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Track loading limits and cross acceptance of vehicle approvals

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Abstract

The requirements for track loading limits are one of the main barriers to simple cross-acceptance of vehicles where rolling stock that is already operating successfully in one (or more) networks has to be re-tested before it can be approved for operation on another network. DynoTRAIN WP 4 studied this area in order to determine whether the additional requirements were justified, or if the process could be made much cheaper and simpler without increasing the risk of track deterioration for the networks.

The review of national requirements identified modified criteria and limit values for track forces in some member states but these can be obtained from additional analysis of the normal test results with no new tests required. The influence of design rail inclination has also been found not to be significant, provided a realistic range of wheel-rail contact conditions are included in the tests. For line speeds greater than or equal to 160km/h the current standards for track construction across the member states appear to be similar. On lower speed lines in some countries, a ‘weaker’ track condition may require a lower limit on one of the vehicle assessment parameters.

Track dynamics modelling has shown that the vehicle assessment parameters used in international standards are suitable for use in cross-acceptance for track forces. The use of multiple regression analysis allows the estimated maximum value for relevant parameters to be evaluated for different target conditions and then compared with the appropriate limit value, or with values for existing, comparable vehicles. Guidance has also been provided on the relevant parameters to consider when developing operating controls for different types of track deterioration.

Keywords: Cross-acceptance, rail vehicle, approvals, track loading, track forces, track dynamics, authorisation, certification, simulation

1. Introduction

The requirements for track loading limits are one of the main barriers to simple cross-acceptance of vehicles where rolling stock that is already operating successfully in one (or more) networks has to be re-tested before it can be approved for operation on another network. DynoTRAIN Work Package 4 (WP4) studied this area in order to determine whether the additional requirements were justified, or if the process could be made much cheaper and simpler without increasing the risk of track deterioration for the networks.

The objectives of WP 4 were to:
• improve the process of cross-acceptance between different countries;
• reduce the number of additional tests required for approval onto non-TSI compliant infrastructure;
• develop limit values related to infrastructure construction and maintenance;
• clearly identify specific local requirements;
• provide proposals for cross-acceptance processes, including the use of simulations;
• provide proposals for operating limits dependent on infrastructure conditions;

A detailed review of the current requirements for track loading in TSIs, ENs, UIC leaflets, other international documents and national rules was carried out to check for differences. Any relationships of these requirements and limit values to track construction or maintenance standards were also noted. Existing track dynamics models were used to investigate relationships between the vehicle parameters assessed during approval tests and the forces and accelerations likely to cause deterioration in the track structure.

Information was gathered from a number of infrastructure managers regarding the construction and maintenance standards for different track categories and line speeds. Any specific approval requirements for rolling stock were also collected. The development work being undertaken by CEN TC256 WG10 on EN14363 was also reviewed to determine if any additional parameters should be considered.

The data from the DynoTRAIN WP 1 tests was analysed to investigate the relationships of the measured track loading parameters with influencing factors such as speed, cant deficiency and curve radius. Using track data from DynoTRAIN WP 1 [1] and WP 2 [2] and wheel and rail profiles from WP 3 [3], a number of vehicle dynamic simulations were also undertaken to confirm the relationships found and investigate others, such as with wheel-rail friction, that were not available from the tests.

2. WP4 activities
The following activities were undertaken by partners in WP4:
• Review of relevant requirements in national, international and bi- / multi-lateral documents
• Review of ORE, ERRI and UIC reports to seek to understand the background to current requirements
• Detailed track dynamics modelling to check the relevance of the identified parameters and sensitivity to track and operating conditions
• Survey of infrastructure construction and maintenance requirements across networks to identify similarities and differences
• Use of data from WP 1 tests to examine the relationships between track and vehicle response
• Vehicle dynamic simulations to extend the range of conditions considered
• Investigations into the use of multiple regression analysis in compatibility assessment

These various activities are explained in more detail below.

3. Review of relevant requirements
3.1 International documents
A range of parameters are applied for vehicle approval, including cross acceptance, and for technical compatibility checks. For track loading assessment the following parameters are used in the international standards ([4] and [5]) and TSIs ([6], [7] and [8]):

- \( Q_{\text{qst}} \) – Quasi-static vertical wheel load
- \( Q_{\text{max}} \) – Maximum vertical wheel force
- \( Y_{\text{qst}} \) – Quasi-static lateral guiding force
- \( \Sigma Y_{\text{max}} \) – Lateral track-shifting force
- \( H_{\text{max}} \) – Sum of lateral axle-box forces

Various additional parameters are used in National documents (e.g. track stresses or different combinations of wheel forces) or have been proposed from other studies. Examples are:

- \( B_{\text{qst}} \) – Quasi-static loading forces
- \( T_{\text{qst}} \) – Rail surface damage indicator

These assessment parameters are measured on the test vehicle, there is no measurement on the track itself, and therefore the parameters are not directly assessing the track behaviour. However it is assumed that the measurements taken give a good indication of the expected effect of the assessed vehicle on the track, in terms of forces in the track structure (rails, pads, sleepers, ballast) and the resulting deterioration.

\( Q_{\text{qst}} \) describes the quasi-static vertical force on the outer rail in plain curve sections of curves below 400m radius and is associated with track (and especially rail) fatigue. It is also probable that \( Q_{\text{qst}} \) could be relevant for assessment of track settlement.

\( Q \) or \( Q_{\text{max}} \) describes the dynamic vertical force on the rail and is associated with deterioration of track components (rails, fastenings, sleepers, ballast).

\( Y_{\text{qst}} \) describes the quasi-static lateral force on the outer rail in plain curve sections of curves below 400m radius and is associated with lateral wear of the outer rail, lateral resistance of rails, welds, joints and fastenings and the resistance of rails to bending stresses.

\( \Sigma Y_{\text{max}} \) is used to control the risk of lateral shifting of the whole track (sleepers and both rails) under the influence of cant deficiency forces. The force is a running average over 2m and modelling shows a rather consistent trend between the sleeper force and the input wheel-rail lateral force.

\( H_{\text{max}} \) is used in place of \( \Sigma Y_{\text{max}} \) in the simplified measuring method when measurement of lateral wheel-rail forces is not carried out and is an indirect alternative. It considers the lateral force at the axle level.

\( B_{\text{qst}} \) describes the resultant force (combined vertical and lateral) on the outer rail in plain curve sections of curves below 400m radius and is associated with track (and especially rail) fatigue.

\( T_{\text{qst}} \) is a proposed new parameter from CEN TC256 WG10 SG8 which is related to rail surface damage and this also appears to be relevant and should be considered as an addition.

The limit values related to these various parameters have been developed over many years and, in some cases, the background for the limits is not clear today. Some limit values are based on scientific studies, while others are based on experience and comparison with
existing vehicles which were at that time believed to be at the limit of acceptability for the track.

3.2 National and multi-national documents

In order to understand more completely the current requirements, in the area of track loading limits, additional national or local requirements were also analysed. This work concentrated on requirements found in the Member States of the European Community as additional to the international requirements in the TSIs. Twenty National Reference Documents were available at the ERA Cross Acceptance unit website and the following clauses from these documents were analysed:

3.2.1 Running safety and dynamics

Including tolerance of vehicle to distortion of track, running on curved or twisted tracks, safe running on points and diamond crossings, etc.

3.2.3 Track loading compatibility parameters

E.g. dynamic wheel force, wheel forces exerted by a wheel set on the track (quasi static wheel force, maximum total dynamic lateral force, quasi static guiding force).

Checklists from bi- or multilateral agreements for cross acceptance and data from the “International Requirement List” IRL were also analysed and some additional requirements were identified.

3.3 Background documents

Where possible, the background documents were reviewed. These included the work of ORE committees B10, D71, C138, ERRIC C209, the UIC group which reviewed UIC518 to produce the 2009 version [5] and the CEN group currently reviewing EN14363[9].

ORE committee B10 ‘Constructional arrangements for improving the riding stability and the guiding quality of electric and diesel locomotives and vehicles’, summary report No.15, October 1974, makes reference to the limit value for $\Sigma Y_{\text{max}}$, but gives no additional information on the background.

ORE committee D71 ‘Stresses in the rails, the ballast and in the formation resulting from traffic loads’, summary report No.13 describes the programme of work which covered:

- Stresses in rails
- Stresses in rail fastening systems
- Stresses in sleepers
- Stresses in ballast and formation

and concentrated on experimental methods. Whilst some of the results are of general interest there are no limit values or vehicle test methods proposed.

ORE committee C138 ‘Permissible maximum values for the Y- and Q- forces as well as the ratio Y/Q’ produced several reports. Rpt No. 2, Rpt No. 6 and Rpt No. 9 have been reviewed for relevant background information. RP2 deals with the limit values for Y and Q forces from the point of view of rail loading. It identifies three potential consequences for the rail under the action of excessive loads:

- Plastic deformation
- Failure due to brittle fracture
• Failure due to fatigue damage.

Plastic deformation was not considered during the study, partly because of the limitations of the calculation methods available at that time, and the rail was assumed to be elastic. For a range of different applied loads it is indicated for what magnitude of Y and Q forces, and at what position on the rail, the stresses reach the ‘standard’ value of $10^8$ N/m$^2$. The results, which were valid for a new UIC60 rail profile, sleeper spacing 0.6m, and loads applied mid-way between two sleepers, showed that the adopted conditions (up to a wheel load of 100kN) were not dangerous for this rail.

RP6 examined the influence on the permissible Y and Q forces of various parameters:

• Horizontal and vertical stiffness of the track
• Sleeper spacing
• Adjacent axles
• Rail profile.

The vertical stiffness appeared to have the most influence on the permissible maximum values for Y and Q forces. An increase in sleeper spacing from 0.6m to 0.7m resulted in an increase of at most 10% in the maximum stresses, the effect of variation in axle spacing was negligible and the change from UIC60 to UIC54 rail gave an increase in stresses by at most 10%. No international limit values were developed in this study, the conclusions state that ‘the limiting values of Y and Q should be determined by each railway on the basis of local conditions and requirements’.

RP9 was the final report and compared the limit values obtained by C138 under specific test conditions with the Y and Q forces actually obtained in service so as to assess:

• The risk of lateral displacement of the track ($\Sigma Y$)
• The stressing of the rails (Q, Y).

The work of C138 considered the $\Sigma Y$ forces acting over a 2m length and this criterion is still used today. However over time there have been changes in the way in which the values are calculated, see C138 RP9 Sec 2.1. This states that in UIC432 the ‘2m’ value was the value which was exceeded for 2m, say 2m(max). This assessment was simple to make visually on a paper trace of the measured signal using a straight-edge of 2m equivalent length. C138 RP9 and more recent documents assess the ‘2m’ value as a running average over the 2m length, say 2m(ave). This assessment was easier when computer assessment of measured signals became the usual process.

The resulting values are often similar but the running average is always greater than or equal to the maximum value, (2m(ave) ≥ 2m(max)) and in some cases the difference can be large, having the effect of making the limit values more difficult to meet.

ERRI C209 was tasked to assess the specifications in UIC518:1995 and for this purpose the railways made available and analysed already existing test results. DT 338 considered vehicles running at speeds between 140 and 200 km/h using tests on locomotives, coaches and a body-tilt power car. Some small exceedance of the limit values for $Q_{\text{max}}$ was noted with heavy locomotives. DT 339 considered vehicles running at speeds below 140 km/h and noted that the $Q_{\text{max}}$ limit of 170 kN with an axle load of 22 t was not met in curves. No specific comments on track force limit values were made in DT 337 (speeds >200 km/h). The C209 Rpt1 proposed a $Q_{\text{lim}}$ criterion related to vehicle speed, a version of which was adopted. Rpt2 made a number of proposals (for example changes to some of the
acceleration limit values), which were incorporated in the February 1999 draft of UIC 518 but the track force limits were unchanged.

3.4 Results of review of requirements

The analysis showed that:

- Rules are sometime widely spread in laws (generic requirements), national standards, national regulations
- Sometimes it is required to follow (not defined) requirements of the Infrastructure Manager
- By far not all documents with requirements are available
- In most cases only the international standards ([4] and [5]) are applied without additional track loading requirements
- The main barriers for mutual recognition (cross acceptance) are differences in rail inclination leading to different contact geometry conditions

Specific national requirements may result from national legal requirements and/or may reflect special conditions of track construction or maintenance. In most cases no scientific justification is given.

4. Detailed track dynamics modelling

To improve the understanding of track damage mechanisms and parameters relevant for vehicle track-interaction, existing track dynamics simulation tools were used. These tools allowed an insight into the interaction between vehicle and track for a range of frequencies not covered in other work. They were aimed at understanding the relationship between track loading limits presently in force within the EU countries and the actual loading applied on different track components, such as rail pad, fastening, sleepers and ballast. These studies concentrated on locomotives and laden freight wagons, as these are generally the most critical vehicles for track forces. The outputs from these simulations were also analysed in terms of track damage considering three aspects; vertical track settlement, track component fatigue and rail surface damage.

This goal was achieved by means of numerical simulations which allow for numerical experiments, where not only the forces in the track elements but also wheel-rail contact forces are estimated. Interesting results were obtained from a sensitivity analysis on track constructions and their behaviour both in tangent track and in curves, assessed in a deterministic way and providing information on the dependence that track forces have on track loading assessment quantities suggested by the standards.

4.1 Finite element track model

To this end, a mathematical model of train-track interaction was developed using a finite element schematisation of the flexible track and a multi-body model of the vehicle. The train-track interaction model reproduces in the time domain the dynamic interaction effects associated with the coupling of the two sub-systems via wheel-rail contact, including the movement of the train relative to the track [10].

The vehicle model is based on a multi-body formulation, according to which the vehicle (or the complete train set) is divided into modules, each one representing one carbody or one bogie, this latter inclusive of the wheelsets [10], [11]. The equations of motion for each
module are then written with respect to a local moving frame travelling along a path defined based on the ideal geometry of the line. The equations of motion for the vehicle system are linearised with respect to kinematics only, by assuming the motion to be a small perturbation around the large motion of the moving references introduced, whereas the non-linear effects associated with wheel-rail contact and with the behaviour of some suspension components are accounted for without introducing simplifying assumptions.

The model considers three-dimensional motion of the system, allowing the analysis of the non-stationary behaviour of a single rail vehicle or of an entire trainset running in tangent and curved track.

In the present study, weak interaction is assumed among the vehicles forming the train set, and hence the mathematical model is confined to one single vehicle, in particular a Locomotive vehicle.

The track model is based on a finite element schematisation of the track, in which the rails are modelled by Euler-Bernoulli beam elements, while the sleepers are considered as concentrated masses spaced by 0.60 cm. The rail pads and fastening devices are represented by linear visco-elastic elements connecting the rails to the sleepers, and finally the ballast is partly modelled as visco-elastic layers, in lateral and vertical direction, connecting the sleeper and the ground. Furthermore approximately half of the ballast mass is considered as rigidly attached to the sleeper. Although this model is a simplified one from the point of view of the ballast/sleeper, it was demonstrated by preliminary analyses to be adequate for the purpose of this work.

The finite element model is completely three dimensional, although the longitudinal displacements of the nodes are decoupled from the vertical and lateral displacement components and are therefore neglected in the analysis. In this work the track model consisted of 300 sleeper spans, corresponding to a track of 180 m length. Figure 1 shows a sectional schematic of the model described above.

![Figure 1: Track model schematisation](image)

The equations of the finite element track model are written with respect to a fixed reference frame, and therefore during the simulation the train travels over the track model.
In particular, wheel-rail contact forces represent the coupling between the track model and the vehicle model. In fact they are functions of both vehicle and track motion, therefore the model used for the simulation of the dynamic interaction between vehicle and track solves, for each integration step, the system of equations iterating at each time-step on the terms of the forces, which are functions of the degrees of freedom of both the systems.

The vehicle and track subsystems are described by two different sets of equations which are simultaneously integrated in the time domain using a modified Newmark time-step procedure. The co-simulation procedure is based on the coupling of train and track dynamics as result of the contact forces exchanged at the wheel/rail interface, as shown in Figure 2.

![Diagram](image)

Figure 2: Numerical model for train/track interaction: vehicle and track as separated and interacting sub-systems

The model of wheel-rail contact used to reproduce the dynamic coupling between the vehicle and the track is a pre-tabulated, multi-Hertzian one. Prior to the simulation, wheel/rail contact geometry is processed starting from measured or theoretical wheel and rail profiles, and contact parameters required to compute wheel/rail contact forces are stored in a contact table. The parameters in the contact table include the contact angle, the variation of the wheel rolling radius with respect to the nominal one, the curvatures of the wheel and rail profiles in the contact point region [11].

Based on wheel and rail displacements and velocities in the contact point, the value of the contact forces is derived. An elastic co-penetration in the normal direction is computed accounting for the relative wheel/rail displacements and contact angle, then the normal force is identified according to the Hertzian formulae. The longitudinal and lateral creepages are then defined, and the creep force components are computed using the heuristic formulae by Shen, Hedrick & Elkins [12].
The contact forces are then transformed to the global reference in which vehicle and track displacements are defined, and the vectors of generalised forces \( F_{cv} \) and \( F_{ct} \) acting respectively along the vehicle and track coordinates.

### 4.2 Combined use of vehicle and track dynamics models

Alongside the modelling technique described above another modelling approach was taken which considers the vehicle-track dynamics interaction and the track forces distribution in two consecutive and independent steps. The main purpose of this approach is to allow the simulation of a high number of curving conditions together with a high sample of representative track irregularities. There is a small loss of accuracy in the process due to the non-interactive prediction of wheel and rail forces, but this mainly affects frequencies higher than those of interest within the frame of EN14363 for which forces are low pass filtered at 20Hz and this approach was also compared to the previous complete system interaction modelling to ensure a valid interpretation of the results.

The various vehicle service conditions tested with the method are the vehicle speed and loading condition but also wheel profile state (i.e. new or worn) and cant deficiency. The cant deficiency in curves generates a non-compensated centrifugal force that loads the outer rail and also increases the lateral cornering forces imposed by the leading outer wheel. The simulation occurs in two steps:

1) the vertical and lateral wheel forces generated at one bogie in curving situations are predicted from the commercial railway vehicle dynamics software Vampire assuming a simplified linear elastic track model and using two non-linear vehicle models: a locomotive (23t axle load) and a container freight wagon (20t).

2) the predicted forces are used as input function onto a fixed length flexible track model. The dynamic load from the leading axle is applied at a fixed location directly above one sleeper, while the load from the second axle is applied at the specified distance from the bogie geometry. The resulting ballast and rail pad forces under the leading axle are used as output.

Figure 3 shows the input load predicted from the first step simulation for the locomotive, across a 12km track site with decreasing curve radius from 8km to 600m (EN14363 test track zone 2). As the curve tightens, the cant deficiency increases and the vehicle speed is adjusted in steps so that a representative set of cant deficiencies are simulated within the limits specified by EN14363, whose upper limit is 1.1 times admissible cant deficiency (with \( cd_{adm} = 130\text{mm} \)). The maximum vehicle speed considered is 1.1 times the vehicle maximum running speed (\( V_{\text{max}} = 200\text{km/h} \)). The vertical forces are thus increasing on the high rail of the curve while reducing on the lower rail. At the same time the lateral forces imposed by the leading axle on the high rail are clearly increasing towards the end part of the track, as well as on the low rail to a lesser extent.

After these input forces are applied to the flexible track model, the corresponding maximum forces on the rail pad and sleeper directly below the input forces can be plotted. Figure 4 shows the flexible track with a snapshot of the dynamics loading and the position of the measured sleeper response. Example outputs of this approach are shown in Figure 6 and Figure 7.
4.3 Results of track dynamics modelling

In the following an example of numerical experiments in tangent track is described together with the main results from the time histories. The numerical experiments were carried out considering a locomotive (based on the BR120 tested in WP1[1]) running at its maximum service speed (i.e. 200 km/h) along a straight track. A preliminary analysis was
carried out neglecting the track irregularity. The sleeper and rail fastening force time histories were compared for a section situated at mid-length of the overall track length. Different track constructions were considered: soft, typical and stiff, being considered homogeneous for all the track length. The values chosen were based on the EUROBALT project [13] and are given below with vertical stiffness in kN/mm and damping in kNs/m (all per sleeper end).

<table>
<thead>
<tr>
<th>Dynamic values</th>
<th>Bed stiffness</th>
<th>Bed damping</th>
<th>Pad stiffness</th>
<th>Pad damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft track</td>
<td>20</td>
<td>50</td>
<td>150</td>
<td>20</td>
</tr>
<tr>
<td>Typical track</td>
<td>80</td>
<td>100</td>
<td>300</td>
<td>30</td>
</tr>
<tr>
<td>Stiff track</td>
<td>200</td>
<td>150</td>
<td>1000</td>
<td>50</td>
</tr>
</tbody>
</table>

Figure 5 shows the sleeper and rail fastening forces for the passage of the locomotive.

![Figure 5: Vertical forces in the track; rail fastening (left) and ballast (right)](image)

The immediate conclusion is that if a reduction of track loading is desired, a soft track configuration or at least a typical configuration is preferable. However, when accelerations at the rail pad and sleeper were analysed instead of forces, it was found that when a stiff track is adopted, a lower level of peak acceleration can be obtained with respect to softer tracks. This raises the question of the dependence of track degradation on different mechanical phenomena induced in the track during train passage. Most track degradation models assume ballast settlement to be caused by the forces generated under the sleeper. However, it is a known fact that too high levels of acceleration can be a cause of accelerated degradation, known as “ballast liquefaction”. Also, the amount of track settlement produced for the same level of forces generated under the sleepers is dependent on the type of ballast, and hence on the vertical stiffness of the track.

The relationships between the measured vehicle parameters and the forces in the track components were also studied.

The aim of this task was to determine the influence of vehicle service parameters on the forces in the track elements. The vehicle service parameters as understood for this work include:

- The vehicle type and its axle load
- The vehicle speed
- The track zone (with a focus on zone 2 – large radii, and zone 4 – tight curves)
- The cant deficiency
- The wheel and rail profiles types and condition
- The wheel-rail friction coefficient

Different infrastructure constructions and levels of track maintenance quality were compared in terms of forces transmitted through the rail pad and through the sleeper to the ballast. Sleeper acceleration was also investigated. This analysis can be used to identify possible margins for an infrastructure type-related increase of the limit values to be used for compatibility checks between vehicle and infrastructure. Additionally, the effect of vehicle service parameters, such as vehicle speed, was investigated and the subsequent variation of track loading considered.

As an example, Figure 6 shows that $Q_{\text{max}}$ has an almost linear relationship with the sleeper-ballast and rail pad forces. Figure 7 shows that $Y_{qst}$ also has a relationship with rail pad forces and sleeper-ballast forces, but in this case the relationship is not linear.

![Figure 6: Relationship between track forces (rail pad and sleeper-ballast) and $Q_{\text{max}}$](image-url)

$F_{\text{ballast}} = f(Q_{\text{max}})$

$F_{\text{pad}} = f(Q_{\text{max}})$
The result of the work only allows assessment of the ‘relative’ behaviour of different track configurations. It does not quantify absolute forces imposed by a specific vehicle on a specific track, nor does it provide absolute force limits to be imposed on a particular track configuration. The main conclusions from this work are:

- The assessment criteria values in the range <20 Hz are a good indicator of dynamic forces on track elements (pad/fastening and sleeper/ballast) in the wider frequency range (including 1st and 2nd mode around 30-50 Hz and 300-500 Hz respectively)
- Sleeper-ballast accelerations are thought to have an influence on track settlement and they are not directly related to <20 Hz vehicle forces measured during test.

### 4.4 Simulations of the track shift limit with non-linear models

Up to now, the virtual homologation exercise has been limited to studying lateral forces on a perfectly elastic track model and comparing them to the Prud'Homme limit, which can be considered as a Coulomb limit adapted to a track structure. Looking to the future, the idea is to simulate directly the permanent deformation with an advanced track model where the elements are highly non-linear.

Classical elements, such as the rails, are modelled as elastic, uniform and straight Timoshenko beams, supported discretely by sleepers or blocks considered as masses, and the railpads and fastenings on the sleepers, modelled by linear springs and dashpots. But the interactions between sleepers and ballast have been divided into three dry friction elements according to the contributions of bottom (SS #2 on Figure 8), shoulders and crib ballast [14].

The under-sleeper stiffness (SS#1) is represented by an exponential expression in order to reproduce a very soft stiffness at small displacement, and a stiffer one for common displacement when the wheelset is passing, corresponding to EUROBALT results.

This unilateral contact is initially loaded by the weight of the elements and can be unloaded by the flexibility of the rails just before the wheelset passes.
a. The track layout

The Prud’Homme trials took place on a curved 800 m radius curve. The track components consisted of continuous welded rails (CWR) of 46 kg/m (ex-U33), timber sleepers and a freshly tamped ballast bed. The cant deficiency, which is unknown, is assumed to be zero.

b. The vehicle and wheel/rail contact models

Two freight wagons have been modelled to represent the Prud'Homme cases. The carbody masses are varied to obtain 6, 12 and 17 t/axle. In each case, a pure lateral force is directly applied on the inner face of the outer wheel at rail level.

A preliminary task was to identify a set of track parameters that closely matched the Prud’Homme results and a “W” measurement (so called because of the characteristic shape of the signal) of a standard track with wooden sleepers, in an optimisation procedure minimising the difference between the prediction and the experiments (Figure 9). Two series of computations, involving a vehicle loaded at 17 t/axle, were performed, with applied lateral forces of 4 t or 6 t.

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**Figure 8: VOCO discrete track structure used to simulate Prud’Homme experiments**

- **a.** The track layout

- **b.** The vehicle and wheel/rail contact models

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Figure 9: Comparison between simulations and two sorts of experiments

The magnitude of the elastic deformations in vertical and lateral are approximately correct at both the first and the second wheelset passage. Both residual deformations are reached. Consequently this model can be used to simulate the lateral permanent shift on timber ties and ballasted CWR track. A similar approach using DB’s results [15] has been used to define the parameters of a concrete sleepered track.

The estimation of the lateral track-shifting force limit ($\Sigma Y_{\text{max}}$) in the same conditions as the Prud’Homme tests is not trivial because, with only a few dry friction elements in the model, some discontinuities are revealed in simulations which are not present in experiments. Some limits are presented in Figure 10.

Figure 10: Simulated lateral resistance versus axle load for concrete and timber sleeper

As observed in the Prud’Homme experiments, numerical results show a linear relationship between the lateral track-shifting limit and the axle load. The slopes are close to the Prud’Homme criterion that proposed 0.33. The inaccuracy is partly due to the way of determining the lateral track-shifting limit and partly to the discretisation. More research on a more complex model could also improve the agreement. This would also need some additional experiments, which can be defined with the model parameter analysis.

With the identified models, the Prud’Homme experiments (wooden sleepers) are quite well reproduced for axle loads between 10 and 16t.

Once the track models are validated, the parametric studies show a small dependency on sleeper spacing and on the rail type for timber sleepers, and a weak dependency for the concrete ones. The model shows less sensitivity than the results compiled in ERRI reports [16].
A first attempt had been made at the simulation of buckling but the sensitivity to temperature needs more development.

From this type of model, it can be proposed that in the future, in the virtual homologation process it will be possible to use non-linear advanced track models to simulate the parameter dependent track shift and not only the reaching of the lateral track-shift limit, used with the present linear models.

5. Survey of infrastructure construction and maintenance requirements

Using a questionnaire it was found that most networks use a track categorisation system based on some combination of line speed and service density. Across the member states who replied to the questionnaire, track design and construction rules vary depending on track category. In most cases the category can be linked back to line speed to allow a comparison between the different track constructions that a train of a given operating speed would be likely to encounter if it did move between different member states. This comparison was carried out based on existing standards in the member states and is not necessarily a direct reflection of the construction of all existing lines, which may have been built to older standards.

For line speeds greater than or equal to 160 km/h, the current standards for track construction across the member states appear to be similar. Differences between construction and maintenance standards are, as expected, more significant for lower speed lines. For example, track with a line speed of 140 km/h will have a 600 mm sleeper spacing in Germany, whilst for the same line speed in GB there could be a sleeper spacing of 700 mm. On lower speed lines in some countries a ‘weaker’ track condition may require a lower limit for one of the vehicle assessment parameters.

6. Relationships between track and vehicle response

The LOC & PAS TSI [7] allows in clause 4.2.3.4.2.2 in certain cases (e.g. when a vehicle exceeds the limit value for the quasi-static guiding force), that the operational performance of the rolling stock (e.g. maximum speed) may be limited by the infrastructure, considering track characteristics (e.g. curve radius, cant and rail height) in order to reduce loading below the limit value. This practice is already used on many networks in cases where the track design or maintenance status is not good and/or the track layout is demanding. The application of this rule is not easy as the dependencies between the relevant quantities and the operating conditions (i.e. the track layout, track geometric quality and friction levels) are not currently well defined.

Multiple regression can be a useful tool to describe the behaviour of a vehicle in terms of these dependencies. For each of the identified track force criteria, the sensitivity to different input parameters was therefore investigated by multiple regressions.

6.1 Vehicle dynamics simulations

To cover a wider range of conditions than those tested in WP1 [1], vehicle dynamic simulations were also used. Simulations were undertaken for three vehicle models, two
locomotives (PRIMA II and BB 26000) and a freight wagon with Y25 bogies, and the regression behaviour compared with results from the WP 1 tests.

The simulation cases cover a wide range of operating conditions observed during test runs according to EN 14363, as well as normal operation of the vehicles (very extreme cases are excluded):

- **Curve Radius** $R$: $150 \text{ m} \ldots \infty$ (Right hand curves only)
- **Cant deficiency** $I$: up to $150 \text{ mm}$
- **Speed** $V$ up to
  - $200 \text{ km/h}$ for BB 26000,
  - $140 \text{ km/h}$ for PRIMA II,
  - $120 \text{ km/h}$ for the freight wagon with Y25 bogies
- **Friction Coefficient** $\mu$: $0.1 \ldots 0.45 \ldots 0.6$
- **Geometric track quality:**
  - Use of measured track irregularities taken from WP 1 covering the range of class D geometry specified in prEN 13848-6:2012 [17] for line speeds between $< 80 \text{ km/h}$ and $300 \text{ km/h}$. The level of irregularities is described by the standard deviation in the longitudinal level (vertical) $s_{LL}$, and lateral, $s_{AL}$ geometry.
- **Contact Geometry:**
  - In curve radii below $600 \text{ m}$:
    - Normal nominal conditions (Rail: UIC 60E1, 1:40, 1435 / 1445 for $R \leq 150$, Wheels: S1002, 1425) complemented with real worn profiles measured in WP 1
  - In straight track and high cant deficiency curves:
    - Normal nominal conditions (Rail: UIC 60E1, 1:40, 1435, Wheels: S1002, 1425) complemented with real worn profiles measured in WP 1

### 6.2 Evaluation of results by multiple regression

The diagrams of the results are presented as two lines of figures, the top line shows the measured output (on the Y axis) against the inputs (on the X axis); whilst the bottom line of figures presents the outputs, against each input parameter, adjusted to remove the influence of all other input parameters, referred to as “added variable plots”. This second set of figures allows a comparison of the output parameter against the defined input parameter only, and removes ‘aliasing’ between two input parameters. For example when presenting the effect of speed on $Q$, results can be distorted because track with a higher line speed also tends to have better track quality; when the influence of track quality is removed a more direct relationship with speed can be demonstrated.

In the bottom line plots, the $\beta$ value gives the coefficient between the input and output variables and the $p$-value (shown as % in brackets after the $\beta$ value) indicates the significance level. A small $p$-value provides evidence that the sample depends on the input variable whilst a larger value suggests that there is no dependence. All output values are normalised against the limit value for that parameter.

As an example, the simulation results for $\Sigma Y_{\text{max}}$ (Figure 11) correlate with speed, lateral track alignment standard deviation and wheel-rail equivalent conicity. The solid red lines are the regression lines, whilst the dashed red lines show a statistical confidence interval; where a second, higher, solid red line is also shown, this is the maximum likely output value. For speed and wheel-rail equivalent conicity there is a wide confidence interval at
the extremities of the input values. This is largely a function of the choice of input parameters, with a lot of cases focused on the 160 km/h speed range and conicity cases tending to be focused toward the lower (more commonly experienced) values. Because of this possibility for a less strong correlation, speed and wheel-rail equivalent conicity are deemed to have a weak correlation with $\sum Y_{\text{max}}$; whilst lateral alignment standard deviation is deemed to have a strong correlation.

Figure 11: Parameters affecting $\sum Y_{\text{max}}$ for the BB26000 locomotive on straight track, plots with: speed; track geometry standard deviation (lateral); equivalent conicity ($\tan \gamma_e$); axleload ($P$); wheel-rail friction ($\mu$)

Figure 12 shows a comparison of the regression of outputs for $\sum Y_{\text{max}}$ for the measured results from the BR120 locomotive and the simulated results from the BB26000 and Prima II-2 locomotives.

BR120 loco measured results
BB26000 loco simulated results

Prima II-2 loco simulated results

Figure 12: Comparison of $\sum Y_{\text{max}}$ locomotive regressions on straight track, plots with: speed; track geometry standard deviation (lateral); equivalent conicity ($\tan \gamma_e$); axleload ($P$); wheel-rail friction ($\mu$)

The locomotive models used for the simulation are not the same as the BR120 locomotive used on the test runs but the vehicles are similar and the results were expected to be comparable.

In the simulations it was possible to control friction levels and also to change the mass of the vehicle; however, in the measured results outputs are only correlated against speed, lateral track alignment standard deviation and wheel-rail equivalent conicity.

The measured results and both simulations all show a positive increase in $\sum Y_{\text{max}}$ with speed and lateral alignment standard deviation. The measured output for the BR120 and simulated output for the BB26000 are relatively similar for the effect of speed; whilst the simulated BB26000 is more sensitive to lateral track alignment standard deviation. The Prima II-2 locomotive simulated results show less of a correlation in $\sum Y_{\text{max}}$ with speed and track alignment standard deviation, there is a general increase with both but it is more difficult to draw a clear correlation.

Results for the effect of wheel-rail equivalent conicity show a clear difference between the measured results and simulated results. The measured results show $\sum Y_{\text{max}}$ to be reasonably independent of conicity; whilst both sets of simulated results show an increase in $\sum Y_{\text{max}}$ with increased conicity.
For all measured results, the confidence interval is much tighter, this is because of the higher number of samples included.

This work has shown the benefit, and applicability of multiple regression analysis for investigating the behaviour of vehicles on different infrastructure conditions, and has demonstrated that this method can be applied with either test data or results from a validated dynamic model. This will enable the amount of testing needed to demonstrate vehicle compatibility with different infrastructure conditions to be reduced.

6.3 Relationships between parameters

The most relevant parameters were identified and summary tables (see below) indicate the relationships between:

- Vehicle assessment parameters and track deterioration effects (Table 1)
- Track installation and maintenance conditions and different track damage mechanisms (Table 2)
- Vehicle assessment parameters and track / operating conditions (Table 3)

Table 1: Relationships between track deterioration effects and vehicle assessment parameters

<table>
<thead>
<tr>
<th>Track deterioration effect</th>
<th>Fatigue / wear of rails</th>
<th>Fastenings</th>
<th>Sleepers</th>
<th>Ballast</th>
<th>Track bed</th>
<th>Track geometry Vertical / Cross-level</th>
<th>Track geometry Lateral</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q_{qst}</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q or Q_{max}</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y_{qst}</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>ΣY_{max}</td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Y_{max}</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_{qst}</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B_{qst}</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B_{max}</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It can be seen that all of the parameters proposed for vehicle assessment have some relationship with one or more deterioration mechanisms of the track. Some of the deterioration mechanisms can be influenced by more than one of the assessment parameters.

Table 2 summarises the sensitivities identified between the track installation and maintenance conditions and the different track damage mechanisms. It should be noted that speed and cant deficiency are clearly related in curves. The Table has attempted to
indicate which of them is the key factor. The symbol (✓) indicates a less strong sensitivity than ✓.

**Table 2: Sensitivities between track installation and maintenance conditions and track damage mechanisms**

<table>
<thead>
<tr>
<th>Operating conditions</th>
<th>Track installation</th>
<th>Track geometry</th>
<th>Track maintenance</th>
<th>Wheel-rail conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Cant deficiency</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Axle load</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Curve radius</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sleeper type and spacing</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Rail type &amp; fastening</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Ballast type &amp; depth</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Track support stiffness</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Track quality - vertical</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Track quality – lateral</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Wheel/Rail profile, lubrication / friction modifier</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

Surface damage of rails | ✓ | ✓ | ✓ | ✓ |

Fatigue of rails | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |

Fastenings | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |

Sleepers | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |

Ballast | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |

Track bed | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |

Track geometry - Vertical | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |

Track geometry – Lateral | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |

From the relationships in Table 2, Table 3 seeks to identify the relationships between the vehicle assessment parameters and the track and operating conditions.

**Table 3: Relationships between vehicle assessment parameters and track / operating conditions**

<table>
<thead>
<tr>
<th>Operating conditions</th>
<th>Track installation</th>
<th>Track geometry</th>
</tr>
</thead>
</table>
These tables allow potential operating controls to be considered that may control the track deterioration if the track is weak, or if certain vehicle performance parameters have high values. In combination with the developed methods for extended evaluation of data from testing and from simulation, additional testing for track loading is no longer necessary in most cases.

### 7. Proposed assessment methods for cross acceptance

#### 7.1 Parameters for assessment

The following track force assessment parameters are used in the international standards [4] and [5]) and TSIs ([6], [7] and [8]):

- **Q**<sub>qst</sub> – Quasi-static vertical wheel load
- **Q**<sub>max</sub> – Maximum vertical wheel force
- **Y**<sub>qst</sub> – Quasi-static lateral guiding force
- **ΣY**<sub>max</sub> – Lateral track-shifting force
- **H**<sub>max</sub> – Sum of lateral axle-box forces
- **B**<sub>qst</sub> – Quasi-static loading forces
- **B**<sub>max</sub> – Maximum loading forces

Various additional parameters are used in National documents (e.g. track stresses or different combinations of wheel forces) or proposed by other studies. They are applied for vehicle cross acceptance and for technical compatibility checks and examples are:

- **B**<sub>qst</sub> – Quasi-static loading forces
• $T_{qst}$ – Rail surface damage indicator

Studies in WP4 have shown that all of these parameters are useful for assessing the influence of a vehicle on track forces and the deterioration of track and track components.

### 7.2 Rail inclination and wheel-rail conditions

A number of documents related to Cross Acceptance which were reviewed by WP4 required additional tests because the nominal rail inclination of the track used for the initial approval was different from that where the new approval was requested. For example, a main issue in the checklist for cross acceptance between France and Germany is the question of rail inclination, because France uses a rail inclination of 1:20 whilst Germany uses 1:40. There is agreement and recognition between the two countries of test methods, assessment parameters and limit values, but tests on the alternative inclination are required.

There is a difference in equivalent conicity and radial steering index for rails and wheels in the design condition, depending on the rail inclination and the design wheel profile. However, the influence of the design inclination is strongly modified by the wear of wheels and rails to obtain the in-service conditions.

As the wheel profiles in normal use on the French network are based on the 1:40 tread angle, which is the same as the normal wheel profiles in use on the German network, the worn shape of the rails is strongly influenced by these wheels. The studies in DynoTRAIN WP3[3] show that for the actual in-service condition there is almost no difference in the characteristics of the two networks (France and Germany) because of the always present wear of the rails related to the wear of the wheels.

The influence of the chosen wheel profiles can also be seen by comparison of the results in WP3 from France (where most wheel profiles are based on 1:40) and from GB (where most wheel profiles are based on 1:20) and the resulting worn rail conditions are very different although the design inclination is the same.

Thus additional tests should not be required simply because of the design inclination of the rails. It should be sufficient to demonstrate that a sufficient range of equivalent conicity has been covered.

In Germany tests are required at a higher value of equivalent conicity, in particular for safety (stability). This is required because the maximum values reached in some parts of the German network are particularly high due to the combination of in-service track gauge and rail shape. There should not be any additional requirement for tests for track forces because of these higher conicity values.

### 7.3 Use of simulations and multiple regression analysis

Figure 13 shows an example for an analysis of the quasi-static track loading parameters taking into account the influences of curvature ($1/R$) and cant deficiency ($I$). The target test condition ($I = 150$ mm, $R_m = 350$ m) is marked by a larger red triangle. In this example the limit for $Y_{qst}$ is slightly above the limit of 60 kN, while the $Q_{qst}$ is clearly below the limit of 145 kN. In that case the combined parameter $B_{qst}$ reflecting the tension in the rail foot remains below the limit of 180 kN that is given by the two separate values ($B_{qst,lim} = Y_{qst,lim} + 0.83 Q_{qst,lim}$).

Note: To avoid the inclusion of test results achieved under exceptional friction condition, for this analysis $Y_{qst}$ values were used, that were “corrected” for the friction conditions by using the formula of UIC 518:2009. This formula normalises
high $Y_{qst}$ - values caused by $Y/Q_i$ – values above 0.40 taking into account 50 % of the assumed effect of the friction.

In this example it can be observed, as in many other cases, that the lateral force $Y_{qst}$ is much less dependent on cant deficiency than the vertical force $Q_{qst}$.

**Figure 13: Example for multiple regression analysis of test results for quasi-static track loading parameters**

For the extension of the curvature range to smaller curve radii, the same approach might be applied to simulation results. In that case the simulation results can be adjusted with the test results if necessary. Using the simulation approach, the influence of design parameter change (e.g. axle load) can also be included as an independent parameter in the analysis.

Results analysed and presented in this way can be used to look at trends to define operating rules depending on local infrastructure conditions. This applies to detailed analysis of the compatibility of a certain vehicle design with the infrastructure as well as to a more rough analysis taking into account the trends of several vehicles, which are typical for the traffic on a network or line.
In the use of multiple regression it is important to consider the following factors:

- Selection of data to ensure a good spread of input conditions
- Identification of key influencing variables
- Consideration of the relevant target conditions
- Exclusion of any ‘outliers’ to avoid very wide confidence intervals

For example, regarding the target conditions, these should be related to the practical operating and maintenance rules on the network concerned, so that it is probably not realistic to combine maximum cant deficiency with the worst track quality.

It is recommended that the process used should be documented. This documentation should include:

- The tests, or simulations, used
- The input variables selected to be included and the reasons
- The range of input conditions covered, including cross-plots
- The statistical properties of the regressions obtained

More detail is given in [18].

8. Conclusions of WP4

Studies in WP 4 have shown that all of the parameters used in international standards are useful for assessing the influence of a vehicle on track forces, and the deterioration of track and track components. The influence tables developed allow potential operating controls to be considered that may control the track deterioration if certain of the vehicle performance parameters have high values, or if some limits are exceeded.

For line speeds greater than or equal to 160km/h the current standards for track construction across the member states appear to be similar. On lower speed lines in some countries, a ‘weaker’ track condition may require a lower limit on one of the vehicle assessment parameters.

The review of national requirements identified modified criteria and limit values for track forces from Austria, Switzerland and Norway. These can be obtained from additional analysis of the normal test results with no new tests required. In GB there are additional requirements for compatibility, but these are assessed using calculations or simulations and do not depend on tests. In Germany tests are required for safety at higher values of equivalent conicity, but it is not clear how this relates to track forces. The influence of design rail inclination has also been found not to be significant, provided a realistic range of wheel-rail contact conditions are included in the tests. In France, the application of the parameter $B_{qst}$ has been studied to determine relevant operating conditions for new vehicles on weaker parts of the network and this method could be extended to other railways.

The use of multiple regression analysis allows the estimated maximum value for relevant parameters to be evaluated for different target conditions and then compared with the appropriate limit value, or with values for existing, comparable vehicles. The target values for the input variables should be selected to represent the real track conditions where the new acceptance is required. Realistic combinations of the conditions should be selected as, for example, it is probably not realistic to consider the maximum uncompensated lateral acceleration at the same time as the worst track quality. This method can be applied to
either test data or results from a validated dynamic model. This will enable the amount of
testing needed to demonstrate vehicle compatibility with different infrastructure conditions
to be reduced. Some recommendations for the application of multiple regression have
been provided.

Guidance has also been provided on the relevant parameters to consider when developing
operating controls for different types of track deterioration.

Acknowledgement

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Programme, contract number: 234079 and was led by UNIFE.

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