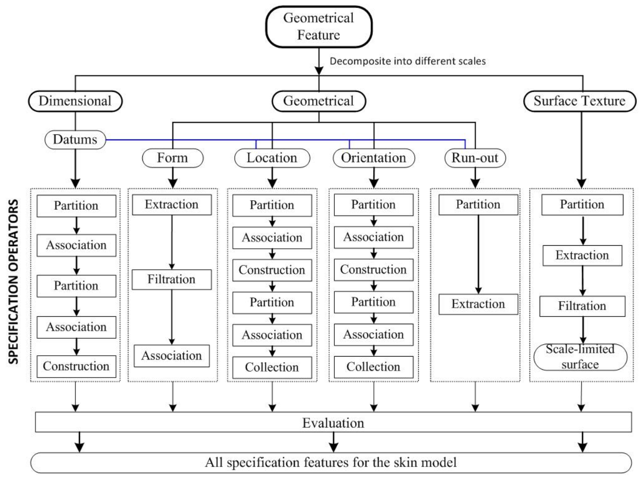



Fig. 3. The top level of hierarchy structure

$\mathcal{SPA}$, and all of the objects in category $\mathcal{PA}$ belong to the only object *partition* in category $\mathcal{SPA}$.

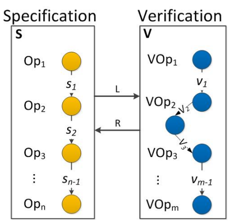



Fig. 4. Adjoint functors $L$ and $R$ between category $\mathcal{S}$ and Category $\mathcal{V}$

After the inheritances of the objects in the high level categories, the pair of adjoint functors $L$ and $R$ will be decompose into a set pairs of adjoint functors between operations in specification and verification respectively.

### 3.4. The Fourth level

The fourth level is the refinement of relationships in the third level. We employ pullbacks, pushouts, limits and colimits to structure more complex relationships between objects.

Pullbacks normally appear between objects in the same category. However, there are often relationships between objects in different categories which appear not as functors but more like pullbacks between different categories. This type of relationships is denoted as 'categories pullbacks'.

Fig.6 gives an example of categories pullback $CP_1$ between categories $\mathcal{EX}$ and $\mathcal{PA}$ for roundness, a determination process of *sampling_space* $\in Ob(\mathcal{EX})$. Category $\mathcal{PA}$ includes objects:*feature_type* (the invariant type of the geometrical feature), *DOF* (degrees of freedom), *diameter* (the diameter of each circumferential section) and a morphsim $s_4$ which indicates that *DOF* is determined by the type of geometrical feature. Category $\mathcal{EX}$ has five objects, *sampling_space*, *sampling_point*, *sampling_number* (number of sampling in each wave), *sampling_length* and *tip_radius*, and a morphsim $s_5$ which indicates that the number of sampling points will be constrained by the number of sampling and the cutoff frequency.

The product of object sampling point in category $\mathcal{EX}$ and object *diameter* in category $\mathcal{PA}$ determines *sampling_space* $\in Ob(\mathcal{EX})$. To form a category pullback, all related objects will be inherit from the original category, and then form subcategories. Category $\mathcal{PAS}$ is a subcategory of $\mathcal{PA}$ consist with object *diameter*, and category $\mathcal{EXS}1$ and $\mathcal{EXS}2$ are subcategories of $\mathcal{EX}$ consist with object *sampling_point* and *sampling_space* respectively.

The category pullback $CP_1$ is the pullback of category $\mathcal{PAS}$ and $\mathcal{EXS}1$ over $\mathcal{EXS}2$, and $\pi_1 p_1 \circ \lambda_1 p_1 = \pi_2 p_1 \circ \lambda_2 p_1$.

$\mathcal{PAS} \times_{\mathcal{EXS}2} \mathcal{EXS}1$ is the subproduct of $\mathcal{PAS}$ and $\mathcal{EXS}1$ over $\mathcal{EXS}2$. It represents the subcategory of the universal product $\mathcal{PAS} \times \mathcal{EXS}1$ that actually occurs for the relationship $\mathcal{EXS}2$ which represents all objects of this type of association between $\mathcal{PAS}$ and $\mathcal{EXS}1$.

### 3.5. The Fifth level

In the fifth level, the properties of morphisms (epic, monic, isomorphic) and properties of objects (initial objects, terminal object) will be addressed. Take the morphism $\pi_1$ in the pullback structure $CP_1$ as an example. The morphism $\pi_1$ is a projection of the subproduct *sampling_point* $\times$ *diameter* over *diameter*. If $\pi_1$ is epic then every *diameter* appears at least once in the subproduct. Thus every *diameter* participates in the relationship and object of *diameter* has mandatory participation in the relationship. If $\pi_1$ is not epic, not every *diameter* participates in the relationship and object of *diamter* has optional participation in the relationship. If $\pi_1$ is monic then each *diameter* appears just once in the subproduct. If $\pi_1$ is not monic, a *diameter* may participate more than once in the relationship. If $\pi_1$ is isomorphic then each *diameter* appears once in the subproduct and object *diameter* has mandatory participation in the relationship.

There are also some typical terminal objects in Category $\mathcal{S}$ and category $\mathcal{V}$. In the category of $\mathcal{S}$, the order set of operations will form an object named *charac_symbol* for each feature characteristic. This *charac_symbol* object is said to be the terminal object in the category $\mathcal{S}$, as shown in Fig.7.

The specification operators are defined according to their design requirements. If the requirements can form an object, it then will be an initial object such that all other objects will be determined or partially determined.

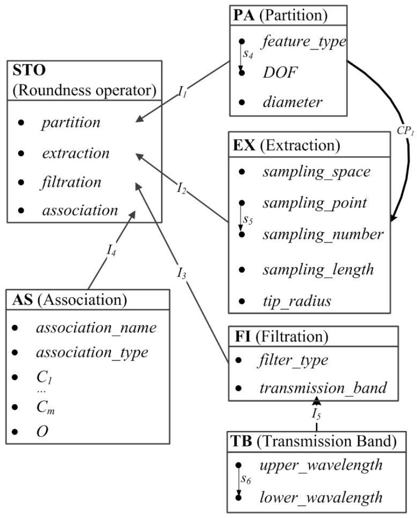



Fig. 5. The third level relationships between operations

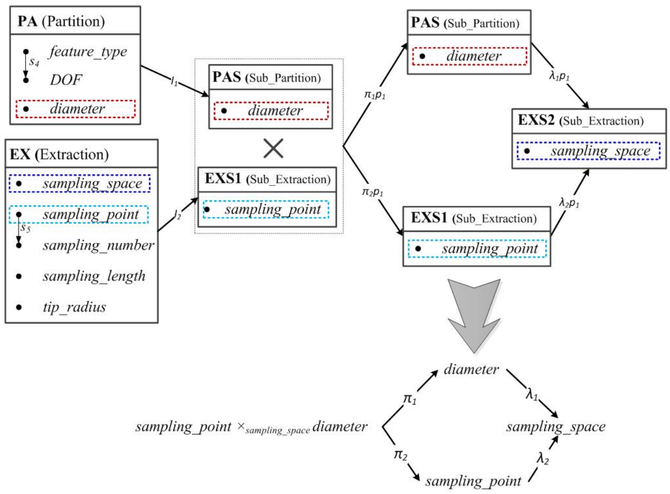
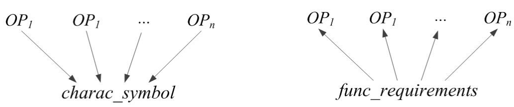



Fig. 6. A category pullback $CP_1$ for roundness

## 4. Towards the automation of specification and verification

So far the hierarchical category model of GPS is abstract and may not be seen as a functional model that will serve the purpose of automation for the design of GPS specification and verification. In this section, how the five levels will be used towards the automation of designing GPS specification and verification will be discussed.




a) Terminal object b) Initial object

Fig. 7. An example of terminal object and initial object in category $\mathcal{S}$

In the hierarchy model, a GPS characteristics' specification can be automatically generated with a certain input elements, i.e. the function requirements and information of the designed component. As the two stable mappings between specification and verification are dual and serve the structure of adjoint functors, if a set of specification operations can be generated automatically, applying the adjoint functors the related complete set of verification operations can then be derived. This process is called a forward mapping from category $\mathcal{S}$ to category $\mathcal{V}$ in the hierarchy model. It uses a Bottom-up approach, in which mappings are started with the fifth level and finished in the top level.

The inverse mapping is however applied with a different approach. It is often used when specified features are known or partially known (feature type in the top level), as the verification operator/operators cannot be formed without knowing the specified feature. Using the Top-down approach, the specified features and related specification operators will be decided from the top level to the fifth level. The derived specification should be the simplest solution in the hierarchy structure, which is a solution with minimal number of categories and objects. To assist a better understanding of the inverse mapping, a test case on a freeform surface for generating its specification operations has been carried out.

A full set of specification operator and verification operator of a freeform surface can be rather complicated. In this case, the operators are simplified which mainly focus on fitting operations and operations that support it. The freeform surface is a bearing surface of a total knee joint replacement bearing couple from Refs [24,25].

**Step 1**: identify the first and second level. The design template is supplied as a CAD model. A set of nominal points is obtained from the model using CMM collateral software. These discrete points are then reconstructed into a continuous representation with a reconstruction method, e.g. NURBS or Radial Basis Function. The workpiece is measured by CMM with spacing $d$=0.5mm. The form error of the workpiece is evaluated by fitting the measurement data with the reconstructed surface.

**Step 2**: identify the third level. This level identifies objects and relationships for three operations of the verification which are Reconstruction, Extraction and Fitting. Categories $\mathcal{IF}$(InitialFitting) and $\mathcal{FF}$(FinalFitting) which represent the two fitting steps are inherited from $\mathcal{F}$(Fitting), and categories $\mathcal{IFP}$(InitialFittingPara) and $\mathcal{IFR}$(InitialFittingResults) which represent the parameters and results respectively of the initial fitting operation are inherited from $\mathcal{IF}$, and so do for $\mathcal{FFP}$(FinalFittingPara) and $\mathcal{FFR}$(FinalFittingResults). The inheritances are indicated as $F_1 - F_6$, and the relationships between categories are abstracted into pullbacks $CP_i$.

**Step 3**: identify the fourth level. This level is mainly the refinement of category pullbacks ($CP_i$). Two category pullbacks examples are $CP_1$ and $CP_2$, where $CP_1$ is the determination process of $\mathcal{IF}$-object $FittingMethod$ from the $\mathcal{FE}$-object $CADModel$. The practical meaning of the pullback is that the fitting method is decided by the CAD model of the designed surface. Pullback $CP_2$ is the determination process of $\mathcal{IFP}$-object $RBFStructure$ from the $\mathcal{EX}$-objects $Samplingstrategy$ and $SamplingSpace$. It indicates that the sampling strategy and sampling space of the surface will decide the structure of the RBF.

**Step 4**: identify the fifth level and derive the specification elements. The properties of morphisms and properties of objects are addressed. As shown in Fig.8, objects that have specified in the design are highlighted as yellow, and codomain objects that are epic and isomorphism are highlighted as grey. The simplification process then remove the 'grey' object such that the inherited categories $\mathcal{IFP}$, $\mathcal{IFR}$, $\mathcal{FFP}$, $\mathcal{FFR}$ are removed. The remaining objects in each operation are then formed the specification elements that are independent.

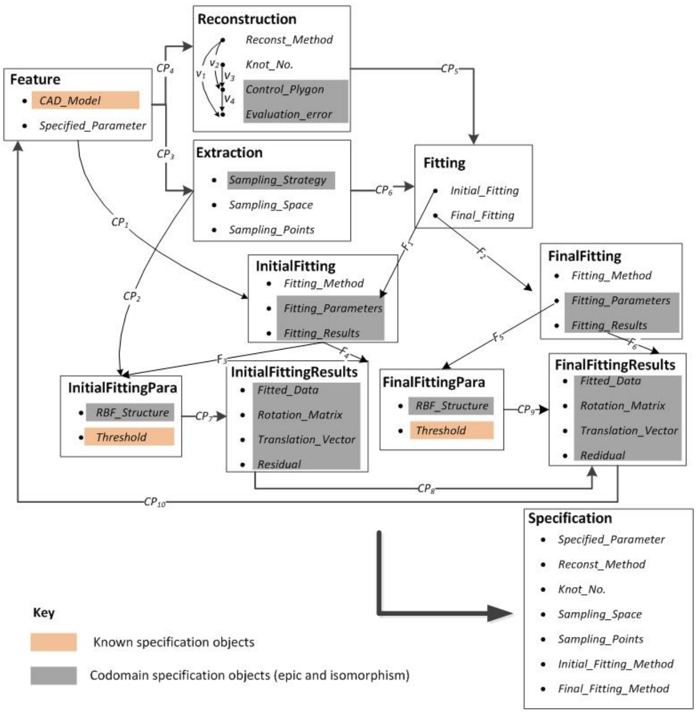



Fig. 8. The derived specification elements of test case

## 5. Conclusion

In this paper, a hierarchical category model is developed to support the decision-making for specification and verification of geometrical products. The model is expected to generate specifications that comply with GPS specification rules but without redundancy. For some geometric products whose specification requirements are still unknown, the model helps with forming a specification structure with minimum/independent specification objects. We can then conclude that the completeness of a specification, is not meant to specify all the specification operator, should be with minimum independent specification objects that can generate a complete specification operator. And the hierarchical category model was developed to facilitate this goal.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## References


[1] Srinivasan, V.. A geometrical product specification language based on a classification of symmetry groups. Computer-Aided Design 1999;31(11):659–668.

[2] Dantan, J.Y., Ballu, A., Mathieu, L.. Geometrical product specification-smodel for product life cycle. Computer-Aided Design 2008;40(4):493–501.

[3] Ballu, A., Mathieu, L., Dantan, J.Y.. Global view of geometrical specifications. In: Geometric Product Specification and Verification: Integration of Functionality. Springer; 2003, p. 13–24.

[4] ISO 14638. Geometrical product specification(GPS)- Masterplan. Standard; International Organization for Standardization; Geneva, CH; 1995.

[5] Rosén, B.G., Ohlsson, R., Westberg, J.. Interactive surface modelling, an implementation of an expert system for specification of surface roughness and topography. International Journal of Machine Tools and Manufacture 1995;35(2):317–324.

[6] Bui, S.H., Muralikrishnan, B., Raja, J.. A framework for internet-based surface texture analysis and information system. Precision engineering 2005;29(3):298–306.

[7] Filippi, S., Cristofolini, I.. The design guidelines (dgls), a knowledge-based system for industrial design developed accordingly to iso-gps (geometrical product specifications) concepts. Research in engineering design 2007;18(1):1–19.

[8] Xu, Y., Xu, Z., Jiang, X., Scott, P.. Developing a knowledge-based system for complex geometrical product specification (gps) data manipulation. Knowledge-Based Systems 2011;24(1):10–22.

[9] Qi, Q., Jiang, X., Liu, X., Scott, P.J.. An unambiguous expression method of the surface texture. Measurement 2010;43(10):1398–1403.

[10] Qi, Q., Jiang, X., Scott, P.J.. Knowledge modeling for specifications and verification in areal surface texture. Precision Engineering 2012;36(2):322–333.

[11] Anselmetti, B., Chavanne, R., Yang, J.X., Anwer, N.. Quick gps: A new cat system for single-part tolerancing. Computer-Aided Design 2010;42(9):768–780.

[12] Eilenberg, S., MacLane, S.. General theory of natural equivalences. Transactions of the American Mathematical Society 1945;:231–294.

[13] Spivak, D.I., Kent, R.E.. Ologs: a categorical framework for knowledge representation. PLoS One 2012;7(1):e24274.

[14] Baianu, I., Brown, R., Glazebrook, J.F.. A category theory and higher dimensional algebra approach to complex systems biology, meta-systems and ontological theory of levels: emergence of life, society, human consciousness and artificial intelligence. Acta Univers Apul 2011;(Suppl 1):176–99.

[15] Healy, M.J., Caudell, T.P.. Ontologies and worlds in category theory: implications for neural systems. Axiomathes 2006;16(1-2):165–214.

[16] Healy, M.J., Olinger, R.D., Young, R.J., Taylor, S.E., Caudell, T., Larson, K.W.. Applying category theory to improve the performance of a neural architecture. Neurocomputing 2009;72(13):3158–3173.

[17] Haruna, T.. Theory of interface: Category theory, directed networks and evolution of biological networks. BioSystems 2013;114(2):125–148.

[18] Diskin, Z., Maibaum, T., Czarnecki, K.. Towards category theory foundations for model management. Tech. Rep.; Tech. Rep. GSDLab-TR 2014-03-03, University of Waterloo/McMaster University, http://gsd. uwaterloo. ca/node/566; 2014.

[19] Qi, Q., Scott, P.J., Jiang, X., Lu, W.. Design and implementation of an integrated surface texture information system for design, manufacture and measurement. Computer-Aided Design 2014;57:41–53.

[20] Spivak, D.I.. Category Theory for the Sciences. 1 edition ed.; Cambridge, Massachusetts: MIT Press; 2014. ISBN 9780262028134.

[21] McLarty, C.. Elementary categories, elementary toposes. Oxford University Press; 1992.

[22] Pierce, B.C.. Basic category theory for computer scientists. MIT press; 1991.

[23] ISO 17450-1:2011. Geometrical product specifications (GPS) General concepts, Part 1: Model for geometrical specification and verification. Standard; International Organization for Standardization; Geneva, CH; 2011.

[24] Zhang, X., Jiang, X., Scott, P.J.. A new free-form surface fitting method for precision coordinate metrology. Wear 2009;266(5):543–547.

[25] Jiang, X., Scott, P., Whitehouse, D.. Freeform surface characterisation-a fresh strategy. CIRP Annals-Manufacturing Technology 2007;56(1):553–556.